**2.1 First order Logic.**

1. You have a dog.

- ∃x Dog(x) ∧ Own (You, x)

2. The person you are looking for buys carrots by the bushel.

- BuyCarrots (ROBIN)

3. Anyone who owns a rabbit hate anything that chases any rabbit.

- ∀x (∃y(Own(x,y)∧Rabbit(y)) → (∀z (∃w (Rabbit(w)∧Chase(z,w)) → Hate(x,z))))

4. Every dog chases some rabbit.

- ∀x Dog(x) → ∃y (Rabbit(y) ∧ Chase(x,y))

5. Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.

- ∀x (BuyCarrots(x) → (∃y (Own (x, y) ∧ Rabbit(y) ∨ Own (x, GroceryStore)))

6. Someone who hates something owned by another person will not date that person.

- ∀x ∀y (∃z (Owns (y, z) ∧ Hates (x, z)) → ¬Date (x, y))

**2.2 Conjunctive Normal Forms**

1. Remove implications.

7. ∀x(¬∃y(Own(x,y) ∧ Rabbit(y))) ∨ (∀z(¬∃w(Rabbit(w) ∧ Chase(z,w)) ∨ Hate(x,z))))

[from 3.]

8. ∀x ¬Dog(x)∨∃y(Rabbit(y) ∧ Chase(x,y)) [from 4.]

9. ∀x ¬ (BuyCarrots(x) ∨ (∃y (Own (x, y) ∧ Rabbit(y) ∨ Own (x, GroceryStore))) [from 5.]

10. ∀x ∀y (¬∃z(Own(y,z) ∧ Hate(x,z))) ∨ ¬Date(x,y)) [from 6.]

1. Minimise negations.

11. ∀x∀y(¬Own(x,y) ∨ ¬ Rabbit(y)) ∨ (∀z∀w (¬Rabbit(w) ∨ ¬Chase(z,w)) ∨ Hate(x,z))

[from 7.]

12. ∀x ∀y ∃z(¬Own(y,z) ∨ ¬Hate(x,z)) ∨ ¬Date(x,y)) [from 10.]

1. Standardise variables.

13. ∀x1∀y1(¬Own (x1, y1) ∨ ¬ Rabbit(y1)) ∨ (∀z1∀w1 (¬Rabbit(w1) ∨ ¬Chase (z1, w1)) ∨ Hate(x1, z1)) [from 11.]

14. ∀x2 ¬Dog(x2) ∨ ∃y2(Rabbit(y2) ∧ Chase (x2, y2)) [from 8.]

15. ∀x3 ¬ (BuyCarrots(x3) ∨ (∃y3 (Own (x3, y3) ∧ Rabbit(y3) ∨ Own (x3, GroceryStore)))

[from 8.]

16. ∀x4 ∀y4 ∃z2(¬Own (y4, z2) ∨ ¬Hate (x4, z2)) ∨ ¬Date(x4,y4)) [from 12.]

1. Skolemise existentials.

17. Dog(D) ∧ Own (YOU, D) [from 1.]

18. ∀x2 ¬Dog(x2) ∨ (Rabbit(A(x2)) ∧ Chase (x2, A(x2))) [from 14.]

*Note: the existential on y2 was in the scope of the universal on x, hence the need for*

*introduction of the function A(x2).*

19. ∀x3 ¬ (BuyCarrots(x3) ∨ (Own (x3, B(x3)) ∧ Rabbit(B(x3)) ∨ Own (x3, GroceryStore))

[from 8.]

*Note: the existential on y3 was in the scope of the universal on x, hence the need for*

*introduction of the function B(x3).*

1. Drop universals.

20. ¬ Own (x1, y1) ∨ ¬ Rabbit(y1) ∨ ¬ Rabbit(w1) ∨ ¬ Chase (z1, w1) ∨ Hate (x1, z1))

[from 13.]

21. ¬Own (y4, z2) ∨ ¬Hate (x4, z2) ∨ ¬Date (x4, y4)) [from 16.]

22. ¬Dog(x2) ∨ (Rabbit(A(x2)) ∧ Chase (x2, A(x2))) [from 18.]

23. ¬ BuyCarrots (x3) ∨ (Own (x3, B(x3)) ∧ Rabbit(B(x3)) ∨ Own (x3, GroceryStore))

[from 19.]

1. Convert to CNF.

24. (¬Dog(x2) ∨ Rabbit(A(x2))) ∧ (¬Dog(x2) ∨ Chase (x2, A(x2))) [from 22.]

25. (¬BuyCarrots(x3) ∨ (Own (x3, B(x3)) ∧ (¬BuyCarrots(x3) ∨ Rabbit(B(x3))) ∨ (¬BuyCarrots(x3) ∨ Own (x3, GroceryStore))) [from 23.]

1. Final set

26. Dog(D) ∧ Own (You,D)

27. BuyCarrots(ROBIN)

28. ¬ Own (x1, y1) ∨ ¬ Rabbit(y1) ∨ ¬ Rabbit(w1) ∨ ¬ Chase (z1, w1) ∨ Hate (x1, z1))

29. (¬Dog(x2) ∨ Rabbit(A(x2))) ∧ (¬Dog(x2) ∨ Chase (x2, A(x2)))

30. (¬BuyCarrots(x3) ∨ (Own (x3, B(x3)) ∧ (¬BuyCarrots(x3) ∨ Rabbit(B(x3))) ∨ (¬BuyCarrots(x3) ∨ Own (x3, GroceryStore)))

31. ¬Own(y4, z2) ∨ ¬Hate(x4,z2) ∨ ¬Date(x4,y4)

**2.3 Goal state**

• If the person you are looking for does not own a grocery store, she will not date you.

* ¬Own(ROBIN,GroceryStore) → ¬Date(ROBIN,YOU)
* Negation: Own(ROBIN,GroceryStore)∧¬¬Date(ROBIN,YOU)
* CNF:

Own(ROBIN,GroceryStore)

Date(ROBIN,YOU)

**2.3 Resolution Proof**

1. Resolve {Own (ROBIN, GroceryStore)} and {¬Own (ROBIN, GroceryStore)} to get {}(empty clause).
2. Date (ROBIN, YOU)} and {¬Date(ROBIN,YOU)} to get {} (empty clause).

Since the resolution led to an empty clause, which indicates a contradiction, it means the conclusion is valid, and Madame Irma's prediction is true. Therefore, you should go see Robin and declare your love to her.