BA64018_Assignment3

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Table representation for the transportation problem

```r
library(kableExtra)
library(lpSolve)

tran_table = matrix(c("$20", "$14", "$25", "$400", 100,
                  "$12", "$15", "$14", "$300", 125,
                  "$10", "$12", "$15", "$500", 150,
                  80, 90, 70, "-", "-"), nrow = 4, byrow = TRUE)

colnames(tran_table) = c("W1", "W2", "W3", "cost", "Supply")
rownames(tran_table) = c("PlantA", "PlantB", "PlantC", "Demand")
tran_table = as.table(tran_table)

tran_table
```

```
##        W1  W2  W3  cost Supply
## PlantA $20 $14 $25 $400 100
## PlantB $12 $15 $14 $300 125
## PlantC $10 $12 $15 $500 150
## Demand 80  90  70  -    -
```

```r
tran_table %>% kable() %>% kable_classic() %>%
  column_spec(2, border_left = TRUE) %>%
  column_spec(6, border_left = TRUE) %>%
  row_spec(3, extra_css = "border-bottom:dotted;")
```

	W1	W2	W3	cost	Supply
PlantA	$20	$14	$25	$400	100
PlantB	$12	$15	$14	$300	125
PlantC	$10	$12	$15	$500	150
Demand	80	90	70	•	•

The supply quantity in this transportation problem is 375, while the demand quantity is 240. There is an imbalance in supply and demand in this transportation problem.The first step in solving the unbalanced transportation problem is to create a dummy variable to turn it into a balanced problem. Since there is less demand than supply in this situation, we will establish a dummy demand (dummy column) with 135 dummy demand quantity and zero transportation cost. Following the creation of dummy demand, the transportation issue now appears as follows:

```r
tran_table1 = matrix(c(420, 414, 425, 0, 100,
                    312, 315, 314, 0, 125,
                    510, 512, 515, 0, 150,
                    80, 90, 70, 135, 375), byrow = TRUE, nrow = 4)

colnames(tran_table1) = c("W1", "W2", "W3", "Dummy", "Supply")
rownames(tran_table1) = c("PlantA", "PlantB", "PlantC", "Demand")
tran_table1 = as.table(tran_table1)

tran_table1
```

```
##         W1  W2  W3 Dummy Supply
## PlantA 420 414 425     0    100
## PlantB 312 315 314     0    125
## PlantC 510 512 515     0    150
## Demand  80  90  70   135    375
```

```
tran_table1 %>% kable() %>% kable_classic() %>%
  column_spec(2, border_left = TRUE) %>%
  column_spec(6, border_left = TRUE) %>%
  row_spec(4, extra_css = "border-bottom:dotted;")
```

	W1	W2	W3	Dummy	Supply
PlantA	420	414	425	0	100
PlantB	312	315	314	0	125
PlantC	510	512	515	0	150
Demand	80	90	70	135	375

Now, demand equals supply. So we can finally solve the Transportation Problem.

Formulate the transportation model

Objective Function:

$$Min = 420W_{11} + 414W_{12} + 425W_{13} + 0W_{14} + 312W_{21} + 315W_{22} + 314W_{23} + 0W_{24} + 510W_{31} + 512W_{32} + 515W_{33} + 0W_{34}$$

Supply constraint:

Plant A:

$$W_{11} + W_{12} + W_{13} + W_{14} = 100$$

Plant B:

$$W_{21} + W_{22} + W_{23} + W_{24} = 125$$

Plant C:

$$W_{31} + W_{32} + W_{33} + W_{34} = 150$$

Demand constraints:

Warehouse 1:

$$W_{11} + W_{21} + W_{31} = 80$$

Warehouse 2:

$$W_{12} + W_{22} + W_{32} = 90$$

Warehouse 3:

$$W_{13} + W_{23} + W_{33} = 70$$

Dummy warehouse:

$$W_{14} + W_{24} + W_{34} = 135$$

Non-negativity of the decision variables:

$$W_{11}, W_{12}, W_{13}, W_{14}, W_{21}, W_{22}, W_{23}, W_{24}, W_{31}, W_{32}, W_{33}, W_{34} >= 0$$

Formulating the transportation model

```
costs = matrix(c(420, 414, 425, 0,
                 312, 315, 314, 0,
                 510, 512, 515, 0), byrow = TRUE, nrow = 3)


row.signs = rep("=", 3)
row.rhs = c(100, 125, 150)
col.signs = rep("=", 4)
col.rhs = c(80, 90, 70, 135)


lptrans = lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)


lptrans
```

```
## Success: the objective function is 88250
```

```
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   10   90    0    0
## [2,]   55    0   70    0
## [3,]   15    0    0  135
```

Dual of the primal transportation model

In constraints, there are two types of classes: demand and supply. As we know, the constraints coefficients will become the coefficients in the objective function for dual. As the primary goal is to reduce transportation costs, the secondary goal is to maximize profit through value addition. That is the profit obtained by selling goods less the cost of producing the goods. The supply constraint coefficients are 100,125, and 150 in this case. The demand constraint coefficients are 80,90,70,135

Objective function:

$$Max = 80x_1 + 90x_2 + 70x_3 - 100y_1 - 125y_2 - 150y_3$$

Constraints:

$$x_j - y_i >= z_{ij}$$

Plant A has to Supply 3 Warehouses

$$x_1 - y_1 \geq z_{11} = 420$$

$$x_1 - y_2 \geq z_{12} = 414$$

$$x_1 - y_3 \geq z_{13} = 425$$

Plant B has to Supply for 3 Warehouses

$$x_2 - y_1 \geq z_{21} = 312$$

$$x_2 - y_2 \geq z_{22} = 315$$

$$x_2 - y_3 \geq z_{23} = 314$$

Plant C has to Supply for 3 Warehouses

$$x_3 - y_1 \geq z_{31} = 510$$

$$x_3 - y_2 \geq z_{32} = 512$$

$$x_3 - y_3 \geq z_{33} = 515$$

Non-Negative Constraints

$$x_1, x_2, x_3, y_1, y_2, y_3, z_{11}, z_{12}, z_{13}, z_{21}, z_{22}, z_{23}, z_{31}, z_{32}, z_{33} >= 0$$

Economic Interpretation of the Dual

1. According to the MR=MC rule, here the dual constraint is

$$x_j - y_i >= z_{ij}$$

. So, here the equation would be

$$x_j >= z_{ij} + y_i$$

To be more specific,

$$x_1 \geq z_{11} = 420 + y_1$$

, The left side is the per-unit revenue generated by selling one unit of the product. This is what we call “MR (marginal revenue) in economics. The right side is the per-unit cost of making and transporting goods. This is called”MC.(marginal cost). Plant A keeps on increasing production and shipping to Warehouse 1 as long as

$$x_1 \geq z_{11} = 420 + y_1$$

when

$$MR >= MC$$

.In Contrast, Plant A reduces production and shipping if

$$x_1 \leq z_{11} = 420 + y_1$$

when

$$MR <= MC$$

Both are dynamic situations where either production increases or decreases. When

$$x_1 = z_{11} = 420 + y_1$$

, when

$$MR = MC$$

, the producer neither increases production nor decreases it. This is what we call equilibrium for profit maximization. Thus, the transportation cost minimization problem is equivalent to profit maximization in the dual, which ends up with

$$MR = MC$$

2. Whether to hire a shipping company or not for shipping goods if

$$x_j - y_i >= z_{ij}$$

Plant A directly supplies goods from Plant A to Warehouse 1. However, if the supplier finds any other shipping company that is able to transport goods from the plants to the warehouses satisfying

$$x_j - y_i <= z_{ij}$$

, then the supplier hires the shipping company rather than directly involved in transporting goods. If the producer finds a shipping company who is willing to ship goods satisfying the constraints rather than >=, then the producer hires the shipping company. So, if

$$x_j - y_i >= z_{ij}$$

, the producer (supplier) and the shipper will be the same. But if

$$x_j - y_i <= z_i j$$

, then the producer (supplier) just produces goods and hires another shipping company for the transportation of goods.