

BA64018_Assignment1

Ruthvick Bulagakula

2023-09-10

Problem 1

1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of \$32. Each Mini requires 40 minutes of labor and generates a unit profit of \$24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.
 - a. Clearly define the decision variables
 - b. What is the objective function?
 - c. What are the constraints?
 - d. Write down the full mathematical formulation for this LP problem.

LP Model for Problem 1

This represents data in the table format.

```
data = matrix(c(3,1000,45, '$32',
                2,1200,40,'$24'), ncol=4, byrow = TRUE)

colnames(data) = c('Square-Feet', 'Sales', 'Time', 'Profit')
rownames(data) = c('Collegiate', 'Mini')

final = as.table(data)

print(final)
```

##	Square-Feet	Sales	Time	Profit
## Collegiate	3	1000	45	\$32
## Mini	2	1200	40	\$24

Decision Variable

Let us assume, x_c for number of collegiate and x_m for number of mini.

Objective Function

Objective for back savers is to increase profits. So, collegiate is making 32 Dollars profit and mini is making 24 Dollars profit.

Objective function for Back savers:

$$Z = 32x_c + 24x_m$$

Constraints

Material Constraint:

Total square foot nylon material used for production cannot exceed 5000 square feet. So, this can be expressed as $3x_c + 2x_m \leq 5000$

Sales Constraint:

Sales forecasts indicate that 1000 collegiate backpacks and 1200 mini backpacks can be sold per week. So, this can be expressed as $x_c \leq 1000$, $x_m \leq 1200$

Labor Constraint:

Back savers has 35 labors and they work 40 hours per week each. Each collegiate need 45 mins of labor and each mini need 40 mins of labor. So, this can be expressed in hours by dividing mins by 60 since we have hours included as 40 per week.

$$0.75x_c + 0.67x_m \leq 35 * 40$$

$$0.75x_c + 0.67x_m \leq 1400$$

Non Negative Constraint:

$$x_c \geq 0 , x_m \geq 0$$

Mathematical Formula

Decision Variable:

$$x_c, x_m$$

Objective Function:

$$Z = 32x_c + 24x_m$$

Constraints:

Material Constraint:

$$3x_c + 2x_m \leq 5000$$

Sales Constraint:

$$x_c \leq 1000 , x_m \leq 1200$$

Labor Constraint:

$$0.75x_c + 0.67x_m \leq 35 * 40$$

$$0.75x_c + 0.67x_m \leq 1400$$

Non Negative Constraint:

$$x_c \geq 0 , x_m \geq 0$$

So, this above mathematical formulas will help back savers to maximize profits.

Problem 2

2. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes—large, medium, and small—that yield a net unit profit of \$420, \$360, and \$300, respectively. Plants 1, 2, and

3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

- a. Define the decision variables
- b. Formulate a linear programming model for this problem.

LP Model for Problem 2

This represents data in the table format.

```
data_plant = matrix(c(750,13000,
                     900,12000,
                     450,5000), ncol=2, byrow = TRUE)

colnames(data_plant) = c('Capacity', 'Square Feet')
rownames(data_plant) = c('P1', 'P2', 'P3')

final_plant = as.table(data_plant)

print(final_plant)
```

```
##      Capacity Square Feet
## P1      750      13000
## P2      900      12000
## P3      450       5000
```

```
data_size = matrix(c(20,900,'$420',
                     15,1200,'360',
                     12,750,'$900'), ncol=3, byrow = TRUE)

colnames(data_size) = c('Square Feet', 'Sales', 'Profit')
rownames(data_size) = c('Large', 'Medium', 'Small')

final_size = as.table(data_size)

print(final_size)
```

```
##      Square Feet Sales Profit
## Large  20      900   $420
## Medium 15     1200   360
## Small  12     750   $900
```

Decision Variable

Let us assume, $p1_l$, $p1_m$, $p1_s$ for plant 1, $p2_l$, $p2_m$, $p2_s$ for plant 2, and $p3_l$, $p3_m$, $p3_s$ for plant 3.

Objective Function

Objective for Weigelt Corporation is to increase profits. So, all large units in all plants are making 420 Dollars profit, all medium units in all plants are making 360 Dollars profit, and all small units in all plants are making 300 Dollars profit

Objective function for Weigelt Corporation:

$$Z = 420(p1_l + p2_l + p3_l) + 360(p1_m + p2_m + p3_m) + 300(p1_s + p2_s + p3_s)$$

Constraints

Capacity Constraint:

Total units produced by plant 1 per week is 750, Total units produced by plant 2 per week is 900, and Total units produced by plant 3 per week is 450.

$$\text{Plant 1: } p1_l + p1_m + p1_s \leq 750$$

$$\text{Plant 2: } p2_l + p2_m + p2_s \leq 900$$

$$\text{Plant 3: } p3_l + p3_m + p3_s \leq 450$$

Storage Constraint:

Total square feet required for large per week is 20, Total square feet required for medium per week is 15, and Total square feet required for small per week is 12

Total square feet required for plant 1 is 13000, Total square feet required for plant 2 is 12000, and Total square feet required for plant 3 is 5000

$$\text{Plant 1: } 20p1_l + 15p1_m + 12p1_s \leq 13000$$

$$\text{Plant 2: } 20p2_l + 15p2_m + 12p2_s \leq 12000$$

$$\text{Plant 3: } 20p3_l + 15p3_m + 12p3_s \leq 5000$$

Same Capacity Constraint:

To ensure same percentage of excess capacity does not exceed at each plant. So, we can use constraint like this

$$p1_l + p1_m + p1_s / 750 = p2_l + p2_m + p2_s / 900 = p3_l + p3_m + p3_s / 450$$

Non Negative Constraint:

$$p1_l, p1_m, p1_s, p2_l, p2_m, p2_s, p3_l, p3_m, p3_s \geq 0$$

Mathematical Formula

Decision Variable:

$$p1_l, p1_m, p1_s, p2_l, p2_m, p2_s, p3_l, p3_m, p3_s$$

Objective Function:

$$Z = 420(p1_l + p2_l + p3_l) + 360(p1_m + p2_m + p3_m) + 300(p1_s + p2_s + p3_s)$$

Constraints:

Capacity Constraint:

$$\text{Plant 1: } p1_l + p1_m + p1_s \leq 750$$

$$\text{Plant 2: } p2_l + p2_m + p2_s \leq 900$$

$$\text{Plant 3: } p3_l + p3_m + p3_s \leq 450$$

Storage Capacity:

$$\text{Plant 1: } 20p_{1l} + 15p_{1m} + 12p_{1s} \leq 13000$$

$$\text{Plant 2: } 20p_{2l} + 15p_{2m} + 12p_{2s} \leq 12000$$

$$\text{Plant 3: } 20p_{3l} + 15p_{3m} + 12p_{3s} \leq 5000$$

Same Capacity Constraint:

$$p_{1l} + p_{1m} + p_{1s}/750 = p_{2l} + p_{2m} + p_{2s}/900 = p_{3l} + p_{3m} + p_{3s}/450$$

Non Negative Constraint:

$$p_{1l}, p_{1m}, p_{1s}, p_{2l}, p_{2m}, p_{2s}, p_{3l}, p_{3m}, p_{3s} \geq 0$$

So, this above mathematical formulas will help Weigelt Corporation to maximize profits.