

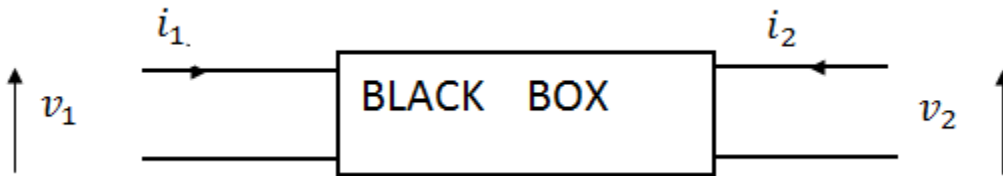
Hybrid Parameters

Hybrid Parameters:-

Every linear circuit having input and output terminals can be analyzed by four parameters (one is measured in ohm, one in mho and two dimensionless)

Since these parameters have mixed dimensions they are called hybrid parameters or h parameters.

Consider the linear circuit having input voltage (v_1) output voltage (v_2), input current (i_1) output current (i_2). Here the input current i_1 and output voltage v_2 are taken as independent variables where output current i_2 and input voltage are taken as dependent variables.



The dependent variables are related to independent variables as,

$$v_1 = h_{11} i_1 + h_{12} v_2 \dots \dots \dots (1)$$

$$i_2 = h_{21} i_1 + h_{22} v_2 \dots \dots \dots (2)$$

$$\begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

In these equations h_{ij} 's are fixed constants for a given circuit and are called h-parameter.

The h-parameter can be defined from equation (i) and (ii) as follows,

$h_{11} = \left. \frac{v_1}{i_1} \right|_{v_2=0} \rightarrow$ Input impedance with output short circuited (h_{11}) [dimension of ohm]

$h_{12} = \left. \frac{v_1}{v_2} \right|_{i_1=0} \rightarrow$ Reverse voltage gain with input open circuited [dimensionless]

$h_{21} = \left. \frac{i_1}{i_2} \right|_{v_2=0} \rightarrow$ Forward current gain with output short circuited. [dimension less]

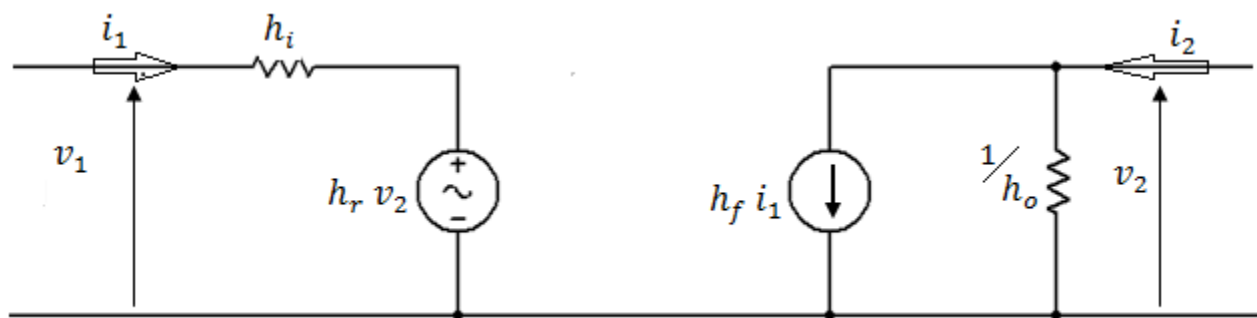
$h_{22} = \left. \frac{i_2}{v_2} \right|_{i_1=0} \rightarrow$ Output admittance with input open circuited. [dimension of ohm⁻¹]

The above h-parameters are also denoted as h_i, h_r, h_f, h_o

h-parameter equivalent circuit:- The h-parameter equivalent circuit is drawn of a two port network on the basis of the following equations.

$$v_1 = h_i i_1 + h_r v_2 \dots \dots \dots (3)$$

$$i_2 = h_f i_1 + h_o v_2 \dots \dots \dots (4)$$



Application of the KVL in input circuit gives the equation (iii) where as application of KCL in output ckt provides equation (iv).

Advantages of h-parameter:- We prefer h-parameter model due to the following advantages.

1. These can be easily measured, can also be calculated from transistor static characteristics curves.
2. The h-parameters are quite suitable for circuit designing and analysis.
3. The h-parameters are real numbers (at low or audio frequencies)
4. The transistor may be replaced by an equivalent circuit and linear theories like Thevenin Norton theorems may be applied.
5. A set of h-parameters is generally specified by the transistor manufacturers.

Hybrid model of a transistor in different mode of connections:-

A transistor is actually a three-terminal device. However any of the terminals is used in common with both output and input. Thus virtually it has two input and two output terminals. Therefore it can be modeled by a two port hybrid model.

The hybrid model can be applied to explain the operation of a transistor, only when it acts as a small-signal low frequency amplifier. The h-parameter model assumes two factors (i) linear variables (ii) absence of reactance. These are satisfied only if the above conditions are maintained.

Generally to the notations of h-parameters h_i, h_r, h_f, h_p a second subscript b, e, c , is induced to designate the ckt configuration.

i.e. h_{ie}, h_{ib}, h_{ic} represents the CE, CB, CC config. of the transistor.

The following diagram shows different transistor configuration and corresponding hybrid model.

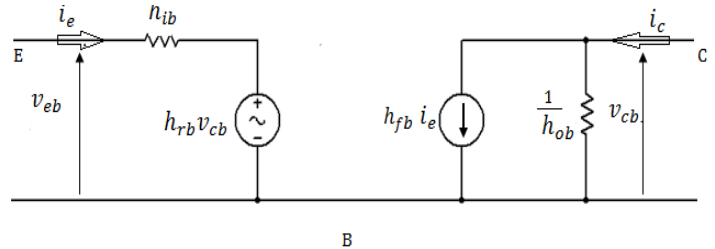
CB – MODE:- Here i_b and v_{ce} are independent variables

i.e. $v_{eb} = v_{eb}(i_e, v_{cb})$; $i_c = i(i_e, v_{cb})$

∴ The basic equations

$$v_{eb} = h_{ib} i_e + h_{rb} v_{cb}$$

$$i_c = h_{ib} i_e + h_{ob} v_{cb}$$



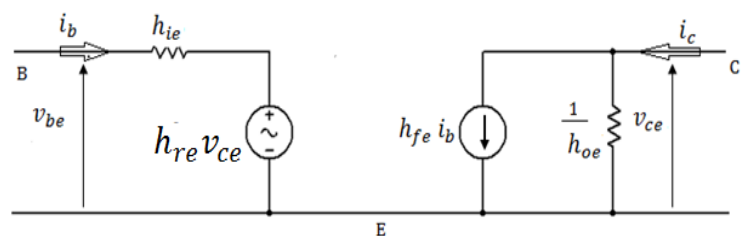
CE- CONFIGURATION :- Here i_b and v_{ce} are independent variables

i.e. $v_{be} = v_{be}(i_b, v_{ce})$; $i_c = (i_b, v_{cb})$

∴ The basic relations with h-parameters

$$v_{be} = h_{ie} i_b + h_{re} v_{ce}$$

$$i_c = h_{fe} i_b + h_{oe} v_{ce}$$



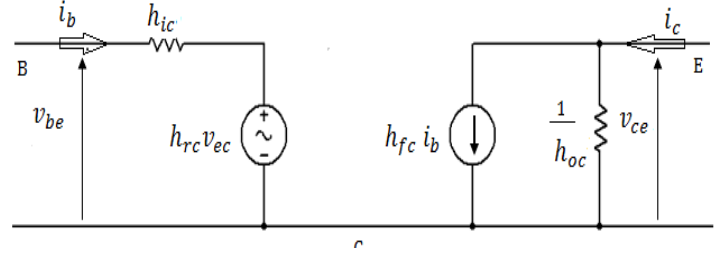
CC- Configuration:- Here i_b and v_{ce} are independent variables

i.e. $v_{bc} = v_{bc}(i_b, v_{ec})$; $i_e = (i_b, v_{cb})$

∴ The basic relations with h-parameters

$$v_{be} = h_{ic} i_b + h_{rc} v_{ec}$$

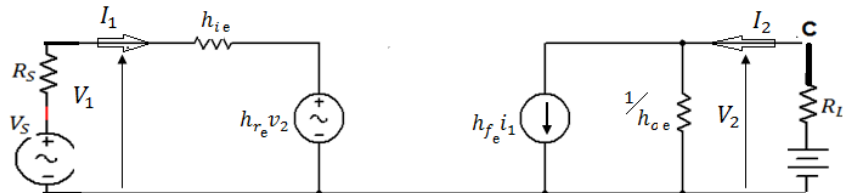
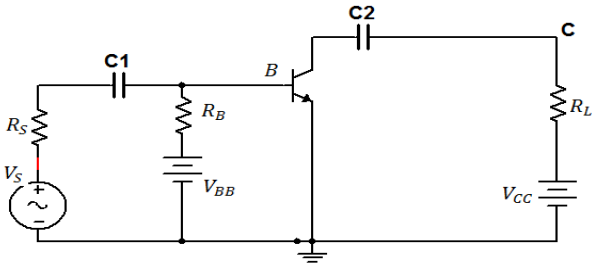
$$i_e = h_{fc} i_b + h_{oc} v_{ec}$$



Analysis of a CE amplifier using h-parameter model:-

We consider the basic circuit of a single stage low frequency CE amplifier using a simple load R_L as shown in the figure; R_S is the internal resistance of the signal source.

To analysis the performance of the amplifier analytically the ckt of fig (a) is replaced by its h equivalent circuit as shown in fig (b)



Current gain:- The current gain or the current amplifier factor A_I of the transistor is defined as the ratio of the output load current I_L to input current I_I .

$$\text{i.e. } A_I = \frac{I_L}{I_I} = -\frac{I_2}{I_1}$$

$$\text{Now } I_2 = h_{fe} I_1 + h_{oe} V_2$$

$$= h_{fe} I_1 + h_{oe} I_L R_L$$

$$\therefore A_I = -\frac{h_{fe} I_1 + h_{oe} I_L R_L}{I_1} = h_{fe} I_1 + h_{oe} R_L A_I$$

$$\therefore (1 + h_{oe} R_L) A_I = -h_{fe} ; \quad A_I = -\frac{h_{fe}}{1 + h_{oe} R_L}$$

Input resistance:- It is defined as the ratio of the input voltage and input current, i.e.

$$R_i = \frac{V_I}{I_1}$$

$$\text{Now } V_I = h_{ie}I_1 + h_{re}V_2$$

$$= h_{ie}I_1 + h_{re}I_L R_L$$

$$\therefore R_i = \frac{V_1}{I_1} = h_{ie}I_1 + h_{re}A_I R_L$$

$$\text{Putting } A_I = -\frac{h_{fe}}{1+h_{oe}R_L}$$

$$R_i = h_{ie} + \frac{h_{re}R_L(-h_{fe})}{1+h_{oe}R_L}$$

$$= \frac{h_{ie} + \Delta h R_L}{1+h_{oe}R_L} \quad \Delta h = h_{ie}h_{oe} - h_{re}h_{fe}$$

Voltage gain:- The voltage gain or voltage amplification A_V is defined as the ratio of output voltage v_2 to input voltage v_1 .

$$\therefore A_v = \frac{V_2}{V_1} = \frac{I_L R_L}{V_1} = \frac{A_I I_1 R_L}{V_1}$$

$$\therefore A_v = \frac{A_I R_L}{R_i}$$

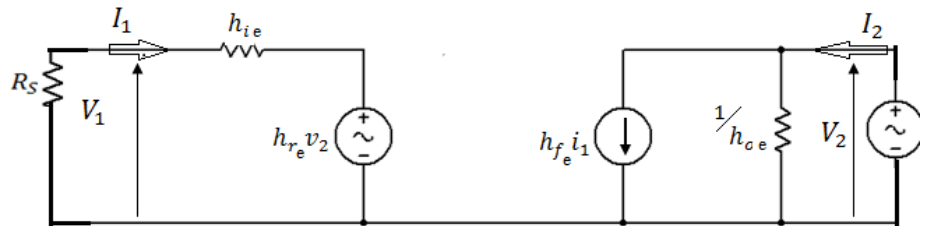
Putting values of A_I , R_i , we can have,

$$A_v = -\frac{h_{fe}R_L}{h_{ie} + \Delta h R_L}$$

Output Resistance:- To find output resistance R_o of the amplifier we replace load resistance R_L by a voltage source v_2 and input voltage source by its internal resistance R_g . If I_2 be output current delivered by voltage V_L ,

$$\text{Then } R_o = \frac{V_L}{I_2}$$

$$\text{Now, } I_2 = h_{fe}I_1 + h_{oe}V_2,$$



From input circuit,

$$I_1(R_s + h_{ie}) + h_{re}v_2 = 0$$

$$\therefore I_1 = -\frac{h_{re}}{R_s + h_{ie}}v_2$$

$$\text{Thus } I_2 = h_{oe}V_2 - \frac{h_{fe} h_{re} v_2}{R_s + h_{ie}}$$

$$R_0 = \frac{V_2}{I_2} = \frac{V_2}{h_{oe}V_2 - \frac{h_{fe} h_{re} v_2}{R_s + h_{ie}}} = \frac{1}{h_{oe} - \frac{h_{fe} h_{re}}{R_s + h_{ie}}} = \frac{R_s + R_{ie}}{R_s h_{oe} + \Delta h}$$

Note R_0 depends on the resistance of the source.

Power gain:- The power gain is defined as the ratio of the output ac power to the input ac power.

$$\text{i.e. } A_P = \frac{v_2 I_L}{v_1 I_1} = \frac{V_2}{V_1} \cdot \frac{I_L}{I_1} = A_v A_I$$

$$A_P = \frac{h_f^2 R_L}{1 + h_{oe} R_L (\Delta h R_L + h_i)}$$

Simplified Hybrid model:- In most of the practical cases instead of making lengthy calculations by using the complete hybrid model, we use a simplified model.

- Since h_{re} is usually very small put $h_{re}v_2 = 0$.
- Since output admittance is very high, $\frac{1}{h_{oe}}$ is a very high resistance and usually ignored comparison to a parallel load resistance.

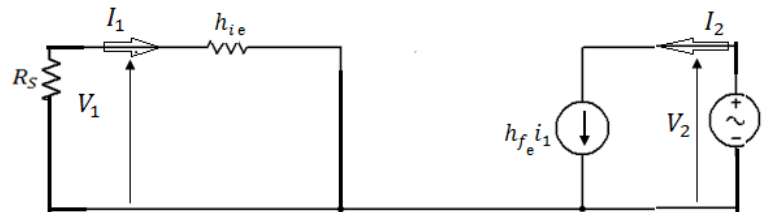
Approximation calculations for CE configuration:- According to the fig (b) we have,

Current gain A_I :- Here output current $-i_2$

$$A_I = -\frac{i_2}{i_1} = -h_{fe}$$

Input resistance R_i :- Here input resistance ,

$$\therefore R_I = \frac{V_1}{I_1} = \frac{i_1 h_{ie}}{I_1} = h_{ie}.$$



Voltage gain A_v :- Here output voltage $v_2 = -i_2 R_L$

$$\therefore \text{Voltage gain } A_v = \frac{V_2}{V_1} = -\frac{i_2 R_L}{i_1 h_{ie}} = \frac{-h_{fe} R_L}{h_{ie}}$$

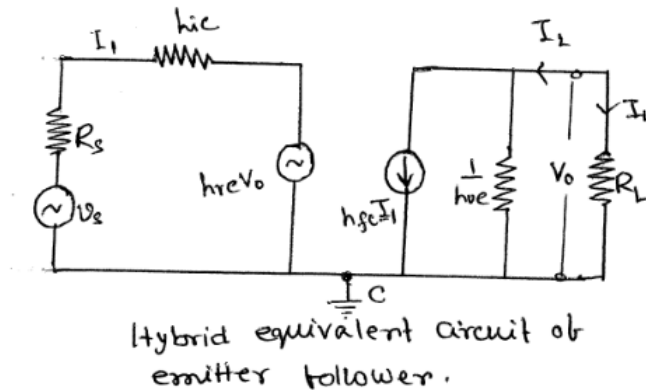
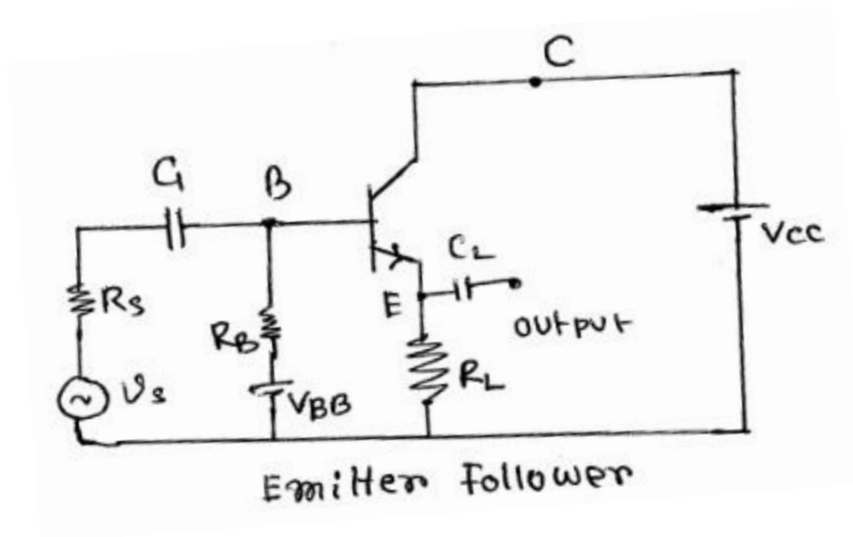
Output impedance :- It is defined as $\frac{V_2}{I_2}$, when source voltage is removed [hence $V_1 = 0$; $I_1 = 0$] by a short circuit and load resistance is removed

$$\therefore I_2 = h_{fe} I_1 = 0, \therefore R_0 \rightarrow \infty$$

Analysis for CB and CC amplifier :- The method of analyzing for CE-mode is also applicable for CB and CC mode. The only difference is that the values of h- parameter will be different. Transistor manufactures usually supply the h-parameters for CE-mode. But when a transistor is used in CB or CC mode it becomes necessary to know the corresponding h-parameters so it is useful to express CB and CC mode h-parameters in terms of CE h-parameters.

CE mode	CB- mode	CC-mode
h_{ie}	$h_{ib} = \frac{h_{ie}}{1+h_{fe}}$	$h_{ic} = h_{ie}$
h_{re}	$h_{rb} = \frac{h_{ie} h_{oe}}{1+h_{fe}} - h_{re}$	$h_{rc} = 1$
h_{fe}	$h_{fb} = -\frac{h_{fe}}{1+h_{fe}}$	$h_{fc} = -(1 + h_{fe})$
h_{oe}	$h_{oc} = \frac{h_{oe}}{1+h_{fe}}$	$h_{oc} = h_{oe}$

The Emitter Follower (CC amplifier):- The fig shows a CC amplifier using n-p-n transistor with an increase of base voltage, the emitter current, hence the v.d. across the load resistance R_L increases leaving the emitter potential more positive. The emitter voltage thus follows the base voltage. That is why the circuit is called an emitter follower.



Making an analysis exactly similar to the case of CE amplifier we can get the following for the performance quantities.

$$A_1 = -\frac{h_{fe}}{1+h_{oe}R_L}, Z_i = h_{ic} + h_{rc} A_1 R_L$$

$$A_v = \frac{A_1 R_L}{Z_i}; Z_0 = \frac{1}{h_{oc} - \frac{h_{fe} h_{rc}}{h_{ic} + R_s}}$$

Now replacing,

$$h_{ic} = h_{ie}; \quad h_{fc} = -(1 + h_{fe}); \quad h_{rc} = 1 - h_{re} \approx 1; \quad h_{oc} = h_{oe}$$

We get,

$$A_1 = \frac{1+h_{fe}}{1+h_{oe}R_L}; Z_i = h_{ie} + A_1 R_L$$

$$A_v = \frac{A_1 R_L}{Z_i} = \frac{Z_i - h_{ie}}{Z_i} = 1 - \frac{h_{ie}}{Z_i}$$

$$Z_0 = \frac{1}{h_{oe} + \frac{1+h_{fe}}{h_{ie} + R_s}}$$