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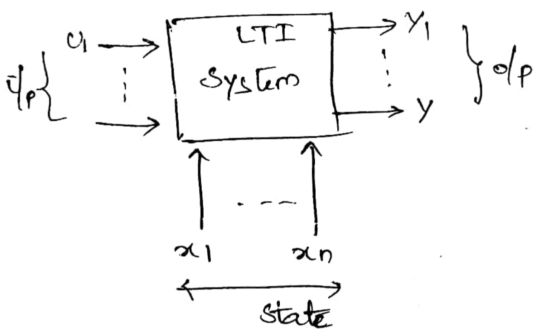
UNIT – V

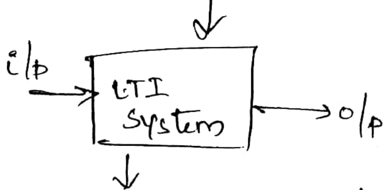
State Space Analysis of Continous System

$$\dot{X} = [A][X] + [B][U]$$
$$Y = [C][X] + [D][U]$$
} state space eqns.

$y(t) = ZIR + ZSR$



Valid for MIMO system



→ Assuming initial conditions are zero

→ not valid for MIMO system

It gives the initial behaviour of the system based on present i/p & past study of the i/p.

Past study of the system is described by State variables. State variable analysis is a direct time domain approach for analysing design of linear, Non-linear, time variant, invariant & MIMO systems.

Limitations of T/F Approach:

→ Initial conditions are assumed to be zero. It is not convenient for analysing the MIMO system. It is applicable for MIMO systems only. It gives analysis of system for specific type of i/p's like step, ramp & parabola.

→ The classical methods like Root locus, polar plot & Bode plot are basically trial & error procedure which fail to give the optimum sol^n required.

→ In modern method, uses the total internal state of the system considering all the initial conditions. This technique is called state space Analysis.

Differences b/w T.F & State Variable Approach:

T.F. Approach	State Variable Approach
→ Initial conditions are not considered	→ Initial conditions are considered.
→ It is based on i/p & o/p relationship	→ It is based on description of system eq^n interms of n first order D.E which is considered in state variat
→ T.F is applicable to LTI System and it is limited to SISO system	→ State variable approach is applicable for linear, Non-linear, TV, TI, SISO, MIMO.
→ classical design methods are based on trial & error procedure will give acceptable solution	→ Design methods is not based on trial & error procedure will give optimum sol^n for the system.
→ only i/p-o/p & error signals are considered. The i/p & o/p variables must be measured	→ The state variable need not represent physical variable. They need not even Measurable, controllable &

→ T.F. of the system is unique → state model of a system is not unique.

State Variable: & state model of a system:

The state eqn of a system are set of 1st order D.E where each 1st derivative of state variable is a linear combination of system states and i/p's.

$$\dot{X} = [A]_{n\times n}\,[X] + [B]_{n\times l}\,[U]$$

$$Y = [C]_{m\times n}[X] + [D]_{m\times l}[U]$$

o/p variable at time 't' are linear combination of i/p & state variables at time 't' is,

$$\dot{x}_1 = a_{11}x_1(t) + a_{12}x_2(t) + \dots + a_{1n}x_n(t) + b_{11}U_1(t) + b_{12}U_2(t) + \dots + b_{1m}U_m(t)$$

$$\vdots \qquad\qquad \vdots$$

$$\dot{x}_n = a_{n1}x_1(t) + a_{n2}x_2(t) + \dots + a_{nn}x_n(t) + b_{n1}U_1(t) + b_{n2}U_2(t) + \dots + b_{nm}U_m(t)$$

$$\dot{X} = AX(t) + BU(t)$$

State model of a system is

$$\dot{X} = [A]_{n\times n}[X] + [B]_{n\times l}[U]$$

$$Y = [C]_{m\times n}[X] + [D]_{m\times l}[U]$$

X – state variable
A – state matrix
B – i/p "
C – o/p "
D – transmission matrix
n – total no. of state variables
l – total " i/p's
m – " " o/p's.

Derivation of state model from the different representation of a system:

1. State model from Differential Eqn (D.E)
2. State model from Transfer funcn (T.F)
   can be derived using signal flow graph method, using Direct decomposition
3. State model by Cascade programming
4. State model by parallel programming (or) Cannonical form (or) ~~Pos~~ Posture form.
5. State model by Jordan's Cannonical form.

State model from D.E:

(a) Find the state model of the

$\ddot{y} + 5\dot{y} + 8y = 4$

~~$\dot{x}_2 + 5x_2 + 6x_1 = U$~~

~~$\dot{x}_2 = -5x_2 - 6x_1 + U$~~

~~$x_1 = y$~~

~~$\dot{x}_1 = x_2$~~

~~$x_1 = y$~~

~~$x_2 = \dot{x}_1 = \dot{y}$~~

~~$x_3 = \dot{x}_2 = \ddot{y}$~~

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y = 0$$

$n = 2$

$x_1 = y$

$x_2 = \dot{x}_1 = \dot{y}$

$x_3 = \dot{x}_2 = \ddot{y}$

$\dot{x}_2 + 5x_2 + 6x_1 = U$

$\dot{x}_2 = -5x_2 - 6x_1 + U$

$\dot{x}_1 = \dot{y}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = -5x_2 - 6x_1 + U$

$y = x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overset{B}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} U$$

$$[Y] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

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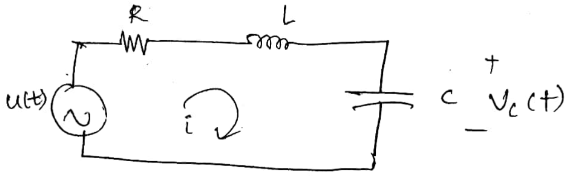
State Model for electrical system:

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$



$$i(t) = x_1$$

$$V_c(t) = X_2$$

No. of state variables = No. of energy ~~storage~~ storage elements

$$u(t) = Ri(t) + L\frac{di(t)}{dt} + V_c(t)$$

$$L\frac{di}{dt} = Ri(t) + V_c(t) - u(t)$$

$$\frac{di}{dt} = \frac{1}{L}Ri(t) + \frac{1}{L}V_c(t) - \frac{1}{L}u(t)$$

$$\boxed{\dot{x}_1(t) = \frac{1}{L}Rx_1 + \frac{1}{L}x_2 - \frac{1}{L}u(t)}$$

$$V_c(t) = \frac{1}{C}\int i_1 dt$$

$$\frac{dv_c(t)}{dt} = \frac{1}{C}i_1(t)$$

$$\dot{X}_2 = \frac{1}{C}x_1$$

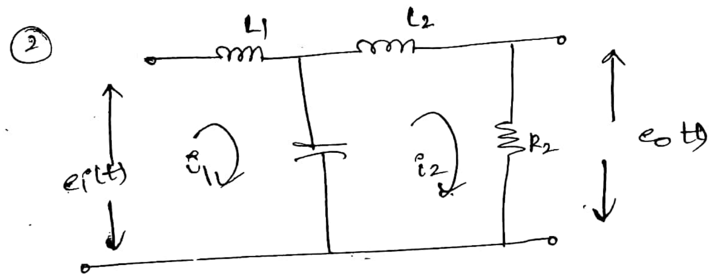
$$y(t) = V_c(t)$$

$$\boxed{y(t) = X_2}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} R/L & 1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1/L \\ 0 \end{bmatrix} u(t)$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

②



$$i_1 = x_1, \quad i_2 = x_2, \quad V_c = x_3$$

$$e_i(t) = L_1 \frac{di_1(t)}{dt} + V_c$$

$$u(t) = L_1 \frac{di_1}{dt} + V_c$$

$$\frac{di_1}{dt} = \frac{1}{L_1} V_c - \frac{1}{L} u(t)$$

$$\boxed{\dot{x}_1 = -\frac{1}{L_1} x_3 - \frac{1}{L} u(t)}$$

$$L_2 \frac{di_2}{dt} + i_2(t) . R_2 + V_c = 0$$

$$\boxed{\frac{di_2}{dt} = \frac{-R_2}{L_2} x_2 - \frac{1}{L_2} x_3}$$

$$\boxed{\dot{x}_2 = \frac{-R_2}{L_2} x_2 - \frac{1}{L_2} x_3}$$

$$V_c(t) = \frac{1}{C} \int (i_1 - i_2)\, dt$$

$$\frac{dV_c(t)}{dt} = \frac{1}{C} (i_1 - i_2)$$

$$\dot{x}_3 = \frac{1}{C}x_1 - \frac{1}{C}x_2$$

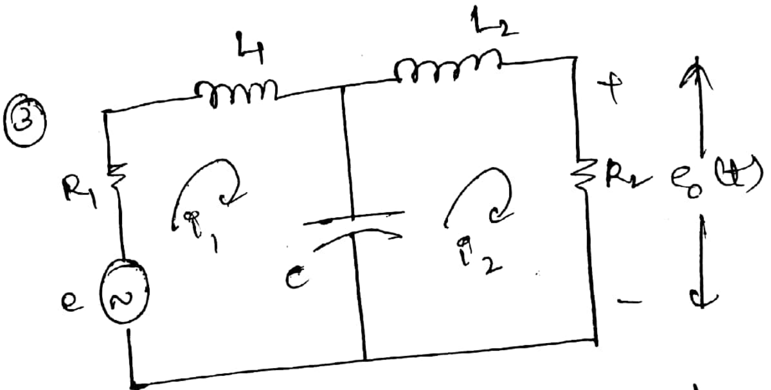
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/L_1 \\ 0 & -R_2/L_2 & -1/L_2 \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix} u(t)$$

$e_0(t) = i_2(t).R_2$

$e_0(t) = x_2 R_2$.

$$e_0(t) = \begin{bmatrix} 0 & R_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

(3)



No. of energy storage elements = 3

→ 3 Eqns.

$i_1(t) = x_1$

$i_2(t) = x_2$

$V_c(t) = x_3$

$$e(t) = R_1 i_1(t) + L_1 \frac{di_1(t)}{dt} + V_c(t)$$

$$L_1 \frac{di_1(t)}{dt} = u(t) - R_1 i_1(t) - V_c(t)$$

$$\boxed{\dot{x}_1 = \frac{-R}{L_1} x_1 - \frac{1}{L_1} x_3 + \frac{1}{L_1} u(t)}$$

$$v_c(t) = \frac{1}{c}\int (i_1 - i_2)\,dt$$

$$\frac{dv_c(t)}{dt} = \frac{1}{c}(i_1 - i_2).$$

$$\boxed{\dot{x}_3 = \frac{1}{c}x_1 - \frac{1}{c}x_2}$$

$$L_2\frac{di_2(t)}{dt} + R_2 i_2(t) + V_c(t) = 0$$

$$\frac{di_2(t)}{dt} = -\frac{R_2}{L_2} i_2(t) - \frac{1}{L_2}V_c(t)$$

$$\boxed{\dot{x}_2 = -\frac{R_2}{L_2}x_2 - \frac{1}{L_2}x_3}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-\frac{R}{L_1} & 0 & \frac{1}{L_2}\\ 0 & -\frac{R_2}{L_2} & -\frac{1}{L_2}\\ \frac{1}{c} & -\frac{1}{c} & 0\end{bmatrix} + \begin{bmatrix}1/L_1\\ 0\\ 0\end{bmatrix}u(t).$$

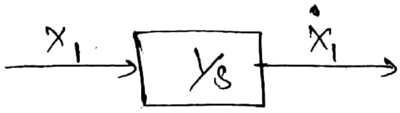
$$y(t) = e_0(t) = R_2 x_2$$

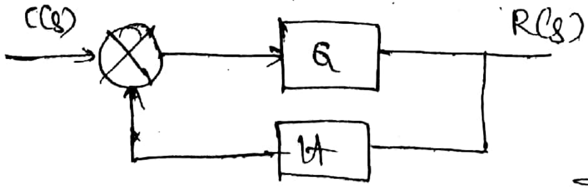
$$= \begin{bmatrix}0 & R_2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

State model from T.F

$$\dot{x} = Ax + Bu$$

$$Y = Cx + Du.$$





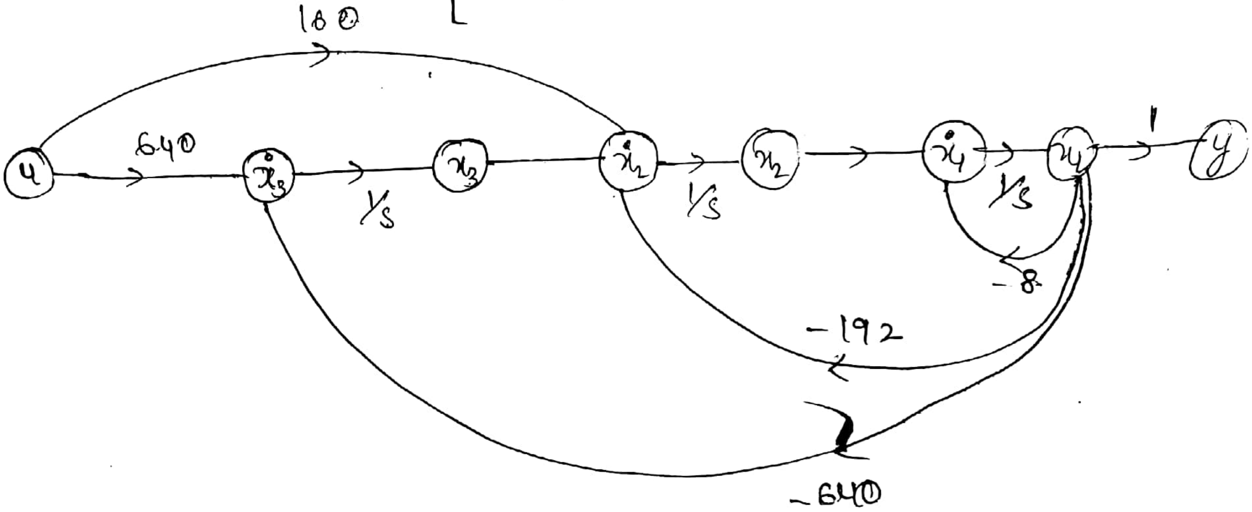
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Q) Derive the state model from T.F

$$\frac{Y(s)}{U(s)} = \frac{160(s+4)}{s^3+8s^2+192s+640}$$

sol. $$\frac{Y(s)}{U(s)} = \frac{160(s+4)}{s^3\left[1+\frac{8}{s}+\frac{192}{s^2}+\frac{640}{s^3}\right]}$$

$$= \frac{160\left[\frac{s}{s^3}+\frac{4}{s^3}\right]}{1-\left[-8\cdot\frac{1}{s}-192\frac{1}{s^2}-640\frac{1}{s^3}\right]}$$



$y = x_4$

$\dot{x}_4 = -8x_1 + x_2$

$\dot{x}_2 = -192x_4 + x_3 + 160u$

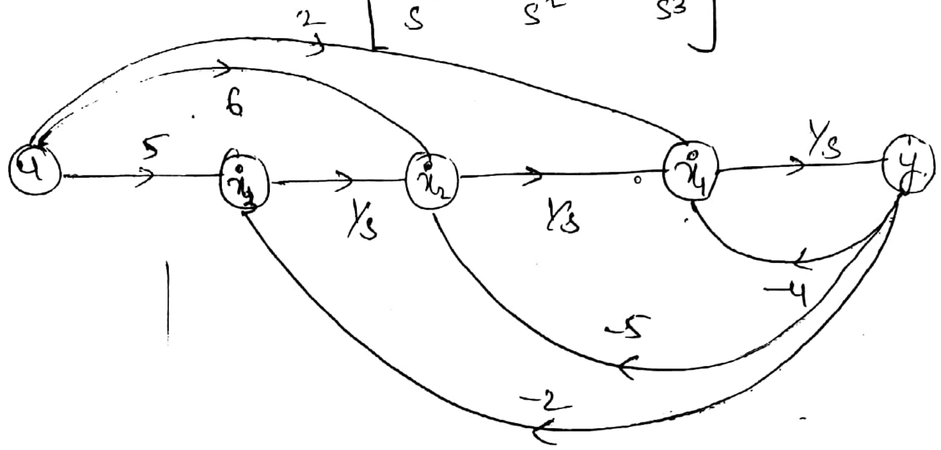
$\dot{x}_3 = 640u - 640x_4$

Q) Derive the state model for the T.F

$$\frac{Y(s)}{U(s)} = \frac{2s^2+6s+5}{s^3+4s^2+5s+2}$$

$$\frac{Y(s)}{U(s)} = \frac{2s^2+6s+5}{s^3\left[1+\frac{4}{s}+\frac{5}{s^2}+\frac{2}{s^3}\right]}$$

$$\frac{Y(s)}{U(s)} = \frac{\frac{2}{s}+\frac{6}{s^2}+\frac{5}{s^3}}{1-\left[\frac{-4}{s}-\frac{5}{s^2}-\frac{2}{s^3}\right]}$$



$y = x_1$

$\dot{x}_1 = -4y + x_2 + 2u$

$\dot{x}_2 = -5x_1 + x_3 + 6u$

$\dot{x}_3 = -2x_1 + 5u$

Derivation of T.F from state Model

$\dot{X} = AX + Bu$

$Y = CX + Du$

$\dot{X}(t) = AX(t) + Bu(t)$

$Y(t) = CX(t) + Du(t)$

Apply L-transform on both sides

$sX(s) - X(0) = AX(s) + BU(s)$

$[sI - A]X(s) = X(0) + BU(s)$

$$X(s) = [SI-A]^{-1} X(0) + [SI-A]^{-1} B U(s) \longrightarrow (1)$$

Assume $X(0) = 0$

$$X(s) = [SI-A]^{-1} B U(s).$$

$$Y(s) = C X(s) + D U(s)$$

$$Y(s) = C [SI-A]^{-1} B U(s) + D U(s)$$

$$\frac{Y(s)}{U(s)} = D + BC [SI-A]^{-1}$$ For MIMO system

$$\frac{Y(s)}{U(s)} = C [SI-A]^{-1} B$$ For SISO system

$$[SI-A]^{-1} = \frac{Adj [SI-A]}{|SI-A|}$$

$$\frac{Y(s)}{U(s)} = C \cdot \frac{Adj [SI-A]}{|SI-A|} \cdot B$$

$$C.E \Rightarrow |SI-A| = 0$$

$$\dot{X} = AX$$

then $$\frac{dx}{dt} = Ax$$

$$\boxed{x(t) = e^{At}}$$ STM (2)

$$\boxed{STM \quad \phi(t) = e^{At}}$$

from (1),

let $$\phi(t) = e^{At}$$ (3)

$$\phi(s) = [SI-A]^{-1}$$

$$X(s) = \phi(s)\, X(0) + \phi(s)\, B U(s)$$

$$L^{-1}[X(s)] = x(t)$$

$$L^{-1}[\phi(s) \cdot X(0)] = \phi(t) \cdot X(0)$$

$$\cancel{L^{-1}[\phi(s) \cdot B \cdot U(s)]} = \int_0^t \cancel{\phi(t-\tau) \cdot e^{A(t-\tau)}} \cdot \cancel{d\tau}\, \cancel{B u(\tau)\, d\tau}$$

$$L^{-1}[\phi(s)\, B U(s)] = \int_0^t \phi(t-\tau)\, B u(\tau)\, d\tau$$

$$x(t) = \phi(t)\, x(0) + \int_0^t \phi(t-\tau)\, B u(\tau)\, d\tau$$

Total response = ZIR + ZSR

$$\phi(-t) = -\phi(t)$$

$$\phi(t) = e^{At}$$

$$\{\phi(t)\}^K = K[\phi(t)]$$

$$\phi(0) = 1 = I$$

~~Properties~~

~~Prop 1:~~

~~2: $\phi(t_1+t_2)$~~

Properties of state transition Matrix

1. $\phi(t) = e^{At}$;
   $\boxed{\phi(0) = 1} = I$
2. $|\phi(t_1+t_2)| = |\phi(t_1)|\,|\phi(t_2)| \cancel{e}\, e^{A(t_1+t_2)}$
   $= e^{At_1} \cdot e^{At_2}$

(2) Proof

$$\phi(t_1+t_2) = e^{A(t_1+t_2)} \quad (\because \phi(t) = e^{At})$$

$$= e^{At_1} \cdot e^{At_2}$$

$$\boxed{\phi(t_1+t_2) = \phi(t_1) \cdot \phi(t_2)}$$

(3) $\boxed{\{\phi(t)\}^K = \phi(Kt)}$

W.K.T $\phi(t) = e^{At}$

$[\phi(t)]^K = e^{KAt}$

$[\phi(t)]^K = \phi(KAt)$

(4) $\phi^{-1}(t) = \phi(-t)$

$\phi^{-1}(t) = \frac{1}{e^{At}} = e^{-At} = \phi(-t)$

(5) $\phi(t_2-t_1)\,\phi(t_1-t_0) = \phi(t_2-t_0)$

$= e^{A(t_2-t_1)} \cdot e^{A(t_1-t_0)}$

$= e^{A(t_2-t_1+t_1-t_0)}$

$= e^{A(t_2-t_0)}$

$= \phi(t_2-t_0)$

Q: A system is represented by state and o/p. Find the o/p eq^n $\dot{X} = \begin{bmatrix} -3 & -2 \\ -1 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$; $Y = [1 \;\; 2] X$

Sol:-

$$\dot{X} = \begin{bmatrix} -3 & -2 \\ -1 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$Y = [1 \;\; 2] X$$

$$T.F = C[SI - A]^{-1} B + D$$

char. eq^n $\Rightarrow [SI - A] = 0$

$$\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ -1 & -2 \end{bmatrix} = 0$$

$$\begin{bmatrix} S+3 & +2 \\ +1 & S+2 \end{bmatrix} = 0$$

$(s+3)(s+2) - 2 = 0$

$s^2+5s+6-2=0$

$s^2+5s+4=0$

$s^2+4s+s+4=0$

$(s+4)(s+1)=0$

$s=-4,-1$

Q: Find the T.F of the system which is represented in the state space model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [0 \quad 1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Sol:- $\frac{Y(s)}{U(s)} = T.F = C[SI-A]^{-1}B + D$

$$[SI-A] = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} s+2 & -1 & 0 \\ 0 & s+3 & -1 \\ +3 & 4 & 5 \end{bmatrix}$$

$\cancel{[SI-A]^{-1}}$ $[SI-A]^{-1} =$

$T.F = \frac{s+2}{s^3+10s^2+ \dots}$

(2) An LTI system characterised by $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \\ 1 \end{bmatrix}$

where u is unitstep func^n. Compute sol^n. ~~Assume~~. $X(0) = X_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$X(t) = L^{-1}\left[\phi(s) * X(0)\right] + L^{-1}\left[\phi(s) \cdot BU(s)\right]$$

$$\phi(s) = [SI - A]^{-1}$$

$$SI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$\phi(s) = [SI - A]^{-1} = \frac{\mathrm{Adj}[SI - A]}{|SI - A|} = \frac{\begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}}{(s-1)^2}$$

$$\phi(s) = \begin{pmatrix} 1/s-1 & 0 \\ 1/(s-1)^2 & 1/s-1 \end{pmatrix}$$

$$\phi(s)\, X(0) = ZIR = \begin{pmatrix} 1/s-1 & 0 \\ 1/(s-1)^2 & 1/s-1 \end{pmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/s-1 \\ 1/(s-1)^2 \end{bmatrix}$$

$$L^{-1}\left[\phi(s)\, X(0)\right] = \begin{bmatrix} e^{t} \\ te^{t} \end{bmatrix}$$

$$\phi(s)\, BU(s) = \frac{1}{s} \begin{bmatrix} 1/s-1 & 0 \\ 1/(s-1)^2 & 1/s-1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{s} \begin{bmatrix} 0 \\ 1/s-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1/s(s-1) \end{bmatrix}$$

$$\left[\phi(s)\, BU(s)\right] = \begin{bmatrix} 0 \\ -1+e^t \end{bmatrix}$$

$$Y(t) = \begin{bmatrix} e^t \\ te^t \end{bmatrix} + \begin{bmatrix} 0 \\ -1+e^t \end{bmatrix} = \begin{bmatrix} e^t \\ e^t(1+t)-1 \end{bmatrix}$$

(8) Solve $Y(t)$ for the following system represented in state space where $u(t)$ is the step funcn.

$$\dot{X} = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -5 & 1 \\ 0 & 0 & -2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \;;\; Y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} X \;;$$

$$X(0) = X_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Y(t) = L^{-1}[\phi(s)\, X(0)]$$

(4) Find state transition matrix for state eqn

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$\text{STM} \Rightarrow \phi(t) = e^{AT}$$

$$\phi(s) = [sI - A]^{-1}$$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} 1/(s-1) & 0 \\ 1/(s-1)^2 & 1/(s-1) \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

(5) Obtain the complete time response of a system given $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} x(t)$; $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $Y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} X(t)$

$$x(t) = L^{-1}\left[\phi(s)\, X(0)\right]$$

$$\phi(s) = [sI - A]^{-1}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s \end{bmatrix}$$

$$\phi(s) = \frac{1}{s^2+2} \begin{bmatrix} s & 1 \\ -2 & s \end{bmatrix}$$

$$\phi(s)\, X(0) = \begin{bmatrix} s/s^2+2 & 1/s^2+2 \\ -2/s^2+2 & s/s^2+2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} s+1/s^2+2 \\ s-2/s^2+2 \end{bmatrix}$$

$$= L^{-1} \begin{bmatrix} s/s^2+2 & 1/s^2+2 \\ -2/s^2+2 & s/s^2+2 \end{bmatrix}$$

$$\Rightarrow X(t) = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}} \sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}$$

$$Y(t) = \frac{3}{\sqrt{2}} \sin\sqrt{2}t$$

...rolability: For a given system is it possible to transfer any initial state to any other desired state in infinite time under the effect of suitable control i/p force is called Controlability.

(or)

It is defined as, it is possible to transfer the system state from initial state $x(t_0)$ to any other desired state $x(t_f)$ in a specified finite interval t_f by a control vector $u(t)$.

A system is completely state controllable if and only iff composite matrix Q_c is

$$Q_c = \left[B : AB : A^2B \text{ ------- } (A)^{n-1}.B\right]$$

A system is completely Controllable if rank of Q_c = Order of matrix / no. of state variables.

Observability: If the o/p is measured at finite time with the knowledge of i/p is it possible to determine initial state of system is called Observability.

(or)

If every state $x(t_0)$ can be completely identified by the measurements of o/p $y(t)$ over a finite time interval.

$$Q_o = \left[C^T \quad A^T C^T \quad (A^T)^2 C^T \text{ ---- } (A^T)^{n-1} C^T\right]$$

A system is completely observable if rank of Q_o = order of matrix

① Evaluate contro of system $A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$n = 2$

$Q_C = [B \,\vdots\, AB]$

$$AB = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\therefore \quad Q_C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0 \quad \text{rank of matrix} = 1$$

$\therefore n \neq r$

$\therefore$ system is not controlable

② Evaluate observability $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

$$Y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$n = 2$

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$Q_0 = [C^T \,:\, A^T C^T]$$

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$|Q_o| = 1 - 0 = 1 \neq 0$

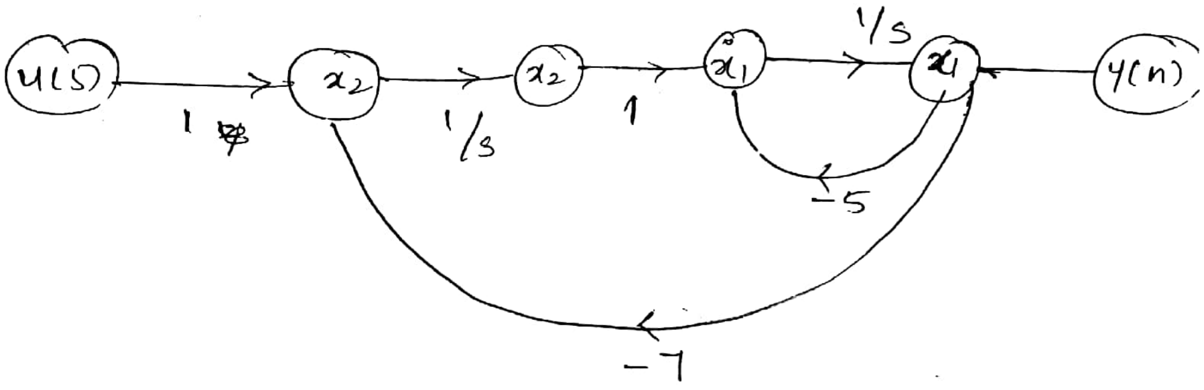
$\therefore r = 2$

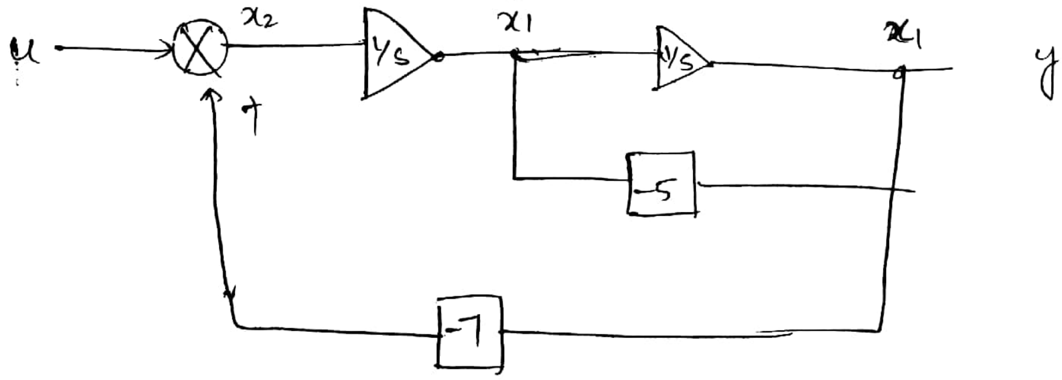
$\therefore$ System is completely observable.

state model from direct decomposition method:

$$T.F = \frac{1}{s^2 + 5s + 7}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2\left[1 + \frac{5}{s} + \frac{7}{s^2}\right]} = \frac{1/s^2}{1 - \left[-\frac{5}{s} - \frac{7}{s^2}\right]}$$





$y = x_1$; $\dot{x}_1 = -5x_1 + x_2$

$\dot{x}_2 = -7x_1 + u$

State Vector & State Space:

In Control engg; A state space representatn is a Mathematical model physical system as a set of i/p's, o/p's & state variables related by 1st order D.E [The state of a System can be represented as a vector within the space → state vector]