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ECE-A

DSP ASSIGNMENT-II

1. Realize Direct form I and Direct form II structures for the system described by the difference equation

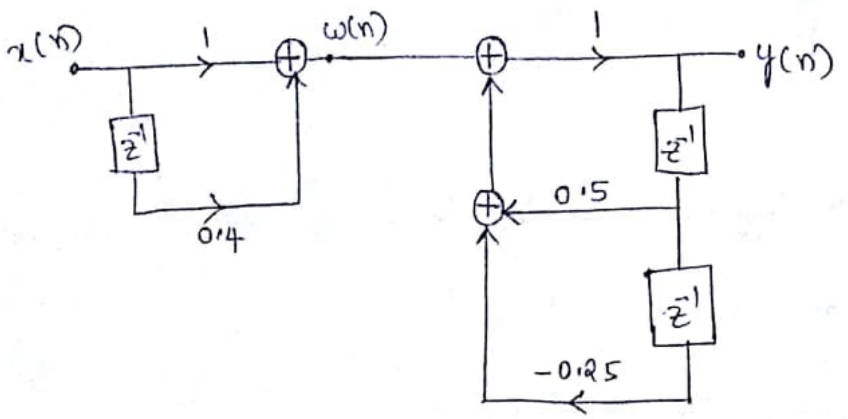
$$y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$$

Ans: (i) Direct form I realization:

Given $y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1) \rightarrow (1)$

Let $w(n) = x(n) + 0.4x(n-1) \rightarrow (2)$

$y(n) = 0.5y(n-1) - 0.25y(n-2) + w(n) \rightarrow (3)$



(ii) Direct form II realization:

Given $y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$

Apply Z.T.

$\Rightarrow Y(z) = 0.5z^{-1}Y(z) - 0.25z^{-2}Y(z) + X(z) + 0.4z^{-1}X(z)$

$\Rightarrow Y(z)\left[1 - 0.5z^{-1} + 0.25z^{-2}\right] = (1 + 0.4z^{-1})X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+0.4z^{-1}}{1-0.5z^{-1}+0.25z^{-2}} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1-0.5z^{-1}+0.25z^{-2}}$$

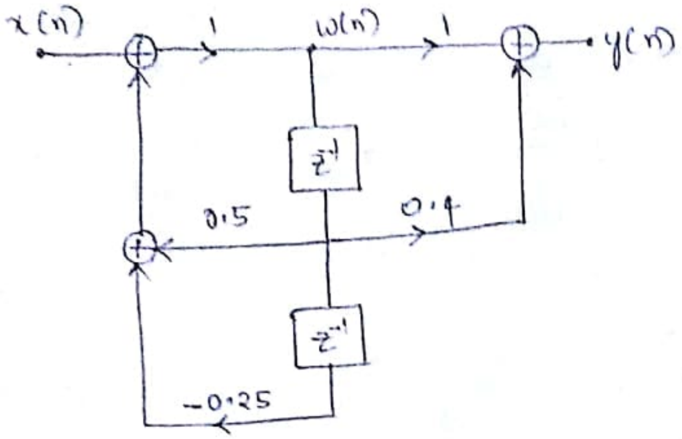
$\Rightarrow W(z)\left[1-0.5z^{-1}+0.25z^{-2}\right] = X(z)$

Applying IZT $\Rightarrow w(n)-0.5\,w(n-1)+0.25\,w(n-2) = x(n)$

$\Rightarrow w(n) = x(n)+0.5w(n-1)-0.25\,w(n-2) \longrightarrow (2)$

$\frac{Y(z)}{W(z)} = 1+0.4z^{-1} \Rightarrow Y(z) = W(z)\left[1+0.4z^{-1}\right]$

$\Rightarrow y(n) = w(n)+0.4\,w(n-1) \longrightarrow (3)$



2. Realize the system with difference equation in cascade and parallel form.

$y(n) = \frac{3}{4}y(n-1)-\frac{1}{8}y(n-2)+x(n)+\frac{1}{3}x(n-1)$

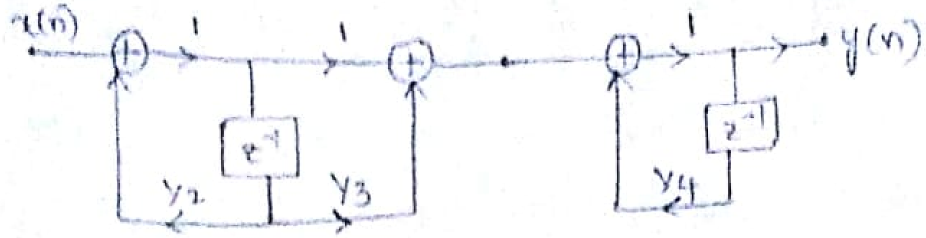
Ans: (i) Cascade form realization:
Given $y(n) = \frac{3}{4}y(n-1)-\frac{1}{8}y(n-2)+x(n)+\frac{1}{3}x(n-1)$

Apply Z.T $\Rightarrow Y(z) = \frac{3}{4}z^{-1}Y(z)-\frac{1}{8}z^{-2}Y(z)+X(z)+\frac{1}{3}z^{-1}X(z)$

$\Rightarrow Y(z)\left[1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right] = X(z)\left[1+\frac{1}{3}z^{-1}\right]$

$$\frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{1+\frac{1}{3}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$$\frac{Y(z)}{X(z)} = \frac{\left(1+\frac{1}{3}z^{-1}\right)}{1-\frac{1}{2}z^{-1}} \times \frac{1}{1-\frac{1}{4}z^{-1}} = H_1(z)H_2(z)$$



(ii) Parallel form realization :

Given $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$

$$\Rightarrow Y(z) = \frac{3}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z) + X(z) + \frac{1}{3}z^{-1}X(z)$$

$$Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = X(z)\left[1 + \frac{1}{3}z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{\frac{1}{3}z^{-1}+1}{\frac{1}{8}z^{-2}-\frac{3}{4}z^{-1}+1} = \frac{\frac{1}{3}z^{-1}+1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$$\cancel{\frac{1}{8}z^{-2}-\frac{3}{4}z^{-1}+1 \;\Big)\; \frac{1}{3}z^{-1}+1 \;\Big(\; -4/9}$$
$$\cancel{+\frac{1}{3}z^{-1} - \frac{4}{9} - \frac{1}{18}z^{-2}}$$
$$\cancel{\frac{1}{18}z^{-2} + 1\frac{4}{9}}$$

P.F : $\frac{\frac{1}{3}z^{-1}+1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{4}z^{-1}}$

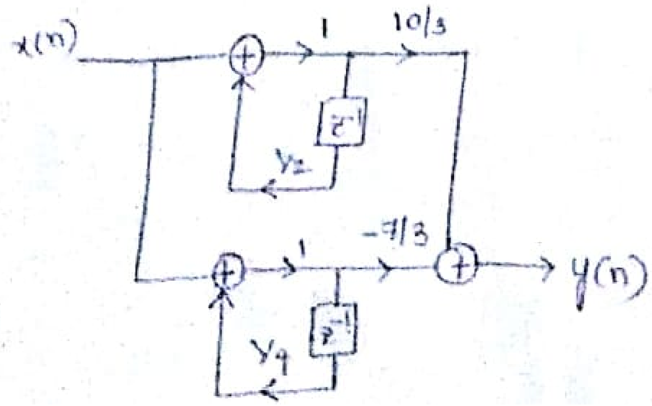
$$= \frac{A\left(1-\frac{1}{4}z^{-1}\right) + B\left(1-\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

Comp Consts $\Rightarrow A + B = 1$

Comp. z^{-1} Coeff $\Rightarrow -\frac{1}{4}A - \frac{1}{2}B = \frac{1}{3}$

$\Rightarrow A = \frac{10}{3}$; $B = -\frac{7}{3}$

$$\Rightarrow H(z) = \frac{10/3}{1-\frac{1}{2}z^{-1}} + \frac{-7/3}{1-\frac{1}{4}z^{-1}}$$



3. For an analog Transfer function $H(s)=\dfrac{2}{(s+1)(s+2)}$. Determine $H(z)$ using impulse invariance method. Assume $T = 1$ second. 4

Ans:

Given $H(s)=\dfrac{2}{(s+1)(s+2)} = \dfrac{A}{s+1}+\dfrac{B}{s+2} = \dfrac{A(s+2)+B(s+1)}{(s+1)(s+2)}$

Comp. Const $\Rightarrow 2A+B=2$

Comp s coff $\Rightarrow \underline{-A\pm B=0}$

$A=2,\ B=-2$

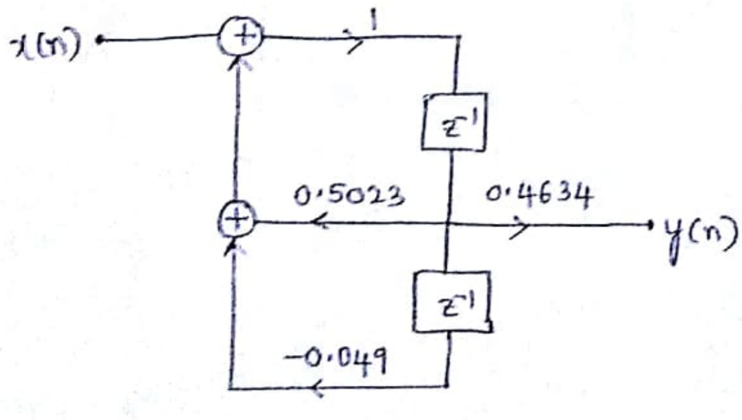
$$\Rightarrow H(s)=\frac{2}{s+1}-\frac{2}{s+2}$$

$$\therefore H(z)=\frac{2}{1-e^{-1}z^{-1}}-\frac{2}{1-e^{-2}z^{-1}}$$

$$\therefore H(z)=\frac{2}{1-0.367z^{-1}}-\frac{2}{1-0.1353z^{-1}}$$

$$H(z)=\frac{2(1-0.1353z^{-1})-2(1-0.367z^{-1})}{(1-0.367z^{-1})(1-0.1353z^{-1})}$$

$$H(z)=\frac{0.4634z^{-1}}{1-0.5023z^{-1}+0.049z^{-2}}$$

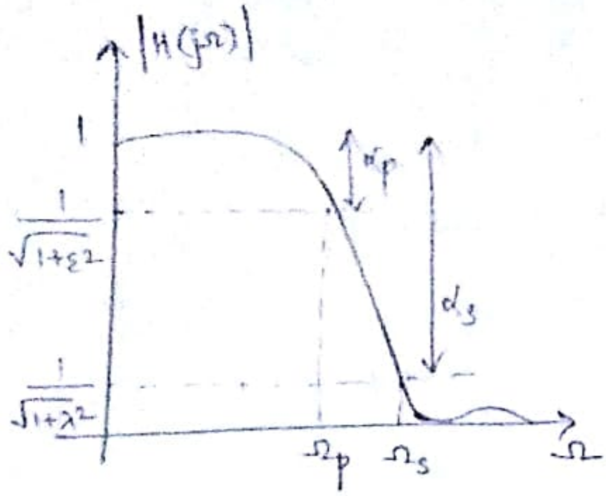


4. Compute the poles of an Analog Butterworth filter TF that satisfies the constraints $0.707 \le |H(j\Omega)| \le 1$; $0 \le \Omega \le 2$

$|H(j\Omega)| \le 0.1$; $\Omega \ge 4$

and determine $H_a(s)$ and hence obtain $H(z)$ using optimum transformation.

Ans:



$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.707 \; ; \; \Omega_p = 2$$

$$\Rightarrow \varepsilon = 1$$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.1 \; ; \; \Omega_s = 4$$

$$\Rightarrow \lambda = 9.95$$

$$N \geq \frac{\log_{10}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log_{10}\left(\dfrac{\Omega_s}{\Omega_p}\right)} = \frac{\log(\lambda/\varepsilon)}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)} = 3.3147$$

$$N = 4$$

To calculate poles $\Rightarrow s_k = e^{j\phi_k}$

where $\phi_k = \frac{\pi}{2} + \frac{(2k-1)}{2N}\pi$

For $N=4$

$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{8} = \frac{5\pi}{8} \Rightarrow s_1 = e^{j\frac{5\pi}{8}} = -0.3827 + j\,0.9239$$

$$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{8} = \frac{7\pi}{8} \Rightarrow s_2 = e^{j\frac{7\pi}{8}} = -0.9239 + j\,0.3827$$

$$\phi_3 = \frac{\pi}{2} + \frac{5\pi}{8} = \frac{9\pi}{8} \Rightarrow s_3 = e^{j\frac{9\pi}{8}} = -0.9239 - j\,0.3827$$

$$\phi_4 = \frac{\pi}{2} + \frac{7\pi}{8} = \frac{11\pi}{8} \Rightarrow s_4 = e^{j\frac{11\pi}{8}} = -0.3827 - j\,0.9239$$

$$\therefore \; H(s) = \frac{1}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)}$$

$$H(s) = \frac{1}{(s^2+0.7656\,s+1)(s^2+1.824\,s+1)}$$

$$\Omega_c = \frac{\Omega_p}{\left(10^{0.1\alpha_p}-1\right)^{1/2N}} = \frac{\Omega_p}{\varepsilon^{1/N}} = 2$$

$$H_a(s) = H(s)\Big|_{s \to \frac{s}{\Omega_c} = \frac{s}{2}}$$

$$H_a(s) = \frac{1}{\left[\left(\frac{s}{2}\right)^2 + 0.765\left(\frac{s}{2}\right)+1\right]} \times \frac{1}{\left[\left(\frac{s}{2}\right)^2 + 1.824\left(\frac{s}{2}\right)+1\right]}$$

$$= \frac{4}{(s^2+1.53s+4)} \times \frac{4}{(s^2+3.648s+4)}$$

$$\therefore H_a(s) = \frac{16}{(s^2+1.53s+4)(s^2+3.648s+4)}$$

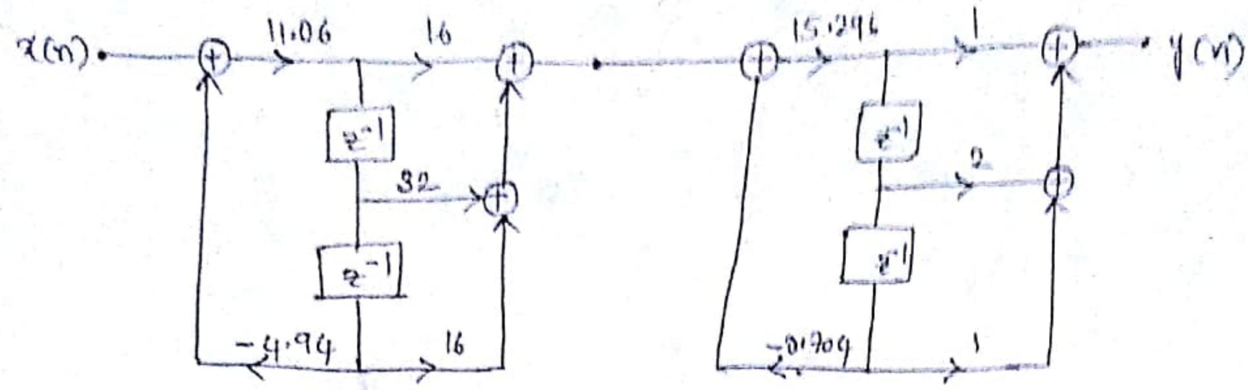
$$H(z) = H_a(s)\Big|_{s \to \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$ (Using bilinear Transformation)

Let $T=1$

$$H(z) = \frac{16}{\left[2^2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 3.06\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+4\right]} \times \frac{1}{\left[2^2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 7.296\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+4\right]}$$

$$= \frac{16(1+z^{-1})^2}{(4+4z^{-2}-\cancel{8z^{-1}}+3.06-3.06z^{-2}+4z^{-2}+\cancel{8z^{-1}}+4)} \times \frac{(1+z^{-1})^2}{(4+4z^{-2}-\cancel{8z^{-1}}+7.296-7.296z^{-2}+4z^{-2}+\cancel{8z^{-1}}+4)}$$

$$H(z) = \frac{16(1+2z^{-1}+z^{-2})}{(z^{-2}4.94+11.06)} \times \frac{(1+2z^{-1}+z^{-2})}{(0.704z^{-2}+15.296)}$$

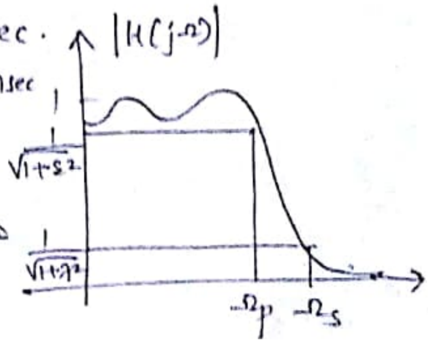


5. Design a digital low pass chebyshev filter with 2 dB cutoff frequency at 100 rad/sec. The attenuation should be atleast 15 dB for frequencies larger than 200 rad/sec. The sampling frequency is 10 kHz. Use bilinear transformation.

Ans: Given $\alpha_p = 2\,dB$; $\alpha_s = 15\,dB$

Digital freq. ← $\omega_p = 100$ rad/sec ; $\omega_s = 200$ rad/sec.

Given Sampling freq = 10 kHz ⇒ $T = \frac{1}{10k} = 0.1\,msec$



WKT

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)} = 1.89$$

$\therefore$ $N = 2$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = \frac{2}{0.1\times10^{-3}}\tan(100)$$
$$= 11744\ \text{rad/sec}$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p}-1} = 0.765$$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{2}{10^{-4}}\tan(50)$$
$$= 54.38\ \text{rad/sec.}$$

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.95$$

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 3086.97$$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 6253$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k\pm1)\pi}{2N} \qquad s_k = a\cos\phi_k + jb\sin\phi_k$$

$\phi_1 = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$; $\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = \frac{5\pi}{4}$

$s_1 = 3087 \cos\frac{3\pi}{4} + j6253 \sin\frac{3\pi}{4} = -2182.84 + j4421.54$

$s_2 = 3087 \cos\frac{5\pi}{4} + j6253 \sin\frac{5\pi}{4} = -2182.84 - j4421.54$

$D_r = (s - s_1)(s - s_2)$

$= (s + 2182.84 - j4421.54)(s + 2182.84 + j4421.54)$

$= (s + 2182.84)^2 + \cancel{(155.477)^2}\ (4421.54)^2$

$N_r = \dfrac{D_r|_{s=0}}{\sqrt{1+\varepsilon^2}} = \dfrac{(2182.84)^2 + (4421.54)^2}{\sqrt{1 + (0.765)^2}} = (4394.53)^2$
(N is even)

$\therefore\ H(s) = \dfrac{(4394.53)^2}{(s + 2182.84)^2 + (4421.54)^2} = \dfrac{(4394.53)^2}{s^2 + 4365.68 s + (2182.84)^2 + (4421.54)^2}$

Substitute $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ in above i.e.,

$H(z) = H(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$

where $T = 10^{-4}$ sec

$$H(z) = \frac{(4394.53)^2}{\left[2\times10^{4}\frac{(1-z^{-1})}{(1+z^{-1})}\right]^2 + 4365.68\left(2\times10^4\frac{(1-z^{-1})}{(1+z^{-1})}\right) + (4931)^2}$$

$$= \frac{(4394.53)^2(1+z^{-1})^2}{4\times10^8(1-2z^{-1}+z^{-2}) + 8730.76\times10^4(1-z^{-2}) + (4931)^2(1+2z^{-1}+z^{-2})}$$

$$= \frac{(4394.53)^2(1+2z^{-1}+z^{-2})}{(22619)^2 - (27411)^2 z^{-1} + (18357.75)^2 z^{-2}}$$

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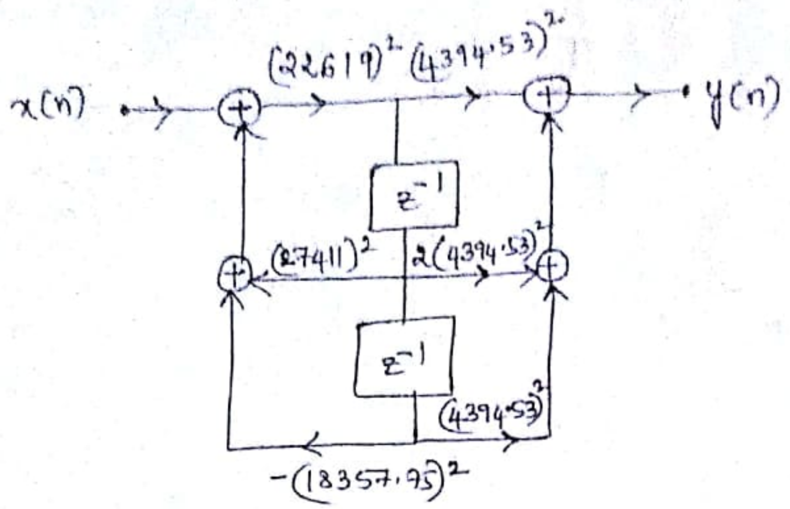


6. Discuss in detail the procedure of designing an analog filter using Butterworth approximation technique.

Ans: <u>steps in designing an analog filter using Butterworth approximation:</u>

1) For the given specifications, find the order of filter N using

$$N \geq \frac{\log_{10}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log_{10}\left(\dfrac{\Omega_s}{\Omega_p}\right)} = \frac{\log_{10}\left(\dfrac{\lambda}{\varepsilon}\right)}{\log_{10}\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

where Ω_s = analog stop band frequency

Ω_p = analog pass band cut-off frequency.

α_s = attenuation in stopband

α_p = attenuation in pass band

2) Round off to the nearest highst integer value.

3) Find the transfer function H(s) for Ω_c = 1 rad/sec for the value of N.

a) First calculate the 'N' no. of poles using the formula

$$s_k = e^{j\phi_k} \quad \text{where} \quad \phi_k = \frac{\pi}{2} + \frac{(2k-1)}{2N}\pi$$

in order to obtain only stable poles and for all N.

b) Then Calculate $H(s) = \prod_{K=1}^{N} \frac{1}{(s - s_K)}$

4) Calculate the value of cutoff frequency Ω_c where (to design LP (or) HP)

$$\Omega_c = \frac{\Omega_p}{\left(10^{0.1\alpha_p} - 1\right)^{1/2N}} \quad (\text{or}) \quad \Omega_c = \frac{\Omega_s}{\left(10^{0.1\alpha_s} - 1\right)^{1/2N}} \quad (\text{or})$$

$$\Omega_c = \frac{\Omega_p}{\varepsilon^{1/N}} \quad (\text{or}) \quad \Omega_c = \frac{\Omega_s}{\lambda^{1/N}}$$

where $\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$ and $\lambda = \sqrt{10^{0.1\alpha_s} - 1}$

5) Find the Transfer function $H_a(s)$ for the above value of Ω_c

a) by substituting $s \rightarrow \frac{s}{\Omega_c}$ in $H(s)$. (for LP)

i.e., $H_a(s) = H(s)\Big|_{s \rightarrow \frac{s}{\Omega_c}}$.

b) For designing HP substitute $s \rightarrow \frac{\Omega_c}{s}$ in $H(s)$ i.e., $H_a(s) = H(s)\Big|_{s \rightarrow \frac{\Omega_c}{s}}$

c) For designing BP with cutoff frequencies Ω_l, Ω_u Can be accomplished by

$$s \longrightarrow \frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)} \quad ; \quad A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1(\Omega_u - \Omega_l)}$$

and $\Omega_r = \min\{|A|, |B|\}$; $B = \dfrac{\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)}$

d) For designing Band Stop filter with Cutoff frequencies Ω_l, Ω_u

$$s \rightarrow \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_l \Omega_u} \quad ; \quad A = \frac{\Omega_1(\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_l \Omega_u}$$

$\Omega_r = \min\{|A|, |B|\}$; $B = \dfrac{\Omega_2(\Omega_u - \Omega_l)}{-\Omega_2^2 + \Omega_l \Omega_u}$

7. Explain how to convert an analog filter transfer function into digital filter transfer function using Bilinear transformation.

Ans: 1. From the given specifications, find pre warping analog frequencies using analog $\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$

where ω = digital frequency (given)

2. Using analog frequencies, find $H_a(s)$ of analog filter by either butterworth (or) chebyshev approximation.

3. Select the sampling rate T of the digital filter.

4. Substitute $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ into the Transfer function

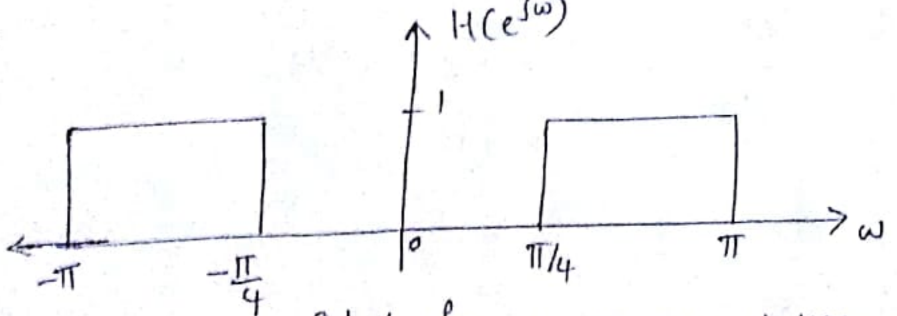
$$H(z) = H_a(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

5. Realize the obtained $H(z)$ using any one of the forms.

8. Design an ideal HPF with frequency response $H_d(e^{j\omega}) = \begin{cases} 1 & \text{for } \frac{\pi}{4} \le \omega \le \pi \\ 0 & \text{otherwise} \end{cases}$

using Hamming window for $N = 9$.

Ans:



Ideal frequency response of HPF.

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} 1 \cdot e^{j\omega n}\, d\omega + \int_{\pi/4}^{\pi} 1 \cdot e^{j\omega n}\, d\omega\right]$$

$$h_d(n) = \frac{1}{2\pi jn}\left[e^{j\omega n}\Big]_{-\pi}^{-\pi/4} + e^{j\omega n}\Big]_{\pi/4}^{\pi} \right]$$

$$= \frac{1}{\pi n(2j)}\left[e^{-j\frac{\pi n}{4}} - e^{-j\pi n} + e^{j\pi n} - e^{j\frac{\pi n}{4}} \right]$$

$$= \frac{1}{\pi n}\left[\frac{e^{j\pi n} - e^{-j\pi n}}{2j} + \frac{-e^{j\frac{\pi n}{4}} + e^{-j\frac{\pi n}{4}}}{2j} \right]$$

$$h_d(n) = \frac{1}{\pi n}\left(\sin \pi n - \sin \frac{\pi}{4} n\right) \quad [\text{for all } n \text{ except } n=0]$$

$N = 9 \Rightarrow$ $h(n) = \begin{cases} h_d(n)\omega_H(n) & \text{for } |n| \le 4 \\ 0 & \text{otherwise} \end{cases}$

Using Hamming window:

$$\omega_H(n) = \begin{cases} 0.54 + 0.46 \cos \frac{2\pi n}{N-1} \; ; & -\frac{(N-1)}{2} \le n \le \frac{N-1}{2} \\ 0 \; ; & \text{otherwise.} \end{cases}$$

$\omega_H(n) = 0.54 + 0.46 \cos \frac{\pi n}{4}$; $-4 \le n \le 4$

$\omega_H(0) = 1$

$\omega_H(-1) = \omega_H(1) = 0.912$

$\omega_H(-2) = \omega_H(2) = 0.682$

$\omega_H(-3) = \omega_H(3) = 0.398$

$\omega_H(-4) = \omega_H(4) = 0.1678$

and

$h_d(1) = h_d(-1) = -0.225$

$h_d(2) = h_d(-2) = -0.159$

$h_d(3) = h_d(-3) = -0.075$

$h_d(4) = h_d(-4) = 0.$

$$h_d(0) = \lim_{n\to 0} \frac{\sin \pi n}{\pi n} - \lim_{n\to 0} \frac{\sin \frac{\pi}{4} n}{\pi n}$$

$$h_d(0) = 1 - \frac{1}{4} = 3/4 = 0.75$$

∴ Filter Co-efficients using Hamming window

$h(0) = h_d(0)\,\omega_H(0) = 0.75$

$h(1) = h_d(1)\,\omega_H(1) = -0.2052 = h(-1)$

$h(2) = h(-2) = h_d(2)\, \omega_H(2) = -0.1084$

$h(3) = h(-3) = h_d(3)\, \omega_H(3) = -0.03$

$h(4) = h(-4) = h_d(4)\, \omega_H(4) = 0$

$$\therefore H(z) = h(0) + \sum_{n=1}^{4} h(n)\left[z^{-n} + z^{n}\right]$$

$$H(z) = 0.75 - 0.2052(z^{-1} + z) - 0.1084(z^{-2} + z^{2}) - 0.03(z^{-3} + z^{+3})$$

Transfer function of a realizable filter

$$H'(z) = H(z)\, z^{-4} \qquad \left[\because H'(z) = H(z)\, z^{-\frac{(N-1)}{2}}\right]$$

$$H'(z) = 0.75 z^{-4} - 0.2052(z^{-5} + z^{-3}) - 0.1084(z^{-6} + z^{-2}) - 0.03(z^{-7} + z^{-1})$$

$$H'(z) = -0.03 z^{-1} - 0.1084 z^{-2} - 0.2052 z^{-3} + 0.75 z^{-4} - 0.2052 z^{-5}$$
$$- 0.1084 z^{-6} - 0.03 z^{-7}$$

$\therefore$ Filter Co-efficients are

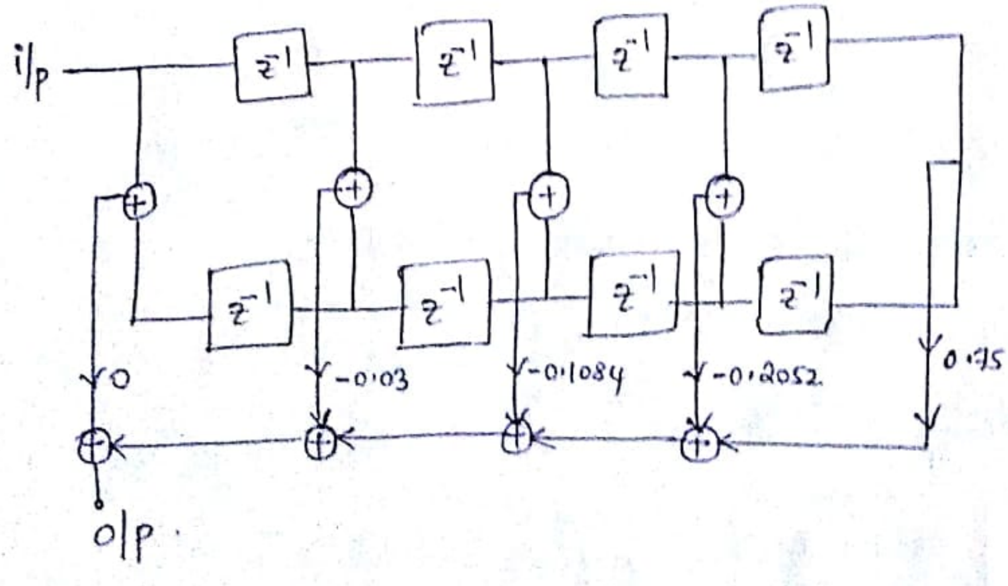
$h(0) = h(8) = 0.$

$h(1) = h(7) = -0.03$

$h(2) = h(6) = -0.1084$

$h(3) = h(5) = -0.2052$

$h(4) = +0.75$



$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{(N-1)/2} a(n)\cos\omega n$$

where $a(0) = h\left(\frac{N-1}{2}\right) = h(4) = 0.75$

$a(n) = 2h\left(\frac{N-1}{2} - n\right)$

$a(1) = 2h(4-1) = 2h(3) = -0.2052 \times 2 = -0.4104$

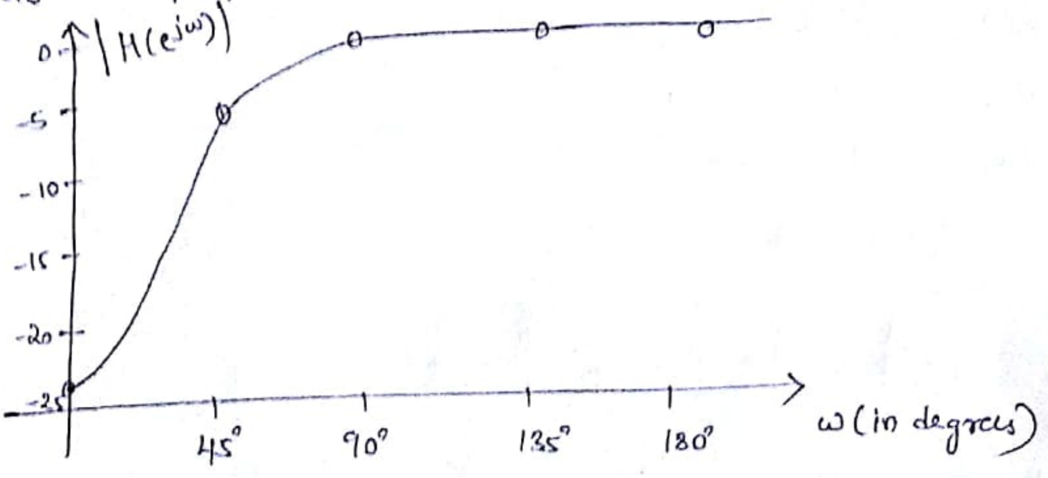
$a(2) = 2h(4-2) = 2h(2) = -0.1084 \times 2 = -0.2168$

$a(3) = 2h(4-3) = 2h(1) = -0.03 \times 2 = -0.06$

$a(4) = 2h(4-4) = 2h(0) = 0.$

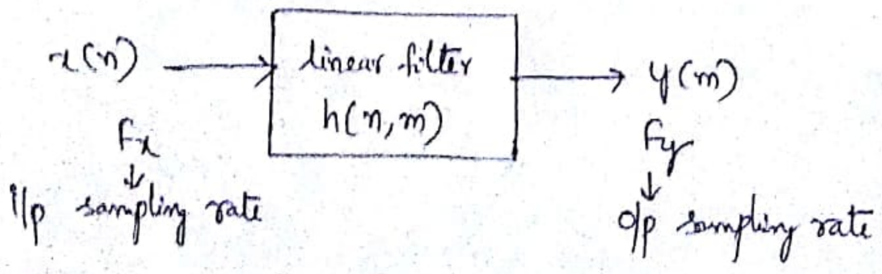
$\bar{H}(e^{j\omega}) = 0.75 - 0.4104\cos\omega - 0.2168\cos 2\omega - 0.06\cos 3\omega$

ω (in degrees)	$0°$	$45°$	$60°$	$90°$	$120°$	$135°$	$180°$
$\bar{H}(e^{j\omega})$	0.0628	0.50	0.7132	0.9668	1.0036	0.99	1.0036
$\bar{H}(e^{j\omega})_{dB}$	-24.04	-6.02	-2.935	-0.293	0.031	-0.087	0.031

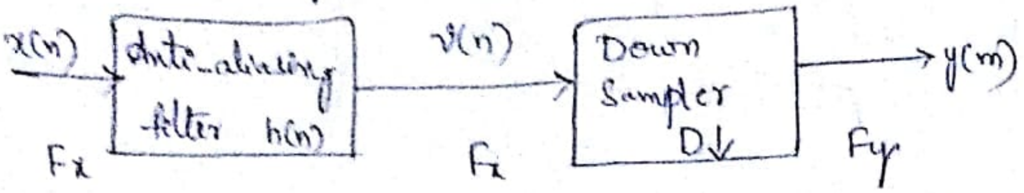


9. Derive the expression for decimation by factor D.

Sol: Decimation is a process of down sampling, reducing the sampling rate by a factor D.



To avoid aliasing, for Band limited $|\omega| \le \frac{\pi}{D}$.



Anti-aliasing filter is perfect LPF with impulse h(n).

$$v(n) = x(n) * h(n) \longrightarrow (1)$$

$$v(n) = \sum_{K=0}^{\infty} h(k)\, x(n-k)$$

$$y(m) = v(mD) = \sum_{K=0}^{\infty} h(k)\, x(mD-k) \longrightarrow (2)$$

Down sampling operation is observed as multiplication of two sequences.

$$\tilde{v}(n) = v(n)\, S(n)$$

$$y(m) = \tilde{v}(mD) \longrightarrow (3)$$

Take/Apply ZT. to eq(3) $\quad Y(z) = \sum_{m=-\infty}^{\infty} y(m)\, z^{-m}$

$$Y(z) = \sum_{m=-\infty}^{\infty} \tilde{v}(mD)\, z^{-m}$$

$\tilde{v}(m) = 0$ except at multiples of D.

$$\therefore\ Y(z) = \sum_{m=-\infty}^{\infty} \tilde{v}(m)\, z^{-\frac{m}{D}} \longrightarrow (4)$$

$$\tilde{v}(m) = v(m)\, S(m)$$

$$Y(z) = \sum_{m=-\infty}^{\infty} v(m)\, S(m)\, z^{-\frac{m}{D}} \longrightarrow (5)$$

S(m) Can be represented by DFS as

$$S(m) = \frac{1}{D} \sum_{K=0}^{D-1} e^{j\frac{2\pi k m}{D}}$$

$$\Rightarrow Y(z) = \sum_{m=-\infty}^{\infty} v(m) \left[\frac{1}{D} \sum_{k=0}^{D-1} e^{j\frac{2\pi k m}{D}} \right] z^{-\frac{m}{D}}$$

$$Y(z) = \frac{1}{D}\sum_{k=0}^{D-1}\left[\sum_{m=-\infty}^{\infty} v(m)\left(e^{-j\frac{2\pi k}{D}} z^{1/D}\right)^{-m}\right]$$

$$Y(z) = \frac{1}{D}\sum_{k=0}^{D-1} V\left(e^{-j\frac{2\pi k}{D}} z^{1/D}\right)$$

WKT $v(n) = h(n) * x(n) \Rightarrow V(z) = H(z)X(z)$

$$\Rightarrow Y(z) = \frac{1}{D}\sum_{k=0}^{D-1} H\left(e^{-j\frac{2\pi k}{D}}. z^{1/D}\right). X\left(e^{-j\frac{2\pi k}{D}}. z^{1/D}\right)$$

Substitute $z = re^{j\omega_y}$; $r=1$

$$Y(e^{j\omega_y}) = \frac{1}{D}\sum_{k=0}^{D-1} H\left(e^{-j\frac{2\pi k}{D}} e^{j\frac{\omega_y}{D}}\right). X\left(e^{-j\frac{2\pi k}{D}}. e^{j\frac{\omega_y}{D}}\right)$$

$$= \frac{1}{D}\sum_{k=0}^{D-1} H\left(e^{j\frac{\omega_y-2\pi k}{D}}\right). X\left(e^{j\frac{\omega_y-2\pi k}{D}}\right)$$

Let $Y(e^{j\omega_y}) = Y(\omega_y)$

$$Y(\omega_y) = \frac{1}{D}\sum_{k=0}^{D-1} H\left(\frac{\omega_y-2\pi k}{D}\right). X\left(\frac{\omega_y-2\pi K}{D}\right)$$

$X\left(\frac{\omega_y-2\pi k}{D}\right)$ indicates replica's of $X\left(\frac{\omega_y}{D}\right)$ at $\omega = 0, 2\pi, 4\pi, \ldots 2\pi(D-1)$

These are due to down sampling operations

$$\therefore Y(\omega_y) = \frac{1}{D} H\left(\frac{\omega_y}{D}\right). X\left(\frac{\omega_y}{D}\right)$$

with a properly designed filter $H_D(\omega)$, the aliasing is eliminated

$$\therefore Y(\omega_y) = \frac{1}{D} X\left(\frac{\omega_y}{D}\right) \quad \text{for } 0 \le |\omega_y| \le \pi$$

$$Y(\omega_y) = \frac{1}{D} X(\omega_x) \qquad \left(\because \omega_x = \frac{\omega_y}{D}\right)$$

The anti-aliasing filter band limits the signal to π/D.

$$H(\omega_x) = \begin{cases} 1 & \text{for } |\omega_x| \le \frac{\pi}{D} \\ 0 & \text{elsewhere} \end{cases}$$

Two sampling rates are related as $F_y = \frac{F_x}{D}$.

$$\omega_x = 2\pi \frac{F}{F_x} \; ; \; \omega_y = 2\pi \frac{F}{F_y}$$

$$\omega_y = 2\pi \frac{F}{F_x/D} = 2\pi \frac{F}{F_x} \cdot D = \omega_x D.$$

$$\boxed{\therefore \; \omega_x = \frac{\omega_y}{D}}$$

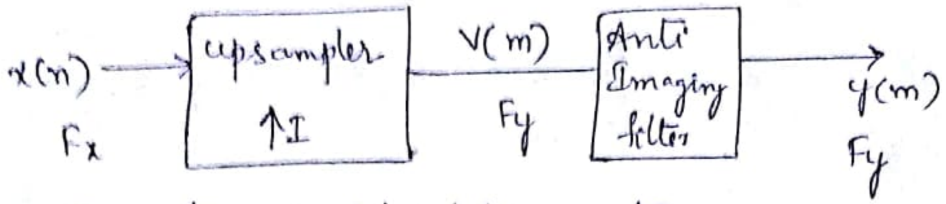
10. Explain Interpolation process with equations and spectrum.

Ans: Interpolation is defined as process of upsampling by a factor I

Interpolator is also called as sampling rate alternation device.

Sampling freq F_x of i/p signal ↑ by $I \Rightarrow F_y = I(F_x)$

Interpolator simply puts $(I-1)$ zeros b/w successive samples of $x(n)$



Derivation of interpolation equation

$$y(m) = V(m) * h(m)$$

$$y(m) = \sum_{k=-\infty}^{\infty} h(m-k)\, V(k)$$

$V(k) \neq 0$ multiples of I

$$\therefore h(m-k)\,V(k) = \begin{cases} 0 & \text{for } k \neq \text{Integral multiple of } I \\ \text{non-zero} & \text{elsewhere} \end{cases}$$

Replace k by kI

$$y(m) = \sum_{k=-\infty}^{\infty} h(m-kI)\, v(kI)$$

$$v(kI) = x(k)$$

$$\therefore y(m) = \sum_{k=-\infty}^{\infty} h(m-kI)\, x(k)$$

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Relation between spectrums of $x(n)$ and $y(m)$ (18)

Step 1: $v(m) = \begin{cases} x\left(\frac{m}{I}\right) & m = 0, \pm I, \pm 2I \\ 0 & \text{otherwise} \end{cases}$

Step 2: z-Transform $V(z) = \sum_{m=-\infty}^{\infty} v(m) z^{-m}$

$$V(z) = \sum_{m=-\infty}^{\infty} v(mI) z^{-mI}$$

Step 3: $v(mI) = x(m)$

$$V(z) = \sum_{m=-\infty}^{\infty} x(m) z^{-mI} = \sum_{m=-\infty}^{\infty} x(m)(z^{I})^{-m}$$

$$V(z) = X(z^{I})$$

Step 4: $V(z)$ is evaluated on unit circle, then its spectrum can be obtained as $z = e^{j\omega_y}$

$$V(e^{j\omega_y}) = X(e^{j\omega_y I}) \Rightarrow V(\omega_y) = X(\omega_y I)$$

Step 5: $F_y = I F_x$

$$\omega_y = 2\pi \frac{F}{F_y} = 2\pi \frac{F}{I F_x} = \frac{\omega_x}{I}$$

$$H(\omega_y) = \begin{cases} C, & \frac{-\pi}{I} \le \omega_y \le \frac{\pi}{I} \\ 0 & \text{otherwise} \end{cases}$$

Step 6: $Y(\omega_y) = \begin{cases} C X(\omega_y I) & \frac{-\pi}{I} \le \omega_y \le \frac{\pi}{I} \\ 0 & \text{otherwise} \end{cases}$

value of scale factor C $\quad y(m) = x\left(\frac{m}{I}\right)$

IFT $\quad y(m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(\omega_y) e^{j\omega m} d\omega_y$

For $m=0 \Rightarrow y(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(\omega_y) d\omega_y = \frac{1}{2\pi} \int_{-\pi/I}^{\pi/I} C X(\omega_y I) d\omega_y$

$\omega_y = \frac{\omega_x}{I}$ (or) $\omega_x = \omega_y I$

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$$y(0) = \frac{c}{2\pi}\int_{-\pi}^{\pi} X(\omega_x)\cdot\frac{1}{I}\,d\omega_x$$

$$y(0) = \frac{c}{I}\cdot\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega_x)\,d\omega_x$$

$$\boxed{y(0) = \frac{c}{I}\,x(0)}$$

$\therefore\ y(0) = x(0)$ if $c = I \longrightarrow$ normalized interpolation.

<u>Spectrum of signals:</u>