Orbital Mechanics & Launchers Unit-II

(1)

- For orbiting satellites atmospheric drag should be minimum (above 400km drag is negligible).
- International Space station (ISS) injected into orbit at an altitude of 397km –
  - 9 June 1999.
  - At end of year, orbital height decreased to 360km
  - Onboard thrusters and sufficient orbital maneuvering fuel is required.

Newton's laws of motion

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2at$ ← neglecting any drag or other perturbing forces

$v = u + at$

$F = ma \rightarrow$ Motion of satellite in a stable orbit

where s = distance travelled from time $t=0$;

u = intial velocity of the object at $t=0$

v = final velocity at time t

(+) or (–) a = acceleration of the object

F = force acting on the object

m = mass of the object

In a stable orbit, two forces acts on Satellite:

1. Centrifugal force → due to KE of satellite → fling the sat. into higher orbit
2. Centripetal force → due to gravitational attraction of planet which attempts to pull sat. toward planet

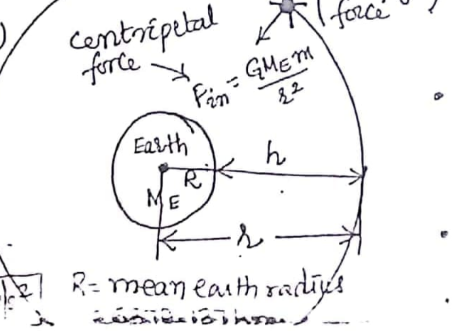
If these two forces are equal, the satellite will remain in a stable orbit.

Satellite in a stable orbit is described as "free fall".

Force (N) = mass (kg) acceleration (m/s^2)

The standard acceleration due to gravity at the earth's surface is

$a = 9.80665 \times 10^{-3}\ km/s^2$



R = mean earth radius

- The centripetal acceleration, a, due to gravity at orbit a distance 'r' from the center of the earth ($r = R + h$ = mean radius of earth + height of the satellite above surface of earth) is:

$$\boxed{a = \frac{\mu}{r^2}\ km/s^2} \rightarrow (1)$$

where $\mu = GM_E$ = kepler's constant

$= 3.986004418 \times 10^5\ km^3/s^2$.

G = Universal gravitational constant

$= 6.672 \times 10^{-11}\ Nm^2/kg^2$ or

$= 6.672 \times 10^{-20}\ km^3/kg \cdot s^2$

- The centripetal force acting on the satellite is given by ($F = ma$):

$$F_{in} = m \times \frac{\mu}{r^2}$$

$$\boxed{F_{in} = m \times \frac{GM_E}{r^2}} \rightarrow (2)$$

- Similarly, the centrifugal acceleration is given by

$$\boxed{a = \frac{v^2}{r}\ km/s^2} \rightarrow (3)$$

- Then the centrifugal force is

$$\boxed{F_{out} = m \times \frac{v^2}{r}} \rightarrow (4)$$

- If the forces on the satellite are balanced

$F_{in} = F_{out}$

$$\Rightarrow \cancel{m} \times \frac{GM_E}{r^{\cancel{2}}} = \cancel{m} \times \frac{v^2}{\cancel{r}} \Rightarrow \boxed{v = \sqrt{\frac{\mu}{r}}} \rightarrow (5)$$

→ Hence the velocity of a satellite in a circular orbit is given by

$$\boxed{v = \left(\frac{\mu}{r}\right)^{1/2}}$$

- If the orbit is circular, the distance travelled by a satellite in one orbit around a planet is $2\pi r$.
- Since Time = $\frac{\text{distance}}{\text{velocity}}$

. Then the period of the satellite's orbit

Table : Various Satellite Systems

Satellite system	orbital height (km)	orbital velocity (km/s)	orbital period h min s
Intelsat (GEO)	35,786.03	3.0747	23 56 4.1
ICO (MEO)	10,255	4.8954	5 55 48.4
Skybird (LEO)	1,469	7.1272	1 55 17.8
Iridium (LEO)	780	7.4624	1 40 27.0

In above table, all orbits are circular and average radius of the earth = 6,378.137 km

Ex: 2.1.1 GEO stationary satellite orbit radius — pp: 29

Ex: 2.1.2 Low Earth Orbit (LEO); pp: 29

Describe the orbit of the satellite around a planet :

Geocentric coordinate system : is a cartesian coordinate system (x, y, z) with the earth at the center and the reference planes (x, y, z) coinciding with the equator and the polar axis (earth rotation axis).

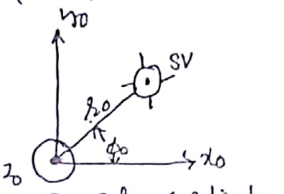
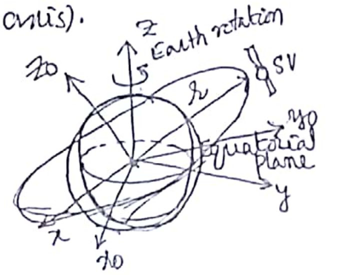


Fig. Polar Coordinate System



The gravitational force F on the satellite is given by

$$\bar{F} = -\frac{GM_E\, m\bar{r}}{r^3} \quad \text{orbit vector distance} \longrightarrow (7)$$

- Force = mass × acceleration

$$\bar{F} = m \cdot \frac{d^2\bar{r}}{dt^2} \longrightarrow (8)$$

- From eq (7) and (8) $\Rightarrow -\frac{(GM_E)\, m\bar{r}}{r^3} = m \cdot \frac{d^2\bar{r}}{dt^2}$

$$\Rightarrow -\frac{\bar{r}}{r^3} \cdot \mu = \frac{d^2\bar{r}}{dt^2} \longrightarrow (9)$$

$\Rightarrow \frac{d^2\bar{r}}{dt^2} + \frac{\bar{r}}{r^3}\mu = 0 \longrightarrow (10)$

(2) • This is a second order linear differential equation and its solution will involve six undetermined constants called the orbital elements.

- The solution to eq (10) is difficult since the second derivative of r involves the second derivative of the unit vector r.
- To remove this dependence, a different set of coordinates can be choosen to describe the location of the satellite such that the unit vectors in the three axes are constant.
- Expressing eq (10) in terms of new coordinate axes x_0, y_0 and z_0 gives

$$\hat{x}_0\left(\frac{d^2x_0}{dt^2}\right) + \hat{y}_0\left(\frac{d^2y_0}{dt^2}\right) + \frac{\mu(x_0\hat{x}_0 + y_0\hat{y}_0)}{(x_0^2+y_0^2)^{3/2}} = 0 \longrightarrow (11)$$

- Eq (11) is easier to solve if it is expressed in a polar coordinate system using the transformations

$$\left.\begin{aligned} x_0 &= r_0\cos\phi_0 \\ y_0 &= r_0\sin\phi_0 \\ \hat{x}_0 &= \hat{r}_0\cos\phi_0 - \hat{\phi}_0\sin\phi_0 \\ \hat{y}_0 &= \hat{\phi}_0\cos\phi_0 + \hat{r}_0\sin\phi_0 \end{aligned}\right\} (12)$$

and equating the vector components of r_0 and ϕ_0 in eq (11) yeilds

$$\frac{d^2r_0}{dt^2} - r_0\left(\frac{d\phi_0}{dt}\right)^2 = -\frac{\mu}{r_0^2} \longrightarrow (13)$$

and $r_0\left(\frac{d^2\phi_0}{dt^2}\right) + 2\left(\frac{dr_0}{dt}\right)\left(\frac{d\phi_0}{dt}\right) = 0 \longrightarrow (14)$

- Using standard mathematical procedure we can develop an equation for the radius of the satellite's orbit, r_0, namely

$$\boxed{r_0 = \frac{p}{1+e\cos(\phi_0-\theta_0)}} \longrightarrow (15)$$

where θ_0 is a constant and 'e' is the eccentricity of an ellipse whose semilatus rectum p is given by where h is the magnitude of the orbital angular momentum of the satellite.

- Eq (16) is the equation of the orbit is an ellipse is first law of planetary motion.

1 (a)

Kepler's Three Laws of planetary Motion

Johannes Kepler (1571–1630) was a German astronomer and scientist who developed his three laws of planetary motion by observations of the behavior of the planets in the Solar System over many years.

Laws:

1. The orbit of any smaller body about a larger body is always an ellipse, with the center of mass of the larger body as one of the two foci.

The orbit of the smaller body sweeps out equal areas in equal time.

- if $t_1 - t_2 = t_3 - t_4$ then $\boxed{A_{12} = A_{34}}$

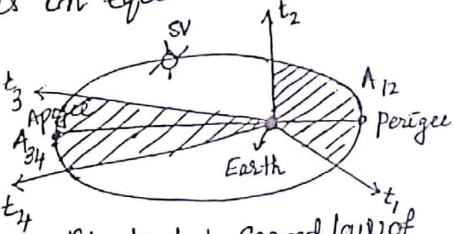


Fig. Kepler's Second law of planetary motion.

Perigee : closest point to the center of the earth

Apogee : farthest point to the " " .

- The square of the period of revolution of the smaller body about the larger body equals a constant multiplied by the third power of the semimajor axis of the orbital ellipse.

i.e $\boxed{T^2 = \frac{4\pi^2}{\mu} \cdot a^3}$

where T = the orbital period

a = semimajor axis of the orbital ellipse

μ = Kepler's constant

NOTE: If the orbit is circular then $a = r$ as in eq. (6)

(3) Describing the orbit of a Satellite

- The quantity θ_0 in eq. (15) serves to orient the ellipse with respect to the orbital plane axes x_0 and y_0.
- The orbit is an ellipse and can always choose x_0 and y_0 so that θ_0 is zero.
- Then eq (15) becomes (if $\theta_0 = 0$) as

$$r_0 = \frac{p}{1 + e\cos\phi_0} \longrightarrow (17)$$

- The path of the satellite in the orbital plane is shown in the fig. below:

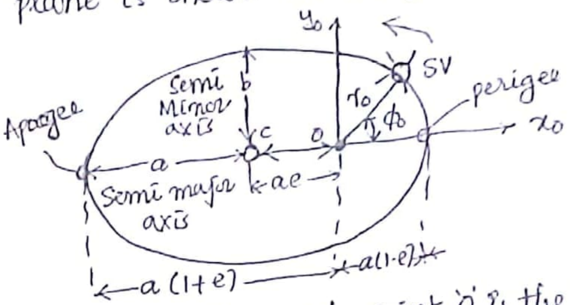


Fig. orbital ellipse. The point 'O' is the center of the earth and the point 'C' is the center of the ellipse. The two centers do not coincide unless the eccentricity, e, of the ellipse is zero (i.e, ellipse becomes a circle and $a = b$).

- The lengths 'a' and 'b' of the semimajor and semiminor axes are given by

$$a = \frac{p}{1 - e^2} \longrightarrow (18)$$

$$b = \sqrt{a(1-e^2)} \longrightarrow (19)$$

- The point in the orbit, where the satellite is closest to the earth is called the Perigee.
- The point in the orbit, where the satellite is farthest from the earth is called the apogee.
- The perigee and apogee are always exactly opposite to each other.
- The differential area swept by the vector r_0 from the origin to the satellite in time dt is given by

= Magnitude of the orbital angular momentum of the satellite, the radius vector of the satellite can be seen to sweep out equal areas in equal times.
This is Kepler's second law of planetary motion.
; equating the area of the ellipse πab to the area swept out in one orbital revolution, we can derive an expression for the orbital period T as

$$\boxed{T^2 = 4\pi^2 a^3/\mu} \longrightarrow (21)$$

this is the mathematical expression of Kepler's third law of planetary motion: ($T^2 \propto a^3$)

Eq.(21) determines the period of the orbit of any satellite, and is used in every GPS receiver in the calculation of the positions of GPS satellites.

Eq.(21) is also used ~~for~~ to find the orbital radius of a GEO satellite, for which the period T must be made exactly equal to the period of one revolution of the earth for the satellite to remain stationary over a point on the equator.

The orbital period 'T' is the time, the orbiting body takes to return to the same reference point in space with respect to the galactic background.

Nearly always, the primary body will also be rotating and so the period of revolution of the satellite may be different from that perceived by an observer who is standing still on the surface of the primary body.

∴ The orbital period of a GEO satellite is exactly equal to the period of rotation of earth, 23h 56 min 4·1sec, but to an observer on ground, the satellite appears to have an infinite orbital period, it

A perfect Geostationary orbit:

needs to have 3 features:

1. It must be exactly circular (i.e. eccentricity $e = 0$)
2. It must be at the correct altitude (i.e to have correct period), and
3. It must be in the plane of the equator (i.e have a zero inclination w.r.t the equator).

Geosynchronous orbit

- If the inclination of the satellite is not zero and/or if the eccentricity is zero, but the orbital period is same as the earth's revolution, then the satellite will be in a geosynchronous orbit.
- The position of a geosynchronous satellite will appear to oscillate about a mean look angle in the sky w.r.t. a station observer on the earth's surface.
- The orbital period of a GEO satellite, 23h 56 min 4·1s, is one sidereal day
- A sidereal day is the time between consecutive crossings of any particular longitude on the earth by any star, other than sun.
- The mean solar day of 24h is the time between any consecutive crossings of any particular longitude by the sun, and is the time between successive sun rises (or sunsets) observed at one location on earth, averaged over an entire year.
- Because the earth moves around the sun once per 365¼ days, the solar day is 1440/365.25 = 3·94 min longer than a sidereal day.

Locating the Satellite in the orbit:

- The equation of the orbit may be rewritten by combining eq (15) and (16) to obtain

$$r_0 = \frac{a(1-e^2)}{1+e\cos\phi_0} \longrightarrow (22)$$

- The angle ϕ_0 is measured from x_0 axis and is called true anomaly

Since we defined the positive x_0 axis so that it passes through the perigee, ϕ_0 measures the angle from the perigee to the instantaneous position of the satellite.

The rectangular coordinates of the satellite are given by

$$x_0 = r_0 \cos\phi_0 \longrightarrow (23)$$

$$y_0 = r_0 \sin\phi_0 \longrightarrow (24)$$

The orbital period T is the time for the satellite to complete a revolution in inertial space, travelling a total of 2π radians.

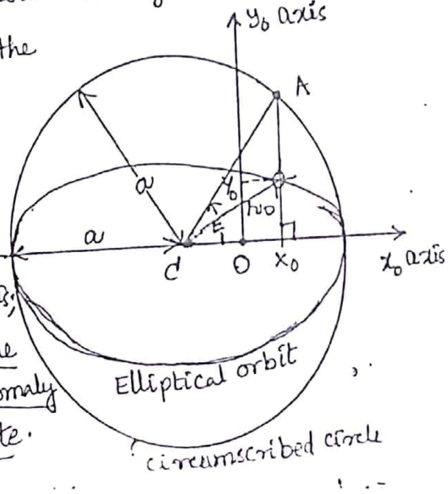
The average angular velocity

$$\boxed{\eta = \frac{2\pi}{T} = (\mu^{1/2}).(a^{-3/2})} \longrightarrow (25)$$

If the orbit is an ellipse, the instantaneous angular velocity will vary with the position of the satellite around the orbit.

consider the geometry of the circumscribed circle as shown in Fig. below.

A line from the center of the ellipse (c) to this point (A) makes an angle E with the x_0 axis; E is called the eccentric anomaly of the satellite.



- E is related to the radius r_0 by

$$r_0 = a(1 - e\cos E) \longrightarrow (26)$$

thus $a - r_0 = ae\cos E \longrightarrow (27)$

- We can also develop an expression that relates eccentric anomaly E to the average angular velocity η, which yields

$$\eta\, dt = (1 - e\cos E)\, dE \longrightarrow (28)$$

- Let t_p be the time of perigee. This is simultaneously the time of closest approach to the earth; – the time when the satellite is crossing the x_0 axis; and the time when E is zero.
- If we integrate both sides of eq (28),

$$\int_{t_p}^{t} \eta\, dt = \int_{t_p}^{t} (1 - e\cos E)\, dE$$

$$\underbrace{\eta(t - t_p)}_{M} = E - e\sin E \longrightarrow (29)$$

- The left of eq (29) is called the mean anomaly M. Thus

$$M = \eta(t - t_p) = E - e\sin E \longrightarrow (30)$$

- The mean anomaly M is the arc length (in radians) that the satellite would have traversed since the perigee passage if it ~~were~~ was moving on the circumscribed circle at the mean angular velocity η
- If we know the time of perigee t_p, the eccentricity e, and the length of the semimajor axis a, we now have the necessary equations to determine the coordinates (r_0, ϕ_0) and (x_0, y_0) of the satellite in the orbital plane.
- The process is as follows:

1. Calculate η using eq (25)
2. Calculate M using eq (30)
3. Solve Eq (30) for E
4. Find r_0 from E using eq (27).

~~5. Find r_0 from E using,~~

6. Solve Eq [illegible] for ϕ_0.

Locating the Satellite w.r.t the Earth

Geocentric equatorial coordinate system

The rotation axis of the earth is the z_i axis, which is through the geographic North pole.

The x_i axis is from the center of the earth toward a fixed location in space called the first point Aries.

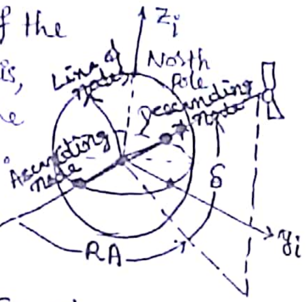


Fig. The geocentric equatorial system

This coordinate system moves through the space; it translates as the earth moves in its orbit around the sun, but it does not rotate as the earth rotates.

The (x_i, y_i) plane contains the earth's equator and is called the equatorial plane.

Angular distance measured eastward in the equatorial plane from the x_i axis is called Right Ascension (RA).

The two points at which the orbit penetrates the equatorial plane are called nodes;
- The satellite move upward through the equatorial plane at the Ascending node, and
- downward through the equatorial plane at a descending node

The right ascension of the ascending node is called Ω.

The angle that the orbital plane makes with the equatorial plane (the planes intersects at the line joining the nodes) is called the inclination, i.

- The variables Ω and i together locate the orbital plane w.r.t the equatorial plane.

- To locate the orbital coordinate system with respect to the equatorial coordinate system, we need ω, the argument of perigee west.
- This is the angle measured along the orbit from the ascending node to the perigee.

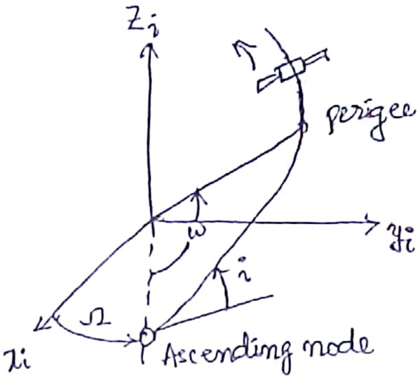


Fig. Locating the orbit in the geocentric equatorial system.

Time:

- Standard time for space operations is Universal Time (UT), also known as Zulu Time (Z).
- This is essentially the mean solar time at the Greenwich observatory near London, England.
- UT is measured in hours, minutes, and seconds or in fractions of a day.
- It is 5h later than Eastern standard Time (EST)

  Ex: 07:00 EST is 12:00:00h UT.
- The civil or calendar day begins at 00:00:00h UT (0h) or previous day mid night (24:00:00)

Orbital Elements

- To specify the absolute (i.e the inertial) coordinates of a satellite at time t, we need to know six quantities.

(1) Eccentricity (e),

(2) Semi major axis (a),

(3) time of perigee (t_p),

(4) Right Ascension of ascending node (Ω),

... and

Ex. 2.1.3 Elliptical orbit; pp: 30 (4)(b) (7)

Look Angle Determination

- Navigation around the earth's oceans became more precise when the surface of the globe was divided up into a gridlike structure of orthogonal lines:

Latitude and 2. Longitude

- Latitude is the angular distance, measured in degrees, north or south of the equator
- Longitude is the angular distance, measured in degrees, from a given reference longitudinal line.
  - ↳ Seafaring nations → England, → France
- England drew its reference zero longitude through Greenwith, a town close to London, England.
- France drew its reference zero longitude through paris, France.
  - British rule ↗ Charged fee for France maps

The use of Greenwich as the zero reference longitude became dominant within few years.

Thus, there are 360° of longitude (measured from 0° at the Greenwich meridian, the line drawn from the North pole to the South pole through Greenwich, England) and ±90° of latitude
- + → North of the equator
- − → South of the equator

Ex: Latitude 90°N (+90°) = North pole
Latitude 90°S (−90°) = South pole

* Look angle: The coordinates to which an earth station antenna must be pointed to communicate with a satellite are called the look angles. These are:

1. Azimuth (Az) .

Azimuth: is measured eastward (clockwise) from geographic north to the projection of the satellite path on a (locally) horizontal plane at the earth station.

Elevation: is the angle measured upwar from the local horizontal plane at the earth station to the satellite paths as shown in Fig. below:

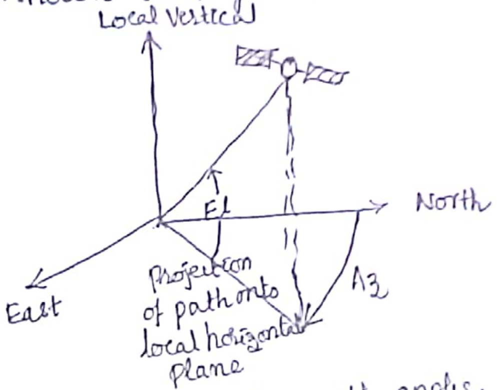


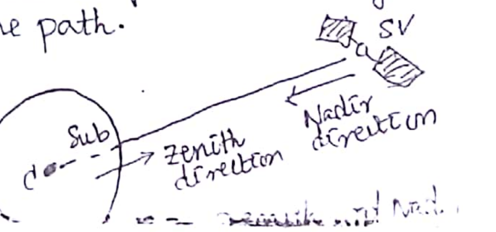
Fig. Elevation and Azimuth angles.

The subsatellite point: is the location on the surface of the earth that lies directly between the satellite and the center of the earth.

— It is the nadir pointing direction from the satellite and, for a satelli in an equatorial orbit, it will alway be located on the equator.

- To an observer of a satellite standing a the subsatellite point, the satellite wi ~~appear~~ appear to be directly overhea in the zenith direction from the obser location.

— The zenith and nadir paths are therefore in opposite directions ~~fro~~ ~~the observing location~~ along the same path.



— Zenith and Nadir

Elevation Angle Calculation

Fig. below shows the geometry of the elevation angle calculation.

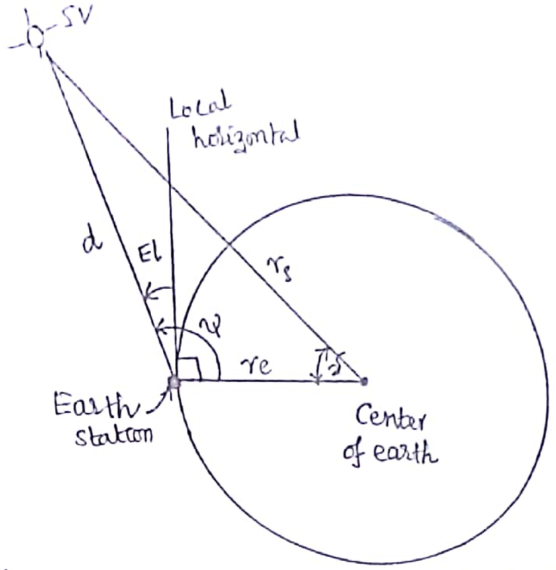


Fig. Geometry of elevation angle calculation

In Fig. above

r_s = the vector from the center of the earth to the satellite

r_e = is the vector from the center of the earth to the earth station

d = vector from the earth station to the satellite

These 3 vectors lie in the same plane and form a triangle.

γ = the central angle measured between r_e and r_s is angle between the earth station and the satellite.

ψ = is the angle (within the triangle). measured from r_e to d (it is non negative)

γ is related to the earth station north latitude L_e and west longitude l_e and the subsatellite point at north latitude L_s and west longitude l_s by → (31)

$$\boxed{\cos(\gamma) = \cos(L_e)\cos(L_s)\cos(l_s - l_e) + \sin(L_e)\sin(L_s)}$$

The law of cosines allows us to relate the magnitudes of the vector joining the center of the earth, the satellite and

(8) Thus,

$$d = r_s\sqrt{\left[1+\left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]} \rightarrow (32)$$

- By law of sines, we have

$$\frac{r_s}{\sin(\psi)} = \frac{d}{\sin(\gamma)} \rightarrow (33)$$

- Since, the local horizontal plane at the earth station is perpendicular to r_e, the elevation angle is related to the central angle ψ by

$$El = \psi - 90^\circ \rightarrow (34)$$

$$\cos(El) = \cos(\psi - 90^\circ)$$

$$= \sin(\psi)$$

$$= \frac{r_s \sin(\gamma)}{d}$$

From Eq (33)

$r_s \sin(\gamma) = d \sin(\psi)$

$\Rightarrow \sin(\psi) = \frac{r_s \sin(\gamma)}{d}$

From Eq (32)

$$\therefore \boxed{\cos(El) = \frac{\sin(\gamma)}{\sqrt{\left[1+\left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]}}} \rightarrow (35)$$

$$\frac{r_s}{r_s} = \frac{1}{\sqrt{1+\left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)}}$$

- Eqs. (35) and (31) permit the elevation angle El to be calculated from knowledge of the subsatellite point and the earth station coordinates, the orbital radius r_s, and earth's radius r_e.

NOTE: Accurate average earth radius $r_e = 6378.137$ km (approximate = 6378)

Azimuth Angle Calculation

- Because the earth station, the center of the earth, the satellite and sub satellite point all lie in the same plane, the azimuth angle Az from the earth station to the satellite is the same as the azimuth from the earth station to the subsatellite point.
- It's calculation is tedious for all other satellites except for G[illegible]

4(b)

Look angle Calculation to (9) Geostationary satellites

- For most geostationary satellites, the subsatellite point is on the equator at longitude l_s; the latitude L_s is 0.

The geosynchronous radius $r_s = 42{,}164.17$ km.

Since $L_s = 0$; from eq. (31)

$$\cos(\gamma) = \cos(L_e)\cos(l_s - l_e) \rightarrow (31)$$

by substituting $r_s = 42{,}164.17$ km and $r_e = 6{,}378.137$ km in Eqs. (32) and (36) gives the following expressions for distance d from the earth station to the satellite and the elevation angle El at the earth station

$$d = 42{,}164.17\sqrt{[1.02288235 - 0.30253825\cos(\gamma)]}\ \text{km} \rightarrow (37)$$

$$\cos(El) = \frac{\sin(\gamma)}{\sqrt{[1.02288235 - 0.30253825\cos(\gamma)]}} \rightarrow (38)$$

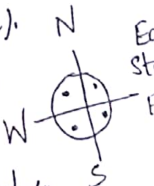
For a geostationary satellite with an $r_s = 42{,}164.17$ km and mean earth radius $r_e = 6378.137$ km

$$\Rightarrow \frac{r_s}{r_e} = 6.6107345$$

$$\boxed{El = \tan^{-1}\left[(6.6107345 - \cos\gamma)/\sin\gamma\right] - \gamma} \rightarrow (39)$$

To find the azimuth angle, an intermediate angle α must be found first

- α permits the correct 90° quadrant to be found for the azimuth since the azimuthal angle can lie anywhere between 0° (true north) and clockwise through 360° (back to true north again).



The intermediate angle is found from

$$\alpha = \tan^{-1}\left[\frac{\tan|(l_s - l_e)|}{\sin(L_e)}\right] \rightarrow (40)$$

If α is known, azimuth (Az) can be found as:

Case (1) Earth station in the Northern hemisphere with

(a) Satellite to the SE of the earth station: $Az = 180° - \alpha$

(b) Satellite to the SW of the earth station: $Az = 180° + \alpha$

Case (2): Earth station in Southern hemisphere with

(c) Satellite to the NE of the earth station: $Az = \alpha$

(d) Satellite to the NW of the earth station $Az = 360° - \alpha$

Ex: 2.2.1; PP: 36

Visibility Test of ~~Geostationary~~ Satellite

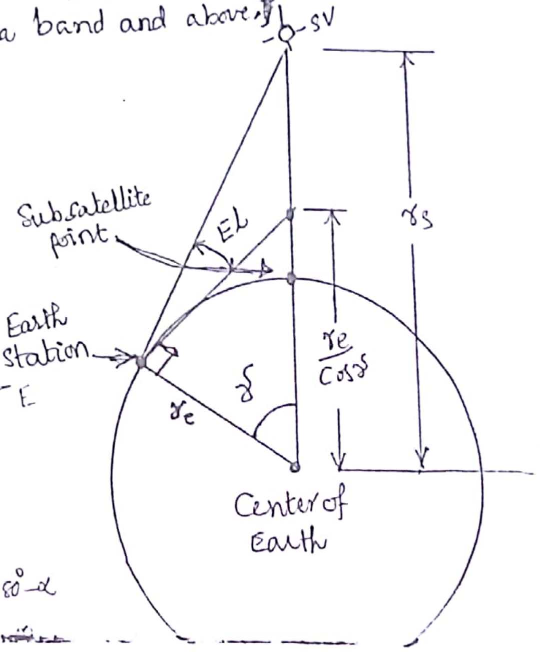
- For a satellite to be visible from an earth station, its elevation angle El must be above some minimum value, which is atleast 0°.
- A positive or zero elevation requires that (See fig. below):

$$\boxed{r_s \geq \frac{r_e}{\cos(\gamma)}} \rightarrow (42)$$

- This means that the maximum central angular seperation between the earth station and the subsatellite point is limited by

$$\boxed{\gamma \leq \cos^{-1}\left(\frac{r_e}{r_s}\right)} \rightarrow (43)$$

- For a nominal geostationary orbit, $\boxed{\gamma \leq 81.3°}$ for the satellite to be visible
- Elevation angle minima is typically 5° at C band, 10° at Ku band and 20° at Ka band and above.



number of geosynchronous orbit satellites have inclinations that are much larger than the nominal 0.05° inclination maximum for current geostationary satellites.

- In general, a geosynchronous satellite with an inclination of $< 0.1°$ may be considered to be geostationary.

in extreme cases, the inclination can be several degrees.

- If the orbit maneuvering fuel of the satellite is almost exhausted and the satellite's position in the nominal location is only controlled in longitude and not in inclination.

- This happens with most geostationary communications satellites toward the end of their operational life time.

• Since the reliability of the payload exceeds that of the lifetime of the maneuvering fuel.

- These satellites that can no longer be maintained in a fully geostationary orbit, but are still used for communications services, are referred to as inclined orbit satellites.

- They need to have tracking antennas at the earth terminals once the inclination becomes too large to allow the satellite to remain within the 1-dB beamwidth of the earth station antennas, substantial additional revenue can be earned beyond the normal life time of the satellite.

- These inclined orbital satellites can also be used to communicate to parts of high latitude regions, but only for a limited part of the day.

(10)

• The exceptional reliability of electronic components in space, once they have survived the launch and deployment sequences.

• The spacecraft designers manufacture satellites with two end of life criteria.

• These are:

(1) End of Design Life (EODL), which refers to the lifetime expectancy of the payload components, and

(2) End of maneuvering life (EOML), which refers to the spacecraft bus capabilities,

— Anticipated life time of the spacecraft with full maneuver capabilities in longitude and inclination.

• Current spacecrafts are designed with fuel tanks that have a capacity more than EODL.

∴ The additional fuel tanks allows to increase on orbit lifetime and orbital perturbation corrections.

(4)(a) Orbital perturbations

• Satellite mass, it is assumed that is affected only by the gravitation of the earth.

— Under these ideal conditions, the Keplerian elliptical orbital properties are constant with time.

• In practice, the satellite and earth respond to many other influences

(1) assymetry of the gravitation field,

(2) the gravitational fields of the sun and the moon, and

(3) solar radiation pressure

(4) Atmospheric drag in case of LEO satellites

• All these interfering forces cause the true orbit to be different from a Keplerian ellipse.

— If it is uncorrected, the subsatellite [illegible] geosynchronous satellite to

- In an osculating orbit is defined for some instant of ~~to~~ time by defining the orbital elements $(a, e, t_p, \Omega, i, \omega)$ by assuming perturbing forces are zero.

The perturbations cause the orbital elements to vary with time.

- The orbit and satellite location at any instant are taken from the osculating orbit calculated with orbital elements corresponding to that time.

→ Assume that the orbital elements vary linearly with time at constant rates given by $(da/dt, \frac{de}{dt}, \ldots$ etc$)$

Assume that osculating orbital elements at time t_0 are $(a_0, e_0, t_p, \Omega_0, i_0, \omega_0)$. The satellite's position at any time t_1 is then calculated from a Keplerian orbit with elements

$$\left[a_0 + \frac{da}{dt}(t_1 - t_0),\ e_0 + \frac{de}{dt}(t_1 - t_0), \ldots \text{etc}\right]$$

This approach is particularly useful in practice because it permits the use of either theoritically calculated derivatives or empirical values based on satellite observations.

NOTE: 1. As the perturbed orbit is not an ellipse, some care must be taken in defining the orbital period.

. Since the satellite does not return to the same point in space once per revolution, the quantity most frequently specified is the so-called anomalistic period: the elapsed time between successive perigee passages.

(II) • In addition to the orbit not being a perfect Keplerian ellipse, there will other influences that will cause the apparent position of a geostationary satellite to change with time.

. These causes:

1. Longitudinal changes
2. orbit inclination changes

1. Longitudinal Changes: Effects of the Earth's oblateness:

- The earth is neither a perfect sphere nor a perfect ellipse.
- Earth can be described as a triaxial ellipsoid[1]. (To be checked)
- The earth is flattened at the poles; the equatorial diameter is about 20km more than the average polar diameter.
- The equatorial radius is not constant although the noncircularity is small the radius does not vary by more than about 100m around the equator.
- In addition to these non regular features of the earth, there are regions where the avg density of the earth appears to be higher.
  - These are referred to as regions of mass concentration or Mascons.
- 1. The non sphericity of earth,
  2. The non circularity of the equatorial radius, and
  3. The Mascons

  → lead to a nonuniform gravitational field around the earth.

∴ The force on an orbiting satellite vary with position.

For LEO satellites, the rapid change in position of satellite w.r.t the earth's surface will lead to an averaging out of the perturbing forces in line with the orbital velocity vector.

For Geostationary/Geosynchronous satellite is weightless when in orbit.

- A smallest force on the satellite will cause it to accelerate and then drift away from its nominal location.
- The satellite is required to maintain a constant longitudinal position over the equator, but there will generally be an additional force toward the nearest equatorial bulge in either an eastward/westward direction along the orbit plane.
- Since this force will rarely be inline with the main gravitational force toward the earth's center, there will be a resultant component of force acting in the same/opposite direction as the satellite's velocity vector, depending on the precise position of the satellite in the GEO orbit.

⇒ This will lead to a resulting acceleration/decceleration component that varies with longitudinal location of the satellite.

Due to the position of the Mascons and equatorial bulges, there are 4 equilibrium points in the Geostationary orbit:
1. Two stable points — 75°E & ~~252°E~~/105°W
2. Two unstable points — 165°E & ~~348°E~~/15°W

Fig. 3.2 see

- If a satellite is perturbed slightly from one of the stable points, it will tend to drift back to the stable point without [illegible] work.

- If a satellite perturbed slightly from one of the unstable points will immediately begin to accelerate its drift toward the nearest stable point and, once it reaches the point, it will oscillate in longitudinal position about this stable point until it stabilizes (centuries later) at that point.
- These stable points are called the graveyard synchronous orbit location

NOTE: 1. The graveyard orbit of a geosynchronous satellite is the orbit to which the satellite is raised once the satel halts / ceases to be useful.
2. Due to the nonsphericity of the earth the stable points are neither 180° apart nor are the stable and unstable point precisely 90° apart.

2. Inclination changes: Effects of the sun and the Moon

- The plane of the earth's orbit around the sun — the ecliptic — is at an incline of 7.3° to the equatorial plane of the Sun.
- The earth is tilted about 23° away from the normal to the ecliptic.
- The moon circles the earth with an inclination of around 5° to the equatorial plane of the earth.
- Due to the fact that the various plane
  1. The sun's equator,
  2. The ecliptic,
  3. The earth's equator, and
  4. The moon's orbital plane around the earth
- are all in different planes.

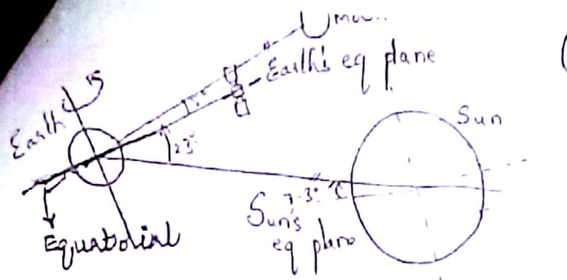


Fig. Relation between orbital planes of earth, sun, moon.

A satellite in orbit around the earth will be subjected to a variety of out of plane forces.

i.e, there will be a net acceleration force that is not in the plane of the satellite's orbit, and this will tend to change the inclination of the satellite's orbit from its intial inclination.

The mass of the sun is significantly larger than that of the moon but moon is considerably closer to the earth than the sun.

Table: comparative data for the sun, Moon and Earth

Sun	Mean radius (Km)	Mass (Units)	Mean orbit radius	earth day Spin period
Sun	6,96,000	3,33,432	30,000 light years	25.04 days
Moon	3,476	0.012	3,84,500 Km	27.3
Earth	6,378.14	1.0	1,49,597,870 km	1

For this reason, the acceleration force induced by the moon on a geostationary satellite is about twice as large as that of the sun.

The net effect of the acceleration forces induced by the moon and the sun on a geostationary satellite to change the plane of the orbit at an intial average rate of change of 0.85°/year from the equatorial plane.

When both the sun and moon are acting on the same side of the satellite's orbit,

(13) geostationary satellite's orbit will be higher than average.

— When both sun and moon are in opposite sides of the orbit, the rate of change of the plane of the satellite's orbit will be less than average.

Examples:

- Maximum rate of change years: are 1988 and 2006 (0.94°/year).
- Minimum rate of change years: are 1997 and 2015 (0.75°/year).

NOTE 1. These rates of change are neither constant with time nor with inclination

2. They are at maximum, when the inclination is zero and rates are zero when the inclination is 14.67°.

i.e, From an intial zero inclination, the plane of the geostationary orbit will change to a maximum inclination of 14.67° over 26.6 years.

— The acceleration forces with then change direction at this maximum inclination and the orbit inclination will move back to zero in another 26.6 years.

— out to −14.67° over further 26.6 years

— to zero over another 26.6 years

— and so on.

- To increase the orbital maneuver lifetime of a satellite, satellites are deliberately placed with an inclination larger than the normal 0.05° for geostationary satellites.

— Necessary professional forces are used to ~~make~~ reduce inclination error (zero) automatically over a required period without use of any thruster firing.

ethods to Correct Orbital perturbations

Under normal operations, ground controllers command spacecraft maneuvers to correct for both the (1) in-plane changes (longitudinal drifts) and (2) out of plane changes (inclination changes) of a satellite so that it remains in the correct orbit.

For a geostationary satellite, the inclination, ellipticity and longitudinal position are controlled so that the satellite is bounded by ±0.05° in latitude and longitude over the subsatellite point.

Methods: 1. single burn method
2. Two burn method.

Some maneuvers are designed to correct for both inclination and longitude drifts simultaneously in one burn of the maneuvering rockets on the satellite.

· The two maneuvers are kept seperate:

• one burn will correct for ellipticity and longitude drift;

• Second burn will correct for inclination changes.

Advantages of two burn method:

1. Much larger velocity increment needed to change the plane of an orbit (the so-called north-south (N-S) maneuvers)

2. Much smaller velocity increment needed to change the longitude/ellipticity of an orbit (the socalled East-West (E-W) maneuvers).

· The difference in energy requirement is about 10:1

(1)(2)(low thrust and high efficiency) are used for N-S maneuvers and liquid propellant thrusters (high thrust and low efficiency) are used for inplane changes and orbit rising.

Ex: 2.3.1 Drift with a GEO; PP: 42

Orbit Determination

• requires to determine the 6 orbital elements needed to calculate the future orbit of the satellite, and calculate the required changes that need to be made to the orbit to keep it within nominal orbital location.

• Six unknowns — orbital elements
— Three angular position measurem are required.
— Each measurement provides two equations (six — total)
w.r.t azimuth, w.r.t. Elevation
as a fn of 6 orbital elements.

• The control earth stations used to measure the angular position of the satellites
— also carry out range measuremen using unique time stamps in the telemetry stream or communicati carrier.

• These earth stations are referred as the TTC&M (telemetry, trackin command and monitoring) stations
— These stations spread around the world.
— Smaller satellite systems contract for TT&C functions from large satellite system operators.

Launches and Launch Vehicles

- A satellite cannot be placed into a stable orbit, unless two parameters ~~if and only if~~ are coupled together
  1. the velocity vector and
  2. orbital height – are simultaneously correct.

For Geostationary satellite
- orbital height of 35,786.03 km above the surface earth (or 42,164.17 km radius from the center of the earth).
- with an inclination of 0°
- with an ellipticity of zero ($e=0$)
- and ~~a~~ with a velocity of 3,074.7 m/s tangential to the earth in the plane of the orbit (which is the earth's equatorial plane).

In any earth satellite launch, the largest fraction of the energy expended by the rocket is used to accelerate the vehicle from rest up to 32 km above the earth.

It is common to get rid off excess mass from the launcher as it moves upward on launch (to make efficient use of the available fuel). This is called ~~tame~~ **Staging**.

Fig. below gives a schematic of a Proton launch from the Russian Baikonur Complex at Kazakhstan, near Tyuratam.

- Most launch vehicles have multiple stages
- As each stage is completed, that part of the launcher is expended until the final stage places the satellite into the desired trajectory. Hence the term: Expendable Launch Vehicle (ELV).

- The space shuttle, called the Space Transportation System (STS) by NASA, is partially reusable.
- The solid rocket boosters are recovered and refurbished for future missions and the shuttle vehicle itself is flown back to earth for refurbishment and reuse. Hence the term: Reusable Launch Vehicle (RLV) for such launch

- The earth spins towards the east.
- At the equator, the rotational velocity of a sea level site in the plane of the equator is $= \dfrac{2\pi \times \text{radius of earth}}{\text{one sidereal day}}$

$$= \frac{2\pi \times 6{,}378.137\ \text{km}}{\cancel{24}\ 23\text{h}\ 56\text{min}\ 4.01\text{s}} = 0.4651\ \text{km/s}$$

$$\simeq 1610\ \text{km/h} \quad (0.4651 \times 60 \times 60\ \text{km/h})$$

- An easterly launch from the equator has a velocity increment of 0.465 km/s imparted by the rotation of the earth.
- A satellite in a circular, equatorial orbit at an altitude of 900 km requires an orbital velocity of about 7.4 km/s tangential to the surface of the earth.
- A rocket launcher from the equator needs to impart an additional velocity of $(7.4 - 0.47)$ km/s $= 6.93$ km/s.
- i.e., the equatorial launch has reduced the energy required about 6%.

f the launch is not to be into an (16) equatorial orbit, the payload capabilities of any given rocket will reduce as he inclination increases.

A satellite launched into a prograde orbit from a latitude of ϕ degrees will enter an orbit with an inclination of ϕ degrees to the equator.

If the satellite is intended for geostationary orbit, the satellite must be given a significant velocity increment to reorient the orbit into the earth's equatorial plane.

∴ A satellite launched from cape canaveral at 28.5°N latitude required a velocity increment of 366 m/s to attain an equatorial orbit from geosynchronous orbit plane of 28.5°.

∴ Ariane is launched from the Guiana Space center in French Guina, located at a latitude of 5°S in South america.

∴ Sea launch can launch from the equator.

NOTE: The lower latitude of these lanch sites results in significant savings in the fuel used by the Apogee Kick Motor (AKM).

Expendable Launch Vehicles (ELVs)

1998 — US — ELVs

Teal Group estimated ~~that~~ (in 1999) that 1447 satellites would be launched world wide between 2000-2009 on 850 to 900 launch vehicles. [Fig. 2.16]

Placing Satellites into Geostationary orbit

- Some of the launch vehicles deliver the spacecraft directly to geostationary orbit (called a direct-insertion lauch) while others inject the spacecraft into a geostationary transfer orbit (GTO)
- Spacecrafts lauched into GTO must carry additional rocket motors and/or propellant to enable the vehic to reach the geostationary orbit.
- There are 3 basic ways to achieve the geostationary orbit:

1. Geostationary Transfer orbit and AKM
2. GTO with slow orbit raising
3. Direct insertion to GEO.

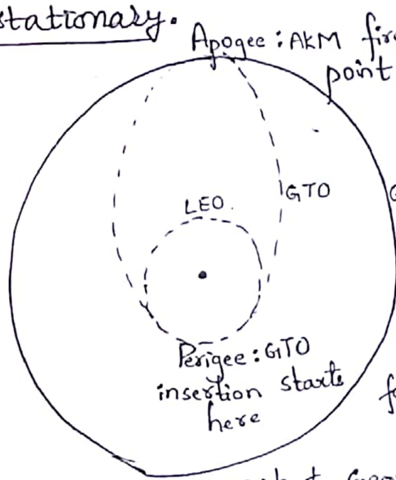
1. GTO and AKM

- The intial approach to launching geostationary satellites was to place the spacecraft into LEO with final rocket stage still attached.
- After a couple of revolutions (during which the orbital elements are measur the final stage is reignited and the spacecraft is launched into a GTO.
- The GTO has a perigee (that is the original LEO altitude) and an apoge (that is the GEO altitude).
- Fig. below illustrates the process.
- The position of the apogee point is close to the orbital longitude that would be the in-orbit test location. the satellite prior to it being moved to its operational position

Again after a few revolutions (17) on the GTO (orbital elements are measured), a rocket motor (usually contained within the satellite itself) is ignited at apogee and the GTO is raised until it is a circular, geostationary orbit.

Since, the rocket motor fires at apogee, it is called as the apogee kick motor (AKM).

The AKM is used both to circularize the orbit at GEO and to remove any inclination error so that the final orbit of the satellite is very close to geostationary.



See also page 21 for Indian GEOSat

Fig. GTO-AKM approach to Geostationary orbit.

GTO with slow orbit Raising

In this procedure, rather than employing an AKM that imparts a vigorous acceleration over a few minutes, the spacecraft thrusters are used to raise the orbit from GTO to GEO over a number of burns.

Since many the spacecraft cannot be spin-stabilized during the GTO, many of the satellite elements are deployed

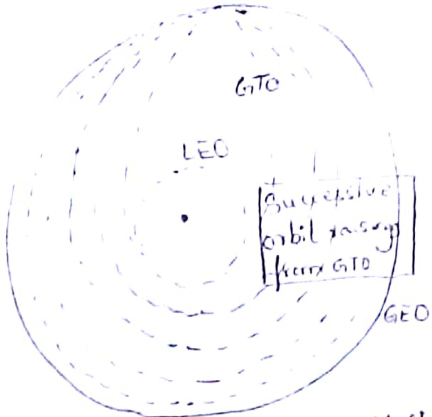


Fig. Illustration of slow orbit raising to the geostationary orbit

- The satellite has two power levels of thruster:
  1. one for More powerful orbit raising maneu
  2. and another for on-orbit (low thrust) maneuvers.
- Since thruster take many hours o operation to achieve the geostationa orbit, the perigee of the orbit is rais gradually over successive thruster firings.
- The thruster firings occur symmetri about the apogee (although they coul occur at the perigee as well).
- The burns are typically 60 to 80 min long on successive orbits and up to six orbits can be used (Fig. above

NOTE: In the above two methods, the GTO may be a modified orbit with the apogee well above the required altitude for GEO.

- The excess energy of the orbit due t the higher than necessary altitude at apogee can be traded for energy required to raise the perigee.
- The net energy to circularize the orbit at GEO is therefore less and

• **Direct Insertion to GEO**

This is similar to the GTO technique but, in this case, the launch service provider contracts to place the satellite into GEO.

(The final stages of the rocket are used to place the satellite directly into GEO rather than the satellite using its own propulsion system to go from GTO to GEO.)

3 (a)

Orbital Effects in Communications Systems performance:

1. Doppler Shift

To a stationary observer, the frequency of a moving radio transmitter varies with the transmitter's velocity relative to the observer.

If the true transmitter frequency (i.e., the frequency that the transmitter would send when at rest) is f_T,

- the received frequency f_R is higher than f_T when transmitter is moving toward the receiver, and
- the received frequency f_R is lower than f_T when the transmitter is moving away from the ~~tr~~ Receiver.

The relationship between the transmitted and received frequencies is

$$\frac{f_R - f_T}{f_T} = \frac{\Delta f}{f_T} = \frac{V_T}{v_p}$$

or $$\boxed{\Delta f = \frac{V_T f_T}{c} = \frac{V_T}{\lambda}} \rightarrow (4.4)$$

where V_T = transmitter velocity component directed toward the Rx,

$v_p = c$ = the phase velocity of light $= 2.9979 \times 10^8 \simeq 3 \times 10^8$ m/s in free space.

λ = wavelength of the transmitted signal.

• If the transmitter is moving away from the receiver, then V_T is negative.

– This change in frequency is called the doppler shift, the doppler effect, or just Doppler.

• For LEO satellites, Doppler shift can be quit pronounced requiring the use of frequency tracking receivers.

• For geostationary satellites, the doppler effect is negligible.

Ex: 2.6.1 Doppler shift for LEO satellites
pp: 50

2. Range Variations

• The position of a satellite w.r.t the earth exhibits a cyclic daily variation (even with the best station-keeping available for geostationary satellites).

• The variation in position will lead to a variation in range between the satellite and user terminals.

• If TDMA is being used, careful attention must be paid to the timing of frame within the TDMA bursts, so that the individual user frames arrive at the satellite in the correct sequence and at the correct time.

• Range variations on LEO satellites can be significant, as can path loss variat

— while guard times between bursts can be increased to help in any rang and/or timing in-accuracies, this reduces the capacity of the transpond

• The on-board capabilities of some satellites permit both timing contro of the burst sequence and power leve control of the individual user streams.

3. Solar Eclipse

- A satellite is said to be in eclipse when the earth prevents sunlight from reaching it, i.e, when the satellite is in shadow of the earth.

For geostationary satellites, eclipses occur during two periods that begin 23 days before the equinoxes (about March 21 and September 23) and end 23 days after the equinox periods.

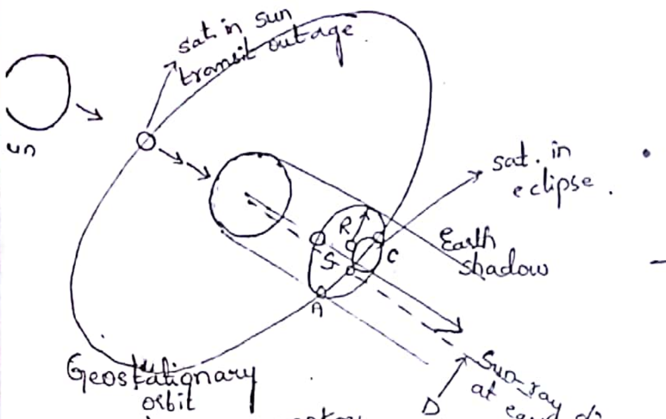


Fig. Eclipse geometry.

Eclipse occurs close to the equinoxes, as these are the times when the sun, the earth and satellite are all nearly in the same plane.

During full eclipse, a satellite receives no power from its solar array and it must operate entirely from its batteries.

- Batteries are designed to operate with a maximum depth of discharge;
- the percentage depth of discharge can be lower for the better battery

If the battery is discharged below its maximum depth of discharge, then the battery may not recover to full operational capacity once recharged.

∴ The depth of discharge sets the power drain limit during eclipse operations.

(17) • Nickel Hydrogen batteries can operate at about a 70% depth of discharge and recover fully, once recharged.

- Ground controllers perform battery-conditioning routines prior to eclipse operations to ensure the best battery performance during eclipse.

— The routines consist of deliberately discharging the batteries until they are close to their maximum depth of discharge, and then fully recharging the batteries just before eclipse season begins.

- The eclipse season is a design challenge for spacecraft builders.

— Not only the main power source withdrawn (the sun) but also the rapidity with which the satellite enters and exits the shadow can cause extreme changes in both power and heating effects over relatively short periods.

— Satellites can suffer many of their component failure under sudden stress situations.

∴ Eclipse periods are monitored careful by ground controllers, as this is when most of the equipment failure are likely to occur.

4. Sun Transit Outage

- During the equinox periods, not only does the satellite pass through the earth's shadow on the "dark" side of the earth, but the orbit of the satellite will also pass directly in front of the sun on the sunlit side of the earth.
- The sun is a hot mi…

(20)

about 6000 to 10,000°K, depending on the time within the 11 years sun spot cycle, at the frequencies used by Communications satellites (4 to 50GHz).

The earth station antenna will therefore receive not only the signal from the satellite but also the noise temperature transmitted by the sun.

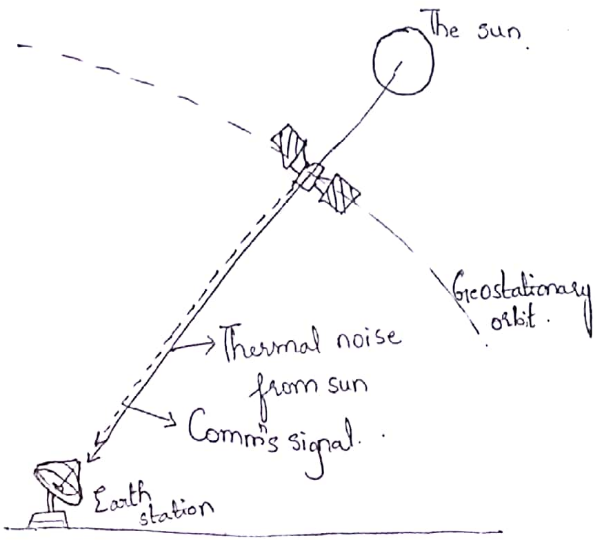
The added noise temperature will cause the fade margin of the receiver to be exceeded and an outage will occur.

· These outages may be precisely predicted.

For satellite system operators with more than one satellite at their disposal, traffic can be off-loaded to satellites that are just out of, or yet to enter, a sun outage.

The outage in this situation can therefore be limited as far as an individual user is concerned.

However, the outages can be determined to operators committed to operations during day light hours.



[illegible] outage conditions.