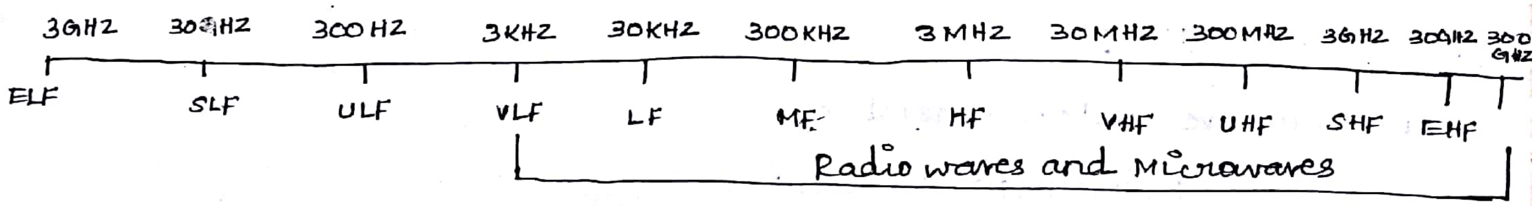
UNIT-I RECTANGULAR WAVEGUIDES AND CAVITY RESONATORS

- Microwave spectrum and bands
- Application of Microwaves
- Rectangular waveguides - solution of wave equation in Rectangular coordinates
- TE/TM Mode analysis
- Expressions for E and H fields.
- cut off frequencies
- Dominant and Degenerate modes
- sketches of TE and TM mode
- Fields in cross section
- Mode characteristics - phase and Group velocities, wavelengths and Impedance relations, Illustrative problems.

CAVITY RESONATORS

- Rectangular guides - power Transmission and power Losses
  - Impossibility of TEM mode
- cavity Resonators - Introduction
  - Rectangular cavities
  - Dominant modes and Resonant Frequencies
  - Q Factor and coupling co-efficients.
- [illegible] Inf[illegible] - [illegible] Die[illegible] constant - [illegible] Q [illegible]

MICROWAVE SPECTRUM AND BANDS:



- μW frequency range - 1 GHz to 300 GHz (1 GHz to 10^6 GHz).
- wavelength of μ wave is in micron ranges

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MICROWAVE BANDS

NEW U.S MILITARY μWAVE BANDS

DESIGNATION	FREQUENCY IN GHZ
A	0.1-0.25
B	0.25 - 0.5
C	0.5 - 1
D	1 - 2
E	2 - 3
F	3 - 4
G	4 - 6
H	6 - 8
I	8 - 10
J	10 - 20
K	20 - 40
L	40 - 60
M	60 - 100

IEEE μWAVE FREQUENCY BANDS

DESIGNATION	FREQUENCY IN GHZ
HF	0.003 - 0.030
VHF	0.030 - 0.300
UHF	0.300 - 1.000
L	1 - 2
S	2 - 4
C	4 - 8
X	8 - 12
Ku	12 - 18
K	18 - 27
Ka	27 - 40
MILLIMETER	40 - 300
SUBMILLIMETER	> 300

MICROWAVE SYSTEM :

- A Microwave system consists of
  - → TRANSMITTER SUBSYSTEM
    - μwave oscillator
    - waveguides
    - Transmitting antenna
  - → RECEIVING SUBSYSTEM
    - Receiving Antenna
    - waveguide
    - μwave amplifier
    - Receiver

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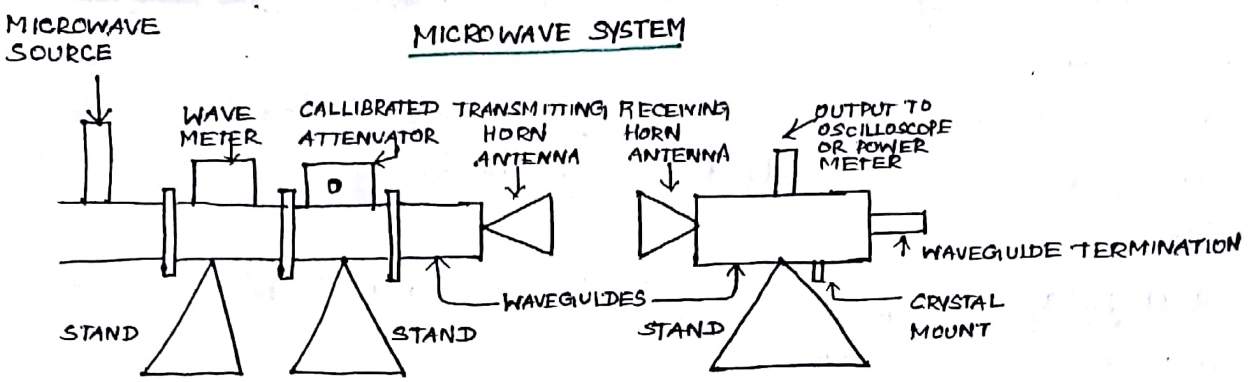
MICROWAVE SYSTEM



APPLICATIONS OF MICROWAVES:

- Communication
  - → used in broadcasting and telecommunication transmissions
  - → As they have shorter wavelengths, they allow to use smaller antennas.
  - → GSM use μ waves of frequency range 1·8 to 1·9 GHZ
  - → satellite communication
- Remote sensing
  - → RADAR and SONAR
  - → RADAR is used to illuminate an object by using a Transmitter and receiver to detect its position and velocity.
  - → Radiometry is also one of the remote sensing applications.
- Heating
  - → μ wave oven to bake
  - → vibration of electrons present in the food particles
- Medical science
  - → used in diagnosis and various Therapies.

RECTANGULAR WAVEGUIDES:

- A waveguide consists of a hollow metallic tube of a rectangular or circular shape used to guide an EM wave.
- Wave guides are used principally at frequencies in the μ wave range
- In waveguides, the electric and magnetic fields are confined to the space within the guides. Thus, no power is lost through radiation and even the dielectric loss is negligible. However, there is some power loss as heat in the walls of the guides, but the loss is very small.

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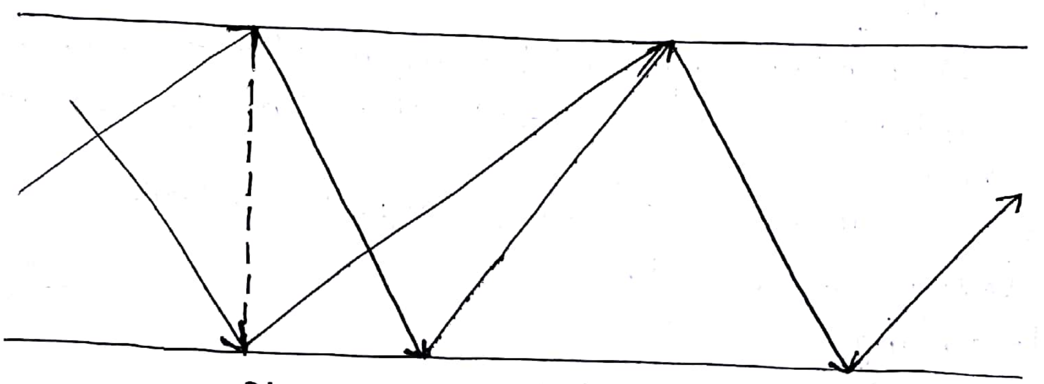
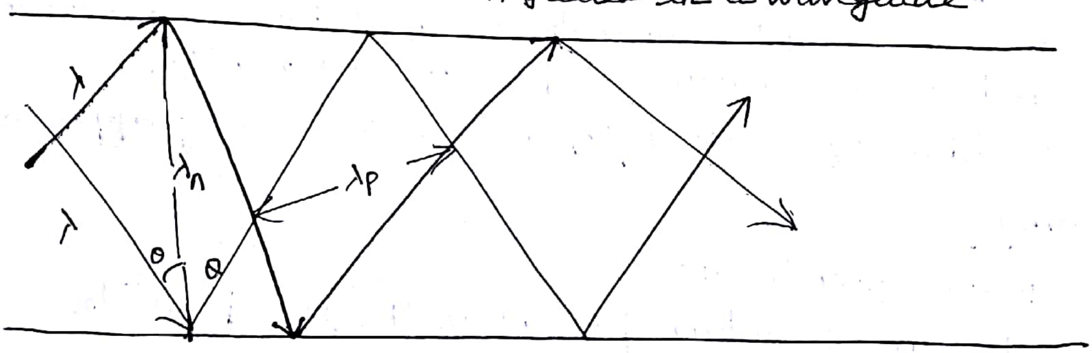
- Rectangular waveguides are one of the earliest type of transmission lines.
- Isolators, Detectors, Attenuators, couplers and slotted lines are available for various standard waveguide between 1 GHz to above 220 GHz.
- A Rectangular waveguide is a hollow metallic tube with a rectangular cross section.
- The conducting walls of the guide confine the electromagnetic fields and thereby guide the EM wave.



Plane wave Reflected in a waveguide.



- The above figure shows that any uniform plane wave in a lossless guide may be resolved into TE and TM waves.
- It is clear that wavelength λ is in the direction of propagation of incident wave, there will be one component λ_n in the direction normal to the reflecting plane and another λ_p parallel to the plane.

These components are

$$\lambda_n = \frac{\lambda}{\cos\theta} \longrightarrow (1)$$

$$\lambda_p = \frac{\lambda}{\sin\theta} \longrightarrow (2)$$

where θ = Angle of incidence
λ = wavelength

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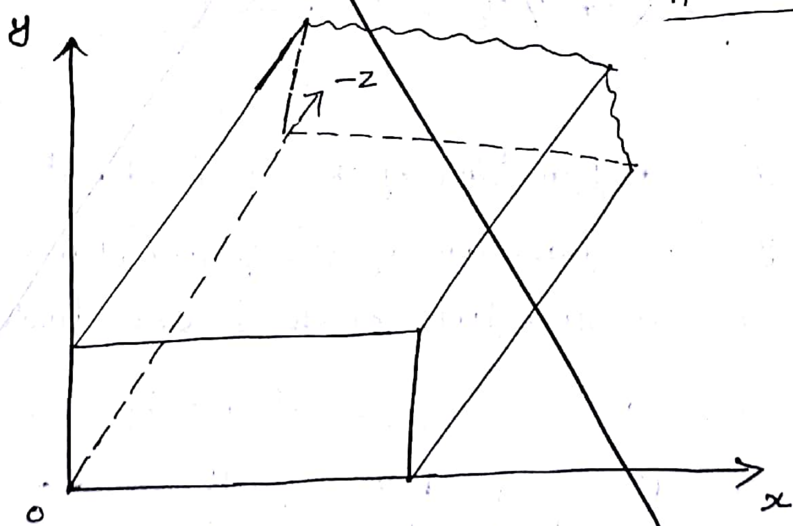
- A plane wave in a waveguide resolves into 2 components
  # one standing wave in the direction normal to the reflecting walls of the guide
  # one Travelling wave in the direction parallel to the reflecting walls.
- In Lossless waveguides, the modes may be classified into
  ⇒ Transverse Electric (TE) mode
  ⇒ Transverse Magnetic (TM) mode
- In Rectangular waveguides, the modes are designated as TE_{mn} or TM_{mn}.
- The integer m denotes number of half waves of electric or magnetic intensity in the x direction
- The integer n denotes number of half waves in y direction, if the propagation of the wave is assumed in positive z direction

SOLUTION OF WAVE EQUATIONS IN RECTANGULAR CO-ORDINATES:

A Rectangular co-ordinate system is shown in following figure.



Rectangular co-ordinates

→ WAVE EQUATION is an important 2nd order Linear Differential equation for the description of the waves arise in fields like Acoustics, and Electromagnetics. ($V = f \cdot \lambda$)

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The Electric and Magnetic wave equations in frequency domain is given by

$$\nabla^2 E = \gamma^2 E \longrightarrow (3)$$

$$\nabla^2 H = \gamma^2 H \longrightarrow (4)$$

where

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta . \longrightarrow (5)$$

These equations are called VECTOR WAVE EQUATIONS.

The Rectangular components of E or H satisfy the complex SCALAR WAVE EQUATION or HELMHOLTZ EQUATION

$$\nabla^2 \psi = \gamma^2 \psi \longrightarrow (6)$$

The Helmholtz equation in Rectangular coordinates is

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \gamma^2 \psi \longrightarrow (7)$$

By method of seperation of variables, the solution is assumed in the form of

$$\psi = X(x)\, Y(y)\, Z(z) \longrightarrow (8)$$

where

$X(x)$ = a function of the x coordinate only

$Y(y)$ = a function of the y coordinate only

$Z(z)$ = a function of the z coordinate only

sub. (8) in (7) and division of resultant by ψ yields

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = \gamma^2 \longrightarrow (9)$$

since the sum of three terms on the L.H.S is a constant and each term is independently variable, it follows that each term must be equal to a constant.

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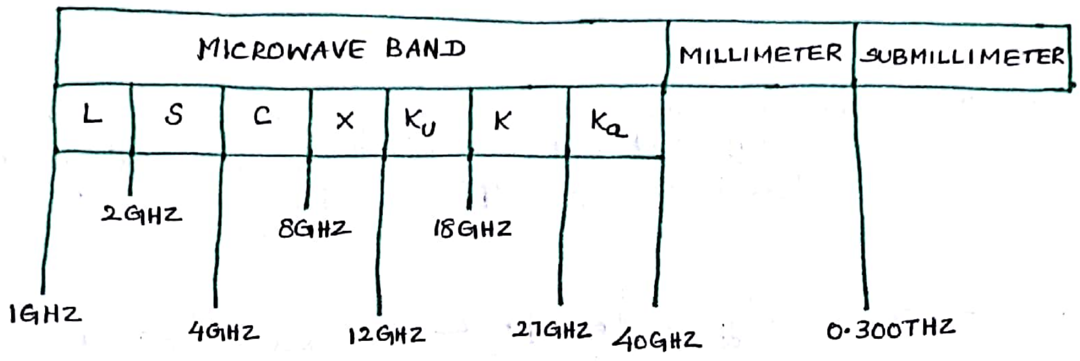
ELECTROMAGNETIC FREQUENCY SPECTRUM

The various available EM spectrum are as follows:



MICROWAVE REGION AND BAND DESIGNATIONS:

Propagation characteristics and Applications of various bands

BAND	FREQUENCY	WAVELENGTH	PROPAGATION CHARACTERISTICS	APPLICATIONS
ELF	30-300HZ	10-1 Mm	Penetration into Earth and sea.	communication with submarines.
VLF	3-30KHZ	100-10Km	→ Surface wave upto 1000 km. → sky wave in the night extends the range. → Low Attenuation during day and night. → very Reliable	Long Distance point-to-point communication
LF	30-300KHZ	10-1 km	→ surface wave and sky wave at night → surface wave Attenuation greater than VHF.	→ point-to-point marine communication → Time standard frequency broadcast.
MF	300-3000KHZ	1000-100m	→ Ground wave during day and in addition sky wave at night. → Attenuation high in ~~night~~ day time and low during night.	Broadcasting and Marine communication
HF	3-30MHZ	100-10m	→ Reflection from Ionosphere and varies as per time of day.	Moderate and Long distance communication of all types.

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BAND	FREQUENCY	WAVELENGTH	PROPAGATION CHARACTERISTICS	APPLICATIONS
VHF	30-300 MHZ	10-1 m	space wave Line of sight	Television, FM service
UHF	300-3000 MHZ	100-10 cm	→ same as VHF. → Affected by tall objects like hills skyscrapers	short Distance communication including RADAR.
SHF	3-30 GHZ	10-1 cm	suffers atmosphere Attenuation above 10 GHz.	RADAR, Microwave and space communication.
EHF	30-300 GHZ	10-1 mm	same as SHF	RADAR, Microwave and space communication.

ADVANTAGES OF MICROWAVES:

1. INCREASED BANDWIDTH AVAILABILITY:

* Microwaves have Large Bandwidth
* More information can be transmitted in Microwave frequency ranges.
* Microwave region contains thousand sections of frequency band (0 to 10^9 HZ), and hence any one of these 1000 sections may be used to transmit all the TV, radio and other communications
* currently Microwaves are employed in

⇒ Telephone Networks

⇒ TV Networks

⇒ space communication

⇒ Telemetry

⇒ Defence

⇒ Railway e.t.c.

⇒ FM

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2. IMPROVED DIRECTIVE PROPERTIES:

As frequency increases, Directivity increases and Beamwidth decreases. Hence the beamwidth of radiation θ is proportional to λ/D.

At Microwave frequencies, antenna size of several wavelength lead to smaller beamwidths.

For (e.g). For a parabolic antenna

$$B = \frac{140^\circ}{(D/\lambda)} \longrightarrow (1)$$

where

D - Diameter in cm.

λ - wavelength in cm.

B - Beamwidth in degrees.

At 30 GHz ; λ = 1cm and 1° beamwidth,

$$D = \frac{140}{B} \times \lambda = \frac{140}{1} \times 1 \Rightarrow D = 140\,cm \longrightarrow (2)$$

At 300 MHz: λ = 100cm and 1° beamwidth,

$$D = \frac{140}{1} \times 100 \Rightarrow D = 140\,m \longrightarrow (3)$$

From above case, it is clear that antenna size is small for Microwave frequencies

3. FADING EFFECT AND RELIABILITY:

Due to Line of sight and high frequency, Microwaves are less prone to fading effect and are more reliable.

4. POWER REQUIREMENTS:

Transmitter/Receiver power requirements are pretty low at Microwave frequencies.

5. TRANSPARENCY PROPERTY OF MICROWAVES:

* Microwave Frequency band are capable of freely propagating through the ionized Layers surrounding the earth as well as the atmosphere.
* The presence of such a transparent window in a Microwave band facilitates the study of Microwave Radiation from sun and stars.
* It also makes it possible for duplex communication and exchange of information between ground station and space vehicles.

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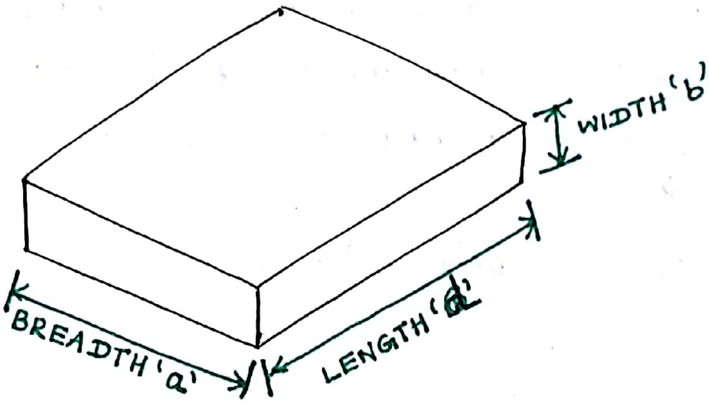
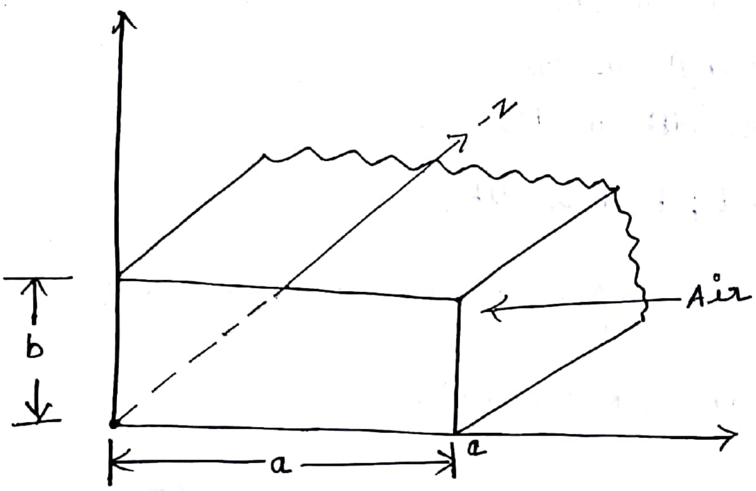
PROPAGATION OF WAVES IN RECTANGULAR WAVEGUIDE:





* The above Rectangular waveguide is situated in the rectangular co-ordinate system with its breadth along x axis, width along y axis and wave is assumed to propogate along the direction of z axis.

The wave Equation for TE and TM waves are given by

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \quad \text{for TE wave } (E_z = 0) \longrightarrow (4)$$

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z \quad \text{for TM wave } (H_z = 0) \longrightarrow (5)$$

Expanding $\nabla^2 E_z$ in rectangular co-ordinate system

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \varepsilon E_z \longrightarrow (6)$$

since the wave is propagating in the z direction, we have the operator

$$\frac{\partial^2}{\partial z^2} = \gamma^2 \longrightarrow (7)$$

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Sub. (7) in (6), we get

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu E_z \longrightarrow (8)$$

(or)

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu) E_z = 0 \longrightarrow (9)$$

Let $\gamma^2 + \omega^2 \mu\varepsilon = h^2$, be a constant, (9) can be written as

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \quad \text{for TE wave} \longrightarrow (10)$$

III^{ly},
$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \quad \text{for TM wave} \longrightarrow (11)$$

By solving the above partial Differential Equations, we get solutions for E_z and H_z.

Using MAXEWELL'S Equation, it is possible to find the various components along x and y directions (E_x, H_x, E_y, H_y).

From MAXEWELL'S 1st equation, we have

$$\nabla \times H = j\omega\varepsilon E \longrightarrow (12)$$

Expanding $\nabla \times H$,

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon \left[\hat{i} E_x + \hat{j} E_y + \hat{k} E_z \right] \longrightarrow (13)$$

Replacing $\frac{\partial}{\partial z} = -\gamma$, we get

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon \left[\hat{i} E_x + \hat{j} E_y + \hat{k} E_z \right] \longrightarrow (14)$$

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Equating co-efficients of $(\hat{i}, \hat{j}, \hat{k})$, we get

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x \longrightarrow (15)$$

$$\frac{\partial H_z}{\partial x} + \gamma H_x = -j\omega\varepsilon E_y \longrightarrow (16)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \longrightarrow (17)$$

similarly, from Maxwell's 2nd equation, we have

$$\nabla \times E = -j\omega\mu H \longrightarrow (18)$$

Expanding $(\nabla \times E)$, we get

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\left[\hat{i} H_x + \hat{j} H_y + \hat{k} H_z\right] \longrightarrow (19)$$

Expanding and equating co-efficients of $\hat{i}, \hat{j}$ & $\hat{k}$, we get

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \longrightarrow (20)$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = +j\omega\mu H_y \longrightarrow (21)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \longrightarrow (22)$$

combining (15) and (21) to eliminate H_y, we get an expression for E_x.

From (21),

$$H_y = \frac{1}{j\omega\mu} \frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu} E_x \longrightarrow (23)$$

sub. (23) in (15), we get

$$\frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu} \frac{\partial E_z}{\partial x} + \frac{\gamma^2}{j\omega\mu} E_x = j\omega\varepsilon E_x \longrightarrow (24)$$

(or)

$$E_x\left[j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right] = \frac{\gamma}{j\omega\mu} \frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y} \longrightarrow (25)$$

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Multiplying by $j\omega\mu$, we get

$$E_x\left[-\omega^2\mu\varepsilon-\gamma^2\right] = \gamma\frac{\partial E_z}{\partial x} + j\omega\mu\frac{\partial H_z}{\partial y} \longrightarrow (26)$$

(or)

$$E_x\left[-(\gamma^2+\omega^2\mu\varepsilon)\right] = \gamma\frac{\partial E_z}{\partial x} + j\omega\mu\frac{\partial H_z}{\partial y} \longrightarrow (27)$$

where $\gamma^2+\omega^2\mu\varepsilon = h^2 \longrightarrow (28)$

Dividing by $-h^2$, throughout, we get

$$\boxed{E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}} \longrightarrow (28)$$

III^ly

$$\boxed{E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}} \longrightarrow (29)$$

and

$$\boxed{H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}} \longrightarrow (30)$$

and

$$\boxed{H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}} \longrightarrow (31)$$

* The above equations give a general relationship for components within a waveguide.

PROPAGATION OF TEM WAVES:

⇒ we know, for a TEM wave, $E_z = 0$ and $H_z = 0$.

* Sub. these values in (28) to (31), we get that all the field components along x and y directions E_x, E_y, H_x, H_y vanish and hence a TEM wave cannot exist inside a waveguide.

TE AND TM MODES:

* The Electromagnetic wave inside a waveguide can have an infinite number of patterns which are called MODES.
* An Electromagnetic wave consists of Electric and magnetic fields which are perpendicular to each other.
* Electric field is always perpendicular to surface at a conductor and Magnetic field is parallel to the surface of conductor.

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* In general, there are 2 kinds of modes in waveguide.
  - → TE Mode
  - → TM Mode

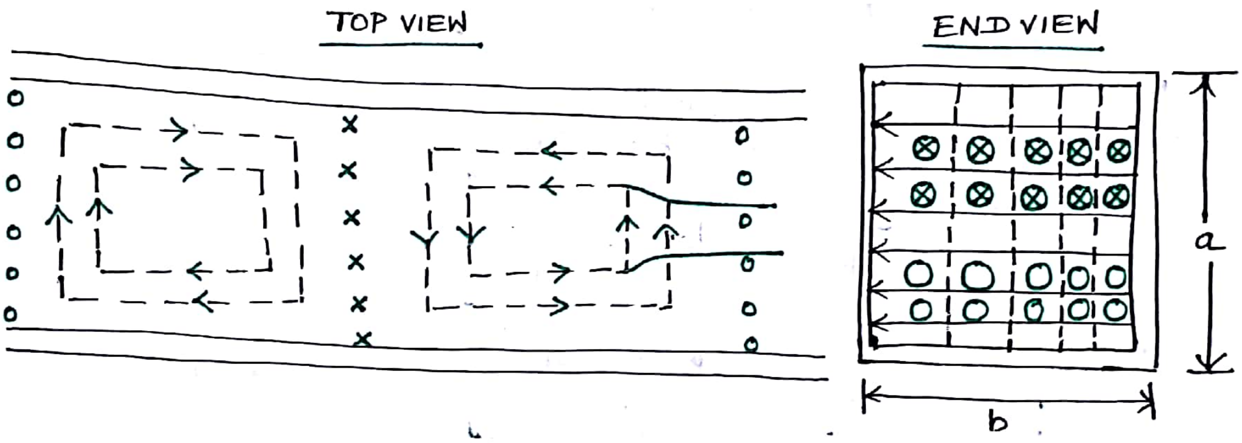
TE MODE:

* The Electric field is always transverse to the direction of propagation and is called Transverse Electric wave. ($E_z = 0$; but ($H_z \neq 0$))

TM MODE:

* The Magnetic field is always transverse to the direction of propagation and is called Transverse Magnetic wave. ($H_z = 0$; but ($E_z \neq 0$)).

FIELD PATTERNS:



* The above figure shows the field pattern of a TE wave.
* Solid Lines depict ELECTRICAL FIELD lines or VOLTAGE lines.
* Dotted Lines depict MAGNETIC FIELD lines.
* We use subscript for designating a particular mode (ie) TE_{mn} or TM_{mn}

  where m ⇒ indicates number of half wave variations of the electric field (or magnetic field in a TM) across the wider dimension 'a'.

  n ⇒ indicates the number across the narrow dimension 'b'.

* Referring to TE pattern shown in above figure, it can be seen that the voltage varies from 0 to maximum and maximum to 0 across the wide dimension 'a'. This is one half variation. Hence $m=1$.
* Across narrow dimension 'b', there is no voltage variation. Hence $n=0$. Therefore this mode is TE_{10} mode.
* The mode with highest cut off wavelength is known as Dominant mode of the waveguide and all other modes are called higher modes.

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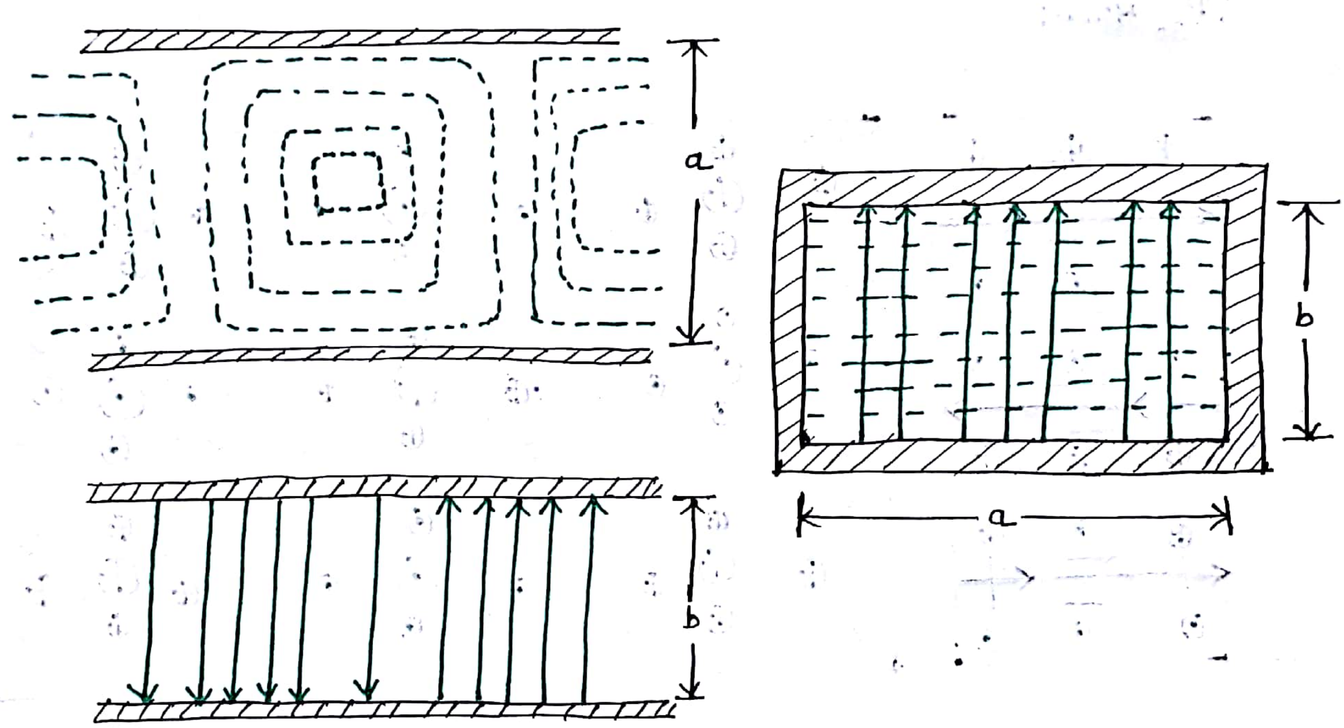
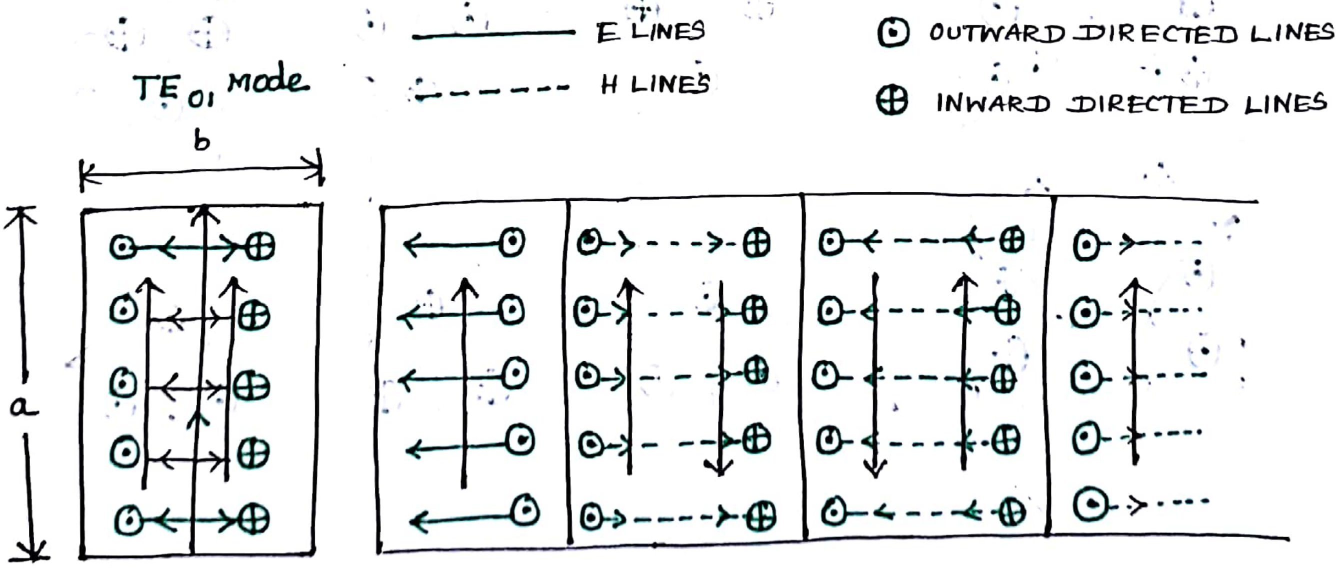
* For example, TE_{10} is the dominant mode for TE waves. It is the mode which is practically used for all electromagnetic transmission in a Rectangular waveguide.
* DOMINANT MODES are almost low-loss, Distortionless transmission modes.
* Higher modes result in significant loss of power.

RADIATION PATTERN OF TE_{10} MODE:



FIELD PATTERN OF HIGHER ORDER MODES:



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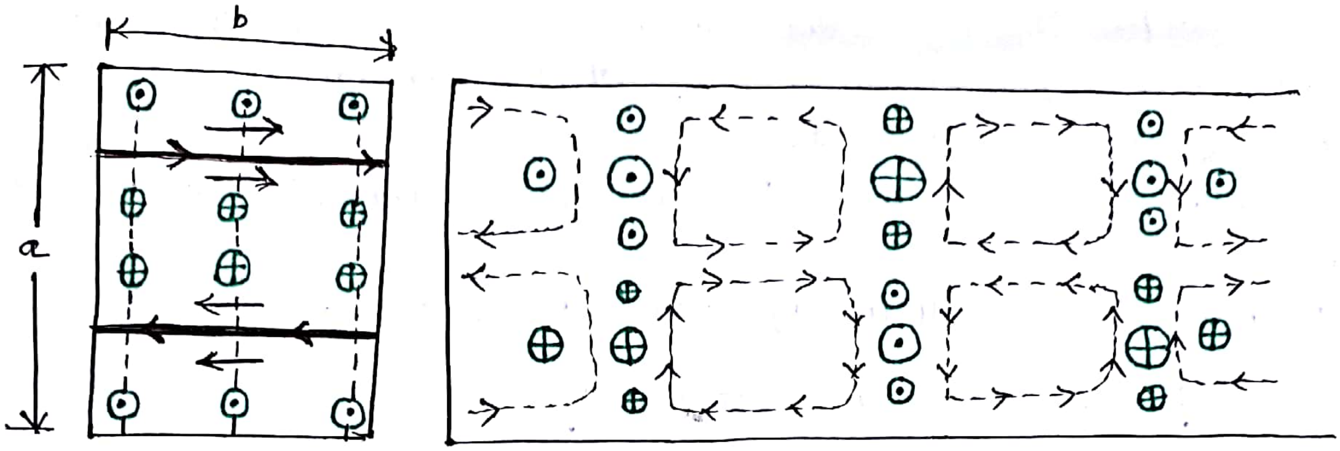
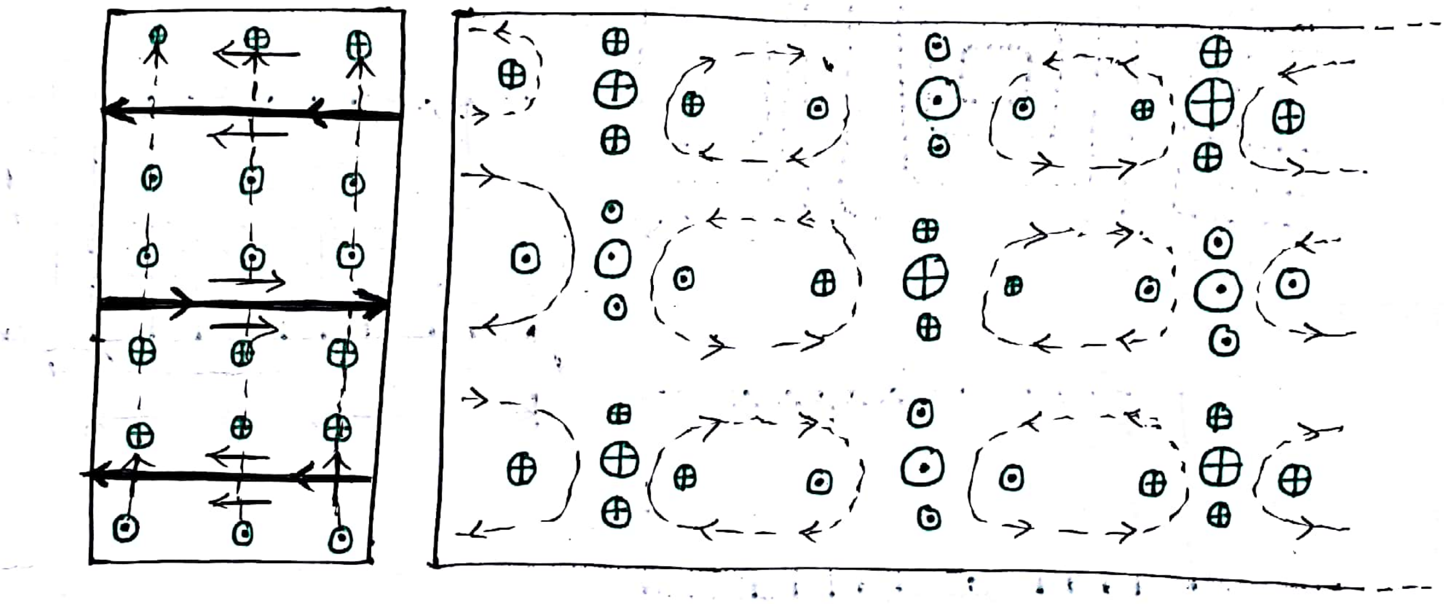
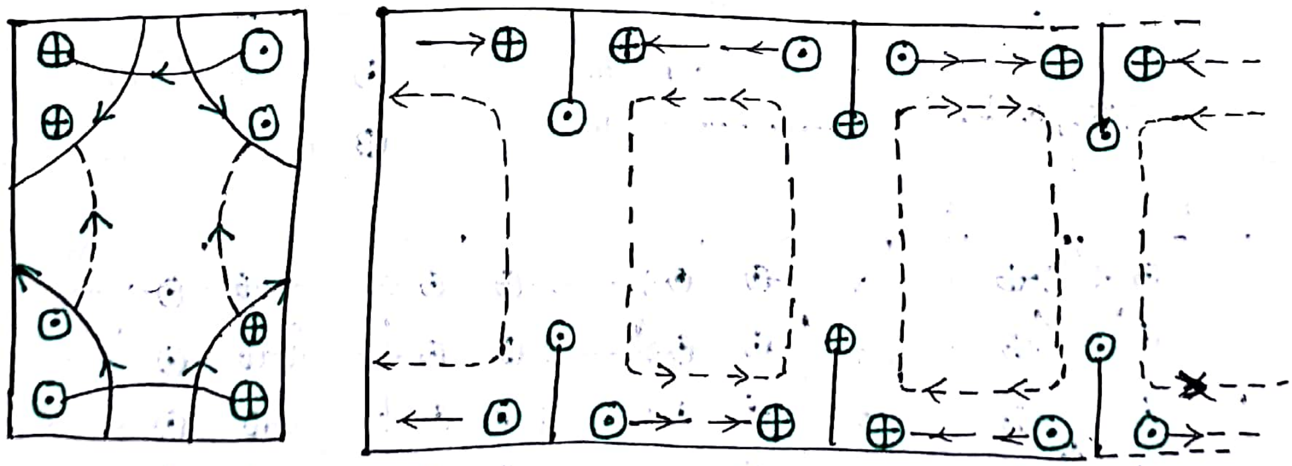
TE_{20} MODE:



TE_{30} MODE:



TE_{11} MODE:



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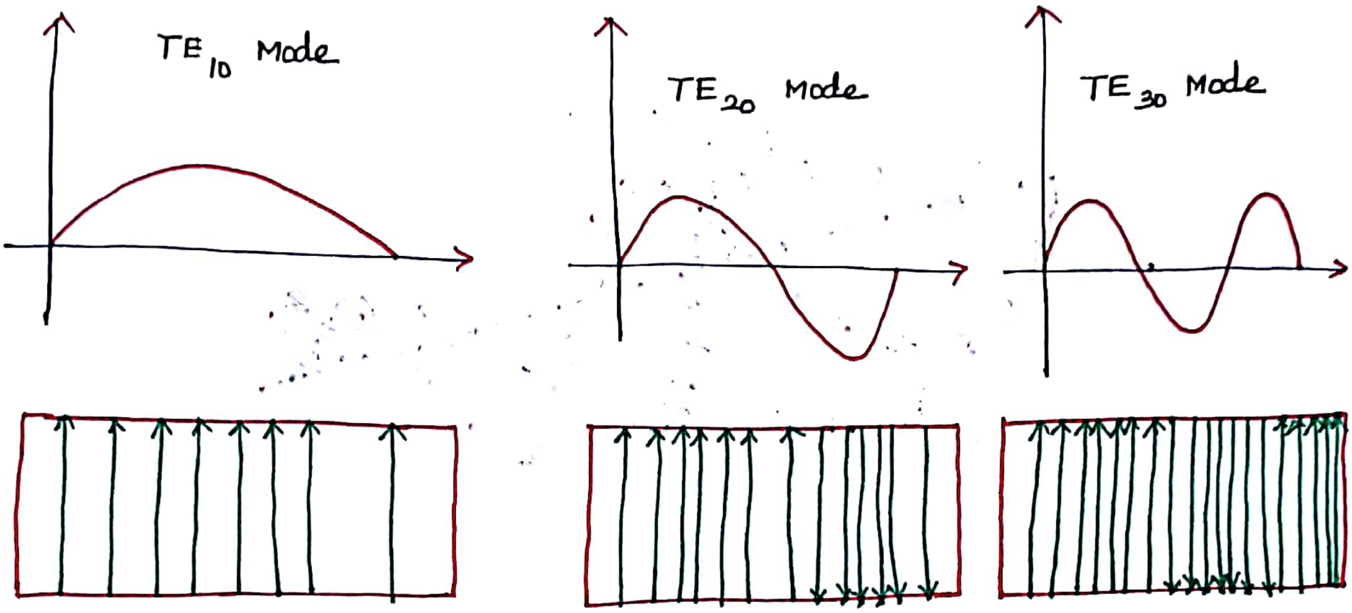
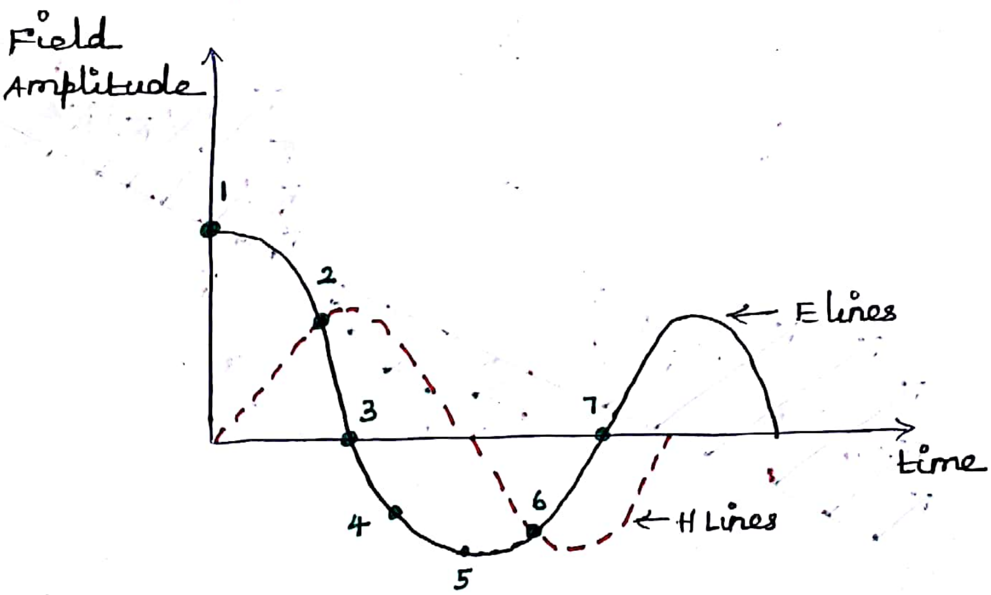
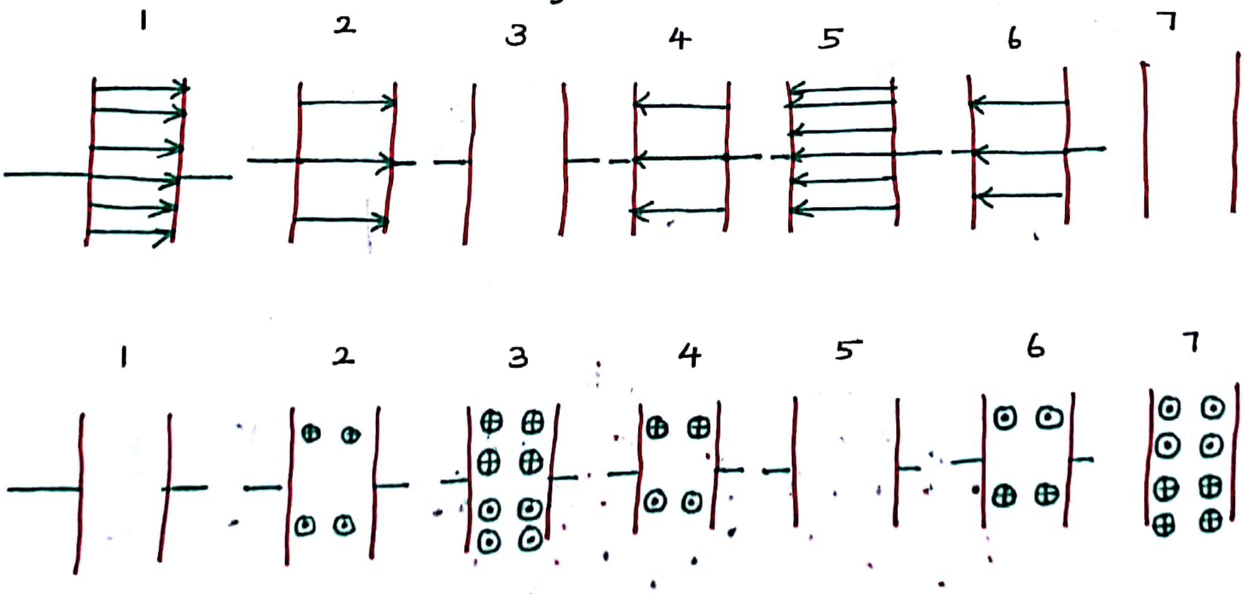
MODES OF PROPAGATION:







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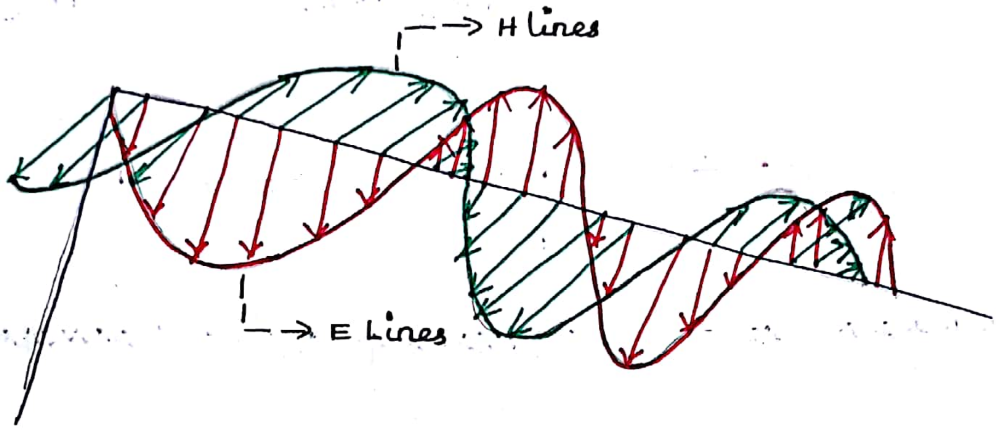

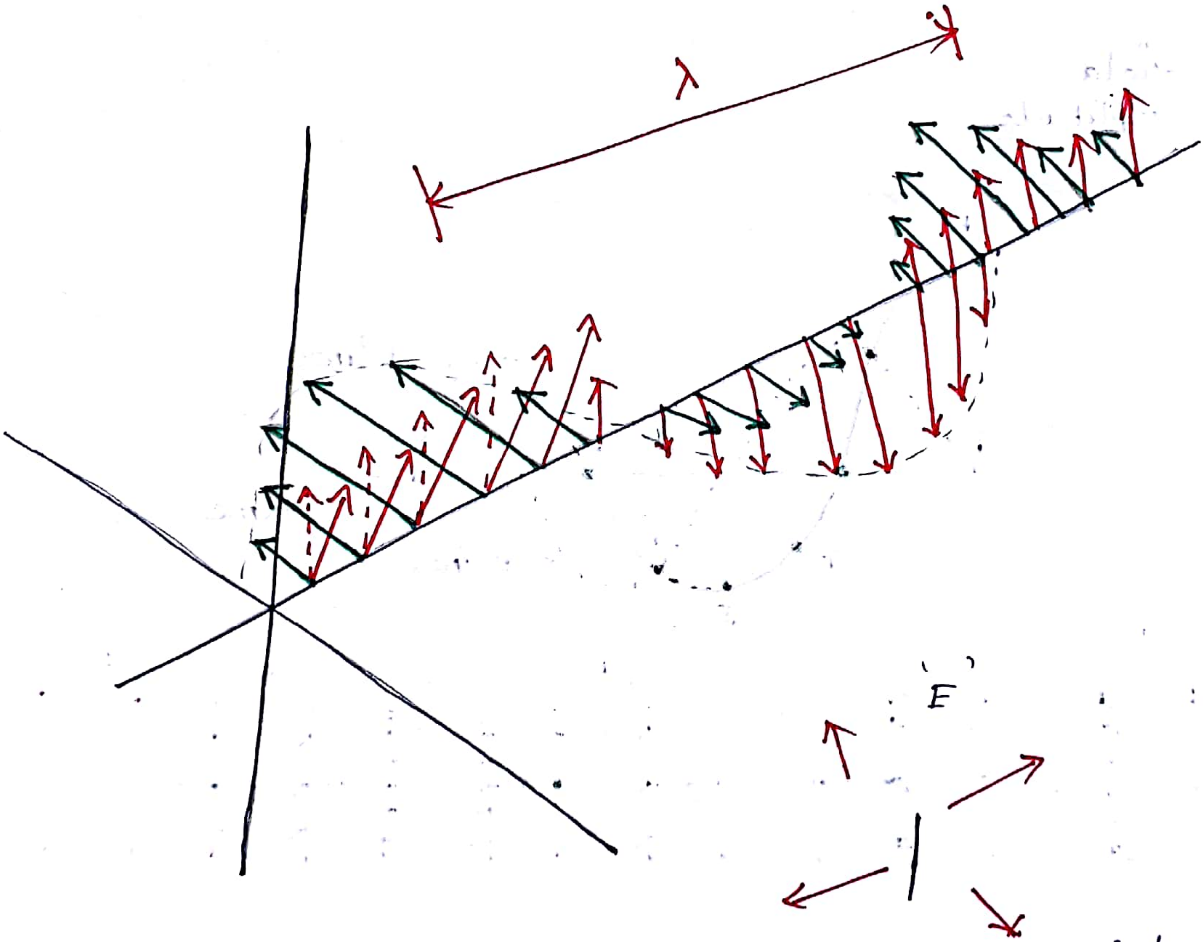

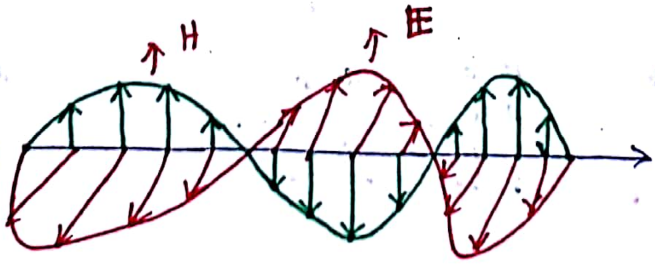

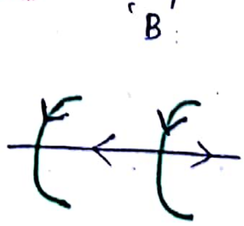

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PROPAGATION OF TM WAVES IN RECTANGULAR WAVEGUIDE:

For TM wave: $H_z = 0$; $E_z \neq 0$

The wave equation of a TM wave is

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \longrightarrow (32)$$

The above partial differential equation can be solved to get the differential field components E_x, E_y, H_x and H_y by 'separation of variables' method.

Let us assume a solution

$$E_z = XY \longrightarrow (33)$$

where

'X' is a pure function of 'x' only

'Y' is a pure function of 'y' only

since 'x' and 'y' are independent variables

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial^2 (XY)}{\partial x^2} = Y \frac{\partial^2 X}{\partial x^2} \longrightarrow (34)$$

$$\frac{\partial^2 E_z}{\partial y^2} = \frac{\partial^2 (XY)}{\partial y^2} = Y \frac{\partial^2 Y}{\partial y^2} \longrightarrow (35)$$

sub (34) and (35) in (32), we get

$$Y \frac{d^2 X}{dx^2} + X \frac{d^2 Y}{dy^2} + h^2 XY = 0 \longrightarrow (36)$$

Dividing throughout by XY, we get

$$\frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} + h^2 = 0 \longrightarrow (37)$$

$\frac{1}{X}\frac{d^2 X}{dx^2}$ is a pure function of x only.

$\frac{1}{Y}\frac{d^2 Y}{dy^2}$ is a pure function of y only.

we use seperation of variables to solve (37)

Let $\frac{1}{X}\frac{d^2X}{dx^2} = -B^2 \longrightarrow$ (38)

and $\frac{1}{Y}\frac{d^2Y}{dy^2} = -A^2 \longrightarrow$ (39)

where $-A^2$ and $-B^2$ are constants.

sub. (38) and (39) in (37), we get

$$-B^2 - A^2 + h^2 = 0 \longrightarrow (40)$$

(or)

$$h^2 = A^2 + B^2 \longrightarrow (41)$$

Eqns. (38) and (39) are ordinary 2nd order differential equations, the solution is given by

$$X = C_1 \cos Bx + C_2 \sin Bx \longrightarrow (42)$$

$$Y = C_3 \cos Ay + C_4 \sin Ay \longrightarrow (43)$$

where C_1, C_2, C_3 and C_4 are constants which can be evaluated by boundary conditions.

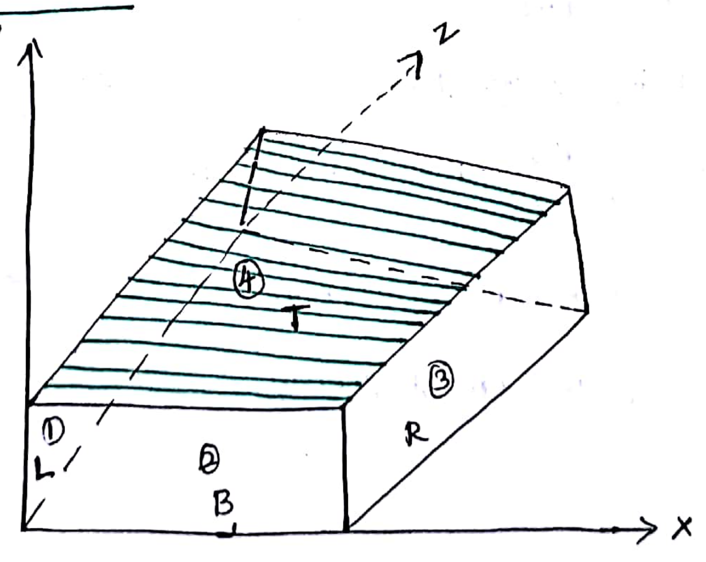
The complete solution is given by (33)

(ie)... $E_z = X \cdot Y \longrightarrow$ (44)

substituting the values of X and Y from (42) and (43), we get

$$E_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay] \longrightarrow (45)$$

Boundary conditions:



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* Since the entire surface of the rectangular waveguide acts as a short circuit or ground for electric field, $E_z = 0$ along all the boundary walls of the guide.
* Since there are 4 walls, there are 4 boundary conditions

1ST BOUNDARY CONDITION:

[Bottom plane or Bottom wall]

We know that $E_z = 0$, all along the bottom wall

(ie) $E_z = 0$ at $y = 0 \; \forall x \rightarrow 0$ to a $\longrightarrow$ (46)

2ND BOUNDARY CONDITION:

[Left side plane or left side wall]

$E_z = 0$ at $x = 0$ for all $y \rightarrow 0$ to b $\longrightarrow$ (47)

3RD BOUNDARY CONDITION:

[Top plane or Top wall]

$E_z = 0$ at $y = b$ for all $x \rightarrow 0$ to a. $\longrightarrow$ (48)

4TH BOUNDARY CONDITION:

[Right side plane or Right side wall]

$E_z = 0$ at $x = a \; \forall y \rightarrow 0$ to b

$\Rightarrow$ sub. 1st boundary condition in (45), we get

$$E_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$$

we have, $E_z = 0$ at $y = 0$ for all $x \rightarrow 0$ to a:

(or)

$$0 = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$$

$$0 = [C_1 \cos Bx + C_2 \sin Bx]\, C_3 \quad \text{as } \cos 0 = 1 \;\&\; \sin 0 = 0 \longrightarrow (49)$$

This is true for all $x \rightarrow 0$ to a

$$\therefore \; C_1 \cos Bx + C_2 \sin Bx = 0 \longrightarrow (50)$$

$$C_3 = 0 \longrightarrow (51)$$

sub. (50) and (51) in 45, we get

$$\boxed{E_z = [C_1 \cos Bx + C_2 \sin Bx][C_4 \sin Ay]} \longrightarrow (52)$$

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$\Rightarrow$ sub. 2nd boundary condition in (52), we get

$$E_z = 0 = C_1 C_4 \sin Ay \;\forall\; y \to 0 \text{ to } b$$

since $\sin Ay \neq 0$ and $C_4 \neq 0$.

$$C_1 = 0 \longrightarrow (53)$$

sub. (53) in (52), we get

$$\boxed{E_z = C_2 C_4 \sin Bx \sin Ay} \longrightarrow (54)$$

$\Rightarrow$ sub. 3rd boundary condition in (54), we get

$$E_z = 0 = C_2 C_4 \sin Bx \sin Ab \; [\text{at } y = b, \forall x \to 0 \text{ to } a]$$

since $\sin Bx \neq 0$, $C_4 \neq 0$, $C_2 \neq 0$,

$$\sin Ab = 0 \longrightarrow (55)$$

(or)

$$Ab = \text{a multiple of } \pi = n\pi \longrightarrow (56)$$

where n is a constant, $n = 0, 1, 2 \ldots$

$$\boxed{A = \frac{n\pi}{b}} \longrightarrow (57)$$

$\Rightarrow$ sub. 4th boundary condition in (54), we get

$$E_z = 0 = C_2 C_4 \sin Ba \sin Ay \; [\text{at } x = a, \forall y \to 0 \text{ to } b]$$

since $\sin Ay \neq 0$, $C_4 \neq 0$, $C_2 \neq 0$

$$\sin Ba = 0 \longrightarrow (58)$$

where m is another constant, $m = 0, 1, 2 \ldots$

$$\boxed{B = \frac{m\pi}{a}} \longrightarrow (59)$$

The complete solution is given by

$$E_z = C_2 C_4 \sin Bx \sin Ay \longrightarrow (60)$$

sub. (57) and (59) in (60), we get

$$E_z = C_2 C_4 \sin\left(\frac{m\pi}{a}\right) x \sin\left(\frac{n\pi}{b}\right) y \cdot e^{-\gamma z} \cdot e^{j\omega t} \longrightarrow (61)$$

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where

$e^{-\gamma z}$ – propagation along z direction

$e^{j\omega t}$ – sinusoidal variation w.r.to t

Let $C = C_2 C_4$,

$$\boxed{E_z = C \sin\left(m\frac{\pi}{a}\right)x \sin\left(n\frac{\pi}{b}\right)y\, e^{j\omega t-\gamma z}} \longrightarrow (62)$$

since E_z is known, E_x, E_y, H_x and H_y are given by the following equations

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \longrightarrow (63)$$

For TM wave, $H_z = 0$,

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} \longrightarrow (64)$$

$$\therefore \boxed{E_x = \frac{-\gamma}{h^2} C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t-\gamma z}} \longrightarrow (65)$$

and

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \longrightarrow (66)$$

$$\boxed{E_y = \frac{-\gamma}{h^2} C\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{j\omega t-\gamma z}} \longrightarrow (67)$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$\boxed{H_x = \frac{j\omega\varepsilon}{h^2} C\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{j\omega t-\gamma z}} \longrightarrow (68)$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \longrightarrow (69)$$

$$\boxed{H_y = \frac{j\omega\varepsilon}{h^2} C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t-\gamma z}} \longrightarrow (70)$$

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TM MODES IN RECTANGULAR WAVEGUIDES:

* Depending on the values of m and n, we have various modes in TM waves.

TM_{00} mode: ($m=0$ and $n=0$)

* If $m=0$ and $n=0$ are substituted in E_x, E_y, H_x and H_y (eqns 65, 68, 69 & 70) all of them vanish and hence TM_{00} mode cannot exist.

TM_{01} mode: ($m=0$ and $n=1$)

* All field components vanish here

TM_{10} mode: ($m=1$ and $n=0$)

* All field components vanish here

TM_{11} mode: ($m=1$ and $n=1$)

* Here we have all 4 components (E_x, E_y, H_x and H_y) (ie) TM_{11} mode exists, and for all high values of m and n, the components (ie) all higher modes exist.

CUT OFF FREQUENCY OF A WAVEGUIDE:

* From equations 57 and 59, we know that

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \longrightarrow (71)$$

$$\therefore \gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon \longrightarrow (72)$$

(or)

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} = \alpha + j\beta \longrightarrow (73)$$

At Lower frequencies,

$$\omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \longrightarrow (74)$$

$\Rightarrow$ γ becomes real and positive, and is equal to the attenuation constant 'α'. (ie) the wave is completely attenuated and there is no phase change. Hence the wave cannot propagate.

At Higher Frequencies,

$$\omega^2 \mu \varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \longrightarrow (75)$$

* γ becomes imaginary, there will be phase change β and hence the wave propagates. At the transition, γ becomes zero and the propagation just starts. The frequency at which γ just becomes zero is defined as CUT OFF FREQUENCY.

* At $f = f_c$, $\gamma = 0$ or $\omega = 2\pi f = 2\pi f_c = \omega_c \longrightarrow (76)$

$$0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2 \mu \varepsilon \longrightarrow (77)$$

(or)

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2} \longrightarrow (78)$$

(or)

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2} \longrightarrow (79)$$

W.K.T $\Rightarrow c = \frac{1}{\sqrt{\mu\varepsilon}}$

$$\therefore \boxed{f_c = \frac{c}{2\pi} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}} \longrightarrow (80)$$

(or)

$$\boxed{f_c = \frac{c}{2} \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}} \longrightarrow (81)$$

The cut-off wavelength (λ_c) is given by

$$\boxed{\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}}} \longrightarrow (82)$$

(or)

$$\boxed{\lambda_{c\,m,n} = \frac{2ab}{\sqrt{m^2 b^2 + n^2 a^2}}} \longrightarrow (83)$$

* All wavelengths greater than λ_c are attenuated and those less than λ_c are allowed to propagate inside the waveguide.

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GUIDE WAVELENGTH (λg) :-

* It is defined as the distance travelled by the wave inorder to undergo a phase shift of 2π radians.
* It is related to phase constant by the relation

$$\lambda g = \frac{2\pi}{\beta} \longrightarrow (84)$$

* The wavelength in waveguide is different from wavelength in free space.

$$\frac{1}{\lambda g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda c^2} \longrightarrow (85)$$

The above equation (85) depicts the relation between Guide wavelength (λg), Free space wavelength (λ_0) and cut off wavelength (λc).

$$\therefore \boxed{\lambda g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda c)^2}}} \longrightarrow (86)$$

PHASE VELOCITY (V_p) :-

* It is defined as the rate at which the wave changes its phase interms of Guide wavelength.

(ie) $$V_p = \frac{\lambda g}{\text{unit time}} \longrightarrow (87)$$

$$V_p = \lambda g \cdot f$$

$$V_p = \frac{2\pi f \cdot \lambda g}{2\pi} \longrightarrow (88)$$

$$V_p = \frac{2\pi f}{2\pi / \lambda g} \longrightarrow (89)$$

$$\boxed{V_p = \frac{\omega}{\beta}} \longrightarrow (90)$$

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where

$$\omega = 2\pi f$$

&

$$\beta = \frac{2\pi}{\lambda g}$$

GROUP VELOCITY: (V_g):

* It is defined as the rate at which the wave propagates and is given by

$$\boxed{V_g = \frac{d\omega}{d\beta}} \longrightarrow (91)$$

EXPRESSION FOR PHASE VELOCITY AND GROUP VELOCITY:

Expression for Phase velocity (V_P):

From (90), we know that $V_p = \frac{\omega}{\beta}$

Also, $h^2 = \gamma^2 + \omega^2 \mu \varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \longrightarrow (92)$

and $\gamma = \alpha + j\beta \longrightarrow (93)$

For wave propagation, $\gamma = j\beta$ ($\because$ attenuation $\alpha = 0$)

$$\gamma^2 = (j\beta)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon \longrightarrow (94)$$

At $f = f_c$, $\omega = \omega_c$, $\gamma = 0$

$$\omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \longrightarrow (95)$$

using (95) in (94), we get

$$\gamma^2 = (j\beta)^2 = \omega_c^2 \mu \varepsilon - \omega^2 \mu \varepsilon \longrightarrow (96)$$

$$\gamma^2 = \beta^2 = \omega^2 \mu \varepsilon - \omega_c^2 \mu \varepsilon \longrightarrow (97)$$

(or)

$$\beta = \sqrt{\omega^2 \mu \varepsilon - \omega_c^2 \mu \varepsilon} \longrightarrow (98)$$

$$\beta = \sqrt{\mu \varepsilon (\omega^2 - \omega_c^2)} \longrightarrow (99)$$

$$\beta = \sqrt{\mu \varepsilon}\,\sqrt{\omega^2 - \omega_c^2} \longrightarrow (100)$$

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$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\mu\varepsilon}\sqrt{\omega^2-\omega_c^2}} \longrightarrow (101)$$

$$V_p = \frac{1}{\sqrt{\mu\varepsilon}} \cdot \frac{1}{\sqrt{1-(\omega_c/\omega)^2}} \longrightarrow (102)$$

(ie) $$\boxed{V_p = \frac{c}{\sqrt{1-(f_c/f)^2}}} \longrightarrow (103)$$

W.K.T $f = \frac{c}{\lambda_0}$ & $f_c = \frac{c}{\lambda_c}$,

$$\frac{f_c}{f} = \frac{c}{\lambda_c} \cdot \frac{\lambda_0}{c} \longrightarrow (104)$$

$$\frac{f_c}{f} = \frac{\lambda_0}{\lambda_c} \longrightarrow (105)$$

sub. (105) in (103), we get

$$\boxed{V_p = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}}} \longrightarrow (106)$$

Expression for Group velocity (V_g) :-

* We know that V_p is greater than the speed of light by the ratio λ_g/λ_0.

$$V_p = \frac{\lambda_g}{\lambda_0} c \rightarrow (107)$$

$$V_g = \frac{\lambda_0}{\lambda_g} c \rightarrow (108)$$

$$V_p V_g = \frac{\lambda_g}{\lambda_0} c \cdot \frac{\lambda_0}{\lambda_g} \cdot c = c^2 \Rightarrow V_p V_g = c^2 \longrightarrow (109)$$

W.K.T $V_p = \lambda_g \cdot f \longrightarrow (110)$

$$V_p = \frac{\lambda_g}{\lambda_0} \cdot c \longrightarrow (111)$$

Also $$V_p = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}} = \frac{c}{\quad} = \frac{\lambda_g}{\lambda_0} \cdot c$$

(or)

$$\boxed{\lambda_g = \frac{\lambda_0}{1-(\lambda_0/\lambda_c)^2}} \longrightarrow (112)$$

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TM MODES IN RECTANGULAR WAVEGUIDES:

TM_{11} Mode : ($m=1$ and $n=1$)

From (83), we get

$$\lambda c_{m,n} = \frac{2ab}{\sqrt{m^2a^2+n^2b^2}} \longrightarrow (113)$$

$$\boxed{\lambda c_{1,1} = \frac{2ab}{\sqrt{a^2+b^2}}} \longrightarrow (114)$$

TM_{12} Mode : ($m=1$ and $n=2$)

$$\boxed{\lambda c_{1,2} = \frac{2ab}{\sqrt{b^2+4a^2}}} \longrightarrow (115)$$

TM_{21} Mode : ($m=2$ and $n=1$)

$$\boxed{\lambda c_{2,1} = \frac{2ab}{\sqrt{4b^2+a^2}}} \longrightarrow (116)$$

PROPAGATION OF TE WAVES IN A RECTANGULAR WAVEGUIDE:

* The TE_{mn} modes in a rectangular waveguide are characterized by $E_z=0$.
* H_z component must exist inorder to have energy transmission in the guide.

The wave equation (Helmholtz equation) for TE wave is given by

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \longrightarrow (117)$$

(ie) $$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \varepsilon H_z \longrightarrow (118)$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z + \omega^2 \mu \varepsilon H_z = 0 \longrightarrow (119) \left[\because \frac{\partial^2}{\partial z^2} = \gamma^2\right]$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) H_z = 0 \longrightarrow (120)$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \longrightarrow (121)$$

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The solution for above partial Differential Equation has been assumed to be of the form

$$H_z = XY \longrightarrow (122)$$

where X is a pure function of 'x' only

Y is a pure function of 'y' only

W.K.T

$$Y \frac{d^2X}{dx^2} + X \frac{d^2Y}{dy^2} + h^2 XY = 0 \longrightarrow (123)$$

Dividing throughout by XY, we get

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + h^2 = 0 \longrightarrow (124)$$

here $\frac{1}{X}\frac{d^2X}{dx^2}$ is purely a function of x

and $\frac{1}{Y}\frac{d^2Y}{dy^2}$ is purely a function of y

Equating, each of the terms to a constant, we get

$$\frac{1}{X}\frac{d^2X}{dx^2} = -B^2 \longrightarrow (125)$$

and $$\frac{1}{Y}\frac{d^2Y}{dy^2} = -A^2 \longrightarrow (126)$$

where $-B^2$ and $-A^2$ are constants

sub. (125) and (126) in (124), we get

$$-B^2 - A^2 + h^2 = 0 \longrightarrow (127)$$

$$h^2 = A^2 + B^2 \longrightarrow (128)$$

solving for X and Y by seperation of variable method,

$$X = C_1 \cos Bx + C_2 \sin Bx \longrightarrow (129)$$

$$Y = C_3 \cos Ay + C_4 \sin Ay \longrightarrow (130)$$

∴ The complete solution is $H_z = XY$

(ie) $$\boxed{H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)} \longrightarrow (131)$$

where C_1, C_2, C_3 and C_4 are constants which can be evaluated by applying boundary conditions.

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Let $C_1 C_3 = C$

$$\therefore \boxed{H_z = C\cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)\cdot y\cdot e^{(j\omega t-\gamma z)}} \longrightarrow (132)$$

$$\therefore \quad E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

Since $E_z=0$, for TM wave,

$$\boxed{E_x = \frac{j\omega\mu}{h^2} c\cdot\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\cdot e^{(j\omega t-\gamma z)}} \longrightarrow (133)$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

Since $E_z=0$ for TM wave,

$$\boxed{E_y = \frac{-j\omega\mu}{h^2} c\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\cdot e^{(j\omega t-\gamma z)}} \longrightarrow (134)$$

Similarly,

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$\boxed{H_x = \frac{\gamma}{h^2} c\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\cdot e^{(j\omega t-\gamma z)}} \longrightarrow (135)$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$\therefore \boxed{H_y = \frac{-\gamma}{h^2} c\cdot\left(\frac{n\pi}{b}\right)^2\cos\left(\frac{m\pi}{a}\right)x\cdot \sin\left(\frac{n\pi}{b}\right)y\cdot e^{(j\omega t-\gamma z)}} \longrightarrow (136)$$

TE MODES IN RECTANGULAR WAVEGUIDES:

TE_{00} MODE: ($m=0$ & $n=0$)

All field components vanish, therefore it cannot exist

TE_{01} MODE: ($m=0$ & $n=1$)

$E_y=0$ $\quad E_x$ - Exist

$H_y=0$ $\quad H_x$ - Exist

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TE_{10} MODE: ($m=1$ & $n=0$)

$E_x = 0$ $\quad$ E_y - exist

$H_y = 0$ $\quad$ H_x - exist

TE_{11} MODE: ($m=1$ & $n=1$)

Exist.

DOMINANT MODE:

Dominant mode is the mode at which cut off wavelength (λc) assumes a maximum value.

W.K.T $\quad \lambda c_{m,n} = \dfrac{2ab}{\sqrt{m^2b^2+n^2a^2}}$

For TE_{01} Mode:	$\lambda c_{01} = \dfrac{2ab}{\sqrt{a^2}} = 2b$	$\longrightarrow$ (137)
For TE_{10} Mode:	$\lambda c_{10} = \dfrac{2ab}{\sqrt{b^2}} = 2a$	$\longrightarrow$ (138)
For TE_{11} Mode:	$\lambda c_{11} = \dfrac{2ab}{\sqrt{a^2+b^2}}$	$\longrightarrow$ (139)

From above equations, λc_{10} has the maximum value, since 'a' is the Larger dimension. Hence TE_{10} mode is the dominant mode in the Rectangular waveguides.

The other expressions remain the same, as for TM waves

$\beta = \dfrac{2\pi}{\lambda g} = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$	$\longrightarrow$ (140)
$V_p = \dfrac{c}{\sqrt{1-(\lambda_0/\lambda c)^2}}$	$\longrightarrow$ (141)
$V_g = c \cdot \sqrt{1-(\lambda_0/\lambda c)^2}$	$\longrightarrow$ (142)
$\lambda g = \dfrac{\lambda_0}{1-(\lambda_0/\lambda c)^2}$	$\longrightarrow$ (143)

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1) A Rectangular waveguide has $a = 4$ cms; $b = 3$ cms; as its sectional dimensions. Find all the modes which will propogate at 5000MHz.

SOLUTION:-

The condition for a wave to propagate along a waveguide is that $\lambda_c > \lambda_0$.

Given $f = 5000\,MHz = 5\,GHz$

W.K.T $\lambda_0 = \frac{c}{f} = \frac{3\times10^8\ m/s}{5\times10^9\ Hz} = 6$ cms.

case: 1: (TE_{01} mode)

For TE waves

$$\lambda_c = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

here $a = 4$ & $b = 3$

For TE_{01}, sub $m=0$ & $n=1$

$$TE_{01} \Rightarrow \lambda_c = \frac{2ab}{\sqrt{a^2}} = 2b = 2\times3 = 6\text{ cms}$$

since λ_c of TE_{01} (6 cms) is not greater than λ_0 (6 cms), the wave will not propagate in TE_{01} mode.

case: 2: (TE_{11} mode)

$$TE_{11} \Rightarrow \lambda_c = \frac{2ab}{\sqrt{a^2+b^2}} = \frac{2\times4\times3}{\sqrt{4^2+3^2}} = \frac{24}{5} = 4.8\text{ cms}$$

$\lambda_c\ TE_{11} < \lambda_0\ TE_{11}$, the wave will not propagate in TE_{11} mode

case: 3 (TE_{10} mode)

$$TE_{10} \Rightarrow \lambda_c = 2a = 2\times4 = 8\text{ cms}$$

$\lambda_c\ TE_{10} > \lambda_0\ TE_{10}$, the wave will propagate in TE_{10} mode

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2.) When the Dominant mode is propagated in an air filled Rectangular waveguide, the guide wavelength for a frequency of 9000 MHz is 4 cm. Calculate breadth of the guide.

SOLUTION:-

For a Rectangular waveguide, TE_{10} mode is the Dominant mode

$$\lambda g = \frac{\lambda o}{\sqrt{1-(\lambda o/\lambda c)^2}}$$

$$\lambda o = \frac{c}{f} = \frac{3\times10^8}{9\times10^9} = 3.333 \text{ cms}$$

~~$\lambda g =$~~

$$4 = \frac{3.333}{\sqrt{1-(3.333/\lambda c)^2}}$$

$$\Rightarrow \quad \lambda c^2 = \frac{11.11111}{0.3055}$$

$$\Rightarrow \quad \lambda c = 6.0302 \text{ cms}$$

$\lambda c > \lambda o$, the wave propagates and

$$\lambda c = 2a \text{ for } TE_{10} \text{ mode}$$

$$a = \frac{\lambda c}{2} = 3.0151 \text{ cms}$$

$$\text{Breadth} = a = 3.0151 \text{ cms}$$

$$b = \frac{\lambda c}{4} \quad \therefore a = 2b$$

$$b = 1.5 \text{ cms.}$$

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3.) Determine the cut off wavelength for the dominant mode in the Rectangular waveguide of breadth 10 cms. For a 2.5 GHz of signal propagated in this waveguide in the dominant mode: calculate the Guide wavelength, Group velocity and phase velocity.

SOLUTION:-

In a Rectangular waveguide, the dominant mode is the TE_{10} mode.

$$\lambda c_{TE_{10}} = 2a = 2 \times 10 = 20 \text{ cms}$$

Given $f = 2.5$ GHz

$$\lambda g = \frac{\lambda o}{\sqrt{1-(\lambda o/\lambda c)^2}}$$

$$\lambda o = \frac{c}{f} = \frac{3 \times 10^8}{2.5 \times 10^9} = 12 \text{ cms}$$

$$\lambda g = \frac{12}{\sqrt{1-(12/20)^2}}$$

$$\lambda g = 15 \text{ cms}$$

$$V_p = \frac{c}{\sqrt{1-(\lambda o/\lambda c)^2}}$$

$$V_p = \frac{3 \times 10^8}{0.8} = 3.75 \times 10^{10} \text{ cms/sec}$$

$$V_p \cdot V_g = c^2$$

$$V_g = \frac{c^2}{V_p} = \frac{(3 \times 10^8)^2}{3.75 \times 10^{10}} = 2.4 \times 10^{10} \text{ cms/sec.}$$

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4. A Rectangular waveguide has dimensions 2.5 X 5 cms. Determine the Guide wavelength, Phase constant β and phase velocity at a wavelength of 4.5 cms for the Dominant mode.

SOLUTION:-

For TE_{10} mode : $\lambda_c = 2a = 2 \times 5 = 10$ cms

Given: $\lambda_0 = 4.5$ cms

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}} = \frac{4.5}{\sqrt{1-(4.5/10)^2}} = \frac{4.5}{0.573}$$

$$\lambda_g = 7.803 \text{ cms}$$

$$V_p = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}} = \frac{3 \times 10^8}{0.573} = 5.22 \times 10^{10} \text{ cms/sec.}$$

$$\beta = \frac{1}{c}\sqrt{\omega^2 - \omega_c^2}$$

$$\omega = \frac{2\pi c}{\lambda_0}$$

$$\omega^2 = \frac{(2\pi c)^2}{(\lambda_0)^2}$$

$$\omega_c^2 = \frac{(2\pi c)^2}{(\lambda_c)^2}$$

$$\beta = \frac{1}{c}\sqrt{\left(\frac{2\pi c}{\lambda_0}\right)^2 - \left(\frac{2\pi c}{\lambda_c}\right)^2}$$

$$\beta = \frac{2\pi c}{c}\sqrt{\left(\frac{1}{\lambda_0}\right)^2 - \left(\frac{1}{\lambda_c}\right)^2}$$

$$\beta = \frac{2\pi}{\lambda_0 \lambda_c}\sqrt{(\lambda_c)^2 - (\lambda_0)^2}$$

$$\beta = \frac{2\pi}{4.5 \times 10}\sqrt{100 - 20.25}$$

$$\beta = 1.246 \text{ Radians}$$

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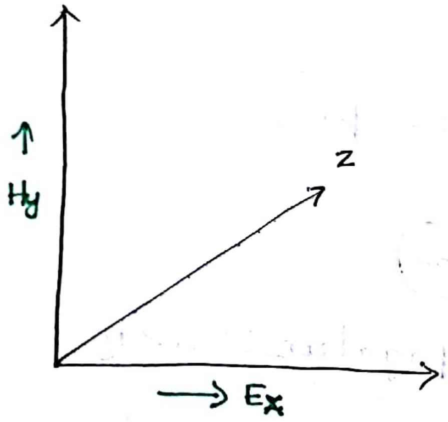
WAVE IMPEDANCE: (Z_z)

It is defined as the ratio of strength of electric field in one transverse direction to the strength of the magnetic field along the other transverse direction



$$Z_z = \frac{E_x}{H_y} = \frac{-E_y}{H_x} \longrightarrow (144)$$

(or)

$$Z_z = \frac{\sqrt{E_x^2 + E_y^2}}{\sqrt{H_x^2 + H_y^2}} \longrightarrow (145)$$

WAVE IMPEDANCE OF TM WAVE IN RECTANGULAR WAVEGUIDE:

$$Z_z = Z_{TM} = \frac{E_x}{H_y} = \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}} \longrightarrow (146)$$

For a TM wave, $H_z = 0$ & $\alpha = j\beta$

$$Z_{TM} = \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}}{-\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}} = \frac{\gamma}{j\omega\varepsilon} = \frac{j\beta}{j\omega\varepsilon} \longrightarrow (147)$$

$$Z_{TM} = \frac{\beta}{\omega\varepsilon} \longrightarrow (148)$$

W.K.T $\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$

$$Z_{TM} = \frac{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}{\omega\varepsilon} \longrightarrow (149)$$

$$Z_{TM} = \frac{\sqrt{\mu\varepsilon}\cdot\sqrt{\omega^2 - \omega_c^2}}{\varepsilon\cdot\omega} \longrightarrow (150)$$

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$$Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{1-(\lambda_0/\lambda_c)^2} \longrightarrow (151)$$

$$\longrightarrow (152)$$

For air, $\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36}\pi \times 10^{-9}}} = \sqrt{4\pi \times 36\pi \times 10^{2}} = 2\times 6\pi \times 10 = 120\pi$

$$\eta = 377\,\Omega \longrightarrow (153)$$

where η is the Intrinsic Impedance of the free space

$$\therefore Z_{TM} = \eta\sqrt{1-(\lambda_0/\lambda_c)^2} \longrightarrow (154)$$

since λ_0 is always less than λ_c, for wave propagation $Z_{TM} < \eta$.

This shows that wave Impedance is ~~d~~ always less than Free space Impedance.

WAVE IMPEDANCE OF TE WAVES IN RECTANGULAR WAVEGUIDES:

$$Z_z = Z_{TE} = \frac{E_x}{H_y} = \frac{\frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{\frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}} \longrightarrow (155)$$

For TE waves, $E_z = 0$ and $\gamma = j\beta$

$$\therefore Z_{TE} = \frac{\frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{\frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y}} = \frac{j\omega\mu}{\gamma} \longrightarrow (156)$$

$$Z_{TE} = \frac{j\omega\mu}{j\beta} \longrightarrow (157)$$

$$Z_{TE} = \frac{\omega\mu}{\beta} \longrightarrow (158)$$

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$$Z_{TE} = \frac{\omega\mu}{\sqrt{\mu\varepsilon}\sqrt{\omega^2-\omega_c^2}} = \frac{\eta}{\sqrt{1-(\omega_c/\omega)^2}} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} \rightarrow (159)$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}} \rightarrow (160)$$

$\therefore$ $Z_{TE} > \eta$ and $\lambda_0 > \lambda_c$ for wave propagation.

This shows that wave Impedance of a TE wave is always greater than free space wave Impedance.

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1) consider a guide of 8x4 cms. Given critical wavelength of $TE_{10} = 16$ cms; $TM_{11} = 7.16$ and $TM_{21} = 5.6$ cms. what modes are propagated at a free space wavelength of (i) 10cm (ii) 5cms.

$\lambda_0 = 10$ cms.

$TE_{10} \rightarrow$

TM_{11} & $TM_{21} \not\rightarrow$

$\lambda_0 = 5$ cms

16 cm = $\lambda_c\, TM_{10} > \lambda_0$: $TE_{10} \rightarrow$

16 cm = $\lambda_c\, TM_{11} > \lambda_0$: $TE_{11} \rightarrow$

~~5~~ 6.6 cm = $\lambda_c\, TM_{21} > \lambda_0$: $TE_{21} \rightarrow$

2. A rectangular w.g with 3x2 cms operates in TM_{11} mode at 10 GHZ. Determine characteristic wave Impedance.

$$Z_{TM} = \eta\sqrt{1-(\lambda_0/\lambda_c)^2}$$

$$\eta = 120\pi$$

$$\lambda_{c_{11}} = \frac{2ab}{\sqrt{a^2+b^2}} = \frac{2\times3\times2}{\sqrt{3^2+2^2}} = 3.328 \text{ cms}$$

$$\lambda_0 = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 3 \text{ cms.}$$

$$Z_{TM} = 120\pi\sqrt{1-(\lambda_0/\lambda_c)^2} \qquad 120\pi\sqrt{\cancel{1-(0.81)^2}\ 1-0.81}$$

$$Z_{TM} = 120\pi\times 0.163 \qquad 120\pi\times 0.435$$

$$Z_{TM} = 61.618\,\Omega$$

$$164.24\,\Omega$$

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3) ~~An air~~ A hollow rectangular waveguide has dimensions $a = 1.5$ cm and $b = 1$ cm. calculate the amount of attenuation if the frequency of the signal is 6 GHz.

soln:

For dominant TE_{10} mode, $\lambda c_{10} = 2a = 2 \times 1.5 = 3$ cm

$$fc_{10} = \frac{c}{\lambda c_{10}} = \frac{3 \times 10^8}{3} = 10 \text{ GHz}.$$

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon}$$

$$\alpha = \sqrt{\left(\frac{\pi}{0.015}\right)^2 + 0 - (2\pi \times 6 \times 10^9)^2 \times 4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}$$

$\alpha = 167.5$ nepars/m

(one neper $Np = \frac{20}{\ln(10)} = 8.685$ dB)

$\alpha = 1453.23$ dB/m

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The TM mode field equations in Rectangular waveguides are

$$E_x = E_{ox} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \longrightarrow (75)$$

$$E_y = E_{oy} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \longrightarrow (76)$$

$$E_z = E_{oz} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \longrightarrow (77)$$

$$H_x = H_{ox} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \longrightarrow (78)$$

$$H_y = H_{oy} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z} \longrightarrow (79)$$

$$H_z = 0 \longrightarrow (80)$$

TM mode characteristic equations are shown below

$f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$	$\longrightarrow$ (81)
$\beta_g = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$	$\longrightarrow$ (82)
$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$	$\longrightarrow$ (83)
$v_g = \frac{v_p}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$	$\longrightarrow$ (84)
$Z_g = \frac{\beta_g}{\omega\varepsilon} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$	$\longrightarrow$ (85)

POWER TRANSMISSION IN RECTANGULAR WAVEGUIDES:

* The power transmitted through a waveguide and power loss in the guide walls can be calculated by means of complex POYNTING Theorem.

The power transmitted through a wave guide is given by

$$P_{tr} = \oint P.ds = \oint \frac{1}{2}(E \times H^*)\, ds \longrightarrow (86)$$

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$$j\beta_g E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y \longrightarrow (64)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \longrightarrow (65)$$

$$\beta_g H_y = \omega\varepsilon E_x \longrightarrow (66)$$

$$-\beta_g H_x = \omega\varepsilon E_y \longrightarrow (67)$$

$$\frac{\partial H_x}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \longrightarrow (68)$$

The above equations can be solved simultaneously for E_x, E_y, H_x and H_y in terms of E_z.

The Resultant field equations for TM modes are

$$E_x = \frac{-j\beta_g}{K_c^2}\frac{\partial E_z}{\partial x} \longrightarrow (69)$$

$$E_y = \frac{-j\beta_g}{K_c^2}\frac{\partial E_z}{\partial y} \longrightarrow (70)$$

$$E_z = E_{oz}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta_g z} \longrightarrow (71)$$

$$H_x = \frac{j\omega\varepsilon}{K_c^2}\frac{\partial E_z}{\partial y} \longrightarrow (72)$$

$$H_y = \frac{-j\omega\varepsilon}{K_c^2}\frac{\partial E_z}{\partial x} \longrightarrow (73)$$

$$H_z = 0 \longrightarrow (74)$$

where $\beta_g^2 - \omega^2\mu\varepsilon = -K_c^2$ is replaced.

Differentiating eqn (62) w.r.to x or y and substituting the results in eqns (69) through (74) yield a new set of field equations.

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For a Lossless Dielectric, the time average power flow through a Rectangular waveguide is given by

$$P_{tr} = \frac{1}{2Z_g}\int_a |E|^2 da = \frac{Z_g}{2}\int_a |H|^2 da \longrightarrow (87)$$

where $Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x} \longrightarrow (88)$

$$|E|^2 = |E_x|^2 + |E_y|^2 \longrightarrow (89)$$

$$|H|^2 = |H_x|^2 + |H_y|^2 \longrightarrow (90)$$

For TE modes, the average power transmitted through a Rectangular waveguide is given by

$$\boxed{P_{tr} = \frac{\sqrt{1-(f_c/f)^2}}{2\eta}\int_0^b\int_0^a (|E_x|^2 + |E_y|^2)\,dx\,dy} \longrightarrow (91)$$

For TM modes, the average power transmitted through a Rectangular waveguide is given by

$$\boxed{P_{tr} = \frac{1}{2\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\int_0^b\int_0^a (|H_x|^2 + |H_y|^2)\,dx\,dy} \longrightarrow (92)$$

where $\eta = \sqrt{\mu/\varepsilon}$ is the Intrinsic Impedance in an unbounded dielectric.

POWER LOSSES IN RECTANGULAR WAVEGUIDES:

* There are 2 types of Losses in a Rectangular waveguide
  - ⇒ Losses in the Dielectric
  - ⇒ Losses in the Guide walls

LOSSES IN DIELECTRIC:

The plane wave travelling in an unbounded Lossy Dielectric is given by

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta\sigma}{2} \longrightarrow (93)$$

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The Attenuation caused by Dielectric in the Rectangular waveguide is given by

$$\alpha_g = \frac{\sigma \eta}{2\sqrt{1-(f_c/f)^2}} \quad \text{for TE mode} \longrightarrow (94)$$

$$\alpha_g = \frac{\sigma \eta}{2}\sqrt{1-(f_c/f)^2} \quad \text{for TM mode} \longrightarrow (95)$$

When the Electric and Magnetic intensities propagate through a Lossy waveguide, their magnitudes may be written as

$$|E| = |E_{oz}| e^{-\alpha_g z} \longrightarrow (96)$$

$$|H| = |H_{oz}| e^{-\alpha_g z} \longrightarrow (97)$$

where E_{oz} and H_{oz} are field intensities at $z=0$.

For a low loss waveguide, the time average power flow decreases to $e^{-2\alpha_g z}$

Hence

$$P_{tr} = (P_{tr} + P_{Loss}) e^{-2\alpha_g z} \longrightarrow (98)$$

$$\frac{P_{Loss}}{P_{tr}} + 1 = 1 + 2\alpha_g z \longrightarrow (99)$$

Finally $$\boxed{\alpha_g = \frac{P_L}{2P_{tr}}} \longrightarrow (100)$$

where P_L is Power Loss per unit length

The surface Resistance of the Guide wall is given by

$$\boxed{R_s = \frac{\rho}{\delta} = \frac{1}{\sigma\delta} = \frac{\alpha_g}{\sigma} = \sqrt{\frac{\pi f \mu}{\sigma}}} \;\Omega/\text{square} \longrightarrow (101)$$

where

ρ - Resistivity of the conducting wall in ohms-meter.

σ - conductivity in mhos per meter

δ - skin depth

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POWER LOSS is ~~defined as~~ obtained by integrating the power density over the surface of the conductor

$$P_L = \frac{R_S}{2} \int_S |H_t|^2 ds \longrightarrow (102)$$

where H_t is the tangential component of magnetic intensity at guide walls.

Sub. (87) and (102) in (101), we get

$$\boxed{\alpha_g = \frac{R_S \int_S |H_t|^2 ds}{2 Z_g \int_a |H|^2 da}} \longrightarrow (103)$$

where

$$|H|^2 = |H_z|^2 + |H_y|^2 \longrightarrow (104)$$

$$|H_t|^2 = |H_{tx}|^2 + |H_{ty}|^2 \longrightarrow (105)$$

P2. A waveguide with a cross section of 2×1 cm transports energy in the TE_{10} mode at the rate of 0.5 hp. The impressed frequency is 30 GHz. What is the peak value of the electric field occuring in the waveguide.

SOLUTION:

The field components of the dominant mode TE_{10} can be obtained by substituting m=1 and n=0. then

$$E_x = 0$$

$$E_y = E_{oy} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_g z}$$

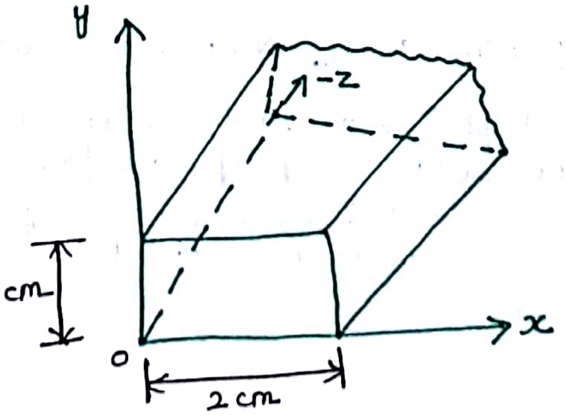
$$E_z = 0$$

$$H_x = \frac{E_{oy}}{Z_g} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_g z}$$

$$H_y = 0$$

$$H_z = H_{oz} \cos\left(\frac{\pi x}{a}\right) e^{-j\beta_g z}$$

where $Z_g = \frac{\omega \mu_0}{\beta_g}$.



The phase constant (βg) can be calculated as follows

$$\beta g = \sqrt{\omega^2 \mu_o \varepsilon_o - \frac{\pi^2}{a^2}} = \pi \sqrt{\frac{(2f)^2}{c^2} - \frac{1}{a^2}}$$

$$\beta g = \pi \sqrt{\frac{4 \times 9 \times 10^{20}}{9 \times 10^{6}} - \frac{1}{4 \times 10^{-4}}}$$

$$\beta g = 193.5\pi$$

$$\boxed{\beta g = 608.81 \text{ rad/m}}$$

The power delivered in the z direction by the waveguide is given by

$$P = Re\left[\frac{1}{2}\int_0^b\int_0^a (E \times H^*)\right] \cdot dx\,dy\,u_z$$

$$P = \frac{1}{2}\int_0^b\int_0^a \left[\left(E_{oy}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta g z}u_y\right)\left(-\frac{\beta g}{\omega\mu_o}E_{oy}\sin\left(\frac{\pi x}{a}\right)e^{j\beta g z}u_x\right)\right] \cdot dx\,dy\,u_z$$

$$P = \frac{1}{2}E_{oy}^2\frac{\beta g}{\omega\mu_o}\int_0^b\int_0^a \left(\sin\left(\frac{\pi x}{a}\right)\right)^2 dx\,dy$$

$$P = \frac{1}{4}E_{oy}^2\frac{\beta g}{\omega\mu_o}ab$$

$$373 = \frac{1}{4}E_{oy}^2 \frac{193.5\pi(10^{-2})(2 \times 10^{-2})}{2\pi(3 \times 10^{10})(4\pi \times 10^{7})}$$

$$\boxed{E_{oy} = 53.87 \text{ kV/m}}$$

The peak value of the Electric Intensity is 53.87 kV/m.

IMPOSSIBILITY OF TEM MODES IN RECTANGULAR WAVEGUIDES :-

* In a parallel plate waveguide, a TEM mode for which both electric and magnetic fields are perpendicular to the direction of propagation. This however, is not true for Rectangular waveguides, because Rectangular waveguide is without an inner conductor.

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* we know that lines of magnetic field (H) are closed loops.
* Since, there is no z component for magnetic field, such loops must lie in the x-y plane.
* However, a loop in the x-y plane, according to Ampere's law, implies an AXIAL CURRENT.
* If there is no inner conductor, there cannot be a real current. The only other possibility is then a DISPLACEMENT CURRENT.
* However, an axial displacement current requires an axial component of the electric field, which is zero for TEM mode. Thus, TEM mode does not exist in a hollow conductor.

P3. A Rectangular waveguide has dimensions 8 cm x 4 cm. Determine all the modes that can propagate when the operating frequency is

a) 1 GHz

b) 3 GHz

c) 8 GHz

SOLN:

The cutoff frequency for a (m, n) mode is given by

$$\frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$= 1.5\times10^{10}\sqrt{\frac{m^2}{64}+\frac{n^2}{16}}.$$

m	n	CUTOFF FREQUENCY
0	1	3.75
0	2	7.5
0	3	11.25
1	0	1.875
1	1	4.192
1	2	7.731
1	3	11.405
2	0	3.75
2	1	5.30
2	2	8.39
3	0	5.625
3	1	6.76
3	2	9.375
4	0	7.5
4	1	8.38
5	0	9.375

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P4. The cut off frequency for a TE_{10} mode in a waveguide is 1.875 GHZ. what would be the cut-off frequency of this mode if the guide were to be filled with a dielectric of permitivity $9\varepsilon_0$?

SOLUTION÷

The cut-off frequency is proportional to $\frac{1}{\sqrt{\mu\varepsilon}}$.

Thus frequency would be reduced by a factor of 3, making it 625 MHZ.

P5. what should be the third dimension of a cavity having a length of 1 cm x 1 cm which can operate in a TE103 mode at 24 GHz?

SOLUTION÷

The operating frequency is given by

$$\omega = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{l\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2+\left(\frac{n\pi}{c}\right)^2\right]^{\frac{1}{2}}$$

sub: $l=1$, $m=0$ & $n=3$ ⇒ we get $d = 2.4$ cm.

P6. Two signals one of frequency 10 GHZ and other of 12 GHZ are simultaneously launched in a rectangular waveguide of dimension 2cm x 1cm. Find the time interval between the arrival of the signals at a distance of 10m from the common place of their launch.

SOLUTION÷

The cut-off Frequency is given by $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = 7.5$ GHZ

$$v_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$f = 10$ GHZ; $v_g = 1.98\times10^8$ m/sec

$f = 12$ GHZ; $v_g = 2.07\times10^8$ m/sec

Thus the difference in speed is 9×10^6 m/s, resulting in a time difference of approximately 10^{-5} sec in travelling 10m.

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CAVITY RESONATORS:

MICROWAVE CAVITIES:

Resonance:

* The increase in amplitude of oscillation of an electric or mechanical system exposed to a periodic force whose frequency is equal or very close to the natural undamped frequency of the system.
* Resonance occurs when the amplitude of an objects oscillations are increased by the matching vibrations of another object.
* In an Electric circuit, the resonance exists, when the Inductive reactance and capacitive reactance are of equal magnitude, causing electrical energy to oscillate between the magnetic field of the inductor and electric field of the capacitor.

Microwave cavity:

* A Microwave cavity or a Radio Frequency cavity is a special type of resonator, consisting of a closed metal structure that confines Electro Magnetic fields in the Microwave region of the spectrum.
* The structure is either Hollow or filled with dielectric material.
* The Microwaves bounce back and forth between the walls of the cavity. At the cavity's resonant frequencies, they reinforce to form standing waves in the cavity.
* These cavities act as a BANDPASS FILTER, allowing Microwaves of a particular frequency to pass while blocking microwaves at nearby frequencies.
* The cavities serve as resonators (TANK CIRCUIT) to determine the frequency of the oscillators.

They are used in

⟹ oscillators and Transmitters to create Microwave signals

⟹ Filters to seperate a signal at a given frequency from other signals.

Cavity Resonator:

* In general, a cavity Resonator is a metallic enclosure that confines the Electromagnetic energy.

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* The stored Electric and Magnetic Energy inside the cavity determine its equivalent Inductance and capacitance.
* The Energy dissipated by the finite conductivity of the walls determine its Equivalent Resistance.

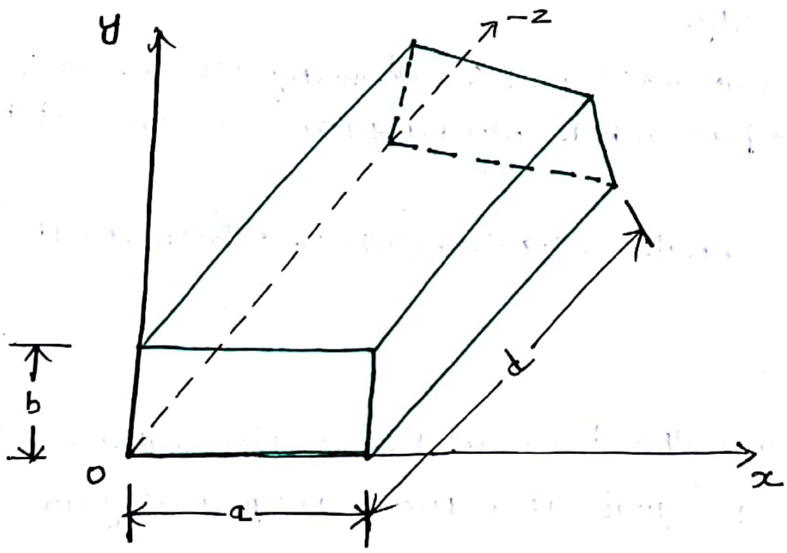
In practice, commonly used Microwave Resonators are

⇒ Rectangular cavity Resonator
⇒ circular cavity resonator
⇒ Reentrant cavity Resonator

* Theoretically, a given Resonator has an infinite number of Resonant modes, and each corresponds to a definite Resonant Frequency.
* When the frequency of an impressed signal is equal to a resonant frequency, a maximum amplitude of the standing wave occurs, and the peak energies stored in electric and magnetic fields are equal.
* The mode with LOWEST RESONANT FREQUENCY is known as the DOMINANT MODE.

RECTANGULAR CAVITY RESONATOR:

CO-ORDINATES OF RECTANGULAR CAVITY



* The Electro Magnetic Field inside the cavity should satisfy Maxwell's Equation: subject to the boundary conditions that the Electric field tangential to the metal and magnetic field normal to the metal walls must vanish.

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* The wave equations in the Rectangular resonator should satisfy the boundary condition of zero tangential Electric Field at four of the walls.

The Harmonic functions can be found if :

$$H_z = H_{oz} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right) \longrightarrow (106)$$

(TE_{mnp})

where

$m = 0, 1, 2, 3 \ldots$ represents half wave periodicity in x direction
$n = 0, 1, 2, 3 \ldots$ represents half wave periodicity in y direction
$p = 0, 1, 2, 3 \ldots$ represents half wave periodicity, in z direction

and

$$E_z = E_{oz} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right) \longrightarrow (107)$$

(TM_{mnp})

where

$m = 1, 2, 3, 4 \ldots$
$n = 1, 2, 3, 4 \ldots$
$p = 0, 1, 2, 3 \ldots$

The seperation Equation for both TE and TM modes are given by

$$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \longrightarrow (108)$$

For a Lossless dielectric, $k^2 = \omega^2 \mu \varepsilon$; therefore the Resonant Frequency is expressed by

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2} \longrightarrow (109)$$

(TE_{mnp}, TM_{mnp})

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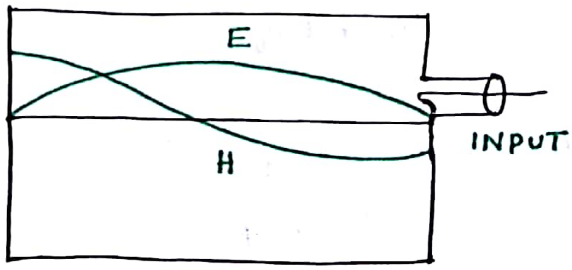
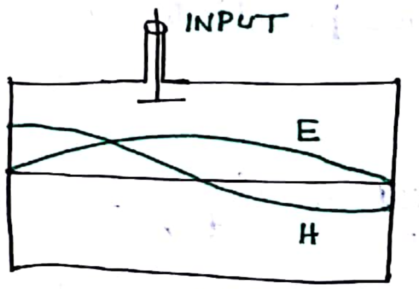
* In general, a straight wire probe inserted at the position of maximum electric intensity is used to excite a desired mode



* The Loop coupling placed at the position of maximum magnetic intensity is utilized to launch a specific mode.



QUALITY FACTOR OF A CAVITY RESONATOR:- (Q)

* The Quality Factor (Q) is a measure of frequency selectivity of a Resonant or Antiresonant circuit.

→ It is defined as

$$Q = 2\pi \frac{\text{maximum energy stored}}{\text{Energy dissipated per cycle}} = \frac{\omega W}{P} \longrightarrow (110)$$

where W is the maximum energy stored
P is the Power loss

* At Resonant frequency, the electric and magnetic energies are equal. When the electric energy is maximum, the magnetic energy is zero and vice versa.

The Total energy stored in the Resonator is obtained by integrating the energy density over the volume of the Resonator

$$W_e = \int_v \frac{\epsilon}{2} |E|^2 dv = W_m = \int_v \frac{\mu}{2} |H|^2 dv = W \longrightarrow (111)$$

where $|E|$ and $|H|$ are peak values of field intensities.

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The Average power Loss in a Resonator is given by

$$P = \frac{R_s}{2}\int_S |H_t|^2\,da \longrightarrow (112)$$

where

H_t is the peak value of the tangential magnetic intensity

R_s is the surface Resistance of the Resonator.

sub: (111) and (112) in (110), we get

$$Q = \frac{\omega\mu\int_v |H|^2 dv}{R_s\int_S |H_t|^2\,da} \longrightarrow (113)$$

since the peak value of the magnetic intensity is related to its tangential and normal components by

$$|H|^2 = |H_t|^2 + |H_n|^2 \longrightarrow (114)$$

where H_n is the peak value of the normal magnetic intensity, the value of $|H_t|^2$ at the resonant walls is approximately twice the value of $|H|^2$ is given by

$$\therefore \boxed{Q = \frac{\omega\mu\,(\text{volume})}{2R_s\,(\text{surface areas})}} \longrightarrow (115)$$

An unloaded Resonator can be represented by either a series or parallel resonant circuit.

The Resonant Frequency and unloaded Q_0 of a cavity Resonator are

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \longrightarrow (116)$$

$$Q_0 = \frac{\omega_0 L}{R} \longrightarrow (117)$$

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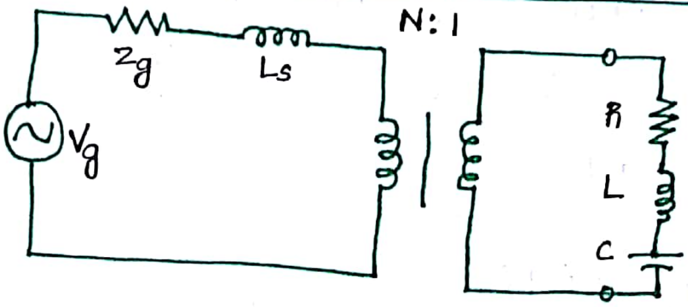
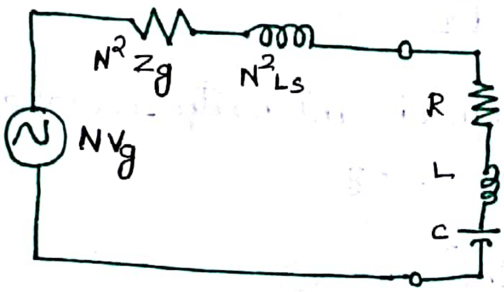
* If the cavity is coupled by means of an ideal N:1 transformer and a series Inductance L_s to a generator having Internal Impedance Z_g, then the coupling circuit and its equivalent is shown as

cavity coupled to a generator: coupling circuit



Equivalent circuit



The Loaded Q_l of the system is given by

$$Q_l = \frac{\omega_0 L}{R + N^2 Z_g} \quad \text{for } |N^2 L_s| << |R + N^2 Z_g| \longrightarrow (118)$$

The COUPLING COEFFICIENT of the system is defined as

$$\boxed{K \equiv \frac{N^2 Z_g}{R}} \longrightarrow (119)$$

∴ Loaded Q_l will become

$$Q_l = \frac{\omega_0 L}{R(1+K)} = \frac{Q_0}{1+K} \longrightarrow (120)$$

Rearrangement of (120) yields

$$\frac{1}{Q_l} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \longrightarrow (121)$$

where $Q_{ext} = \frac{Q_0}{K} = \frac{\omega_0 L}{KR}$ is the external Q. $\longrightarrow$ (122)

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There are 3 types of coupling coefficients

1. critical coupling :

* If the Resonator is matched to the Generator, then $K=1$

⇒ then loaded Q_l is given by

$$Q_l = \frac{1}{2} Q_{ext} = \frac{1}{2} Q_0 \longrightarrow (123)$$

2. Over coupling : $(K>1)$

* The cavity terminals are at a voltage maximum :

* Normalized Impedance at the voltage maximum is the standing wave Ratio (ρ). That is

$$K = \rho \longrightarrow (124)$$

⇒ then loaded Q_l is given by

$$Q_l = \frac{Q_0}{(1+\rho)} \longrightarrow (125)$$

3. under coupling : $(K<1)$

* The cavity terminals are at a voltage minimum

* Input terminal impedance is equal to the reciprocal of standing wave Ratio (ρ). that is

$$K = \frac{1}{\rho} \longrightarrow (126)$$

then loaded Q_l is given by

$$\Rightarrow Q_l = \frac{\rho}{\rho+1} Q_0 \longrightarrow (127)$$

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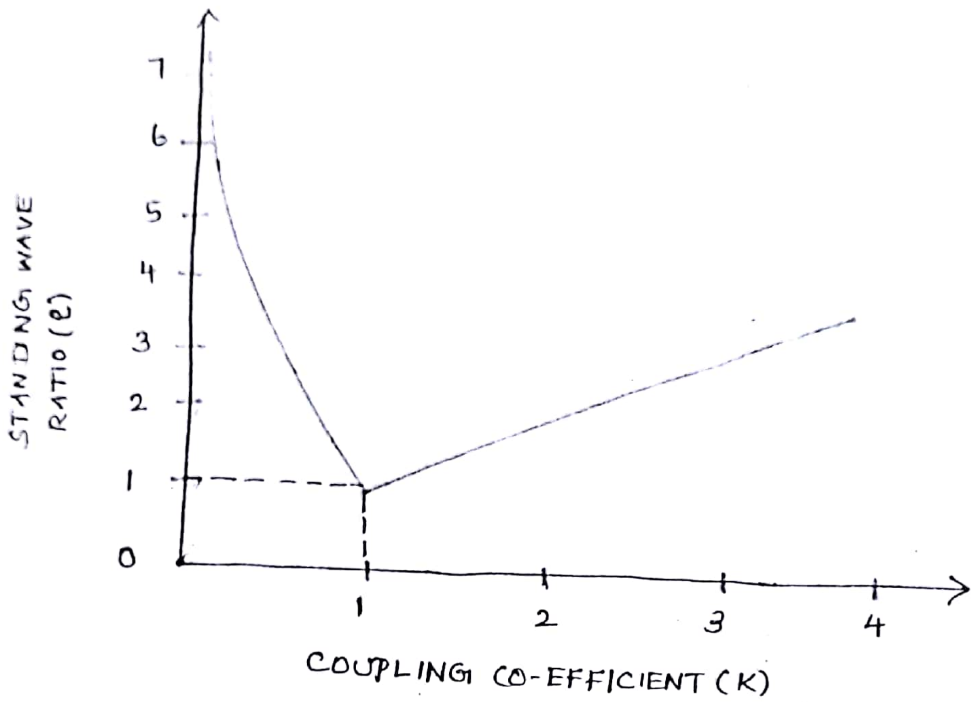
coupling co-efficient vs standing wave Ratio



MICROSTRIP LINES:

Microstrip:

* Microstrip is a type of electrical transmission line which can be fabricated using PCB technology and is used to convey microwave signals.

* It consists of a conducting strip seperated from a Ground plane by a dielectric Layer known as substrate.

* Microwave components such as

→ Antennas
→ couplers
→ Filters
→ power Dividers e.t.c can be formed from microstrip

* Microstrip is much less expensive than traditional waveguide technology, as well as being far lighter and compact

cross-section of MICROSTRIP Geometry

B
A
C
D

A - conductor
B - Dielectric layer (Typically air)
C - Dielectric substrate
D - Ground plane

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) A Rect wg has a dimension of 1.5 × 0.6 cm, $\sigma = 0$, $\mu = \mu_0$ & mag field comp is gn as $\epsilon_r = 4\epsilon_0$

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi}{b}y\right)\sin\left(\pi\times10^{11}t - \beta z\right)\ \text{A/m}.$$

Det a) mode of opn b) cut off freq c) phase vel. d) prop. cons e) wave Imp

soln.

a) $$H_x = \frac{j\omega\epsilon}{h^2} \cdot C \cdot \frac{n\pi}{b}\sin\left(\frac{m\pi}{a}\right)x\cos\frac{n\pi}{b}\cdot y \cdot e^{j\omega t - \gamma z}$$

$m = 1,\ n = 3$ & cons = 2

$\omega = \pi\times10^{11} \Rightarrow 2\pi f = \pi\times10^{11} \Rightarrow f = 0.5\times10^{11}\ Hz$

$\boxed{f = 50\,GHz}$, $\lambda_0 = \frac{c}{f} = \frac{3\times10^{10}}{50\times10^9}$ $\boxed{= 0.6\ cm}$

TE_{13} or TM_{13} mode

b) cut off freq: $f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{1}{2\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{1}{2\sqrt{\mu_0\epsilon_0 4}}\left[\left(\frac{1}{1.5}\right)^2 + \left(\frac{3}{0.6}\right)^2\right]^{1/2}$$

$$f_c = \frac{1}{4\sqrt{\mu_0\epsilon_0}}(3.805) = \frac{0.952}{4\pi\times10^{-7}\times8.854\times10^{-12}} = 285.4\times10^{?}$$

$\boxed{f_c = 0.285\,GHz}$

$$\lambda_c = \frac{c}{f_c} = \frac{3\times10^{10}}{0.28\times10^9} = 107.14\ cm$$

1st unit
→ TE, TM } E_x, E_y, H_z, H_x, H_z

2nd unit
{ Power trans & losses
{ Losses in μ striplines
{ Q factor &
{ C co-eff of cavity resonator

3rd unit
1) Var. attenuator
2) variable Phase shifter

4th unit
1) Magic Tee

$$V_p = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} \cdot \frac{1}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

$$\left(\frac{\lambda_0}{\lambda_c}\right)^2 = \left(\frac{0.6}{107.14}\right)^2 = 0.000081 \Rightarrow \sqrt{1-(\lambda_0/\lambda_c)^2} \simeq 1$$

$$\therefore V_p = \frac{1}{\sqrt{4\pi\times10^{-7}\times8.854\times10^{-12}\times4}} = \frac{2999.79\times10^6}{2} \text{ cm/s} = 149.8\times10^6 \text{ cm/s}$$

$$\beta = \sqrt{\mu\varepsilon(\omega^2-\omega_c^2)} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} = \pi\times10^{11}\sqrt{\mu_0\varepsilon_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\boxed{\beta = 2094.36 \text{ rad/m}}$$

$$\gamma = j\beta = j\,2094.57\,m^{-1}$$

$$Z_{TM} = \eta\sqrt{1-(\lambda_0/\lambda_c)^2}$$

Wave Imp for $TM_{13} = 154.64\,\Omega$

$$Z_{TE} = \frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

&

$TE_{13} = 229.45\,\Omega$

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2) A rectangular W guide is filled by dielectric material of $\varepsilon_r = 9$ and has dimensions of 7×3.5 cm. It operates in dominant TE mode.
(i) cut off freq (ii) phase vel at 2 GHz (iii) λ_g at 2 GHz (iv) Z_{TE} (v) V_g

(i) $\lambda_c = 2a = 14$ cms ; $f_c = \frac{c}{\lambda_c} = \frac{3\times10^{10}}{14} = 0.214\times10^{10} = 2.14$ GHz

(ii) $V_p = \frac{c}{\sqrt{\varepsilon_r-(f_c/f)^2}} \Rightarrow V_p = \frac{3\times10^{10}}{\sqrt{9-\left(\frac{2.14}{2}\right)^2}} \Rightarrow V_p = 10.7\times10^9$ cm/s

(iii) $\lambda_0 = \frac{c}{f} = \frac{3\times10^{10}}{2\times10^9} = 15$ cm $\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_r-(\lambda/\lambda_c)^2}}$

v) $V_g = c\cdot\sqrt{1-(\lambda_0/\lambda_c)^2}$ $\lambda_c = 14$ cm $\left(\frac{\lambda_0}{\lambda_c}\right)^2 = \left(\frac{15}{14}\right)^2 = 1.147$

$$\lambda_g = \frac{15}{\sqrt{9-1.147}} = 4.35 \text{ cm}$$

(iv) $Z_{TE} = \frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}} = \frac{377}{\sqrt{1-\left(\frac{15}{14}\right)^2}}$

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