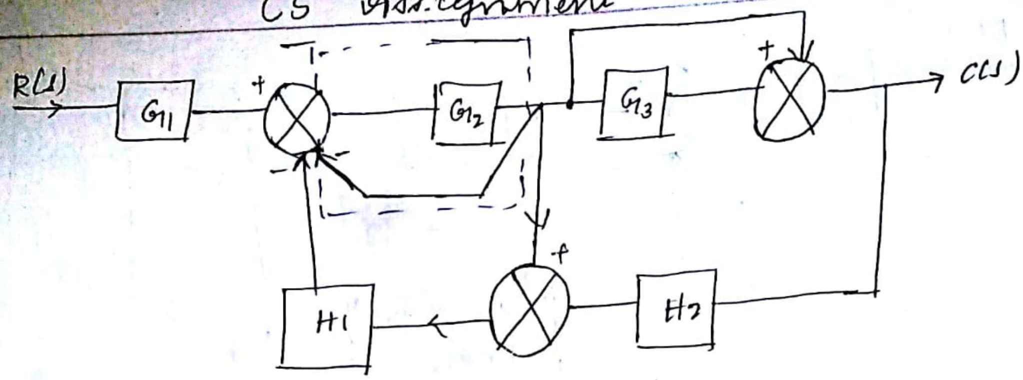
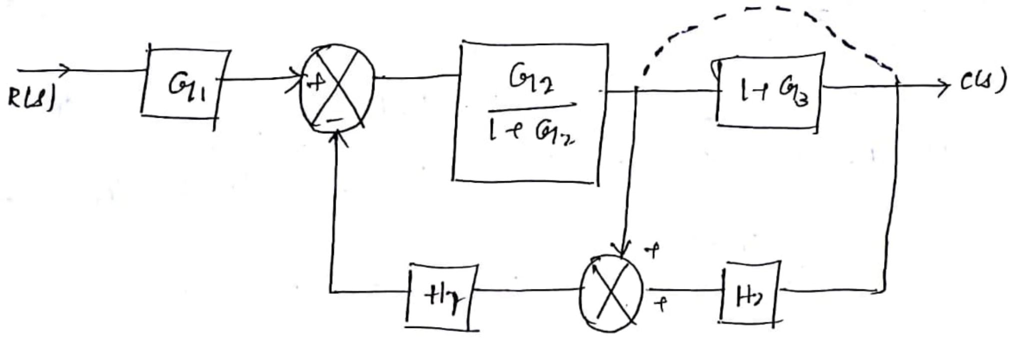
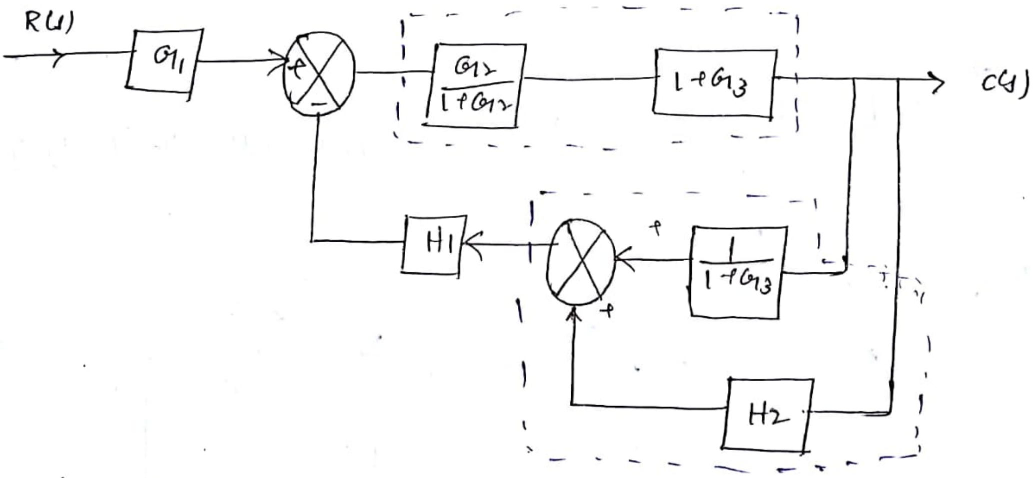
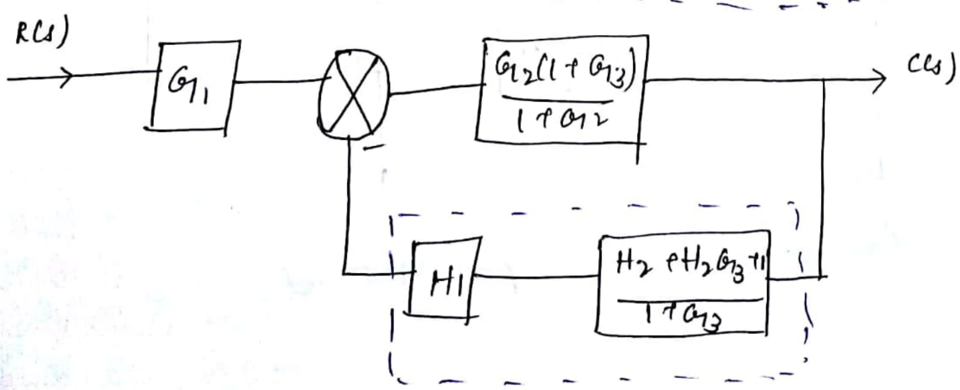
CS Assignment

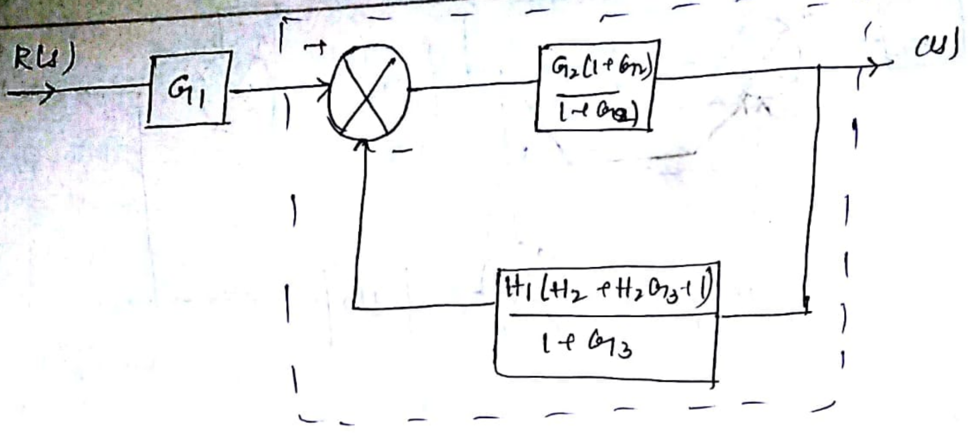
1.









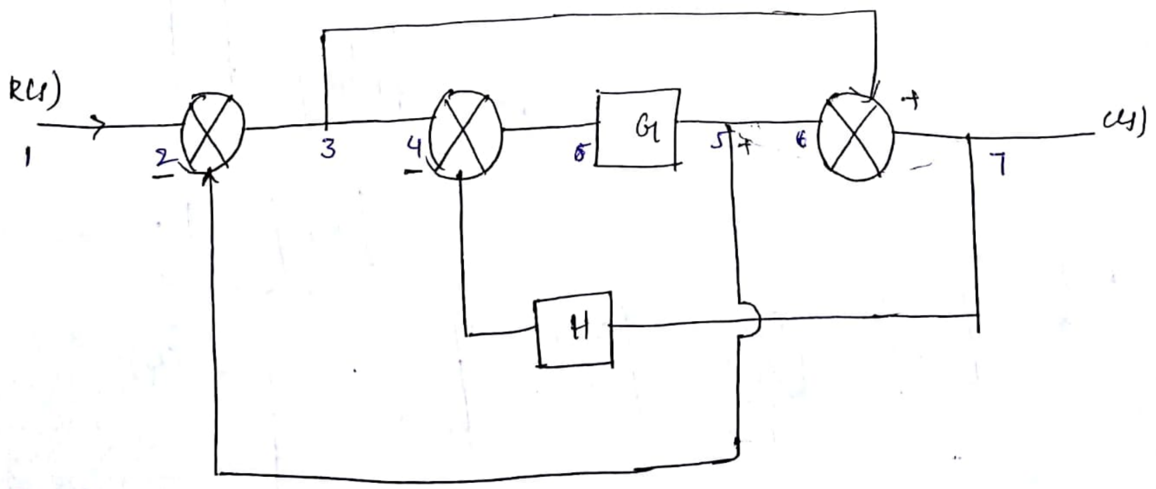


$$\frac{\dfrac{G_2(1+G_3)}{1+G_2}}{\dfrac{1+H_1(H_2+H_2G_3+1)(1+G_3)G_2}{(1+G_2)(1+G_3)}} = \frac{G_2(1+G_3)}{1+G_2}\times\frac{(1+G_2)(1+G_3)}{(1+G_3)[(1+G_2)+H_1(H_2+H_2G_3+1)G_2]}$$

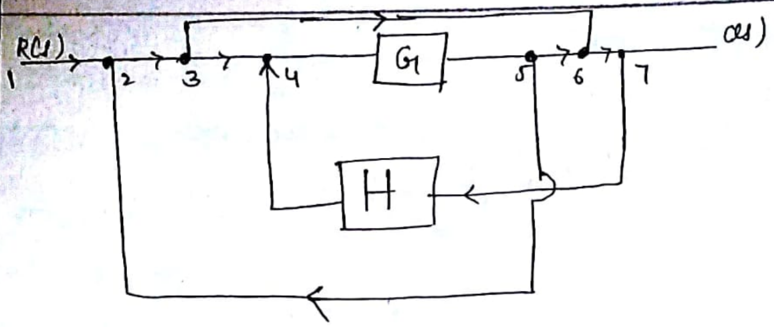
$$= \frac{G_2(1+G_3)}{1+G_2+H_1(H_2+H_2G_3+1)G_2}$$

Transfer function $\frac{C(s)}{R(s)} = \frac{G_1G_2(1+G_3)}{1+G_2+H_1(H_2+H_2G_3+1)G_2}$

2.



Using signal flow graph method



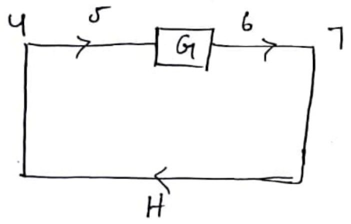
Path-1 : 

$= P_1 = G$

Path-2

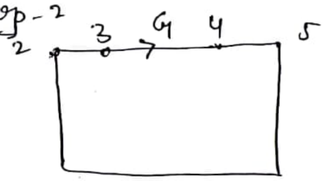
$= P_2 = 1$

Loop-1



$L_1 = -GH$

Loop-2



$L_2 = -G$

$$\text{Transfer function} = \frac{1}{\Delta} \sum_{k=1}^{n} P_k \Delta_k$$

$\Delta_1 = 1, \; \Delta_2 = 1$

$$= \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\Delta = 1 - [GH - G] = 1 + GH + G.$$

$$T = \frac{G + 1}{1 + GH + G}$$

5. Properties of signal flow graph

(i) The algebraic equns which are used to construct signal flow graph (SFG) must be in the form of cause & effect relationship.

(ii) SFG is applicable to linear systems only.

(iii) A node in the SFG represents the variable (s) signal.

(iv) A node adds the signals of all incoming branches & transmits the sum to all outgoing branches.

(v) A mixed node which has both incoming & outgoing signals can be treated as an o/p node by adding an outgoing branch of unity transmittance.

(vi) A branch indicates functional dependence of one signal on the other.

(vii) The signals travel along branches only in the marked direction & when it travels it gets multiplied by gain of the branch.

6.

Block diagram reduction	Signal flow graph
1. Primary importance is given to elements & T.F.	1. Primary importance is given to variables in the system.
2. Each unit in the system is represented by block	2. Each unit is represented by a line segment.
3. The summing points & take-off points are seperately represented	3. The summing point & the take-off points have same representation.
4. T.F cannot be solved easily	4. Using Mason's Gain formula the T.F can be easily solved.
5. Self loop doesn't exist.	5. In SFG there is a possibility of self loops.

6. It is applicable to LTI systems only.	6. It is applicable to LTI systems

8. Response of second order system for underdamped case of step i/p.

The closed loop T.F of second order system is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad -(1)$$

for under damped system $0 < \xi < 1$

The char eqn is $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

On solving, we get

$$s = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

The roots of the char eqn are complex conjugate

$$s = -\xi\omega_n \pm \omega_n\sqrt{(-1)(1-\xi^2)}$$

$$= -\xi\omega_n \pm j\omega_n\sqrt{(1-\xi^2)} = -\xi\omega_n \pm j\omega_d$$

from eqn - (1)

$$C(s) = R(s)\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

for unit step i/p $u(t) = 1 \Rightarrow R(s) = \frac{1}{s}$.

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$= \frac{A}{s} + \frac{Bs + C}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad -(2)$$

$$\omega_n^2 = A(s^2 + 2\xi\omega_n s + \omega_n^2) + Bs + C\,(s) \quad -(3)$$

$$A = sC(s)\Big|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

put A in - (3) we get

$$\omega_n^2 = s^2 + 2\xi\omega_n s + \omega_n^2 + (Bs + C)s$$

equating coeff of $s^2 \Rightarrow 0 = 1 + B$

$$B = -1$$

equating coeff of $s \Rightarrow 0 = 2\xi\omega_n + C$

$$C = -2\xi\omega_n$$

$$c(s) = \frac{1}{s} - \frac{(s + 2\xi\omega_n)}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{(s + 2\xi\omega_n)}{s^2 + 2\xi\omega_n s + \omega_n^2 + \xi^2\omega_n^2 - \xi^2\omega_n^2}$$

$$= \frac{1}{s} - \frac{(s + \xi\omega_n)}{(s + \xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2}$$

$$= \frac{1}{s} - \frac{(s + \xi\omega_n)}{(s + \xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n}{\omega_d} \cdot \frac{\omega_d}{(s + \xi\omega_n)^2 + \omega_d^2}$$

Taking inverse laplace transform

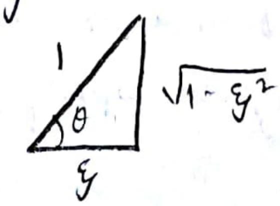
$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t}\sin\omega_d t$$

$$= 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}\sin\omega_d t\right]$$

$$= 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t\right]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\omega_d t\,\xi + \cos\omega_d t\sqrt{1-\xi^2}\right]$$

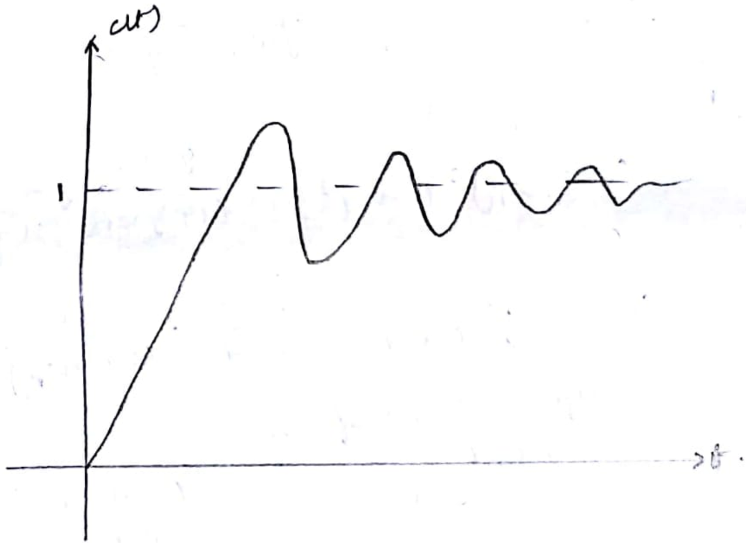


$\sin\theta = \sqrt{1-\xi^2}$, $\cos\theta = \xi$

$$\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} [\sin\omega_d t \cos\theta + \cos\omega_d t . \sin\theta]$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$



8. Time domain specifications

The desired performance characteristics of c.s are specified in terms of time domain specifications. Systems with energy storage elements cannot respond simultaneously & will exhibit transient responses, whenever they are subjected to i/ps.

The desired performance characteristics of a sm in any order may be specified in terms of the transient response to a unit step i/p signal.

The transient response of a system to a unit step i/p depends on initial conditions. The most practical standard is to start with system at rest i.e. output & all time derivatives before t = 0 will be zero. The transient response of a practical c.s often exhibits damped oscillation before reaching steady state.

The following are the time domain specifications :-

1. Delay time (td)
2. Rise time (tr)
3. Peak time (tp)
4. Maximum overshoot (Mp)

5. Settling time (t_s)

1. Delay time (t_d) :- It is the time taken for response to reach 50% of the final value, for first time

2. Rise time (t_r) :- It is the time taken for response to raise from 0 to 100% for the very first time. For underdamped system, the rise time is calculated from 0 to 100%. But for overdamped system it is the time taken by the response to raise from 10% to 90%.

3. Peak time (t_p) :- It is the time taken for the response to reach the peak value the very first time.

4. Peak overshoot (M_p) :- It is defined as the ratio of the maximum peak value to the final value, where the max peak value is measured from final value

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)}$$

5. Settling time :- It is defined as the time taken by the response to reach & stay within a specified error.

9. Advantages of R-H Criteria