22/1/18

UNIT-III

ROOT LOCUS.

Stability of Analysis of cs

→ Absolute stable system
→ Relative " "
→ Conditionally " "
→ Asymptotically " "

Stability: If there is small change in i/p conditions;(or) system parameter (or) initial conditions; then the o/p ~~cannot~~ doesnot change in the system.

(or)

→ A system is said to be stable; iff

(i) for bounded (finite) i/p and.

(ii) system produces bounded o/p.

⇒ BIBO.

(OR)

→ Under the absence of i/p; o/p should be zero

Absolute stable system: A system is absolutely stable for all the parameters, over the entire range, The system o/p is constant.

Conditionally stable system: For a particular parameter; & for a particular range; the system o/p is constant then it is conditionally stable system.

Relative stability: A system is said to be stable in accordance with another system as a relative parameter (settling time), the system o/p is measured, then it is called Relative stability.

Asymptotically stable: A system is said to be asymptotically stable only when its o/p should tend to zero under the absence of i/p (or) zero i/p.

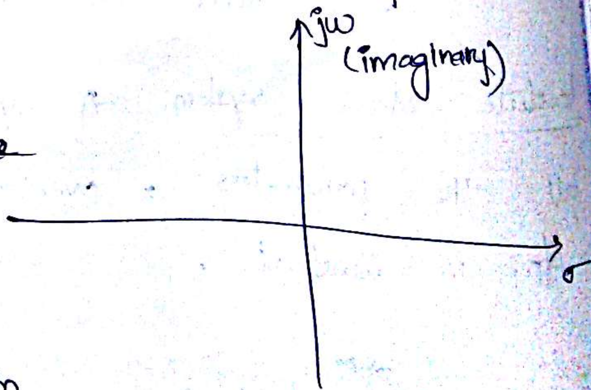
Marginally stable system: If a system is neither absolutely stable nor unstable system; it is Marginally stable system.

Concept of location of poles & stability:

→ If all the poles are on left half of s-plane; then system is Absolutely stable

→ If non-repeated poles are on Imaginary axis; then Marginally stable.

→ If repeated poles are on Imaginary axis; then, Unstable System.



→ If any one pole is on right half of S-plane; System is Unstable.

→ If repeated poles are at origin; ~~then~~ conditionally stable.

→ ~~If only~~

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Mathematical expression

$C(s) = \frac{1}{s+a}$

$C(t) = e^{-at}$

$C(s) = \frac{1}{s-}$

$C(t) = e$

$C(s) = \frac{1}{s+(a-j\beta)}$

$C(t) = e^{-at}$

$C(s) = \frac{1}{s+j\beta}$

$= 2\cos$

$C(s) =$

$C(t) =$

$C(s) =$

$C(t)$

$C(s) =$

$C($

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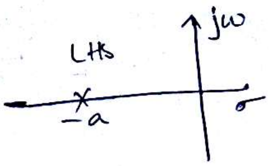
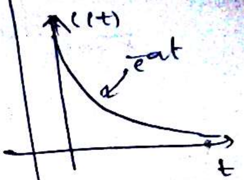
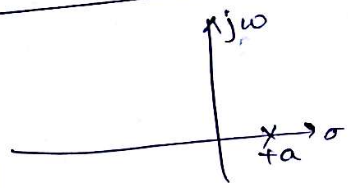
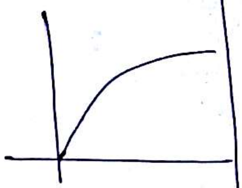
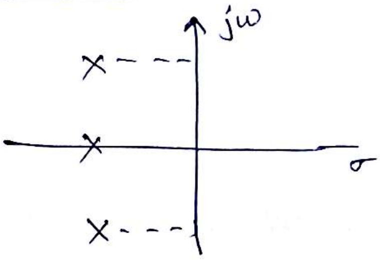
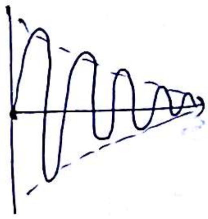
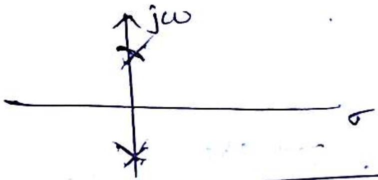
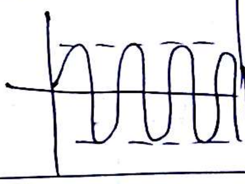
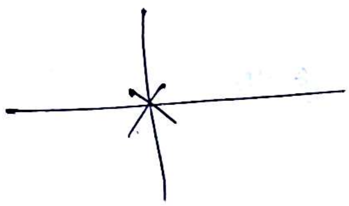
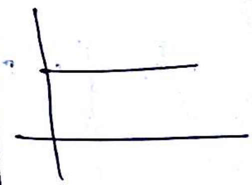
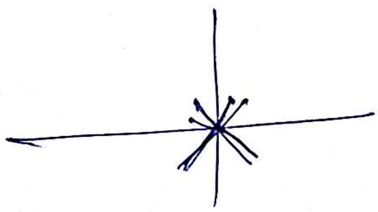
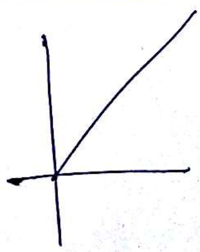
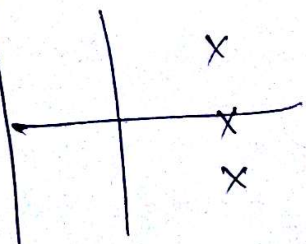
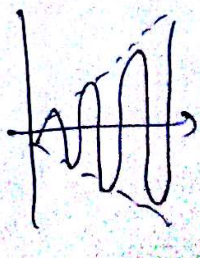
Mathematical expression	location of poles on s-plane	Step Response	Stability
$C(s) = \frac{1}{s+a}$ $c(t) = e^{-at}$			Stable
$C(s) = \frac{1}{s-a}$ $c(t) = e^{at}$			Unstable
$C(s) = \frac{1}{s+(a-jb)} + \frac{1}{s+(a+jb)}$ $c(t) = e^{-at}\cos(bt)$			stable
$C(s) = \frac{1}{s+jb} + \frac{1}{s+(-jb)}$ $= 2\cos bt$			Marginally stable.
$C(s) = \frac{A}{s}$ $c(t) = u(t)$			stable.
$C(s) = \frac{A}{s^2}$ $c(t) = t\,u(t)$			unstable
$C(s) = \frac{1}{s-(a+jb)} + \frac{1}{s-(a-jb)}$ $c(t) = 2e^{at}\cos bt$			Unstable.

(1) Determine stability of the System with T.F.

$$C\dot{E} = s^6 + 2s^5 + 2s^4 + 3s^3 + 5s^2 + 6s + 1$$

~~Rotth~~ by Routh Hurwitz Table.

s^6	1	2	5	1
s^5	2	3	6	
s^4				
s^3				
s^2				
s^1				
s^0				

$$b_1 = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \frac{3\times1 - 2\times2}{2} = -0.5$$

$$b_2 = \begin{bmatrix} 1 & 5 \\ 2 & 6 \end{bmatrix} = \frac{6-10}{2} = -2$$

30/1/18 R-H Criteria

$$C\cdot E = s^{n-1}a_{n-1} + s^{n-2}a_{n-2} + s^{n-3}a_{n-3} + \cdots + a_1 s + a_0$$

$$\begin{array}{c|ccccc}
s^{n-1} & a_{n-1} & a_{n-3} & - - - - & a_1 \\
s^{n-2} & a_{n-2} & a_{n-4} & - - - & a_0 \\
s^{n-3} & b_{n-1} & b_{n-2} & - - - & b_0 \\
s^{n-4} & c_{n-1} & c_{n-2} & - - - - & c_0
\end{array}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \frac{bc-ad}{c}$$

If no changes; Roots are on left half of s-plane

If any changes; " " " right " " "

Ex; $s^5+4s^4+8s^3+8s^2+7s+4=0$

$$\begin{array}{c|ccc}
s^5 & 1 & 8 & 7 \\
s^4 & 4 & 8 & 4 \\
s^3 & 6 & 6 & \\
s^2 & 4 & 4 & \\
s^1 & 2 & 2 & \\
s^0 & 4 & 0 &
\end{array}$$

LHS.

$$c_1 = \begin{vmatrix} 1 & 8 \\ 4 & 8 \end{vmatrix} = \frac{32-8}{4} = 6$$

$$C_2 = \begin{vmatrix} 1 & 7 \\ 4 & 4 \end{vmatrix} = \frac{28-4}{4} = \frac{24}{4} = 6$$

Q: $s^6+2s^5+8s^4+12s^3+20s^2+16s+16$

$s^6+2s^5+8s^4+4s^3+20s^2+16s+16=0$

s^6	1	8	20	16
s^5	2	12	16	0
s^4	$2=b_1$	18	16	
s^3	0	0	0	
s^2				
s^1				
s^0				

$b_1=\begin{vmatrix}1 & 8\\ 2 & 12\end{vmatrix}=\frac{16-12}{2}=2$

$b_2=\begin{vmatrix}1 & 20\\ 2 & 16\end{vmatrix}=\frac{2\times20-16\times1}{2}=12$

$b_3=\begin{vmatrix}1 & 16\\ 2 & 0\end{vmatrix}=16$

Case (i):

when all the elements in one row of Routh array is zero; (when a row of zeros appear in Routh array), then Ch. eqn is analysed using following steps.

Steps:

(i) Find the auxilary eqn $A(s)=0$ using the coefficients from the row just above the row of zeros.

(ii) Differentiate A.E w.r.to 's' to get $\frac{d}{ds}A(s)$

(iii) Replace row of zeros with the coefficients of $\frac{d}{ds}A(s)$

(iv) Continue the construction of array in the usual manner for remaining rows

(v) Interpret the sign changes in the first column of Routh array.

(vi) Obtain the roots of AE if there are roots on imaginary axis and no other sign changes in other rows, then system is Marginally / limited / conditional stable system.

If the roots of A.E has a root on RHS of S-plane, system is Unstable.

No repeated root on imaginary axis, two roots are located on left plane of s-plane

$\Rightarrow$ System is Marginally stable.

$$A(s) = s^4 + 6s^2 + 8$$

$$\frac{d}{ds} A(s) = 0 \Rightarrow 4s^3 + 12s = 0$$

$$s^3 + 3s = 0$$

Put $s^2 = x$ in $A(s)$

$$x^2 + 6x + 8 = 0$$

$$x = -2, -4$$

$$s^2 = -2, \quad s^2 = -4$$

$$s = \pm\sqrt{2}j, \quad \pm 2j$$

$\Rightarrow$ System is Marginally stable.

Q: $s^4 + s^3 + 5s^2 + 4s + 4 = 0$

s^4	1	5	4
s^3	1	4	0
s^2	1	4	
s^1	0	0	
s^0	[illegible]		

$s^2 + 4 = 0 \Rightarrow \frac{d}{ds} A(s) = 0$

$\Rightarrow 2s = 0$

$s = 0$

$s = \pm 2j$

$\therefore$ System is Conditionally stable.

Q: $s^6 + 2s^5 + 8s^4 + 15s^3 + 20s^2 + 16s + 16 = 0$.

Q: $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$

s^5	1	8	7
s^4	4	8	4
s^3	6	6	0
s^2	4	4	
s^1	(0) 8	0	
s^0	4		

$b_1 = \frac{32-8}{4} = 6$

∴ System is Marginally stable system.

$b_2 = \frac{28-4}{4} = 6$

$c_1 = \frac{48-24}{6} = 4$

$c_2 = \frac{24-0}{6} = 4$

$c_1 = \frac{48-24}{6} = 4$

$A(s) = 4s^2 + 4 = 0$

$s^2 + 1 = 0$

$s = \pm j$

$c_2 = \frac{24-0}{6} = 4$

$4s^2 + 4 = 0$

$\Rightarrow 8s = 0 \Rightarrow s = 0.$

Q: $s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3 = 0$.

Case (ii)

(i) the first element of a row is zero (since division by '0' occurs in next row) we simply replace the zero by ~~value~~ very small +ve integer 'ε'. (ii) The value of ε is ~~allocated~~ allowed to approach zero than we have to examine the sign of the entries in the first column.

Q: $s^6 + 4s^5 + 3s^4 - 16s^2 - 64s - 48 = 0$.

+	s^6	1	3	-16	-48
+	s^5	4	0	-64	0
+	s^4	3	0	-48	
+	s^3	(0) 12	0		
+	s^2	(0) ε	-48		
+	s^1	$\frac{12\times48}{\varepsilon}$	0		
−	s^0	-48			

$$e_2 = \frac{3(-64) - 4(-48)}{3} = 0$$

$$A(s) = 3s^4 - 48 = 0$$

$$s^4 - 16 = 0$$

$$s = \pm 2, \pm 2j$$

RHS → S − 1 +ve

2 Im

LHS → 3 − ve.

∴ System is Unstable.

Q: For a unity f/b system having forward T.F. $G(s) = \frac{k}{s(1+0.6s)(1+0.4s)}$. Determine the range of values of 'K', Marginal value of k, freq. of sustained oscillations:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$CE \Rightarrow 1 + G(s)H(s) = 0$$

$$1 + \frac{k}{s(1+0.6s)(1+0.4s)} = 0$$

$$0.24 s^3 + s^2 + s + k = 0$$

s^3	0.24	1
s^2	1	k
s^1	$1-0.24k$	0
s^0	k	

$$1-0.24k > 0$$

$$1-0.24k = 0$$

$$k = \frac{1}{0.24}$$

$$k = 4.16$$

$$0 < k < 4.16$$

$$s^2 + k = 0$$

$$s^2 = -k$$

$$s = \pm j2.04$$

$$\therefore \omega = \pm j\, 2.04 \text{ rad/s}$$

Q: UFS having open loop TF given by

$$G(s) = \frac{K}{(s+2)(s+2)(s^2+6s+25)}$$

. By Applying, RH criteria, Discuss the stability of closed loop T.F. as function of K. Determine the values of K which will cause sustained oscillations. in closed loop system. what are the corresponding oscillating frequency?

$$G(s) = \frac{K}{(s+2)(s+4)(s^2+6s+25)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$1+G(s)H(s)=0$$

$$1+\frac{K}{(s+2)(s+4)(s^2+6s+25)}=0$$

$$s^4+12s^3+69s^2+228s+200+k=0$$

s^4	1	69	200+k
s^3	12	228	0
s^2	52.5	200+k	
s^1	$\frac{7995-12K}{52.5}$	0	
s^0	200+K		

$$7995-12k=0$$

$$k=666.25$$

$$0<K<666.25$$

$$52.5\, s^2+200+k=0$$

$$52.5\, s^2+866.25=0$$

$$s^2=-16.5$$

$$s_1=+j4.06$$

$$s_2=-j4.06.$$

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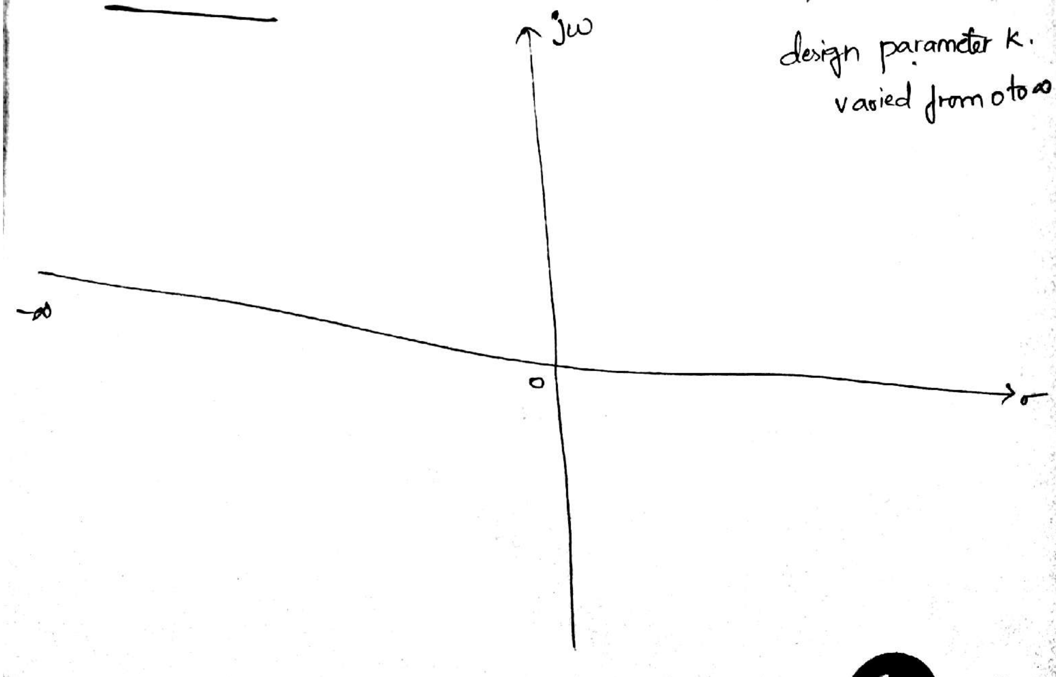
III. Stability Analysis in S-domain

Root locus.

design parameter K.
varied from 0 to ∞



system paramter gain - k.

→ Root locus: For adjusting the location of closed loop poles to achieve the desired system performance by varying one (or) more system parameters.

⇒ As system parameter is varied from 0 to ∞; Root locus is a powerful method of analysis & design for stability & transient response of the system. It gives information about absolute stability as well as Relative stability of a system. It clearly shows the range of stability; range of instability & conditions that cause a system to break into oscillations.

** Rules for construction of root locus:

1. Symmetry
2. No. of branches
3. Starting & ending points
4. locus on Real axis
5. Asymptotes as $S \to \infty$
6. Break away | Break in points
7. Angle of Departure | Angle of Arrival
8. locus crosses on imaginary axis
9. Given gain 'K', determine location of poles.
10. Given pole location, determine value of K.
11. Centroid.

1. Symmetry: Root locus is symmetrical about Real axis

2. No. of branches: The no. of Root locus branches is equal to (or) No. of branches = no. of poles of O.L.T.F

3. Starting & ending points: The origin of Root locus is at a O.L pole & ends at O.L zero

3. ~~Starting & ending points~~

~~Segment~~

4. Locus on Real Axis:

segment of Real axis having odd no. of open loop poles (or) zeros to their right part of the root locus.

5. Asymptotes: The 'n-m' root locus branches that tends to '∞' along a straight line asymptotes making angles with the real axis is given by;

(n - no. of poles
m - no. of zeros)

$$\phi_A = \frac{\pm(2q+1)}{n-m} \times 180^\circ \quad ; q = 0, 1, \ldots, (n-m)-1$$

n - no. of poles
m - no. of zeros.

6. Centroid: The pt. of intersection of the asymptotes with the real axis is centroid;

$$\text{Centroid} = \sigma_A = \frac{R.P - R.P.Z}{n-m}$$

$$\text{i.e.,}\quad \sigma_A = \frac{\text{Sum of real part of poles} - \text{Sum of real part of zeros}}{n-m}$$

7. Break away & Break In pt. (BWP / BIP):

These are determined from the roots of eqn

$$\frac{dk}{ds} = 0$$

8. **Angle of Departure (ADP):**

It is determined ~~to~~ from a complex open loop pole is given by;

$$\phi_p = \pm 180^\circ (2q+1) + \phi \;;\; q = 0, 1, \ldots$$

ϕ = net angle contribution at the pole by all other open loop poles & zeros.

9. **Angle of Arrival:** (AI)

At complex open loop zero is calculated

$$\phi_z = \pm 180^\circ (2q+1) + \phi \;;\; q = 0, 1, \ldots$$

ϕ = net angle contribution at the zero by all other open loop poles & zeros.

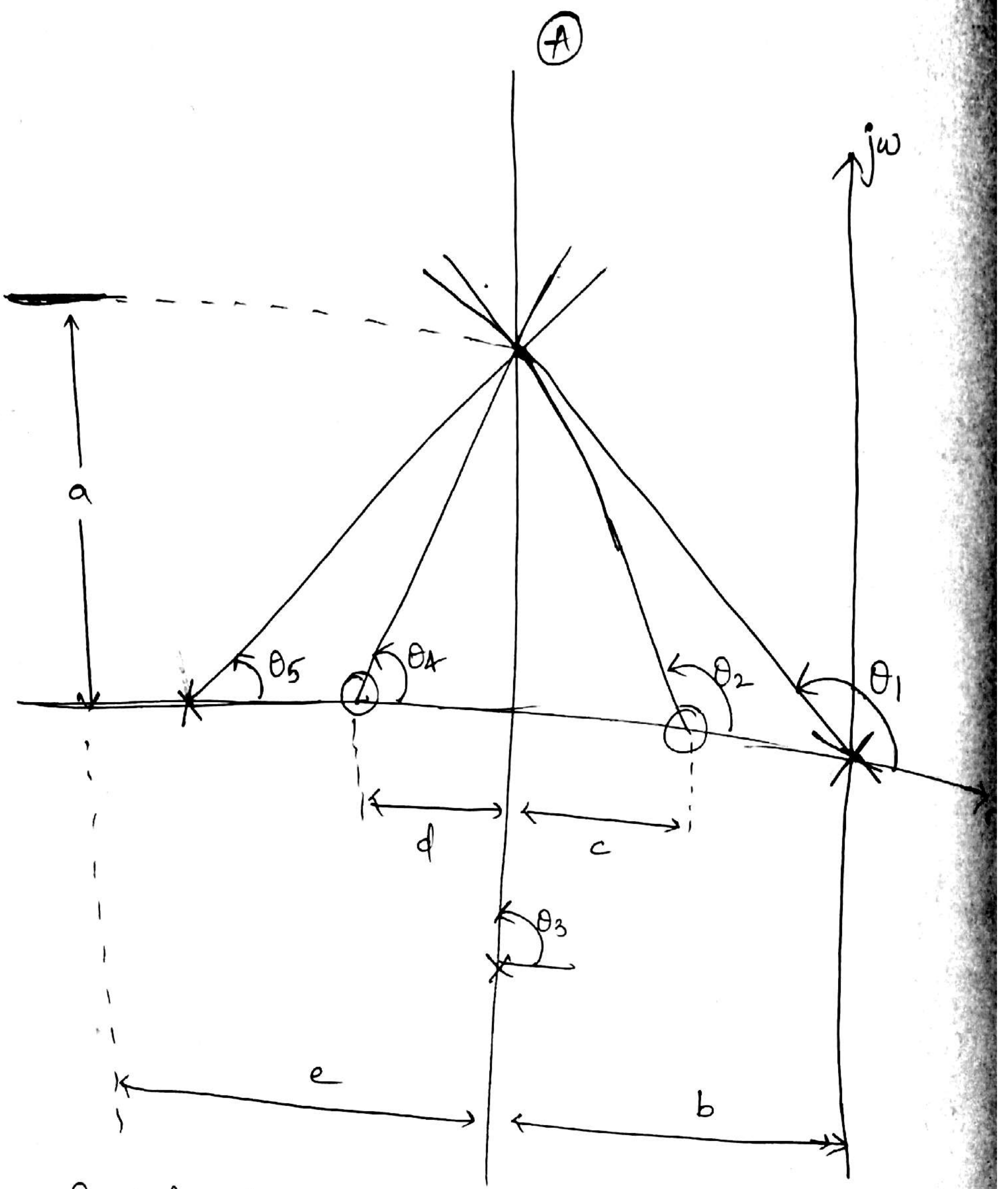
10. **Intersection of Root locus branches with the imaginary axis** can be determined by the use of Routh Criteria (or) by letting $s = j\omega$ in the characteristic eqn & equating real part & imaginary part to zero to solve 'ω' & 'K'.

The value of 'ω' is intersection pt. on the imaginary axis ; 'K' is the value of the gain at the intersection point.

11. Open loop gain 'K' at any pt. $s = s_a$ on the Root locus is given by ;

$$K = \frac{\prod_{i=1}^{n} |s_a + p_i|}{\prod_{i=1}^{m} |s_a + z_i|}$$

$$= \frac{\text{product of vector lengths from open poles to the point } S_a.}{\text{product of vector lengths from open loop zeros to the point } S_a.}$$



$\theta_1 = 180^\circ - \tan^{-1}\frac{a}{b}$

$\theta_2 = 180^\circ - \tan^{-1}\frac{a}{c}$

$\theta_3 = 90^\circ$

$\theta_4 = \tan^{-1}\frac{a}{d}$

$\theta_5 = \tan^{-1}\frac{a}{e}$

Angle of Departure (ADP) $= 180^\circ - (\theta_1+\theta_3+\theta_5) + (\theta_2+\theta_4)$

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Q: Sketch the Root locus of the given T.F.

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+3)}$$

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+3)} \rightarrow \text{O.L.T.F.}$$

C.L.T.F → 1. poles; $n=0, n=-1, n=-3$

zeros; $m=0$

$S=0,\ S=-1,\ S=-3$

2. ~~Real Axis represented~~ ~~on~~ Root locus is;

from $S=0$ to -1 & $S=-3$ to $-\infty$.

∵ from pt., -1; only one pole+zero (odd no)

& from pt., -3; ~~and three of~~ two poles&zeros (even).

∴ No Root locus

& from pt; $-\infty$; three poles & zeros (odd no.)

∴ Root locus exists.

3. Asymptotes $= \dfrac{(2q+1)}{n-m} \times \pm 180°$; $q = 0, 1, 2, \dots (n-m)-1$

$q=0$, $\Rightarrow \dfrac{2\times 0+1}{3-0} \times \pm 180° = \dfrac{1}{3} \times 180° = \pm 60°$

$q=1 \Rightarrow \dfrac{2\times 1+1}{3-0} \times 180° = \dfrac{3}{3} \times 180° = \pm 180°$

$q=2 \Rightarrow \dfrac{2\times 2+1}{3-0} \times 180° = \dfrac{5}{3} \times 180° = \pm 300°$

Step-1: The ~~no. of~~ poles; $n = 0, -1, -3$ & zeros; $m=0$. are from given eqn.

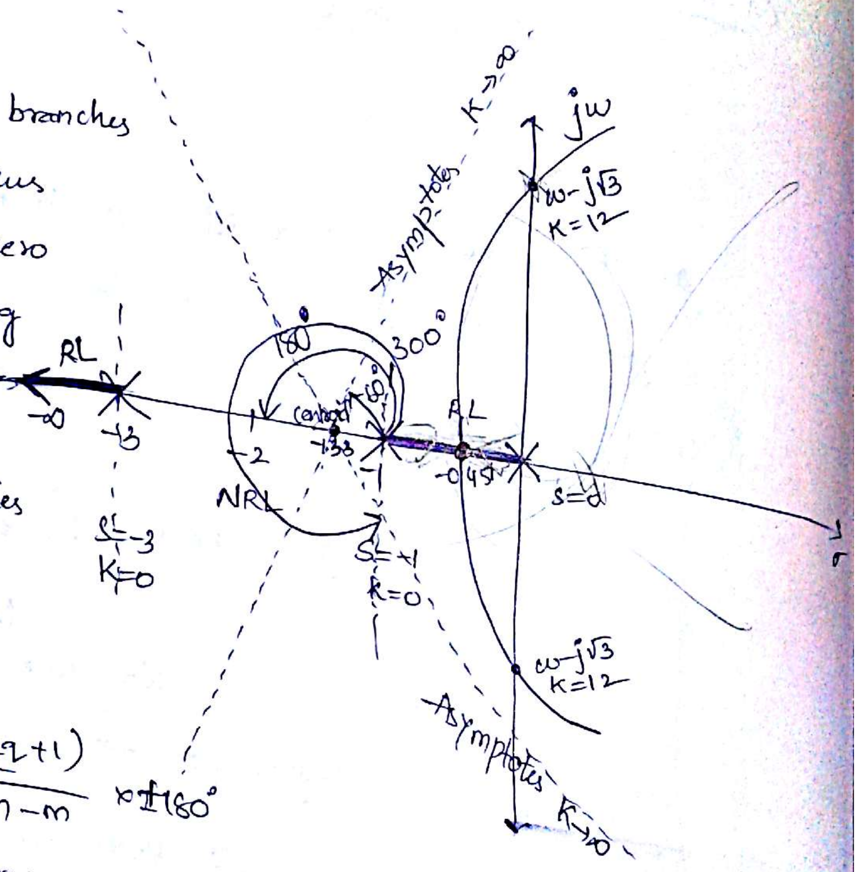
Step-2: There are 3 poles, ⇒ 3 branches exists. Th
3 branches start at $s=0, s=-1, s=-3$; where
$K=0$ & terminate open loop zeros at ∞ where $K=\infty$.

Step-3: Root segment on Real axis ; Root locus exists
on Real axis b/w

$$s=0 \text{ to } s=-1$$

and from $s=-3$ to $s=-\infty$.

Step-4: The 3 branches of Root locus go to ~~the~~ zero at '∞' along st. lines, Asymptotes making angles of



Angle of Asymptotes $= \dfrac{(2q+1)}{n-m} \times \pm 180^\circ$

where; $q = 0, 1, 2, \ldots, (n-m)-1$

$q=0 \Rightarrow \pm 60^\circ$; $q=1 \Rightarrow \pm 180^\circ$; $q=2 \Rightarrow \pm 300^\circ$.

Step-5: The pt. of intersection of Asymptotes on the Real Axis is Called Centroid.

$$\text{Centroid}(\sigma) = \frac{\Sigma \text{R.P of poles} - \Sigma \text{R.P of zeros}}{n-m}$$

$$= \frac{0-1-3-0}{3-0} = \frac{-4}{3} = -1.33$$

Now, locate Centroid on Real axis.

From the centroid; draw asymptotes for each angle (i.e., for 60°, 180°, 300°.)

~~Step-6:~~
Step-6:
~~Here,~~ Root locus is between two poles;

⇒ Break-Away point.

Break away Point
↳ BWP ⇒ $\frac{dK}{ds} = 0$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{k}{s(s+1)(s+3)} = 0$$

$$K + s(s+1)(s+3) = 0$$

$$K = -(s^3 + 4s^2 + 3s)$$

Now; $\frac{dK}{ds} = -(3s^2 + 8s + 3)$

$$\Rightarrow \frac{dK}{ds} = 0$$

$$\Rightarrow -(3s^2 + 8s + 3) = 0$$

$$3s^2 + 8s + 3 = 0$$

$$\cancel{3s^2 + 9s - s + 3 = 0}$$

$$\cancel{3s(s+3) - 1(s+3) = 0}$$

$$\cancel{s = 1/3,\ s = -3.}$$

$$\Rightarrow s_1 = -0.451,\ s_2 = -2.28.$$

→ −0.451 is located on Root locus;

Now, draw an arc line which ~~touches~~ follows Asymptotes

Step-7: → using R-H criteria; 'ω' → pt. that; Curves touches imaginary axis is Calculated as;

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$s^3+4s^2+3s+K=0$

R-H Criteria

s^3	1	3
s^2	4	K
s^1	$\frac{12-K}{4}$	0
s^0	K	

→ Breakaway from the ... enter the Complex plane, then one branch moves to ... along 60° asymptotes; and ... ∞ along 300° asymptotes. The branch which represent complex roots are known as Complex ... root branch

$K=12 \quad \left(\because \left(\frac{12-K}{4}\right)s=0\right)$

$4s^2+K=0$

$s=\sqrt{-3}=\pm j\sqrt{3}$

2. Find value of 'K' from $G(s)H(s)=\frac{K}{s(s+2)(s+4)}$ so that, damping ratio of closed loop system is 0.5.

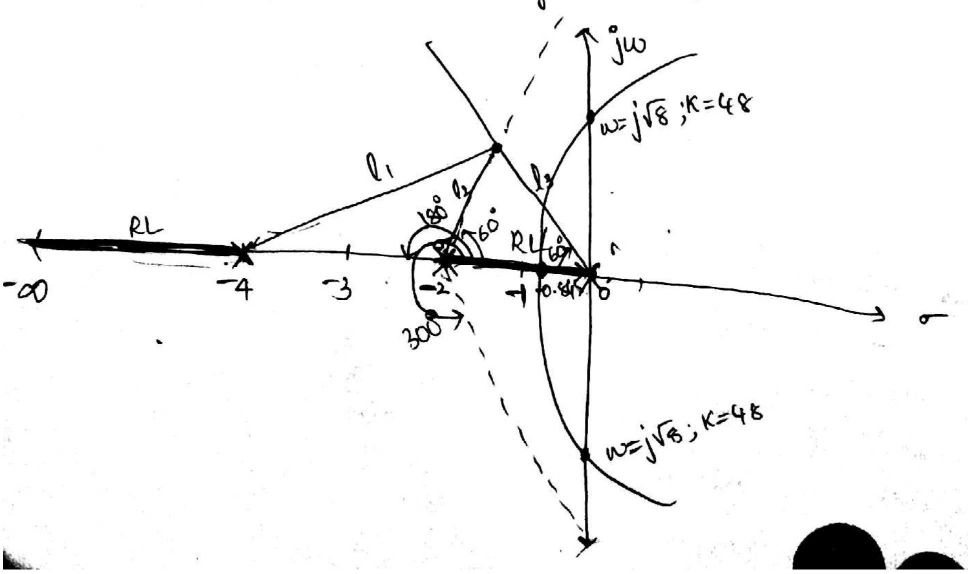
$$G(s)H(s)=\frac{K}{s(s+2)(s+4)}$$

poles; $n=0,\ n=-2,\ n=-4$

$\Rightarrow$ no. of poles $=3$

zeros; $m=0$

$\Rightarrow$ no. of zeros $=0$.



Root locus exists from $s=-2$ to 0 & $s=-\infty$ to -4

Asymptotes $= \dfrac{2q+1}{n-m} \times \pm 180^\circ.$

$q = 0, 1, \ldots (n-m)-1 = 0, 1, 2$

$q=0, \Rightarrow \pm 60^\circ$

$q=1, \Rightarrow \pm 180^\circ$

$q=2 \Rightarrow \pm 300^\circ.$

$$\text{Centroid } (\sigma) = \frac{\Sigma \text{ R.P. of poles} - \Sigma \text{ R.P. of zeros}}{n-m}$$

$$= \frac{0-2-4-0}{3-0}$$

$$= \frac{-6}{3} = -2$$

BWP $\Rightarrow \dfrac{dk}{ds} = 0.$

$1 + G(s)H(s) = 0$

$$1 + \frac{k}{s(s+2)(s+4)} = 0$$

$$K = -(s^3 + 6s^2 + 8s)$$

$$\frac{dK}{ds} \Rightarrow -3s^2 - 12s - 8$$

$$\Rightarrow \frac{dk}{ds} = 0 \Rightarrow 3s^2 + 12s + 8 = 0$$

$S_1 = -0.845, \quad S_2 = -3.154$

R-H.

$s^3 + 6s^2 + 8s + k = 0$

s^3	1	8
s^2	6	k
s^1	$\frac{48-k}{6}$	0 ; $k=48$
s^0	k	

$6s^2 + k = 0$

~~$s = \pm j\sqrt{8}$.~~

$K = 3.08, \quad K = -3.08$

$s^2 = \pm j\sqrt{8}.$

$\theta = \cos^{-1}(\xi)$

$\theta = \cos^{-1}(0.5)$

$\theta = 60^\circ.$

12/2/18.

Q: $G(s) = \dfrac{K}{s(s^2+4s+13)}$

$G(s) = \dfrac{K}{s(s^2+4s+13)}$

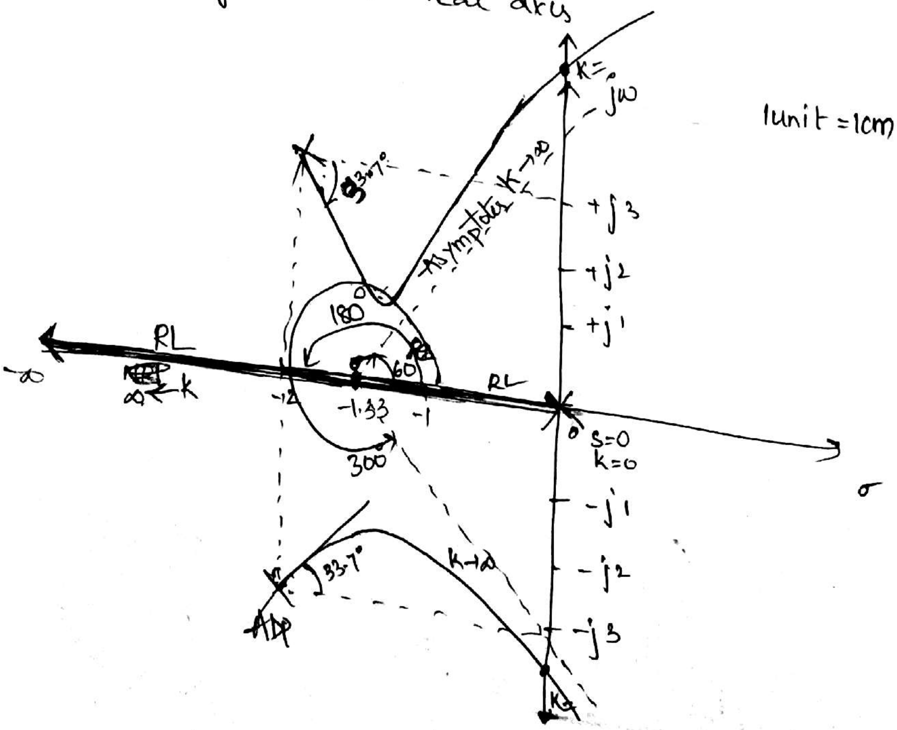
(i) $s=0,\ s^2+4s+13=0$

$s = -2 \pm j3 \Rightarrow -2+j3,\ -2-j3.$

~~n=0, m=3~~ ~~zeros~~ ~~poles~~ $n=3$ (poles), $m=0$ (zeros)

(ii) no. of branches = no. of poles = 3

(iii) Segment on real axis



4. ~~Angle~~ of Asymptotes

$$\frac{(2q+1)}{(n-m)} \times 180^\circ \; ; \; q = 0, 1, 2, 3, \ldots, (n-m)-1$$

$$n - m = 3.$$

$$q = 0, 1, 2$$

$q = 0$; $\Rightarrow 60^\circ$

$q = 1$; $\Rightarrow 180^\circ$

$q = 2$; $\Rightarrow 300^\circ$.

Centroid; $$\sigma = \frac{\Sigma \text{R.P. of poles} - \Sigma \text{R.P. of zeros}}{n-m}$$

$$= \frac{0 - 2 - 2 - 0}{3 - 0} = \frac{-4}{3} = -1.33$$

BWP $\Rightarrow$ $$\frac{dK}{ds} = 0$$

Ch. eqn: $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s^2+4s+13)} = 0.$$

$$K = -s(s^2+4s+13) = -(s^3+4s^2+13s) \quad (1)$$

$$\frac{dK}{ds} = -(3s^2+8s+13)$$

$$\frac{dK}{ds} = 0 \Rightarrow 3s^2+8s+13 = 0.$$

$$s_1 = -1.33 + 1.598j$$
$$s_2 = -1.33 - 1.598j$$

~~R-H: $s^3+4s^2+13s+K$~~ Substitute 's' in eq. (1)

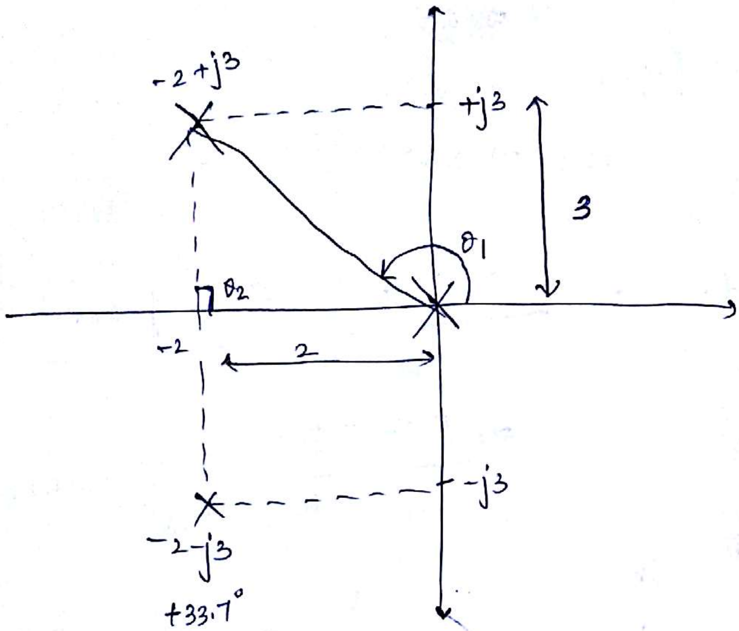
$$K =$$

$\because$ Complex roots $\Rightarrow$ Angle of Departure (ADP).

$$\theta_1 = 180^\circ - \tan^{-1}(3/2)$$

$$\theta_2 = 90^\circ$$

$$\phi_{ADP} = 180^\circ - (\theta_1 + \theta_2)$$



$$\phi_{ADP} = 180^\circ - (\theta_1 + \theta_2)$$

$$\phi_{ADP} = 180^\circ - (123.7^\circ + 90^\circ)$$

$$\phi_{ADP} = -33.7^\circ$$

$$G(s) = \frac{K}{s(s^2+4s+13)}$$

Crossing on Imaginary axis.

C.E. $\rightarrow$ $s^3 + 4s^2 + 13s + K = 0$

s^3	1	13	0
s^2	4	K	0
s^1	$\frac{52-K}{4}$	0	
s^0	K.		

$\boxed{K=52}$

$4s^2 + K = 0$

$s^2 = \frac{-K}{4}$

$s^2 = -\frac{52}{4}$

$s = \pm j3.6$

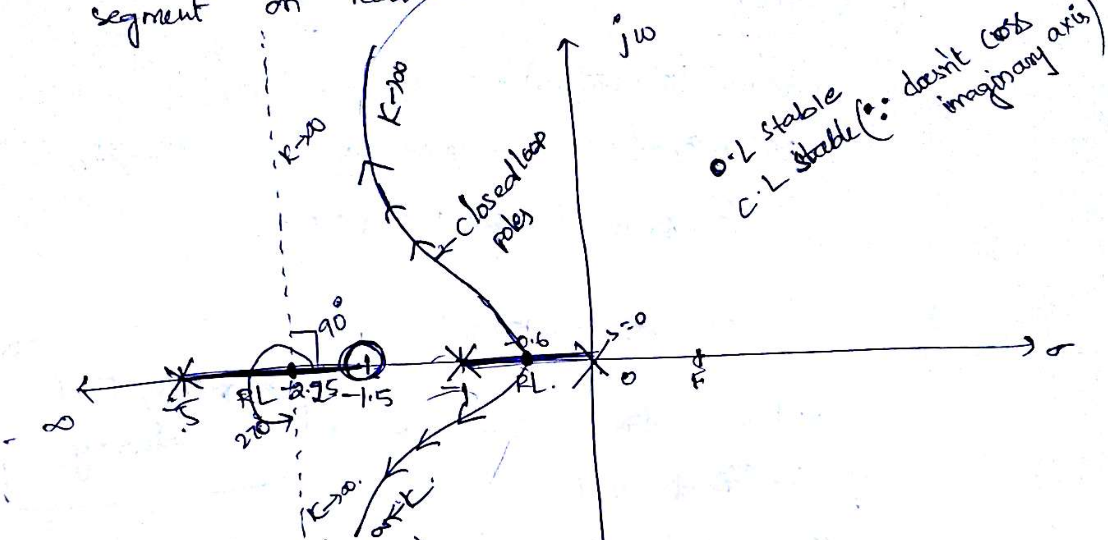
Q: $G(s) = \dfrac{k(s+1.5)}{s(s+1)(s+5)}$

poles: $n = 3$; $s = 0, -1, -5 \longrightarrow$ open loop poles.

Zeros: $m = 1$; $s = -1.5$.

$n - m = 3 - 1 = 2$.

segment on real axis.



Asymptotes: $\dfrac{(2q+1)}{(n-m)} \times 180°$

$q = 0, 1, \ldots (n-m-1)$

$q = 0, 1$

$q = 0; \Rightarrow \dfrac{180°}{2} = 90°$

$q = 1; \Rightarrow \dfrac{3}{2} \times 180° = 270°$.

Centroid: $\sigma = \dfrac{\Sigma \text{ R.P. of poles} - \Sigma \text{ R.P. of zeros}}{n-m}$

$\sigma = \dfrac{0 - 1 - 5 - (-1.5)}{2}$

$\sigma = \dfrac{-4.5}{2} = -2.25$

BWP

$1 + G(s)H(s) = 0.$

$1 + \dfrac{k(s+1.5)}{s(s+1)(s+5)} = 0.$

$\Rightarrow \quad s(s+1)(s+5) + k(s+1.5) = 0$

$$K(s+1.5) = -\left[s(s+1)(s+5)\right]$$

$$K = -\frac{(s^3+6s^2+5s)}{(s+1.5)} \quad \text{—— (1)}$$

$$\frac{dK}{ds} = 0.$$

$$-\left[\frac{(s+1.5)(3s^2+12s+5) - (s^3+6s^2+5s)(1)}{(s+1.5)^2}\right] = 0.$$

$$(s+1.5)(3s^2+12s+5) - s^3 - 6s^2 - 5s = 0.$$

$$3s^3 + 12s^2 + \cancel{5s} + 4.5s^2 + 18s + 7.5 - s^3 - 6s^2 - \cancel{5s} = 0$$

$$2s^3 + 10.5s^2 + 18s + 7.5 = 0.$$

$$s = -0.6, \quad -2.321 \pm 0.890j$$

Substitute s roots in (1)

if K = +ve real value;

then that root is valid

here, only -0.6 is valid.

$\therefore$ BWP $= -0.6$

$K \to$ +ve, real always

(1) $\Rightarrow$ $s^3 + 6s^2 + s(K+5) + 1.5K = 0$.

$\Rightarrow$ R-H

s^3	1	$K+5$	0
s^2	6	$1.5K$	0
s^1	$\frac{(K+5)6-1.5K}{6}$	0	
s^0			

$$= \frac{6K+30-1.5K}{6}$$

$$= \frac{4.5K+30}{6}$$

$\Rightarrow$

$$\left(\frac{4.5K+30}{6}\right)s = 0$$

$\Rightarrow$ $4.5K = -30$

$$K = \frac{-30}{4.5}$$

$6s^2 + 1.5K = 0.$

$6s^2 \div 6.66(1.5) = 0 \Rightarrow s^2 = \frac{6.66 \times 1.5}{6} \Rightarrow s = \pm 5/3$

∴ K is -ve, ~~No need to draw the~~

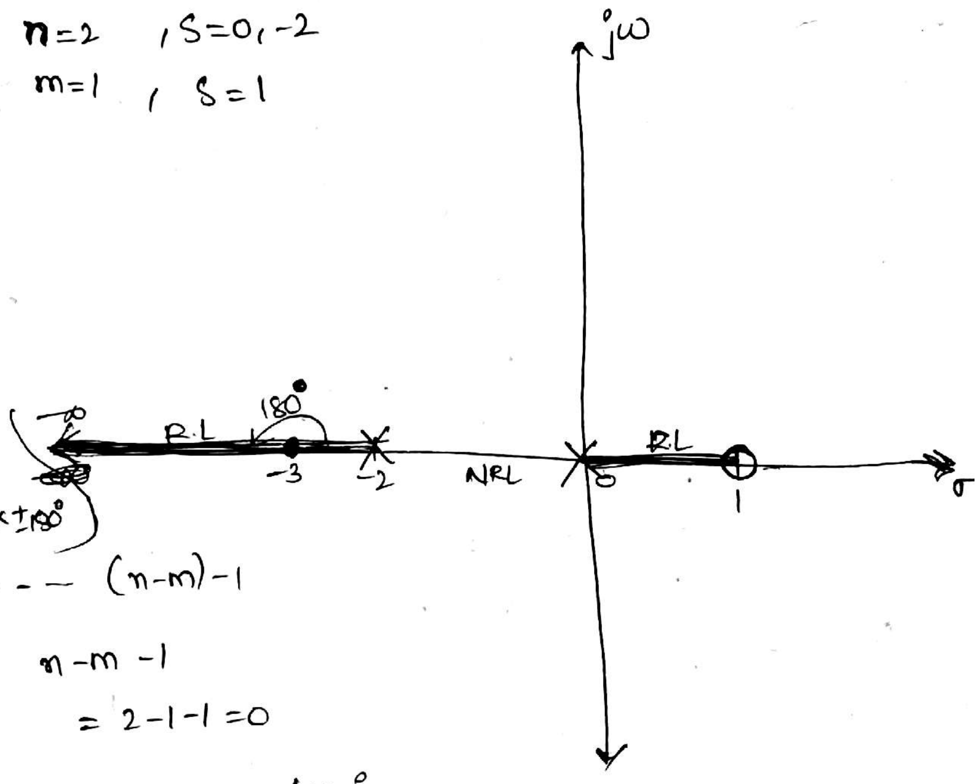
there is no crossing pts on imaginary axis

Q: $G(s) = \frac{K e^{-s}}{s(s+2)}$

$G(s) = \frac{K(1-s)}{s(s+2)}$

$n = 2$, $s = 0, -2$

$m = 1$, $s = 1$



Asymptotes $= \frac{(2q+1)}{n-m} \times \pm 180°$

$q = 0, 1, \ldots (n-m)-1$

$n - m - 1$

$= 2 - 1 - 1 = 0$

$\Rightarrow q = 0$; $\Rightarrow \pm 180°$

$\sigma = \frac{0-2-1}{2-1} \Rightarrow \sigma = -3$

Frequency Response Analysis: (17/2/18)

→ In time domain analysis various standard test i/p's like unitstep, ramp signals are used to study the performance of C.S.

→ From the step Response of 2nd order system we can find time domain specifications, T_r, T_p, T_s, T_d, $\%M_p$ & e_{ss} but the extraction of transfer function from the step response is very difficult.

→ On the other hand using freq. response method we can easily obtain the T.F. from the experimental data

→ freq. response is the steady state response (o/p) of a system when the i/p to the system is sinusoidal.

$$x(t) \longrightarrow \underset{h(t)}{\boxed{LTI}} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t)$$
$$Y(s) = X(s) \cdot H(s)$$
$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

The magnitude & phase relationship b/w the sinusoidal i/p & steady state o/p of a given system is termed as frequency response.

The freq. response of a system is normally obtained by varying the freq. of i/p signal & keeping the mag. of i/p signal constant value.

The freq. response of a system can be directly obtain from the sinusoidal T.F. of the system.

Frequency domain Specification:

The performance & characteristics of a system in f domain are measured interms of freq. domain specifica

The requirement of a system to be designed are usually specified interms of the following specifications.

1. Resonant peak (Mr)
2. Resonant frequency (ω_r)
3. Cut off rate
4. Bandwidth (BW)
5. Gain Margin (GM, $\angle\phi gc$)
6. phase Margin (PM), (γ)

1. Resonant Peak (Mr):

The max^m value of magnitude of C.L.T.F is Called resonant peak. A large value of Mr indicates large overshoot in transient response.

2. Resonant frequency (ω_r):

The freq. at which resonant peak occurs is Called resonant frequency.

3. Bandwidth (BW):

The range of freq. over which the system gain is Constant.

4. Cut-off rate:

The slope of log magnitude curve nearer to the cut-off freq. is Called Cut-off rate. Cut off rate indicates ability of the system to distinguish signal from noise.

5. Gain cross over frequency (GCF):

The freq. at which the magnitude of the system becomes unity (or) 0 dB is Called gain cross over freq. (GCF)

$$G(j\omega) H(j\omega) = -1 + j0$$

$$|G(j\omega) H(j\omega)| = 1 \text{ (or) } 0\,dB$$

$|H| = 1$ or $0\ dB$

6. Phase Cross Over freq (PCF):

The freq. at which the phase angle of the system is 180° (or) phaseplot crosses 180° is called phase cross over frequency (PCF)

7. Gain Margin (GM, ϕ_{pc}):

It is defined as reciprocal of the magnitude of at ~~phase~~ gain phase cross over freq.

$$GM \text{ (or) } \phi_{pc} = \frac{1}{|M|_{w=w_{pc}}}$$

Gain margin is defined as amount of additional gain that should be added at phase cross over freq. to bring the system to verge of instability.

Gain margin is always +ve for stable C.S.

8. Phase Margin (PM):

phase margin is the amount of additional phase lag at gain cross over freq. to bring the system to become verge of instability.

phase margin is always +ve for stable C.S.

$$PM \text{ (or) } \gamma = \pm 180^\circ + \angle\phi_{w=w_{gc}}$$

The values of GM & PM gives inform about relative Stability. large values of GM & PM system more stable.

Small values of GM & PM the systems are less stable.

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frequency domain specifications of 2nd order system:

CLTF of 2nd order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

let $s=j\omega$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2+j2\xi\omega_n\omega+\omega_n^2}$$

$$T.F. = T(j\omega) = \frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{-\omega^2+j2\xi\omega_n\omega+\omega_n^2}$$

$$= \frac{\omega_n^2}{\omega_n^2\left[1-\left(\frac{\omega}{\omega_n}\right)^2+\left(j2\xi\frac{\omega}{\omega_n}\right)\right]}$$

$$T(j\omega) = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+j2\xi\left(\frac{\omega}{\omega_n}\right)}$$

$$\frac{\omega}{\omega_n} = U$$

$$T(j\omega) = \frac{1}{(1-U^2)+2\xi jU}$$

Bode Plot:

$$M = \text{Magnitude} = \frac{1}{\sqrt{(1-U^2)^2+(2\xi U)^2}} \quad —①$$

$$\phi = \text{phase angle} = -\tan^{-1}\frac{2\xi U}{1-U^2} \quad —②$$

Resonant Peak (M_r):

Max^m peak $= f_r$.

$$\frac{d}{dU}\left[\frac{1}{\sqrt{(1-U^2)^2+(2\xi U)^2}}\right] = 0$$

$$\frac{2(1-U^2)(-2U)+4\xi^2(2U)}{\left((1-U^2)^2+(2\xi U)^2\right)^{3/2}}=0$$

$$2(1-U^2)(-2U)+8\xi^2U=0$$

$$-4U(1-U^2)+8\xi^2U=0$$

$$1-U^2=2\xi^2$$

$$U=\sqrt{1-2\xi^2}$$

$$\boxed{U_r=\sqrt{1-2\xi^2}}$$

Sub. Mr in. (1)

$$M=\text{Magnitude}=\frac{1}{\sqrt{\left(1-\left(\sqrt{1-2\xi^2}\right)^2\right)^2+\left(2\xi\sqrt{1-2\xi^2}\right)^2}}$$

$$=\frac{1}{\sqrt{4\xi^4+\xi^2-(4\xi^4)^2}}$$

$$=\frac{1}{\sqrt{4\xi^2-4\xi^4}}$$

$$\boxed{M_r=\frac{1}{2\xi\sqrt{1-\xi^2}}}\rightarrow \text{Resonant peak.}$$

2. Resonant frequency (ω_r):

$$U_r=\frac{\omega_r}{\omega_n}$$

$$\omega_r=\omega_n U_r$$

$$\boxed{\omega_r=\omega_n\sqrt{1-2\xi^2}}\rightarrow \text{Resonant freq.}$$

3. Bandwidth (BW):

$$M=\frac{1}{\sqrt{(1-U^2)^2+(2\xi U)^2}}=\frac{1}{\sqrt{2}}$$

Q: The ...

ω_n^2

$s^2+2\xi\omega$...

not ex...

limitin...

Square on both sides.

$$\frac{1}{(1-u^2)^2+(2\xi u)^2} = \frac{1}{2}$$

$$(1-u^2)^2+(2\xi u)^2 = 2$$

$$u^4+2u^2-(2\xi^2-1)-1=0$$

$$u^2=x$$

$$x^2+2x(2\xi^2-1)-1=0$$

$$x = \frac{-2(2\xi^2-1) \pm \sqrt{(2(2\xi^2-1))^3-4\times1(-1)}}{2\times1}$$

$$u^2=x$$

$$u=\frac{\omega}{\omega_n}$$

$$BW = \omega_n\left[\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}}\right]$$

Q: The specifications of 2nd order system with CLTF.

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$ & that the max. peak overshoot must not exceed 30%. & rise time less than 0.2 sec. Find the limiting value of max. resonant peak & BW.

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.03 \rightarrow \xi = ?, \quad \theta = \cos^{-1}\xi.$$

$$t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\theta}{\omega_n\sqrt{1-\xi^2}} = 0.2 \Rightarrow \omega_n = ?$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = ?$$

8/9/18

4. Phase Margin of 2nd order System:

$$G(s) = O.L.T.F. = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s}$$

$$s = j\omega$$

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + j2\xi\omega_n\omega}$$

Magnitude ; $|M| = \dfrac{\omega_n^2}{\sqrt{\omega^4 + (2\xi\omega_n\omega)^2}}$

phase angle ; $\phi = -\tan^{-1}\left[\dfrac{2\xi\omega_n\omega}{\omega^2}\right]$

$$\frac{\omega_n^2}{\sqrt{\omega^4 + 4\xi^2\omega_n^2\omega^2}} = 1$$

$|M| = 1$ at GCF

$$\omega_n^2 = \sqrt{\omega^4 + 4\xi^2\omega_n^2\omega^2}$$

square on both sides

$$\omega_n^4 = \omega^4 + 4\xi^2\omega_n^2\omega^2$$

$$\omega^4 + 4\xi^2\omega_n^2\omega^2 - \omega_n^4 = 0$$

put $\Rightarrow \omega^2 = x$

$$x^2 + 4\xi^2\omega_n^2 x - \omega_n^4 = 0$$

ω_n = natural freq.
= constant.

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$$x = \frac{-4\xi^2\omega_n^2 \pm \sqrt{(4\xi^2\omega_n^2)^2 - 4\times1\times(-\omega_n^4)}}{2\times1}$$

$$x = -2\xi^2\omega_n^2 \pm \sqrt{4\xi^4\omega_n^4 + \omega_n^4}$$

$$x = -2\xi^2\omega_n^2 \pm \omega_n^2\sqrt{1+4\xi^4}$$

$$x = \omega_n^2\left[-2\xi^2 \pm \sqrt{1+4\xi^4}\right].$$

$\therefore$

$$\boxed{\omega_{gc} = \sqrt{-2\xi^2\omega_n^2 + \omega_n^2\sqrt{4\xi^4+1}}}$$

$$PM = 180^\circ + \angle\phi\Big|_{\omega=\omega_{gc}}$$

$$PM = -\tan^{-1}\left[\frac{2\xi\omega_n\omega}{-\omega^2}\right]\Bigg|_{\omega=\omega_{gc}} + 180^\circ.$$

5. GM for 2nd order systems

Bode Plot: → Magnitude plot (M)
→ phase plot (ϕ)

Factors:

(i) Constants 'K': Present either in numerator / denominator.

$|M| = K$

$M = 20 \log K$

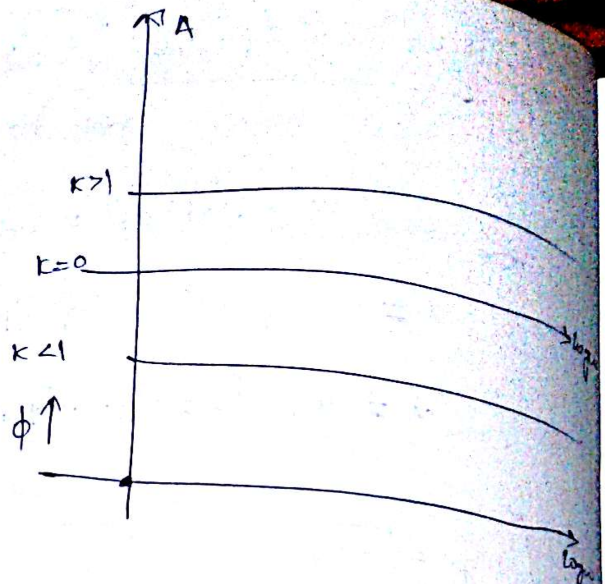
$K = 0 \longrightarrow M = \infty$

$K = 1 \longrightarrow M = 0\,dB$

$K < 1 \longrightarrow M = -ve$

$K > 1 \longrightarrow M = +ve$

$\phi = 0^\circ;\ 0\,dB/dec.$



(ii) Integral factor K/s:

$G(s) = K/s$

$G(j\omega) = K/j\omega$

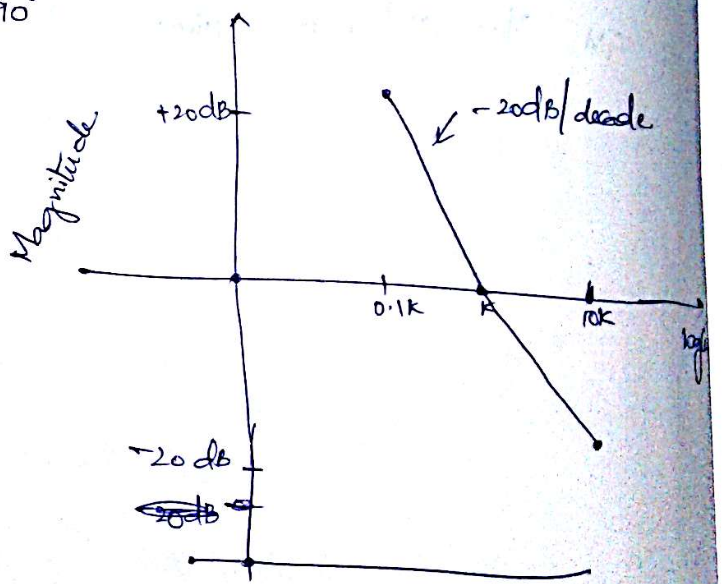
$M = K/\omega = 20 \log \frac{K}{\omega}$

$\phi = -90^\circ$

$\omega = 0.1K \quad M = +20\,dB$

$\omega = K \quad M = 0\,dB$

$\omega = 10K \quad M = -20\,dB.$



(iii) Differentiator factor:

$G(s) = Ks$

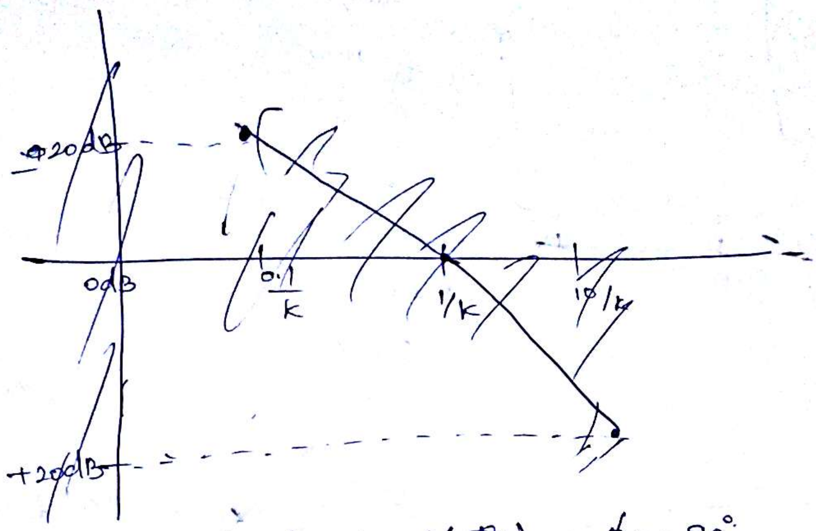
$G(j\omega) = Kj\omega$

$|M| = |G(j\omega)| = 20 \log (K\omega)$

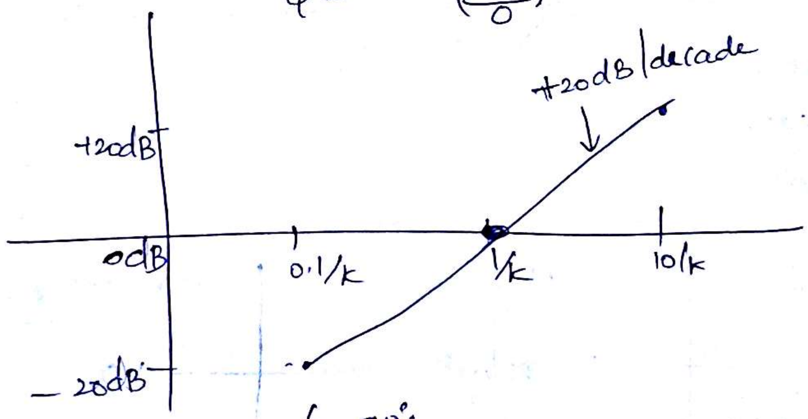
$\omega = \frac{0.1}{K}$, $M = -20\, dB$

$\omega = 1/K$, $M = 0\, dB$

$\omega = 10/K$, $M = +20\, dB/decade$



$\phi = -\tan^{-1}\left(\frac{K\omega}{0}\right) = \phi = -90°$



$\phi = -90°$

Q: $G(s) = \frac{1}{1+sT}$

$G(j\omega) = \frac{1}{1+j\omega T}$; $\phi = -\tan^{-1}\left(\frac{\omega T}{1}\right)$

$M = 20\log\frac{1}{\sqrt{1+(\omega T)^2}}$

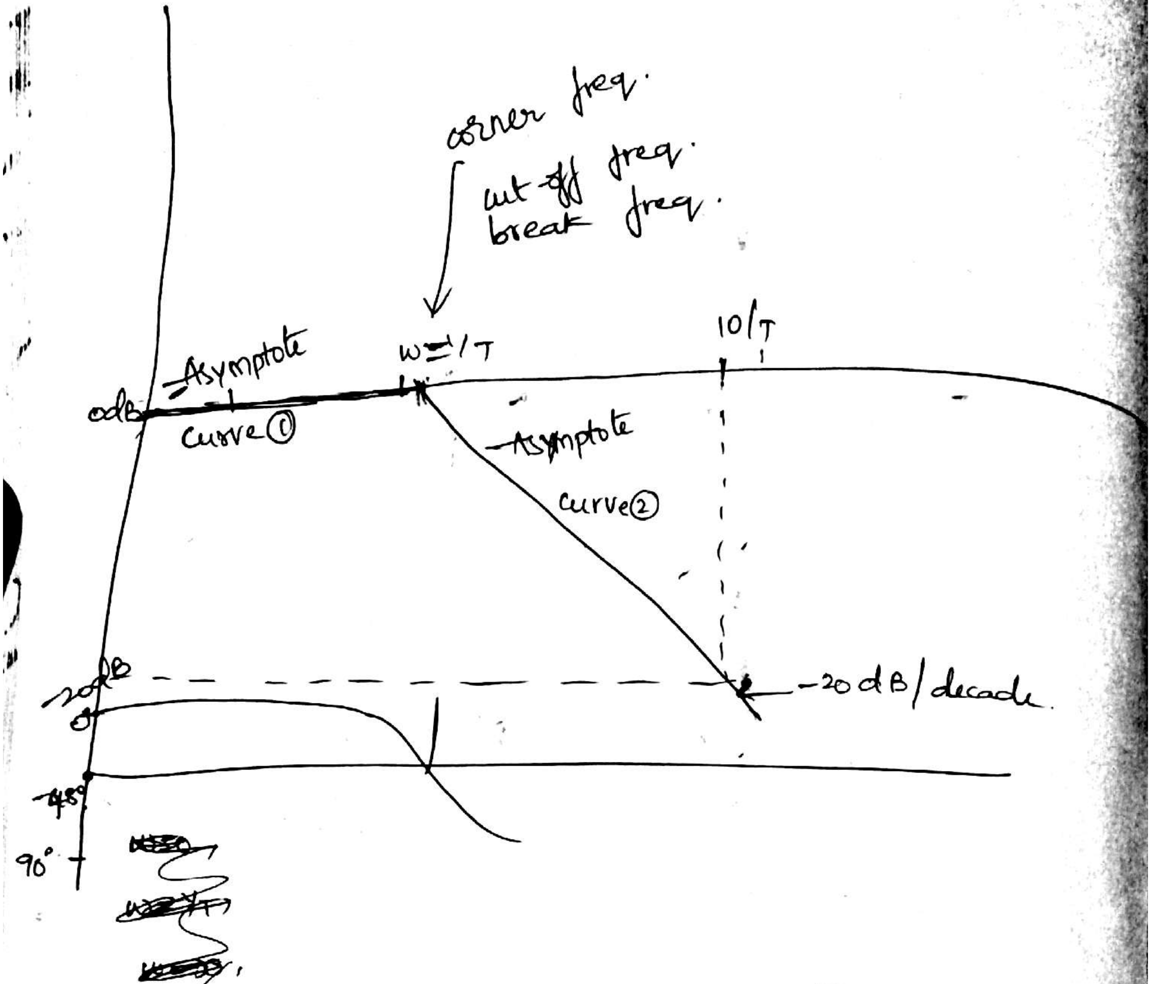
$M = -20\log\left(\sqrt{1+(\omega T)^2}\right)$

$\omega T << 1$ —— ; $M = 0$

$\omega = 1/T$ —— ; $M = -3.010\, dB.$

$\omega T >> 1$, $M \cong -20 \log(\omega T)$

$M = -20 \, dB$



Factor / term	Magnitude	slope (dB/decade)
K →	$20 \log K$ →	0
$1/j\omega$ →	$-20 \log \omega$ →	-20
$j\omega$ →	$20 \log \omega$ →	$+20$
$(j\omega)^n$ →	$20 \log (\omega)^n$ →	$+20 \times n$
$1/(j\omega)^n$ →	$-20 \log (\omega)^n$ →	$-20 \times n$
$1/(1+j\omega T)$ →	$M = 0 \quad 0 \le \omega \le \frac{1}{T}$ $M = -20 \log(\omega T)$; $\frac{1}{T} < \omega \le \infty$	0

$\frac{1}{(1+j\omega T)^n}$	$M=0$; $\omega \le \frac{1}{T}$ $M = -20\log(\omega T)^n$; $\frac{1}{T} \le \omega \le \infty$	$-20\times n$
$(1+j\omega T)$	$M=0$; $\omega \le 1/T$ $M = +20\log(\omega T)$; $\omega > 1/T$	$+20$
$(1+j\omega T)^n$	$M=0$; $\omega \le 1/T$ $M = +20\log(\omega T)^n$; $\omega > 1/T$	$+20\times n$
$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	$M=0$, $\omega < \omega_n$ $M = -40\log\frac{\omega}{\omega_n}$, $\omega > \omega_n$	-40 dB/decade

Q: Plot the Bode diagram for T.F. & obtain GCF & PCF.

20/2/18

$$G(s) = \frac{10}{s(1+0.4s)(1+0.1s)}$$

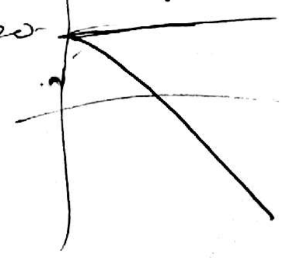
$$\frac{1}{T}$$

$$G(j\omega) = \frac{10}{j\omega(1+j0.4\omega)(1+j0.1\omega)}$$

$$\frac{1}{s+\frac{1}{T}}$$

Magnitude eqn;

$$20\log_{10} 10 - 20\log(\omega) - 20\log(0.4\omega) - 20\log(0.1\omega)$$



Term	Corner freq.	Slope
10	—	0
$\frac{1}{j\omega}$	—	-20 dB/decade
$\frac{1}{(1+j0.4\omega)}$	$\omega_{c_1} = 1/T = 1/0.4 = 2.5$ rad/s	-20 dB/decade
$\frac{1}{(1+j0.1\omega)}$	$\omega_{c_2} = 1/T = 1/0.1 = 10$	-20 dB/decade

0.1   1   10 10^2 10^3

$\longrightarrow \omega$ (rad/s)

$0 \le \omega \le 2.5$

$M = 20\log(10) - 20\log(\omega)$

slopes.
$0 - 20 = -20$ dB/decade

$\omega = 0.1$; $M = +40$ dB

$\omega = 2.5$; $M = +12.04$ dB $\approx +12$ dB

$2.5 \le \omega \le 10$

$M = 20\log 10 - 20\log\omega - 20\log(0.4\omega)$

slopes.
$0 - 20 - 20$
$= -40$ dB/decade

$\omega = 10$; $M = -12.04$ dB ≈ -12 dB

$10 \le \omega \le \infty$

$\omega = 50$

$M = 20\log 10 - 20\log\omega - 20\log(0.4\omega) - 20\log(0.1\omega)$

$M = -54$ dB.

slopes.
$0 - 20 - 20 - 20 = -60$ dB/decade

$\phi = -90^\circ - \tan^{-1}(0.4\omega) - \tan^{-1}(0.1\omega)$

ω	-90	$-\tan^{-1}(0.4\omega)$	$-\tan^{-1}(0.1\omega)$	ϕ
0.1	-90°	-2.29	-0.57	$-92.86 \approx -92^\circ$
1	-90°	-21.8	-5.7	$-117.5 \approx -118^\circ$
2.5	-90°	-45	-14.03	$-149.03 \approx -150^\circ$
4	-90°	-57.9	-21.8	$-169.79 \approx -170^\circ$
10	-90°	-75.9	-45	$-210.96 \approx -210^\circ$
20	-90°	-82.8	-63.4	$-236.30 \approx -236^\circ$

freq.	Magnitude	slope
$0 \le \omega \le 2.5$ $\omega = 0.1$ $\omega = 2.5$	$M = +40$ dB $M = +12$ dB	-20 dB/dec
$2.5 < \omega \le 10$ $\omega = 10$	$M = -12$ dB	-40 dB/dec
$10 < \omega \le \infty$ $\omega = 50$	$M = -54$ dB	-60 dB/dec

ω_{gc} → freq. at which Magnitude $= 1$, (or) 0dB.

$\therefore$ @ $\omega = 2.5$; ~~slop~~ Magnitude $= +12$ dB.

@ $\omega = 10$; $M = -12$ dB

in b/w ⇒ $2.5 < \omega \le 10$; $M = 0$ dB

@ $2.5 < \omega \le 10$;

$$M = 20\log_{10} 10 - 20\log_{10}(\omega) - 20\log_{10}(0.4\omega) = 0.$$

$$20 - \left[20\log\omega + 20\log 0.4\omega\right] = 0.$$

$$20 = 20\log\left[\omega(0.4\omega)\right]$$

$$20 = 20\log(0.4\omega^2)$$

$$\log(0.4\omega^2) = 1$$

$$0.4\,\omega^2 = 10$$

$$\omega^2 = \frac{10}{0.4}$$

$$\omega^2 = 25$$

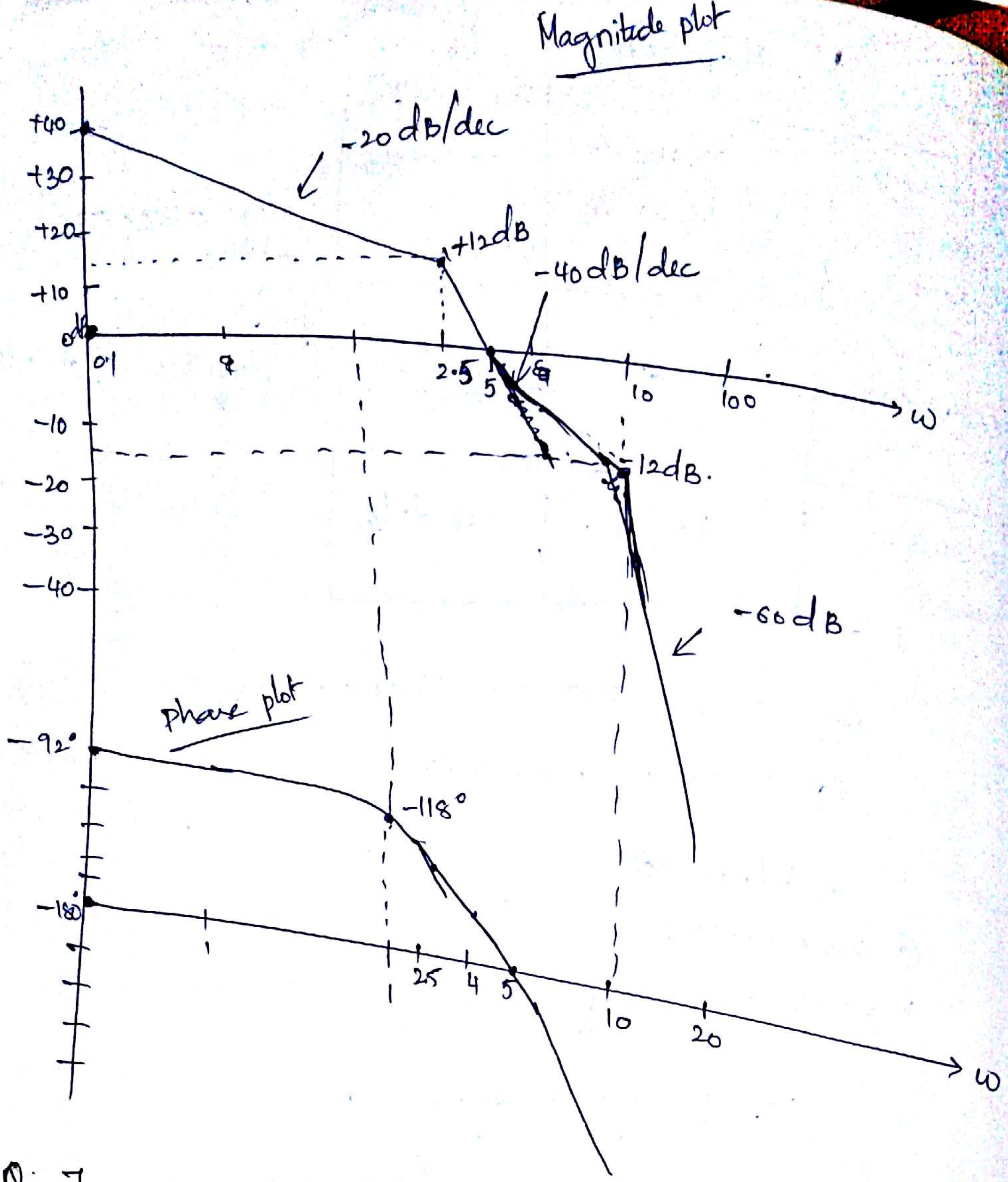
$$\omega = 5 \text{ rad/sec.}$$

$$\Rightarrow \boxed{\omega_{gc} = 5 \text{ rad/s}}$$

@ $\omega = 4$; $\phi \approx -180°$.

$\omega_{pc} = 4$

(phase margin at which $\phi = -180°$ → phase of T.F.)



Q: For the following T.F ; Draw the Bode plot

$$G(s) = \frac{20}{s(1+3s)(1+4s)}$$

$$G(j\omega) = \frac{20}{j\omega(1+3j\omega)(1+4j\omega)}$$

$$M = 20\log 20 - 20\log\omega - 20\log(3\omega) - 20\log(4\omega)$$

term	corner freq.	slope
20	—	0
$1/j\omega$	—	-20
$1/(1+j3\omega)$	$\omega_{c_1} = \frac{1}{3} = 0.33$	-20
$1/(1+4j\omega)$	$\omega_{c_2} = 1/4 = 0.25$	-20

$\phi = -90^\circ - \tan^{-1}(3\omega) - \tan^{-1}(4\omega)$

$0 \le \omega \le 0.25$

$M = 20\log 20 - 20\log\omega$

$0-20 = -20\,dB/dec.$

ω

$\omega = 0.1, \quad M = +46\,dB$

$\omega = 0.25, \quad M = +38\,dB$

$0.25 \le \omega \le 0.33$

$0-20-20 = -40\,dB/dec$

$M = 20\log 20 - 20\log\omega - 20\log 4\omega.$

$\omega = 0.33, \quad M = +33\,dB.$

$0.33 \le \omega \le \infty$

$M = 20\log 20 - 20\log\omega - 20\log 3\omega - 20\log 4\omega$

$\omega = 1, \quad M = 4\,dB$

$0-20-20-20 = -60\,dB/dec$

$\rightarrow \omega$

$20\log 20 - 20\log\omega - 20\log 3\omega - 20\log 4\omega = 0.$

$20\log 20 - 20[\log(12\omega^3)] = 0.$

$\log_{10} 20 = \log_{10}(12\omega^3)$

plot

~~20 = 12ω³~~

~~ω³ = 20~~

$12\omega^3 = 10^{\log_{10} 20}$

$12\omega^3 = 10^{1.3}$

~~12ω³ = 20~~

~~ω³ = 20/12~~

~~ω = (5/3)^{1/3}~~

$\omega^3 = \dfrac{19.95}{12}$

$g(4\omega)$

$\omega^3 = 1.6$

$\omega_{gc} = 1.18\,r/s$

ω	ϕ
0	
0.1	
0.25	
0.3	
0.33	
1	

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Q: Sketch the Bode plot for the following T.F. Determine phase margin & Gain margin.

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}$$

$$G(j\omega) = \frac{75(1+0.2j\omega)}{j\omega(-\omega^2+16j\omega+100)}$$

$$= \frac{75(1+j0.2\omega)}{j\omega\,100\left(1+j\frac{16}{100}\omega - \frac{\omega^2}{100}\right)}$$

$$G(j\omega) = \frac{0.75\,(1+j0.2\omega)}{j\omega\left(1+j0.16\omega - \frac{\omega^2}{100}\right)}$$

for $(1+j\omega T) \rightarrow \omega_{c1} = 1/T_1 = 1/0.2 = 5$ rad/s

for $(s^2+2\xi\omega_n s+\omega_n^2) \rightarrow \omega_{c2} = \omega_n = 10$ rad/s

$(\because \omega_n^2 = 100)$

Term/factor	Corner freq.	slope (dB/dec)
0.75	—	0
$1/j\omega$	—	-20
$1+j0.2\omega$	$\omega_4 = 1/0.2 = 5$	$+20$
$\dfrac{1}{\left(1+j\omega 0.16-\frac{\omega^2}{100}\right)}$	$\omega_{c_2} = \omega_n = 10$	-40

$(1+j\omega T)^2$ $20\log\left(\frac{\omega}{\omega_n}\right)$

$= (1-\omega^2T)^2 + 2j\omega T$

$M = 20\log 0.75 - 20\log\omega + 20\log 0.2\omega - 40\log\left(\frac{\omega}{10}\right)$

for $0 \le \omega \le 5$;

$M = 20\log 0.75 - 20\log\omega$

slope $= 0-20$ dB $= -20$ dB

$\omega = 0.1$, $M = 17.5$

$\omega = 5$, $M = -16.47$

$5 \le \omega \le 10$

$M = 20\log 0.75 + 20\log 0.2\omega - 20\log\omega$

slope $= 0-20+20 = 0$ dB

$\omega = 10$, $M = -16.47$

$10 \le \omega \le \infty$

$M = 20\log 0.75 + 20\log 0.2\omega - 20\log\omega - 40\log\left(\frac{\omega}{10}\right)$

$\omega = 100$, $M = -56.47$

slope $= 0-20+20-40$
$= -40$ dB

~~20 log 0.75 - 20 log ω = 0~~

$$20\log\left(0.75\times 0.2\omega\right) - 20\log\left(\frac{\omega^2}{10}\right) = 0$$

$$20\log(0.75\times 0.2\omega) = 20\log\left(\frac{\omega^2}{10}\right)$$

$$0.75\times 0.2\omega = \frac{\omega^2}{10}$$

$$0.75\times 0.2\times 10 = \omega$$

$\omega = 1.5$ rad/s.

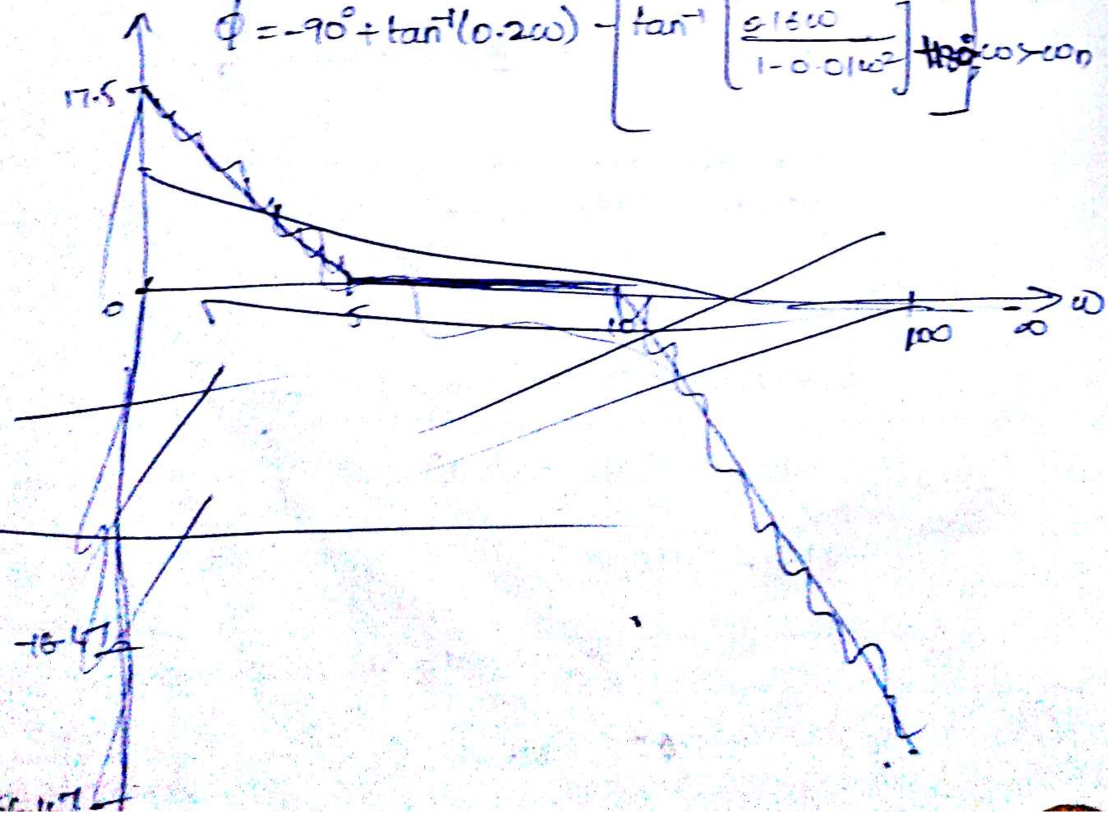
$$20\log 0.75 - 20\log\omega = 0$$

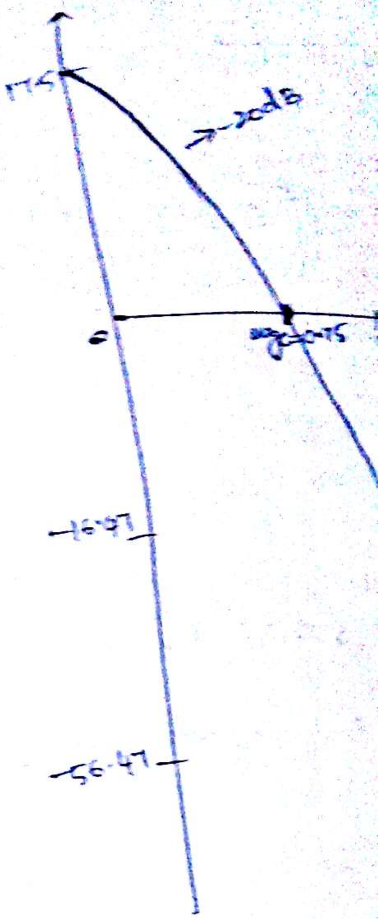
$$\boxed{0.75 = \omega}$$

$\omega_{gc} = 0.75$ rad/s.

$$\phi = -90^\circ + \tan^{-1}(0.2\omega) - \tan^{-1}\left[\frac{0.16\omega}{1-0.01\omega^2}\right] \; ; \; \omega \le \omega_n$$

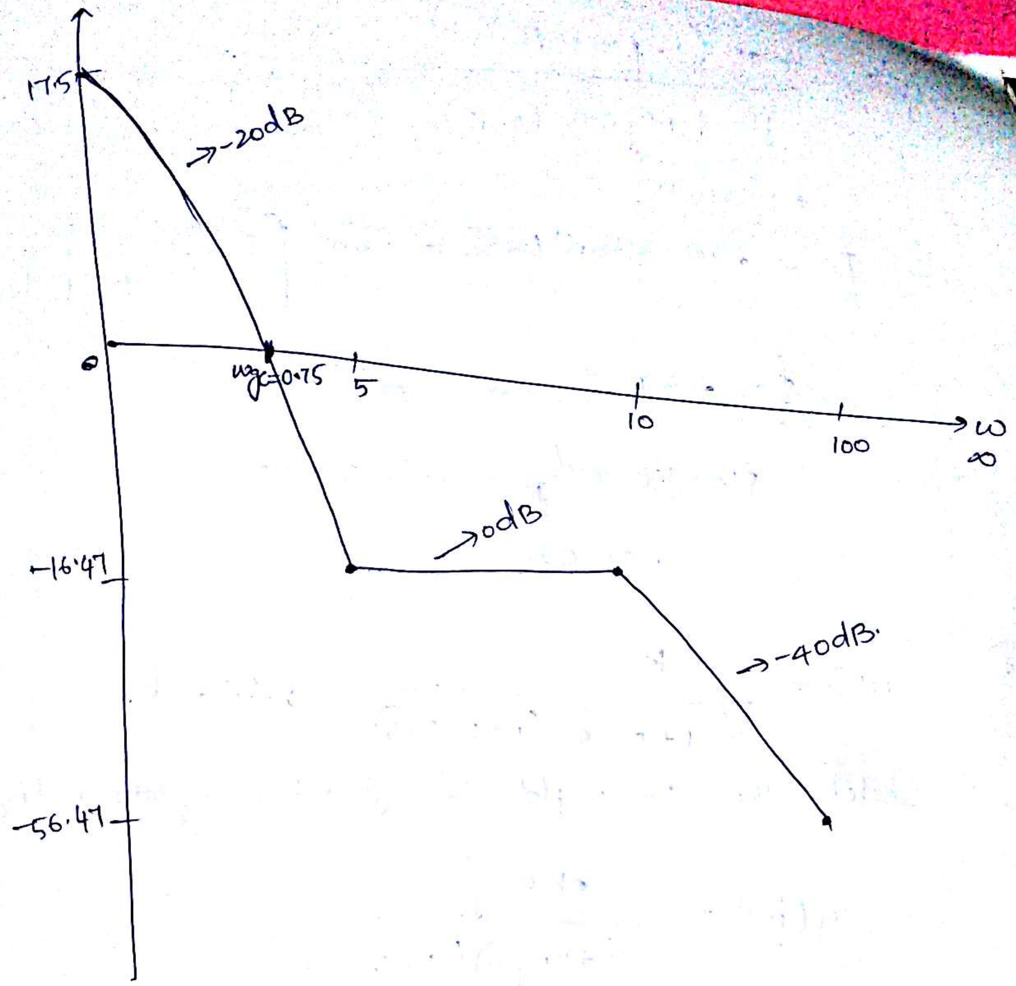
$$\phi = -90^\circ + \tan^{-1}(0.2\omega) - \left[\tan^{-1}\left[\frac{0.16\omega}{1-0.01\omega^2}\right] + 180^\circ\right] \; \omega > \omega_n$$





ω	φ
0	-90
2.5	-86.5
5	-9
7.5	-103
10	-1
50	-192.
100	-186.

Phase m



ω	ϕ
0	-90
2.5	-86.54
5	-91
7.5	-103.65
10	-116.
50	-192.72
100	-186.31

ω	ϕ
0.5	-88
1	-88
5	-92
10	-116
20	-148
50	-168
100	-194
5×10^{12}	-180

~~Gain margin~~

$\omega_{pc}=\infty$

$GM = \frac{1}{\frac{1}{\infty}} = \infty$

$GM = \left|\frac{1}{G(s)}\right|_{\omega=\omega_{pc}}$

Phase margin; $PM = 180^\circ + \phi_{gc}$

$\phi_{gc} = \angle G(s)H(s) \Big|_{\omega=0.75}$

$$G(j\omega) = \frac{0.75\left(1+j(0.2)(0.75)\right)}{j\omega\left(1+j0.16(0.75) - \frac{(0.75)^2}{100}\right)}$$

~~At $\omega=0.75$~~

$$\phi = +90^\circ + \tan^{-1}(0.2\omega) - \tan^{-1}\left[\frac{0.16\omega}{1-0.01\omega^2}\right]$$

$$\phi = -89.97$$

$$PM = 180 + \phi_{gc} = 180^\circ - 89.97$$

$$= 90.02$$

(2)

$$G(s) = \frac{ks^2}{(1+0.2s)(1+0.02s)}$$; find k.

Sketch the Bode plot & find k; $\omega_{gc} = 5$ rad/s.

$$G(j\omega) = \frac{k\omega^2 j^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

$$M = 20\log k + 20\log\omega^2 - 20\log 0.2\omega - 20\log 0.02\omega$$

let $k=1$;

$$M = +40\log\omega - 20\log 0.2\omega - 20\log 0.02\omega.$$

~~let k=1;~~

$$M = +40\log\omega - 20\log 0.2\omega - 20\log 0.02\omega$$

terms	corner freq.	slope
k	—	0
$(j\omega)^2$	—	$20\times 2 = +40$ dB
$\frac{1}{1+0.2j\omega}$	$\omega_{c1} = 1/0.2 = 5$	-20 dB
$\frac{1}{1+0.02j\omega}$	$\omega_{c2} = 1/0.02 = 50$	-20 dB

$0 \le \omega \le 5$

$M = 40 \log \omega$ ~~+20 log 0.04~~ slope = 40dB

~~M=0,~~ $\omega = 0.1$, $M = -40$

~~M=5,1~~ $\omega = 5$, $M = 27.95$

$\omega = 1$, $M = 0$

$5 < \omega \le 50$

$M = 40 \log \omega - 20 \log 0.2\omega$ ~~+20 log 0.04~~

slope = 20 dB

$\omega = 25$, $M = 41.9$

$\omega = 50$, $M = 47.9$

$50 < \omega \le \infty$

$M = 40 \log \omega - 20 \log 0.2\omega - 20 \log 0.02\omega$

slope = 0 dB.

$\omega = 100$, $M = 47.95$

At $\omega = 1$, $M = 0$

$\Rightarrow \omega_{gc} = 1$

$\Rightarrow$ @ $0 \le \omega \le 5$;

~~40 log ω = 0~~ ~~M = 40 log kω~~

at $\omega_{gc} = 5$; Magnitude should be zero.

~~40 log kω = 0~~

~~kω = 1~~

~~k =~~

$\Rightarrow \omega_{gc} = 5 \Rightarrow M = -27.95$

$20 \log k = -27.95$

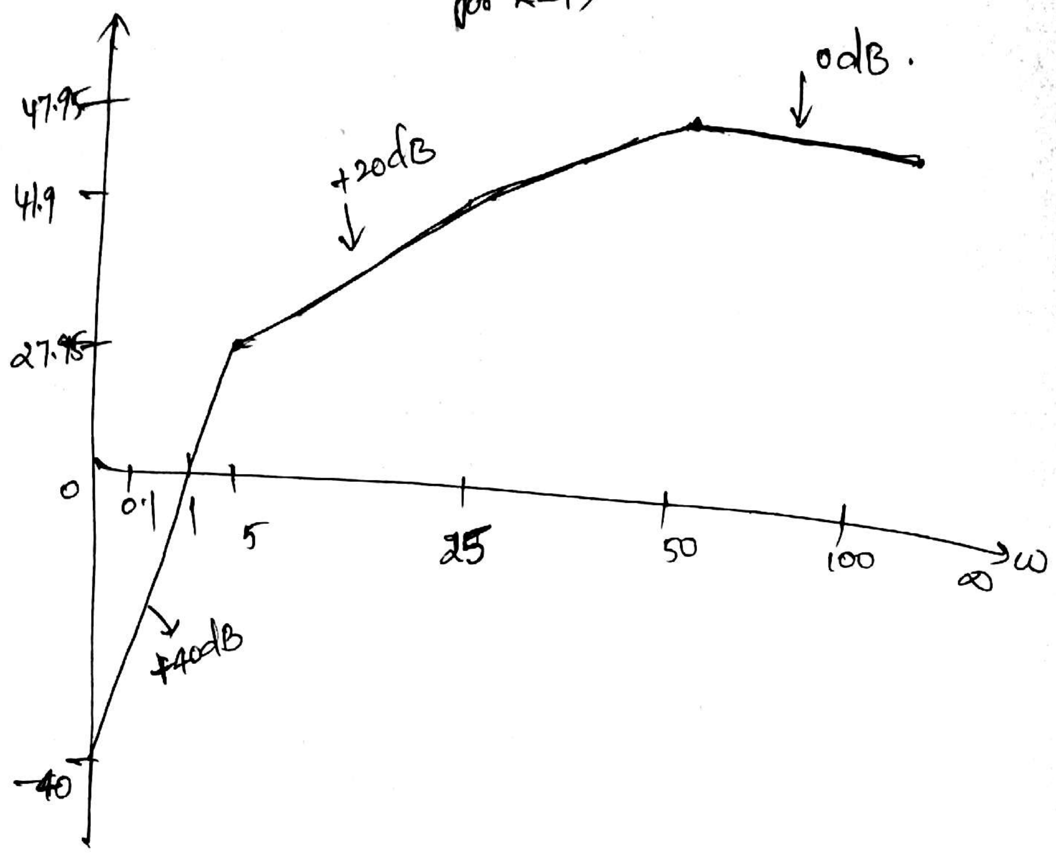
$\log k = \frac{-27.95}{20}$

$k = 10^{-27.95/20}$

$\boxed{k = 0.04}$

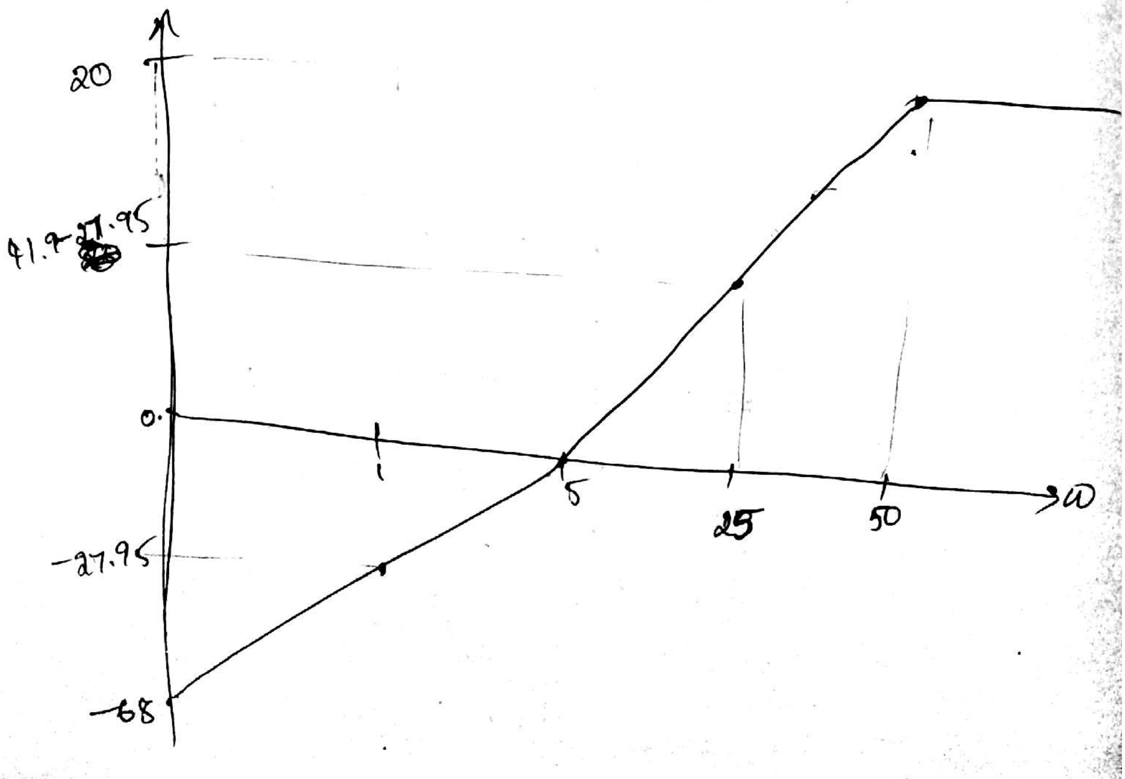
⇒ ~~Magnitude~~

for $k=1$,



for $k=0.04$, $20\log k = -27.95$.

⇒ shift above plot by 27.95 dB down.



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UNIT-IV

Polar plot.

→ polar plot is used to draw the freq. response of open loop T.F. to find the closed loop system stability. polar plot is a plot which can be drawn b/w the magnitude & phase angle of ~~G(jω)H~~ G(jω)H(jω) by varying 'ω' from 0 to ∞

→ The Complete freq. response plot is nyquist plot as 'ω' → −∞ to +∞.

→ polar graph consisting of concentric circles & radial lines. Concentric circle represents magnitudes. Radial lines represents phase angle

→ Angles are +ve value in anti-clockwise direcn.

→ Angles " −ve " " Clockwise ".

→ Angle 270° in anticlockwise direcn = −90° in Clockwise

Rules for drawing polar plot:

1. Substitute $s=j\omega$ in the O.L.T.F.
2. Write the magnitude & phase expression for $G(j\omega)\,H(j\omega)$
3. Find the starting magnitude & phase of $G(j\omega)$ & $H(j\omega)$ by substituting $\omega=0$, So, the polar plot Start with this magnitude & phase angle
4. Find ending magnitude & phase angle $G(j\omega)$ & $H(j\omega)$ by

Substituting $\omega = \infty$; So the polar plot ends with this magnitude & phase angle.

5. Check whether the polar plot intersects the real axis by making the imaginary term of $G(j\omega)H(j\omega)$ find the value of ω:

6. Check whether the polar plot intersects the imaginary axis by making real term=0 & find the value of ω:

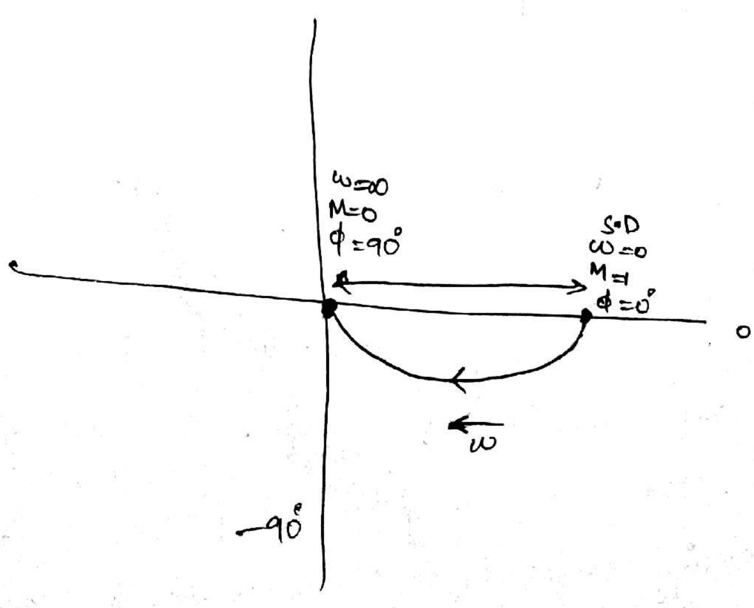
Q: $G(s)\,H(s) = \dfrac{1}{s+1}$

$$G(j\omega)H(j\omega) = \frac{1}{j\omega+1}$$

$$M = \frac{1}{\sqrt{1+\omega^2}} \; ; \quad \phi = -\tan^{-1}(\omega)$$

Starting; $\omega=0$, $M=1$, $\phi = 0^\circ$

ending; $\omega = \infty$, $M=0$, $\phi = -90^\circ$



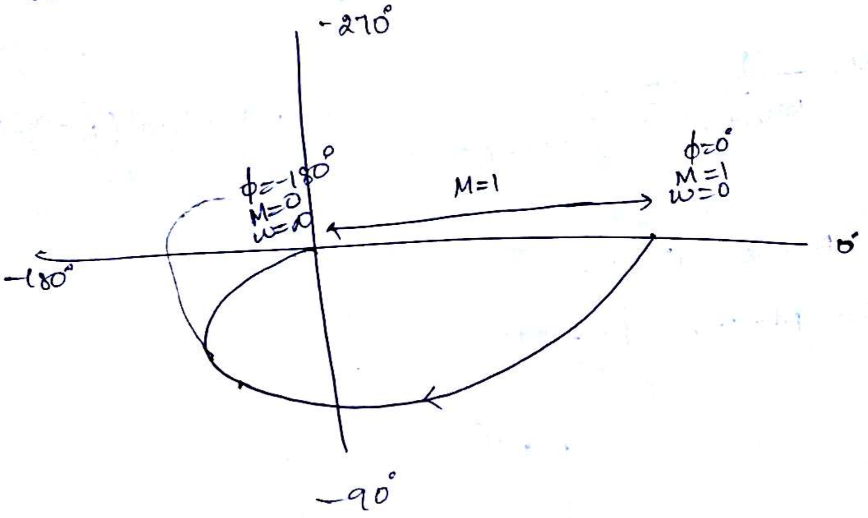
2. $G(s)H(s) = \frac{1}{(1+sT_1)(1+sT_2)}$

$$G(j\omega)H(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$M = \frac{1}{\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}} \quad ; \phi = -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

S.D; $\omega = 0$, $M = 1$, $\phi = 0$

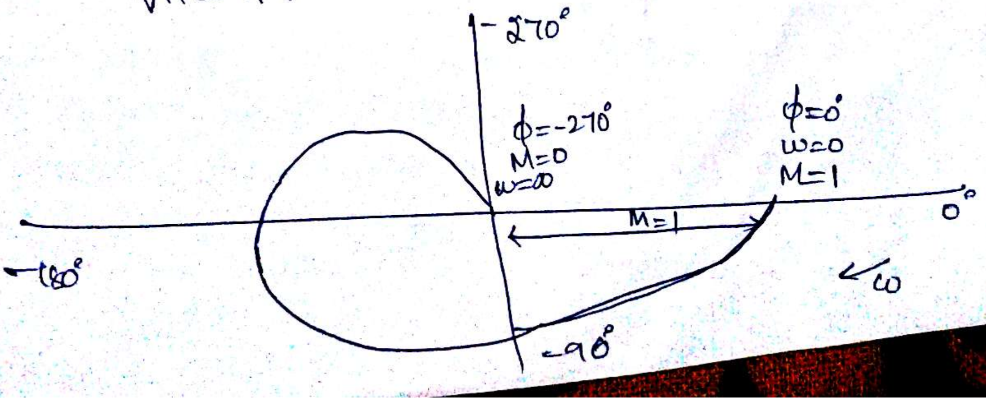
E.D; $\omega = \infty$, $M = 0$, $\phi = -180^\circ$



3. $G(s)H(s) = \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$

$$G(j\omega)H(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$$

$$M = \frac{1}{\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}\sqrt{1+\omega^2T_3^2}} \quad ; \phi = -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2) - \tan^{-1}(\omega T_3)$$



Note:

→ when a pole is added to the system, the polar plot end point is shifted by -90°.

→ when zero is added to the system, the polar plot end point is shifted by $+90^\circ$.

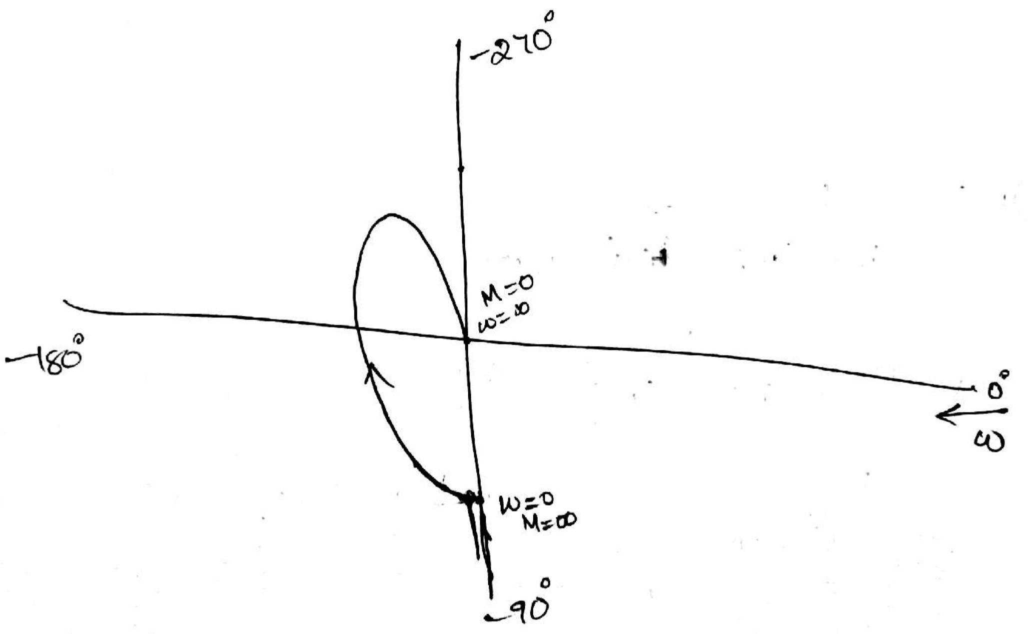
Q: $G(s)H(s) = \dfrac{1}{S(1+ST_1)(1+ST_2)}$ → Type-1, order-3

$$G(j\omega)H(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)\, j\omega}$$

$$M = \frac{1}{\omega\sqrt{1+\omega^2T_1^2}\,\sqrt{1+\omega^2T_2^2}}, \quad \phi = -90^\circ - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

$\omega = 0 \Rightarrow M = \infty$, $\phi = -90^\circ$

$\omega = \infty \Rightarrow M = 0$, $\phi = -270^\circ$.



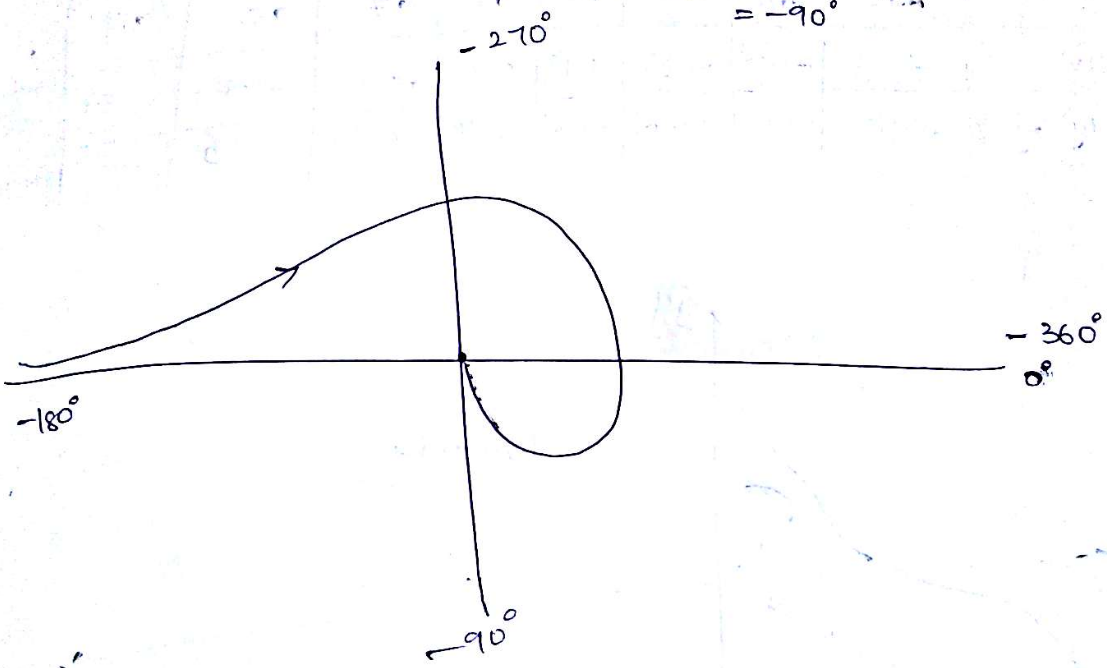
Q: $G(s)\cdot H(s) = \dfrac{1}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$ → Type-2, Order-5

$$M = \frac{1}{\omega^2\sqrt{1+\omega^2T_1^2}\,\sqrt{1+\omega^2T_2^2}\,\sqrt{1+\omega^2T_3^2}}$$

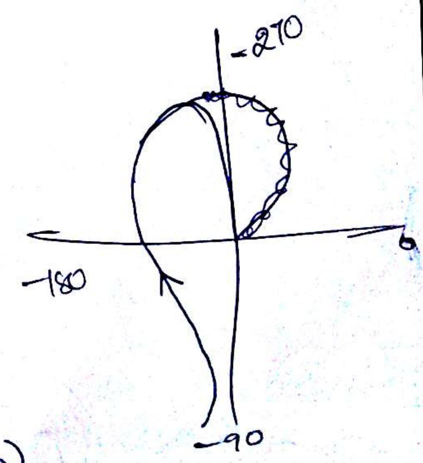
$$\phi = -90^\circ - 90^\circ - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2) - \tan^{-1}(\omega T_3)$$

$\omega = 0$, $M = \infty$, $\phi = -180^\circ$

$\omega = \infty$, $M = 0$, $\phi = -180^\circ - 270^\circ = -360^\circ - 90^\circ$
$= -90^\circ$



Q: $G(s)\,H(s) = \dfrac{1}{s(1+s)(1+2s)}$



$$G(j\omega)\,H(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

$$M = \frac{1}{\omega\sqrt{1+\omega^2}\,\sqrt{1+4\omega^2}}$$

$$\phi = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega).$$

$\omega = 0$, $M = \infty$, $\phi = -90^\circ$

$\omega = \infty$, $M = 0$, $\phi = -270^\circ$

$r = M + j\phi$. $\quad r\angle\theta \rightarrow$ polar Coordinates

ω	0.35	0.4	0.45	0.5	0.6	0.7	0.8	0.9	1
M	2.2	1.8	1.5	1.26	0.9	0.7	0.5	0.4	0.3
ϕ	−144.4	−150	−156	−161	−171	−179	−186	−192	−198

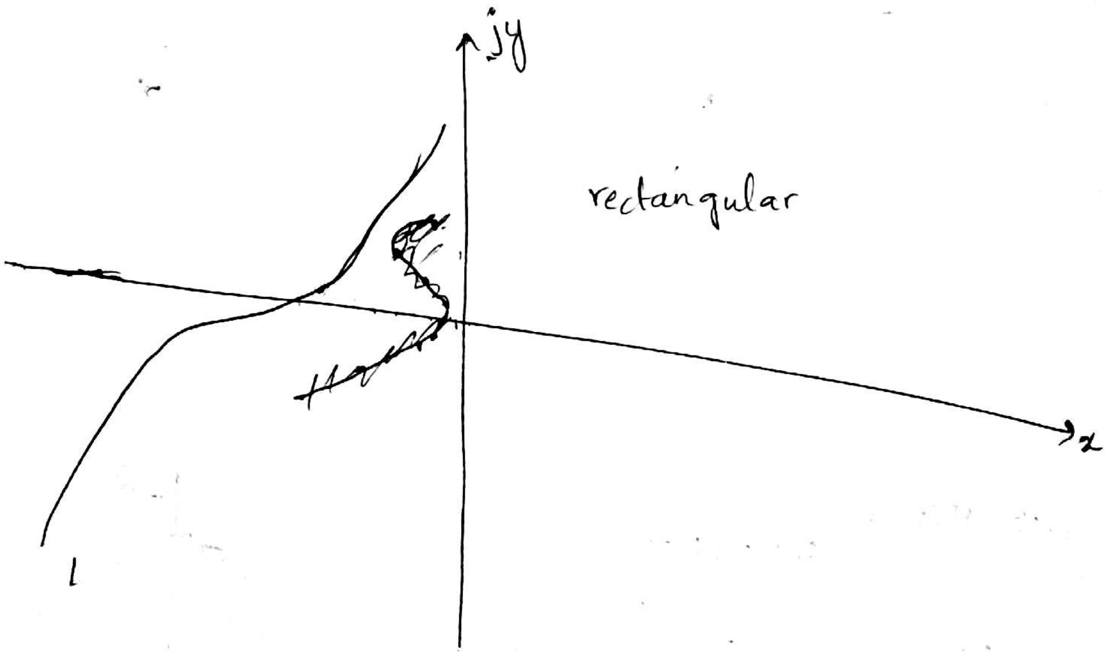
Rect = $x + jy$

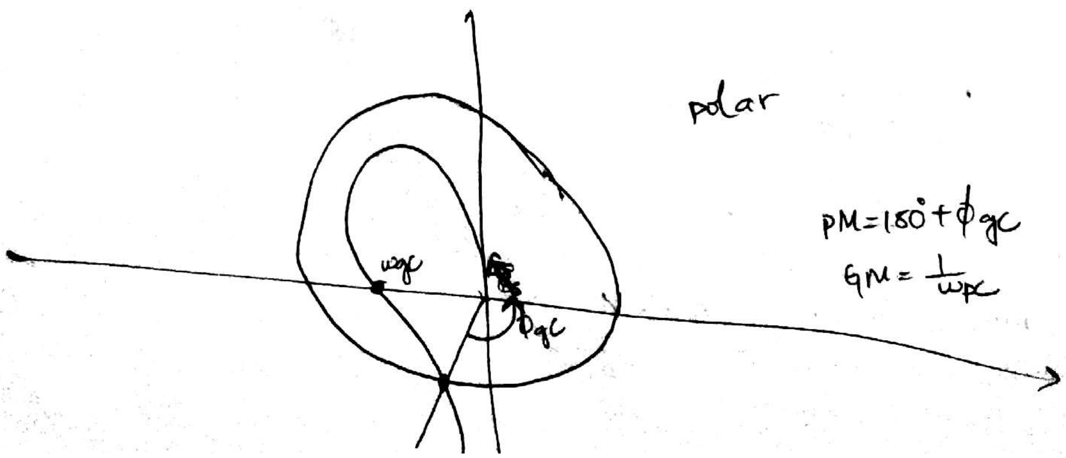
$= M(x) + j\,\phi(y)$

Rect

Rectangular Coordinates.

ω	0.35	0.4	0.45	0.5	0.6	0.7	0.8	0.9	1
M(x)	−1.78	−1.55	−1.37	−1.19	−0.88	−0.69	−0.498	−0.39	−0.28
$\phi(y) = j\theta$	−1.28	−0.9	−0.61	−0.41	−0.14	−0.012	0.051	0.08	0.097





$PM = 180^\circ + \phi gc$

$GM = \frac{1}{\omega pc}$

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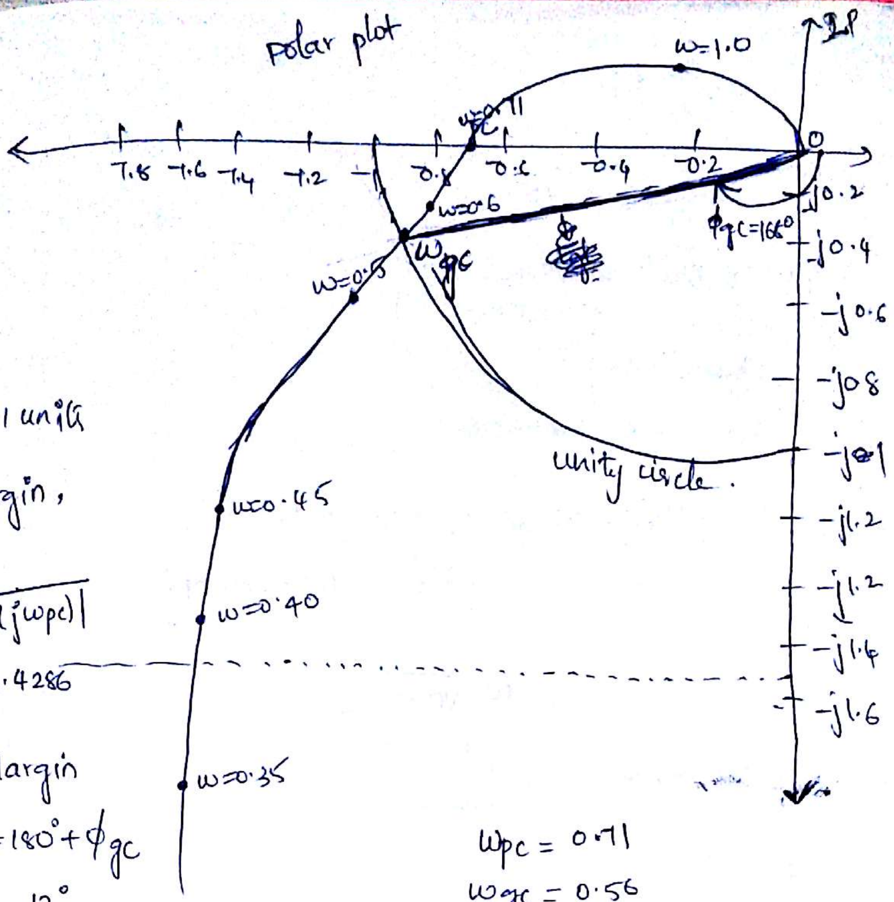


scale:

1 cm = 0.1 units

Gain Margin,

$$k_g = \frac{1}{|G(j\omega_{pc})|}$$

$$= 1.4286$$

phase Margin

$$\Rightarrow \gamma = 180^\circ + \phi_{gc}$$

$$\gamma = 12^\circ$$

$\omega_{pc} = 0.71$

$\omega_{gc} = 0.56$

$$GM = \left(\frac{1}{\cancel{\omega_{gc}}} \cancel{= \frac{1}{0.6}}\right) \quad \frac{1}{\omega_{pc}} = \frac{1}{0.71} = \frac{1}{\text{Touches real axis}}$$

$$PM = 180^\circ + \phi_{gc} = 180^\circ - 168^\circ = 12^\circ.$$

ω_{gc} = unit circle Tangent

PM = 180° + Tangent angle

Theoritical:

$$-90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega) = -180^\circ$$

$$\tan^{-1}\left(\frac{2\omega}{1-2\omega^2}\right) = 90^\circ$$

$$1 - 2\omega^2 = 0$$

$$\omega_{pc} = \frac{1}{\sqrt{2}} = 0.707$$

$$|G(j\omega)| = \frac{1}{\frac{1}{\sqrt{2}}\sqrt{1+\frac{1}{2}}\sqrt{1+4\times\left(\frac{1}{2}\right)^2}}$$

$$|G(j\omega)| = 2/3 \Rightarrow GM = 3/2$$

$$\omega\sqrt{1+\omega^2}\ \sqrt{1+4\omega^2} = 1$$

$$\omega^2(1+\omega^2)(1+4\omega^2)=1$$

$$4\omega^6+5\omega^4+\omega^2-1=0$$

$$x=\omega^2$$

$$4x^3+5x^2+x-1=0$$

$$\omega_1^2=0.32,\quad \omega_2^2=-0.78+0.37j$$

$$\omega_3^2=-0.78-0.37j$$

$$\omega=\sqrt{0.32}$$

$$\omega=0.56.$$

27/2/18

Sketch the polar plot of

$$G(s)=\frac{(1+0.2s)(1+0.025s)}{s^3(1+0.005s)(1+0.001s)}$$

& find the ph[ase] margin.

$$G(j\omega)=\frac{(1+0.2j\omega)(1+0.025j\omega)}{(j\omega)^3(1+0.005j\omega)(1+0.001j\omega)}$$

$$M=\frac{\sqrt{1+(0.2\omega)^2}\ \sqrt{1+(0.025\omega)^2}}{\omega^3\sqrt{1+(0.005\omega)^2}\ \sqrt{1+(0.001\omega)^2}}$$

$$\phi=\tan^{-1}(0.2\omega)+\tan^{-1}(0.025\omega)-270^\circ-\tan^{-1}(0.005\omega)-\tan^{-1}(0.00[1\omega])$$

$\omega=0,\ M=\infty,\ \phi=-270^\circ$

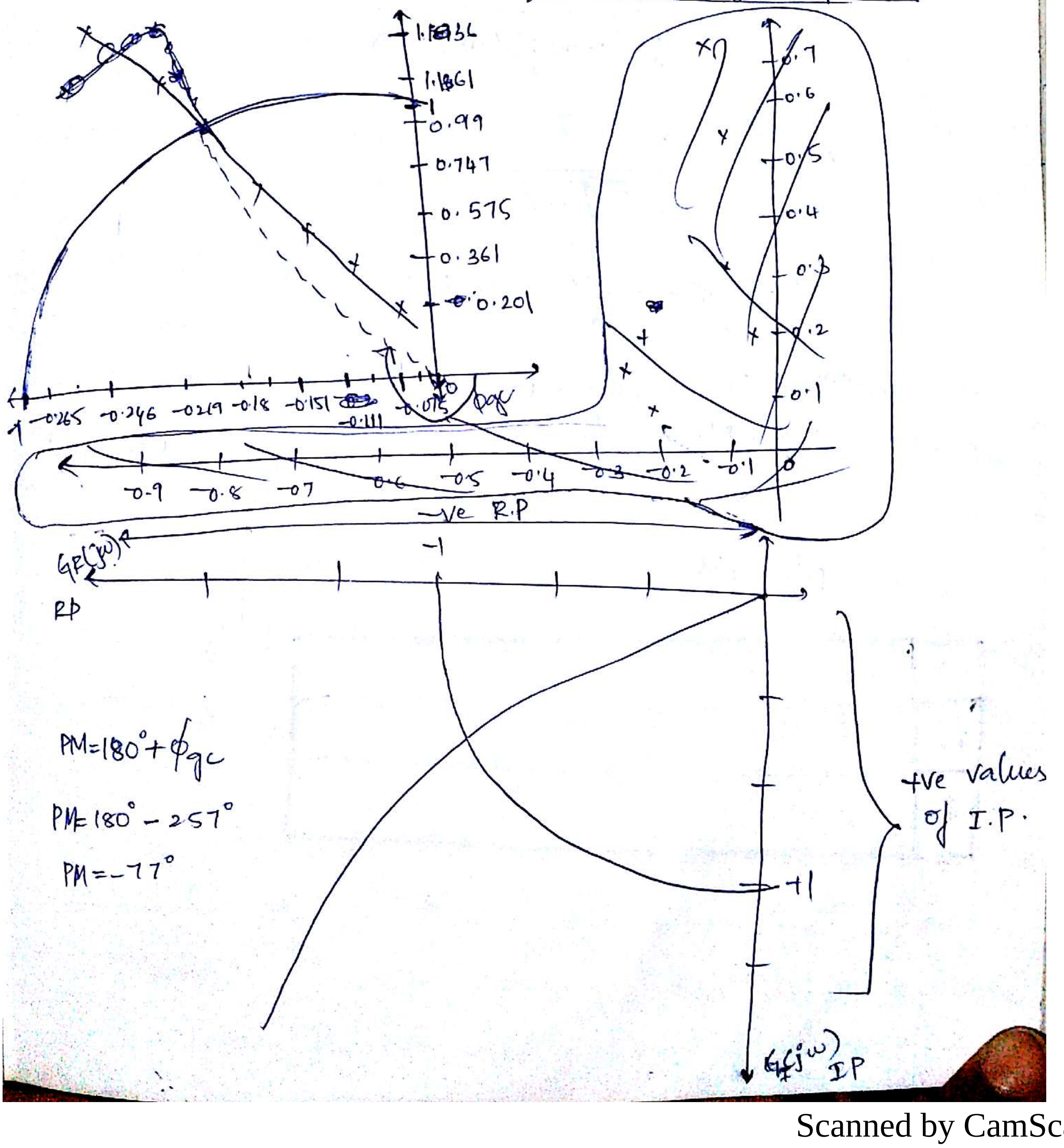
$\omega=\infty,\ M=0,\ \phi=-270^\circ$

Magnitude vs Phase angle.

ω	0.9	0.95	1	1.1	1.2	1.4	1.7
$\lvert M \rvert$	1.39	1.187	1.02	0.769	0.595	0.378	0.215
ϕ (deg)	-258.8	-258.2	-257.6	-256.39	-255.19	-2[illegible].82	-249.31

Real & Imaginary part of $G(j\omega)$ → Rec

ω	0.9	0.95	1	1.1	1.2	1.4	1.7
$G_R(j\omega)$ x	-0.265	-0.246	-0.219	-0.18	-0.151	-0.111	-0.075
$G_I(j\omega)$ y	1.36	1.161	0.99	0.747	0.575	0.361	0.201



$PM = 180^\circ + \phi_{gc}$

$PM = 180^\circ - 257^\circ$

$PM = -77^\circ$

Q: Consider a unity f/b system having open loop T.F.

$$G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$$. Sketch the polar plot.

Determine the value of 'k' so that ; Gain Margin = 18 dB & phase Margin = 60°.

Assume $k = 1$;

$$G(s) = \frac{1}{s(1+0.2s)(1+0.05s)}$$

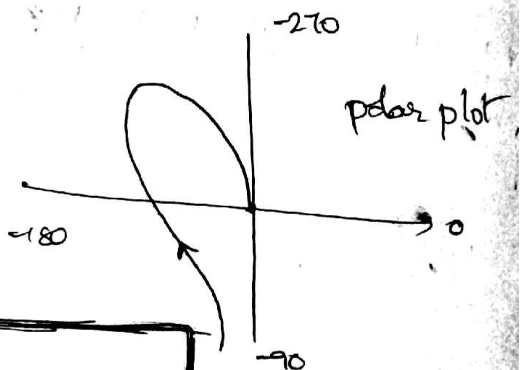
$$G(j\omega) = \frac{1}{j\omega(1+0.2j\omega)(1+0.05j\omega)}$$

$$M = \frac{1}{\omega\sqrt{1+(0.2\omega)^2}\sqrt{1+(0.05\omega)^2}}$$

$$\phi = -90° - \tan^{-1}(0.2\omega) - \tan^{-1}(0.05\omega)$$

$\omega = 0$, $M = \infty$, $\phi = -90°$

$\omega = \infty$, $M = 0$, $\phi = -270°$.

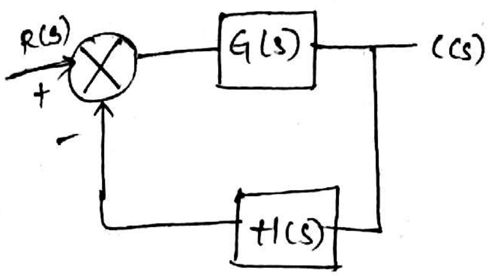


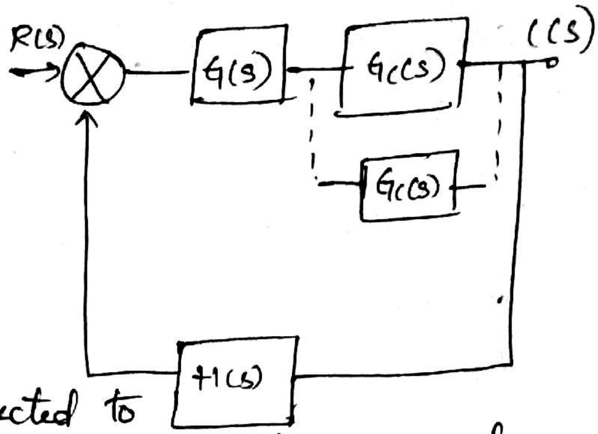
polar plot

polar plot

ω	
M	
ϕ	

Compensators & Controllers:





Compensator:

The n/w which is connected to system (which contains finite no of poles & zeros) to obtain stability is called Compensator.

- HPF – lead compensator – Transient Response Improvement
- LPF – lag " – steady state " "
- BSF – lag-lead " – Both tr & SSR " "
- BPF – lead-lag " – (practically, not available)

→ Compensators are corrective sub-system introduced ~~into~~ the system to compensate for the deficiency in performance of the plant (G(s)) (or) system.

→ So, Given a plant & a set up specifications, suitable Compensators are to be designed so that, the overall system will meet the given specifications.

→ Proper selection of performance specifications is the most important step in the design of compensators.

→ The desired behaviour of a system is specified in terms of transient response & Steady state Response (SSR) and steady state error.

→ The transient response measures relative stability & speed of a response ~~System can be defined by transient Response~~. may be specified in time (or) freq. domain

→ In Time domain, the measure of relative stability is; damping ratio (ξ) (or) peak overshoot time. The speed of response is measured in rise time (t_r), Settling time (t_s) & natural freq. (ω_n).

In freq. domain, measure of relative stability is; phase Margin, Gain Margin & Resonant peak. The speed of response is measured Resonant freq., Bandwidth

Once, set of performance specifications are selected, next step is to choose the compensator. Usually, electrical Compensators are preferred even though Mechanical, hydraulic & chemical (or) other types can be used the compensation may be cascade (or) Series Combination.

GM & PM $= \begin{cases} >0 \rightarrow \text{stable} \\ <0 \rightarrow \text{Unstable} \\ =0 \rightarrow \text{Marginally stable.} \end{cases}$

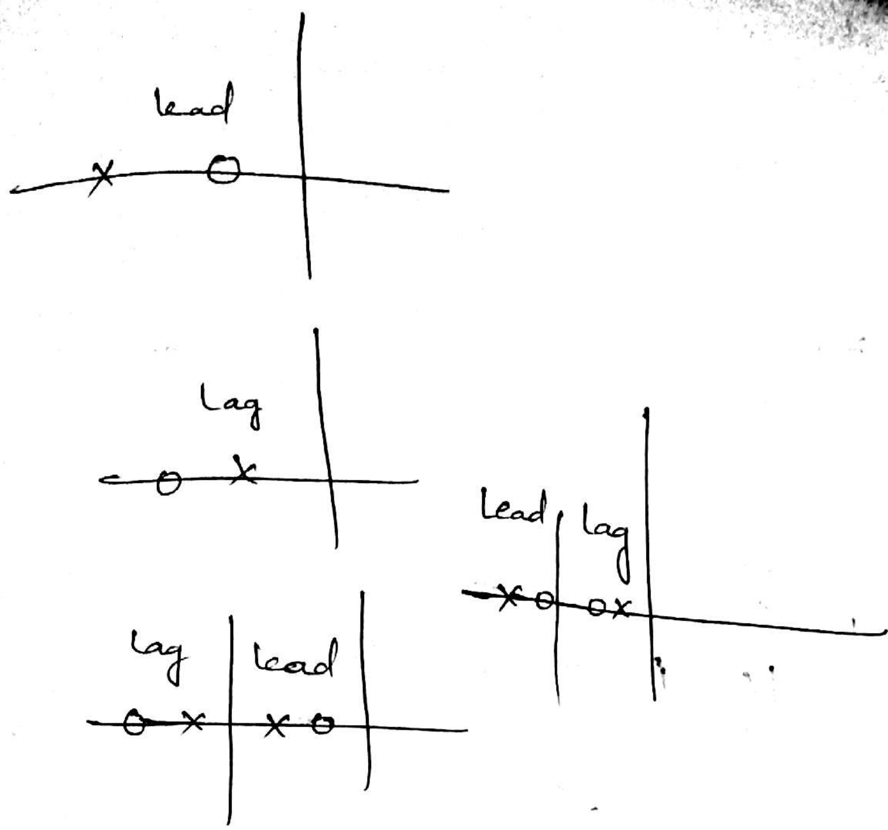
Types of Compensators:

→ A Compensator is an electrical n/w which has finite pole & finite zero to the system. There are 3 types:

HPF – 1. Lead compensator

LPF – 2. Lag "

BSF – 3. Lag-lead "

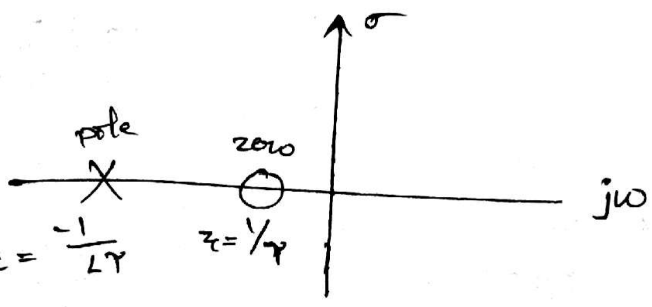


Lead Compensator:

when; a sinusoidal i/p is applied to a n/w, to produce the sinusoidal steady state o/p having phase lead w.r.to i/p, then compensator is called lead Compensator.

lead compensator improves the transient response & increases margin of stability of a system and also ~~(improves)~~ increase the system error constant through a limited range.

Pole-zero diagram of lead Compensator:

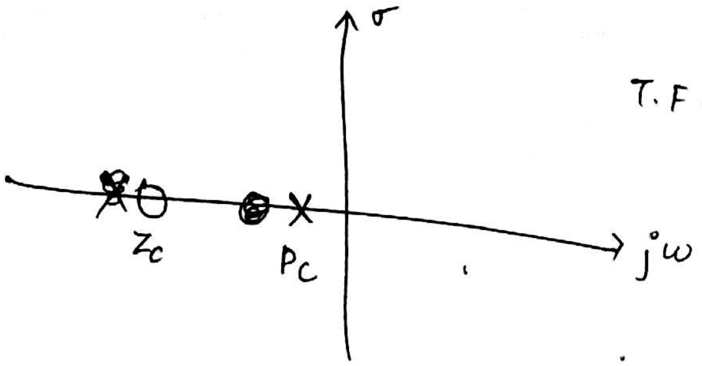


T.F; $\frac{s+Z_c}{Z+P_c}$

$Z_c > P_c$

$\Rightarrow \frac{Z_c}{P_c} > 1$

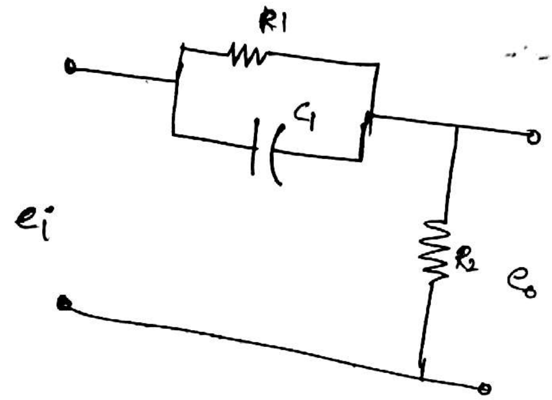
Lag Compensator:



T.F. $\dfrac{s+z_c}{z+P_c}$

$\dfrac{z_c}{P_c} < 1$

Q:



Lead compensator.

$$\frac{e_o(s)}{e_i(s)} = \frac{R_2}{R_2 + R_1 \| \frac{1}{CS}}$$

$$= \frac{R_2}{R_2 + \dfrac{R_1 \times 1/CS}{R_1 + 1/CS}}$$

$$= \frac{R_2(1 + SCR_1)}{R_1 + R_2 + SCR_1R_2}$$

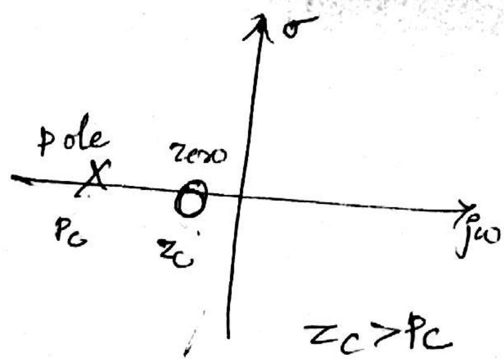
$$= \frac{R_2}{R_1+R_2} \frac{(1 + SCR_1)}{\left(1 + SC\left(\frac{R_1R_2}{R_1+R_2}\right)\right)}$$

let $T = R_1C$

$\alpha = \dfrac{R_2}{R_1+R_2}$

$$T.F = \alpha\left[\frac{1+ST}{1+\alpha ST}\right]$$

$$T.F(j\omega) = \alpha\left[\frac{1+j\omega T}{1+\alpha j\omega T}\right]$$



$$M = \frac{\alpha\sqrt{1+(\omega T)^2}}{\sqrt{1+(\alpha\omega T)^2}}$$

$$\phi = \tan^{-1}(\omega T) - \tan^{-1}(\alpha\omega T)$$

$$\frac{d\phi}{d\omega} = 0$$ Max^m phase lead occurs.

$$\tan\phi = \frac{\omega T - \alpha\omega T}{1+\omega^2\alpha T^2}$$

$$\frac{d}{d\omega}\left[\frac{\omega T - \alpha\omega T}{1+\omega^2\alpha T^2}\right] = 0.$$

$$\frac{d\phi}{d\omega} = \frac{(1+\omega^2\alpha T^2)(1-\alpha)T - \omega T(1-\alpha)\cdot 2\omega\alpha T^2}{1+\omega^2\alpha T^2} = 0$$

$$\boxed{\omega = \frac{1}{T\sqrt{\alpha}}}$$

OR $\omega_m = \dfrac{1}{T\sqrt{\alpha}}$

$$\omega_{c_1} = \frac{1}{T}$$

$$\omega_{c_2} = \frac{1}{\alpha T}$$

$$\omega_m^2 = \omega_{c_1}\cdot\omega_{c_2} = \frac{1}{\sqrt{T\alpha T}} = \frac{1}{T\sqrt{\alpha}}$$

$$\tan\phi_m = \frac{\omega_m T(1-\alpha)}{1+\omega_m^2\alpha T^2}$$

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$$\tan \phi_m = \frac{\frac{1}{T\sqrt{\alpha}} \; T(1-\alpha)}{1 + \frac{1}{T^2\alpha} \; \alpha T^2}$$

$$\tan \phi_m = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\sin \phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\sin\phi_m (1+\alpha) = 1-\alpha$$

$$\sin\phi_m + \sin(\phi_m)\alpha = 1-\alpha$$

$$\sin\phi_m - 1 = -\alpha\,[1+\sin\phi_m]$$

$$\cancel{\sin\phi_m + 1 = -\alpha + \alpha \sin\phi_m}$$

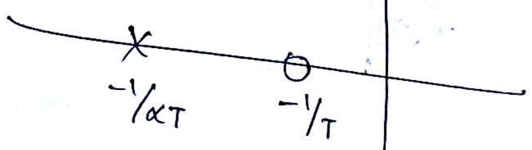
$$-(1-\sin\phi_m) = -\alpha\,[1+\sin\phi_m]$$

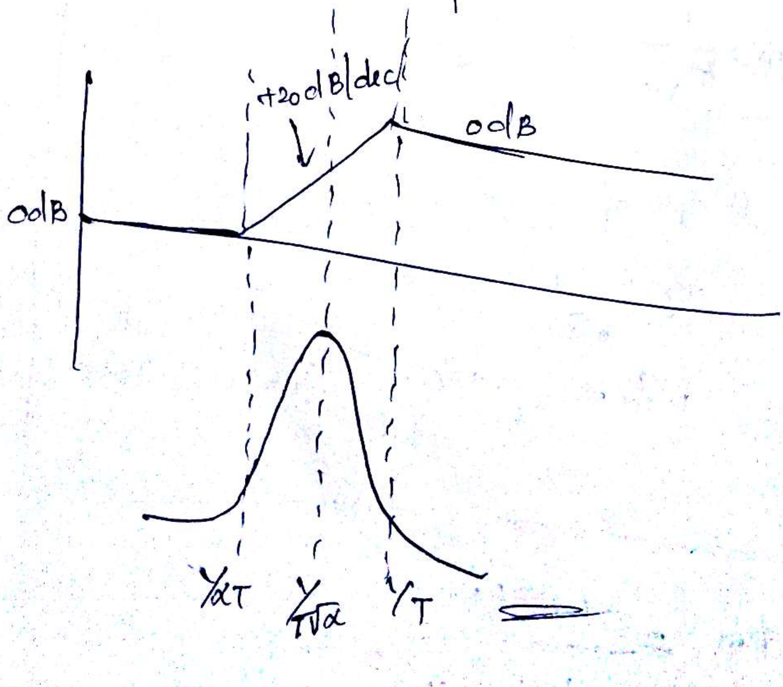
$$[1+\sin\phi_m] = -\alpha\,[1-\sin\phi_m]$$

$$|\alpha| = \left| \frac{1-\sin(\phi_m)}{1+\sin(\phi_m)} \right|$$

$\downarrow$ lead constant

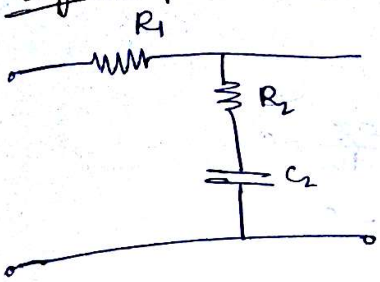
Lead compensator





→ 'α' value should not be less than 0.07, optimal value of $\alpha = 0.5$.

Lag compensator:



When a sine i/p is applied to a n/w to produce sine steady state o/p having phase lag w.r.t i/p then it is called lag compensator.

$$\frac{e_o(s)}{e_i(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{sR_2C_2 + 1}{sR_1C_2 + sR_2C_2 + 1}$$

$$= \frac{1}{\frac{R_1+R_2}{R_2}} \left[\frac{s + \frac{1}{R_2C}}{s + \frac{1}{\left(\frac{R_1+R_2}{R_2}\right) R_2 C}} \right]$$

$$T = R_2 C, \quad \beta = \frac{R_1 + R_2}{R_2} > 1$$

$$G_c(s) = \frac{1}{\beta}\left[\frac{s + \frac{1}{T}}{s + \frac{1}{T\beta}}\right] \therefore \frac{1}{\beta}\left[\frac{s + \frac{1}{T}}{s + \frac{1}{T\beta}}\right] = \frac{Ts+1}{\beta Ts+1}$$

sinusoidal T.F of lag n/w is given by

$$G_c(j\omega) = \frac{1 + j\omega T}{1 + j\beta\omega T}$$

Since $\beta > 1$, the SSR has a lagging phase angle w.r.t sine i/p and hence the name 'lag n/w'

$$M = |G_c(j\omega)| = \left|\frac{(1+j\omega T)}{1+j\omega\beta T}\right| = \frac{\sqrt{1+\omega^2T^2}}{\sqrt{1+\omega^2\beta^2T^2}}$$

$$\phi = \tan^{-1}\omega T - \tan^{-1}\omega\beta T$$

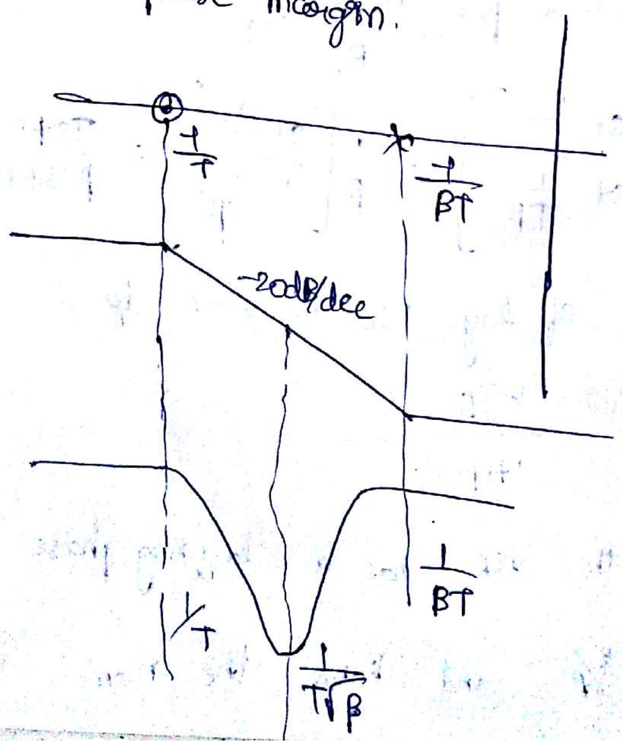
To find max. phase angle $\frac{d\phi}{d\omega} = 0$

$$\frac{d}{d\omega}\left[\tan^{-1}\omega T - \tan^{-1}\omega\beta T\right] = 0$$

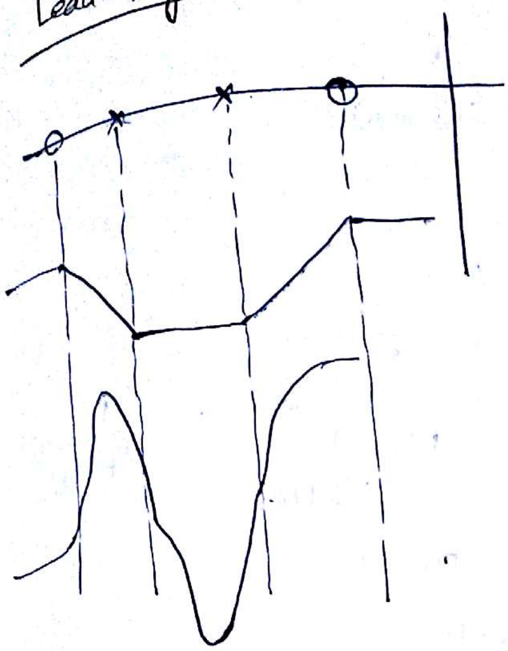
$$\frac{\omega T - \beta\omega T}{1-\omega T.\omega\beta T} \Rightarrow 0$$

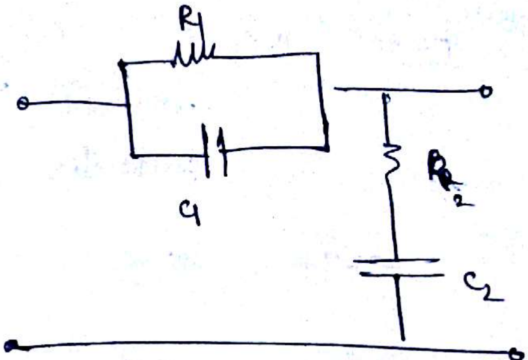
$$\omega \Rightarrow \omega_m = \sqrt{\frac{1}{T}\cdot\frac{1}{T\beta}} = \frac{1}{T\sqrt{\beta}}$$

This is frequency at which the phase lag is at its max. Thus ω_m is the geometric mean of two corner frequencies. Note that the primary f_n of a lag compensator is to provide attenuation in the high frequency range to give a system sufficient phase margin.



Lead-lag compensator:

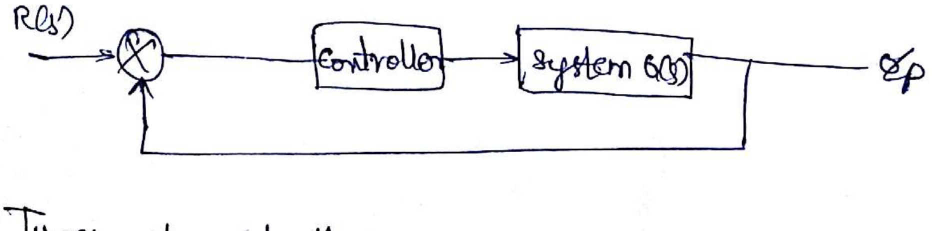




Lag-lead.

Controllers:- A controller is a device which is used to controlled transient of SSR. The best system is the one having smallest rise time, settling time, smallest s.s error and smallest peak overshoot.

To meet the above requirement we require to add ~~a~~ controller to the system



Types of controller:

(i) **Proportional controller (k_p):**

Used to change the TR as per the requirement of the T.F of P controller is k_p

OLTF $G(s)$ / without controller $= \frac{1}{s(s+10)}$

~~CLTF = G(s)~~

$$CLTF = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{s(s+10)}}{1+\frac{1}{s(s+10)}} = \frac{1}{s^2+10s+1}$$

$\omega_n = 1$

$2\xi\omega_n = 10$

$\Rightarrow \xi = 5$

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