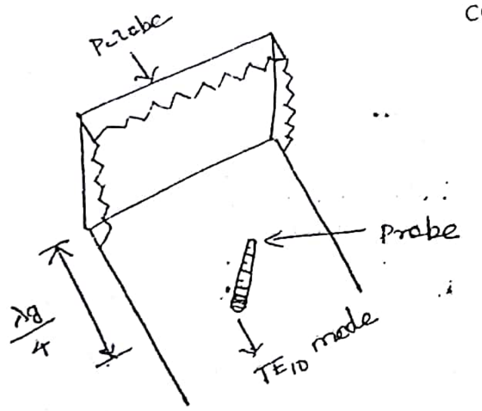
UNIT - III WAVEGUIDE COMPONENTS

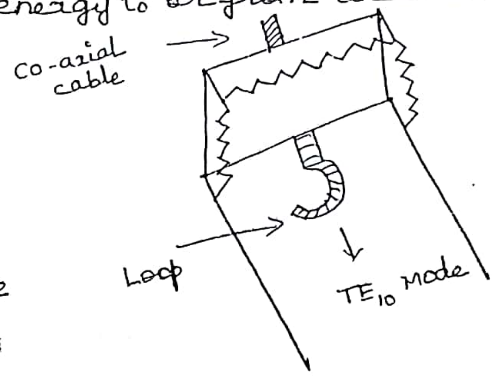
* Coupling Mechanisms - probe, loop, Aperture types
* Waveguide Discontinuties - waveguide windows
* Tuning screws, posts, Matched Loads
* Waveguide Attenuators - Different types - Resistive card and Rotary vane Attenuators
* Waveguide phase shifters - Types, Dielectric and Rotary vane phase shifters

COUPLING MECHANISMS

PROBES AND LOOPS:

* A coupling probe is positioned in a waveguide or cavity Resonator with the goal to transfer energy to or from an external circuit.



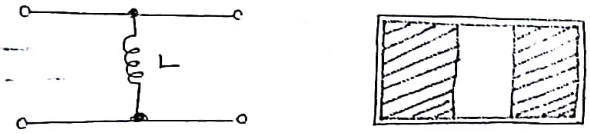


* When a short antenna in the form of a probe or a loop is inserted into a waveguide, it will radiate and if it is placed correctly, the desired mode will be set up.
* From above figure, the probe is placed at a distance of $\lambda g/4$ from the shorted end of the waveguide and the centre of broader dimension of the waveguide, because at that point electric field is maximum.
* This probe will now act as an antenna, which is polarized in the plane parallel to the electric field.
* The coupling loop placed at the centre of shorted end plate of the wave guide is used to launch TE_{10} mode.
* coupling is achieved by means of a loop antenna located in a plane perpendicular to the plane of the probe.

WAVEGUIDE IRISES -

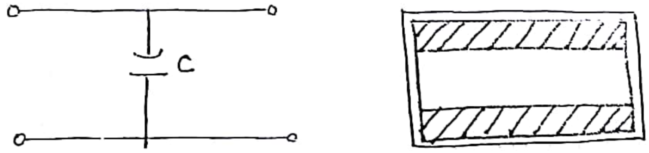
* In any waveguide system, when there is a mismatch, there will be reflections.
* In waveguide, some discontinuities are made use of for some matching purposes.

Inductive Iris:



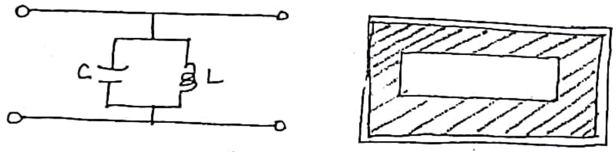
* The Iris is placed in a position where magnetic field is strong and electric field is weak.
* Since the plane of polarization of Electric field is parallel to the Iris, the current flow due to Iris causes a magnetic field to set up.

Capacitive Iris:



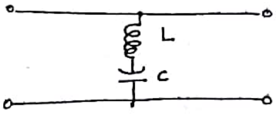
* The Iris is placed in a position where electric field is strong and magnetic field is weak.
* The potential existing between the top and the bottom of the waveguide now exists between the surfaces which are closer and therefore the capacitance has increased at that point.

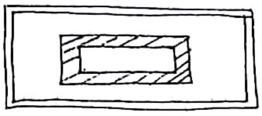
Inductive and capacitive Iris: (Parallel)



* The portion across both magnetic and electric planes form an parallel LC circuit.
* For dominant mode, the iris offers high impedance and for other modes it completely attenuates the wave.
* It acts as a Band Pass Filter to supress noise.

Inductive and capacitive Iris (series):

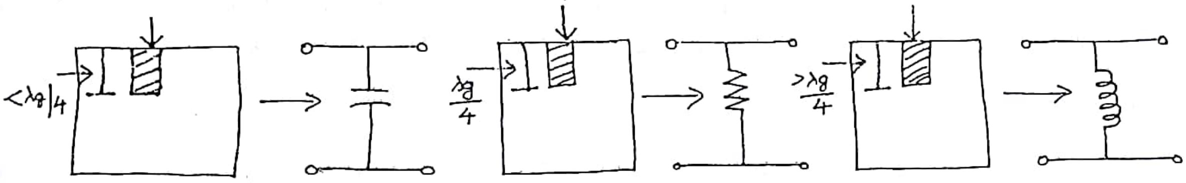


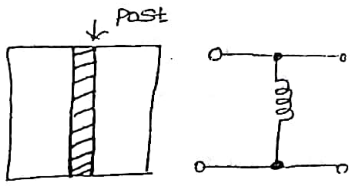


* It is supported by a non metallic material and is transparent to the flow of Microwave energy.

POSTS AND TUNING SCREWS:

POSTS:

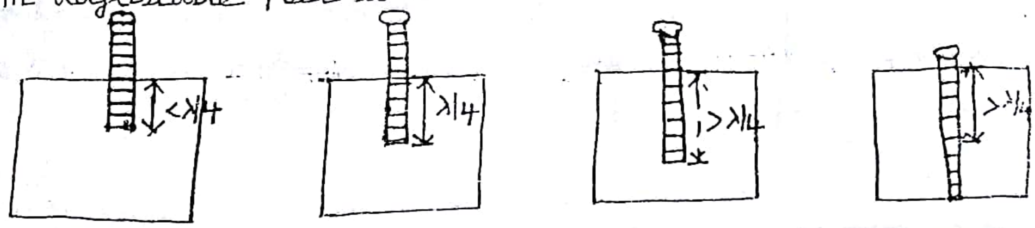




* If the post extends only a short distance $(<\lambda g/4)$, it behaves CAPACITIVE in nature.
* when the depth is equal to $(\lambda g/4)$, the post acts as a SERIES RESONANT circuit.
* when the depth is greater than $(\lambda g/4)$, it behaves INDUCTIVELY.
* when the post is extended completely across the waveguide, the post becomes INDUCTIVE.
* The big advantage of a post over an IRIS is that it is readily adjustable.

SCREWS:

* An adjustable post is known as a SCREW.



* Depending upon the depth of penetration, the tuning screw may introduce inductive or capacitive susceptance.

MICROWAVE ATTENUATORS:

* For perfect matching, we require that the μ wave power in a waveguide be absorbed completely without any reflection. For this we make use of Attenuators.

* Attenuators are commonly used for

  ⇒ Measuring power Gain or Loss in decibels

  ⇒ Reducing power input to a particular stage to prevent overloading

  ⇒ Providing the signal Generators with means of callibrating their outputs accurately so that precise measurement could be made.

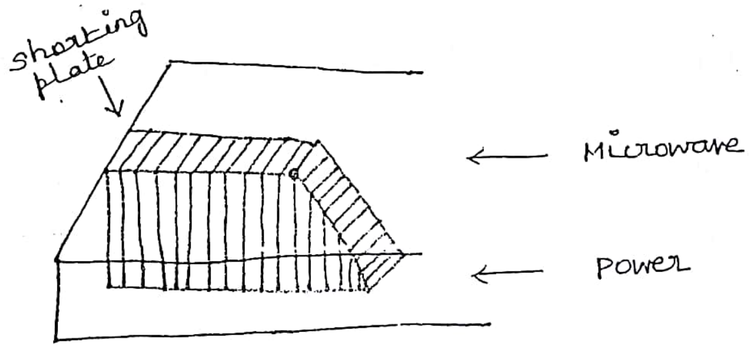
* Attenuators can be classified as

  ⇒ Fixed Attenuators

  ⇒ Variable Attenuators

Fixed Attenuators:

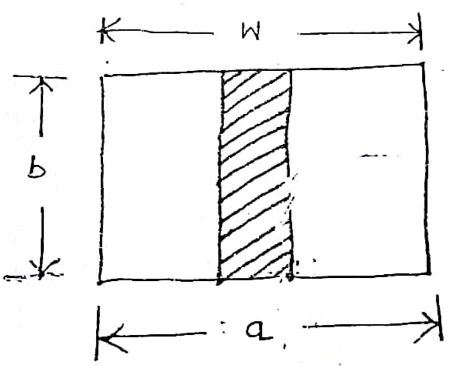
* These are used where fixed amount of attenuation is to be provided. If such a ~~waveguide~~ Fixed Attenuator absorbs all the energy entering into it, we call it as a waveguide Terminator.

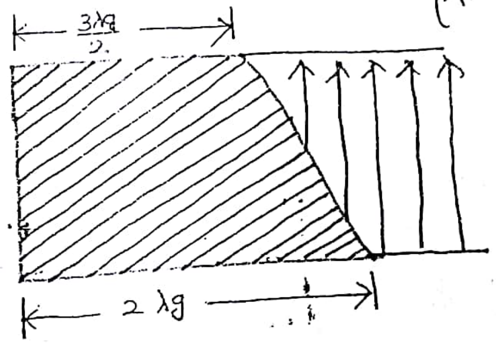


(Transverse Electric Mode TE mode).

$E_x = E \times e^{-\alpha z}$

$\alpha = \sigma + j\beta$

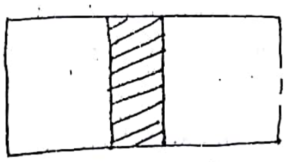




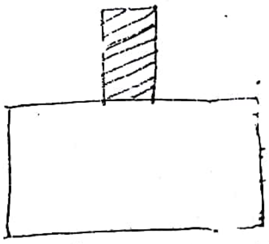
* It is a waveguide with a tapered plug of absorbing material at the end.
* The tapering is done for providing a gradual transition from the waveguide medium to the absorbing medium thus reducing the reflection occuring at the media interface.

Variable Attenuators:

* Provide continuous or step wise variable attenuation.
* For Rectangular waveguides, these attenuators can be flap type or vane type.
* Flap type attenuator consists of a
  → Resistive element
  → slot
  → Locking screw
  → Adjusting knob
* The degree of attenuation is determined by the depth of the insertion of flap.

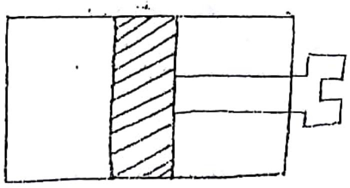


Maximum Attenuation

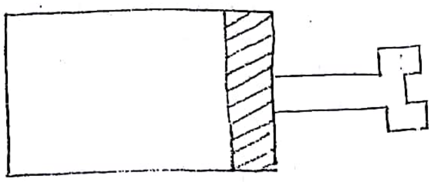


Minimum Attenuation

Vane Type Attenuator:



Maximum Attenuation

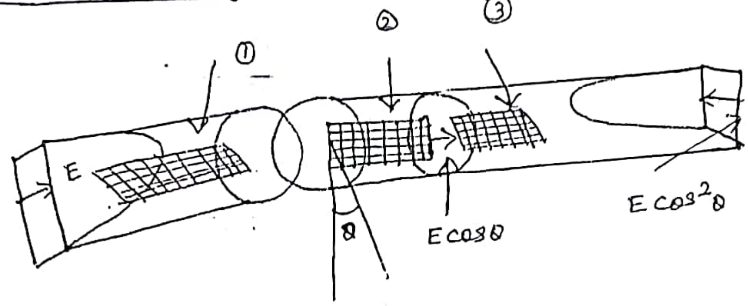


Minimum Attenuation

* The vane type attenuator consists of a glass vane with a coating of carbon
* If the vane used at the center is made movable, it can be used as a variable attenuator.

* The vane positioned at the centre of the waveguide can be moved laterally from the centre, where it provides maximum attenuation to the edges. where the attenuation is considerably reduced since the electric field lines are concentrated at the centre of the waveguide.
* An adequate match is obtained if the taper length is made equal to $\lambda g/2$.

Resistive Rotary vane:



* A Resistive rotary vane attenuator provides precision attenuation with an accuracy of ± 2.1%, over the operating frequency range.
* It consists of 3 vanes. The central vane rotating type placed in the central section of the waveguide arrangement and is tapered at both ends.
* when all 3 vanes have their planes aligned 90° to the direction of the Electric field, there is no attenuation.
* vane 1 prevents any horizontal polarization and hence electric field at the output of vane 1 is vertically polarised.
* The centre vane 2 is rotating type and if it is rotated by an angle θ, the E sin θ componenent is attenuated and E cos θ component is present at the output of vane 2 and the final output of the attenuator becomes $E\cos^2\theta$, which has the same polarisation as the input wave.

PHASE SHIFTERS:

* Phase shifters are used to change the transmission angle of a network.
* Many applications require phase shifts to be introduced between two given positions in a waveguide system.
* The phase shift required may be Fixed or variable.
* Since phase constant β is inversely proportional to Guide wavelength (λg), the magnitude of λg could be changed to obtain variable amounts of phase shift.

* Fixed amount of phase shift can be obtained by

Use of capacitive / Inductive Iris in the waveguide

Inserting dielectric rods across the diameters of a circular wavegui

Reducing wider dimension of a Rectangular waveguide

UNIT - IV

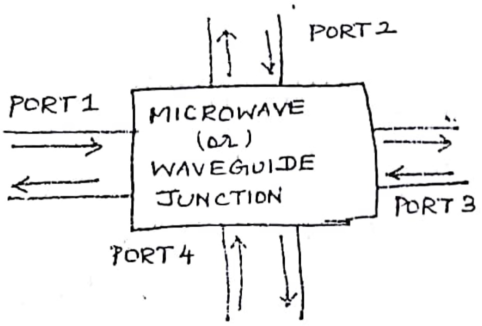
S MATRIX OF MULTIPORT DEVICES

* Scattering Matrix - significance, Formulation and properties
* Directional coupler - 2 hole and Bethel hole.
* E plane Tee, H plane Tee and MAGIC TEE
* Ferrite composition and properties, Faraday Rotation.
* Ferrite components - Gyrator, Isolator, circulator and

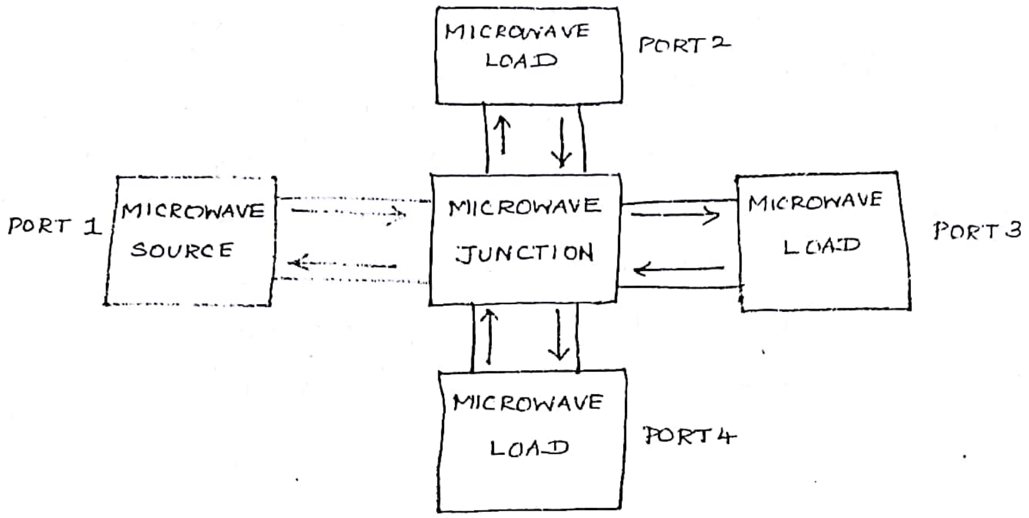
INTRODUCTION TO MICROWAVE COMPONENTS:

* Microwave systems normally consist of several microwave components including source and Load, connected to each other by waveguide or co-axial or Transmission Line systems.
* All these components must be built with
  - ⟹ Low standing wave Ratio (SWR)
  - ⟹ Low Attenuation
  - ⟹ Low Insertion loss

WAVEGUIDE MICROWAVE JUNCTIONS:

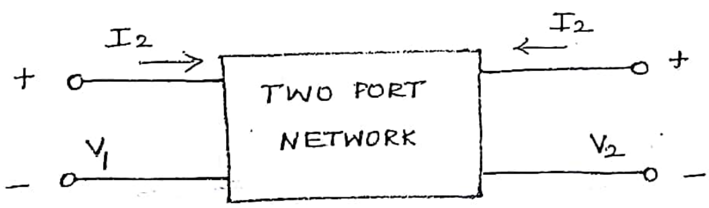


* In a waveguide system, many a times, there arise a need to split all or part of microwave energy into a particular direction.
* It is achieved by waveguides or Microwave junctions. These are combined to form coupler units that direct the energy as required.
* Even the same junction may be used to combine 2 or more signals. The following figure shows a Microwave source at port-1 and Microwave Load at ports 2, 3 & 4.

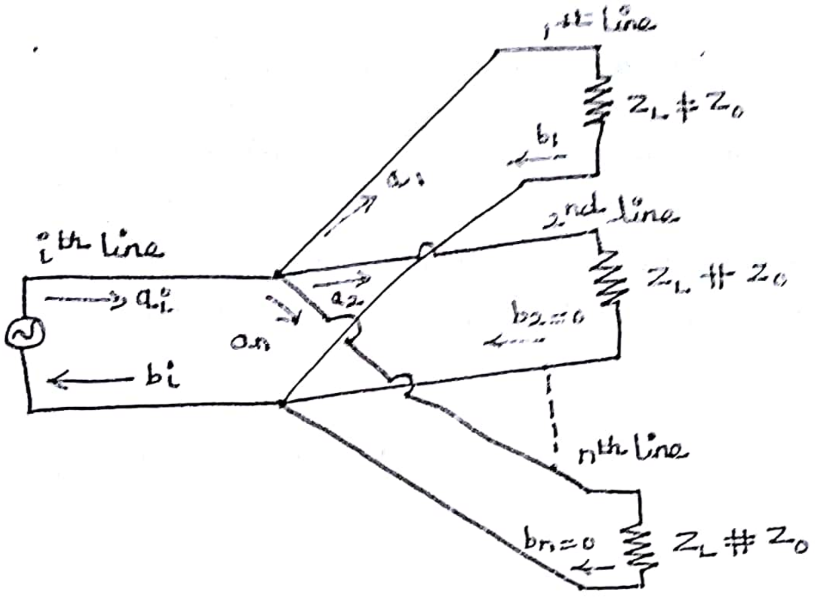


* When Input from Microwave source is applied at port 1, a part of it comes out of port 2, another part out of port 3, some part out of port 4 and the remaining part may come out of port 1 itself, due to mismatch between port 1 and Microwave junction.

SCATTERING PARAMETERS: (S Parameters):



* The Microwave junction can be defined by S parameters
* It is known that, for an input at one port, 4 outputs are possible. Similarly, if we apply input to all ports, 16 combinations are possible, leading to formulation of a Matrix called S MATRIX.
* S Matrix is a square matrix which gives all the combinations of power relationship between various input and output ports of a Microwave junction.
* The elements of this S Matrix are called SCATTERING CO-EFFICIENTS or SCATTERING PARAMETERS.
* To obtain the relationship between the scattering Matrix and the Input/output powers at different ports, consider a junction of 'n' number of transmission lines where i^{th} line is terminated by a source (where i can be any line between 1 to n).



Case:1 Let the first line be terminated in an impedance other than characteristic Impedance (ie) $Z_L \neq Z_0$) and all the remaining lines (2nd line to nth line) in an impedance equal to Z_0 (ie $Z_L = Z$

* If a_i be the incident wave at the junction due to the source at ith line, then it divides itself among (n-1) number of lines as $a_1, a_2 \ldots a_n$ as shown in above figure.
* There will be no reflections from 2nd to nth line and the incident wave are absorbed, since their impedances are equal to characteristic Impedance (Z_0).
* But there is a mismatch at the 1st line and hence there will be a reflected wave b_1 going back into the junction.

b_1 is related to a_1 by

$$b_1 = (\text{Reflection co-efficient}) \cdot a_1 \longrightarrow (1)$$

$$b_1 = S_{i1} \cdot a_1 \longrightarrow (2)$$

where

S_{i1} is the Reflection coefficient of the 1st line

1 is the reflection from first line

a_i is the source connected at ith line

$\therefore \; b_i = S_{i1} \cdot a_1 \quad [\because b_2 = b_3 = b_n = 0] \longrightarrow (3)$

Case 2: Let all $(n-1)$ lines be terminated in an impedance other than Z_0 (ie) $Z_L \neq Z_0$ for all lines).

* Then there will be reflections into junction from every line and hence the total contribution to the outward travelling wave in the i^{th} line is given by:

$$b_i = S_{i1} \cdot a_1 + S_{i2} \cdot a_2 + \cdots S_{in} \cdot a_n \longrightarrow (4)$$

i varies from 1 to n, so

$$\left.\begin{aligned} b_1 &= S_{11} a_1 + S_{12} a_2 + \cdots S_{1n} a_n \\ b_2 &= S_{21} a_1 + S_{22} a_2 + \cdots S_{2n} a_n \\ &\vdots \\ b_n &= S_{n1} a_1 + S_{n2} a_2 + \cdots S_{nn} a_n \end{aligned}\right\} \longrightarrow (5)$$

In Matrix form,

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \longrightarrow (6)$$

Column Matrix [b] corresponding to Reflected waves or output

scattering column Matrix [S] of order $n \times n$

Matrix [a] corresponds to Incident waves or Input

$$\therefore \boxed{[b] = [S][a]} \longrightarrow (7)$$

When a Junction of n number of waveguides are considered

a's represent inputs to particular ports

b's represents outputs out of various ports

S_{ij} corresponds to scattering coefficients resulting due to input at i^{th} port and output taken at j^{th} port.

S_{ii} denotes how much amount of power is reflected back from the junction into the i^{th} port, when the input power is applied at the i^{th} port itself.

PROPERTIES OF S-MATRIX:

1) S Matrix is always a square matrix of order $(n \times n)$.

2) S Matrix is a symmetric matrix

(ie) $S_{ij} = S_{ji}$ $\longrightarrow$ ⑥

3) S Matrix is a unitary matrix

$$[S][S]^* = [I] \longrightarrow ⑦$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{21}^* & S_{22}^* & S_{23}^* \\ S_{31}^* & S_{32}^* & S_{33}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where

[I] – unit Matrix or Identity matrix

4) Zero property

* The sum of the products of each term of any column (or row) multiplied by the complex conjugate of the corresponding terms of any other column (or row) is zero and it is

$$S_{11} S_{12}^* + S_{21} S_{22}^* + S_{31} S_{32}^* = 0 \longrightarrow ⑩$$

* Hence

$$S_{13} S_{23}^* = 0 \longrightarrow ⑪$$

* This means that either S_{13} or S_{23}^* or both should be zero

MICROWAVE TEE JUNCTIONS:

* A Tee junction is an intersection of three waveguides in the form of English alphabet T. There are several types of Tee Junctions

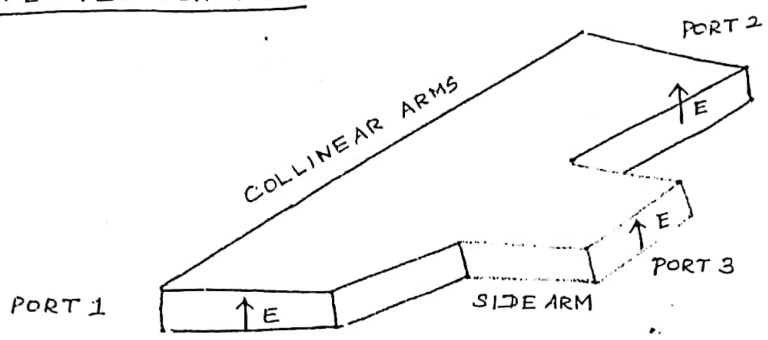
⇒ H plane Tee Junction

⇒ E plane Tee Junction

⇒ MAGIC Tee Junction

⇒ Rat Race Junction

H PLANE TEE JUNCTION:



* A H plane Tee Junction is formed by cutting a rectangular slot along the width of the main guide and attatching another waveguide as side arm, called the H-ARM as shown in above figure.
* Port 1 and 2 of the main guide is called collinear arms and port 3 is the H arm or side arm.
* H. plane Tee is so called because the axis of the side arm is parallel to the planes of the main transmission line
* As all the 3 arms of the H plane Tee lie in the plane of the magnetic field the magnetic field divides itself into the arms. Therefore it is also called as current Junction.
* The order of [S] matrix of H plane Tee is 3X3, since there are 3 possible inputs and 3 possible outputs.

[S] Matrix of H plane Tee is given by

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \longrightarrow (12)$$

⇒ By applying the properties,

* Because of the plane of symmetry of the junction, $S_{13} = S_{23}$.
* From symmetric property, $S_{ij} = S_{ji}$

(i.e.) $S_{12} = S_{21}$, $S_{13} = S_{31}$; $S_{23} = S_{32} = S_{13}$

* since the port is perfectly matched to the junction, $S_{33} = 0$

* with these properties, (12) becomes

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \longrightarrow (13)$$

* From unitary property,

$$[S][S]^* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & \cancel{S_{33}^*}\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow (14)$$

R_1C_1 : $S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = 1 \longrightarrow (15)$

$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \longrightarrow (16)$

R_2C_2 : $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \longrightarrow (17)$

R_3C_3 : $|S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow (18)$

R_3C_1 : $S_{13}S_{11}^* + S_{13}S_{12}^* = 0 \longrightarrow (19)$

from (18), $\Rightarrow$ $2|S_{13}|^2 = 1$ or $\boxed{S_{13} = \frac{1}{\sqrt{2}}} \rightarrow (20)$

comparing (15) & (16) $\Rightarrow$ $|S_{11}|^2 = |S_{22}|^2 \Rightarrow \boxed{S_{11} = S_{22}} \longrightarrow (21)$

From (19), $\Rightarrow$ $S_{13}(S_{11}^* + S_{12}^*) = 0$

since $S_{13} \neq 0$; $S_{11}^* + S_{12}^* = 0$ (or) $S_{11}^* = -S_{12}^*$

(or)

$$\left.\begin{array}{l} S_{11} = -S_{12} \\ \text{(or)} \\ S_{12} = -S_{11} \end{array}\right\} \rightarrow (22)$$

using (22) in (16), we get

$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$

(or)

$2|S_{11}|^2 = \frac{1}{2}$ or $\boxed{S_{11} = \frac{1}{2}} \longrightarrow (23)$

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From 21 and 22, we get

$$\boxed{S_{12} = -\frac{1}{2}} \longrightarrow 24$$

and

$$\boxed{S_{22} = \frac{1}{2}} \longrightarrow 25$$

sub. 21, 23, 24 & 25 in 13, we get

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \longrightarrow 26$$

W.K.T $[b] = [S][a]$

$$\therefore \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \longrightarrow 27$$

(ie)

$$\boxed{b_1 = \frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3} \longrightarrow 28$$

$$\boxed{b_2 = -\frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3} \longrightarrow 29$$

$$\boxed{b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{2}}a_2} \longrightarrow 30$$

Case: 1 Input is given at port 3 and no inputs at port 1 and port 2, we get

$$b_1 = \frac{a_3}{\sqrt{2}} \;;\; b_2 = \frac{a_3}{\sqrt{2}} \;\&\; b_3 = 0 \longrightarrow 31$$

Let P_3 be the input at port 3; then P_3 divides equally between ports 1 and 2.

(ie) $P_1 = P_2$;

But $P_3 = P_1 + P_2 = 2P_1 = 2P_2 \longrightarrow 32$

The amount of output power at port 1 or port 2 due to Input at port 3 is given by $\longrightarrow 33$

$$10 \log \frac{P_1}{P_3} = 10 \log \frac{P_1}{2P_1} = 10 \log_{10}\left(\frac{1}{2}\right) = -10 \log_{10}^{2} = -10 \log(0.3010) = -3dB$$

Hence power output at port 1 or 2 is 3 dB down with respect to input power at port 3. Hence H plane Tee is called as 3 dB SPLITTER.

Case: 2 Input is applied at ports ① and ②, and no input at port ③, we get

$$a_1 = a_2 = a \;;\; a_3 = 0 \longrightarrow 34$$

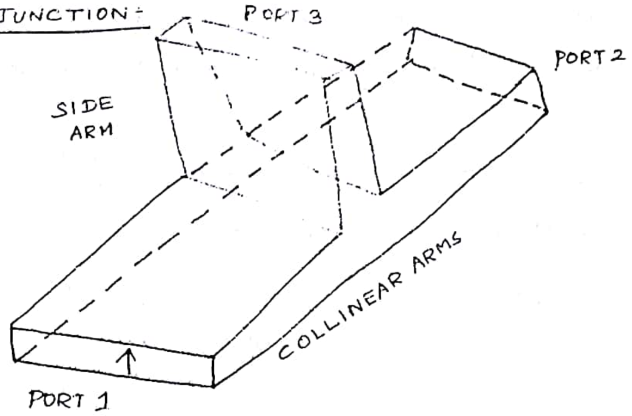
$$\therefore b_1 = \frac{a}{2} - \frac{a}{2} + \frac{1}{\sqrt{2}}a_3 = \frac{a_3}{\sqrt{2}} = 0 \longrightarrow 35$$

$$b_2 = -\frac{a}{2} + \frac{a}{2} + \frac{1}{\sqrt{2}}a_3 = \frac{a_3}{\sqrt{2}} = 0 \longrightarrow 36$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} \longrightarrow 37$$

The output at port 3 is addition of 2 inputs at port 1 and 2 and these are added in phase.

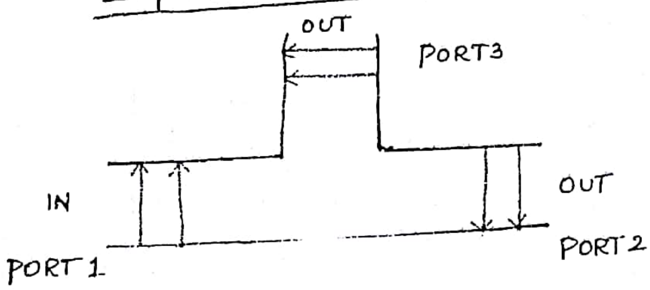
E PLANE TEE JUNCTION:-



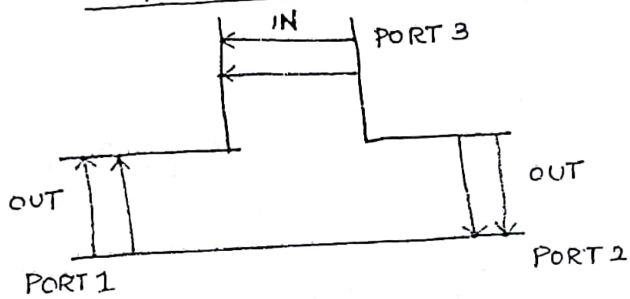
* An E plane Tee is a waveguide tee in which the axis of its side arm is parallel to the Electric field of the main waveguide.
* ports 1 and 2 are collinear arms and port 3 is the E arm.
* when TE_{10} mode is made to propogate into port 3, the two outputs at port 1 and port 2 will have a phase shift of 180°. since the Electric field change their direction when they come out of port ① and port ②, it is called a E-plane Tee.
* E plane Tee is a series junction symmetrical about central arm.

TWO WAY TRANSMISSION OF E PLANE TEE

Input Through Main arm



Input from side arm



* When the waves are fed into side arm (PORT 3), the waves appearing at port 1 and port 2 of the collinear arm will be in opposite phase and in the same magnitude.

S Matrix of E plane Tee is 3x3, since there are 3 ports,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

* The scattering coefficient $S_{23} = -S_{13} \longrightarrow$ (38)

since outputs at port 1 and port 2 are out of phase by 180° with an inp. at port 3.

* If port 3 is perfectly matched to the junction, $S_{33} = 0 \longrightarrow$ (39)

* From symmetric property $S_{ij} = S_{ji}$

$$\left.\begin{aligned} S_{12} &= S_{21} \\ S_{13} &= S_{31} \\ S_{23} &= S_{32} \end{aligned}\right\} \longrightarrow (40)$$

With above properties, [S] matrix of E plane Tee can be represented as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \longrightarrow (41)$$

* From unitary property, $[S][S^*] = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow (42)$$

R_1C_1 : $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \longrightarrow$ (43)

R_2C_2 : $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \longrightarrow$ (44)

R_3C_3 : $|S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow$ (45)

R_3C_1 : $S_{13}S_{11}^* - S_{13}S_{12}^* = 0 \longrightarrow$ (46)

Equating (43) and (44), we get

$$\boxed{S_{11} = S_{22}} \longrightarrow (47)$$

From (45) $\Rightarrow$ $\boxed{S_{13} = \frac{1}{\sqrt{2}}} \longrightarrow$ (48)

From (46) $\Rightarrow$ $S_{13}(S_{11}^* - S_{12}^*) = 0$

$\Rightarrow S_{11} = S_{12} = S_{22} \longrightarrow$ (49)

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using (47), (48) & (49) in (43), we get

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$2|S_{11}|^2 = \frac{1}{2}$$

$$\boxed{S_{11} = \frac{1}{2}} \longrightarrow (50)$$

sub. (48), (49) & (50) in (41), we get

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \longrightarrow (51)$$

W.K.T $[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \longrightarrow (52)$$

$$\boxed{b_1 = \frac{1}{2} a_1 + \frac{1}{2} a_2 + \frac{1}{\sqrt{2}} a_3} \longrightarrow (53)$$

$$\boxed{b_2 = \frac{1}{2} a_1 + \frac{1}{2} a_2 - \frac{1}{\sqrt{2}} a_3} \longrightarrow (54)$$

$$\boxed{b_3 = \frac{1}{\sqrt{2}} a_1 - \frac{1}{\sqrt{2}} a_2} \longrightarrow (55)$$

Case: 1

$$a_1 = a_2 = a \; ; \; a_3 = 0$$

substituting above values in (53), (54) & (55), we get

$$\boxed{b_1 = \frac{a}{2} + \frac{a}{2}} \longrightarrow (56)$$

$$\boxed{b_2 = \frac{a}{2} + \frac{a}{2}} \longrightarrow (57)$$

$$\boxed{b_3 = \frac{1}{\sqrt{2}} a - \frac{1}{\sqrt{2}} a = 0} \longrightarrow (58)$$

Equal inputs at port 1 and port 2 result in no output at port 3.

Case: 2

$a_1 = 0;\ a_2 = 0;\ a_3 \neq 0$

$b_1 = \frac{1}{\sqrt{2}} a_3$	$\rightarrow$ (59)
$b_2 = -\frac{1}{\sqrt{2}} a_3$	$\rightarrow$ (60)
$b_3 = 0$	$\rightarrow$ (61)

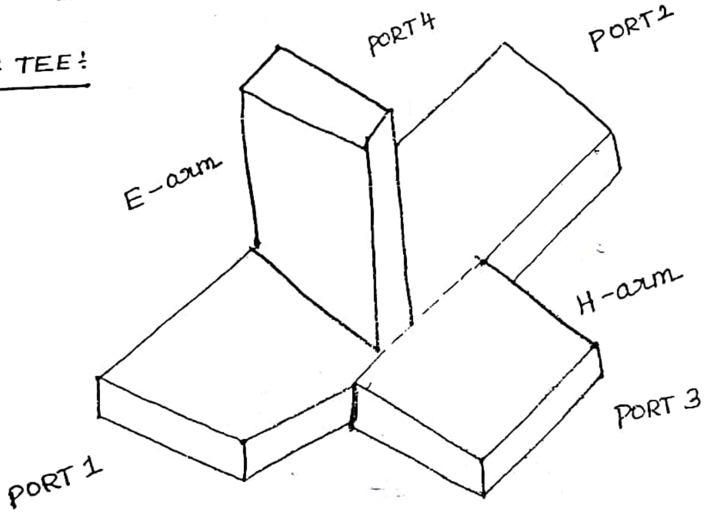
An Input at Port 3 equally divides between ports (1) and (2), but introduces a phase shift of 180° between 2 outputs. Hence E plane TEE also acts as 3dB SPLITTER.

Case: 3

$a_1 \neq 0,\ a_2 = 0,\ a_3 = 0$

$b_1 = \frac{a_1}{2}$	$\rightarrow$ (62)
$b_2 = \frac{a_1}{2}$	$\rightarrow$ (63)
$b_3 = -\frac{a_1}{\sqrt{2}}$	$\rightarrow$ (64)

MAGIC TEE:



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* A MAGIC TEE is a combination of E Plane TEE and H plane TEE.
* Here the rectangular slots are cut along the width and breadth of a lor waveguide and side arms are attached as shown in above figure.
* Ports 1 and 2 are collinear arms.
* Port 3 is H arm and port 4 is E arm.

* [S] is a 4x4 matrix since there are 4 ports

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \longrightarrow (65)$$

* Due ~~to H arm~~

$$S_{23} = S_{13} \longrightarrow (66)$$

* Due to E arm

$$S_{24} = -S_{14} \longrightarrow (67)$$

* Because of the geometry of the junction an input at port 3 cannot come out of port 4 since they are isolated ports and vice versa

$$S_{34} = S_{43} = 0 \longrightarrow (68)$$

* From symmetric property, $S_{ij} = S_{ji}$

$$S_{12} = S_{21}$$
$$S_{13} = S_{31}$$
$$S_{23} = S_{32} \longrightarrow (69)$$
$$S_{34} = S_{43}$$
$$S_{24} = S_{42}$$
$$S_{41} = S_{14}$$

* If ports 3 and 4 are perfectly matched to the junction

$$S_{33} = S_{44} = 0 \longrightarrow (70)$$

sub. 66 to 70 in 65, we get

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \longrightarrow 71$$

From unitary property, $[S][S^*] = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \longrightarrow 72$$

$R_1 C_1$: $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow 73$

$R_2 C_2$: $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow 74$

$R_3 C_3$: $|S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow 75$

$R_4 C_4$: $|S_{14}|^2 + |S_{14}|^2 = 1 \longrightarrow 76$

From 75 and 76

$$\boxed{S_{13} = \frac{1}{\sqrt{2}}} \longrightarrow 77$$

$$\boxed{S_{14} = \frac{1}{\sqrt{2}}} \longrightarrow 78$$

$74 \Rightarrow |S_{12}|^2 + |S_{22}|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$0 + |S_{22}|^2 + 1 = 1$

$|S_{22}|^2 = 0$

comparing 73 and 74

$$S_{11} = S_{22} \longrightarrow 79$$

using 77 and 78 in 73, we get

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$\boxed{S_{11} = S_{12} = 0} \longrightarrow 80$$

From 74, $\boxed{S_{22} = 0} \longrightarrow 81$

* This means that ports 1 and 2 are perfectly matched to the junction. Hence in any 4 port junction, if any 2 ports are perfectly matched to the junction, then remaining ports are automatically matched to the junction. Such a junction in which all 4 ports are matched to the junction is called a MAGIC TEE.

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \longrightarrow (82)$$

W.K.T $[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \longrightarrow (83)$$

$b_1 = \frac{1}{\sqrt{2}}(a_3 + a_4)$	$\longrightarrow$ (84)
$b_2 = \frac{1}{\sqrt{2}}(a_1 + a_2)$	$\longrightarrow$ (85)
$b_3 = \frac{1}{\sqrt{2}}(a_3 - a_4)$	$\longrightarrow$ (86)
$b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2)$	$\longrightarrow$ (87)

Case : 1

$a_3 \neq 0$; $a_1 = a_2 = a_4 = 0$

$$b_1 = \frac{a_3}{\sqrt{2}}$$

$$b_2 = \frac{a_3}{\sqrt{2}}$$

$$b_3 = 0$$

$$b_4 = 0$$

* This is the property of H plane Tee Junction.

Case : 2

$a_4 \neq 0$; $a_1 = a_2 = a_3 = 0$

$$b_1 = \frac{a_4}{\sqrt{2}} \; ; \; b_2 = -\frac{a_4}{\sqrt{2}} \; ; \; b_3 = b_4 = 0$$

* This is the property of E plane Tee Junction

Case : 3

$a_1 \neq 0$; $a_2 = a_3 = a_4 = 0$

$$b_1 = 0; \; b_2 = 0; \; b_3 = \frac{a_1}{\sqrt{2}} \; ; \; b_4 = \frac{a_1}{\sqrt{2}}$$

* When power is fed into port 1, there is no output at port 2. Hence ports 1 and 2 are Isolated ports.
* Similarly input at port 2 will never produce an output at port 1
* ∴ E and H ports are Isolated ports.

Case : 4 $a_3 = a_4$; $a_1 = a_2 = 0$

$$\therefore \; b_1 = \frac{1}{\sqrt{2}}(2a_3) \; ; \; b_2 = 0; \; b_3 = 0; \; b_4 = 0$$

* This is additive property. Equal inputs at ports 3 and 4 result in an output at port 1 (Inphase and equal in Amplitude).

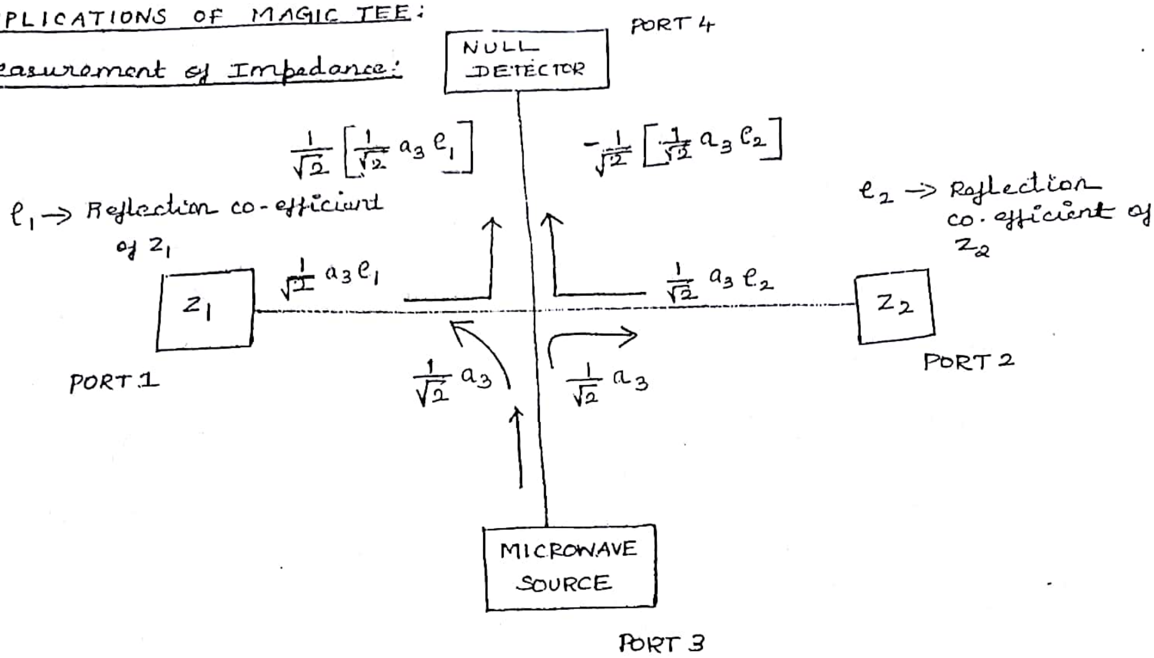
Case : 5 $a_1 = a_2$; $a_3 = a_4 = 0$

$$b_1 = 0 = b_2 = b_4 \; ; \; b_3 = \frac{1}{\sqrt{2}}(2a_1)$$

* Equal inputs at port 1 and 2 results in an output at Port 3 (additive) and no outputs at ports 1, 2 and 4.

APPLICATIONS OF MAGIC TEE:

Measurement of Impedance:



* A MAGIC TEE has been used in the form of a bridge as shown in the above figure for measuring Impedance.
* Microwave source is connected in port 3.
* A Null Detector is connected in port 4.
* The unknown Impedance is connected in port 2.
* standard, known variable Impedance is connected in port 1.
* using the properties of MAGIC Tee, the power from Microwave source (a_3) gets divided equally between ports 1 and 2 ($a_3/\sqrt{2}$). These Impedances are not equal to the characteristic Impedance (Z_0) and hence there will be reflections from ports 1 and 2.
* If e_1 and e_2 are the Reflection coefficients, powers $\frac{e_1 a_3}{\sqrt{2}}$ and $\frac{e_2 a_3}{\sqrt{2}}$ enter the Magic Tee junction from ports 1 and 2 as shown in above figure.
* The Resultant wave into port 4 (ie) the Null Detector can be calculated as follows
* The net wave reaching the Null Detector is given by

$$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} a_3 e_1\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} a_3 e_2\right) = \frac{1}{2} a_3 (e_1 - e_2) \longrightarrow (88)$$

For perfect balancing of the bridge,

$$\text{Equate} \Rightarrow \frac{1}{2} a_3 (e_1 - e_2) = 0 \longrightarrow (89)$$

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$$e_1 - e_2 = 0 \longrightarrow (90)$$

$$e_1 = e_2 \longrightarrow (91)$$

$$\frac{Z_1 - Z_2}{Z_1 + Z_2} = \frac{Z_2 - Z_2}{Z_2 + Z_2}$$

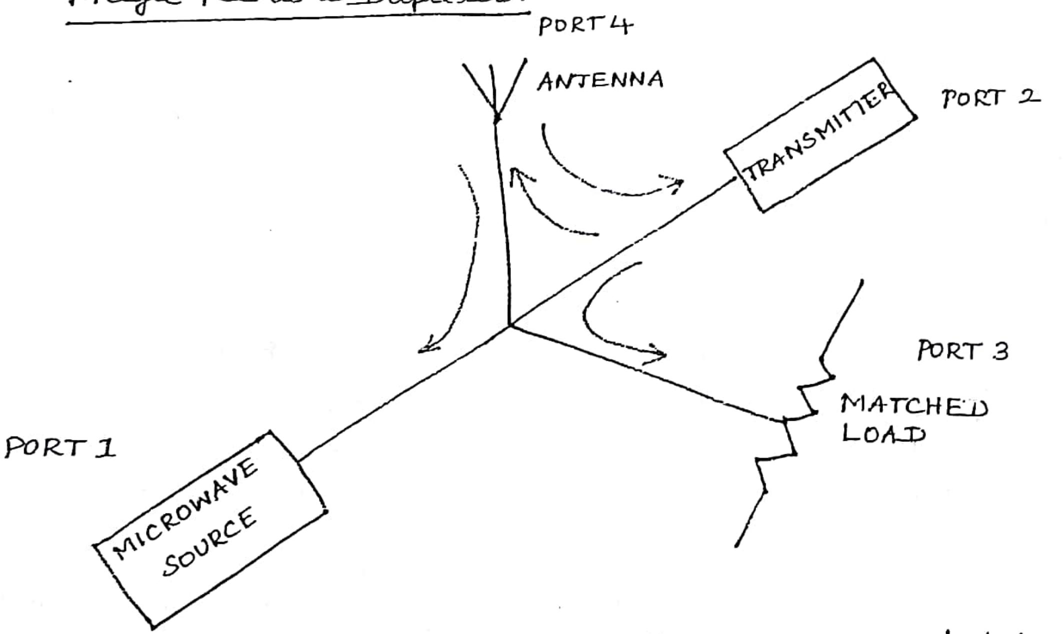
$$Z_1 = Z_2 \longrightarrow (93)$$

(i.e) $R_1 + jX_1 = R_2 + jX_2 \longrightarrow (94)$

(or) $R_1 = R_2$ and $X_1 = X_2 \longrightarrow (95)$

* Thus unknown Impedance can be measured by adjusting the standard variable Impedance till the bridge is balanced and both Impedance become equal.

Magic Tee as a Duplexer:



* The Transmitter and the receiver are connected to ports 1 and 2 as shown in above figure.
* An Antenna is connected to the E arm of the device.
* Port 3 of Magic Tee is terminated in a Matched Load.
* During Transmission, half the power reaches the antenna from where it is radiated into the space. other half reaches the matched load, where it is absorbed without reflections.
* No Transmitter power reaches the receiver, since ports 1 and 2 are isolated ports in the Magic Tee.
* During reception, half of the received power goes to the receiver and the other half to the transmitter are isolated during reception as well as during transmission.

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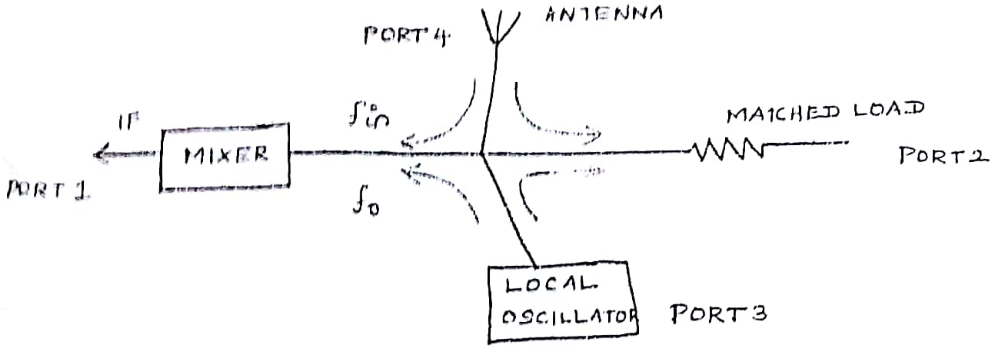
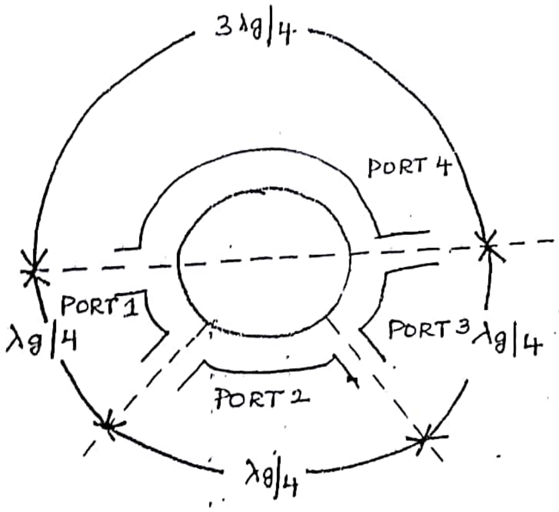
Magic Tee as a Mixer:



* Half of the Local oscillator power and half of the received power from antenna goes to the Mixer where they are mixed to generate the IF.

$$IF = f_{in} \sim f_0$$

RAT RACE JUNCTION



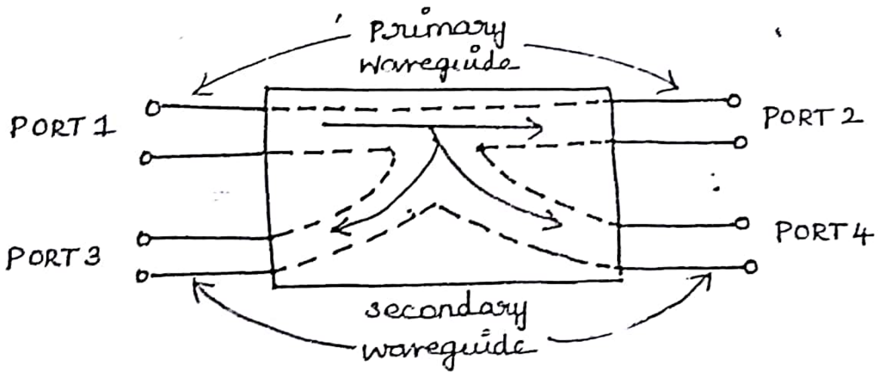
* It is a 4 port junction, where the fourth port being added to a normal 3 port Tee junction.
* The four ports are connected in the form of an angular ring at proper intervals by means of series or parallel junctions.
* These ports are seperated by proper electrical lengths to sustain standing waves.
* For proper operation,
  - ⟹ It is necessary that the mean circumference of the total race be $1.5\ \lambda g$
  - ⟹ Each of the 4 ports seperated from its neighbour by a distance of $\lambda g/4$.

* When power is fed into port 1, it splits equally into ports 2 and 4 and nothing enters port 3.
* At ports 2 and 4 the powers combine in phase but at port 3 cancellation occurs due to $\lambda g/2$ path difference.
* If Input is applied at port 3, it is equally divided between ports 2 and 4 but the output at port 1 will be zero.
* The Rat Race can be used for combining 2 signals or dividing a single signal into 2 equal halves.
* The [S] matrix of Rat Race Junction is as follows

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix} \longrightarrow (96)$$

DIRECTIONAL COUPLERS:

Schematic of Directional coupler



Directional coupler Indicating powers