DSP ASSIGNMENT

1. Check the whether the following system, $y(n) = x(n) + \frac{1}{x(n-1)}$, is
a) linear b) Time invariant c) Causal d) Stable

a) linear or Non-linear

$$y(n) = x(n) + \frac{1}{x(n-1)}$$

$$y_1(n) = x_1(n) + \frac{1}{x_1(n-1)} \qquad y_2(n) = x_2(n) + \frac{1}{x_2(n-1)}$$

$$y_3(n) = TF\left[a_1x_1(n) + a_2x_2(n) + \frac{1}{a_1x_1(n-1)} + \frac{1}{a_2x_2(n-1)}\right]$$

$$a_1y_1(n) + a_2y_2(n) = a_1x_1(n) + a_2x_2(n) + \frac{a_1}{x_1(n-1)} + \frac{a_2}{x_2(n-1)}$$

$$y_3(n) \neq a_1y_1(n) + a_2y_2(n)$$

$\therefore$ The system is Non-linear

b) Time Variance (or) Time invariance

$$y(n) = x(n) + \frac{1}{x(n-1)}$$

$$y(n,k) = x(n-k) + \frac{1}{x(n-1-k)} \qquad y(n-k) = x(n-k) + \frac{1}{x(n-k-1)}$$

$$\therefore y(n,k) = y(n-k)$$

$\therefore$ The system is Time invariant

c) Causal (or) Non-causal

$$y(n) = x(n) + \frac{1}{x(n-1)}$$

$$n=0 \Rightarrow y(0) = x(0) + \frac{1}{x(0-1)} = x(0) + \frac{1}{x(-1)}$$

$$n=1 \Rightarrow y(1) = x(1) + \frac{1}{x(0)}$$

$\therefore$ The System is Causal.

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d) Stable or unstable

$$y(n) = x(n) + \frac{1}{x(n-1)}$$

let $x(n) = \delta(n)$, $\therefore h(n) = \delta(n) + \frac{1}{\delta(n-1)}$

$$\sum_{n=-\infty}^{\infty} h(n) = \cdots \delta(-2) + \delta(-1) + \delta(0) + \delta(1) + \delta(2) + \cdots$$
$$+ \frac{1}{\delta(-2)} + \frac{1}{\delta(-1)} + \frac{1}{\delta(0)} + \frac{1}{\delta(1)} + \frac{1}{\delta(2)} + \cdots$$

$$= \infty$$

$\therefore$ The system is stable

② Find the total response of the system described by the difference equation $y(n) - 0.2y(n-1) - 0.03y(n-2) = x(n) + 0.4x(n-1)$ with i/p $\Rightarrow x(n) = (1/5)^n u(n)$ and initial conditions: $y(-2) = 0$ & $y(-1) = 0.5$

Sol: Natural Response

$$y(n) - 0.2y(n-1) - 0.03y(n-2) = x(n) + 0.4x(n-1)$$

In natural response, input conditions $= 0$ ~~i.e., $x(n) = 0$~~

$$y(n) - 0.2y(n-1) - 0.03y(n-2) = 0$$

Assume $y_h(n) = \lambda^n$

$$\lambda^n - 0.2\lambda^{n-1} - 0.03\lambda^{n-2} = 0$$

$$\lambda^{n-2}\left[\lambda^2 - 0.2\lambda - 0.03\right] = 0$$

$$\lambda^2 - 0.2\lambda - 0.03 = 0$$

$$\lambda_1 = 0.3 \quad \cancel{\lambda_2 = 0.1} \quad \lambda_2 = -0.1$$

$$y_h(n) = c_1(0.3)^n - c_2(0.1)^n$$

Substitute initial conditions, $y(-2) = 0$ & $y(-1) = 0.5$

$$n = -2 \Rightarrow y(-2) = c_1(0.3)^{-2} - c_2(0.1)^{-2} = \frac{c_1}{0.09} - \frac{c_2}{0.01}$$

$$0 = \frac{c_1}{0.09} - \frac{c_2}{0.01} \Rightarrow \frac{c_1}{0.09} = \frac{c_2}{0.01} \Rightarrow \frac{c_1}{9} = \frac{c_2}{1} \Rightarrow \boxed{c_1 = 9c}$$

$n=1 \Rightarrow y(-1) = c_1(0.3)^{-1} - c_2(0.1)^{-1} = \frac{c_1}{0.3} - \frac{c_2}{0.1} \Rightarrow 0.5 = \frac{9c_2}{0.3} - \frac{c_2}{0.1}$

$2c_2 = 0.05 \Rightarrow c_2 = \frac{5}{200} = 1/40$

$\boxed{c_2 = 1/40}$ $\boxed{c_1 = 9/40}$

$\therefore y_h(n) = (0.3)^n (9/40) u(n) - (0.1)^n (1/40) u(n)$

Forced response

$y_f = y_h(n) + y_p(n)$

Initial conditions = 0

$y_h(n) = (0.3)^n c_1 - (0.1)^n c_2$

Input: $x(n) = (1/5)^n u(n)$

$\therefore y_p(n) = k(1/5)^n u(n)$

$k(1/5)^n u(n) - 0.2k(1/5)^{n-1} u(n-1) - 0.03k(1/5)^{n-2} u(n-2) = (1/5)^n u(n) + 0.4(1/5)^{n-1} u(n-1)$

$n=2$

$k \cdot \frac{1}{25} - 0.2 \times \frac{1}{5} k - 0.03k = \frac{1}{25} + \frac{0.4}{5}$

$\frac{k}{25} - \frac{1}{25} k - 0.03k = \frac{1}{25} + \frac{2}{25} \Rightarrow -0.03k = \frac{3}{25}$

$\boxed{k = -4}$

$\therefore \boxed{y_p(n) = -4(1/5)^n u(n)}$

WKT, $y_f(n) = y_h(n) + y_p(n)$

$\boxed{y_f(n) = c_1(0.3)^n - c_2(0.1)^n - 4(1/5)^n u(n)}$

Zero state.

$n=0 \Rightarrow y(0) = c_1 - c_2 - 4$

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$n=1$

$$y(1) = c_1(0.3) - c_2(0.1) - \frac{4}{5}$$

$$n=0 \Rightarrow y(0) - 0.2y(-1) - 0.03y(-2) = x(0) + 0.4x(-1)$$

$$y(0) - 0.2\times0.5 - 0 = 1 + 0.4\times(1/5)^{-1}$$

$$y(0) = 1 + 2 + 0.2\times0.5 = 3.1$$

$$y(0) = 3.1$$

$$n=1 \rightarrow y(1) - 0.2y(0) - 0.03y(-1) = x(1) + 0.4x(0)$$

$$y(1) - 0.2\times3.1 - 0.03\times0.5 = 1/5 + 0.4$$

$$y(1) = 1.23$$

$$y(0) = c_1 - c_2 - 4 \Rightarrow 3.1 = c_1 - c_2 - 4 \Rightarrow c_1 - c_2 = 7.1 \quad -①$$

$$\cancel{y(1) = \frac{c_1\times3}{10}} -$$

$$y(1) = c_1(0.3) - c_2(0.1) - \frac{4}{5} \Rightarrow 1.23 + \frac{4}{5} = c_1(0.3) - c_2(0.1)$$

$$c_1(0.3) - c_2(0.1) = 2.03 \quad -②$$

from ① & ②, $\boxed{c_1 = 6.6}$ $\boxed{c_2 = -0.5}$

$$\boxed{y_f(n) = (6.6)(0.3)^n u(n) + (0.5)(0.1)^n u(n) - 4(1/5)^n u(n)}$$

Total response,

$$y(n) = y_h(n) + y_f(n)$$

$$= (0.3)^n (9/40) u(n) - (0.1)^n (1/40) u(n) + (6.6)(0.3)^n u(n)$$

$$+ (0.5)(0.1)^n u(n) - 4(1/5)^n u(n)$$

$$= 6.82(0.3)^n u(n) + 0.47\,(0.1)^n u(n) - 4(1/5)^n u(n)$$

$$\therefore \boxed{y(n) = 6.82(0.3)^n u(n) + 0.47(0.1)^n u(n) - 4(1/5)^n u(n)}$$

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③ Determine the frequency response of an LTI system with an impulse Response, $h(n) = \frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)$

Sol:

$$h(n) = \frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)$$

Apply DTFT

$$H(e^{j\omega}) = \frac{1}{2} + e^{-j\omega} + \frac{1}{2}e^{-2j\omega} = \frac{1}{2} + \cos\omega - j\sin\omega + \frac{1}{2}\left[\cos 2\omega - j\sin 2\omega\right]$$

$$H(e^{j\omega}) = \frac{1}{2} + \cos\omega - j\sin\omega + \frac{1}{2}\cos 2\omega - \frac{1}{2}j\sin 2\omega$$

$$H(e^{j\omega}) = \frac{1}{2} + \cos\omega + \frac{1}{2}\cos 2\omega - j\left[\sin\omega + \frac{1}{2}\sin 2\omega\right]$$

$$|H(e^{j\omega})| = \sqrt{\left(\frac{1}{2} + \cos\omega + \frac{1}{2}\cos 2\omega\right)^2 + \left(\sin\omega + \frac{1}{2}\sin 2\omega\right)^2}$$

$$\theta(\omega) = \tan^{-1}\left(\frac{\sin\omega + \frac{1}{2}\sin 2\omega}{\frac{1}{2} + \cos\omega + \frac{1}{2}\cos 2\omega}\right)$$

ω	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
$\lvert H(e^{j\omega})\rvert$	2	1.86	1.5	0	0.5	0.13	0
$\theta(\omega)$	0	0.52	1.04	0	-1.04	-0.52	0

④ Determine the frequency response and sketch the magnitude and phase response of the, $y(n) = [x(n) + x(n-2)]\frac{1}{2}$

Sol:

$$y(n) = \frac{1}{2}\left[x(n) + x(n-2)\right]$$

Apply DTFT

$$Y(e^{j\omega}) = \frac{1}{2}X(e^{j\omega}) + \frac{1}{2}e^{-2j\omega}X(e^{j\omega})$$

$$Y(e^{j\omega}) = X(e^{j\omega})\left[\frac{1}{2} + \frac{1}{2}e^{-2j\omega}\right]$$

$$H(e^{j\omega}) = \frac{1}{2}\left[1 + e^{-2j\omega}\right]$$

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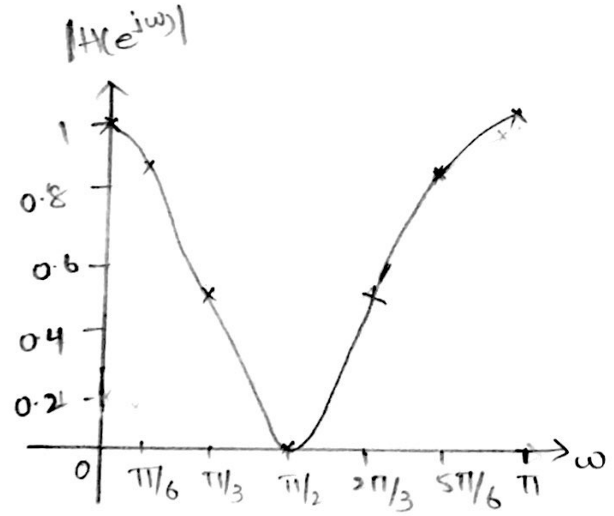
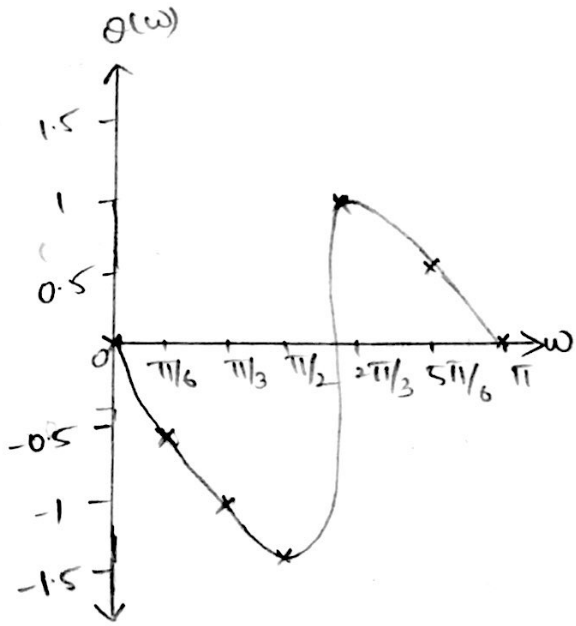
$$H(e^{j\omega}) = \frac{1}{2}\left[1+\cos 2\omega - j\sin 2\omega\right]$$

$$|H(e^{j\omega})| = \frac{1}{2}\sqrt{(1+\cos 2\omega)^2+(\sin 2\omega)^2}$$

$$\theta(\omega) = \tan^{-1}\left(\frac{-\sin 2\omega}{1+\cos 2\omega}\right)$$

ω	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
$\lvert H(e^{j\omega})\rvert$	1	0.86	0.5	0	0.5	0.86	1
$\theta(\omega)$	0	-0.52	-1.04	-1.57	1.04	0.52	0





5. Determine the impulse response and unit step response of the system described by the difference equation,

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n)$$

Sol: Impulse Response

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n)$$

Apply z-transform,

$$Y(z) - 0.6z^{-1}Y(z) + 0.08z^{-2}Y(z) = X(z)$$

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If i/p is impulse, the pratical solution $y_p(n)=0$ hence $h(n)$ can be computed from the homogeneous solution with the initial conditions zero

$$x(n)=\delta(n) \qquad y_p(n)=0$$

$$y_h(n)=y(n)$$

$$y(n)-0.6y(n-1)+0.08y(n-2)=x(n)$$

$$y(n)=\lambda^n \qquad \underline{\underline{i/p=0}}$$

$$\lambda^n-0.6\lambda^{n-1}+0.08\lambda^{n-2}=0$$

$$\lambda^{n-2}[\lambda^2-0.6\lambda+0.08]=0$$

$$\lambda^2-0.6\lambda+0.08=0$$

$$\lambda_1=0.4 \qquad \lambda_2=0.2$$

$$y_h(n)=c_1(0.4)^n+c_2(0.2)^n \quad n\geq 0$$

$n=0 \Rightarrow y(0)=c_1+c_2$ $\qquad$ $n=1 \Rightarrow y(1)=0.4c_1+0.2c_2$

from the difference equations,

$n=0$, $y(0)-0.6y(-1)+0.08y(-2)=x(0) \Rightarrow y(0)=1$

$n=1$, $y(1)-0.6y(0)+0.08y(-1)=x(1) \Rightarrow y(1)=0.6$

$$c_1+c_2=1 \Rightarrow 0.4c_1+0.2c_2=0.6$$

$$4c_1+2c_2=16$$

$$2c_1+c_2=8 \quad -①$$

$c_1+c_2=1$

$2c_1+c_2=8$

$$+c_1=+7$$

$$\boxed{c_1=7} \qquad \boxed{c_2=-6}$$

$$\boxed{y(n)=[7(0.4)^n u(n)-6(0.2)^n u(n)]}$$

Unit step response:

$$y(n)-0.6y(n-1)+0.08y(n-2)=x(n)$$

$$x(n)=u(n) \Rightarrow y_p(n)=K[u(n)]$$

$y(n) = y_h(n) + y_p(n)$

$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n)$

i/p = 0

$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0 \Rightarrow y(n) = \lambda^n$

$\cancel{\lambda^n - \lambda^{n-1} - 0.}$ $\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0 \Rightarrow \lambda^{n-2}[\lambda^2 - 0.6\lambda + 0.08] = 0$

$\lambda^2 - 0.6\lambda + 0.08 = 0 \Rightarrow \lambda_1 = 0.4 \quad \lambda_2 = 0.2$

$\cancel{y_h(n) = c(0.4)^n u(n) + (0.2)^n}$. $\Rightarrow y_h(n) = C_1(0.4)^n + C_2(0.2)^n$

$n=0 \Rightarrow y(0) = C_1 + C_2$ $\quad n=1 \Rightarrow y(1) = C_1(0.4) + C_2(0.2)$

From diff. equation,

$n=0 \Rightarrow y(0) - 0.6y(0-1) + 0.08y(0-2) = x(0) \Rightarrow y(0) = 1$

$n=1 \Rightarrow y(1) - 0.6y(0) + 0.08y(-1) = x(1) \Rightarrow y(1) - 0.6 = 1 \Rightarrow y(1) = 1.6$

$y(0) = C_1 + C_2 \Rightarrow 1 = C_1 + C_2$ $\quad y(1) = 0.4C_1 + 0.2C_2$

$1.6 = 0.4C_1 + 0.2C_2$

$8 = 2C_1 + C_2$

$2C_1 + C_2 = 8$

$C_1 + C_2 = 1$

$\boxed{C_1 = 7}$ $\quad C_1 + C_2 = 1$ $\quad \boxed{C_2 = -6}$

$\boxed{y_h(n) = 7(0.4)^n - 6(0.2)^n}$

for, $y_p(n) = K u(n)$ & $x(n)$ in diff. eqn; $K u(n) - 0.6K u(n-1) + 0.08K u(n-2) = u(n)$

$K - 0.6K + 0.08K = 1$

$0.48K = 1$

$\boxed{y_p(n) = 2.08u(n)}$ $\quad \boxed{K = 2.08}$

$y(n) = C_1(0.4)^n + C_2(0.2)^n + 2.08u(n)$

zero state

$n=0 \Rightarrow y(0) = C_1 + C_2 + 2.08$ $\quad n=1 \Rightarrow y(1) = 0.4C_1 + 0.2C_2 + 2.08$

From diff.,

$n=0 \Rightarrow y(0) - 0.6y(-1) + 0.08y(-2) = x(0) \Rightarrow y(0) = 1$

$n=1 \Rightarrow y(1) - 0.6y(0) + 0.08y(-1) = x(1) \Rightarrow y(1) - 0.6 = 1 \Rightarrow y(1) = 1.6$

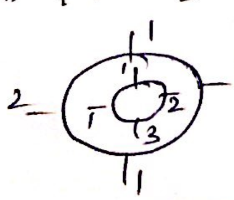
$C_1 + C_2 + 2.08 = y(0) \Rightarrow C_1 + C_2 + 2.08 = 1 \Rightarrow C_1 + C_2 = -1.08$

$0.4C_1 + 0.2C_2 = 1.6 - 2.08 \Rightarrow 0.4C_1 + 0.2C_2 = -0.48 \Rightarrow 2C_1 + C_2 = -2.4$

$\boxed{C_1 = -1.32}$ $\quad \boxed{C_2 = 2.4}$ $\quad \boxed{y(n) = -1.32(0.4)^n + 2.4(0.2)^n + 2.08u(n)}$

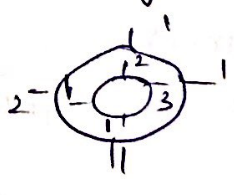
⑥ Obtain the circular convolution b/w the sequences using concentric circle method

(a) $x_1(n) = \{1,1,2,1\}$ $\quad x_2(n) = \{2,3,1,1\}$ The length is same



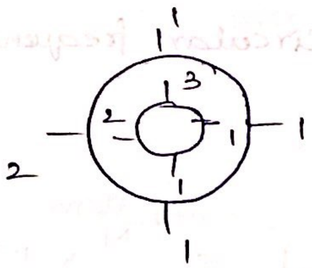
$x_3(0) = 1 + 2 + 3 + 1$

$x_3(0) = 8$



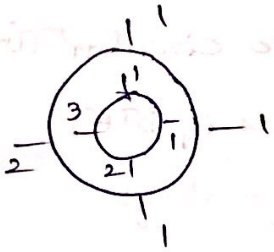
$x_3(1) = 1 + 2 + 2 + 3$

$x_3(1) = 8$



$x_3(2) = 1+4+3+1$

$x_3(2) = 9$



$x_3(3) = 2+6+1+1$

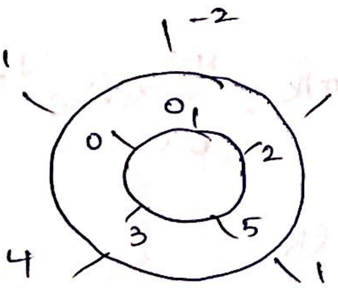
$x_3(3) = 10$

$\therefore \; x_3(n) = \{8, 8, 9, 10\}$

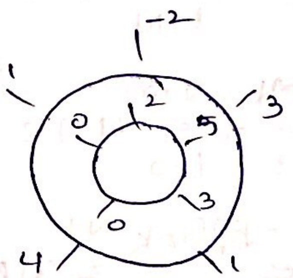
b) $x_1(n) = \{3, -2, 1, 4, 1\}$ $\quad x_2(n) = \{2, 5, 3\}$

As the lengths are not equal so, in $x_2(n)$ we add two zeros.

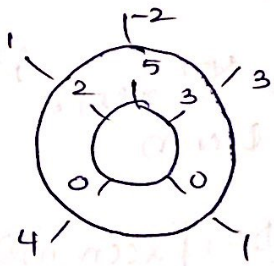
then, $x_1(n) = \{3, -2, 1, 4, 1\}$ $\quad x_2(n) = \{2, 5, 3, 0, 0\}$



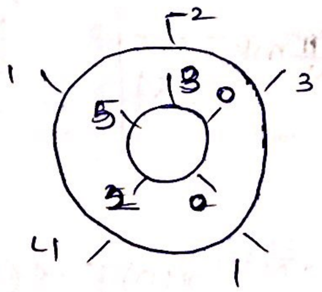
$x_3(0) = 23$



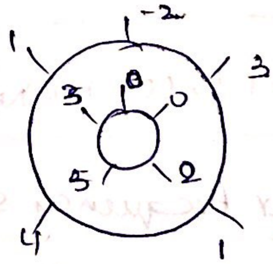
$x_3(1) = 14$



$x_3(2) = 1$



$x_3(3) = 7$



$x_3(4) = 25$

$x_3(n) = \{23, 14, 1, 7, 25\}$

7. State and prove circular Time shift and circular frequency shift properties of DTFT

Circular Time shift

If $DFT[x(n)] = X(K)$, then $DFT[x((n-m))_N] = e^{-j\frac{2\pi}{N}km} X(K)$

Proof:

$\{x((n-m))_N = x(n-m+N)\}$

$\rightarrow DFT[x((n-m))_N] = \sum_{n=0}^{N-1} x((n-m))_N e^{-j\frac{2\pi}{N}kn}$

$$= \sum_{n=0}^{m-1} x((n-m))_N e^{-j\frac{2\pi}{N}kn} + \sum_{n=m}^{N-1} x((n-m))_N e^{-j\frac{2\pi}{N}kn}$$

$(n-m+N = L)$

$$= \sum_{L=N-m}^{N-1} x(l) e^{-j\frac{2\pi}{N}(l+m-N)k} + \sum_{l=N}^{2N-m-1} x(l) e^{-j\frac{2\pi}{N}(l+m-N)}$$

$$= \sum_{l=N-m}^{N-1} x(l) e^{-j\frac{2\pi}{N}(l+m)k} + \sum_{l=0}^{N-m-1} x(l) e^{-j\frac{2\pi}{N}(l+m)k} = \sum_{l=0}^{N-1} x(l) e^{-j\frac{2\pi}{N}(l+m)k}$$

$$DFT[x((n-m))_N] = e^{-j\frac{2\pi}{N}mk} \sum_{l=0}^{N-1} x(l) e^{-j\frac{2\pi}{N}lk}$$

$$\boxed{DFT[x((n-m))_N] = e^{-j\frac{2\pi}{N}mk} X(k)}$$

Circular frequency shift

If $DFT[x(n)] = X(K)$ then, $DFT\left[e^{-j\frac{2\pi}{N}ln} x(n)\right] = X((K-l))_N$

Proof:

$$DFT[x(n)] = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$DFT\left[e^{j\frac{2\pi}{N}ln} x(n)\right] = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} \cdot e^{j\frac{2\pi}{N}ln} = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}(k-l)n}$$

$$\boxed{DFT\left[e^{j\frac{2\pi}{N}ln} x(n)\right] = X((K-l))_N}$$

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8. find the DFT of the sequence $x(n) = 1$ for $0 \le n \le 2$
0 Otherwise

for (i) $N=4$ and (ii) $N=8$. Plot magnitude and phase responses and Comment on the result.

Sol: (i) $N=4$ $\quad x(n) = \{1,1,1,0\}$ $\quad N-L = 4-3 = 1$

$x(n) = \{1,1,1\}$
$\Rightarrow L=3$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk}$$

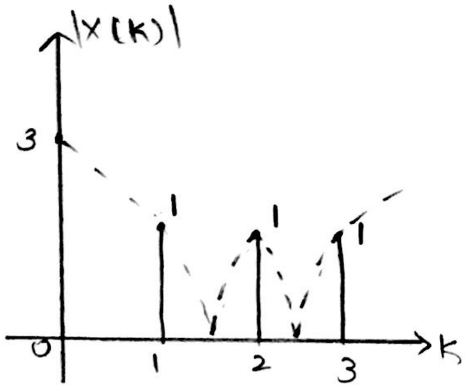
$$X(0) = 1 + 1\cdot e^{-j\cdot\frac{2\pi}{4}\cdot 1\cdot 0} + 1\cdot e^{-j\frac{2\pi}{4}\cdot 2\cdot 0} + 0 = 1+1+1 = 3 \Rightarrow X(0) = 3 = 3\angle 0$$

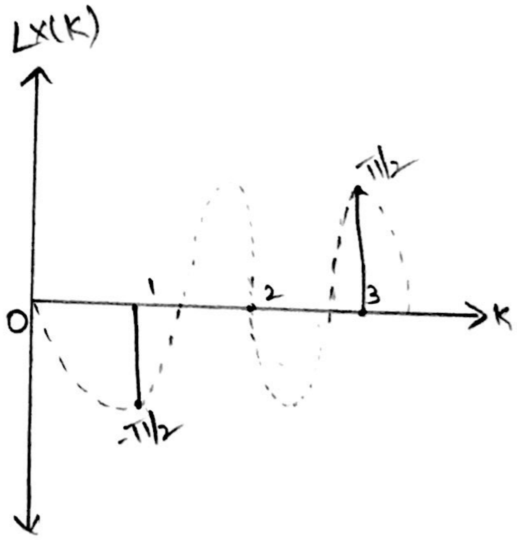
$$X(1) = 1\cdot e^{-j\frac{2\pi}{4}\cdot 0\cdot 1} + 1\cdot e^{-j\frac{2\pi}{4}\cdot 1\cdot 1} + e^{-j\frac{2\pi}{4}\cdot 2\cdot 1} + 0 = 1+(-j)+(-1) = -j$$
$$\Rightarrow X(1) = -j = 1\angle\frac{-\pi}{2}$$

$$X(2) = 1 + 1\cdot e^{-j\frac{2\pi}{4}\cdot 1\cdot 2} + 1\cdot e^{-j\frac{2\pi}{4}2\cdot 2} + 0 = 1+(-1)+1 = 1 \Rightarrow X(2) = 1 = 1\angle 0$$

$$X(3) = 1 + 1\cdot e^{-j\frac{2\pi}{4}\cdot 1\cdot 3} + e^{-j\frac{2\pi}{4}\cdot 2\cdot 3} + 0 = 1+j+(-1) = j \Rightarrow X(3) = j = 1\angle\pi/2$$

$$X(k) = \{3, -j, 1, j\}$$





(ii) $N=8$ $\quad N-L = 8-3 = 5$

$$\therefore x(n) = \{1,1,1,0,0,0,0,0\}$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk}$$

$$X(0) = 1+1+1+0+0+0+0+0 = 3 \Rightarrow X(0) = 3 = 3\angle 0$$

$$X(1) = 1 + 1\cdot e^{-j\frac{2\pi}{8}\cdot 1\cdot 1} + 1\cdot e^{-j\frac{2\pi}{8}\cdot 2\cdot 1} + 0+0+0+0+0 = 1 + \frac{1}{2} - \frac{j}{\sqrt{2}} + (-j)$$

$$X(1) = 1.707 - j(1.707)$$

2.41

$X(2) = 1 + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 1 \cdot 2} + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 2 \cdot 2} \Rightarrow X(2) = -j$ 1

$X(3) = 1 + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 1 \cdot 3} + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 2 \cdot 3} \Rightarrow X(3) = 0.293 + j(0.293)$ 0.41

$X(4) = 1, X($

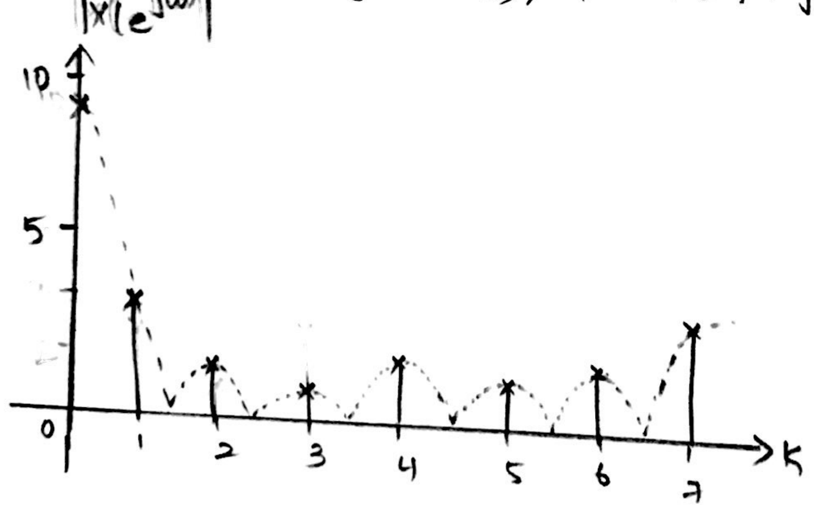
$X(4) = 1 + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 1 \cdot 4} + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 2 \cdot 4} \Rightarrow X(4) = 1$ 1

$X(5) = 1 + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 1 \cdot 5} + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 2 \cdot 5} \Rightarrow X(5) = 0.293 - j(0.293)$ 0.41

$X(6) = 1 + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 1 \cdot 6} + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 2 \cdot 6} \Rightarrow X(6) = 1$ 1

$X(7) = 1 + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 1 \cdot 7} + 1 \cdot e^{-j\frac{2\pi}{8} \cdot 2 \cdot 7} \Rightarrow X(7) = 1.707 + j(1.707)$ 2.41

$\therefore X(k) = \{3, 1.707 - j(1.707), -j, 0.293 + j(0.293), 1, 0.293 - j(0.293), 1, 1.707 + j(1.707)\}$



(9) Perform linear convolution of the two sequences,

$x(n) = \{ \overset{0}{1}, \overset{1}{3}, \overset{2}{-2}, \overset{3}{1}, \overset{4}{3}, \overset{5}{6}, \overset{6}{4}, \overset{7}{3}, \overset{8}{2}, \overset{9}{1}, \overset{10}{1}, \overset{11}{5} \}$ and $h(n) = \{1,2,1,1\}$ using

Over-lap Save method.

Sol. $L_s = 12$ $M = 4$ $N = 5$ given $N = L+M-1 \Rightarrow L = 5-3 = 2$ (New data pts)

$M-1 = 3$ (previous pts).

$x_1(n) = \{0,0,0,x(0),x(1)\} = \{0,0,0,1,3\}$ $\quad \frac{12+3}{2} = \frac{15}{2}$

$x_2(n) = \{0,x(0),x(1),x(2),x(3)\} = \{0,1,3,-2,1\}$

$x_3(n) = \{x(1),x(2),x(3),x(4),x(5)\} = \{3,-2,1,3,6\}$ $\quad = 7.$

$x_4(n) = \{x(3),x(4),x(5),x(6),x(7)\} = \{1,3,6,4,3\}$ $\quad 8//$

$x_5(n) = \{x(5),x(6),x(7),x(8),x(9)\} = \{6,4,3,2,1\}$ $\quad 12+4-1 = 15$

$x_6(n) = \{x(7),x(8),x(9),x(10),x(11)\} = \{3,2,1,1,5\}$

$x_7(n) = \{x(9),x(10),x(11),x(12),x(13)\} = \{1,1,5,0,0\}$

$x_8(n) = \{x(11),x(12),x(13),x(14),x(15)\} = \{5,0,0,0,0\}$

$x_1(n) = \{0,0,0,1,3\}$ $\quad h(n) = \{1,2,1,1,1\}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 3 \\ 1 \\ 5 \end{bmatrix}$$

$y_1(n) = \{7,4,3,1,5\}$

$x_2(n) = \{0,1,3,-2,1\}$ $\quad h(n) = \{1,2,1,1\}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 6 \\ 5 \\ 1 \end{bmatrix}$$

$y_2(n) = \{3,0,6,5,1\}$

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$x_3(n) = \{3, -2, 1, 3, 6\}$ $\quad h(n) = \{1, 2, 1, 1\}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 19 \\ 7 \\ 6 \\ 6 \\ 11 \end{bmatrix}$$

$y_3(n) = \{19, 7, 6, 6, 11\}$

$x_4(n) = \{1, 3, 6, 4, 3\}$ $\quad h(n) = \{1, 2, 1, 1\}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 6 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 17 \\ 12 \\ 16 \\ 20 \\ 20 \end{bmatrix}$$

$y_4(n) = \{17, 12, 16, 20, 20\}$

$x_5(n) = \{6, 4, 3, 2, 1\}$ $\quad h(n) = \{1, 2, 1, 1\}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 13 \\ 19 \\ 18 \\ 18 \\ 12 \end{bmatrix}$$

$y_5(n) = \{13, 19, 18, 18, 12\}$

$x_6(n) = \{3, 2, 1, 1, 5\}$ $\quad h(n) = \{1, 2, 1, 1\}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 15 \\ 14 \\ 13 \\ 8 \\ 10 \end{bmatrix}$$

$y_6(n) = \{15, 14, 13, 8, 10\}$

$x_7(n) = \{1, 1, 5, 0, 0\}$ $\quad h(n) = \{1, 2, 1, 1\}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 8 \\ 12 \\ 6 \end{bmatrix}$$

$y_7(n) = \{6, 3, 8, 12, 6\}$

$x_8(n) = \{5, 0, 0, 0, 0\}$

$h(n) = \{1, 2, 1, 1\}$ $\quad$ wp(t)

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 5 \\ 5 \\ 0 \end{bmatrix}$$

$y_8(n) = \{5, 10, 5, 5, 0\}$

$\cancel{y(n) = \{3, 1, 5, 6, 5, 1, 6, 6, 11, 16, 20, 20, 18, 18, 12, 13, 8, 10, 8, 12, 8\}}$

$y(n) = \{1, 5, 5, 1, 6, 11, 20, 20, 18, 12, 8, 10, 12, 6, 5\}$

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(10) Perform linear convolution of the two sequences.

$x(n) = \{\overset{0}{3}, \overset{1}{-1}, \overset{2}{0}, \overset{3}{1}, \overset{4}{3}, \overset{5}{2}, \overset{6}{0}, \overset{7}{1}, \overset{8}{2}, \overset{9}{1}\}$ & $h(n) = \{1,1,1\}$ using overlap ~~a~~add method.

Sol. $L_s = 10$, $N = 5$, $M = 3$

$M-1 = 2$ (previous pts)

$N = L+M-1$

$L = 5-2 = 3$ (new pts)

$x(n) = \{L \text{ data points}, M-1 \text{ zeros}\}$

$x_1(n) = \{3,-1,0,0,0\}$; $x_2(n) = \{1,3,2,0,0\}$

$x_3(n) = \{0,1,2,0,0\}$; $x_4(n) = \{1,0,0,0,0\}$

add L-1 zeros to h(n), $h(n) = \{1,1,1,0,0\}$

$$y_1(n) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 2 \\ -1 \\ 0 \end{bmatrix} \qquad y_1(n) = \{3,2,2,-1,0\}$$

$$y_2(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 5 \\ 2 \end{bmatrix} \qquad y_2(n) = \{1,4,6,5,2\}$$

$$y_3(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 3 \\ 2 \end{bmatrix} \qquad y_3(n) = \{0,1,3,3,2\}$$

$$y_4(n) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad y_4(n) = \{1,1,1,0,0\}$$

$\Rightarrow y(n) = \{3,2,2,0,4,6,5,3,3,4,3,1\}$

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