UNIT-I

*1. Natural C.S

*2. Man made C.S

*3. Combinational C.S

*4. linear & non-linear C.S

*5. lumped & Distributed C.S

*6. Time variant & invariant C.S

*7. Continous & non-Continous C.S.

*8.

*9. open-loop C.S.

*10. Closed-loop C.S.

* UNIT-I *

* System :-

* System is a Combination of Elements or Components which are Connected in a sequence to Perform specific function is called a "System."

* Every physical object is a system.

* Ex :-

* A class room is a good Example of physical System.

* A room along with the Combination of benches, lights, fans, black boards Etc. Can be called a Class room which acts as an Elementary system.

* A lamp is made up of glass, filament is a physical System.

* Similarly, Systems can be any type i.e. Physical, Biological, mechanical, Electrical. Etc.

* Control System :-

* Control means to regulate, to direct or to command

* Control system is an arrangement of diff physical Elements connected in such a manner.

So as to regulate direct or Command it self or Some other System.

or

* In a system output Quantity is Controlled by Varying its input Quantity, the system is called C.S.

* o/p Quantity is called Controlled variable or response.

* i/p Quantity is called Command signal or Excitation or Stimulus.

* Classification / Types of C.S :-

*1. Natural C.S :-

* The biological system., Systems inside human being is called Natural C.S.

*Ex :-

* Respiratory system inside human being, this system activate Certain glands & regulates temperature of human body.

*2. Manmade C.S :-

* The systems that are designed & manufactured by human beings., Such systems like ~~Vehicl~~ Vehicles, Switches, Robots Etc.

*Ex :- * An automobile system with gears, accelerator, breaking system is a good Example

*3. Combinational C.S :-

* It is one having Natural & Man-made C.S.

* i.e.. driver driving a vehicle.

*4. Time varying & Time - Invarying C.S :-

*If the system's o/p changes with time then time variant C.S.

→ * In which parameters of the system are varying with time.

*Ex :- Space vehicles whose mass decrease with time as it leaves Earth mass is a parameter that decreases.

*Ex :- Rocket aero dynamic ~~dyna~~ damping can change with time as their air density changes with altitude.

→* The parameters ~~are~~ of the system are not changing w.r.t time.

Ie.. i/p's & o/p's are function of time but parameters are independant of time.

* Ex :- All Electrical networks consists R, L, C are time invarient.

Because, Values of R, L, C are Constant.

*5 linear & non-linear systems:-

* If a system obeys Homoginity principle of Satifies Super position principle {Homogenity & additivity} then it is linear system.

* Super position theorem States that the response Produced by Simultaneous application of two different forcing functions is sum of individual systems

*Ex:-

$y(t) = at^2 \rightarrow$ Non-linear.

*6. Continous time & Discrete time systems:-

* All System Variables are functions of continous variable time "t", at any time "t" depends on time.

*Ex:- Speed Control of a DC Motor using Tachometer.

* One or more system variables are known only at Certain descrete interval of time, they are not Continously depend on time.

*Ex:- MP or MC system.

*7. Deterministic or Random/Stocastic Systems :-

*. A C.S is deterministic when its response to I/p as well as behaviour to External disturbances is Predictable.

* If such response is unpredictable then it is said to be Random Systems.

*8. Lumped parameter or Distributed parameter C.S :-

* A C.S can be described by ordinary D.Eq is called lumped paramter C.S.

*Ex:- Electrical N/w with diff parameter R, L, C.

* A C.S that can be described by partial D.Eq

*Ex:- Transmission line.

*9. SISO & MIMO Systems:-

* A System having only single I/p & o/p is called SISO

* A System having multi I/p & o/p is called MIMO

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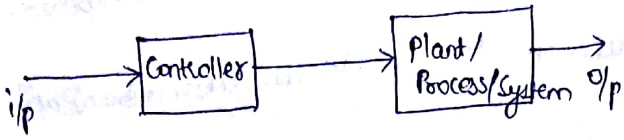
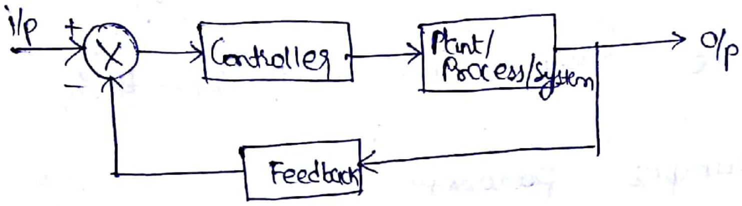
* Open loop & Closed loop C.S:→



* Controlling action is not dependant on o/p.

* o/p is Independant on i/p.

* Systems are more stable.



* Controlling action is dependant on i/p.

* o/p Can be Controlled by changing the i/p.

* Systems are not stable but more accurate & reliable.

* user can Control the o/p i.e.. desired o/p can be obtained.

* O.L.C.S:→

* A System in which o/p depends on i/p but Controlling action is independant of i/p i.e.. o/p quantity has no Effect upon on i/p quantity. is Called an O.L.C.S.

* o/p is not fed back to the i/p action

*Ex:→

* 1. Automatic washing machines

* 2. Fans with Blades.

*3. TV without Remote.

*4. Automatic toaster.

*5. Automatic door system.

*6. Traffic light controller without measuring density of traffic.

*7. ~~Sy~~ Sprinkler.

*8. Stepper motor positioning system.

*Advantages :-

→*) O.L. Systems are simple & Economical.

→*) Easier to Construct.

→*) more stable.

*Disadvantages :-

→*)1. O.L. Systems are inaccurate & unreliable.

→*)2. the changes in the o/p due to the External disturbances are so not Corrected automatically.

→*) An physical System which doesn't automatically track the Variation in its o/p is Called O.L. System.

*C.L.C.S :- / {Automatic C.S}

* A System in which the o/p has an Effect upon i/p Quantity in such a manner to maintain desired o/p Value is called C.L.C.S

*Ex :-

*1. Fan with blades & Regulator.

*2. manual washing machines.

*3. Automatic tunning Control

*4. T.V with remote

*5. Stabilizer

*6. missile launching, Vehicle System.

* Advantages:→

→*) The closed loop systems are accurate.

→*) These systems are less affected by noise

→*) Sensitivity of the system may be ~~(small)~~ made small) to make the system more stable.

* Disadvantages:→

→*) less stable due to the F/B, The system may lead to oscillatory response.

→*) F/B Reduces the overall gain of the system.

→*) Expensive

→*) Complex to design.

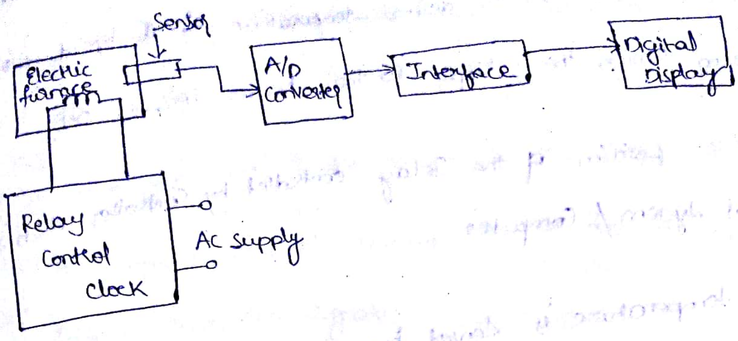
* Examples of C.S:→

*1. Temperature Control System:→
(T.C.S)

→ * Open loop system:→

* Electric furnace is an open loop CS

* The o/p in this system is desired temperature.
The temperature of the system is raised by heat generated by heating Element {coil}, the o/p ~~of~~ temperature ~~of~~ depends on time during which the supply to the heater remains "on".



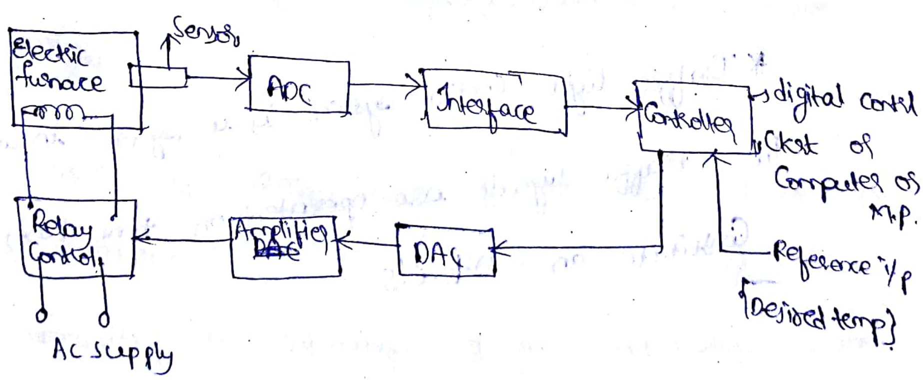
* The ON & OFF supply is Controlled by time setting of the relay

the temperature is measured by a sensor which gives an analog voltage corresponding to the temperature of the furnace.

* This analog is converted to digital {using ADC} & is given to LCD or display system. which displays the temperature of furnace.

* In this system if their is an change in the o/p temperature then the time setting of the relay is not altered automatically.

* Closed loop TC system:-



* The o/p of the system is the desired temperature & it depends on the time during which the supply to the heater remains ON.

* ON & OFF position of the Relay controlled by controller which is a digital system / Computer.

* Actual temperature is sensed by sensor & converted into digital by ADC, the computer or controller reads the actual temperature & compares desired / required temperature if it finds any difference then it sends a signal to switch ON or OFF the Relay through DAC & amplifier.

* Thus, the system automatically acts any changes in the o/p. (required temperature)

* Hence, it is a C.L.C.S.

* TRAFFIC light CONTROL SYSTEM :-

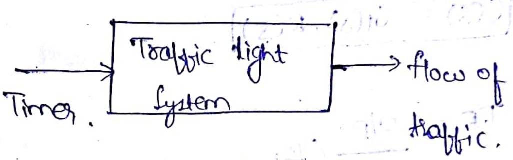
*① open loop T.L.C.S :-

* Traffic light control system is a system in which the traffic signals are operated on time bases constitute an O.L.C.S

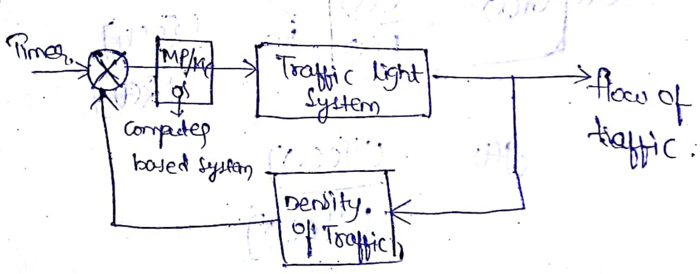
* The sequence of control signals are based on time slot given for each signal.

* The time slots are decided based on the traffic study, the system will not measure the density of the traffic before giving the signals.

* Since, time slot doesn't changes acc to traffic density the system is O.L. system.



*2. Closed loop T.L.C.S :-



* If the time slot of the signals are decided based on density of Traffic

* In closed loop Traffic light C.S. the density of the traffic is measured on all sides & information is fed to a computer, the timings of the control signals are decided by the computer based on density of traffic.

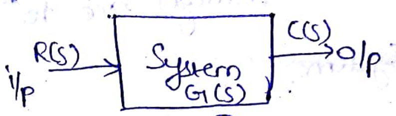
* Since, the C.L.S dynamically changes the timmings, the flow of vehicles will be better than O.L.S.

* Differences b/w OLCS & CLCS are in cam scanner.

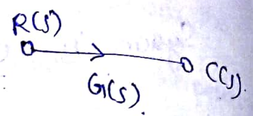
F/B characteristics.

* ~~Effect of feedback in CS~~ *

* O.L :-



(1)



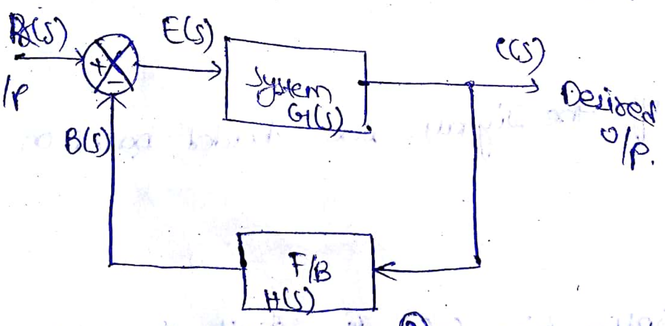
$$\boxed{C(s) = G(s) \cdot R(s)}$$

$$T.F = \frac{o/p}{i/p} \text{ (for the system)}$$

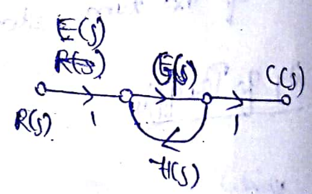
i.e., $\boxed{G(s) = \frac{C(s)}{R(s)}}$ i.e., $\frac{L[c(t)]}{L[R(t)]}$

$$g(t) = \frac{L^{-1}[C(s)]}{L^{-1}[R(s)]}$$

* CL :-



(2)



$$\frac{C(s)}{R(s)} = ?$$

*

* Feed back systems plays an imp role in modern Engineering practice because they have the possibility for being adapted to perform their assigned tasks automatically.

* A non feed back {O.L. System} represented by block diagram & signal flow graph, is shown in figure ①.

* There is no provision within the system for supervision of o/p & no mechanism is provided to crck the system behaviour.

* F/B Control system or C.L.C.S or Automatic C.S is represented by Block & signal flow graphs are shown in ②. figure.

* It is having two signals one is i/p signal & other is F/B signal derived from the o/p of the system.

* the F/B signal gives system capability to act as self crcting mechanism.

* $C(s) = E(s) \cdot G(s)$

$B(s) = C(s) \cdot H(s)$

$E(s) = R(s) - B(s)$.

$\Rightarrow C(s) = G(s)\left[R(s) - B(s)\right]$

$\Rightarrow C(s) = G(s).R(s) - G(s).B(s)$

$\Rightarrow C(s) = G(s)\,R(s) - G(s).C(s).H(s)$

$\Rightarrow C(s) = G(s)\left[R(s) - C(s).H(s)\right]$

$\Rightarrow C(s)\left[1 + G(s).H(s)\right] = G(s).R(s)$

$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}}$$

* gain can be Controlled

* Sensitivity of the system can be reduced by " $\frac{1}{1+G(s).H(s)}$ "

F/B Characteristics:-

* Effect of F/B in C.S:-

* F/B means automatic regulation & Control

→ ① Controlled Variable accurately follows the desired o/p.

→ ② Effect on the Controlled Variable on the External disturbances other than those associated with the F/B Sensor are greatly reduced.

→ ③ Effect on ...

on ...

lev...

* There ...

& ...

F/...

of ...

* F/B ...

C...

* Effect ...

*1.

*2.

③ Effect of variation in controller & processor parameters: {Forward Path}

on the system performance is reduced to acceptable levels.

* These variations are caused due to component age & environmental change, temperature etc i.e..

F/B in control loop allows accurate control of the o/p even when process are or controlled parameters are not known accurately.

* F/B in C.S greatly improves the speed of its response compared to the speed of the response in O.L system.

* Effect of F/B on C.S:→

*1. Effect of F/B on gain.

*2. * Consider an O.L system with overall transfer function as $C(S) = \frac{G(S) \cdot R(S)}{\cancel{1+G(S) \cdot H(S)}}$

* If the F/B H(s) is introduced then its overall gain becomes $\frac{G(S)}{1 \pm G(S) \cdot H(S)}$

For a -ve F/B C.S the gain is reduced by a factor of $\frac{1}{1+G(S) \cdot H(S)}$

* due to the Negative F/B overall gain of the System is reduce.

*2. Reduction of parameter variation by -ve F/B :-

* Parameters of the system may vary with age, with changing Environment

The main purpose of F/B in C.S is to reduce the Sensitivity of the System to parameter Variations.

* Sensitivity is a measure of Effectiveness of F/B in reducing the influence of these Variations on System performance.

* let us define sensitivity, on a quantitative bases.

* for open loop case

$$C(s) = R(s) \cdot G(s)$$

* Suppose due to parameter Variations G(s) is changed to

$$\cancel{G(s)} \quad G(s) \pm \Delta G(s)$$

where $G(s) >>> \Delta G(s)$

* The o/p of O.L system changes to

$$C(s) + \Delta C(s) = [G(s) + \Delta G(s)] R(s).$$

$$\boxed{\Delta C(s) = \Delta G(s)\cdot R(s)} \rightarrow (1)$$

* For closed loop case the o/p is

$$C(s) = \frac{G(s)}{1+G(s)\cdot H(s)}.$$

But $G(s) = G(s) + \Delta G(s)$; $G(s) >>> \Delta G(s)$

$$\Rightarrow C(s) + \Delta C(s) = \frac{G(s) + \Delta G(s)}{1+[G(s)+\Delta G(s)]\cdot H(s)} = \frac{G(s)+\Delta G(s)}{1+G(s)\cdot H(s)}$$

$$\Rightarrow \boxed{\Delta C(s) = \frac{\Delta G(s)}{1+G(s)\cdot H(s)}} \rightarrow (2).$$

* From Eq (1) & (2), seen that in comparision to O.L & C.L due to variation in G(s) is reduced by a factor of $1 + G(s)\cdot H(s)$.

* **Effect of F/B on Time Constant of CS :-**

* Consider an O.L system with overall T.F of

$$G(s) = \frac{K}{1+sT}.$$

with this system is subjected to unit step i/p the response can be obtained as

$$\frac{C(s)}{R(s)} = G(s) \; ; \; R(s) = 1/s \text{ unit step.}$$

$$\Rightarrow C(s) = \left(\frac{K}{1+ST}\right)\cdot\frac{1}{S}.$$

$$\Rightarrow C(s) = \frac{K}{S(1+ST)} = \frac{A}{S} + \frac{B}{1+ST}$$

$$C(t) = L^{-1}[C(s)]$$

$$\Rightarrow A(1+ST) + SB = K.$$

* for $S=0 \Rightarrow A=K$.
* for $S=-1/T \Rightarrow B=-KT.$

$$\Rightarrow C(s) = \frac{K}{S} - \frac{KT}{1+ST} = K\left[\frac{1}{S} - \frac{T}{1+ST}\right]$$

$$\Rightarrow C(t) = K\left[1 - e^{-t/T}\right] \longrightarrow (1)$$

* C.L.C.S:→

$$C(s) = \frac{G(s)\,R(s)}{1+G(s)H(s)} = \frac{\frac{K}{1+ST}}{1+\left(\frac{K}{1+ST}\right)h}\cdot\frac{1}{S}$$

$$C(s) = \frac{K}{(1+ST)+Kh}\cdot\frac{1}{S}$$

$$= \frac{1}{S}\cdot\frac{K/T}{S+\left(\frac{1+Kh}{T}\right)} = \frac{A}{S} + \frac{B}{S+\left(\frac{1+Kh}{T}\right)}$$

$$A\left[S+\left(\frac{1+Kh}{T}\right)\right] + BS = K/T$$

at $S=0 \Rightarrow A = \frac{K}{1+Kh}.$

at $S=-\left(\frac{1+Kh}{T}\right) \Rightarrow B = \frac{-K}{1+Kh}.$

$$C(s) = \frac{K}{1+Kh}\left[\frac{1}{S} - \frac{1}{S+\left(\frac{1+Kh}{T}\right)}\right]$$

$$c(t) = \frac{K}{1+Kh}\left[1 - e^{-t/\left(\frac{T}{1+Kh}\right)}\right]$$

* From the F/B the new time constant is "$\frac{T}{1+Kh}$" for +ve values of "h" & $K > 1$. the time constant is $< T$

$\therefore$ In O.L system the time constant is T

In C.L system the time constant is $\frac{T}{1+Kh}$.

* So, less Time constant in C.L system, if the time constant is less, the system gives faster response.

* Hence, F/B improves time response of a system.

*4 Effect of F/B on Stability :-

* F/B reduces the Time constant & makes the system response more fast. hence, transient response decays more quickly.

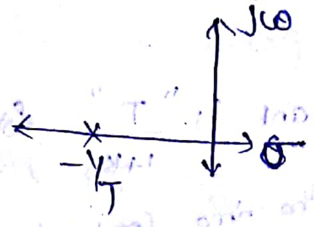
* Consider the O.L T.F of a system as $\frac{C(S)}{R(S)} = \frac{K}{1+ST}$.

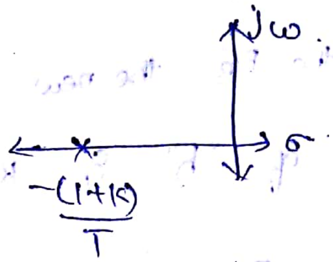
Open loop pole is located at $S = \frac{-1}{T}$.

* Consider unity -ve F/B is introduced in the system then overall T.F of C.L system becomes

* $$\frac{C(s)}{R(s)} = \frac{\frac{K}{1+sT}}{1+\left(\frac{K}{1+sT}\right)(1)} = \frac{K}{1+sT+K} = \frac{K/T}{s+\left(\frac{1+K}{T}\right)}$$

* Thus, the closed loop pole is located at $s = -\left(\frac{1+K}{T}\right)$





* F/B Controls the "Time response, Stability of a system depends on located of poles in s-plane. thus, it can be concluded that the F/B effects the stability & F/B may be improve the unstable system response.

* Effect of F/B on disturbance :-

* Every C.S has some non-linearities present in it it effects on output of the system.

* Some External disturbance signal {noise} also make the system output inaccurate.

* The Ex of External disturbances are high frequency noise in Electronic application, thermal noise in amplifier, C.R.O's etc.

$\frac{K/T}{S+\left(\frac{1+K}{T}\right)}$

$= -\left(\frac{1+K}{T}\right)$

ystem depends

concluded that

improve

in it

o make

h frequency

se in

*The disturbances may be present in Forward paths as well as in F/B paths. {read these two from F/B}

07/12/17

* Mathematical modeling of a control system:→

*Any system can be described by math. Eq's ~~But~~ the math Eq's are Diff Eq or P.D.E

*Types:→

*1. Mechanical translational system. {M.T.S}

*2. Mechanical Rotational System. {M.R.S}

Def:→

* The set of math Eq's describing the dynamic chars of a system is called math. modeling of a system.

*These math Eq's may be D.Eq's or P.D.Eq's.

*The response (or) o/p of the system can be studied by solving the DiffEq for various i/p conditions.

*Two types mech. systems are in C.S. they are ① & ②

*① M.T.S:→

*Consider a Mech. system in which motion takes place along a st.line, such systems are translational.

*These systems are characterised by

ⓐ linear displacement

ⓑ " Velocity

ⓒ " acceleration.

* There are 3 basic Elements in Translational system, they

* (i) Mass (m), * (ii) Dashpot or Friction (B), (iii) Spring (K).

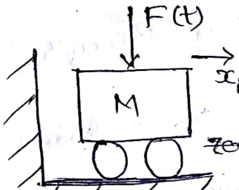
* Newton's 2nd law :-

* For any body the sum of applying forces is & the force resulting the motion in any given direction is Equal to zero.

(or)

* The sum of applying forces is Equal to the Sum of resulting forces.

* (i) Mass (m) :-



F(t) → applied force. ; x_1 = displacement.

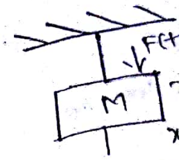
* Consider an ideal mass Element which has negligible friction & Elasticity, let a force is applied on it, which Produces a displacement "X(t)" in the direction of applied force "F(t)". The mass (m) will offer an opposing force

* Which is proportional to acceleration of the body.

$F_M \propto a$. {opposing force due to mass element F_M}

$F_M = Ma$

$\boxed{F_M = M.\frac{dx^2}{dt^2}}$ {M = Co-efficient}



F_M = M.

* (ii) Dashpot

Can be

* Dashpot

air

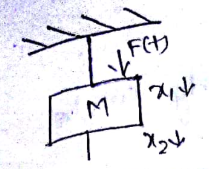
* Friction

betw

* let

forc

Both the End are free but above one End is fixed



$$\boxed{F_M = M.\frac{d^2(x_1-x_2)}{dt^2}}$$

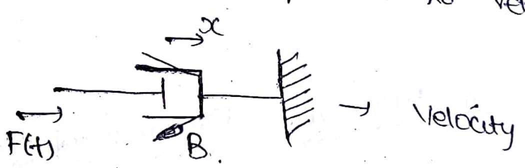
*(ii) Dashpot of Friction (B) :-

* The Friction Existing in mechanical system. Can be represented by Dashpot

* Dashpot is a piston moving in a cylinder filled with air fluid

* Friction may be between moving Element & fixed support or between two moving surfaces.

* let force "F" applied on it, the dashpot will offer an opposing force which is proportional to velocity of the body.



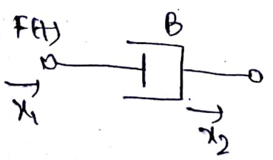
$F_B \propto V.$

$F_B = B.V.$

F_B = opposing force due to friction of Dashpot.

$$\boxed{F_B = B.\frac{dx}{dt}}$$

Both the Ends are free but above one End is fixed

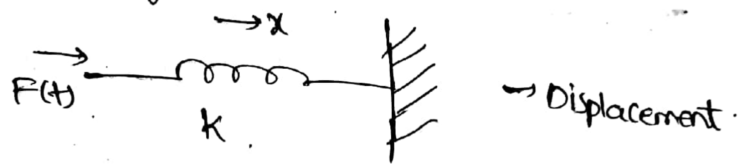


$$\boxed{F_B = B.\frac{d}{dt}(x_1-x_2).}$$

*(iii) Spring (K):→

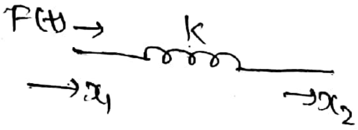
* The ~~Electrice~~ Elastic deformation of the body can be represented by spring.

* Let a force is applied on it, the spring will offer an opposing force, which is proportional to displacement of the body.



$F_K \propto x$. { F_K = opposing force due to spring (K) }

$$\boxed{F_K = Kx}$$



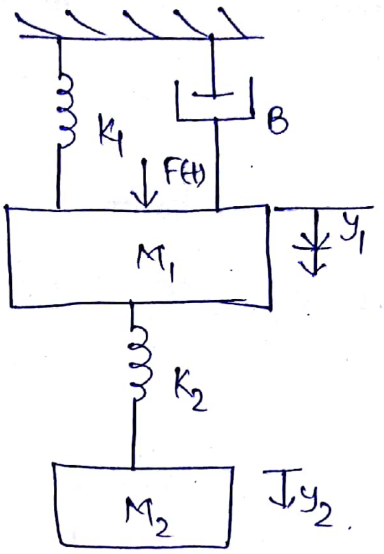
$$\boxed{F_K = K(x_1 - x_2)}$$

11/12/17

* Determination of T.F of mechanical Translational system:→

* write the D.E of mechanical system & T.F. $\frac{Y_2(s)}{F(s)}$.

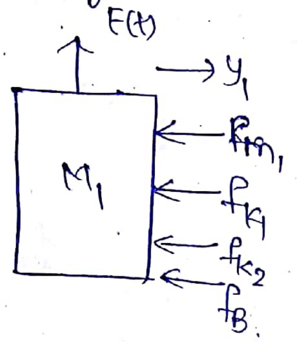
*Q. (1)



*sol

*Free body diagram :-

*for M_1



$$F(t) = f_{m_1} + f_{K_1} + f_{K_2} + f_B.$$

$$F(t) = M_1 \cdot \frac{d^2y_1}{dt} + K_1 y_1 + B \cdot \frac{dy_1}{dt} + K_2(y_1 - y_2)$$

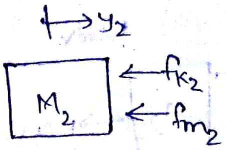
* Apply LT on B.S

$$\Rightarrow F(s) = M_1 s^2 \cdot Y_1(s) + K_1 Y_1(s) + B. S. Y_1(s) + K_2(Y_1(s) - Y_2(s))$$

$$\Rightarrow F(s) = Y_1(s)\left[M_1 s^2 + K_1 + SB + K_2\right] - Y_2(s)\left[K_2\right] \rightarrow (1).$$

Eq TF · $\dfrac{Y_2(s)}{F(s)}$.

* for M_2 is



$$f_{k_2} + f_{m_2} = 0.$$

$$\Rightarrow K_2(y_2 - y_1) + M_2 \cdot \frac{d^2 y_2}{dt^2} = 0.$$

$$\Rightarrow K_2 Y_2(s) - K_2 Y_1(s) + M_2 \cdot s^2 \cdot Y_2(s) = 0.$$

$$\Rightarrow K_2 Y_1(s) = K_2 \cdot Y_2(s) + M_2 s^2 \cdot Y_2(s).$$

$$\Rightarrow Y_1(s) = Y_2(s) + \frac{M_2}{K_2} \cdot s^2 \cdot Y_2(s). \rightarrow ②$$

* from ① & ② we get.

$$\Rightarrow F(s) = Y_2(s)\left[1 + \frac{M_2 s^2}{K_2}\right]\left[M_1 s^2 + K_1 + sB + K_2\right]$$

$$- Y_2(s)\,[K_2]$$

$$\Rightarrow \frac{Y_2(s)}{F(s)} = \frac{1}{(M_1 s^2 + K_1 + sB + K_2)\left[1 + \frac{M_2 s^2}{K_2}\right] - K_2}.$$

$$\Rightarrow \frac{Y_2(s)}{F(s)} = \frac{K_2}{\cancel{M_1 s^2 K_2} + \cancel{K_1 K_2} + \cancel{sBK_2} + K_2^2 + M_1 s^4 M_2 + M_2 s^2 K_1 + s^3 BM_2 + \cancel{M_2 s^2 K_2} - K_2}.$$

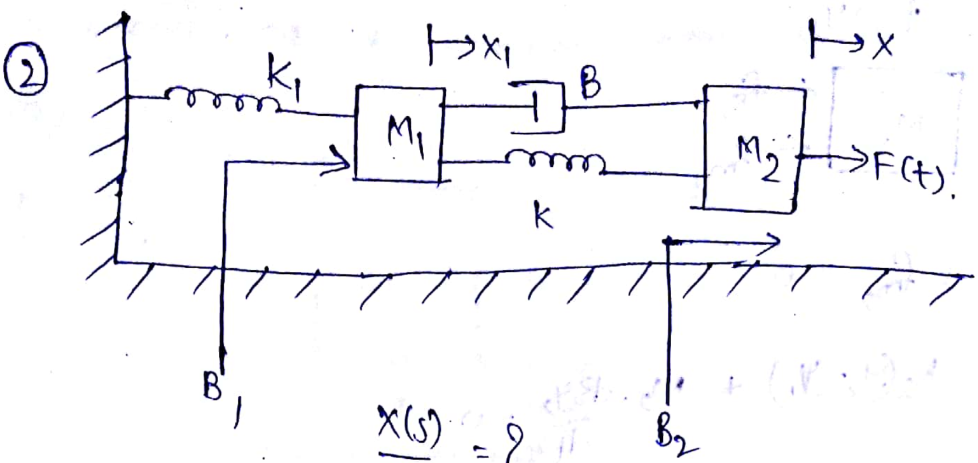
$$\Rightarrow \frac{Y_2(s)}{F(s)} = \frac{K_2}{(K_2 + M_2 s^2)(M_1 s^2 + Bs + K_1 + K_2) - K_2^2}$$

$(y_1 - y_2)$

$(s) + K_2(Y_1(s) - Y_2(s))]$

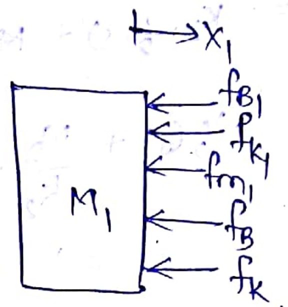
$s)\,[K_2]. \rightarrow ①.$

②



$$\frac{X(s)}{F(s)} = ?$$

*Sol) *. 2 Mass Elements ⇒ 2 D.Eq's.

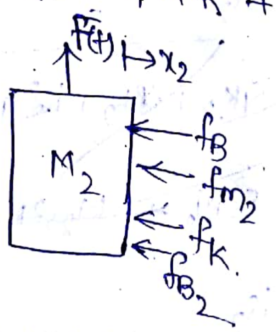


$$* \quad f_{k_1} + f_B + f_k + f_{m_1} + f_{B_1} = 0.$$

$$\Rightarrow B.\frac{d(x_1-x_2)}{dt} + K_1 x_1 + K(x_1 - x_2) + m_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} = 0.$$

$$\Rightarrow BSX_1(s) + K_1.X_1(s) + K[X_1(s)] - K\,X_2(s)$$
$$-BSX_2(s)$$
$$+ m_1.S^2.X_1(s) + B_1 S X_1(s) = 0$$

$$\Rightarrow \begin{bmatrix} K X_2(s) \\ +BX_2(s) \end{bmatrix} = X_1(s)\,[B + K_1 + K + m_1 S^2 + B_1]$$



$$\Rightarrow X_1(s) = \frac{(K + BS)\,X(s)}{BS + K_1 + K + m_1 s^2 + B_1 S} \rightarrow ①$$

$f_B + f_{m_2} + f_K + f_{B_2} = F(t)$.

$$\Rightarrow B.\frac{d(x_0 - x_1)}{dt} + m_2 . \frac{d^2 x_0}{dt^2} + K.(x_0 - x_1) + B_2 . \frac{dx}{dt} = F(s)$$

$$\Rightarrow BSX(s) - BSX_1(s) + m_2 . s^2 . X(s) + K.X(s) - KX_1(s)$$

$$+ B_2 s X(s) = F(s).$$

$$\Rightarrow \cancel{X(s)\left[B + m_2 s^2 + K + B_2\right]} - \frac{\cancel{\left[(K+B)X(s)\right]}}{\cancel{\left[B + K_1 + K + m_1 s^2 + B_1\right]}}$$

$$\cancel{= F(s)}.$$

$$\Rightarrow -X_1(s)\left[+K + Bs\right] + X(s)\left[Bs + m_2 s^2 + K + B_2 s\right] = F(s)$$

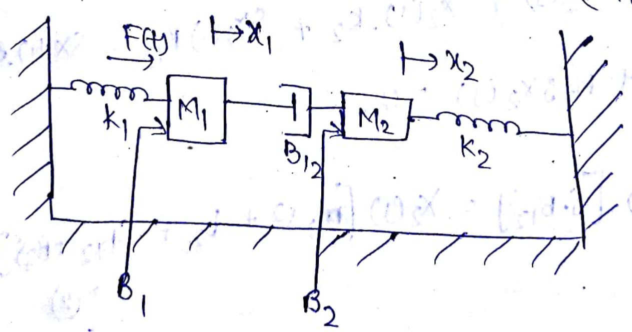
$$\Rightarrow \frac{-(K+Bs)^2 . X(s)}{Bs + K_1 + K + m_1 s^2 + B_1 s} + X(s)\left[Bs + m_2 s^2 + K + B_2 s\right] = F(s)$$

$$\Rightarrow X(s)\left[\frac{-(K+Bs)^2}{Bs + K_1 + K + m_1 s^2 + B_1 s} + \left[Bs + m_2 s^2 + K + B_2 s\right]\right] = F(s)$$

$$\Rightarrow \frac{X(s)}{F(s)} = \frac{Bs + K_1 + K + m_1 s^2 + B_1 s}{(Bs + m_2 s^2 + K + B_2 s)(Bs + K_1 + K + m_1 s^2 + B_1 s) - (K+Bs)^2}$$

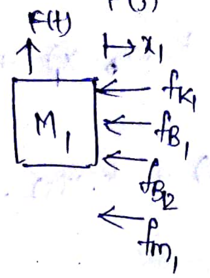
$$B_1 \frac{dx_1}{dt} = 0.$$

③



$\frac{X_1(s)}{F(s)} = ?$; $\frac{X_2(s)}{F(s)} = ?$

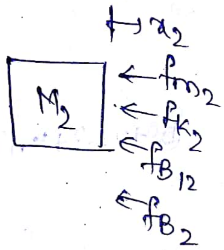
*Sol)



$$F(t) = K_1 . x_1 + B_1 . \frac{dx_1}{dt} + B_{12} . \frac{d(x_1 - x_2)}{dt} + m_1 . \frac{d^2 x_1}{dt^2}$$

$$F(s) = K_1 X_1(s) + B_1 S . X_1(s) + B_{12} . S X_1(s) - B_{12} S X_2(s) + m_1 . S^2 . X_1(s).$$

$$F(s) = X_1(s) [K_1 + B_1 S + B_{12} S + m_1 S^2] - (B_{12} . S) X_2(s) \quad \to (1)$$



$$\Rightarrow m_2 . \frac{d^2 x_2}{dt^2} + K_2 . x_2 + B_{12} . \frac{d(x_2 - x_1)}{dt} + B_2 \frac{dx_2}{dt} = 0.$$

$$\Rightarrow m_2 . S^2 . X_2(s) + X_2(s) . K_2 + S X_2(s) B_{12} - S X_1(s) . B_{12} + B_2 S X_2(s) = 0.$$

$$\Rightarrow X_1(s) [S . B_{12}] = X_2(s) [m_2 S^2 + K_2 + S B_{12} + B_2 S] = 0. \quad \to (2)$$

from ① & ② we get

$$\Rightarrow F(s) = X_1(s)\left[K_1 + B_1 s + B_{12}s + m_1 s^2\right] - B_{12}s\left[\frac{sB_{12}\cdot X_1(s)}{K_2 + sB_{12} + sB_2 + m_2 s^2}\right]$$

$$\Rightarrow F(s) = X_1(s)\left[K_1 + B_1 s + B_{12}s + m_1 s^2 - \frac{s^2 B_{12}^2}{K_2 + sB_{12} + sB_2 + m_2 s^2}\right]$$

$$\frac{X_1(s)}{F(s)} = \frac{K_2 + sB_{12} + sB_2 + m_2 s^2}{(K_1 + B_1 s + B_{12}s + m_1 s^2)(K_2 + sB_{12} + sB_2 + m_2 s^2) - s^2 B_{12}^2}.$$

$\Rightarrow$ from ① & ② we get.

$$\Rightarrow F(s) = (K_1 + B_1 s + B_{12}s + m_1 s^2)\left[\frac{m_2 s^2 + K_2 + sB_{12} + B_2 s}{sB_{12}}\right]\cdot X_2(s)$$

$$- sB_{12}\cdot X_2(s).$$

$$\Rightarrow F(s) = X_2(s)\left[\frac{(K_1 + B_1 s + B_{12}s + m_1 s^2)(m_2 s^2 + K_2 + sB_{12} + B_2 s) - s^2 B_{12}^2}{sB_{12}}\right]$$

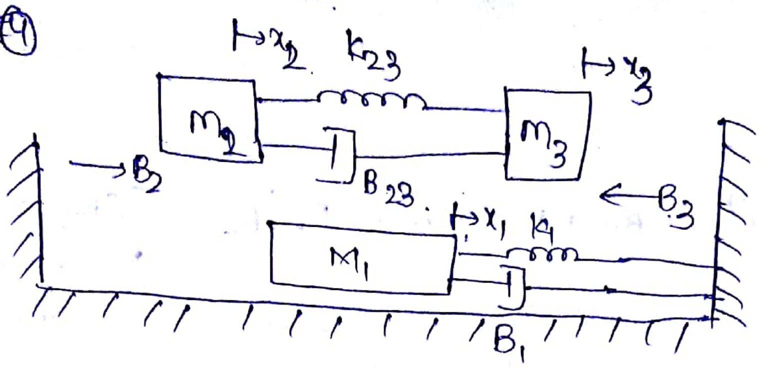
$$\Rightarrow \frac{X_2(s)}{F(s)} = \frac{sB_{12}}{(K_1 + B_1 s + B_{12}s + m_1 s^2)(m_2 s^2 + K_2 + sB_{12} + B_2 s) - s^2 B_{12}^2}.$$

0.

B_{12}

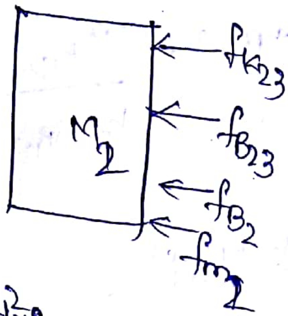
$] = 0.$

*9

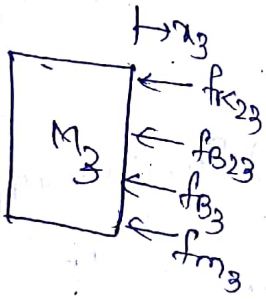


{ B_2 & B_3 are for Both }

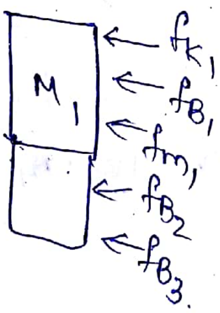
Sol)



$$\Rightarrow m_2 \cdot \frac{d^2x_2}{dt^2} + K_{23}(x_2-x_3) + B_{23}\cdot\frac{d(x_2-x_3)}{dt} + B_2\cdot\frac{d(x_2-x_1)}{dt} = 0$$



$$\Rightarrow m_3\cdot\frac{d^2x_2}{dt^2} + K_{23}\cdot(x_3-x_2) + B_{23}\cdot\frac{d(x_3-x_2)}{dt} + B_3\cdot\frac{d(x_3-x_1)}{dt} = 0.$$



$$\Rightarrow B_3\cdot\frac{d(x_1-x_3)}{dt} + B_2\cdot\frac{d(x_1-x_2)}{dt} + m_1\cdot\frac{dx_1^2}{dt} + k_1(x_1) + B_1\cdot\frac{dx_1}{dt} = 0$$

B_2 & B_3 are ~~Bg~~ Both

$+ B_2 \cdot \frac{d(x_2-x_1)}{dt} = 0.$

$+ B_3 \cdot \frac{d(x_3-x_2)}{dt} = 0.$

$B_1 \cdot \frac{dx_1}{dt} = 0$

* Mechanical Rotational System *

* Motion about fixed axis in such systems force gets replaced by movement about fixed axis (i.e.. force ~~into~~ * distance about fixed axis) which is called Torque.

* Sum of Torques applied on a body (or) system must be Equal to sum of torques consumed by the different Elements of the system in order to produce angular displacement (θ), angular velocity (ω), & angular acceleration (α).

* weight of rotational mechanical system is represented by movement of inertia of the mass, Elastic deformation of the body can be represented by spring {Torsional spring}.

* Friction Existing in mechanical rotational system can be represented by Dashpot / Damper. (B)

θ = angular displacement → radians

$\omega = \frac{d\theta}{dt}$ → Velocity → $\frac{\text{radians}}{\text{second}}$.

$\alpha = \frac{d^2\theta}{dt^2}$ → angular acc → $\frac{\text{Radians}}{\text{Sec}}$.

T = applid Torque → Newton/metre.

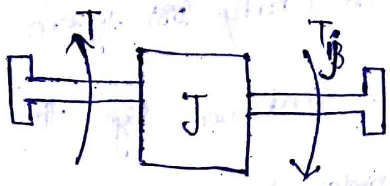
J = moment of inertia → ~~kg/m~~ kg-m²/radians

B = Rotational friction co-efficient → $\frac{\text{Newton/meter}}{\text{radians}}$.

K = Stiffness of the spring → $\frac{N-m}{r}$.

* Rotational system has three basic Elements

① moment of inertia :-



$$T_j \propto \frac{d^2\theta}{dt^2}$$

$$T_j = j \cdot \frac{d^2\theta}{dt^2}$$

* let Torque "T" is applied on it, the opposing torque due to moment of inertia is proportional to angular acceleration of the body.

② Dashpot / Damper :-

* let Torque "T" is applied on it, the opposing torque due to frictional co-efficient is proportional to angular velocity of the body.

Newton-meter

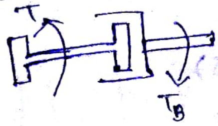
radians

Opposing torque

...tional to angular

the Opposing torque

Proportional to

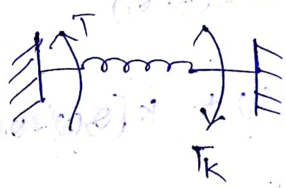


$T_B \propto \frac{d\theta}{dt}$

$T_B = B \cdot \frac{d\theta}{dt}$.

* Torsional Spring :-

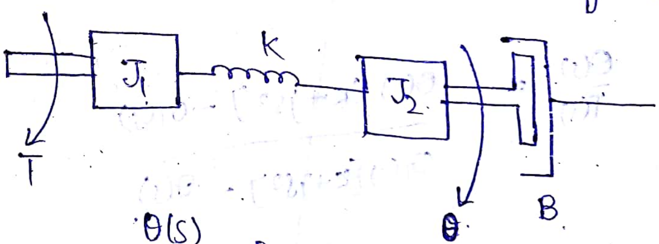
* Let torque "T" is applied on it ; the opposing torque due to Spring is proportional to angular displacement (θ)



* $T_K \propto \theta$.

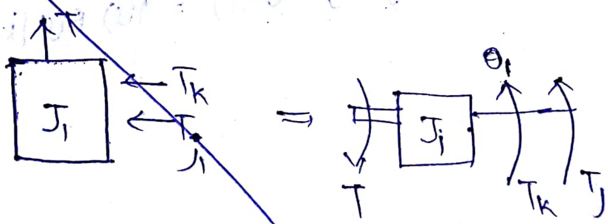
$T_K = K\theta$.

*Find the TF of the mechanical rotational system.



$\frac{\theta(s)}{T(s)} = ?$

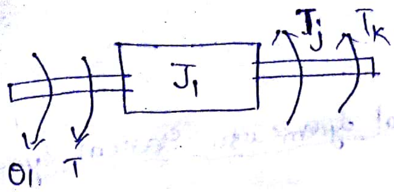
*sol)



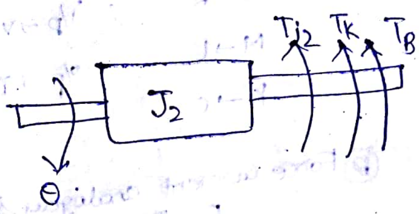
~~$T = T_K + T_{J_1}$~~

~~$T = K.\theta + J\, d^2\theta/dt$.~~ $K \cdot (\theta_2 - \theta_1) + J \cdot \frac{d^2\theta_1}{dt}$.

Sol)



$$T = T_j + T_K = J\frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta)$$



$$T_{j_2} + T_K + T_B = 0.$$

$$J_2 \cdot \frac{d^2\theta}{dt^2} + B \cdot \frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

Solve we get $\dfrac{\theta(s)}{T(s)} = \dfrac{K}{(K + J_2 s^2 + Bs)(K + J_1 s^2) - K^2}.$

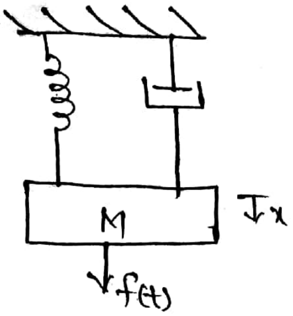
Take 9 pages xerox

* 19/12/17 *

* Convert the given mechanical systems into Electrical systems

* ①

(I)



ⓐ Force voltage analogous system

$F \to V$

$B \to R$; i/p → V

$M \to L$ o/p → I.

$K \to C$

ⓑ Force current analogous system

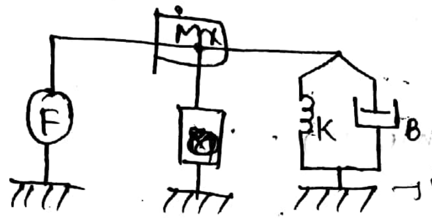
$F \to I$

$B \to 1/R$

$M \to C$

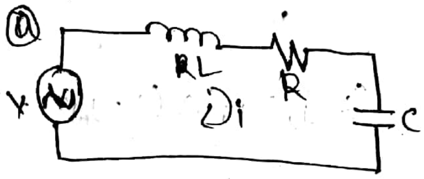
$K \to L$.

⇓

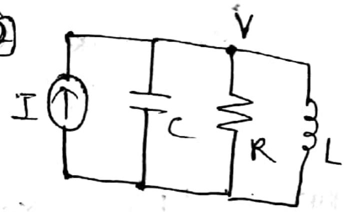


→ {no need of this}

⇒ ⓐ



ⓑ



* for (I)

$$f(t) = M.\frac{d^2x}{dt^2} + Kx + B.\frac{dx}{dt}.$$

for ⓐ $F(s) = X(s)\,[Ms^2 + Bs + K]$

$$V = L.\frac{di}{dt} + R.i + \frac{1}{C}\int i\,dt.$$

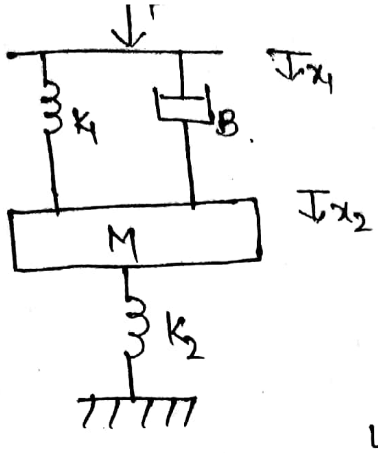
$$V(s) = L\,I(s).s + R.I(s) + \frac{1}{sC}I(s) \Rightarrow V(s) = I(s)[Ls + R + \tfrac{1}{sC}$$

for ⓑ

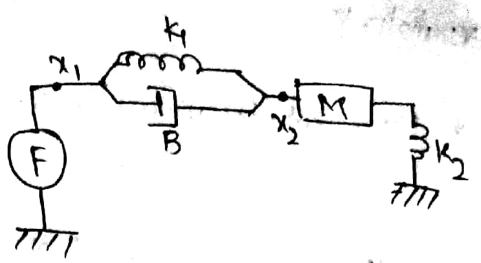
$$I = C.\frac{dv}{dt} + \frac{V}{R} + \frac{1}{L}\int v.dt.$$

$$I(s) = V(s)\left[sC + \frac{1}{R} + \frac{1}{sL}\right].$$

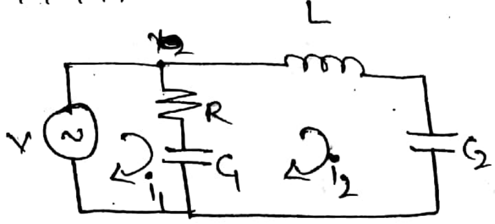
Sol



$\Rightarrow$



(a)



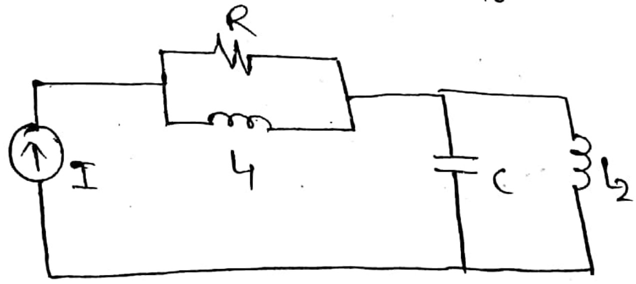
$$V = R(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)\,dt$$

$$0 = L\frac{di_2}{dt} + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int (i_2 - i_1)\,dt + R(i_2 - i_1).$$

$$V(s) = R.[I_1(s) - I_2(s)] + \frac{1}{C_1 s}[I_1(s) - I_2(s)]$$

$$0 = L.I_2(s).s + \frac{1}{C_2 s} I_2(s) + \frac{1}{C_1 s}[I_2(s) - I_1(s)] + R[I_2(s) - I_1(s)]$$

(b)



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29/12/17

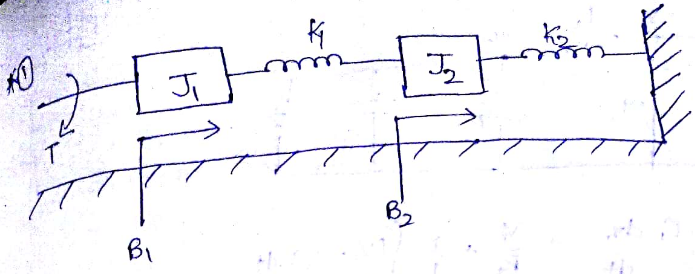
* Mechanical Rotational System:→

Parameter	T→V	T→I
Torque (T)	V	I
Inertia (J)	L	C
Dashpot (B)	R	1/R
Torsional Spring (K)	C	L
θ	q	φ
Junction	loop	node
Newton's law	KVL	KCL

* Mechanical Translational System:→

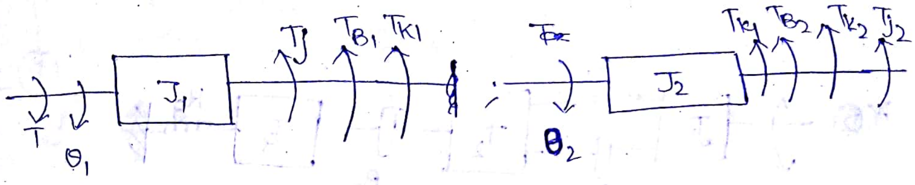
Parameter	F→V	f→I
Force (F)	Voltage (V)	Current (I)
mass (m)	Inductance (L)	Capacitance (C)
Dashpot (B)	Resistor (R)	Conductance/ ~~Resistor~~ (1/R)
Spring (K)	Capacitance (C)	Inductance (L)

1



* write differential Eq's.

* Free body diagram :->

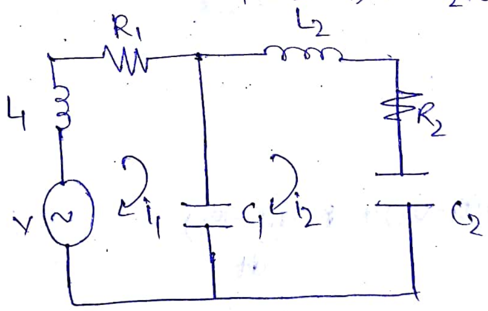


$$* \; T = T_{J_1} + T_{B_1} + T_{K_1}$$

$$\Rightarrow T = J_1 \cdot \frac{d^2\theta_1}{dt^2} + B_1 \cdot \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2)$$

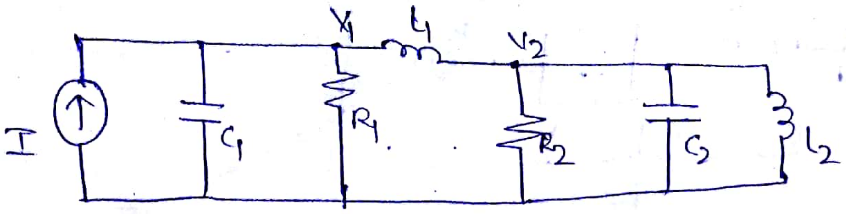
$$* \; T = T_{K_1} + T_{K_2} + T_{B_2} + T_{J_2}$$

$$\Rightarrow 0 = K_1(\theta_2 - \theta_1) + K_2 \cdot \theta_2 + B_2 \cdot \frac{d\theta_2}{dt} + J_2 \cdot \frac{d^2(\theta_2)}{dt^2}$$



$$V = L_1 \cdot \frac{di_1}{dt} + i_1 R_1 + \frac{1}{C_1} \int (i_1 - i_2)\,dt.$$

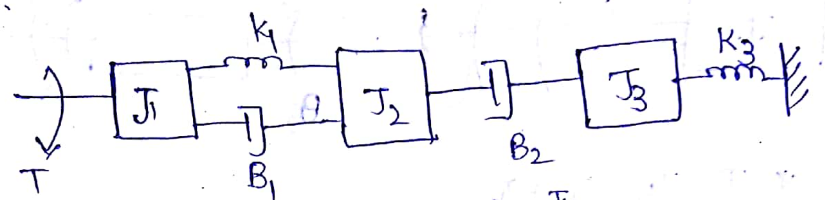
$$0 = L_2 \cdot \frac{di_2}{dt} + \frac{1}{C_2} \int i_2\,dt + \frac{1}{C_1} \int (i_2 - i_1)\,dt + i_2 R_2$$



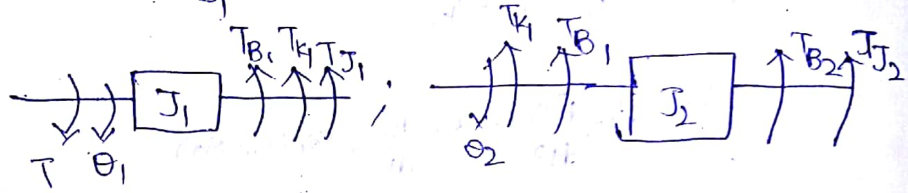
$$I = C_1\frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int (V_1 - V_2).dt$$

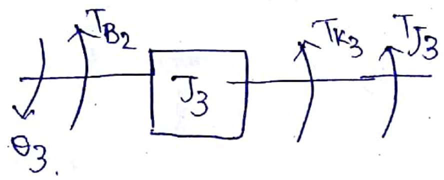
$$0 = \frac{1}{L_1}\int (V_2 - V_1).dt + \frac{V_2}{R_2} + C_2.\frac{dV_2}{dt} + \frac{1}{L_2}\int V_2\,dt.$$

*②



*J(θ1)





$$T = T_{J_1} + T_{K_1} + T_{B_1}.$$

J_1

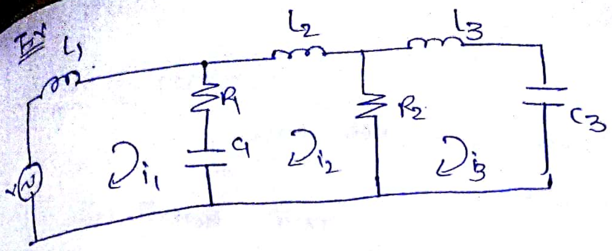
$$\Rightarrow T = J_1.\frac{d^2\theta_1}{dt} + K_1(\theta_1 - \theta_2) + B_1\frac{d(\theta_1 - \theta_2)}{dt}$$

J_2

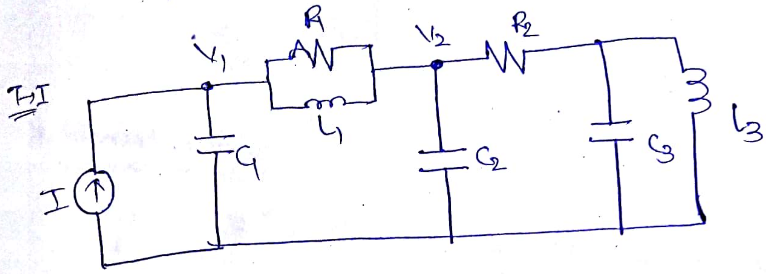
$$\Rightarrow 0 = J_2.\frac{d^2\theta_2}{dt} + K_1(\theta_2 - \theta_1) + B_2\frac{d(\theta_2 - \theta_3)}{dt} + B_1\frac{d(\theta_2 - \theta_1)}{dt}$$

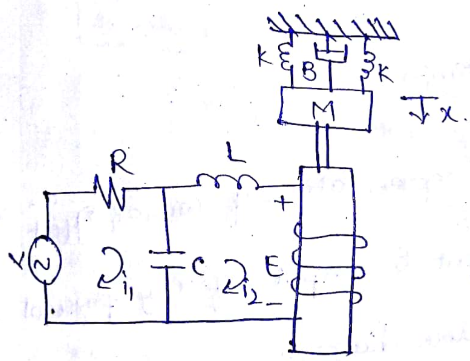
J_3

$$0 = J_3.\frac{d^2\theta_3}{dt} + K_3\theta_3 + B_2.\frac{d(\theta_3 - \theta_2)}{dt}$$



{write Equation for both ckts}





*

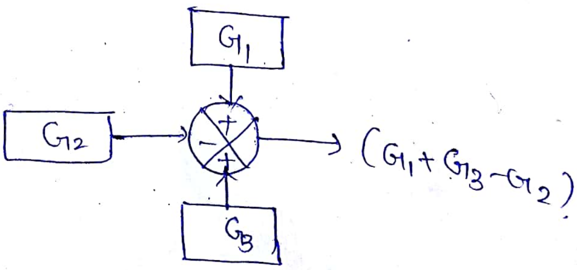
$$\frac{X(s)}{V(s)} = ?$$

*

*Block diagram reduction system:→

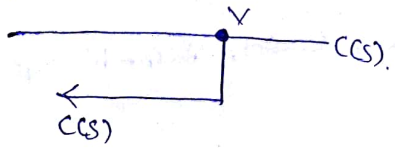
*A short hand pictorial representation of cause & effect relationship between input & output of any physical system is called as block diagram.

*A point at which two or more blocks are added or subtracted is known as summing point.



* **Take off point:-**

  * A point at which output signal is sampled & given to some other block.



* **Forward path:-**

  * The direction of signals from i/p to o/p is known as F.P.

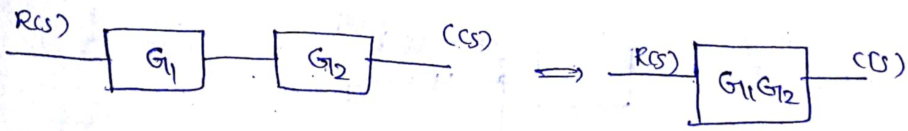
* **F/B Path:-**

  * The direction of signal flow from o/p to i/p is known as F/B path.

* **BDR Rules:-**

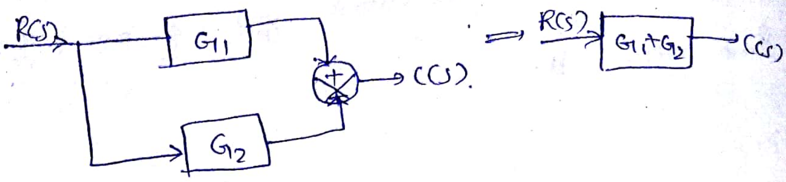
  *1. Blocks connected in series./cascade.

  * If two blocks are connected in cascade, then the overall Transfer function is the product of individual TF's.



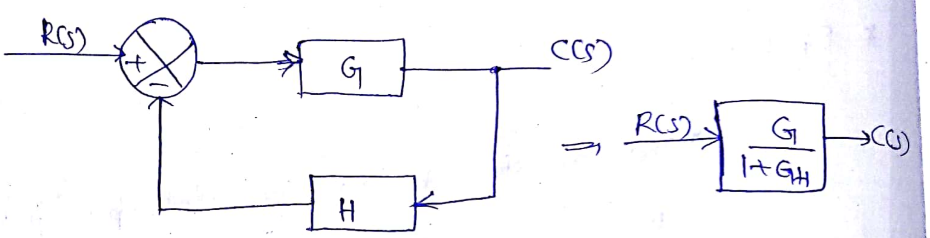
* If a no. of blocks ~~can~~ are in cascade/ series, it can be replaced by its product.

Blocks Connected in parallel :→



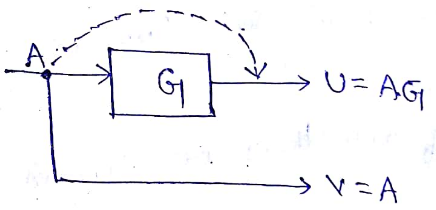
* If a no. of blocks are in parallel, it can be replaced by its Summation.

*3. Elimination of Single loop -ve F/B :→.

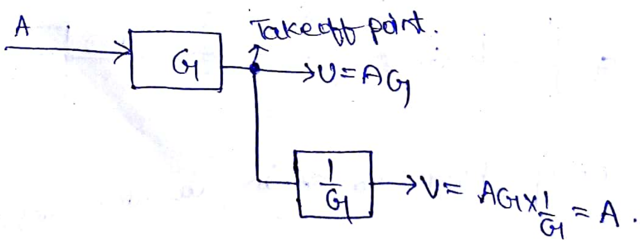


*4. moving the take off point after the block :→

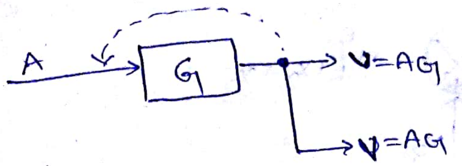
~~moving the~~

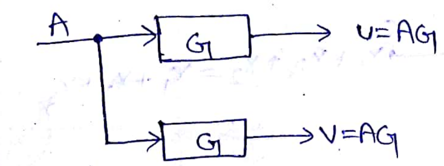


*after moving the taking point after the block it is given by

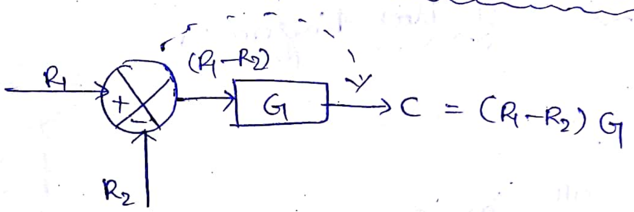


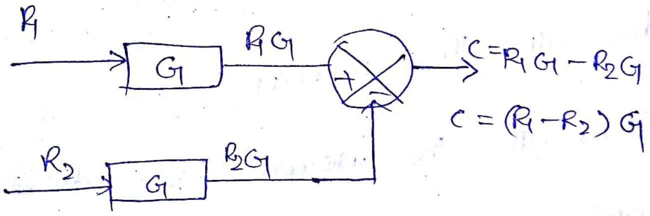
*5 moving the take-off point before the block :-



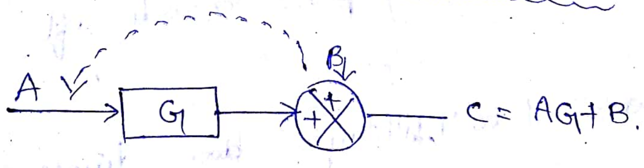


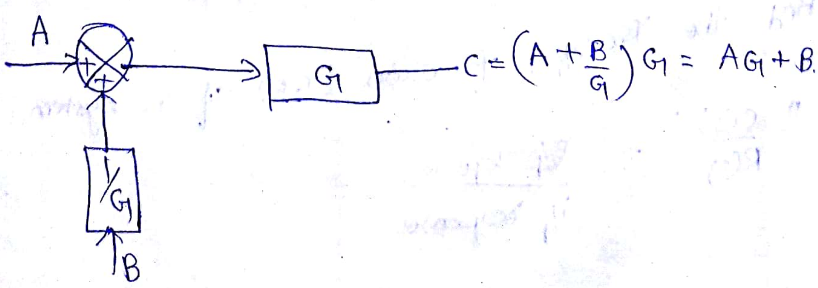
*6. moving the summing point after the block :-



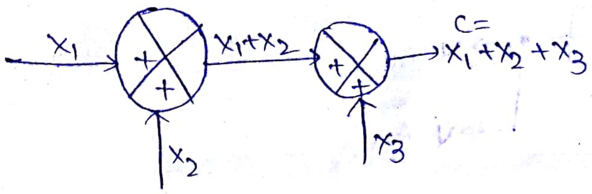


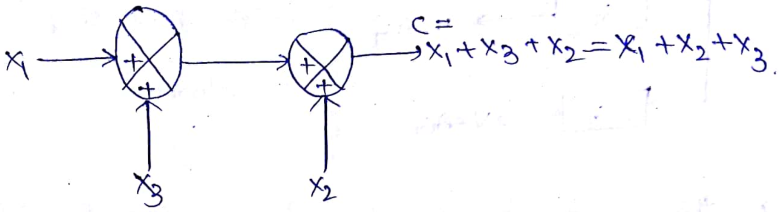
*7. moving the summing point before the block :-





*8. Interchanging of summing points :-





* Procedure for reducing block diagram :-

*Steps:-

*1. Reduce all the cascaded blocks

*2. Reduce all the parallel blocks.

*3. Reduce all the internal F/B ~~series~~ loops.

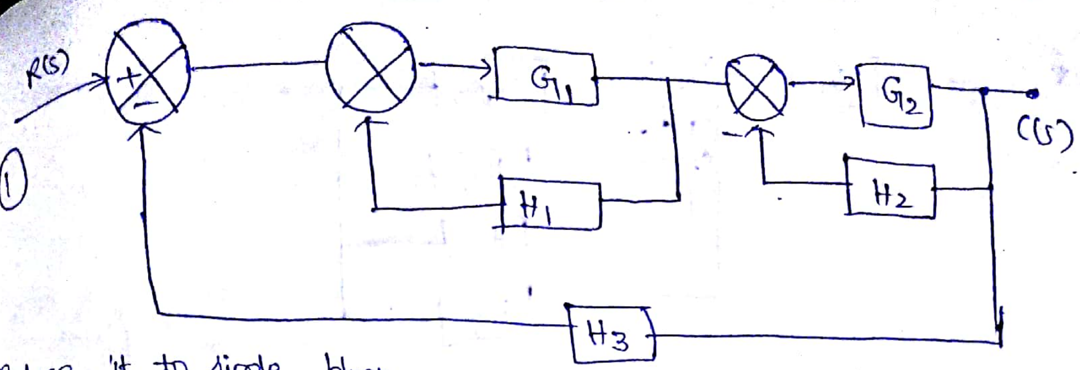
*4. always try to shift take off points towards right & summing points towards left.

*5. Repeat 1 to 4, till simple block diagram is obtained

*6. Find the Transfer function (T.F) of the system

$$\text{"}\frac{C(s)}{R(s)}\text{"} = \frac{\text{O/p response}}{\text{I/p response}}.$$

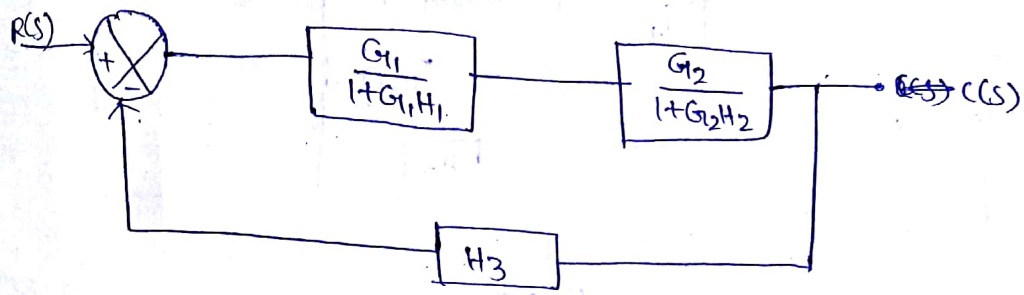
(1)

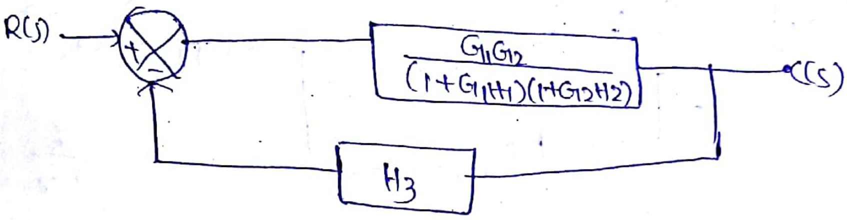


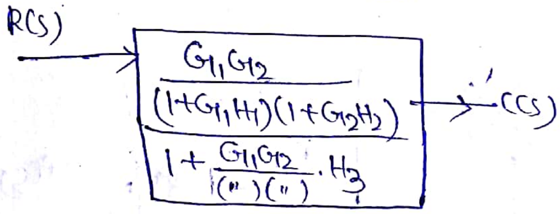
Reduce it to single block.

$X(s)$

$_2 + X_3$.







. loops.

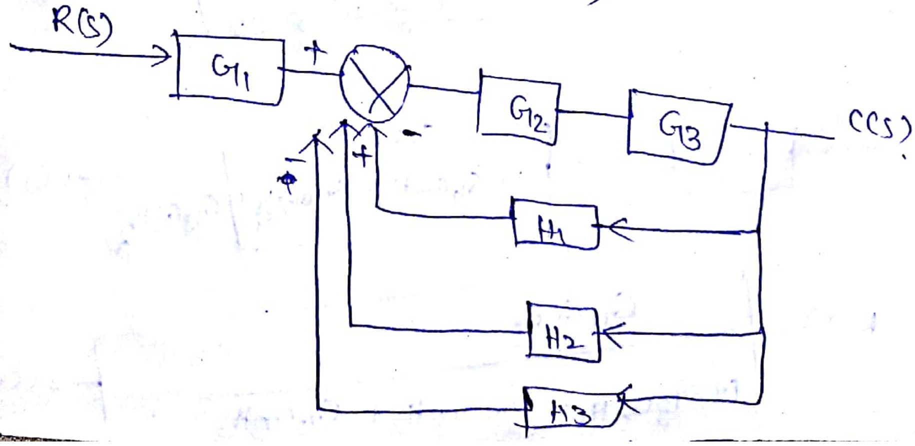
towards

ft.

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2}{(\quad)(\quad)}}{1+\dfrac{G_1G_2H_3}{(\quad)(\quad)}} = \frac{G_1G_2}{(1+G_1H_1)(1+G_2H_2)+G_1G_2H_3}$$

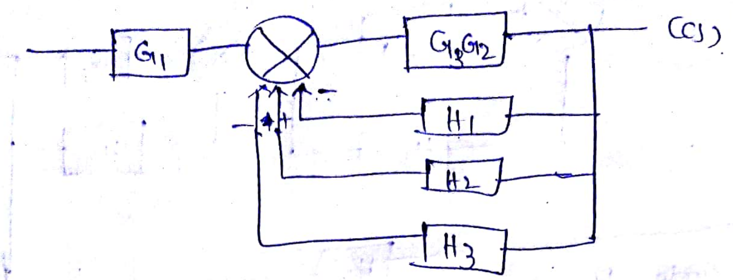
ram is obtained

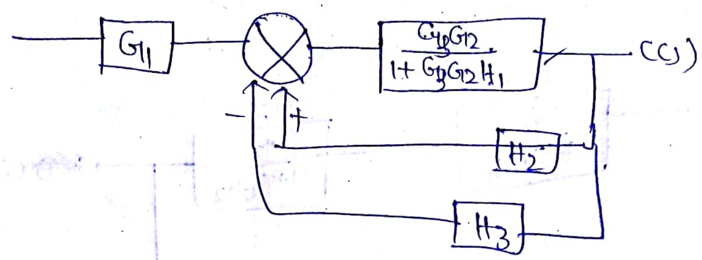
(2)

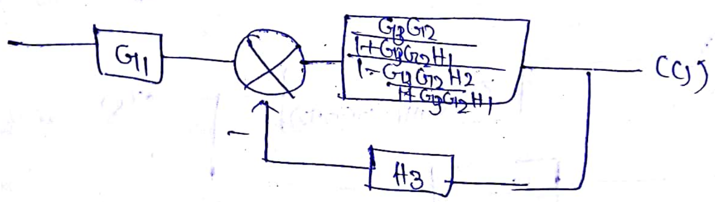


the system

*5(a)

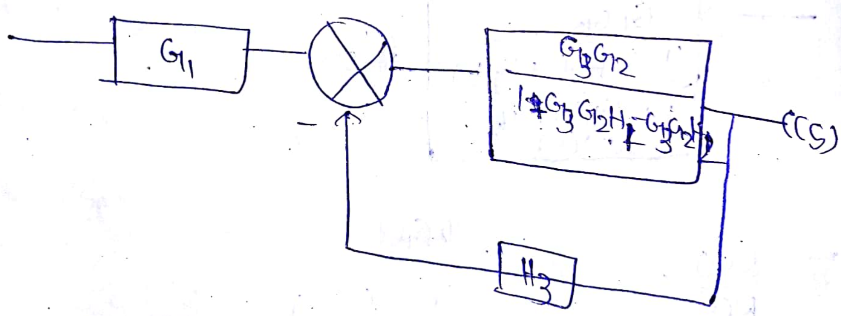


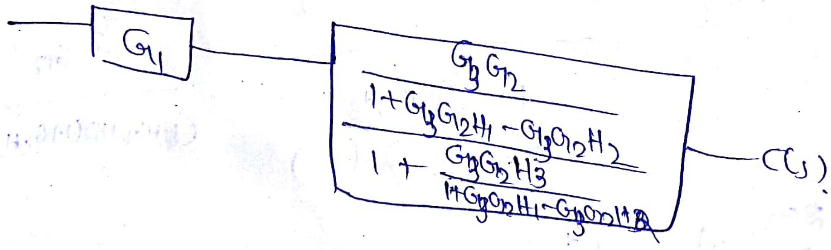


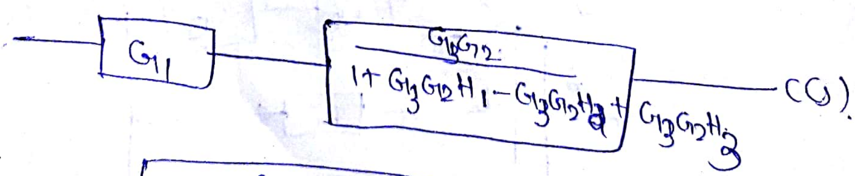


$\frac{x}{1-xH_2}$

$\frac{y}{1-y}$







$$R(s) \rightarrow \boxed{\frac{G_1G_2G_3}{1+G_3G_2H_1-G_2G_3H_2+G_2G_3H_3}} \rightarrow C(s)$$

$\Rightarrow \frac{C(s)}{R(s)} =$

R(s)

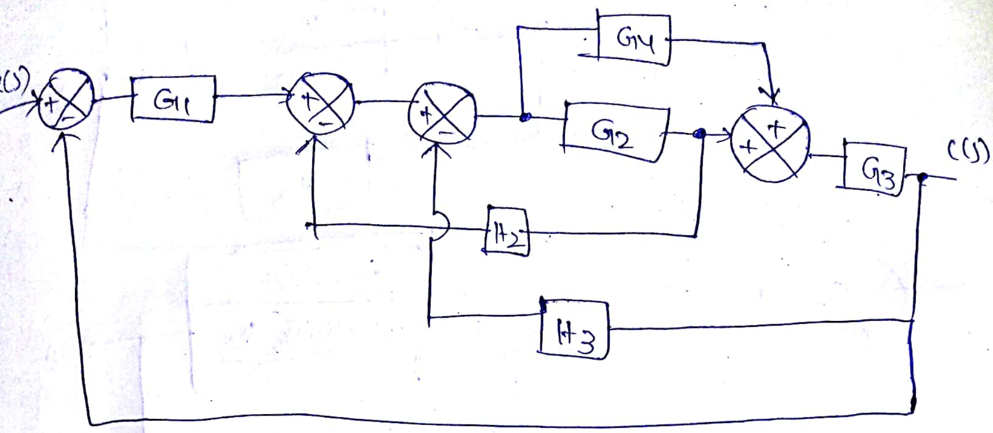
③

*5(e) R(s)

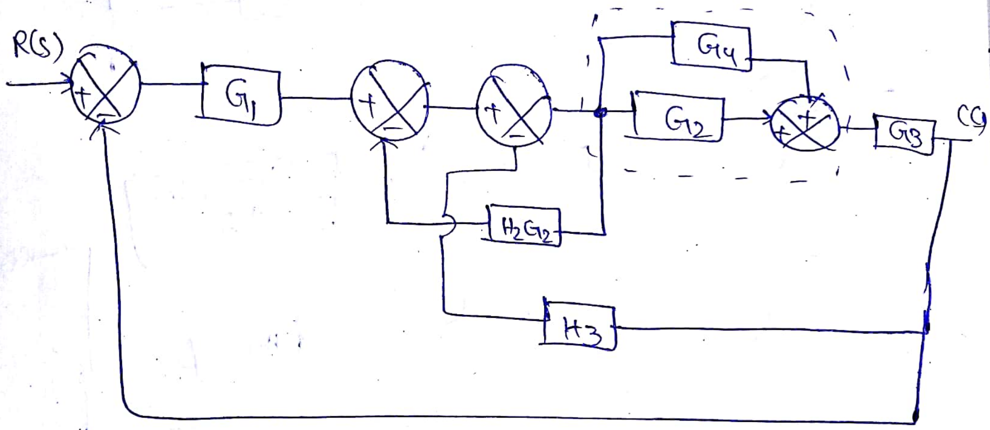
R(s)

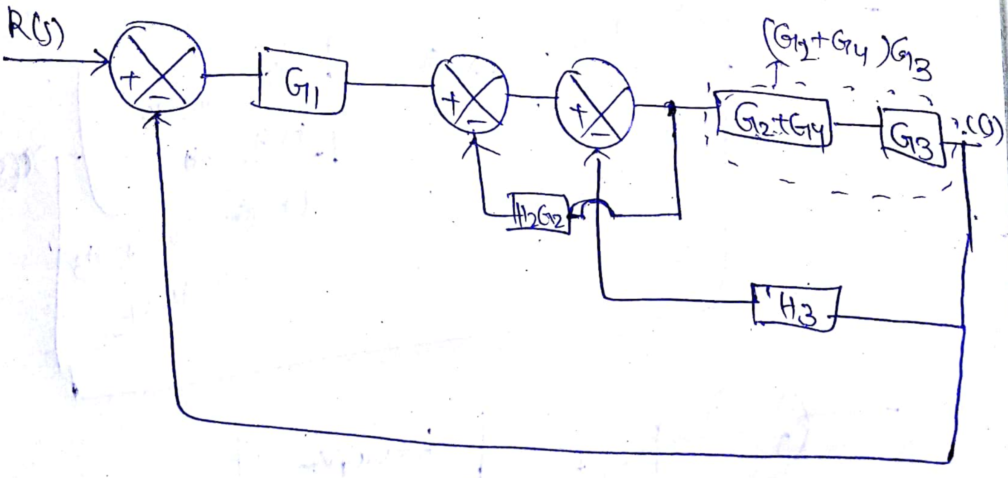
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1 + G_2G_3H_1 - G_2G_3H_2 + G_2G_3H_3}$$

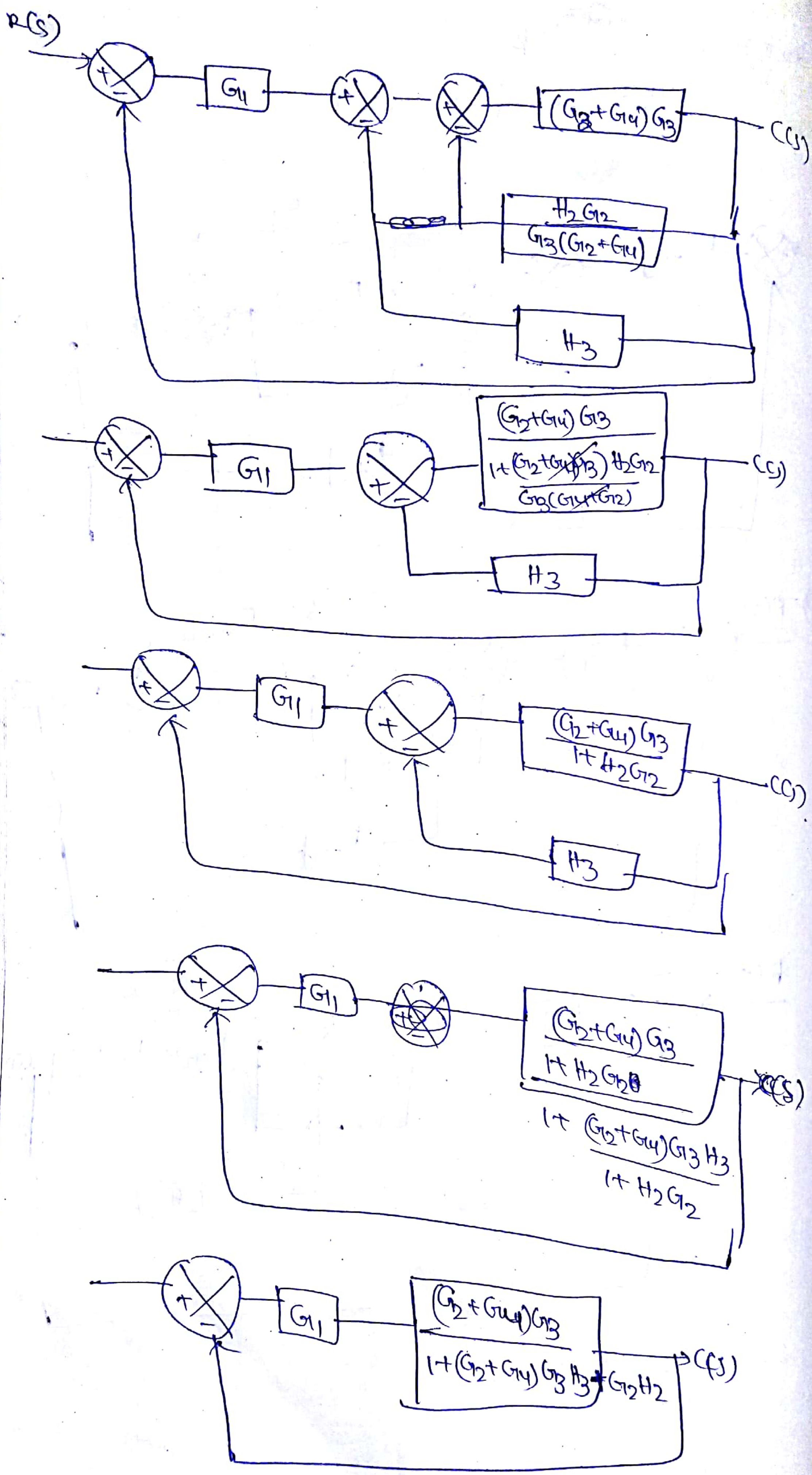
(3)



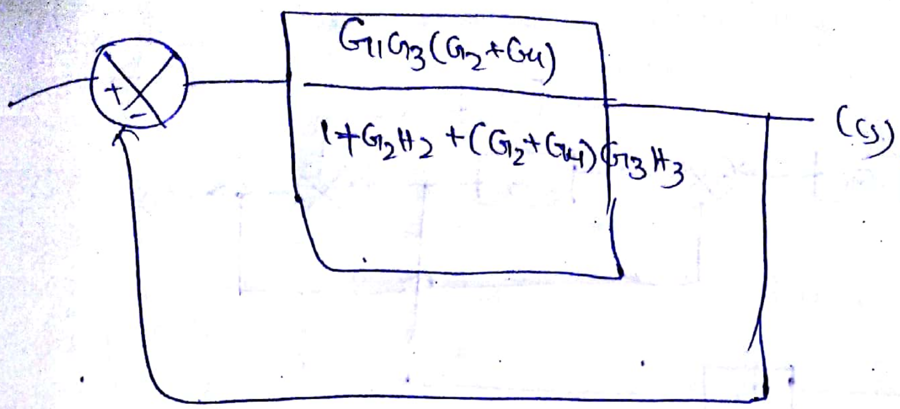
*5(a)

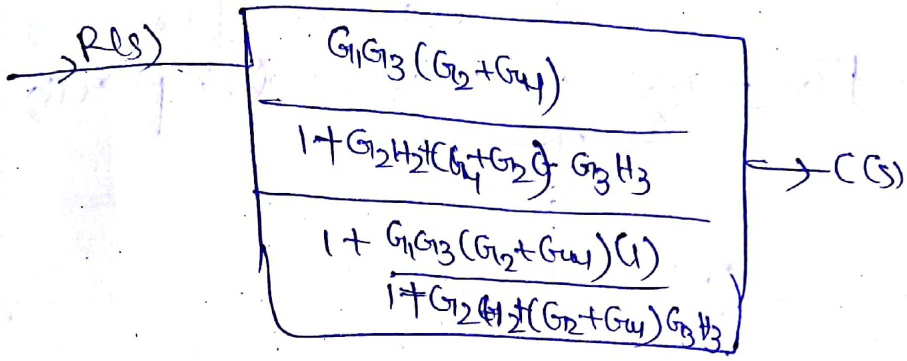






R(S)
G1
(G2+G4) G3
C(s)
H2G2 / G3(G2+G4)
H3
(G2+G4) G3 / 1+ (G2+G4)G3 H2G2 / G3(G4+G2)
(G2+G4) G3 / 1+ H2G2
(G2+G4) G3 / 1+ H2G2 / 1+ (G2+G4)G3 H3 / 1+ H2G2
(G2+G4)G3 / 1+(G2+G4) G3 H3 + G2H2

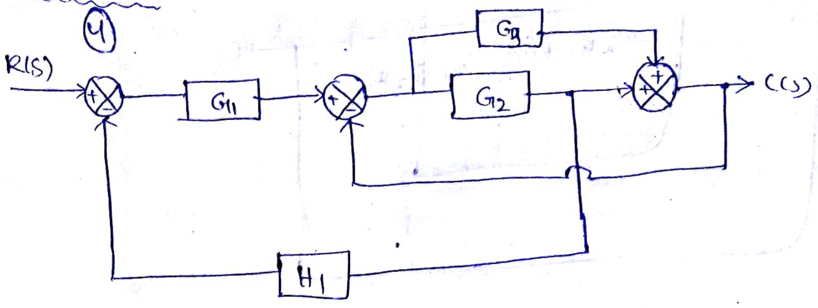




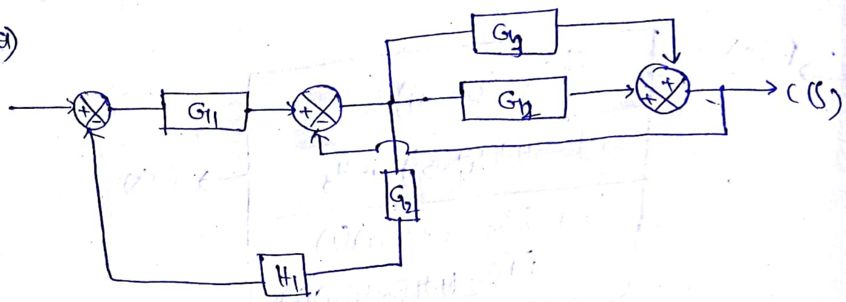
$$\frac{C(s)}{R(s)} = \frac{G_1G_3(G_2+G_4)}{1+G_2H_2+G_2G_3H_3+G_4G_3H_3+G_1G_3G_2+G_1G_3G_4}$$

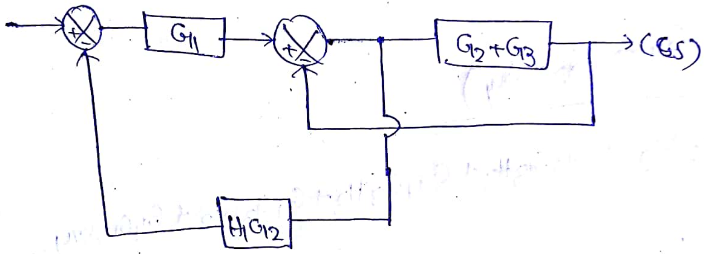
05/01/18

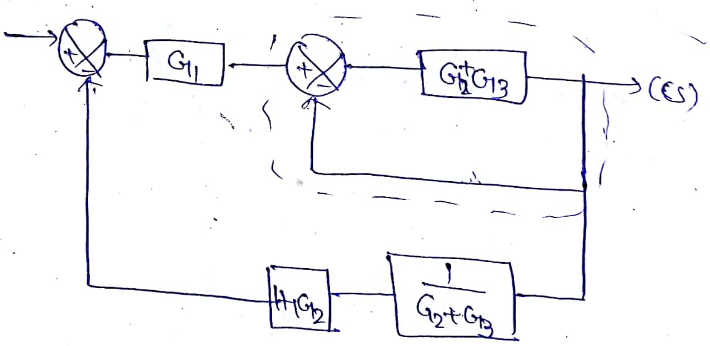
(4)

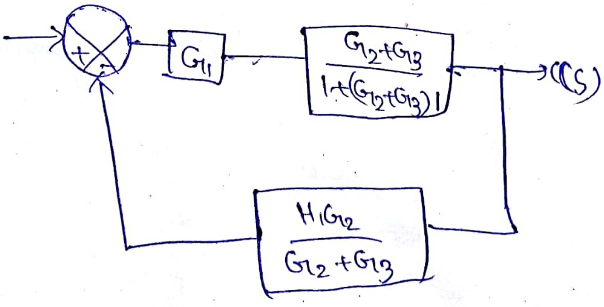

R(S)
G1
G3
G2
H1
C(S)

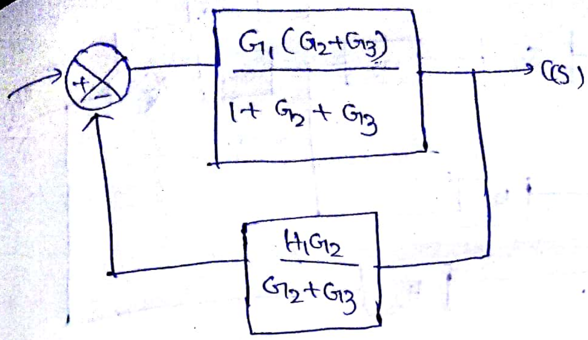
*5(a)

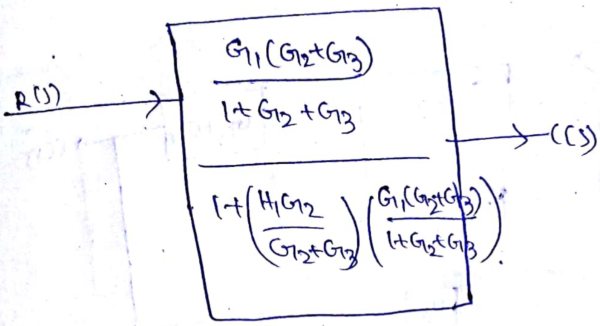

G1
G3
G2
G2
H1
C(S)


G1
G2+G3
H1G2
C(S)


G1
G2+G3
H1G2
1 / G2+G3
C(S)


G1
G2+G3 / 1+(G2+G3)1
H1G2 / G2+G3
C(S)



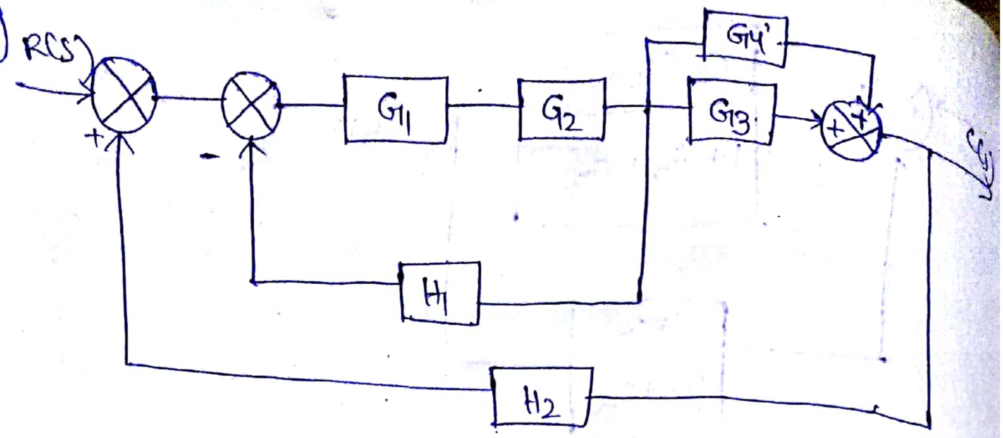


$$\frac{C(S)}{R(S)} = \frac{\dfrac{G_1(G_2+G_3)}{1+\cancel{G_2}+G_3}}{\dfrac{(G_2+G_3)(1+G_2+G_3)+(H_1G_2)(G_1(G_2+G_3))}{(G_2+G_3)\,(1+\cancel{G_2}+G_3)}}$$

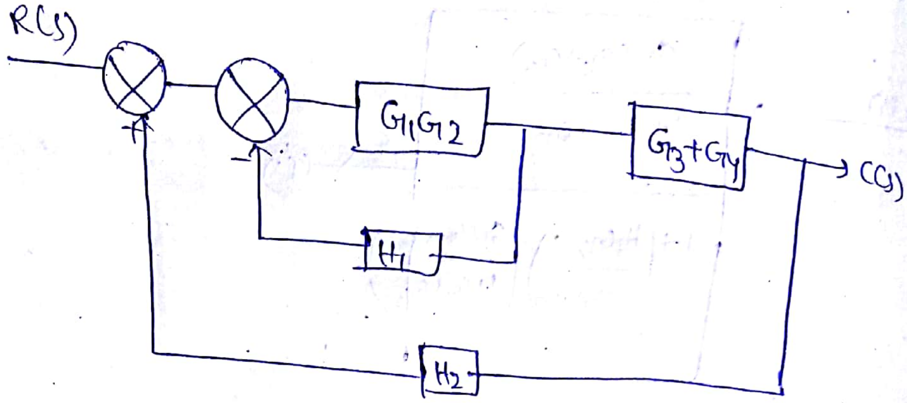
$$= \frac{G_1(G_2+G_3)}{\dfrac{\cancel{G_2+G_3}\left[(1+G_2+G_3)+H_1G_2G_1\right]}{\cancel{G_2+G_3}}}$$

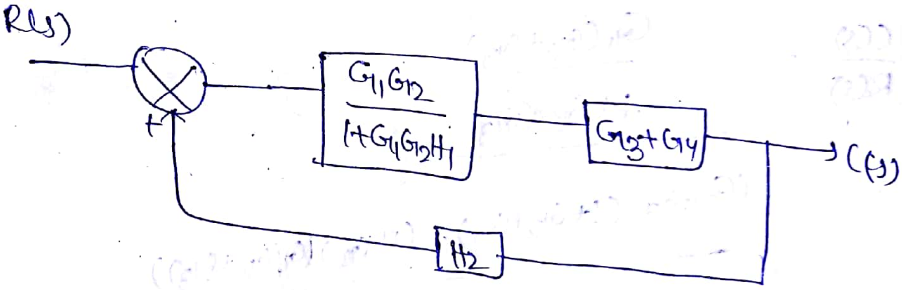
$$\boxed{\frac{C(S)}{R(S)} \Rightarrow \frac{G_1(G_2+G_3)}{1+G_2+G_3+G_1G_2H_1}}$$

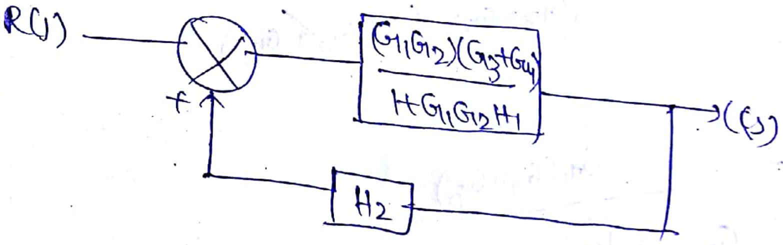
*⑤



*Sol)



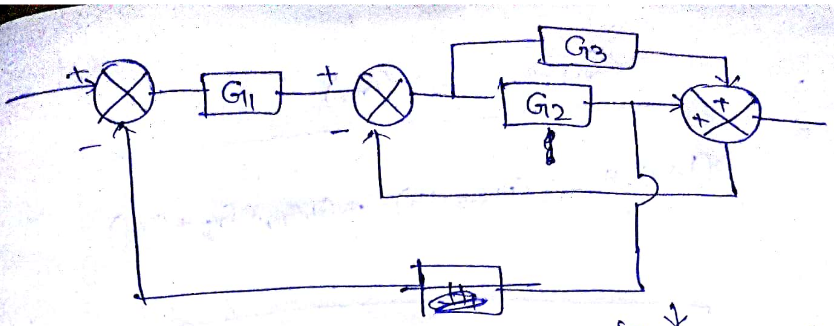




$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2(G_3+G_4)}{1+G_1G_2H_1}}{1 - \dfrac{(G_1G_2)(G_3+G_4)}{1+G_1G_2H_1} \cdot H_2}$$

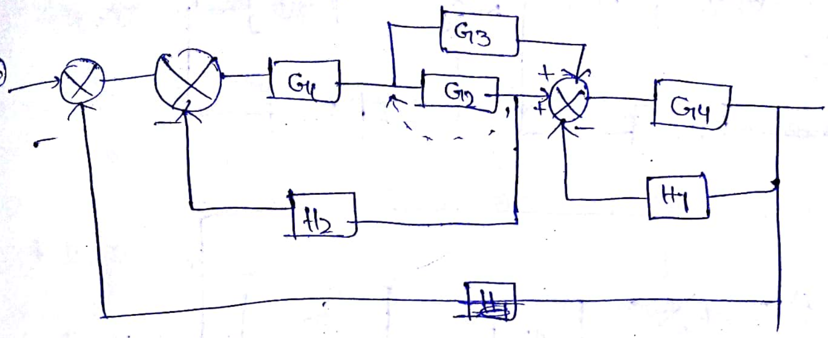
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G_1G_2(G_3+G_4)}{1+G_1G_2H_1 - G_1G_2(G_3+G_4)H_2}$$

*(6)

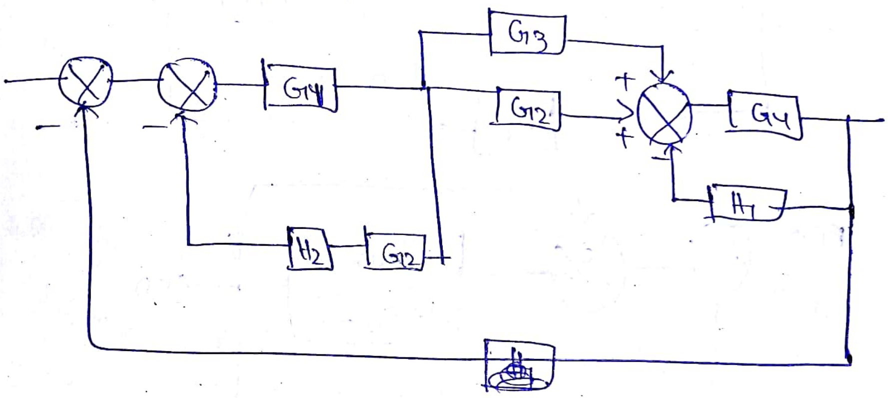


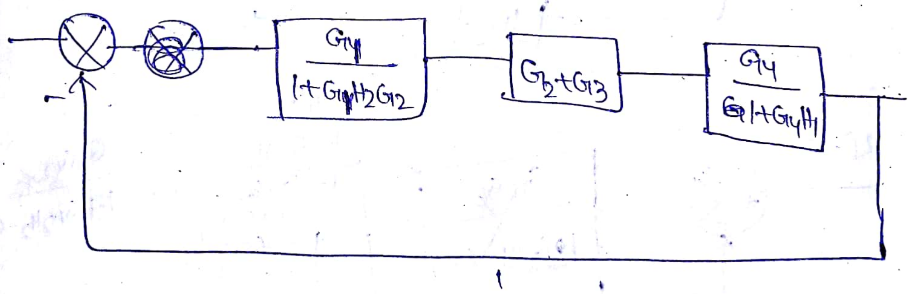
{Same prob 4}

*(6)



*10)

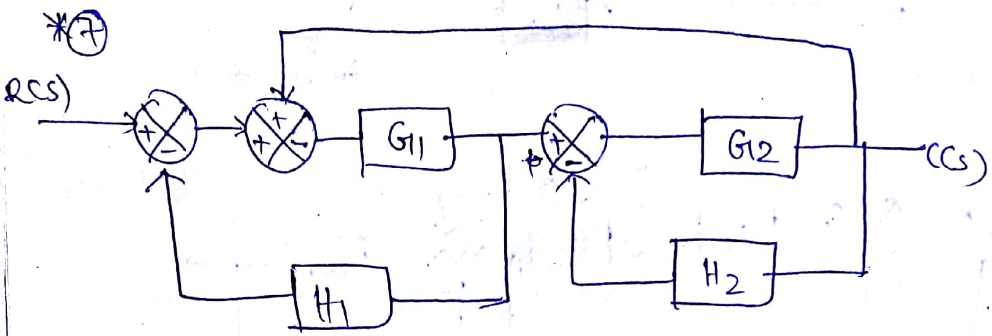




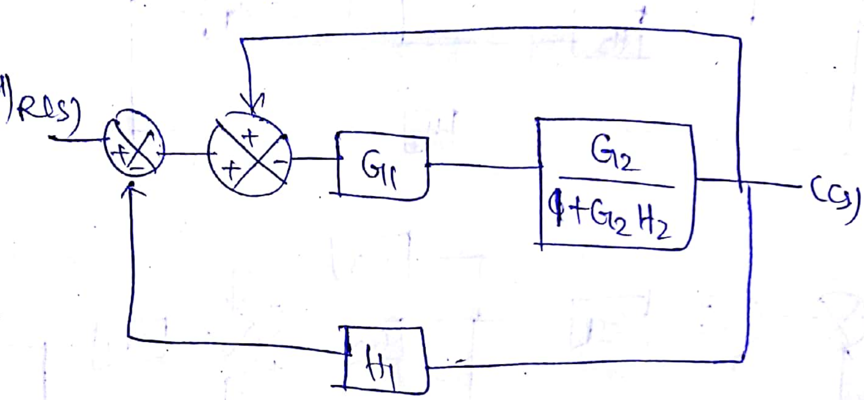
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{\left(\frac{G_1}{1+G_1H_2G_2}\right)(G_2+G_3)\left(\frac{G_4}{1+G_4H_1}\right)}{1+\frac{(G_1)(G_2+G_3)(G_4)}{(1+G_1H_2G_2)(1+G_4H_1)}(1)}$$

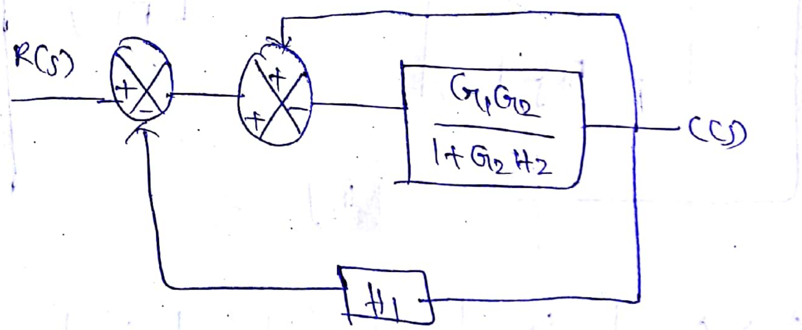
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{(G_1)(G_2+G_3)(G_4)}{(1+G_1H_2G_2)(1+G_4H_1) + G_1G_4(G_2+G_3)}$$

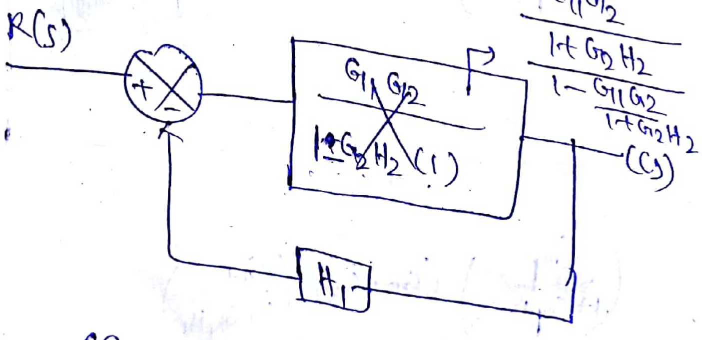
*(7)



*Sol)







$$\frac{\frac{G_1G_2}{1+G_2H_2}}{1-\frac{G_1G_2}{1+G_2H_2}} = \frac{G_1G_2}{1+G_2H_2-G_1G_2}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{G_1G_2}{1+G_2H_2-G_1G_2}}{1+\frac{G_1G_2}{1+G_2H_2-G_1G_2}\cdot H_1} = \frac{G_1G_2}{1+G_2H_2+G_1G_2H_1-G_1G_2}$$

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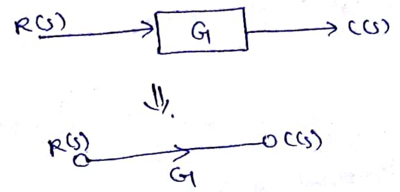
*Signal flow graph :→



* It is nothing but the graphical representation of relationship between the variables of a set of linear algebraic Eq written in the form of Cause & Effect relationship

* It consists of a Network in which nodes represent each of the System variables are connected by directed branches, the directed branches have associated branch gains.

*A signal can be transmitted through a branch only in the direction of arrow.

*Nodes :→

* A node (or) Junction point is a point represents System Variable (or) signal.

* Branch :→

*A Branch is a direct line segment joining two nodes, the arrow on the branch indicates the direction of signal flow & the gain of the branch is transmittance

* Transmittance / gain :→

*The gain acquired by the signal when

it travels from one node to another is called transmittance or gain

* It can be either real or complex value.

*Path:-

*A path is a traversal of connected branches in the direction of the branch arrows.

*The path should not cross a node more than once

*Forward path:-(F.P)

*It is a path from an input node to an output node that doesn't cross any node more than once.

*F.P gain:-

*The product of the branch gains describe in traversing a F.P is called F.P gain.

*loop:-

*A loop is a path which originates & terminates at the same node.

*Non-touching loops:-

*loops which doesn't have common node then the loops are non-touching loops.

* Self loop :-

* A loop consisting of single branch is called self loop.

* Signal flow graph {S.F.G} Reduction :-

* S.F.G of a system can be reduced either using rules of signal flow graph algebra i.e.. by writing Equation at every node & then rearranging these eq's to get the ratio of o/p to i/p.

* To find the T.F of a system using SFG S.J mason has developed a formula called by this name mason's gain formula which is used to find T.F as given below.

$$T.F = \frac{o/p}{i/p} = \frac{1}{\Delta} \sum_{i=1} P_i \Delta_i$$

* P_i = F.P gain of ith F.P.

P_1 = 1st FP gain ; P_2 = 2nd F.P gain ; ......

* Δ = 1 − [Sum of gain product of Individual loop gains]

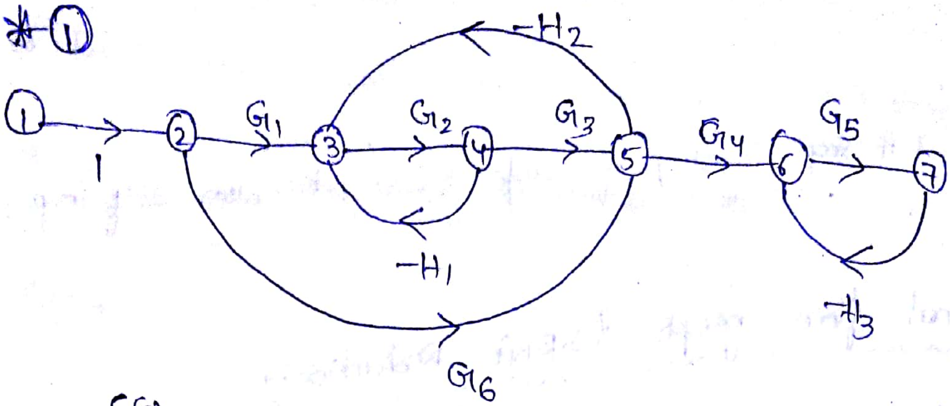
+ [Sum " " " Two non-touching " "]

− [" " " " Three " " " "]

+ [" " " " Four " " " "] ......

Δ_i = Δ for that part of the graph which is not touching ith F.P.

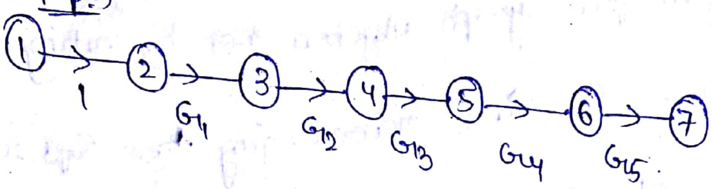
*(1)



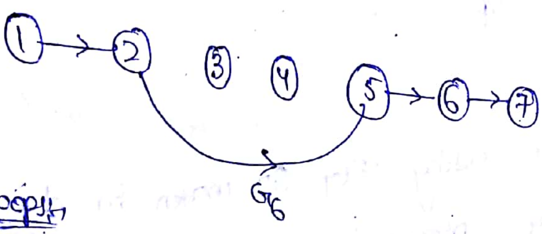
$$\frac{C(s)}{R(s)} = ?$$

*Sol)

* F.P:-



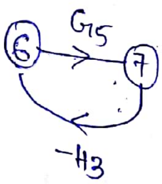
$P_1 = G_1 G_2 G_3 G_4 G_5$.



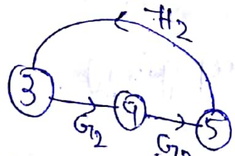
$P_2 = 1 \cdot G_6 \cdot G_4 \cdot G_5$.

*loops:-

$L_1 = -G_2 H_1$



$L_2 = -G_5 H_3$



$L_3 = -G_2 H_2 G_3$.

* Non-touching Loops:-

L_1 & L_3 are non touching (no nodes in common)

L_2 & L_3 " " "

$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1L_3 + L_2L_3]$

$\Delta_1 = 1$

$\Delta_2 = 1 - [L_1]$

$$\Rightarrow T.F = \frac{(G_1G_2G_3G_4G_5)(1) + (G_4G_5G_6)(1 + G_2H_1)}{1 + (G_2H_1 + G_5H_3 + G_2H_2G_3) - (G_5H_3G_2H_1 + G_2H_2G_3G_5H_3)}$$

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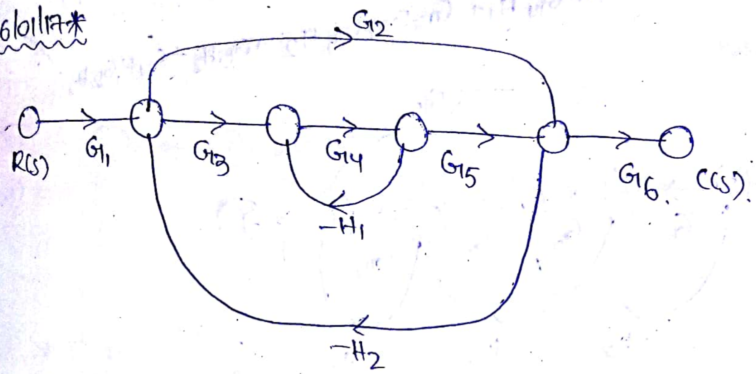
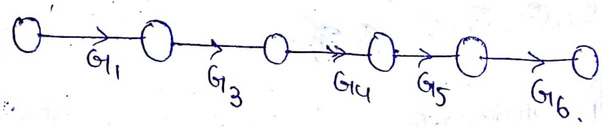
(2)



*Sol. * F.P :-

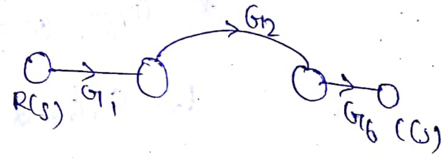
(1)



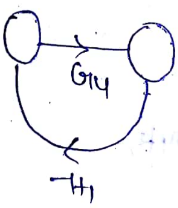
$P_1 = G_1G_3G_4G_5G_6$

$P_2 = G_1G_2G_6$

(2)

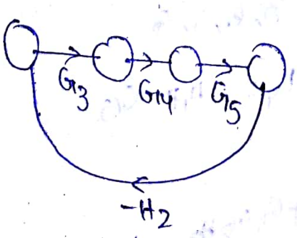


*loops :-

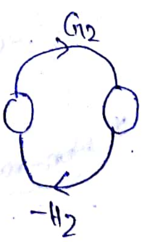


$L_1 = -G_4H_1$

$$T.F = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$



$L_2 = -G_3G_4G_5H_2$;



$L_3 = -G_2H_2$

* Non-Touching loops $= L_1L_3$

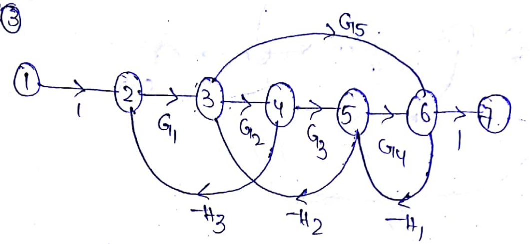
$\Delta = 1 - [L_1 + L_2 + L_3 + L_1 L_3] = 1 + G_4 H_1 + G_3 G_4 G_5 H_2 + G_2 H_2 + G_4 G_2 H_1 H_2$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - L_1 = 1 + G_4 H_1$

$\Rightarrow \quad T.F = \dfrac{(G_1 G_3 G_4 G_5 G_6)(1) + (G_1 G_5 G_6)(1 + G_4 H_1)}{1 + G_4 H_1 + G_3 G_4 G_5 H_2 + G_2 H_2 + G_4 H_1 G_2 H_2}$

*(3)



*Sol) * F.P:-

* 1, 2, 3, 4, 5, 6, 7

$P_1 = G_1 G_2 G_3 G_4$

* 1, 2, 3, 6, 7.

$P_2 = G_1 G_5$

*Loops:-

* 2, 3, 4, 2

$L_1 = -G_1 G_2 H_3$

* 3, 4, 5, 3

$L_2 = -G_2 G_3 H_2$

* 5, 6, 5

$L_3 = -G_4 H_1$

* 3, 6, 5, 3

$L_4 = H_1 H_2 G_5$

* Non-touching loops:- $L_1 L_3 = +G_1 G_2 H_3 G_4 H_1$

$* \Delta = 1 - [L_1 + L_2 + L_3] + L_1 L_3 + L_4.$

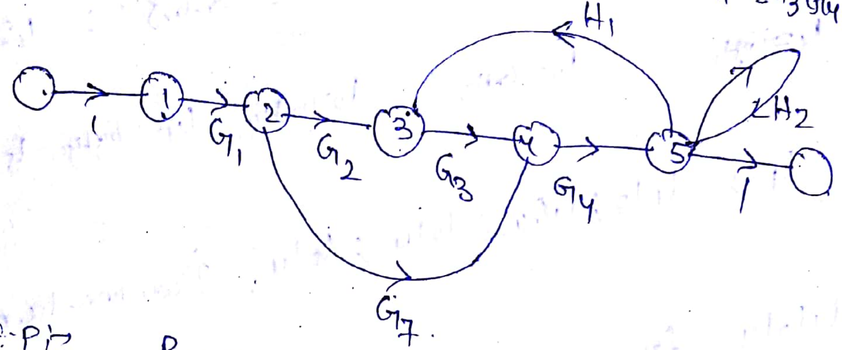
$\Delta_1 = 1$

$\Delta_2 = 1$

$$T.F = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4 + G_1G_5}{1 + [G_1G_2H_3 + G_4H_1 + G_2G_3H_2 + H_1H_2G_5] + G_1G_2H_3G_4H_1}$$

*(4)

*(01)



* F.P :- $P_1 = G_1G_2\cancel{G_5}G_3G_4.$

$P_2 = G_1G_7G_4.$

* loops :- $L_1 = G_3G_4H_1$; $L_2 = H_2.$

* Non-touching :- 0.

$\Delta_1 = 1 - 0$

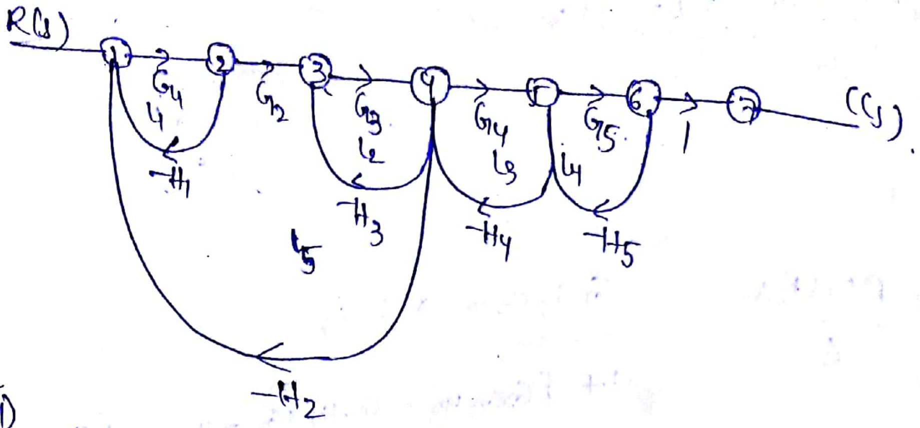
$\Delta_2 = 1 - 0.$

$\Delta = 1 - [L_1 + L_2]$

$$\Rightarrow T.F = \frac{G_1G_2G_3G_4 + G_1G_7G_4}{1 - (G_3G_4H_1 + H_2)}$$

2)

(5)



*sol)

* 1,2,1 ; 3,4,3 ; 4,5,4 ; 5,6,5 ; 1,2,3,4,1.

$L_1 = -G_1H_1$ $L_2 = -G_3H_3$ $L_3 = -G_4H_4$ $L_4 = -G_5H_5$ $L_5 = -G_1G_2G_3H_2$

* L_1L_2 ; L_1L_3 ; L_1L_4 ; L_2L_4 ; L_5L_4. {Two non-touching loops}

* $L_1L_2L_4$ {Three non-touching loops}.

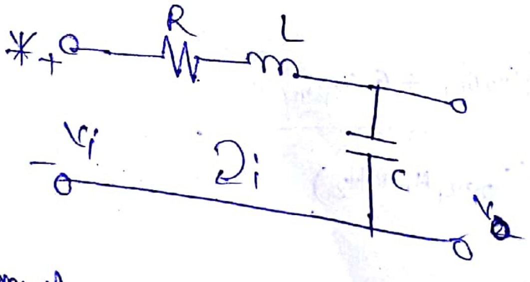
$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5] + [L_1L_2 + L_1L_3 + L_1L_4 + L_2L_4 + L_4L_5] - [L_1L_2L_4]$$

$$\Delta_1 = 1 - 0.$$

$$P_1 = G_1G_2G_3G_4G_5.$$

$$T.F = \frac{P_1\Delta_1}{\Delta}.$$ Substitute

*



*sol)

$$V_i = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt.$$

$$V_o = \frac{1}{C}\int i\,dt.$$

$\circledcirc$ —— $C(s)$.

; 1,2,3,4,1.

$-G_5H_5$ $\quad l_5 = -G_1G_2G_3H_2$

non-touching loops}

$_4 + l_2 l_4 + l_4 l_5]$

18) L.T on B.T for, 2 eqs.

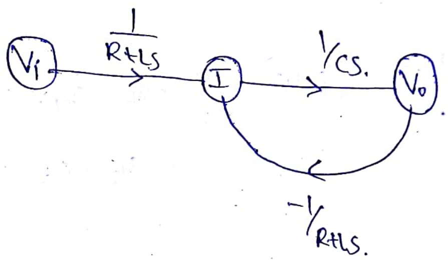
$\Rightarrow V_i(s) = R\,I(s) + L.I(s)\,s + \frac{1}{sC} I(s)$; $\quad V_o(s) = \frac{1}{sC} I(s)$

$\Rightarrow V_i(s) = I(s)\left[R + sL + \frac{1}{sC}\right]$ ↳ ① ; $\quad \boxed{V_o(s) = I(s)\left[\frac{1}{sC}\right]}$ ↳ ②

* from ①, ② we get.

$V_i(s) = I(s)\,[R+sL] + V_o(s)$.

$$\Rightarrow \boxed{\frac{V_i(s) - V_o(s)}{R+Ls} = I(s)}$$



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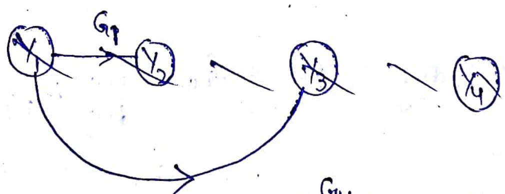
*19

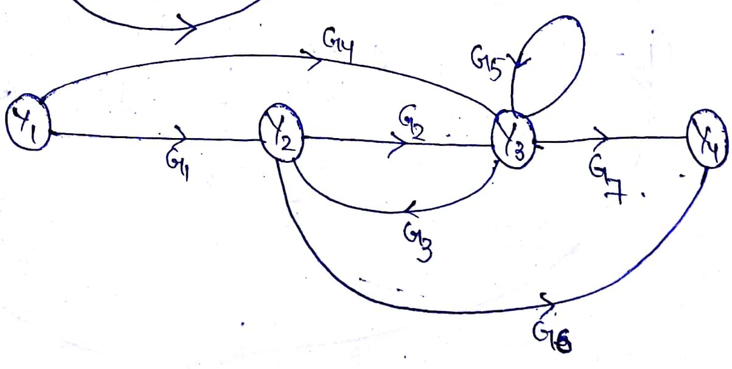
$Y_2 = G_1Y_1 + G_3Y_3$

$Y_3 = G_4Y_1 + G_2Y_2 + G_5Y_3$ $\qquad \frac{Y_4}{Y_1} = ?$

$Y_4 = G_6Y_2 + G_7Y_3$



*Sol



*FP:-

$P_1 = G_1G_2G_7$; $P_2 = G_4G_7$; $P_3 = G_4G_6$; $P_4 = G_4G_3G_6$

*Loops:-

$l_1 = G_2G_3$; $l_2 = G_5$

* $\Delta = 1 - [G_2G_3 + G_5] - (0)$.

$\Delta_1 = 1 - 0$

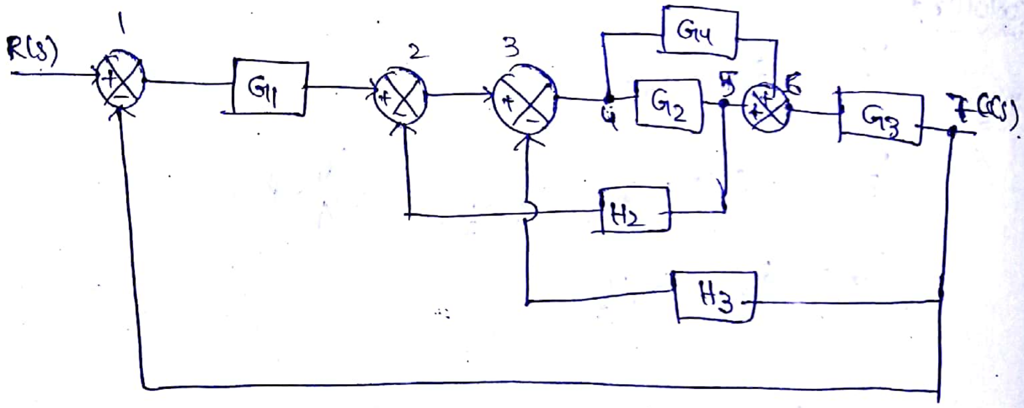
$\Delta_2 = 1 - 0$

$\Delta_3 = 1 - 0$

$\Delta_4 = 1 - 0$

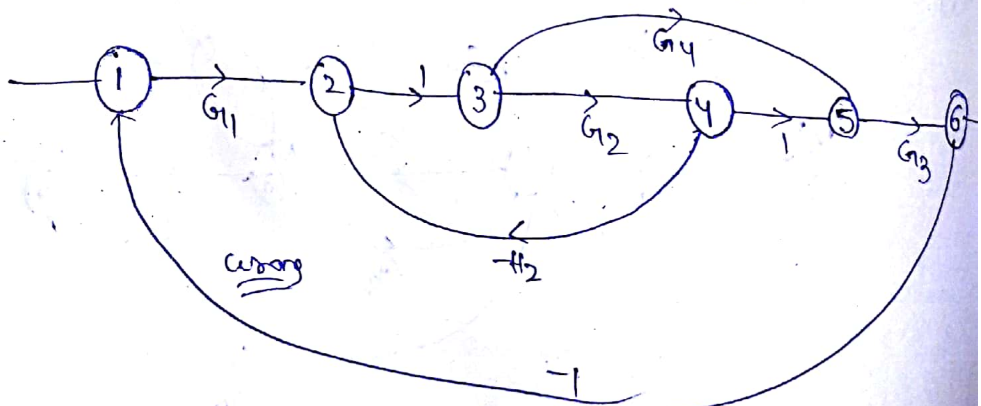
* $\frac{Y_4}{Y_1} = T.F = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta} = \frac{G_1G_2G_7 + G_4G_7 + G_1G_6 + G_4G_3G_6}{1 - G_2 - G_3 - G_5}$

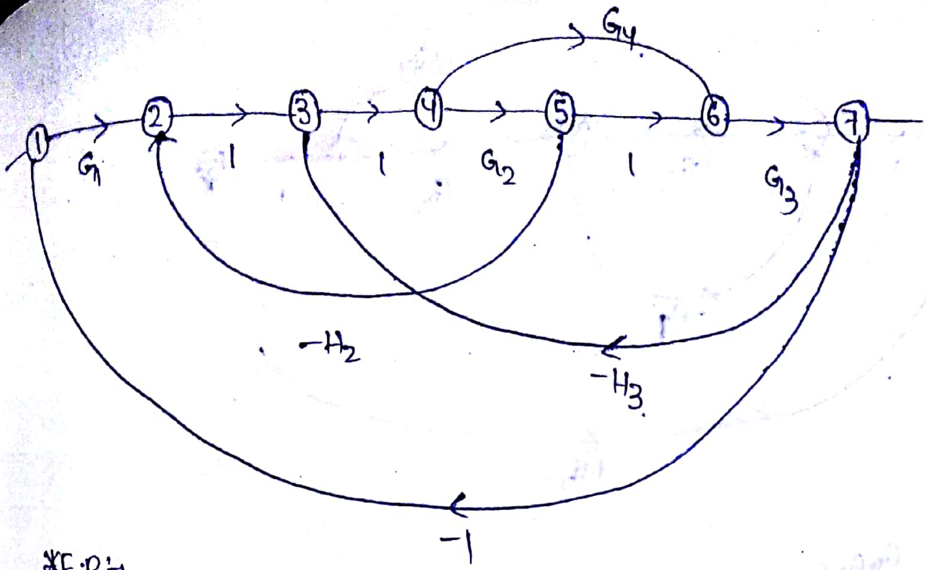
*Q



* Draw the corresponding SFG & find $\frac{C(s)}{R(s)}$ {T.F}

*Sol.





*F.P:-

$P_1 = G_1G_2G_3$; $P_2 = G_1G_4G_3$

$L_1 = -G_2G_3H_3$

$L_2 = -G_4G_3H_3$

$L_3 = -G_1G_2G_3$

$L_4 = -G_1G_4G_3$

$L_5 = -G_2H_2$.

$\Delta = 1 - (L_1+L_2+L_3+L_4+L_5) - 0$

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$\Delta_1 = 1-0$

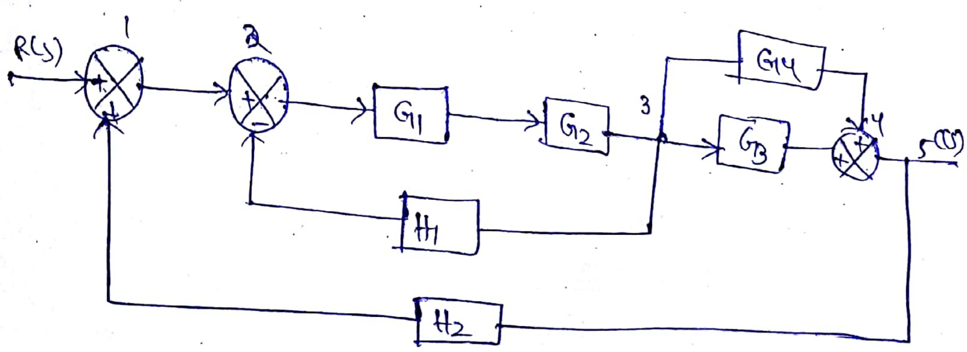
$\Delta_2 = 1-0$

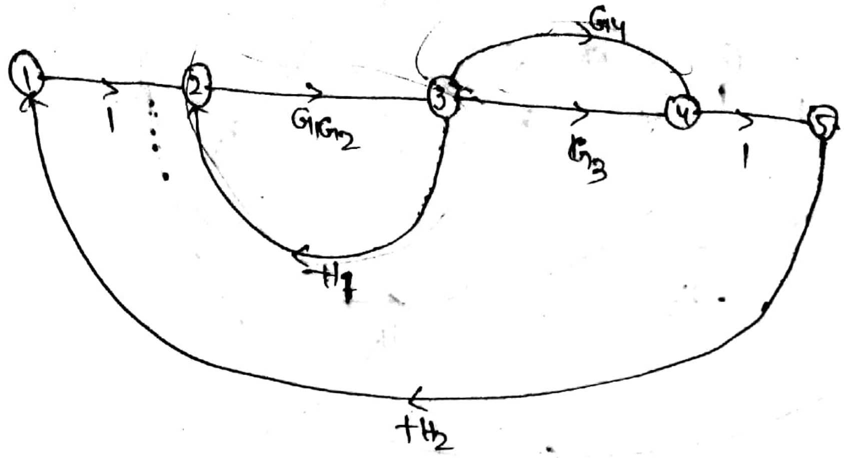
$\cancel{\Delta_3 = 1-0}$

$\cancel{\Delta_4 = 1-0}$

$\cancel{\Delta_5 = 1-0}$

*3.





$P_1 = G_1G_2G_3$

$P_2 = G_1G_2G_4$ $\Delta = 1 - 0$

$L_1 = +G_1G_2G_3H_2$

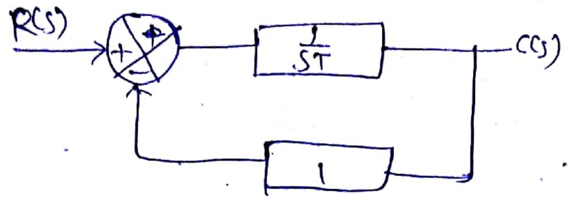
$L_2 = +G_1G_2G_4H_2$

$L_3 = -G_1G_2H_1$

L_4 G_3

*First order Time response Analysis:-

① *Step response:-



$$\frac{C(s)}{R(s)} = \frac{\frac{1}{sT}}{1+\frac{1}{sT}} = \frac{1}{1+sT}$$

$$C(s) = \frac{1}{1+sT} \cdot R(s)$$

if i/p = u(t) ⇒ $R(s) = 1/s$.

$$\Rightarrow C(s) = \frac{1}{s} \cdot \frac{1}{1+sT} = \frac{A}{s} + \frac{B}{1+sT}$$

$$\Rightarrow \cancel{\text{fs}}\ C(s) = \frac{1}{s} - \frac{T}{1+sT} = \frac{1}{s} - \frac{\cancel{T}}{\cancel{T}(s+1/T)}$$

$$\Rightarrow c(t) = \cancel{u(t)}\,1 - \cancel{T}.\,e^{-t/T}$$

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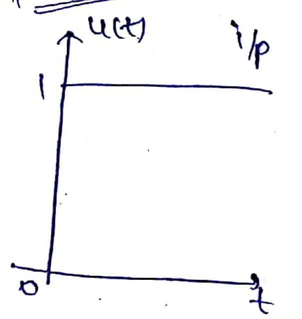
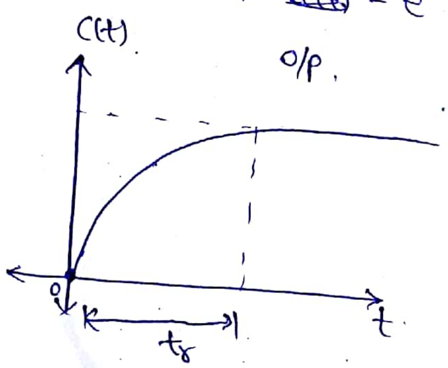
$$\Rightarrow c(t) = \cancel{u(t)}\,1 - e^{-t/T} = 1 - e^{-t/T}$$





② Ramp response:-

$$R(s) = 1/s^2.$$

$$\Rightarrow C(s) = \frac{1}{s^2} \cdot \frac{1}{1+sT} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{1+sT}.$$

$$\Rightarrow A\cdot(1+sT)\cdot s + B(1+sT) + C(s^2) = 1$$

at $s = 0$.

$\Rightarrow B = 1$

at $s = -1/T$

$C(1/T^2) = 1$

$C = T^2$

$A + C = 0$

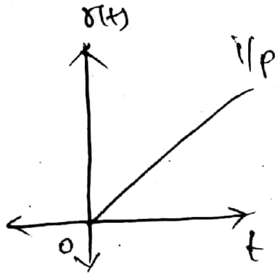
$A = -C = \frac{-T^2}{T} = -T.$

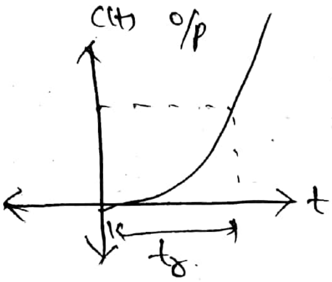
$$\Rightarrow C(s) = \frac{-T}{s} + \frac{1}{s^2} + \frac{T^2}{1+sT}$$

$$c(t) = -T + t + T.e^{t/T}$$

$$c(t) = t - T\left[1 - e^{t/T}\right]$$

$$c(t) = t - T\left[1 - e^{-t/T}\right] \;//$$





③ Impulse :-

$R(s) = 1$

$$C(s) = \frac{1}{1+sT}\cdot R(s)$$

$$= \frac{1}{1+sT} = \frac{1}{T(s+1/T)} = \frac{1}{T}\cdot e^{-t/T}.$$

* Time response of 2nd order System:-

* CLTF of a system $= \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$\omega_n = \dfrac{\text{Critical damping}}{\text{under damping}} = \text{damping ratio.}$

Char. Eq = C.E = $s^2 + 2\xi\omega_n s + \omega_n^2$ {roots of GE = System poles}

*1.	$\xi = 0$	$\Rightarrow s_1 s_2 = \pm j\omega_n$ Purely img	undamped system
*2	$\xi = 1$	$\Rightarrow s_1 = s_2 = -\omega_n$ Real Equal & -ve.	Critical damped system
*3.	$0 < \xi < 1$	$\Rightarrow$ Complex conjugate with -ve real part.	under damped system
*4.	$\xi > 1$	$\Rightarrow$ Real & unequal	over damped system

* unit step:-

$$C(s) = R(s) \cdot \frac{\omega_n^2}{s^2 + 2\omega_n s\xi + \omega_n^2}$$

$\xi = 0$ undamped system.

$R(s) = 1/s$

$$\Rightarrow C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$\frac{\omega_n^2}{s(s^2+\omega_n^2)} = \frac{A}{s} + \frac{Bs + C}{s^2 + \omega_n^2}$$

$$\Rightarrow A(s^2 + \omega_n^2) + Bs^2 + Cs = \omega_n^2$$

at $s = 0$ ~~at s = -ωn²~~ $A + B = 0$

$\Rightarrow A = 1;$ $B = -1.$

at $s^2 = -\omega_n^2$

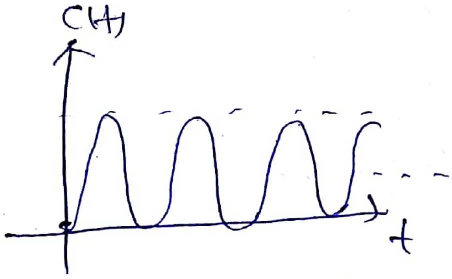
$\Rightarrow -B\cdot\omega_n^2 + C(j\omega_n) = \omega_n^2$

$\Rightarrow \quad C(j\omega_n) = \frac{\cancel{\omega_n^2}\,0}{\cancel{\omega_n}}$

$\Rightarrow C = 0.$

$\Rightarrow \quad C(s) = \frac{1}{s} - \frac{s}{s^2+\omega_n^2} = 1 - \frac{1}{\omega_n^2}\cancel{\sin}(\omega_n t).$

$\Rightarrow C(s) = 1 - \cos(\omega_n t)$ //



* The response of undamped 2nd order system for the unit step i/p is Completely oscillatory

* Every practical system has some amount of damping only hence, the undamped system doesn't exist in practise.

12/01/18

* Time response of 2nd order system:-

*** Case (ii):-** underdamped system ($0 < \xi < 1$). Roots are complex conjugate.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$R(s) = 1/s.$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$C(s) = \frac{A}{s} + \frac{Bs + C}{(\quad " \quad)} = \frac{\omega_n^2}{s(\quad " \quad)}$$

$$A(\quad " \quad) + Bs^2 + Cs = \omega_n^2$$

$$A + B = 0; \quad A = 1$$

$$B = -1$$

$$C = -2\xi\omega_n$$

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + (\xi\omega_n)^2 - (\xi\omega_n)^2 + \omega_n^2}.$$

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)}$$

$$\omega_d^2 = \omega_n^2(1 - \xi^2)$$

$$\omega_d = \omega_n\sqrt{1 - \xi^2}$$ → damping freq.

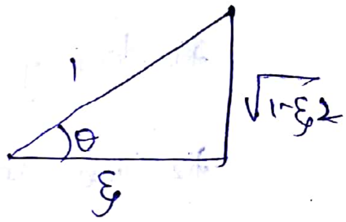
$$\Rightarrow C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$\Rightarrow C(s) = \frac{1}{s} - \left[\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} + \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}\right]$$

$$\Rightarrow C(t) = 1 - e^{-\xi\omega_n t}\cdot\cos(\omega_d t) + \frac{\xi\omega_n}{\omega_d}\cdot\sin(\omega_d t)\cdot e^{-\xi\omega_n t}$$

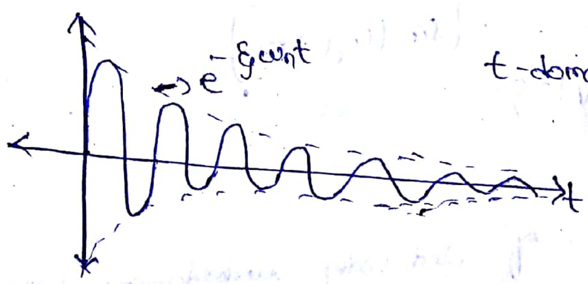
$$\Rightarrow C(t) = 1 - e^{-\xi\omega_n t}\cdot\cos(\omega_d t) + \frac{\xi\omega_n}{\omega_n(1-\xi^2)^{1/2}}\sin(\omega_d t)\, e^{-\xi\omega_n t}.$$

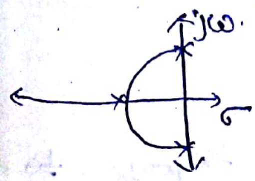
$$\Rightarrow C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sqrt{(1-\xi^2)}\cdot\cos(\omega_d t) + \xi\sin(\omega_d t)\right]$$



$$\Rightarrow C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\theta\cos(\omega_d t) + \cos\theta\sin(\omega_d t)\right]$$

$$\Rightarrow C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t+\theta)\right].$$





Roots are complex conjugate.

locus of the roots of the C.E forms a semi-circle then the response is

~~is~~ undamped response.

→* When $\xi < 1$ output reaches its steady state in moderate time i.e.. neither too quick nor too sluggish

the 90% of CS designs, type of damping is underdamping.

→* If we plot all the poles & zeros of 2nd order system in underdamped system the locus of these poles gives semi-circle

→* Characteristics of 2nd order undamped response:-

*1. damping coefficient $(\xi \omega_n)$

*2. Time Constant $\{1/\xi\omega_n\}$

*3. damping frequency $\{\omega_d\}$ $[\omega_d = \omega_n\sqrt{1-\xi^2}]$

*4. damping angle $\{\theta\}$ $[\cos\theta = \xi,$ ~~$\theta = \cos^{-1}(\theta)$~~$]$

$\theta = \cos^{-1}(\xi)$

$$* \; C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t + \theta)\right]$$

→* Specification of 2nd order underdamped system:-

*1. Delay time:- $\{t_d\}$

* Time taken by the response to reach 50% of the final value of steady state value.

*2 Rise time

of its

*3. peak

*4. Se

*5.

*6. S

*2 Rise time :- $\{t_r\}$

* Time taken by the response to reach 10 to 90% of its final value

*3. peak time :- $\{t_p\}$

* Time taken by the response to reach its first Peak of the response.

*4. Settling time :-

* The time taken by the response to reach its steady state & stay with in the 2% or 5% of tolerance band is known as settling time.

*5. Peak overshoot :-

* The maximum amount of over shoot taken by the response is known as peak over shoot.

it is also defined as difference b/w max value & desired o/p. or steady state value.

$$\% \text{ Max peak over shoot } (M_p) = C(t)\Big|_{t=t_p} - C(t)\Big|_{\text{desired o/p}}.$$

*6. Steady state Error :-

* it is defined as Error b/w actual o/p to desired o/p.

$$e_{ss} = \cancel{c(t)} - \text{actual o/p} - \text{desired o/p}.$$

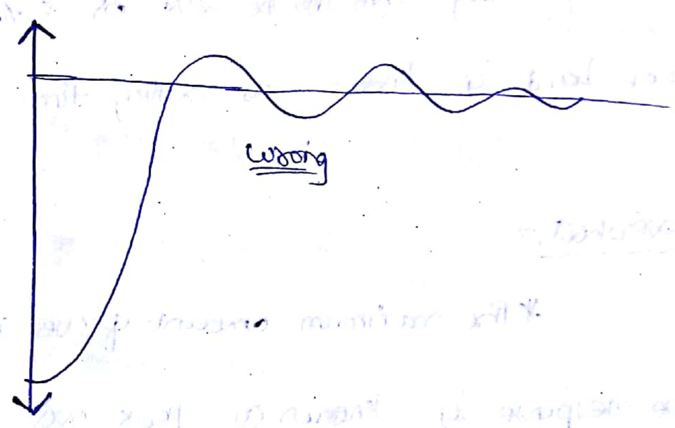
$$= c(t)\Big|_a - c(t)\Big|_d .$$

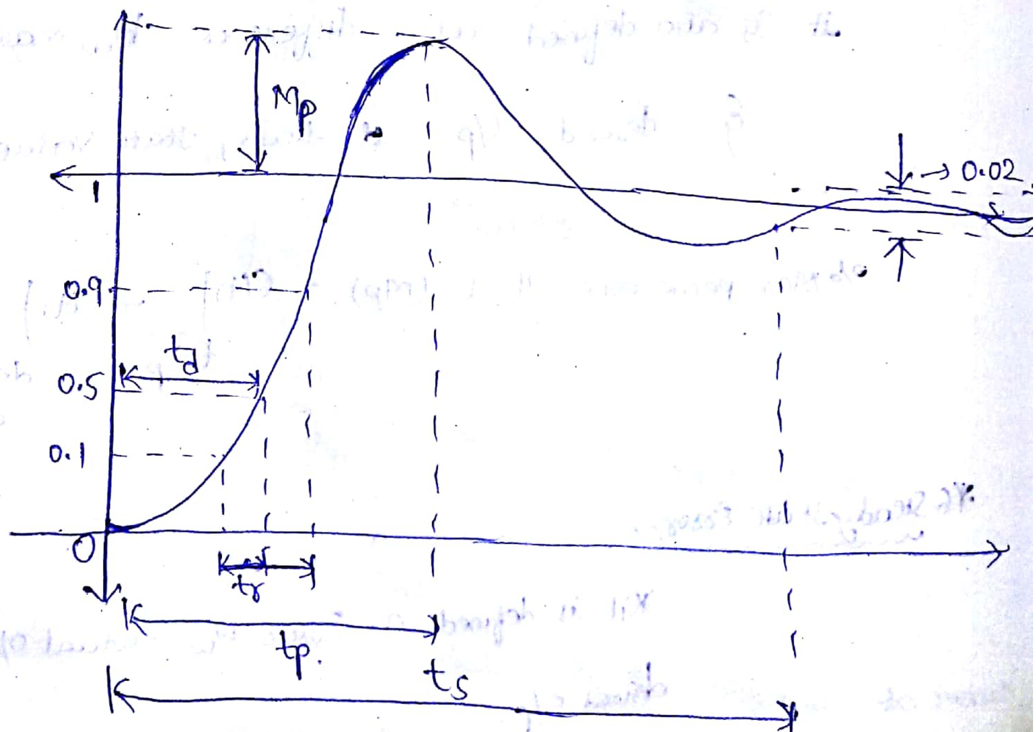
* ~~Bbila~~ *

* TIME DOMAIN SPECIFICATIONS FOR UNDER DAMPED SYSTEM*

$$c(t) = 1 - \frac{e^{-\xi t \omega_n}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

%Max. peak overshoot time $\{\% M_p\} = \dfrac{c(t_p) - c(\infty)}{c(\infty)}$.





* For undamped system $t_r = 10\%$ to 90%.
* For under damped system $t_r = 0\%$ to 100%.
* For Critically damped system $t_r = 5\%$ to 95%.

*Step response is characterised by the following performance indices :

*1. How fast the system moves to follow its input.

*2. How oscillatory it is {indicative of damping}.

*3. How long does it take practically reached to its final value.

*~~Definition~~ of

* Derivation of time domain specifications of 2nd order (underdamped system)

* ① Rise time:→ for underdamped system t_r is 0% to 100%.

* $$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t + \theta) \rightarrow \text{(I)}$$

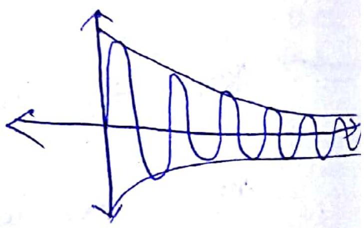
at $t = t_r \Rightarrow C(t_r) = 1$

$$\Rightarrow 1 = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t_r + \theta)$$

$$\Rightarrow \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0.$$

$$\Rightarrow \omega_d t_r + \theta = n\pi = \pi$$

$$\Rightarrow \boxed{t_r = \frac{\pi - \theta}{\omega_d}}$$



Here * $\theta = \cos^{-1}(\xi)$

* $\pi = 3.14$

* $\omega_d = \omega_n\left(\sqrt{1-\xi^2}\right)$

*(2) Peak time :- $\{t_p\}$

* Differentiate (I) on B.S we get.

$$\Rightarrow \left.\frac{dc(t)}{dt}\right|_{t=t_p} = 0.$$

$$\Rightarrow -\left[\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[-\xi\omega_n\right]\left[\sin(\omega_d t+\theta)\right] + \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}.\right.$$

$$\left.\cos(\omega_d t+\theta).\ \omega_d\right] = 0.$$

$$\Rightarrow \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\omega_d\cos(\omega_d t+\theta). - \xi\omega_n \sin(\omega_d t+\theta)\right] = 0.$$

$$\Rightarrow \omega_d.\cos(\omega_d t+\theta) - \xi\omega_n.\sin(\omega_d t+\theta) = 0.$$

$$\Rightarrow \omega_n.\sin\theta\ \cos(\omega_d t+\theta) - \cos\theta.\omega_n\ \sin(\omega_d t+\theta) = 0.$$

$$\left\{\sqrt{1-\xi^2} = \sin\theta;\quad \xi = \cos\theta;\quad \omega_d = \omega_n\sqrt{1-\xi^2}\right\}$$

$$\Rightarrow \sin\theta\cos(\omega_d t+\theta) - \cos\theta\sin(\omega_d t+\theta) = 0.$$

$$\Rightarrow \sin(\omega_d t + \cancel{\theta} - \cancel{\theta}) = 0$$

$$\Rightarrow \omega_d t_p = \theta.n\pi = \pi \quad \{at\ t = t_p\}.$$

$$\Rightarrow \boxed{t_p = \frac{\pi}{\omega_d}}$$

*3 %MP :-

$$\frac{C(t_p) - C(\infty)}{C(\infty)}$$

* %MP $C(t_p) = 1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$

* $C(\infty) = 1 - 0 = 1$.

* $t_p = \frac{\pi}{\omega_d}$.

$$\Rightarrow C(t_p) = 1 - \frac{e^{-\xi\omega_n \cdot \frac{\pi}{\omega_d}}}{\sqrt{1-\xi^2}} \cdot \sin\left(\cancel{\omega_d} \cdot \frac{\pi}{\cancel{\omega_d}} + \theta\right)$$

$$\Rightarrow C(t_p) = 1 - \frac{e^{-\xi\frac{\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin(\pi + \theta).$$

$$\Rightarrow C(t_p) = 1 + \frac{e^{-\xi\frac{\pi}{\sqrt{1-\xi^2}}}}{\cancel{\sqrt{1-\xi^2}}} \cancel{\sin(\theta)} \quad \{\sin\theta = \sqrt{1-\xi^2}\}$$

$$\Rightarrow \%MP = \frac{\cancel{1} + e^{-\xi\frac{\pi}{\sqrt{1-\xi^2}}} - \cancel{1}}{1}$$

$$\Rightarrow \boxed{\%MP = e^{-\xi\frac{\pi}{\sqrt{1-\xi^2}}}}$$

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* Setting time :- (ts).

* The response of 2nd order system has 2 components

(i) decaying Exponential component. $\left\{ \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \right\}$

(ii) Sine Component $\{ \sin(\omega_d t + \theta) \}$

* In this the decaying Exp term reduces the oscillations by ~~Sine~~ Produced by Sine Component. hence, Setting time is decided by Exp term

* The setting time can be calculated by Equating Exp Component to the % of tolerance

* For 2% tolerance band

$$e^{-\xi\omega_n t_s} = 0.02$$

$$\Rightarrow -\xi\omega_n t = \ln(0.02)$$

$$\Rightarrow \boxed{t_s = \frac{4}{\xi\omega_n}}$$

* For 5% tolerance band

$$\Rightarrow e^{-\xi\omega_n t_s} = 0.05$$

$$\Rightarrow \boxed{t_s = \frac{3}{\xi\omega_n}}$$

ie.. $t_s = \frac{\ln[\text{tolerance band}]}{\xi\omega_n}$ or $\frac{\ln[\text{Error}]}{\xi\omega_n}$

$= \frac{\ln[\text{tolerance band}]}{1} \times T$; $T = \frac{1}{\xi\omega_n}$ {Time constant of order 2 system}

Problems

*1) A unit step signal is applied to the unity F/B C.S for which OLTF is $G(s) = \frac{16}{s(s+8)}$ find its CLTF, ω_n, & ξ, ω_d.

*Sol.

$$CLTF = \frac{OLTF}{1+OLTF} = \frac{G(s)}{1+G(s).1} \quad \{H(s)=1\}$$

$$= \frac{\frac{16}{s(s+8)}}{1+\frac{16}{s(s+8)}} = \frac{16}{s^2+8s+16}.$$

w.k.T. $$CLTF = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\Rightarrow \omega_n = 4$$

$$2\xi\omega_n = 8$$

$$2\xi \cancel{\omega_n} 4 = 8$$

$$\Rightarrow \xi = 1$$

$$\Rightarrow \omega_d = \omega_n\sqrt{1-\xi^2} = 4\sqrt{1-1} = 0.$$

*2) The CLTF of a 2nd order system is given by $T(s) = \frac{100}{s^2+10s+...}$ determine & ξ, ω_n, t_r, t_s, & MP.

*Sol)

$$\Rightarrow \omega_n^2 = 100 \Rightarrow \omega_n = 10.$$

$$\Rightarrow 2\xi\omega_n = 10 \quad ; \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\Rightarrow \xi = \tfrac{1}{2}. \quad = 10\sqrt{1-\tfrac{1}{4}} = \frac{\cancel{10}^{5}\sqrt{3}}{\cancel{2}}$$

$$t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi - \cos^{-1}(\xi)}{5\sqrt{3}} = \frac{\pi - 60}{5\sqrt{3}} = 0.24\ \text{sec}$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{(\frac{1}{2})(10)} = \frac{4}{5} = 0.8\ \text{sec} \quad \{\text{for } 2\% \text{ Tolerance band}\}$$

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = e^{-\frac{\frac{\pi}{2}}{\sqrt{1-\frac{1}{4}}}} = e^{-\frac{\frac{\pi}{2}}{\frac{\sqrt{3}}{2}}} = e^{-\pi/\sqrt{3}}$$

$$= 0.16303$$

$$\Rightarrow \%M_p = 16.3\%.$$

*(3) Obtain the response of unity f/B system whose OLTF is $G(s) = \frac{4}{s(s+5)}$ & when the i/p is unit step.

* $$CLTF = \frac{\frac{4}{s(s+5)}}{1+\frac{4}{s(s+5)}} = \frac{4}{s^2+5s+4}.$$

$$\Rightarrow CLTF = \frac{4}{s^2+5s+4}.$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{4}{s^2+5s+4}$$

$$\Rightarrow C(s) = \frac{R(s).4}{s^2+5s+4} \quad \{R(s) = 1/s \Rightarrow \text{unit step} = u(t)\}.$$

$$\Rightarrow C(s) = \frac{4}{s(s^2+5s+4)} = \frac{4}{s(s^2+4s+s+4)}$$

$$\Rightarrow C(s) = \frac{4}{s(s(s+4)+1(s+4))} = \frac{4}{s(s+1)(s+4)}.$$

$$\Rightarrow C(s) = \frac{1}{s} + \frac{(-4/3)}{s+1} + \frac{1/3}{s+4}.$$

$A(s+1)(s+4) + B(s)(s+4) + C(s)(s+1) = 4.$

at $s=0$; at $s=-4$

$\Rightarrow A = 1$

$\Rightarrow C(-4)(-3) = 4$

$\Rightarrow C = 1/3$

at $s=-1$

$\Rightarrow B(-1)(3) = 4 \Rightarrow -4/3$

$\Rightarrow c(t) = u(t) - \frac{4}{3} e^{-t} u(t) + \frac{1}{3} \cdot e^{-4t} u(t).$

$\Rightarrow c(t) = u(t) - \frac{1}{3}\left[4e^{-t} u(t) - e^{-4t} \cdot u(t)\right]$ //

* Steady state error constants:-

* Order & type of a system:-

* Type number indicates no. of poles at origin

* Type of order indicates co-eff power of "s"

in general if N is the no. of poles at origin then the type number is N

Ex:-

* if N=0 then the system is type zero system.

* if N=1 then the system is type one system

* if N=2 then the system is type two system

* Steady State Error is a value of Error signal "e(t)" when $t \to \infty$ Steady State is measure of system accuracy. those Errors are occur from the nature of i/p's type of system & non-linearity in system components.

* The steady state performance of stable control system is generally specified by its steady state Error to unit step, ramp, parabola inputs.

* 20/01/18 * → Take xerox.

Monday
* 22/01/18 *

* For a unit f/b system OLTF = $G(s) = \frac{K(s+2)}{s(s^2+7s+12)}$

① find order & type of a system

② Error Co-efficients.

③ Steady state Error when i/p to the system is $\frac{R}{2}t^2$.

$$r(t) = \frac{R}{2}t^2 \Rightarrow R(s) = \frac{R}{s^3}$$

* Sol

* ①. Type = 1 {1 pole at origin}

order = 3 {Total 3 poles}

* ② $\{H(s) = 1\}$

$$K_p = \lim_{s \to 0} G(s).H(s) = \infty$$

$$K_v = \lim_{s \to 0} s.G(s).H(s) = \frac{K}{6}$$

$$K_a = \lim_{s \to 0} s^2 G(s).H(s) = 0.$$

* ③ $$e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1+G(s).H(s)} = \frac{\cancel{s}.\frac{R}{s^{\cancel{3}2}}}{1+\frac{K(s+2)}{s(s^2+7s+12)}} = \frac{\cancel{R/s}}{\frac{s(s^2+7s+12)+K(s+2)}{\cancel{(s^2+7s+12)}}}$$

$$= \lim_{s \to 0} \frac{\cancel{R(s^2+7s+12)}}{s(s^2+7s+12)+K(s+2)} = \frac{\cancel{12R}}{2k} = \cancel{\frac{6R}{K}}$$

$$e_{ss} = \underset{s\to 0}{lt} \frac{S \cdot \frac{R}{s^3}}{\frac{s(s^2+7s+12)+K(s+2)}{s(s^2+7s+12)}} = \underset{s\to 0}{lt} \frac{\frac{R}{s^2} \times \cancel{s}(s^2+7s+12)}{s(s^2+7s+12)+K(s+2)}$$

$$\Rightarrow e_{ss} = \infty. \quad \text{at } s=0.$$

*Q) loop T.F of a f/B c-s is given by $G(s).H(s) = \frac{100}{s^2(s+4)(s+12)}$ determine static error coefficients also determine steady state error for the i/p $r(t) = 2t^2 + 5t + 10$.

*Sol)

* $K_p = \underset{s\to 0}{lt}\ G(s).H(s) = \infty.$

* $K_v = \underset{s\to 0}{lt}\ s\,G(s).H(s) = \infty.$

* $K_a = \underset{s\to 0}{lt}\ s^2 G(s).H(s) = \frac{100}{48} = 2.0833$ //

* $e_{ss} = \underset{s\to 0}{lt} \frac{S.R(s)}{1+G(s).H(s)} = \underset{s\to 0}{lt} \frac{S.\left[\frac{4}{s^3}+\frac{5}{s^2}+\frac{10}{s}\right]}{1+\frac{100}{s^2(s+4)(s+12)}}$

$$= \underset{s\to 0}{lt} \frac{s^3\left[\frac{4}{s^3}+\frac{5}{s^2}+\frac{10}{s}\right](s+4)(s+12)}{s^2(s+4)(s+12)+100}$$

$$= \underset{s\to 0}{lt} \frac{(4+5s+10s^2)(s+4)(s+12)}{s^2(s+4)(s+12)+100}$$

$$= \frac{4(4)(12)}{100} = \frac{\cancel{16}^{4}\times 12}{\cancel{100}_{25}} = \frac{48}{25} = 1.92$$

*(3) For a system $G(s)\cdot H(s) = \dfrac{K}{s^2(s+2)(s+3)}$ find the value of K to limit the steady state error is 10. When input to the system is $1 + 10t + \frac{40}{2}t^2 \Rightarrow R(s) = \frac{1}{s} + \frac{10}{s^2} + \frac{40}{2}\times\frac{2}{s^3}$

*Sol)
$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)\cdot R(s)} = \lim_{s\to 0} \frac{s\left[\frac{1}{s} + \frac{10}{s^2} + \frac{40\times\cancel{2}}{\cancel{2}\,s^3}\right]}{1 + \frac{K}{s^2(s+2)(s+3)}}$$

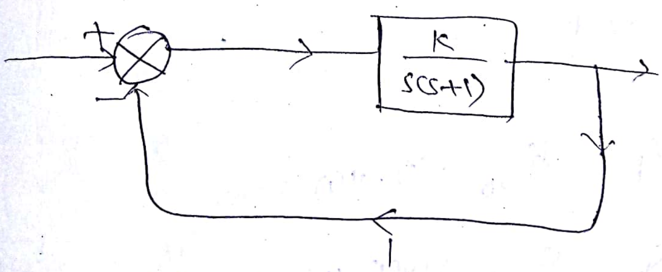
$$\Rightarrow e_{ss} = \lim_{s\to 0} \frac{s^3(s+2)(s+3)\left(\frac{1}{s} + \frac{10}{s^2} + \frac{40}{s^3}\right)}{s^2(s+2)(s+3) + K}$$

$$\Rightarrow 10 = \lim_{s\to 0} \frac{(s+2)(s+3)(s^2+10s+40)}{s^2(s+2)(s+3)+K}$$

$$\Rightarrow \cancel{10} = \frac{(2)(3)(4\cancel{0})}{K}$$

$$\Rightarrow K = 24$$

*(4) Assume $r(t) = 0.1t$ & it is desired that steady state error is ≤ 0.005 $\{e_{ss} \leq 0.005\}$ find the range of values of "K" for error to be specified limit, for the given system is



*Sol)
$$e_{ss} = \lim_{s\to 0} \frac{s\cdot R(s)}{1+G(s)\cdot H(s)} = \lim_{s\to 0} \frac{s\cdot\frac{0.1}{s^2}}{1 + \frac{K}{s(s+1)}\cdot 1}$$

$$e_{ss} = \lim_{s\to 0} \frac{\frac{0.1}{\cancel{s}}\cdot\cancel{s}(s+1)}{(s+1)s+K} = \frac{0.1}{K}$$

$$e_{ss} = \frac{0.1}{k}$$

to limit e_{ss} to 0.005

$$\Rightarrow 0.005 \geq \frac{0.1}{k} \quad \Rightarrow \quad \frac{k}{0.1} \geq \frac{1}{0.005}$$

$$\Rightarrow k \geq \frac{0.1}{0.005} \geq \frac{1}{10} \times \frac{1000}{5} \geq 20$$

$$\Rightarrow k \geq 20.$$

i.e.. for any value of $k \geq 20$; $e_{ss} \leq 0.005$

i.e.. $\boxed{20 \leq k < \infty}$

*(5) For a unity f/B system having $G(s) = \frac{35(s+4)}{s(s+2)(s+5)}$

find (a) Type of the system.

(b) all Error Co-Eff $\{k_p, k_v, k_a\}$

(c) steady state Error $\{e_{ss}\}$ for ramp input with magnitude of 5.

*sol) (a) Type = 1

(b) * $k_p = \lim_{s \to 0} G(s) \cdot H(s) = \infty$

* $k_v = \lim_{s \to 0} s\,G(s) \cdot H(s) = \frac{35 \times 4}{10} = \frac{28}{2} = 14.$

* $k_a = \lim_{s \to 0} s^2 G(s) \cdot H(s) = 0$

(c) $e_{ss} = \lim_{s\to0} \dfrac{s\cdot R(s)}{1+G(s)H(s)} = \dfrac{s\cdot\frac{5}{s^2}}{1+\frac{35(s+4)}{s(s+2)(s+5)}}$

$\Rightarrow e_{ss} = \lim_{s\to0} \dfrac{\frac{5}{\cancel{s}}\,(\cancel{s})(s+2)(s+5)}{s(s+2)(s+5)+35(s+4)}$

$\Rightarrow e_{ss} = \dfrac{5\times\cancel{2}\times\cancel{5}}{\cancel{35}_{7}\times\cancel{4}_{2}} = \dfrac{5}{14}$

*(6) For the following system determine

*(a) Type of the system

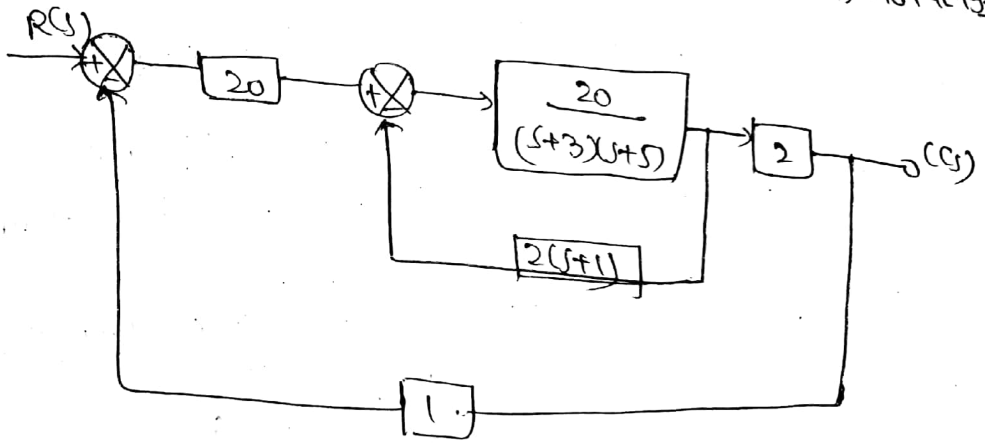
*(b) Error Co-eff

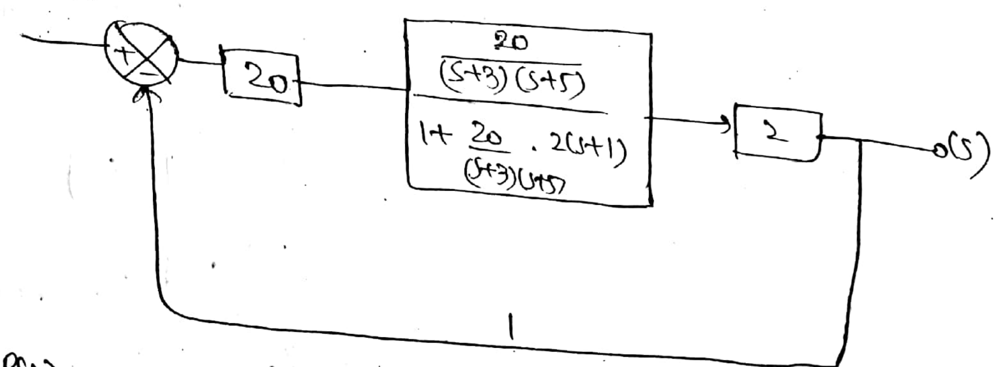
*(c) e_{ss} for i/p's $r(t) = 6$

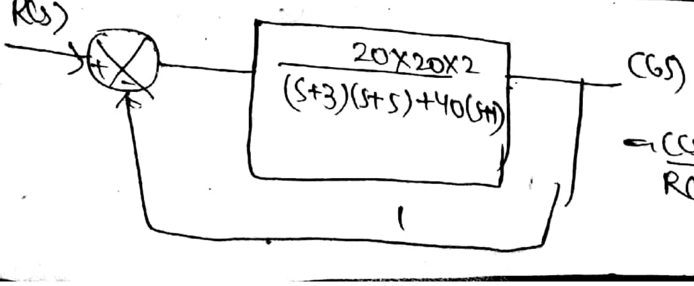
$r(t) = 8t$

$r(t) = 10+4t+\frac{3t^2}{2}$

*Sol)







$\dfrac{C(s)}{R(s)} = \dfrac{\dfrac{800}{(s+3)(s+5)+40(s+1)}}{1+\dfrac{800}{(s+3)(s+5)+40(s+1)}}$

$$\rightarrow \frac{C(s)}{R(s)} = \frac{800}{(s+3)(s+5)+40(s+1)+800}$$

* (a) type = 0.

* (b) $K_p = \lim_{s\to 0} G(s).H(s) = \lim_{s\to 0} \frac{800}{(s+3)(s+5)+40(s+1)}$

$$= \frac{800}{(3)(5)+(40)} = \frac{800}{15+40} = \frac{800}{55}$$

$K_v = K_a = 0.$

* (c) * (i) $r(t) = 6$; $R(s) = \frac{6}{s}$.

$$e_{ss} = \lim_{s\to 0} \frac{s.R(s)}{1+G(s).H(s)} = \frac{\cancel{s}.\frac{6}{\cancel{s}}}{1+\frac{800}{(s+3)(s+5)+40(s+1)}}$$

$$\Rightarrow \lim_{s\to 0} \frac{6(s+3)(s+5)+40(s+1)}{(s+3)(s+5)+40(s+1)+800}$$

$$\Rightarrow \frac{6[(3)(5)+(40)]}{(3)(5)+(40)+800}$$

$$= \frac{90+40}{15+840}$$

$= \cancel{0.3859}$. 0.3859

$5) + 40(s+1)$

$= \frac{800}{15+40} = \frac{800}{55}$

$\frac{5.6/5}{+\frac{800}{(s+3)(s+5)+40(s+1)}}$

$3)(s+5)+40(s+1)$

$).(s+5)+40(s+1)+800$

$5)+(40))] \quad = \frac{90+40}{15+840}$

$5)+(40)+800$

$\cancel{859}$. 0.3859

*(ii). $r(t) = 8t \Rightarrow R(s) = \frac{8}{s^2}$

$$\Rightarrow e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{8}{s^2}}{1 + \frac{800}{(s+3)(s+5)+40(s+1)}} = \frac{8}{s[\;\;]}$$

$= \infty$.

*(iii) $r(t) = 10 + 4t + \frac{3t^2}{2} \Rightarrow R(s) = \frac{10}{s} + \frac{4}{s^2} + \frac{3(\cancel{2})}{\cancel{2}\cdot s^3}$

$$\Rightarrow e_{ss} = \lim_{s\to 0} \frac{s\left[\frac{10}{s} + \frac{4}{s^2} + \frac{3}{s^3}\right]}{1 + \frac{800}{(s+3)(s+5)+40(s+1)}}$$

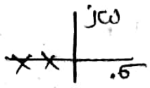
$= \infty$

UNIT – III

3. Root locus

2/01/18

* Stability analysis of a C.S

* Absolute Stable System  Poles

* Relative " "  System o/p varies with variation in system parameters.

* Conditionally " "  a±jb are the roots.
if a = +ve then unstable
a = −ve then stable
a = 0 then stable.

* Asymptotically " "
↳ Neither Relative nor Conditional systems are Asymptotically stable systems.

* Stability :-

* If there is small change in input conditions or system parameters "or" initial conditions then the o/p should not be changed ~~in the system~~.

or

* A system is said to be stable if & only if for bounded input (finite) if the system produces bounded output.

* under the absence of i/p, the o/p of the system should be zero.

* Absolutely stable system :- * A system is said to Absolutely stable for all the parameters over the Entire range $(-\infty < t < \infty)$ the system output is constant.

* Conditionally stable system:→

* For a particular parameter, & for a particular range the system output is constant.

* Relative stable system:→

* A system is said to be Relative stable in accordance with another system with the relative parameter (settling time), the system output measured then it is called relative stability.

* Asymptotically stable system:→

* A system is said to be asymptotically stable, when its output should tend to "0" under the absence of i/p or zero input.

* Marginally stable system:→

* If a system is neither absolutely stable nor unstable system, is known as marginally stable system.

* Concept of location of poles & stability:→

① * If all the poles are lying on left half of s-plane then the system is absolutely stable system.

② * If non-repeated poles are lying of Img axis then the.

System is marginally stable.

*③ If repeated poles are lying on Img axis then the System is unstable.

*④ If any one pole is lying on Right half of s-plane the system is unstable.