UNIT- III

IIR Filters: (Realisation and design of IIR)

Based on impulse Response

IIR:

If the impulse response is of infinite duration, it is IIR filter. It is digital filter.

FIR

If the impulse response is of finite duration, it is FIR filter.

→ There are various forms of analog filters which are then used to construct digital IIR filter.

Realisation of filter:

IIR is recursive filter. → present o/p depends on present i/p, past i/p & past o/p.

FIR is Nonrecursive "

Nonrecursive → does not depend on past o/p

LTI system o/p: $y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$ → FIR

which is a IIR filter. (~~de~~ recursive)

FIR: $y(n) = \sum_{k=0}^{M} b_k\, x(n-k)$

IIR: $Y(z) = -\sum_{k=1}^{N} a_k z^{-k} Y(z) + \sum_{k=0}^{M} b_k z^{-k} X(z)$

$$\boxed{\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}} = \sum_{n=0}^{\infty} h(n) z^{-n}$$

causal.

IIR and FIR filters are realisable only when $h(n)=0 \quad n\leq 0$ & $\sum_{n=0}^{\infty} |h(n)| < \infty$ (Causal)

→ IIR consists of both zeros and poles.

FIR: $y(n) = \sum_{k=0}^{m} b_k x(n-k)$

$$\frac{Y(z)}{X(z)} = \sum_{k=0}^{m} b_k z^{-k}$$

→ T.F consists of only zeros (FIR)

→ To design a filter, filters coefficients are calculated and T.F is calculated and using T.F, Impulse Response is calculated.

Analog (Butterworth ~~etc~~) $\xrightarrow[\text{techniques}]{\text{using Transformation}}$ Digital filter.

Realisation of ~~IIR~~ Digital filters.

Realisation of IIR filters:

Digital filters

↓ IIR

↓ FIR

1) Direct form - I

2) " " - 2

3) Cascade form.

4) Parallel form.

N - 1 .

M

M + N - 1 + 1

M + N

M + N

Direct form - I

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

(Implementation of filter in understandable way - realisation)

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) - a_3 (y(n-3)) \ldots - a_N y(n-N) +$$

$$\underbrace{b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M)}_{w(n)} \quad \text{--- (1)}$$

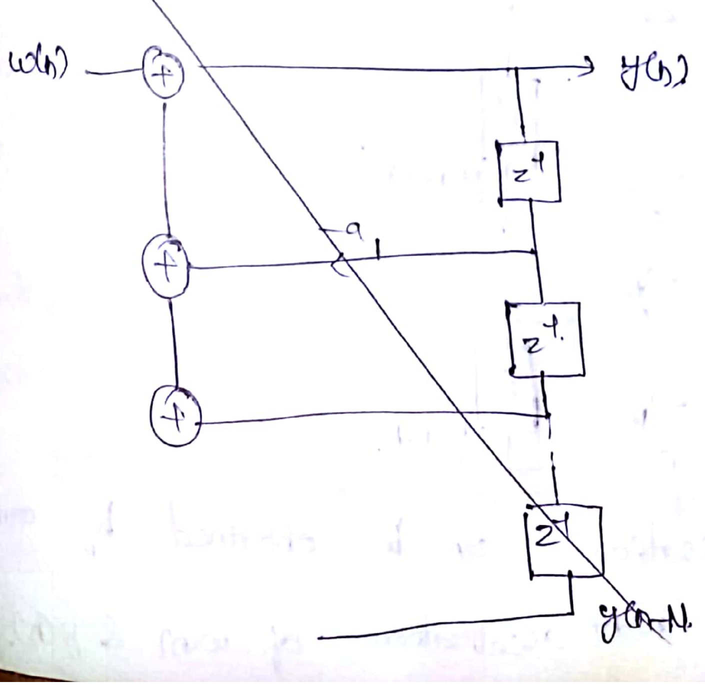
$$\cancel{w(n) = a_1 y(n-1) - a_2 y(n-2) \ldots - a_N y(n-N)}$$

$$\cancel{y(n) = b_0 x(n) + b_1 (x-n) + \ldots b_M x(n-M) + w(n)}$$

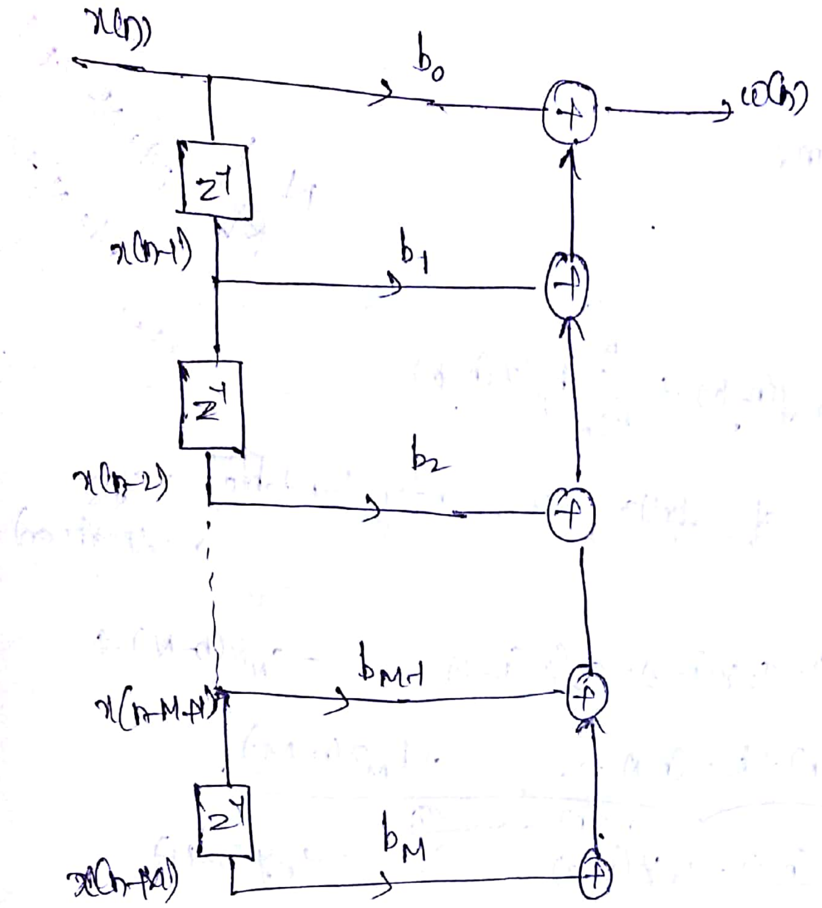
$$y(n) = -a_1 y(n-1) - a_2 y(n-2) \ldots - a_N y(n-N) + w(n) \quad \text{(2)}$$

Realisation using direct form I,

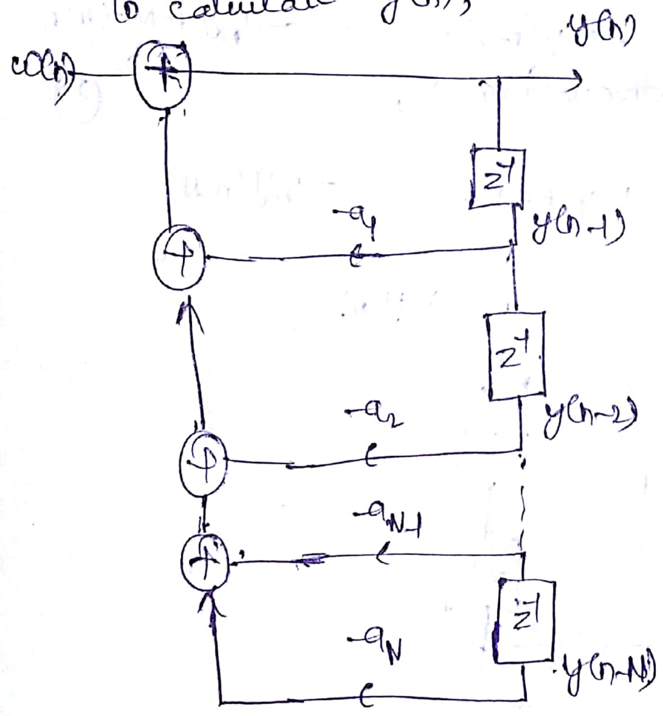
$$\cancel{w(n) = -a_1 y(n-1) - a_2 y(n-2) \ldots - a_N y(n-N)}$$



$w(n) = b_0 x(n) + b_1 x(n-1) \ldots\ldots b_M x(n-M)$ —— (1)



To calulate y(n),



Total realisation can be obtained by combining both ~~w(n)~~ realisation of w(n) & y(n).

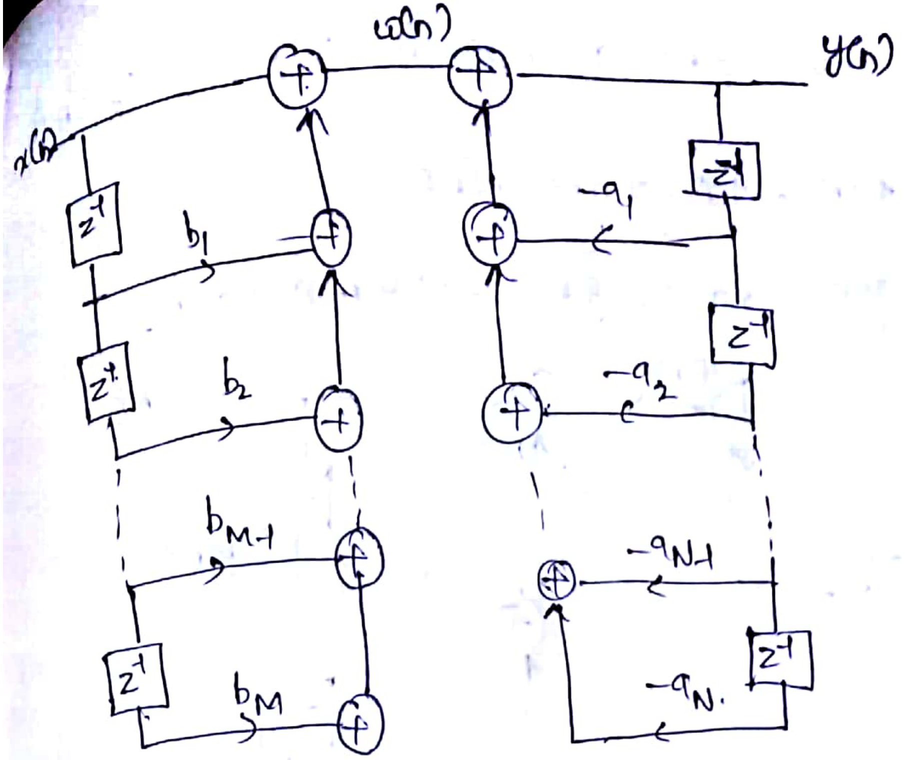


Fig: Direct form I realz

→ The no. of multiplications ~~of~~ and additions are $N+M+1$ and $N+M$ respectively and total no. of delays $= N+M+1$ ($\because x(0)$ also needs a storage location)

→ No. of delays ~~&~~ or storage locations $(N+M+1)$ are more in Direct form-I

→ To over this, direct form-2 is used.

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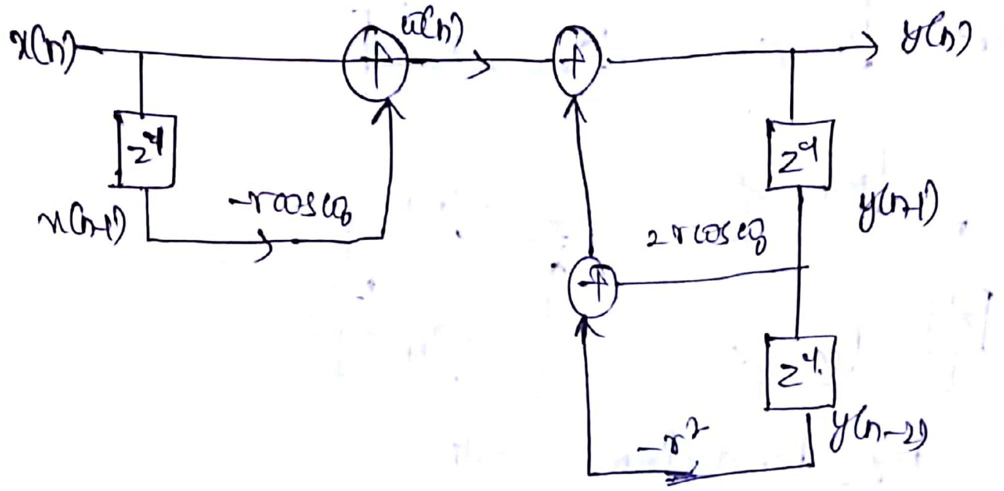
① Realize the 2nd order eqn using Direct form-II realization

Sol:
$$y(n) = 2r\cos\omega_0\, y(n-1) - r^2 y(n-2) + x(n) - r\cos\omega_0\, x(n-1)$$

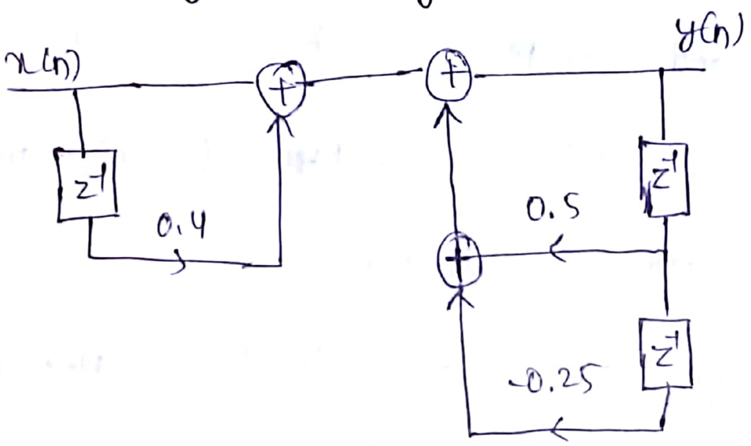
Sol. $y(n) = 2r\cos\omega_0\, y(n-1) - r^2 y(n-2) + x(n) - r\cos\omega_0\, x(n-1)$

Let $\omega(n) = x(n) - r\cos\omega_0\, x(n-1)$.

$y(n) = 2r\cos\omega_0\, y(n-1) - r^2 y(n-2) + \omega(n)$.



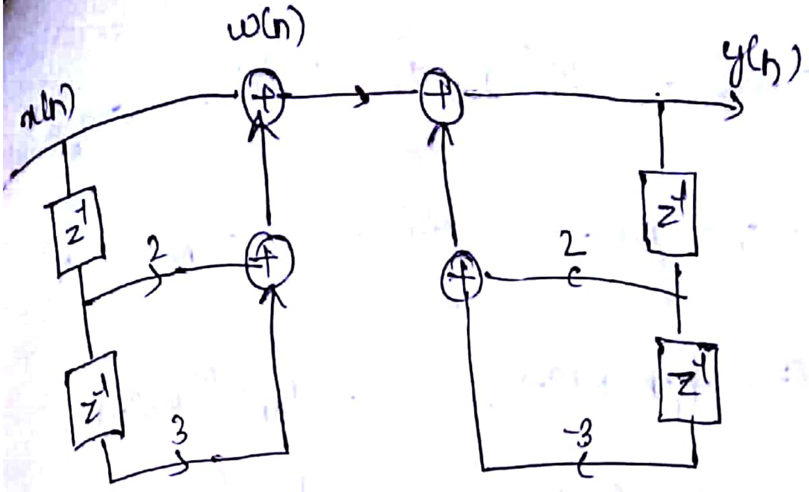
② $y(n) = 0.5\, y(n-1) - 0.25\, y(n-2) + x(n) + 0.4\, x(n-1)$



$\omega(n) = x(n) + 0.4\, x(n-1)$

$y(n) = 0.5\, y(n-1) - 0.25\, y(n-2) + \omega(n)$

③ $y(n) = 2y(n-1) + 3y(n-2) + x(n) + 2x(n-1) + 3x(n-2)$



Direct Form-II Realisation:

Here, storage locations are reduced from N+M+1 to Max(M, N)

common delay is used for both x(n) & y(n)

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k) = w(n)$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

$$\frac{Y(z)}{W(z)}, \frac{W(z)}{X(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

$$\frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

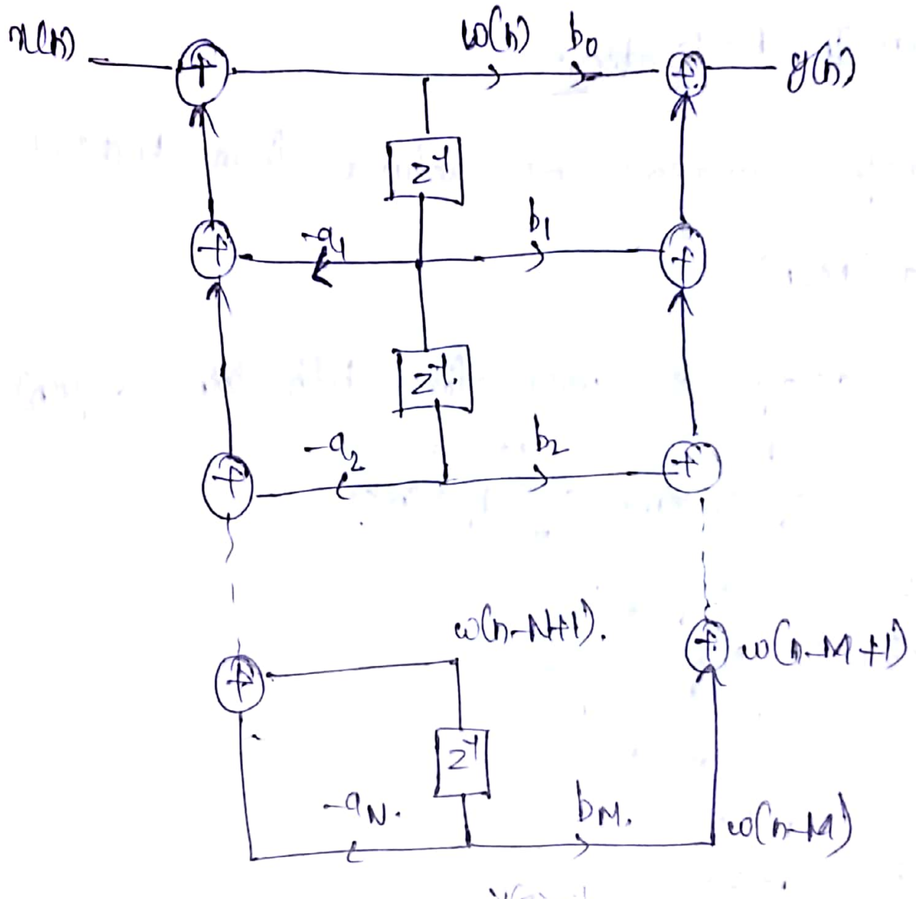
$$W(z)\left(1+\sum_{k=1}^{N} a_k z^{-k}\right) = X(z)$$

$$W(z) = X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) \dots - a_N z^{-N} W(z)$$

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2) \dots - a_N w(n-N)$$

$$Y(z) = b_0 W(z) + b_1 z^{-1} W(z) \dots + b_M z^{-M} W(z)$$

$$y(n) = b_0 w(n) + b_1 w(n-1) \dots b_M w(n-M)$$



(Q) $y(n) = 2r\cos\omega_0\, y(n-1) - r^2 y(n-2) + x(n) + r\cos\omega_0\, x(n-1)$

Realize the second order system (Poth a) using Direct form-II Realisation.

Sol: $w(n) = x(n) + 2r\cos\omega_0\, y(n-1) - r^2 w(n-2)$

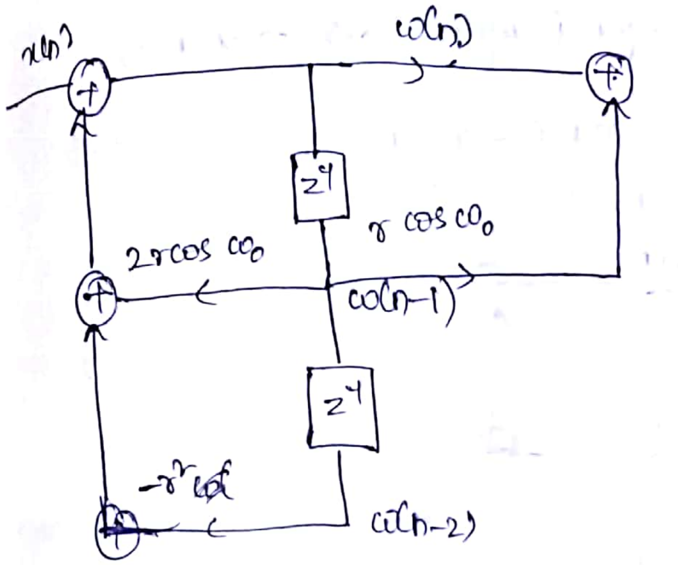
~~$y(n) = w(n)$~~

$$Y(z) = 2r\cos\omega_0 Y(z) z^{-1} - r^2 Y(z) \cdot z^{-2} + X(z) + r\cos\omega_0 z^{-1} X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1 + r\cos\omega_0 z^{-1}}{1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}}$$

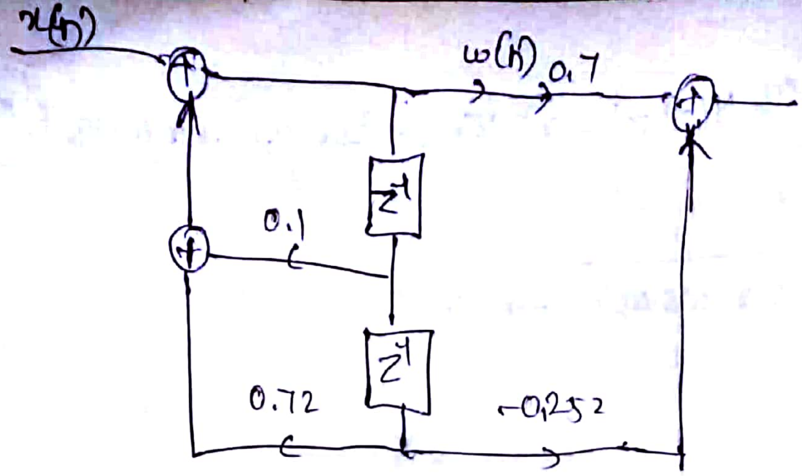
$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$y(n) = w(n) + r\cos\omega_0 \, w(n-1)$$



(2) $y(n) = 0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.252\, x(n-2)$

$$w(n) = x(n) + 0.1\, w(n-1) + 0.72\, w(n-2)$$
$$y(n) = 0.7\, w(n) - 0.252\, w(n-2)$$

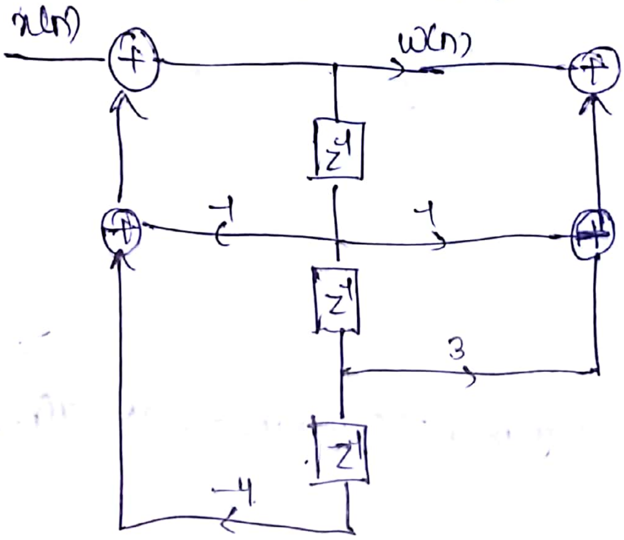


Q) $y(n) + y(n-1) + 4y(n-3) = x(n) + 3x(n-2)$. Realise using Direct form II Realisation.

Sol: ~~$w(n) = x(n)$~~ $y(n) = -y(n-1) - 4y(n-3) + x(n) + 3x(n-2)$

$w(n) = x(n) \cancel{- y(n}\; - w(n-1) - 4w(n-3)$

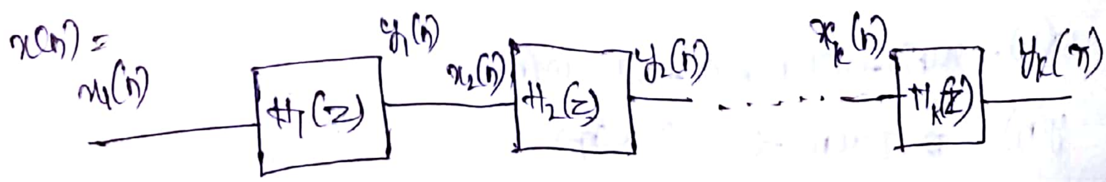
$y(n) = w(n) + 3w(n-2)$



Cascade form Realisation:

T.F $H(z)$ is replaced as; $H(z) = H_1(z)\, H_2(z) \ldots H_k(z)$

Realise all $H_1(z), \ldots H_k(z)$ using direct form: and cascade them



T.F

$$H(z) = \frac{(b_{k_0} + b_{k_1} z^{-1} + b_{k_2} z^{-2})(b_{m_0} + b_{m_1} z^{-1} + b_{m_2} z^{-2})}{(1 + a_{k_1} z^{-1} + a_{k_2} z^{-2})(1 + a_{m_1} z^{-1} + a_{m_2} z^{-2})}$$

$$H(z) = H_1(z) \cdot H_2(z)$$

$$H_1(z) = \frac{b_{k_0} + b_{k_1} z^{-1} + b_{k_2} z^{-2}}{1 + a_{k_1} z^{-1} + a_{k_2} z^{-2}} \qquad H = \frac{Y}{X} = \frac{Y}{W} \cdot \frac{W}{X}$$

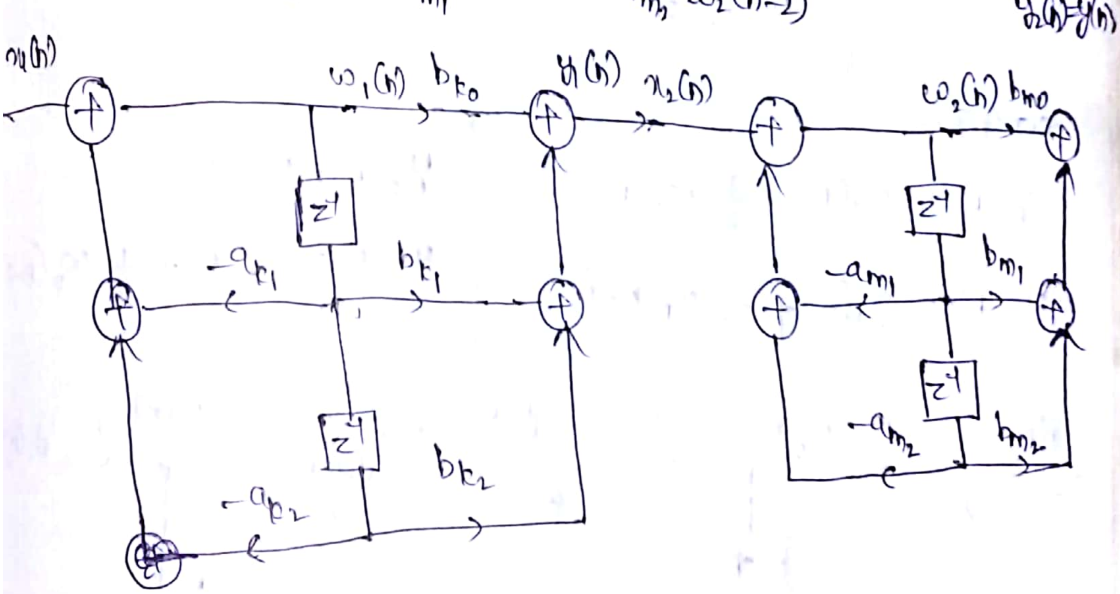
$$H_2(z) = \frac{b_{m_0} + b_{m_1} z^{-1} + b_{m_2} z^{-2}}{1 + a_{m_1} z^{-1} + a_{m_2} z^{-2}}$$

$$y_1(n) = b_{k_0} w_1(n) + b_{k_1} w_1(n-1) + b_{k_2} w(n-2)$$

$$w_1(n) = x(n) \cancel{+} a_{k_1} \cancel{z^{-1}} w_1(n-1) - a_{k_2} w(n-2)$$

$$y_2(n) = b_{m_0} w_2(n) + b_{m_1} w(n-1) + b_{m_2} w_2(n-2)$$

$$w_2(n) = x_2(n) - a_{m_1} w(n-1) - a_{m_2} w_2(n-2)$$



② $y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) + \frac{1}{3} x(n-1)$. Realise this system using cascade form realisation.

~~sol~~

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$$

$$\frac{Y(z)}{X(z)} = \frac{\cancel{\frac{3}{4}z^{-1} - \frac{1}{8}z^{-2}}}{\cancel{1+\frac{1}{3}z^{-1}}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

$$= \frac{1+\frac{1}{3}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\cancel{\left(1-\frac{1}{2}z\right)}\left(1-\frac{1}{2}z^{-1}\right)}$$

$(a-b)^2$

$\frac{-b}{a} = \frac{3}{4}$

$$H_1(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{4}z^{-1}}, \quad H_2(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

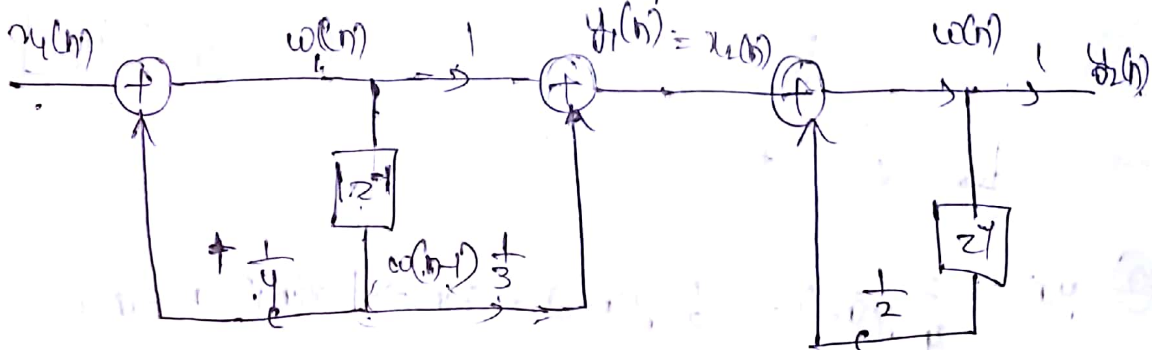
~~$w_1(n) =$~~

$y_1(n) = w_1(n) + \frac{1}{3}w_1(n-1)$

$w_1(n) = x_1(n) + \frac{1}{4}w_1(n-1)$

$y_2(n) = w_2(n)$

$w_2(n) = x_2(n) + \frac{1}{2}w_2(n-1)$



Q) $$H(z) = \frac{\left(1+\frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}\right)\left(1-\frac{3}{2}z^{-1}+z^{-2}\right)}{\left(1+z^{-1}+\frac{1}{4}z^{-2}\right)\left(1+\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}\right)}$$

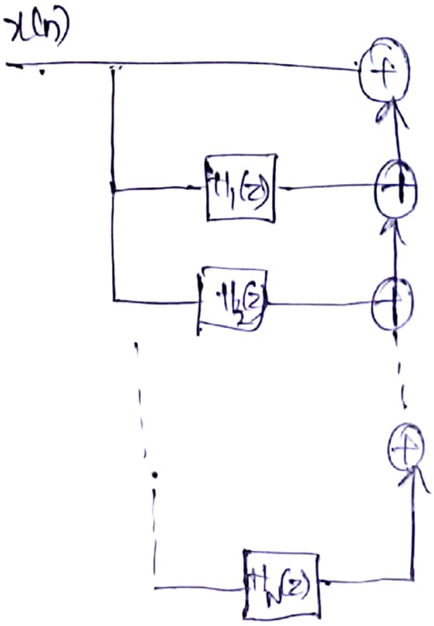
Parallel form Realisation :-

A parallel form realisation of an ~~IIR~~ system can be obtained by ~~formal~~ performing a partial expansion of H(z)

$$H(z) = c + \sum_{k=1}^{N} \frac{c_k}{1-p_k z^{-1}}$$

$$= c + \frac{c_1}{1-p_1 z^{-1}} + \frac{c_2}{1-p_2 z^{-1}} + \cdots \frac{c_N}{1-p_N z^{-1}}$$

$$Y(z) = cX(z) + H_1(z)X(z) + H_2(z)X(z) \ldots + H_N(z)X(z)$$



(Q) $y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.252\, x(n-2)$ using parallel form

Sol: $$H(z) = c + \sum_{k=1}^{N} \frac{c_k}{1-p_k z^{-1}}$$

$$H(z) = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

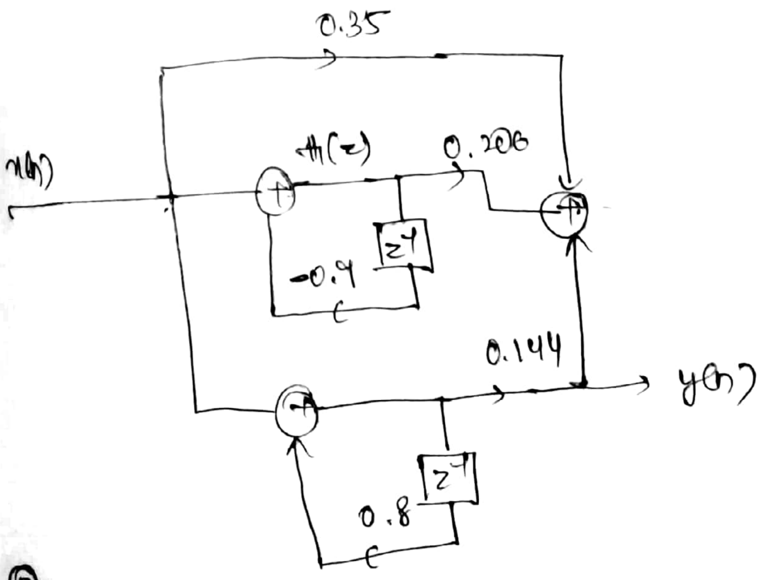
$$\begin{array}{r} 0.35 \\ -0.72z^{-2} + 0.1z^{-1} + 1 \overline{\big) -0.252z^{-2} + 0.7} \\ -0.252z^{-2} + 0.035z^{-1} + 0.35 \\ \hline -0.035z^{-1} + 0.35 \end{array}$$

$$= \cancel{0.35 + \frac{0.35z^{-1} - 0.35}{(1+0.1z^{-1})(0}}$$

$$= 0.35 + \frac{(-0.035z^{-1} + 0.35)}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$$= 0.35 + \frac{(-0.035z^{-1} + 0.35)}{(1+0.9z^{-1})(1-0.8z^{-1})}$$

$$= 0.35 + \frac{0.206}{1+0.9z^{-1}} + \frac{0.144}{1-0.8z^{-1}}$$



(Q) $H(z) = \dfrac{1 + z^{-1} + z^{-2}}{(1 + \frac{1}{2}z^{-1})(1 + \frac{1}{6}z^{-1})}$ using ~~ll~~ Kel form.

Q) $y(n) = -0.7\,y(n-1) + 0.2\,y(n-3) + 3\,x(n) + 3.6\,x(n-1) + 0.6\,x(n-$

Realise second order IIR system with direct form

direct form II, cascade and ||el form.

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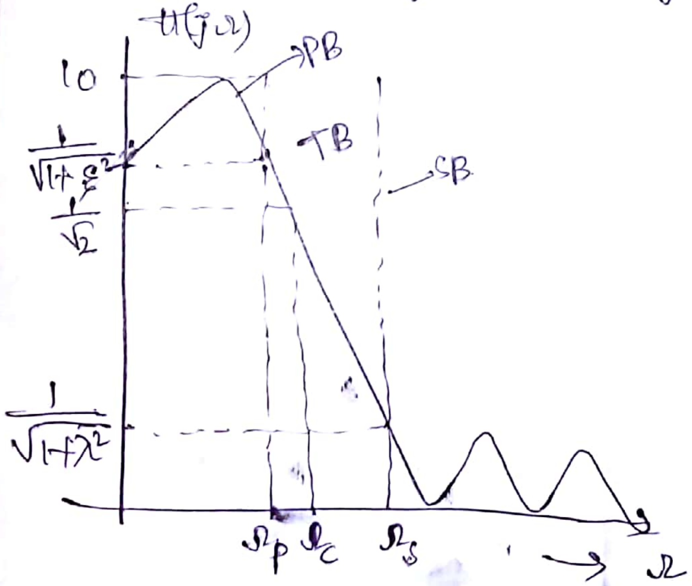
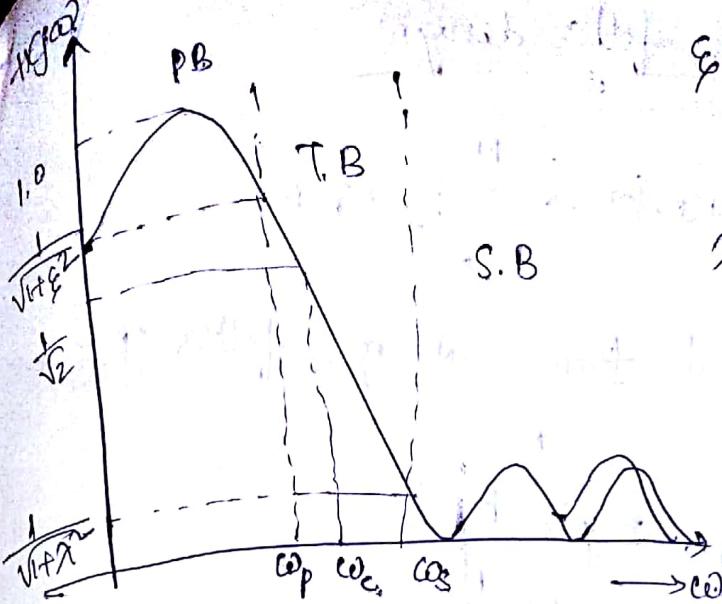
Design of digital filter from analog filter:

1) Design analog filter for given specifications
2) Find the transfer function of filter.
3) Convert analog to digital filter using transformations





$$\varepsilon = \frac{2\sqrt{\varepsilon_p}}{1-\varepsilon_p}$$

$$\lambda = \frac{\sqrt{(1+\delta_p)^2 - \delta_s^2}}{\delta_s}$$

δ_p - PB ~~tolerance~~ (or max. allowable mag. in pass band)

δ_s = max. allowable magnitude in the stop band.

$\Omega \rightarrow$ analog

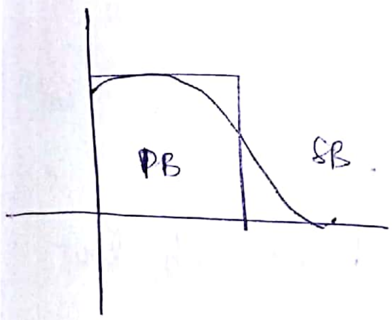
$\omega \rightarrow$ digital.

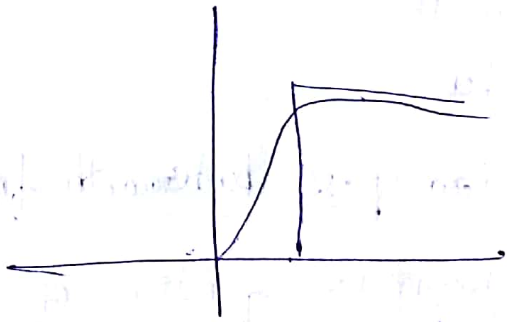
ω_p - maximum allow

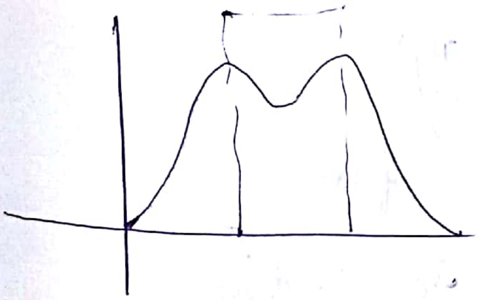
$\varepsilon - \delta_p$

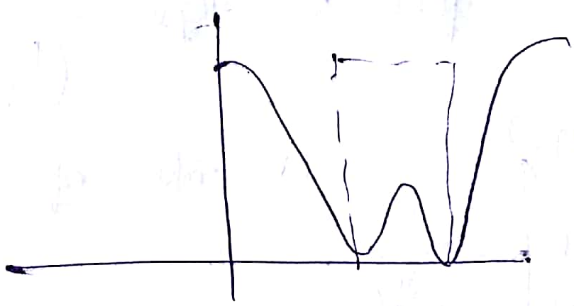
$\lambda - \delta_s$

Practically, there exists a TB(transmission band) b/w PB & SB.









Analog low pass filter design:

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

The most general form analog filter T.F is

$$H(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{M} b_i s^i}{1+\sum_{i=1}^{N} a_i s^i}$$

$$H(s) = \int_{-\infty}^{\infty} h(t) \cdot e^{-st} dt$$

For a stable analog filter, the poles of $H(s)$ lies in the left half of s plane

There are two types of analog filter

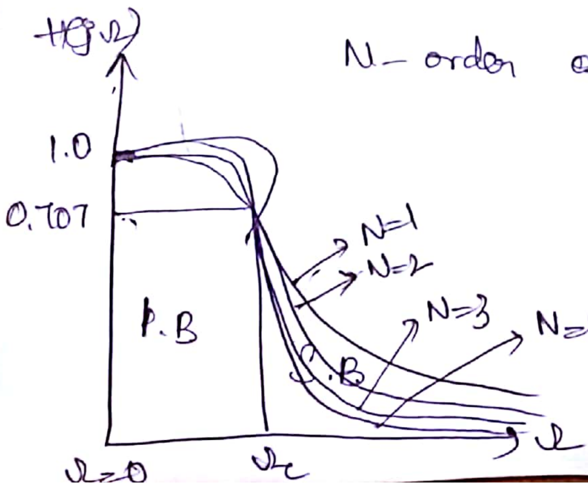
1) Butterworth
2) chebyshev

*** Analog low pass butterworth filter.

The magnitude function of a butterworth low pass filter is $H(j\Omega) = \frac{1}{\left(1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right)^{1/2}}$

N – order of the filter.

$\Omega_c = 1$ rad/sec



The magnitude square function of a normalised butterworth ~~filter~~.

$$|H(j\Omega)|^2 = \frac{1}{1+(\Omega)^{2N}}$$

Transfer function of stable ~~filter~~ can be obtained by substituting $\omega = \frac{s}{j}$

$$|H(j\Omega)|^2 = H(\Omega^2)$$

$$= H\left(\left(\frac{s}{j}\right)^2\right)$$

$$= H(-s^2)$$

$$= H(s) \cdot H(-s)$$

$$H(s)H(-s) = \frac{1}{1+\left(\frac{s}{j}\right)^{2N}}$$

$$= \frac{1}{1+(-1)^N s^{2N}}$$

The above relation tells us ~~that~~ this ~~function~~ has poles in LHP & RHP because of presence of 2 factors $H(s)$ & $H(-s)$

To find roots of eqn, ~~$1+H(s)^2$~~

$$1+(-s^2)^N = 0$$

~~If~~ N-odd

$$s^{2N} = 1.$$

$$s^{2N} = e^{j2\pi k}$$

$$\boxed{S_k = e^{j\pi k/N}} \quad k=1,2,\ldots 2N$$

~~If~~ N-even

~~$s^{2N} = 1$~~

$$s^{2N} = -1$$

$$s^{2N} = e^{j(2k-1)\pi}$$

$$\boxed{s_k = e^{j(2k-1)\pi/2N}} \quad k=1, \dots 2N$$

Let $N=3$

$$s^{2N} = 1 \Rightarrow s^6 = 1$$

$$s_k = e^{j\pi k/3} \quad k=1, 2, \dots 2N$$

$$s_1 = e^{j\pi/3} = 0.5 + j0.866$$

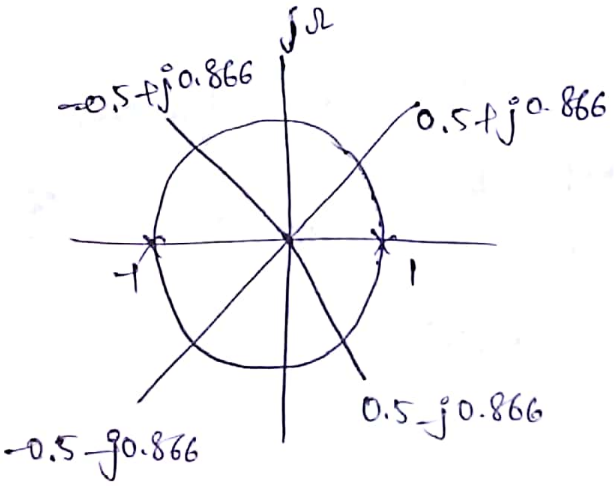
$$s_2 = e^{j2\pi/3} = -0.5 + j0.866$$

$$s_3 = e^{j3\pi/3} = -1$$

$$s_4 = e^{j4\pi/3} = -0.5 - j0.866$$

$$s_5 = e^{j5\pi/3} = 0.5 + j0.866$$

$$s_6 = e^{j6\pi/3} = 1$$



Angle of seperation b/w 2 poles is $\frac{360}{2N}$

Here, poles lie on a circle.

$$(s+1)\left[(s+0.5)^2 - (j0.866)^2\right] = (s+1)\left[(s+0.5)^2 + (0.866)^2\right]$$

$$= (s+1)\left[(s^2 + 0.5s^2 + s) + (0.75)\right]$$

$$= (s+1)(s^2+s+1)$$

$$\boxed{H(s) = \frac{1}{(s+1)(s^2+s+1)}}$$

T.F of 3rd order stable filter.

N	Denominator of H(s)
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.76537s+1)(s^2+1.8477s+1)$
5	$(s+1)(s^2+0.61803s+1)(s^2+1.61803s+1)$
6.	$(s^2+1.931855s+1)(s^2+\sqrt{2}s+1)(s^2+0.51764s+1)$
7.	$(s+1)(s^2+1.80194s+1)(s^2+1.247s+1)(s^2+0.445s+1)$

Pole location of butterworth filter for $\Omega_c = 1$ rad/sec are known as normalised poles in general the unnormalised poles are given by $\boxed{s_k' = \Omega_c s_k}$

The T.F of this type of Butterworth filter can be obtained by substituting $s = \frac{s}{\Omega_c}$ in T.F

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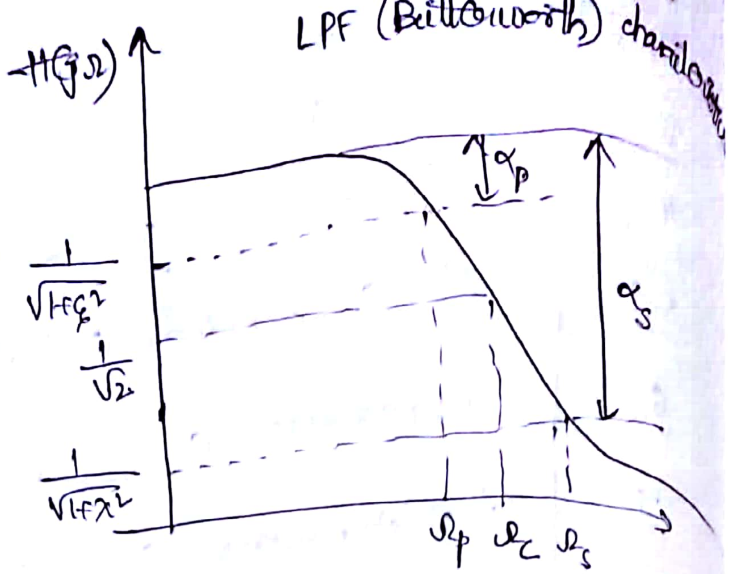
Order of Butterworth filter:



α_p – passband attenuation at pass band frequency Ω_p

α_s – stopband " " stop " " Ω_s

Magnitude function of Butterworth filter:

$$H(j\Omega) = \frac{1}{\left(1+\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}\right)^{1/2}} \quad \text{—— (1)}$$

$$\& \; H(j\Omega) = \frac{1}{\left(1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right)^{1/2}}$$

$$|H(j\Omega)|^2 = \frac{1}{\left(1+\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}\right)}$$

Apply log

$$20\log|H(j\Omega)| = 10\log 1 - 10\log\left[1+\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}\right]$$

At $\Omega = \Omega_p$, $\alpha = \alpha_p$

$$+20\log|H(j\Omega)| = -10\log\left[1+\varepsilon^2\right]$$

$$-\alpha_p = -10\log\left[1+\varepsilon^2\right]$$

$$0.1\alpha_p = \log[1+\varepsilon^2]$$

$$\left[\begin{array}{l} 20\log(|H(j\Omega_p)|) = -\alpha_p \\ 20\log(|H(j\Omega_s)|) = -\alpha_s \end{array}\right]$$

$$\therefore 1+\varepsilon^2 = 10^{0.1\alpha_p}$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p}-1}$$

At $\Omega = \Omega_s$, $\alpha = \alpha_s$

$$20\log|H(j\Omega_s)| = 10\log 1 - 10\log\left[1+\varepsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}\right]$$

$$-\alpha_s = -10\log\left[1+\varepsilon^2\left(\frac{\Omega_s}{\omega_p}\right)^{2N}\right]$$

$$\cancel{1+\varepsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = 10^{0.1\alpha_s}}$$

$$1+\varepsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = 10^{0.1\alpha_s}$$

$$\cancel{\varepsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = 10^{0.1\alpha_s}-1}$$

$$\therefore \varepsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = 10^{0.1\alpha_s}-1$$

Substitute ε^2 in the Eqn

$$\cancel{\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = \frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha}}}$$

$$\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = \frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}$$

Apply log on b.s.

$$2N\log\frac{\Omega_s}{\Omega_p} = \log\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}$$

$$\boxed{N = \frac{\log\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}}$$

~~It~~ is always a ~~not~~ non-integer value.

$\therefore$ N is always approximated to next highest value.

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$$\boxed{N \geq \frac{\log\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}}$$

Finding Ω_c

Equating two magnitude functions

$$|H(j\Omega)|^2 = \frac{1}{\left(1+\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}\right)} \quad \& \quad |H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

$$\frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}} = \frac{1}{1+\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}}$$

$$\cancel{1+}\,\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N} = \cancel{1}+\left(\frac{\Omega}{\Omega_c}\right)^{2N}$$

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = \varepsilon^2$$

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = 10^{0.1\alpha_p}-1$$

$\cancel{\Omega_c =}$

$$\Omega_c = \frac{\Omega_p}{\left(10^{0.1\alpha_p}-1\right)^{1/2N}}$$

$$\Omega_c = \frac{\Omega_s}{\left(10^{0.1\alpha_s}-1\right)^{1/2N}}$$

$$N \geq \frac{\log A}{\log\left(\frac{1}{k}\right)}$$

$$A = \frac{\lambda}{\epsilon} = \frac{\sqrt{10^{0.1\alpha_s}-1}}{\sqrt{10^{0.1\alpha_p}-1}}$$

$$K = \frac{\Omega_p}{\Omega_s}$$

<u>Steps to design an analog butterworth LPF:</u>

1) From the given specifications, find the order of the ~~filter~~ 'N'

2) Round off to the next higher integer value.

3) Find the T.F $H(s)$ for $\Omega_c = 1$ rad/sec. for the value of 'N'

4) calculate the value of cut-off frequency 'Ω_c'.

5) Find the T.F $H_a(s)$ for the above value of Ω_c by substituting ~~$s = \Omega_c$~~ $s = \frac{s}{\Omega_c}$ in $H(s)$

① The given specifications of analog butterworth LPF is $\alpha_p = 1\,dB$, $\alpha_s = 30\,dB$, $\Omega_p = 200$ rad/sec, $\Omega_s = 600$ rad/sec. Find the value of N.

<u>Sol:</u>

$$N \geq \frac{\log\sqrt{\frac{10^{0.1\times30}-1}{10^{0.1\times1}-1}}}{\log\left(\frac{600}{200}\right)} \qquad N \geq \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$N \geq 3.75$

$N = 4$

② Design an analog BW filter that has ~~$\Omega_p =$~~ $\alpha_p = 2\,dB$, $\alpha_s = 10\,dB$, $\Omega_p = 20$ rad/sec and $\Omega_s = 30$ rad/sec.

Sol:

$$N \geq \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{\alpha_p}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

≥ 3.37

$N = 4$

$$\Omega_c = \frac{\Omega_p}{\left(10^{0.1\alpha_p}-1\right)^{1/2N}}$$

$$\Omega_c = \frac{20}{\left(10^{0.1\times 2}-1\right)^{1/8}}$$

$= 21.386$

~~$H(s)$~~ at $\Omega_c = 1$

$S_k = e^{j(2k-1)\pi/2N}$ $\quad k = 1, \ldots 8$

$S_1 = 0.92 + j0.38$

$S_2 = 0.3 + 0.92j$

$S_3 = -0.3 + 0.92j$

$S_4 = -0.9 + 0.38j$

$S_5 = -0.9 - 0.38j$

$S_6 = -0.3 - 0.92j$

$S_7 = 0.3 - 0.92j$

$S_8 = 0.92 - 0.38j$

$$H(s) = \frac{1}{\left((s+0.3)^2+(0.92)^2\right)\left((s+0.9)^2+(0.38)^2\right)}$$

3) For the given specification, design analog Bw filter.

$0.9 \le |H(j\Omega)| \le 1$ for $0 \le \Omega < 0.2\pi$, $H(j\Omega) \le 0.2$

for $0.4\pi \le \Omega \le \pi$

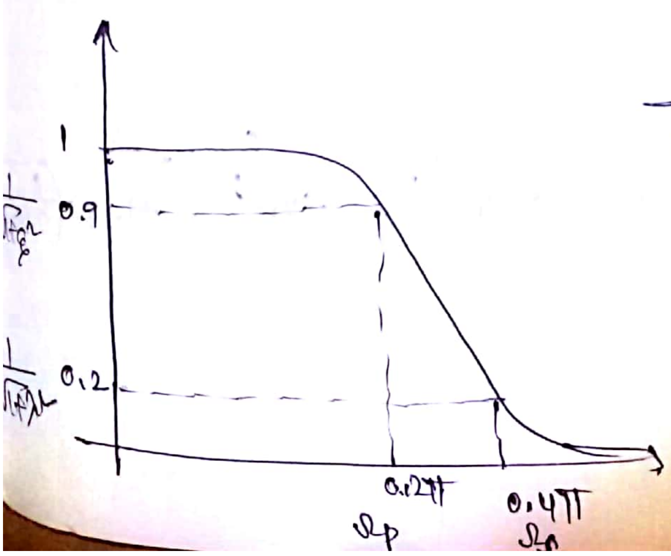
Sol: $0.9 \le |H(j\Omega)| \le 1$

for $0 \le \Omega \le 0.2\pi$

$|H(j\Omega)| \le 0.2$, $0.4\pi \le \Omega \le \pi$

$\Omega_p = 0.2\pi$

$\Omega_s = 0.4\pi$



$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.9$$

$\Rightarrow \varepsilon = 0.48$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$\Rightarrow \lambda = 4.89$

$$N \geq \frac{\log A}{\log\left(\frac{1}{k}\right)}$$

$$A = \frac{\lambda}{\varepsilon} = 10.18$$

$$k = \frac{\Omega_p}{\Omega_s} = \frac{0.2\pi}{0.4\pi} = 0.5$$

$N \geq 3.347$

$N = 4$

From table

$$H(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.84775s+1)}$$

$$\Omega_c = \frac{\Omega_p}{\left(10^{0.1\alpha_p}-1\right)^{1/2N}} = \frac{\Omega_p}{\varepsilon^{1/2N}}$$

$$= \frac{0.2\pi}{(0.48)^{1/8}}$$

$\Omega_c = 0.688$

$$s \rightarrow \frac{s}{0.68}$$

$$H_a(s) = \frac{1}{\left(\left(\frac{s}{0.68}\right)^2 + 0.76537\left(\frac{s}{0.68}\right)+1\right)\left(\left(\frac{s}{0.68}\right)^2+1.847\left(\frac{s}{0.68}\right)\right.}$$

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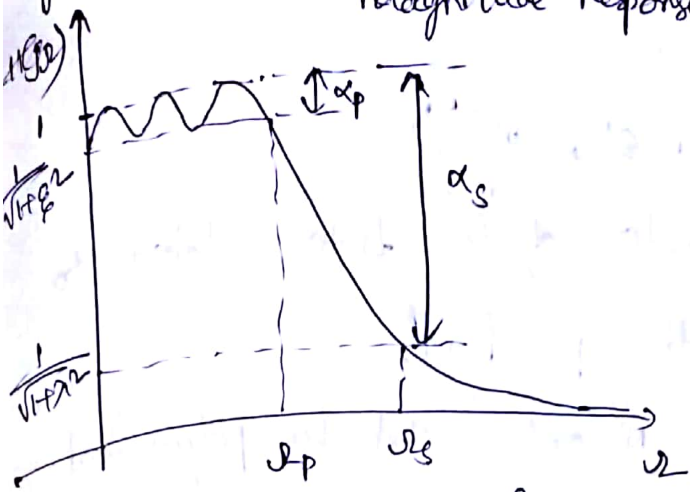
Chebyshev filters

It is of 2 types.

Type-1 → all pole filter,

Magnitude Response

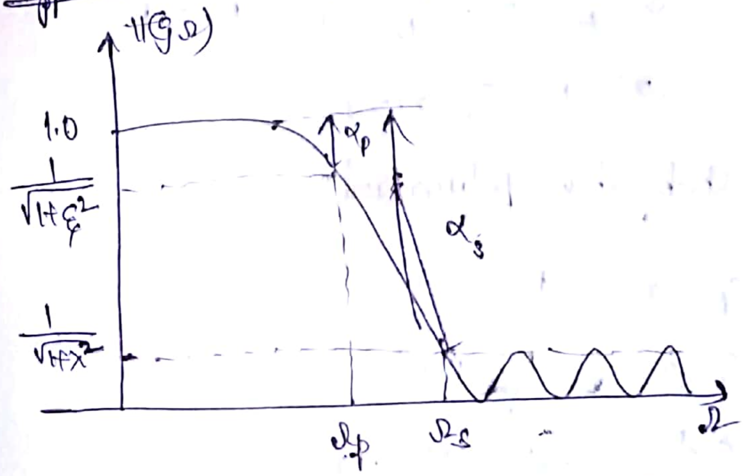


Monotonic in S.B & equiripple behaviour in P.B

N is odd ⇒ ripple starts from maximum value

N " even ⇒ " " " $\frac{1}{\sqrt{1+\varepsilon^2}}$

Type 2 : All pole-zero filter



Monotonic in P.B & equiripple behaviour in S.B.

Design of Chebyshev Type-1 filter:

The magnitude square response of N^{th} order Type-1 filter is $|H(j\Omega)|^2 = \frac{1}{\left(1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)\right)}$

$N = 1, 2, \ldots$

$$H(j\Omega) = \frac{1}{\left(1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)\right)^{1/2}}$$

ε is a parameter of the filter related to the ripple in the pass band.

$C_N(x)$ is N^{th} order chebyshev polynomial.

It is defined as $C_N(x) = \cos(N \cos^{-1}(x))$ for $|x| \leq 1$

$C_N(x) = \cosh(N \cosh^{-1}(x))$ $|x| > 1$ (SB)

Chebyshev polynomial is defined by the recursion formula;

$$C_N(x) = 2x\, C_{N-1}(x) - C_{N-2}(x) \quad N > 1$$

$$C_0(x) = 1, \quad C_1(x) = x$$

Properties of Chebyshev polynomial:

(i) $C_N(x) = -C_N(-x)$ N - odd

$C_N(x) = C_N(-x)$ N - even

$C_N(0) = (-1)^{N/2}$ for N - even

$C_N(0) = 1 \rightarrow$ N - odd

$C_N(1) = 1 \quad \forall N$

$C_N(-1) = 1$. N - even

$C_N(-1) = -1$ N - odd.

(ii) $C_N(x)$ oscillates with equal ripple b/w ± 1

~~for the~~ $|x| \leq 1$

(iii) For all values of 'N', $0 \leq |C_N(x)| \leq 1$ for $0 \leq |x| \leq 1$

$|C_N(x)| > 1$ for $|x| > 1$

(iv) $C_N(x)$ is monotonically increasing, for $|x| > 1$, for all values of 'N'

Order:

$$|H(j\Omega)|^2 = \frac{1}{\left(1 + \varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)\right)}$$

Apply log on both sides

$$20 \log|H(j\Omega)| = 10 \log 1 - 10 \log\left(1 + \varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)\right)$$

At $\Omega = \Omega_p$, $\alpha = \alpha_p$

$$-\alpha_p = 0 - 10 \log\left(1 + \varepsilon^2 C_N^2(1)\right)$$

$$\alpha_p = 10 \log(1 + \varepsilon^2)$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

At $\Omega = \Omega_s$

$$20 \log|H(j\Omega_s)| = 10 \log 1 - 10 \log\left(1 + \varepsilon^2 C_N^2\left(\frac{\Omega_s}{\Omega_p}\right)\right)$$

$$+\alpha_s = 10 \log\left(1 + \varepsilon^2 C_N^2\left(\frac{\Omega_s}{\Omega_p}\right)\right)$$

$$= 10 \log\left(1 + \varepsilon^2\left(\cosh\left(N \cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)\right)\right)^2\right)$$

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$$10^{0.1\alpha_s} - 1 = \varepsilon^2 \left[\cosh\left(N\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)\right)\right]^2$$

$$\sqrt{\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}} = \cosh\left(N \cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)\right)$$

$$\boxed{N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}}$$

N is a non-integral value and it is rounded off to next higher value.

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$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 \cancel{\cos} C_N^2\left(\frac{\Omega}{\Omega_p}\right)}$$

$$\Omega = \frac{s}{j}$$

$$1+\varepsilon^2 C_N^2\left(\frac{-js}{\Omega_p}\right) = 0$$

$$C_N^2\left(\frac{-js}{\Omega_p}\right) = \frac{-1}{\varepsilon^2}$$

$$C_N\left(\frac{-js}{\Omega_p}\right) = \frac{\pm j}{\varepsilon}$$

$$\cos\left[N \cos^{-1}\left(\frac{-js}{\Omega_p}\right)\right] = \pm \frac{j}{\varepsilon}$$

we define $\cos^{-1}\left(\frac{-js}{\Omega_p}\right) = \phi - j\theta$ —— (1)

$$\cos\left[N\phi - jN\phi\right]$$

$= \cos N\phi \cos j(N\theta) + \sin N\phi \sin j(N\theta)$

$= \cos N\phi \cosh(N\theta) + j \sin N\phi \, j \sinh N\theta = \pm \frac{j}{\varepsilon}$

$\cos N\phi \cosh N\theta = 0$ — ①

$\sin N\phi \sinh N\theta = \pm \frac{1}{\varepsilon}$ — ②

$\cosh N\theta > 0$ if θ is real

$\cos N\phi = 0$

$N\phi = \frac{(2k-1)\pi}{2}$ $\quad k = 1, 2, \dots N$

$\phi = \frac{(2k-1)\pi}{2N}$

Substitute ϕ in eqn ②

$\sin \cancel{N}\left(\frac{(2k-1)\pi}{2\cancel{N}}\right) \sinh N\theta = \pm \frac{1}{\varepsilon}$

$\sinh N\theta = \pm \frac{1}{\varepsilon}$

$N\theta = \sinh^{-1} \frac{1}{\varepsilon}$

$$\boxed{\theta = \pm \frac{1}{N} \sinh^{-1}\left(\frac{1}{\varepsilon}\right)}$$

From (i)

$\frac{-js}{\Omega_p} = \cos(\phi - j\theta)$

$s = j\Omega_p(\cos(\phi - j\theta))$

$= j\Omega_p(\cos\phi \cosh\theta + j \sin\phi \sinh\theta)$

$s_k = \Omega_p(-\sin\phi \sinh\theta + j \cos\phi \cosh\theta)$ —, $k = 1, 2, \dots N$

Eqn s_k is simplified using the identity

$\sinh^{-1} x = \ln\left(x + \sqrt{1+x^2}\right)$

$$\sinh^{-1}(\epsilon^{-1}) = \ln\left(\epsilon^{-1} + \sqrt{1+\epsilon^{2}}\right)$$

$$\mu = e^{\sinh^{-1}(\epsilon^{-1})} = \epsilon^{-1} + \sqrt{1-\epsilon^{-2}}$$

$$\sinh\theta = \sinh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right)$$

$$= \frac{e^{\frac{1}{N}\sinh^{-1}(\epsilon^{-1})} - e^{\frac{-1}{N}\sinh^{-1}(\epsilon^{-1})}}{2} \qquad \frac{e^{\theta}+e^{-\theta}}{2}$$

$$\sinh\theta = \frac{\mu^{1/N} - \mu^{-1/N}}{2}$$

$$\cosh\theta = \frac{\mu^{1/N} + \mu^{-1/N}}{2}$$

$$S_k = \Omega_p\left[-\sin\phi\left(\frac{\mu^{1/N}-\mu^{-1/N}}{2}\right) + j\cos\phi\left(\frac{\mu^{1/N}+\mu^{-1/N}}{2}\right)\right]$$

$$S_k = -a\sin\phi + jb\cos\phi$$

$$a = \Omega_p\left(\frac{\mu^{1/N}-\mu^{-1/N}}{2}\right)$$

$$b = \Omega_p\left(\frac{\mu^{1/N}+\mu^{-1/N}}{2}\right)$$

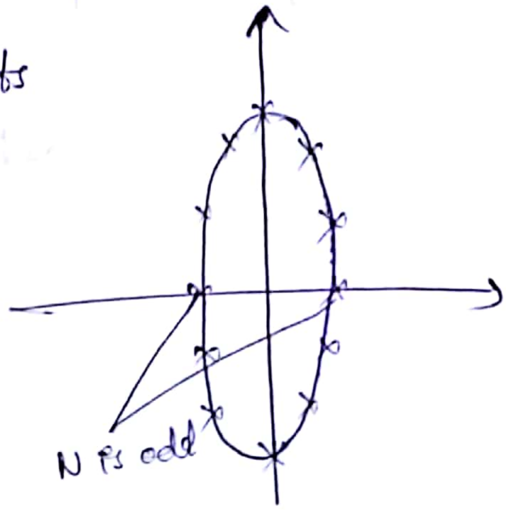
$$S_k = a\left[-\sin\frac{(2k-1)}{2N}\pi\right] + jb\cos\frac{(2k-1)\pi}{2N}$$

$$= a\cos\left(\frac{\pi}{2} + \frac{(2k-1)}{2N}\pi\right) + jb\sin\left(\frac{\pi}{2} + \frac{(2k-1)\pi}{2N}\right)$$

$$\boxed{S_k = a\cos\phi_k + jb\sin\phi_k}$$

The poles of the chebyshev T.F are located on ellipse in the s-plane. represented by .(a & b are major and minor axis of ellipse)

If N is odd, one real pole exists as only left side poles are considered.



Chebyshev type-2 filter:

The magnitude square T.F of chebyshev type-2 filter is $|H(j\Omega)|^2 = \dfrac{1}{1+\varepsilon^2 C_N^2 \left(\dfrac{C_N^2\left(\frac{\Omega_s}{\Omega_p}\right)}{C_N^2\left(\frac{\Omega_s}{\Omega}\right)} \right)}$

$C_N \rightarrow$ Chebyshev polynomial

The zero's are located on imaginary axis given by

$$S_k = \frac{j\Omega_s}{\sin\phi_k} \quad , k = 1, 2, \ldots N.$$

$\hookrightarrow$ same as in type-1 , $\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}$

The poles are located at points (x_k, y_k) where

$$x_k = \frac{\Omega_s \sigma_k}{\sigma_k^2 + \Omega_k^2} \quad k = 1, 2, \ldots k$$

$$y_k = \frac{\Omega_s \Omega_k}{\sigma_k^2 + \Omega_k^2}$$

$$\mu = \lambda + \sqrt{1+\lambda^2}$$

Order of the filter is given by

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{\cosh^{-1}\left(\frac{\lambda}{\varepsilon}\right)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$= \frac{\cosh^{-1} A}{\cosh^{-1}\left(\frac{1}{k}\right)}$$

Steps to design a chebyshev-1 filter:

1) From the given specifications, find the order 'N'

2) Round-off it to the next higher integer value.

3) ~~Step~~ use the formulae and find the values of 'a' & 'b' which are major and minor axes of ellipse

$$a = \Omega_p \left(\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right)$$

$$b = \Omega_p \left(\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right)$$

$\cancel{\mu = \lambda}$ $\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}}$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

4) For normalised chebyshev filter ~~at~~ $\Omega_p = 1$ rad/sec

4) Calculate the poles of the chebyshev filter which lie on the eclipse using the formula

$$S_k = a\cos\phi_k + b\sin\phi_k$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)}{2N}\pi, \quad k = 1, 2, \ldots N$$

5) Find the denominator polynomial of the T.F using above poles.

6) The numerator of the T.F depends on the value of 'N'

N is odd → substitude $s=0$ in the denominator polynomial and find the value. This value = numerator of the T.F (Magnitude response starts from unity for N-odd)

N → even → substitute $s=0$ in the denominator polynomial and divide the result by $\sqrt{1+\epsilon^2}$. This value = numerator of T.F.

Comparison b/w Butterworth and chebyshev:

Butterworth	Chebyshev
→ The frequency response decreases monotonically as freq increases from 0 to ∞	→ The frequency response consists of ripples in either PB or SB
→ poles lie on circle	→ pole lie on eclipse
→ Transition band is more in Butterworth	→ T.B is less than Butterworth
→ ~~Poles~~ No. of poles are more in Butterworth ($\because k = 1$ to $2N$)	→ Poles are less than Butterworth ($\because k = 1$ to N)

→ Order of T.F is more → Order of T.F reduces

(∵ poles are more) (∵ poles are less)

① Given the specifications $\alpha_p = 3dB$, $\alpha_s = 16dB$, $f_p = 1\,kHz$, $f_s = 2kHz$, Design Chebyshev type-I filter.

Sol:

$\Omega_p = 2\pi f_p = \cancel{2\pi\, rad/sec}\ 2\pi \times 10^3\ rad/sec$

$\Omega_s = 2\pi f_s = \cancel{4\pi\, rad/sec}\ 4\pi \times 10^3\ rad/sec$

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

$N \geq 1.91$, $\alpha_s = 16dB$, $\alpha_p = 3dB$

$N = 2$

$\varepsilon = \sqrt{10^{0.1\alpha_p}-1}$

$\cancel{\varepsilon = 1.80}$

$\varepsilon = 0.99$

$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}}$

$\mu = 2.414$

$a = \Omega_p\left(\dfrac{\mu^{1/N} - \mu^{-1/N}}{2}\right)$ $b = \Omega_p\left(\dfrac{\mu^{1/N} + \mu^{-1/N}}{2}\right)$

$= 2\pi \times 10^3\left(\dfrac{(2.414)^{1/2} - (2.414)^{-1/2}}{2}\right)$, $\cancel{b = 2\pi \times 10^3\left(\dfrac{(2.414)^{1/2} + (2.414)^{-1/2}}{2}\right)}$

$a = 2865.3$ $\cancel{b = 6912.59}$ $b = \cancel{6905.65}$

$$b = 2\pi \times 10^{3} \left(\frac{(2.414)^{1/2} + (2.414)^{-1/2}}{2} \right)$$

$$b = 6905.6563$$

4) $$S_k = a \cos \phi_k + j b \sin \phi_k$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \qquad k = 1, 2, \ldots N$$

$$k = 1, 2$$

$$\phi_1 = \frac{3\pi}{4}, \quad \phi_2 = \frac{5\pi}{4}$$

$$S_1 = -643.46\pi + j1554\pi$$

$$S_2 = -643.46\pi - j1554\pi$$

denominator of $H(s) \rightarrow \left((s+643.46\pi)^2 + (1554\pi)^2 \right)$

6) $N=2$

$S=0$ (even) $\Rightarrow \dfrac{(643.46\pi)^2 + (1554\pi)^2}{\sqrt{2}} = (1414.38\pi)^2$

divide with $\dfrac{1}{\sqrt{1+\varepsilon^2}}$

$$H(s) = \frac{(1414.38\pi)^2}{s^2 + 1286\pi s + (1682\pi)^2}$$

② Design an analog chebyshev filter that satisfies

$$\frac{1}{\sqrt{2}} \le |H(j\Omega)| \le 1 \quad \text{for} \quad 0 \le \Omega \le 2$$

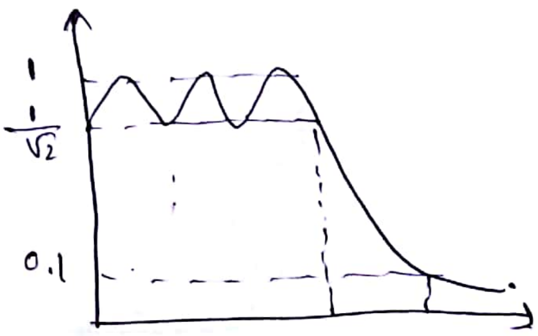
$$|H(j\Omega)| \le 0.1 \quad \text{for} \quad \Omega \ge 4$$

Sol:

$$\frac{1}{\sqrt{1+\varepsilon^2}} = \frac{1}{\sqrt{2}}$$

$$\varepsilon = 1$$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.1$$



$$1+\lambda^2 = 100$$

$$\lambda^2 = 99$$

$$\lambda = 9.94$$

Q) Determine the order and poles of type-1 Low pass chebyshev filter that has 1 dB ripple in the pass band $\Omega_p = 1000\pi$ and $\Omega_s = 2000\pi$ and stop band attenuation $\alpha_s = 40$ dB.

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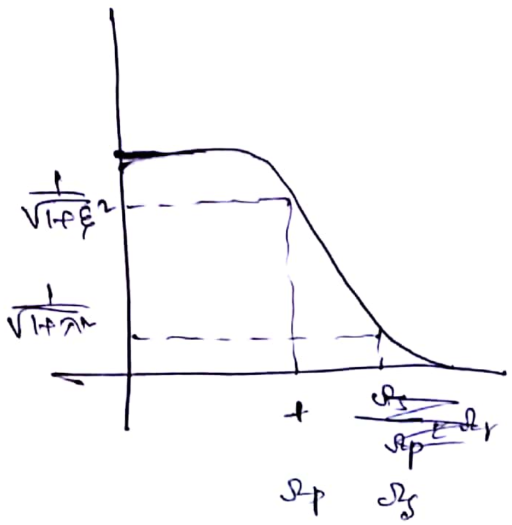
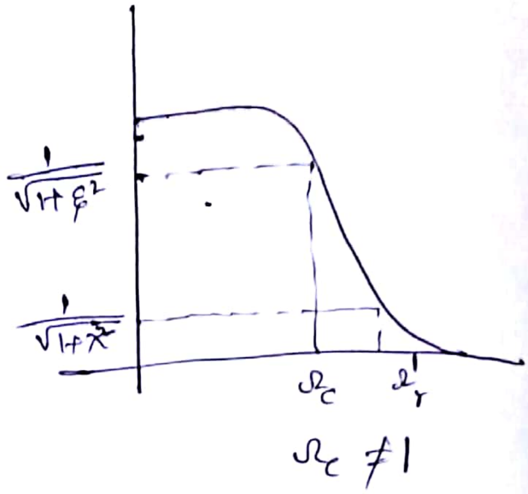
Frequency Transformation in Analog domain

Lowpass to Lowpass filter:





$\Omega_c \neq 1$

$$\left(\Omega_r = \frac{\Omega_s}{\Omega_p}\right)$$

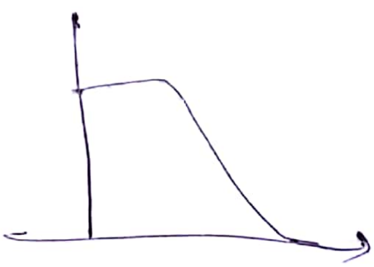
$$H_a(s) = H(s)\Big|_{s \to \frac{s}{\Omega_c}}$$

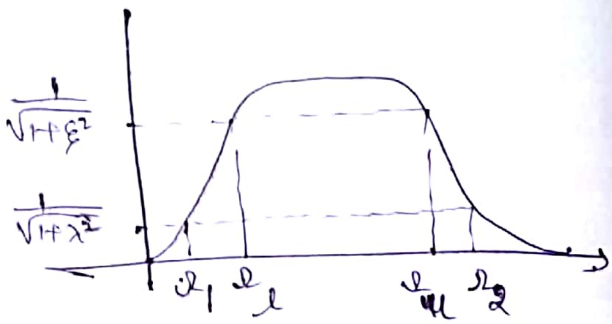
Low to High

$H(s)$

$$H_a(s) = H(s)\Big|_{s \to \frac{\Omega_c}{s}}$$

Lowpass and Bandpass:





$$s \to \frac{s^2+\Omega_l\Omega_u}{s(\Omega_u-\Omega_l)}$$

$$\Omega_r = \min\{|A|, |B|\}$$

$$A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1(\Omega_u - \Omega_l)}$$

$$B = \frac{\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)}$$

Lowpass to Band reject:

$$s \to \frac{s(\omega_u - \omega_l)}{s^2 + \Omega_l \Omega_u}$$

$$\Omega_r = \min\{|A|, |B|\}$$

$$A = \frac{\Omega_1(\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_l \Omega_u}$$

$$B = \frac{\Omega_2(\Omega_u - \Omega_l)}{-\Omega_2^2 + \Omega_l \Omega_u}$$

Q) Given the specifications $\to$ $\alpha_p = 3\,dB$, $\alpha_s = 10\,dB$, $\Omega_p = 1000$ rad/s, $\Omega_s = 500$ rad/s, design a HP Butterworth filter.

Sol: # For : low pass,

$\Omega_p = 500$ rad/sec, $\Omega_s = 1000$ rad/sec

~~$N \geq \log\sqrt{0.1}$~~

$$N \geq \frac{\log\sqrt{\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

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$N \geq 2.64$

$\Omega_c = \Omega_p = 1000 \quad \left(\dfrac{\omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}}\right)$

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$H_a(s) \longrightarrow s \rightarrow \dfrac{\Omega_c}{s}$

$$H_a(s) = \frac{s^3}{(1000+s)((1000)^2 + 1000s + s^2)}$$

Q) Design a chebyshev Type-I filter with a maximum PB attenuation of 2.5 dB at $\Omega_p = 20$ rad/sec and SB attenuation of 30 dB at $\Omega_s = 50$ rad/sec

Sol
$\alpha_p = 2.5$ dB

$\alpha_s = 30$ dB

$$N = \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)} = \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\times 30} - 1}{10^{0.1\times 2.5} - 1}}}{\cosh^{-1}\left(\dfrac{50}{20}\right)}$$

$N \geq 2.72$

$N = 3$

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$\varepsilon = \sqrt{10^{0.1\alpha_p}-1} = \sqrt{10^{0.1\times 2.5}-1} = 0.8822$

$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.645$

$\cancel{\Omega_p}$

$a = \Omega_p\left(\frac{\mu^{1/N}-\mu^{-1/N}}{2}\right)$, $b = \Omega_p\left(\frac{\mu^{1/N}+\mu^{-1/N}}{2}\right)$

$= 20\left(\frac{(2.645)^{1/3}-(2.645)^{-1/3}}{2}\right)$, $b = 20\left(\frac{(2.645)^{1/3}+(2.645)^{-1/3}}{2}\right)$

$a = 6.598$

$b = 24.088$

$\cancel{s_1 = 6.5}$ $s_k = a\cos\phi_k + jb\sin\phi_k$

$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}$, $k = 1, 2, \ldots N.$

$\phi_1 = \frac{2\pi}{3}$, $\phi_2 = \pi$, $\phi_3 = \frac{4\pi}{3}$

$s_1 = 6.598\cos\frac{2\pi}{3} + j\,24.088\sin\left(\frac{2\pi}{3}\right)$

$s_2 = 6.598\cos\pi + j\,24.088\sin\pi$

$s_3 = 6.598\cos\frac{4\pi}{3} + j\,24.088\sin\left(\frac{4\pi}{3}\right)$

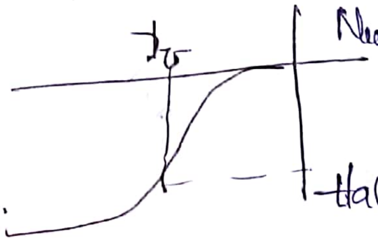
$s_1 = -3.299 + j20.8608$, $s_2 = -6.598$

$s_3 = -3.299 - j20.8608$

denominator $\Rightarrow H(s) \rightarrow (s-6.598)\left((s+3.299)^2+(20.8608)^2\right)$ —(1)

Numerator, $N=3$ odd $\Rightarrow$ $s=1$ in (1)

$\Rightarrow (6.598)\left((3.299)^2+(20.8608)^2\right)$



Numer = -2943.07998

$$H_a(s) = \frac{-2943.07998}{(s-6.598)\left((s+3.299)^2+(20.8608)^2\right)}$$

$$x(t) = 2t^2 - 3t + 6$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{\cancel{2t^2} - 3t + 6 - (\cancel{2t^2} + 3t + 6)}{2}$$

$$= \frac{-6t}{2} \quad (1)$$

$$= -3t$$

27/2/18

Design of digital IIR filters from Analog filters:

Four techniques:

1) Approximation of derivatives
2) Impulse invariant transformation.
3) Bilinear transform.
4) Matched z-transform.

These are used to convert H(s) (analog) to H(z) (discrete)

H(s) H(z)

→ If the poles are on the $j\Omega$ axis, they are mapped on the ~~onto~~ unit circle.

→ If the poles lie in LH s-plane, they are mapped onto inside ~~unite~~ unit cle

→ " " " " RH " , " " " outside the unit cle

→ If the poles lie on LH of splane or $j\Omega$ axis, analog filter is stable

→ " " " " unit cle or inside unit cle, then digital filter is stable.

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1) Approximation of derivatives:

→ Approximate ~~D.t~~ differential eqn into an equivalent difference eqn.

For the derivative

$$\left.\frac{dy(t)}{dt}\right|_{t=nT} = \frac{y(nT) - y(nT-T)}{T}$$

$$y(n) = y(nT)$$

$$\left.\frac{dy(t)}{dt}\right|_{t=nT} = \frac{y(n) - y(n-1)}{T}$$

$$y(t) \longrightarrow \boxed{s} \longrightarrow \frac{dy(t)}{dt}$$

$$y(n) \longrightarrow \boxed{\frac{1-z^{-1}}{T}} \longrightarrow \frac{y(n) - y(n-1)}{T}$$

$$s = \frac{1-z^{-1}}{T}$$

$$H(z) = H(s)\Big|_{s = \frac{1-z^{-1}}{T}}$$

The system function of digital IIR filter obtained as a result of the approximation of derivatives by finite difference eqn is

$$H(z) = H(s)\Big|_{s = \frac{1-z^{-1}}{T}}$$

$$S = \frac{1-z^{-1}}{T}$$

$$z = \frac{1}{1-ST} = \frac{1}{1-j\Omega T}$$

$$z = \frac{1+j\Omega T}{1+\Omega^2 T^2}$$

$$z = \frac{1}{1+\Omega^2 T^2} + \frac{j\Omega T}{1+\Omega^2 T^2}$$

→ It is suitable for low pass and band pass frequencies (i.e. LPF & BPF)

∴ Only LPF & BPF can only be designed using this method.

→ To design a digital filter, first design analog filter and then substitute $S = \frac{1-z^{-1}}{T}$

$$H(z) = H(s)\Big|_{S=\frac{1-z^{-1}}{T}}$$

2) Impulse invariant Transformation:

$\delta(t)\Big|_{n=nT} = \delta(n)$, $\delta(t)$ is sampled at interval 'T'

$$h(n) = \sum \delta(n-T)$$

The z-transform of infinite impulse response is

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$H(z)\Big|_{z=e^{sT}} = \sum_{n=0}^{\infty} h(n)\, e^{-sTn}$$

Let us consider the mapping of points from s-plane to z-plane implied by the relation $z = e^{+sT}$

$$s = \sigma + j\Omega$$

$$z = e^{(\sigma + j\Omega)T}$$

$$r.e^{j\omega} = \left(e^{\sigma t}\right) e^{j\Omega T}$$

$$r = e^{\sigma T} \qquad \omega = \Omega T$$

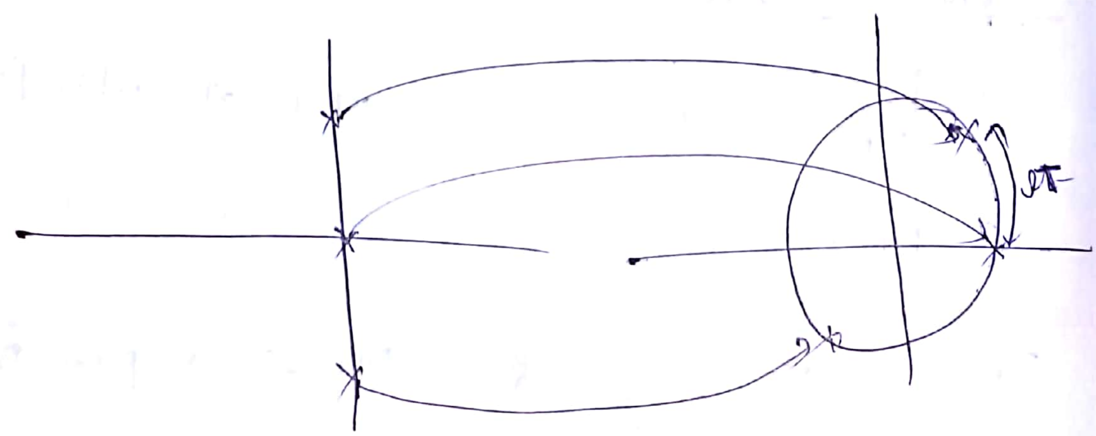
Real part gives radius

Imag. " " angle.

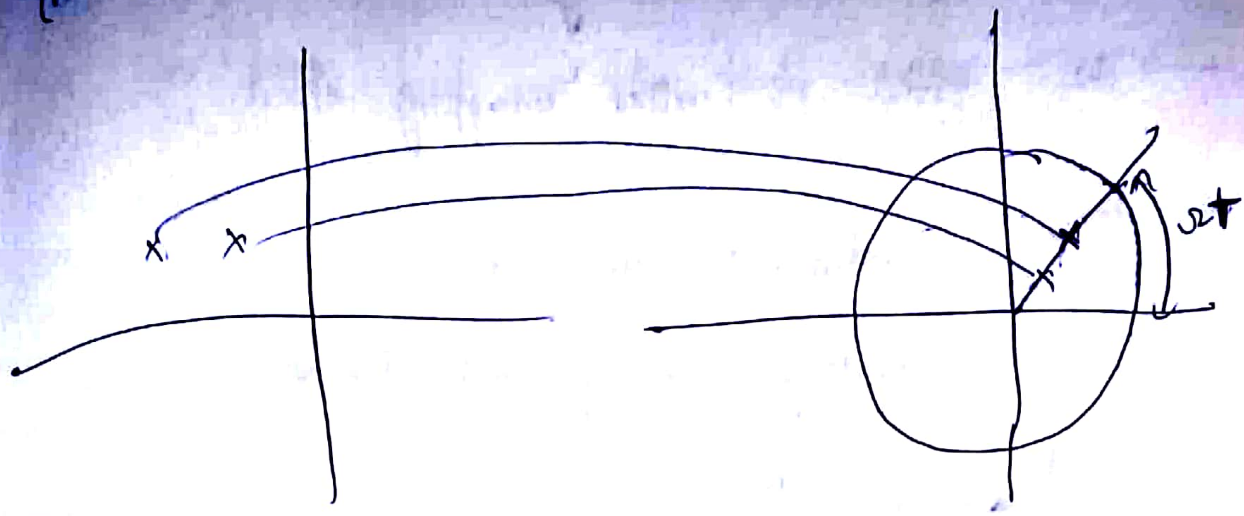
→ If the poles are present on Imag. axis,

(i) $\underline{\sigma = 0}$

$$r = e^{\sigma t} = e^{0t} = 1$$

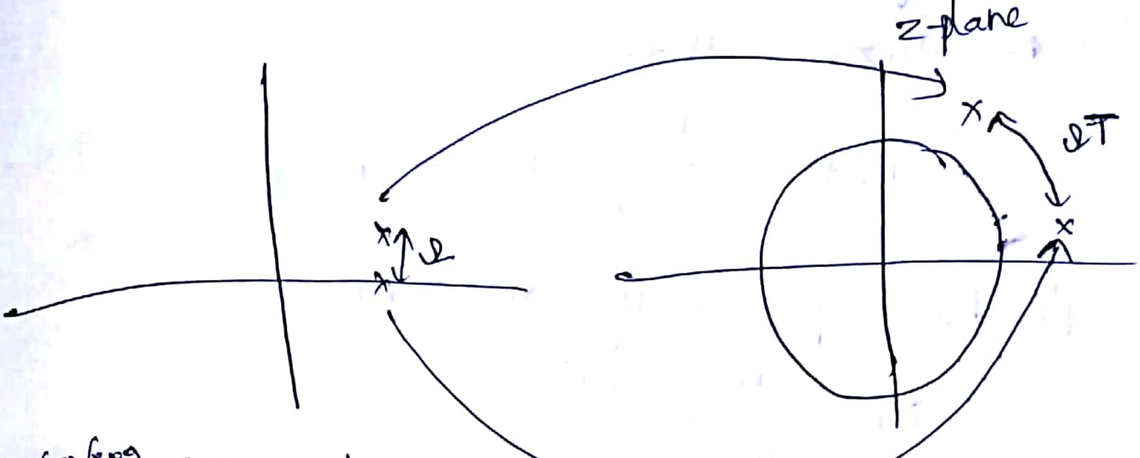


(ii) $\sigma < 0$



(iii) $\sigma > 0 \rightarrow$ poles are on RHS of s-plane

(outside the circle)

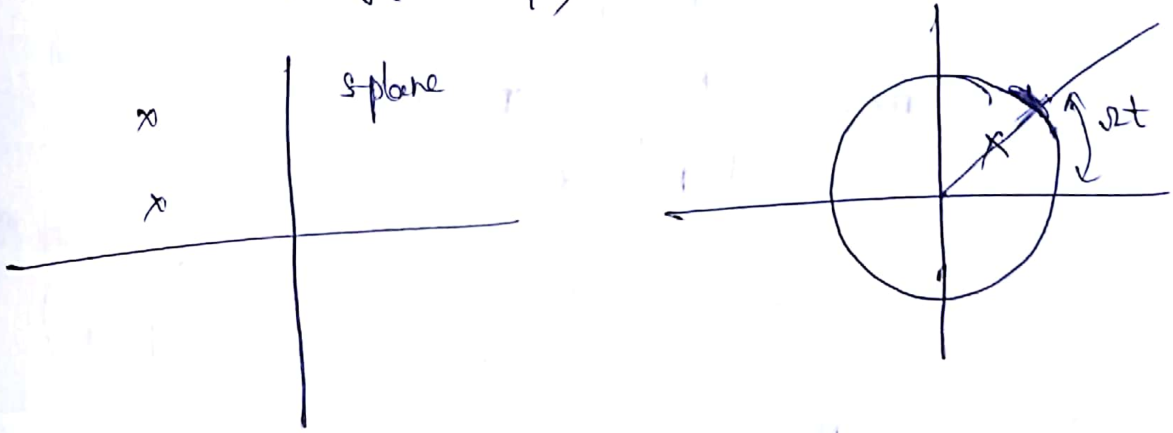


(i), (ii), (iii) are one to one mapping

(iv) If s_1 & s_2 are two poles (if seperated by $\frac{2\pi}{T}$)

$s_1 = \sigma_1 + j\Omega T$

$s_2 = \sigma_1 + j\left(\Omega + \frac{2\pi}{T}\right) T$



. It is many to one mapping.

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→ This is the disadvantage in this method.

→ This effect is called 'wrapping effect'

→ $H(s) \rightarrow$ analog ~~I.F~~ filter T.F

$H_a(s) \rightarrow$ dividing into partial fractions.

$$\begin{array}{ccc} h_a(t) & \xleftrightarrow{ILT} & H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s-p_k} \\ \downarrow t=nT & & \downarrow \\ h(n) & \longleftrightarrow & H(z) \end{array} \quad \cancel{H(z)}$$

$$h_a(t) = \sum_{k=1}^{N} c_k \, e^{p_k t} \qquad t \geq 0$$

Sample $h_a(t)$ periodically at $t = nT$

$$h(n) = \sum_{k=1}^{N} c_k \, e^{p_k nT}$$

$$H(z) = \sum_{n=0}^{\infty} h(n) \, z^{-n}$$

$$= \sum_{n=0}^{\infty} \sum_{k=1}^{N} c_k \, e^{p_k nT} \, z^{-n}$$

$$H(z) = \sum_{k=1}^{N} c_k \sum_{n=0}^{\infty} \left(e^{p_k T} z^{-1}\right)^n$$

$$= \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}} \qquad \left(\frac{a}{1-r}\right)$$

$$\boxed{H(s) = \sum_{k=1}^{N} \frac{c_k}{s-p_k} \longrightarrow H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}}$$

for high sampling rate, the digital filter gain is also high ~~and~~ and for that digital filter is

$$H(z) = \sum_{k=1}^{N} \frac{T c_k}{1 - e^{p_k t} z^{-1}}$$

Steps to design

→ For the given specifications, find $H_a(s)$ i.e. T.F of the analog filter.

→ Select the sampling rate of the digital filter 'T'

→ Express the analog filter T.F as the sum of single pole filter $H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s-p_k}$

→ compute the z-Transform of the digital filter by using the formula

$$H(s) = \sum_{k=1}^{N} \frac{c_k}{s-p_k} \Rightarrow H(z) = \sum_{k=1}^{N} \frac{c_k}{1-e^{p_k T} z^{-1}}$$

① For the analog filter T.F $H(s) = \frac{2}{(s+1)(s+2)}$. Transform into digital form with sampling interval T=1 sec using impulse invariant

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transform.

$$H(s) = \frac{2}{(s+1)(s+2)}$$

$T=1$

$$H_a(s) = \frac{2}{s+1} - \frac{2}{s+2}$$

$$H_a(s) = \frac{2}{s-(-1)} - \frac{2}{s-(-2)}$$

$$H(z) = \frac{2}{1-e^{-1(1)}z^{-1}} - \frac{2}{1-e^{-2}z^{-1}}$$

$$H(z) = \frac{2}{1-0.368z^{-1}} - \frac{2}{1-0.135z^{-1}}$$

Q) Design a 3rd order BW filter using impulse invariant technique, assuming sampling period T=1 sec.

Sol: $H(s) = \frac{1}{(s+1)(s^2+s+1)}$ (from table)

$T = 1$ sec

$$H(s) = \frac{1}{(s+1)(s+0.5-j0.866)(s+0.5+j0.866)}$$

$$= \cancel{\frac{-0.49+0.866j}{s+1} + -140.}$$

Partial fractions.

$A = -0.49 + j\,0.866$, $C = -0.49 - j\,0.866$

$$H(z) = \frac{1}{1 - 0.368 z^{-1}} + \frac{-1 + 0.66 z^{-1}}{1 - 0.786 z^{-1} + 0.368 z^{-2}}$$

3) Design of ~~IIR~~ filter using Bilinear transformation:

→ Mapping is done from splane to zplane same as method ① and method ② ~~usin~~

→ The disadvantage of Impulse invariant transform i.e many to one is overcome in bilinear transformation. using pre-warping.

Let $H(s) = \frac{b}{s+a}$ T.F of Analog filter.

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a}$$

$$sY(s) + aY(s) = bX(s)$$

$$\frac{dy(t)}{dt} + a\,y(t) = b\,x(t)$$

$$y'(t) = -a\,y(t) + b\,x(t) \quad \text{--- (1)}$$

* $y(t)$ is approximated by the trapezoidal formula

$$y(t) = \int_{t_0}^{t} y'(\tau)\,d\tau + y(t_0)$$

derivative of $y(t)$

~~deriv~~

The approximation of the integral by the trapezoidal formula at $t = nT$ ~~ant~~ and $t_0 = nT - T$ is

$$y(nT) = \frac{T}{2}\left[y'(nT) - y'(nT - T)\right] + y(nT - T)$$

From (1)

$$y(nT) = \frac{T}{2}\left[-a\,y(nT) + b\,x(nT) \mp a\,y(nT - T) + b\,x(nT - T)\right] + y(nT - T)$$

$$y(nT) = \frac{-aT}{2}\,y(nT) \mp \frac{aT}{2}\,y(nT - T) + y(nT - T) + \frac{bT}{2}\,x(nT) + \frac{bT}{2}\,x(nT - T)$$

~~$= \frac{-aT}{2}\,y(nT)$~~

$$y(nT) + \frac{aT}{2}\,y(nT) - \left(1 - \frac{aT}{2}\right)y(nT - T) = \frac{bT}{2}\left[x(nT) + x(nT - T)\right]$$

$y(n) = y(nT)$, $x(n) = x(nT)$ (Continuous-discrete

$$\left(1+\frac{aT}{2}\right) y(n) - \left(1-\frac{aT}{2}\right) y(n-1) = \frac{bT}{2}\left[x(n) + x(n-1)\right]$$

Apply z - Transform

$$\left(1+\frac{aT}{2}\right) Y(z) - \left(1-\frac{aT}{2}\right) z^{-1} Y(z) = \frac{bT}{2}\left[1+z^{-1}\right) X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{\frac{bT}{2}(1+z^{-1})}{\left(1+\frac{aT}{2}\right) - \left(1-\frac{aT}{2}\right)z^{-1}}$$

$$H(z) = \frac{\frac{bT}{2}(1+z^{-1})}{\cancel{(1-z^{-1})} (1-z^{-1}) + \frac{aT}{2}(1+z^{-1})}$$

Multiply and divide with $\frac{T}{2}(1+z^{-1})$

$$\cancel{H(z) = \frac{bT^2}{}}$$

$$H(z) = \frac{b\frac{T}{2}(1+z^{-1})}{(1-z^{-1}) + \frac{aT}{2}(1+z^{-1})}$$

**

$$\boxed{H(z) = \frac{b}{\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + a}}$$

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

** This relation b/w s & z is known as Bilinear transform $\boxed{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

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Let $z = r.e^{j\omega}$

$$= \frac{2}{T}\left[\frac{re^{j\omega}-1}{re^{j\omega}+1}\right]$$

$$= \frac{2}{T}\left[\frac{r\cos\omega + jr\sin\omega - 1}{r\cos\omega + jr\sin\omega + 1}\right]$$

$$\Rightarrow \frac{2}{T}\frac{((r\cos\omega - 1) + jr\sin\omega))\ [(r\cos\omega + 1) - jr\sin\omega]}{(r\cos\omega+1)^2 + r^2\sin^2\omega}$$

$$= \frac{2}{T}\left[\frac{r^2-1}{1+r^2+2r\cos\omega} + j\frac{2r\sin\omega}{1+r^2+2r\cos\omega}\right]$$

$\sigma + j\omega$

$$\sigma = \frac{2}{T}\left[\frac{r^2-1}{1+r^2+2r\cos\omega}\right], \quad \Omega = \frac{2}{T}\left[\frac{2r\sin\omega}{1+r^2+2r\cos\omega}\right]$$

$r<1 \rightarrow \sigma < 0$ (-ve) (LHS)

$r>1 \rightarrow \sigma > 0$ (+ve) (RHS)

$r=1 \rightarrow \sigma = 0$

$r=1$

$$\Rightarrow \Omega = \frac{2}{T}\left[\frac{2\sin\omega}{2(1+\cos\omega)}\right]$$

$$= \frac{2}{T}\left[\frac{2\sin\omega/2\ \cos\omega/2}{2\cos^2\omega/2}\right]$$

$$= \frac{2}{T}\tan\omega/2$$

For small values of θ

$\tan\theta = \theta$

For small values of ω

$$\Omega = \frac{\cancel{2}}{T}\frac{\omega}{\cancel{2}}$$

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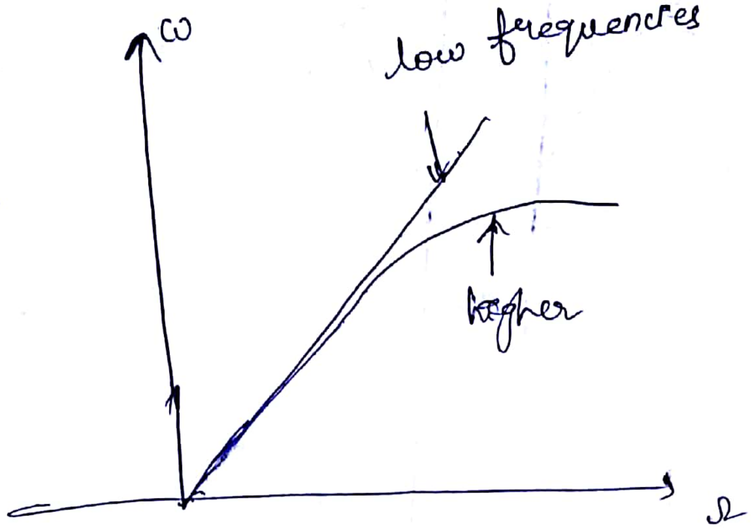
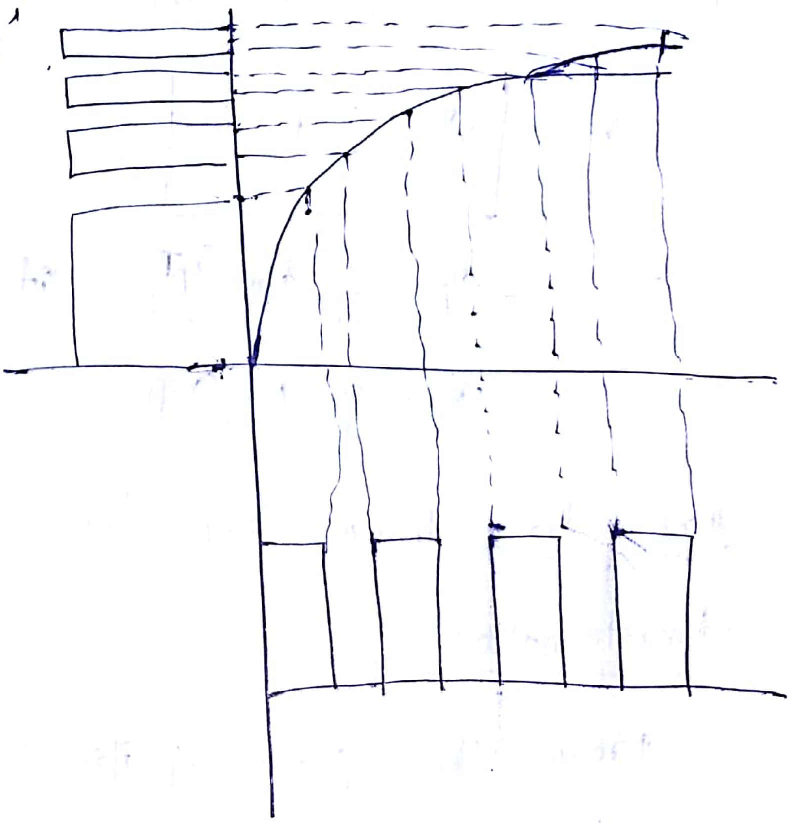
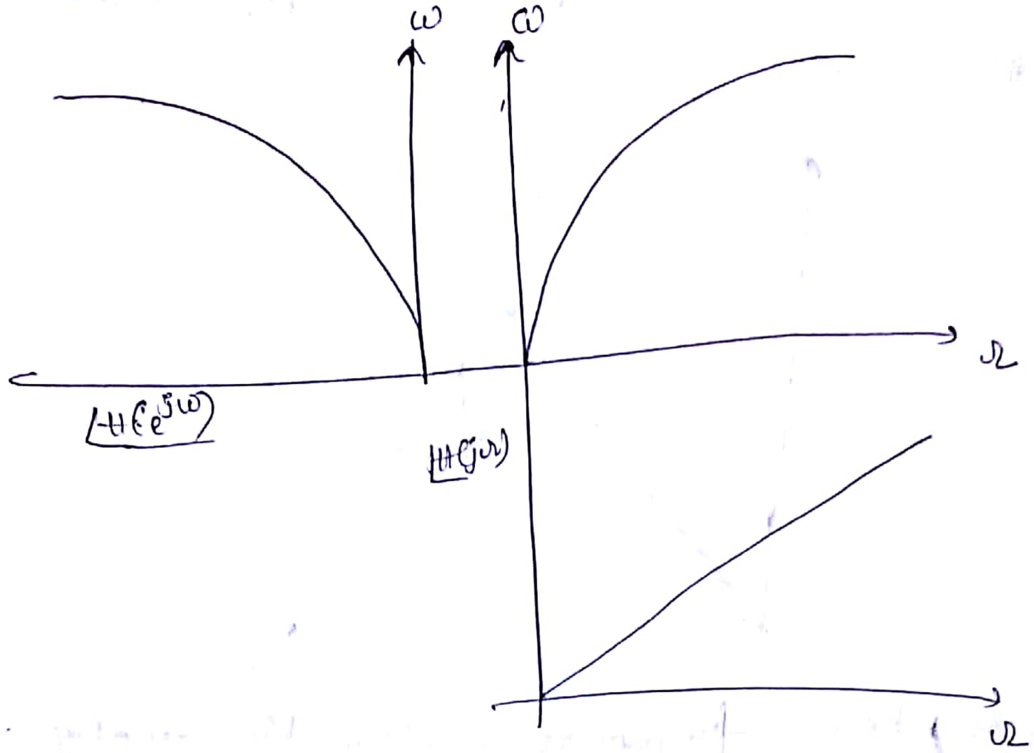
$\boxed{\omega = \Omega T}$

At low frequencies, the relation b/w analog and digital filter frequencies is linear



→ At higher frequencies, relation b/w ~~analog~~ Ω & ω becomes nonlinear and the distortion is introduced in the frequency scale of digital filter to that of analog filter. This is known as 'Wrapping effect'.





The wrapping effect can be eliminated by pre-wrapping the analog filter. This can be done by analog frequencies using the formula

$$\boxed{\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)}$$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p T}{2}\right) \quad \text{if } T \text{ is not given}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s T}{2}\right) \qquad \Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

<u>Steps to design IIR filter using bilinear transformation</u>

→ From the given specifications, find prewrapping analog frequencies using the formulas

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right)$$

→ Using analog frequency, find H(s) of the analog filter.

→ Select the sampling rate of the digital filter 'T' T - sec/sample

→ ~~Sub~~ Substitute $s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ (or) $s = \left(\frac{z-1}{z+1}\right)$ in H(s)

Q) Design a digital filter using bilinear transformation for the T.F $H(s) = \frac{2}{(s+1)(s+2)}$ with $T = 1$ sec.

Sol: $H(s) = \frac{2}{(s+1)(s+2)}$ $T = 1$ sec

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{2}{\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right)\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right)}$$

$$H(z) = \frac{(1+z^{-1})^2}{6-2z^{-1}}$$

② Using a bilinear transformation, design a high pass ~~filter~~ monotone in P.B with $f_c = 1000$ & down 10dB at 350Hz & $f_s = 5000$Hz

Sol:

$f_s = 350$ Hz at $\alpha_s = 10$dB

$f_p = 1000$Hz $\alpha_p = 3$dB

$f_s = 5000$Hz

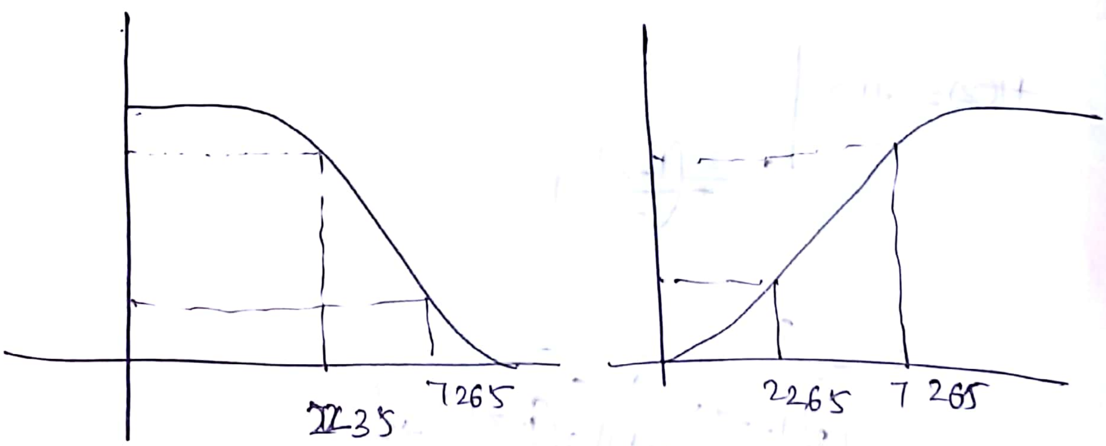
$$T = \frac{1}{f_s} = \frac{1}{5000} = 0.2 \times 10^{-3}$$

$\omega_p = 2\pi \times 1000 = 2000\pi$, $\omega_s = \overset{350}{\cancel{5000}} \times 2\pi = 10000\pi$

$$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p T}{2}\right) = 7265 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s T}{2}\right) = \frac{2}{0.2 \times 10^{-3}} \tan\left(\frac{2\pi \times 350}{2}\right)$$

$= 2235$ rad/sec



For low pass filter,

$\Omega_p = 2235$ rad/sec

$\Omega_s = 7265$ rad/sec

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$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$N = 1$

$H(s) = \frac{1}{s+1}$ Here $\Omega_c = \Omega_p = 2235$ (∵ No need of $\frac{s}{\Omega_c}$)

$$H_a(s) = H(s)\Big|_{s = \frac{\Omega_c}{s} = \Omega_p}$$

$$= H(s)\Big|_{\frac{7265}{s}}$$

$$H_a(s) = \frac{1}{\left(\frac{7265}{s}\right) + 1}$$

$$= \frac{s}{s + 7265}$$

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{\frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$= \frac{0.5792(1-z^{-1})}{1-0.158z^{-1}}$$

Q) Design a digital butterworth filter for the specifications $0.707 \le H(e^{j\omega}) \le 1$ for $0 \le \omega \le \frac{\pi}{2}$
↳ digital

$|H(e^{j\omega})| \le 0.2$ $\quad \frac{3\pi}{4} \le \omega \le \pi$ $\quad T = 1$ sec using

a) bilinear transformation.

b) Impulse invariant "

Sol:

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.707 \qquad \omega_p = \frac{\pi}{2}$$

$$\varepsilon = 1 \qquad \omega_s = \frac{3\pi}{4}$$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$$\lambda = 2$$

$$\sqrt{1+\lambda^2} = \frac{1}{0.2} = 5$$

$$1+\lambda^2 = 25$$

$$\lambda = 4.89$$

$$N \ge \frac{\log \frac{\lambda}{\varepsilon}}{\log \frac{\Omega_s}{\Omega_p}}$$

$$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right) = 2$$

$$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right) = 2\tan\left(\frac{\frac{3\pi}{4}}{2}\right) =$$

~~cos~~ $\frac{\Omega_s}{\Omega_p} = 2.414$

$$N \geq \frac{\log(4.89)}{\log(2.414)}$$

$N \geq 1.8$

$N = 2$

$$H(s) = \frac{1}{s^2 + \sqrt{2}\, s + 1}$$

$$\Omega_p = \frac{2}{T} \tan \frac{\pi}{4} = 2$$

$$\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/2}} = \frac{2}{(1)^{1/2}} = 2 \text{ rad/sec}$$

$$H_a(s) = H(s) \Big|_{s \to \frac{s}{\Omega_s} \to \frac{s}{2}}$$

$$= \frac{1}{\left(\frac{s}{2}\right)^2 + \sqrt{2}\left(\frac{s}{2}\right) + 1}$$

$$= \frac{4}{s^2 + 2\sqrt{2}s + 4} = \frac{4}{s^2 + 2.82s + 4}$$

$$H(z) = H(s) \Big|_{s \to \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)}$$

$$H(z) = \frac{4}{4\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)^2 + 2.828 \times 2\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) + 4}$$

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(11)

$\varepsilon = 1$

$\lambda = 4.89$

$\omega = \Omega T$

$\Omega_s = \frac{\omega_s}{T}$, $T=1$

$\Omega_p = \frac{\omega_p}{T}$

$$N \geq \frac{\log\left(\frac{\lambda}{\varepsilon}\right)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\log\left(\frac{4.89}{1}\right)}{\log\left(\frac{\frac{\pi}{2}}{\frac{3\pi}{4}}\right)}$$

$N \geq 3.9$

$N = 4$

$$H(s) = \frac{1}{(s^2 + 0.76537s + 1)(s^2 + 1.8477s + 1)}$$

$$\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/N}} = \frac{\pi}{2} = \cancel{1.0}\ 1.57$$

$$H_a(s) = H(s)\Big|_{s \rightarrow \frac{s}{1.57}}$$

$\frac{A}{} + A^* + \frac{B}{1} + B^*$

$\frac{C_k}{s - p_k} \rightarrow \frac{C_k}{\ldots}$

(Q) Design a Chebyshev LPF with the specifications

$\alpha_p = 1dB$, ripple in the pass band, $0 \le \omega \le 0.2\pi$,

$\alpha_s = 15dB$ and $0.3\pi \le \omega \le \pi$ using a) bilinear.

b) Impulse invariant

Sol:

$\alpha_p = 1dB$ $\qquad \omega_p = 0.2\pi$

$\alpha_s = 15dB$ $\qquad \omega_s = 0.3\pi$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = 0.649$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = 1.019$$

Chebyshev type-2

$$N \ge \frac{\cancel{\log}\ \cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$N \ge 3.009$

$N = 4$

$$a = \Omega_p\left(\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right), \quad b = \Omega_p\left(\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right)$$

$$\mu = \varepsilon^{-1} + \sqrt{1 - \varepsilon^{-2}} = 4.017$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = 0.508$$

$a = 0.237$ $\qquad b = 0.691$

$\cancel{\phi_2 =}$ $\phi = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}$, $k = 1, 2, 3, 4$

$\phi_1 = \frac{5\pi}{8}$

$\phi_2 = \frac{7\pi}{8}$

$\phi_3 = \frac{9\pi}{8}$

$\phi_4 = \frac{11\pi}{8}$

$s_k = a \cos\phi_k + jb \sin\phi_k$

$\cancel{s_1 = 0.23 + j0.047}$

$\cancel{s_2 =}$ $s_1 = -0.091 + j0.6381$

$s_2 = -0.2185 + j0.2643$

$s_3 = -0.2185 - j0.2643$

$s_4 = -0.091 - j0.6381$

denum $H(s) = \left[(s+0.091)^2 + (0.6381)^2\right]\left[(s+0.2185)^2 + (0.2643)^2\right]$

If N-even ⇒ substitute $s=0$ in denum of $H(s)$

& divide result by $\frac{1}{\sqrt{1+\varepsilon^2}}$

$\cancel{H(s) = 0.0489}$

$\frac{}{\sqrt{1+0.}}$

$$H(s) = \frac{\frac{0.0489}{\sqrt{1+(0.508)^2}}}{(s^2+1.814s+0.4165)(s^2+0.4378s+0.118)}$$

$$= \frac{0.0438}{(s^2+1.814s+0.4165)(s^2+0.4378s+0.118)}$$

Now, $s \to \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$

$H(z) =$

b) Impulse invariant

$$\Omega_p = \frac{\omega_p}{T} = \omega_p = 0.2\pi$$

$$\Omega_s = \frac{\omega_s}{T} = \omega_s = 0.3\pi$$

$$N \geq \frac{\cosh^{-1}\left(\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}\right)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$\mu = \epsilon^{-1} + \sqrt{1+\epsilon^{-2}}$

$N = 4$, $\epsilon = \sqrt{10^{0.1\alpha_p}-1} = 0.568$, $\mu = 4.17$

$$a = \Omega_p\left(\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right), \quad b = \Omega_p\left(\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right)$$

$a = 0.364$ $\qquad$ $b = 1.064$

$$\phi = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad , k = 1, 2, 3, 4$$

$$\phi_1 = \frac{5\pi}{8}, \quad \phi_2 = \frac{7\pi}{8}, \quad \phi_3 = \frac{9\pi}{8}, \quad \phi_4 = 11\pi/8$$

$$S_k = a\cos\phi_k + j\, b\sin\phi_k$$

$$S_1 = -0.0876 + j0.619$$

$$S_2 = -0.2115 + j\,0.2564$$

$$S_3 = -0.2115 - j0.2564$$

$$S_4 = -0.0876 - j0.619$$

denominator of

$$H(s) = \left[(s+0.0876)^2 + (0.619)^2\right]\left[(s+0.2115)^2 + (0.2564)^2\right]$$

If N is even, substitute $s=0$ and divide by $\sqrt{1+\varepsilon^2}$

num of $H(s) = 0.03834$ $\qquad \dfrac{0.0431}{\sqrt{1+(0.508)^2}} = 0.03834$

$$H(s) = \frac{0.03834}{(s^2+0.175s+0.391)(s^2+0.423s+0.11)}$$

$$H(s) = \frac{A}{s-(-0.0876+j0.619)} + \frac{B}{s-(-0.2115+j0.2564)}$$

$$+ \frac{B^*}{s-(-0.2115-j0.2564)} + \frac{A^*}{s-(-0.0876-j0.6\ldots)}$$

$A = -0.0143 + j\,0.0814$

$B = 0.0143 - j\,0.2166$

$$\Rightarrow \frac{c_k}{s-p_k} \rightarrow \frac{c_k^*}{1-e^{p_k T} z^{-1}}$$

$$H(z) = \frac{A}{1 - e^{-0.0876 + j0.619} z^{-1}} + \frac{B}{1 - e^{-0.2115 + j0.2564} z^{-1}}$$

$$+ \frac{A^*}{1 - e^{-0.0876 - j0.619} z^{-1}} + \frac{B^*}{1 - e^{-0.2115 - j0.2564} z^{-1}}$$

5/3/18

UNIT-V

Multirate signal processing:

→ Converting from one sampling rate to another sampling rate is called multirate signal processing ~~or~~ when our application requires systems with more than one sampling rate.

Basic operations in Multirate:

→ Increasing sampling rate → up sampling &

→ Decreasing " " → down sampling

Down sampling:

~~If~~ To decrease the sampling rate of $x(n)$ by M, every M^{th} sample of $x[n]$ is taken and ~~is~~ $M-1$ samples are removed

$x[n] \longrightarrow M$

Taking every M^{th} sample

$$y(n) = x(Mn)$$

Let $M = 2$

$M-1 = 1$

Ex: $x(n) = \{1, \underbrace{2, -1}_{\times}, 4, \underbrace{5, 6}_{\times}, 7, \underbrace{6, -3}_{\times}\}$

$M = 3$

$y(n) = \{1, 4, 7\}$

$\downarrow M \Rightarrow$ down sampling.

$$x(n) \longrightarrow \boxed{\downarrow M} \longrightarrow y(n) = x(Mn)$$

Spectrum of the down sampled signal:

Let $x(n)$ be a sequence

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

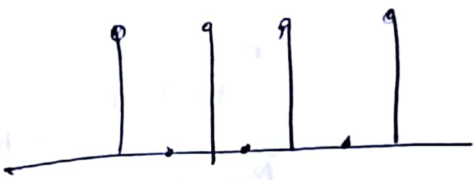
To obtain the down sampled signal, multiply $x(n)$ with train of impulses with period M

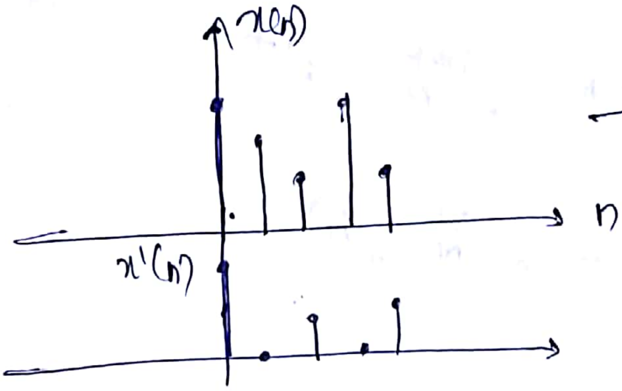
$p(n) \rightarrow$ with period M

$\hookrightarrow x(n)$

$$x'(n) = x(n) p(n)$$

Let $M=2$





$$y(n) = x'(nM) \quad (\because \text{to remove zeros})$$

$$= x(nM)\, p(nM)$$

$\rightarrow p(n)$ is a periodic train of impulse represented by

$$p(n) = \begin{cases} 1 & n = 0, \pm M, \pm 2M \ldots \\ 0 & \text{otherwise} \end{cases}$$

$$\cancel{p(n)}\; p(n) = \frac{1}{M} \sum_{k=0}^{M-1} e^{\frac{j2\pi kn}{M}}$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x'(nM)\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x'(n)\, z^{-n/M}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, p(n)\, z^{-n/M}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \frac{1}{M} \sum_{k=0}^{M-1} e^{j2\pi k \frac{n}{M}}\, z^{-n/M}$$

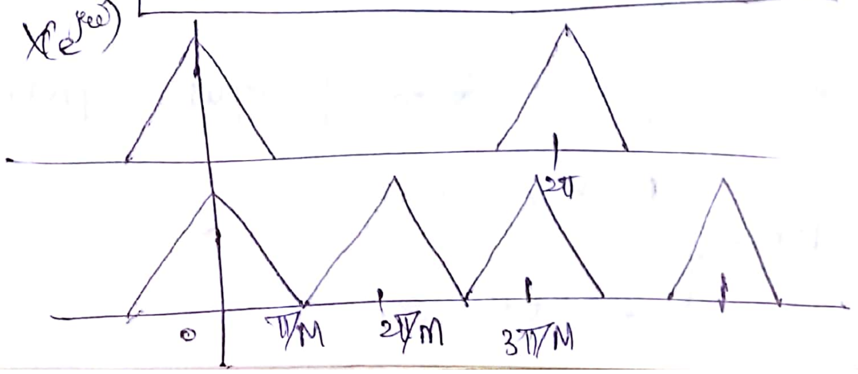
$$= \sum_{n=0}$$

$$= \frac{1}{M} \sum_{k=0}^{M-1} \sum_{n=-\infty}^{\infty} x(n) \left(e^{\frac{-j2\pi k}{M}}\, z^{1/M} \right)^{-n}$$

$$= \frac{1}{M} \sum_{k=0}^{M-1} X\left(e^{\frac{-j2\pi k}{M}}\, z^{1/M} \right)$$

$$z = e^{j\omega}$$

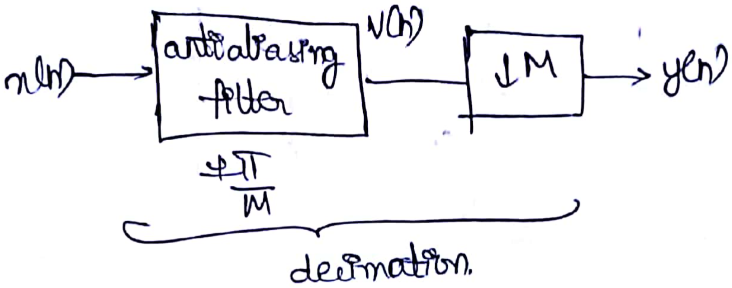
$$\boxed{Y(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} X\left(e^{j\left(\frac{\omega - 2\pi k}{M}\right)} \right)}$$



The i/p signal must be bandlimited to ±π/M before down sampling to avoid overlapping spectrum of samples (Aliasing effect)

∴ Filter is added before down sampling.

This is called as decimation



$F_y = MF_x$

Rate ↓ ⇒ Frequency ↑

Upsampling:

Increasing the sampling rate.

Let $x(n) \to L$

$y(n) = x\left(\frac{n}{L}\right)$

Every L^{th} sample is original sample and remaining samples are zero (in between)

Ex:- $x(n) = \{1, 2, 3, 4, 5\}$, $L = 2$

$y(n) = \{1, 0, 2, 0, 3, 0, 4, 0, 5, 0\}$

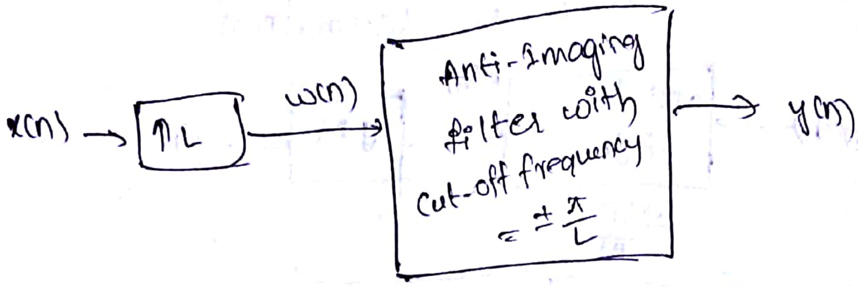
In practice, '0' has some amplitude.

Those additional spectrum are called as "Images"

To remove Imaging spectrum;

"Anti-Imaging filter is used after the up-sampling

Called as Interpolation.



This is called Interpolation.

Spectrum of the upsampled signal:-

$$y(n) = \begin{cases} x\left(\frac{n}{L}\right), & n = 0, \pm L, \pm 2L \ldots \\ 0 & \text{otherwise} \end{cases}$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n}$$

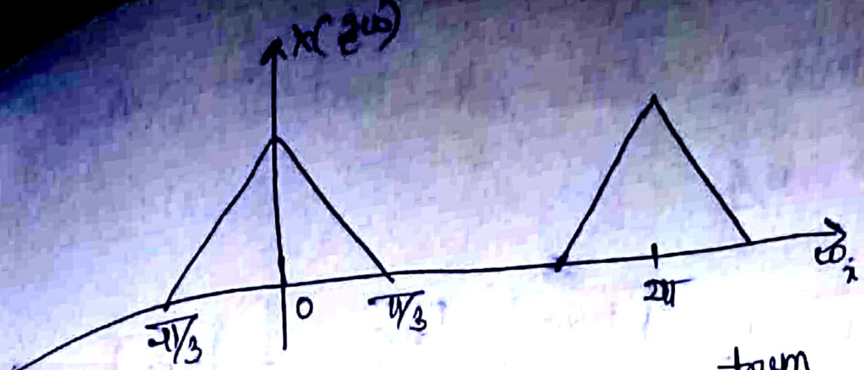
$$= \sum_{n=-\infty}^{\infty} x\left(\frac{n}{L}\right) z^{-n}$$

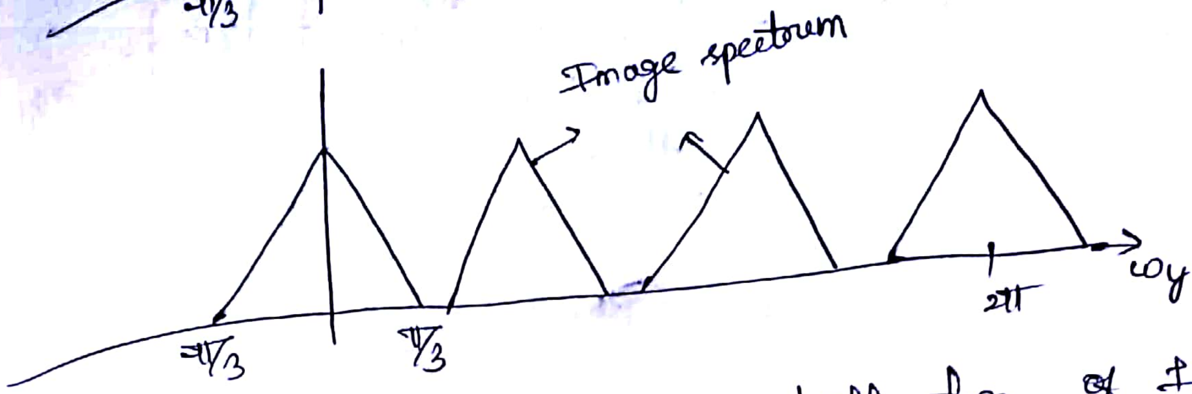
$$= \sum_{n=-\infty}^{\infty} x(n) z^{-Ln}$$

$$Y(z) = X(z^L)$$

$$z = e^{j\omega}$$

$$Y(e^{j\omega}) = X(e^{j\omega L})$$





It is sent to filter with cutoff freq of $\pm\pi/L$

Then, image spectrums are removed.

$$x(n) \rightarrow \boxed{\uparrow L} \rightarrow \underbrace{\boxed{\text{filter}}}_{\pm\pi/L} \rightarrow y(n)$$

Interpolation.

cascading of sampling rate converters:

→ If the factor is non-integer value, cascading is done (i.e. Interpolation followed by Decimation or vice versa.)

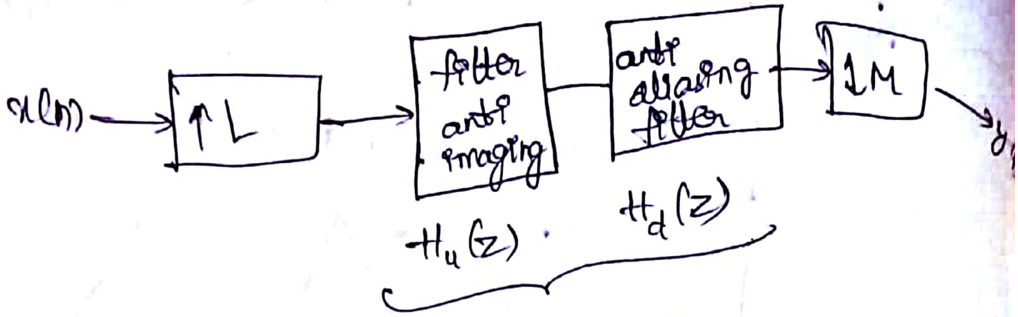
Ex: 44 KHz → 48 KHz

$\frac{48}{44}$

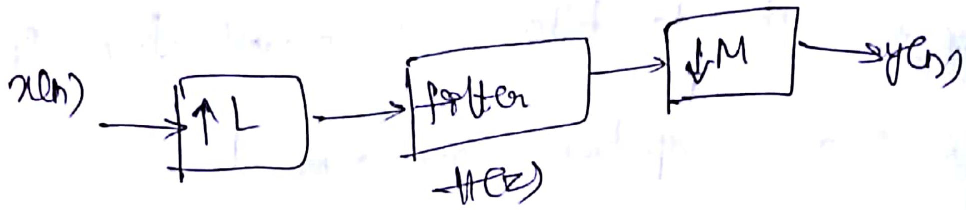
3 → 5

$\frac{5}{3}$

More information is ~~lost~~ not lost in I/D compared to D/I



$$\omega_c = \min\left(\frac{\pi}{L}, \frac{\pi}{M}\right)$$



~~Multi~~

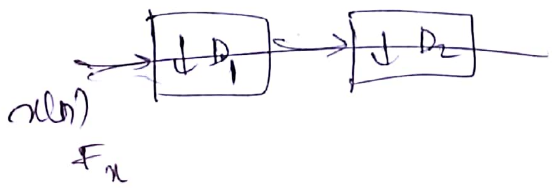
~~Multip~~

<u>Multistage implementation of sampling rate converter</u>

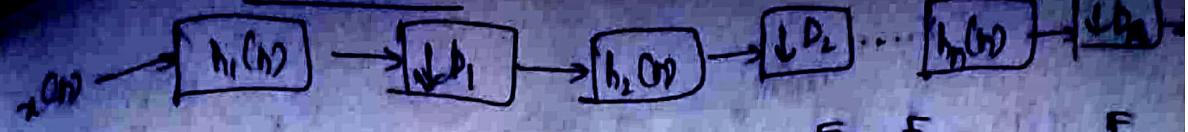
It is used when I (or) D are very high.

Filter design is complex when I (or) D is high

<u>Eg:</u> For $I=100$, two stages of $I=10$ can be cascaded to get $I=100$



Decimation:

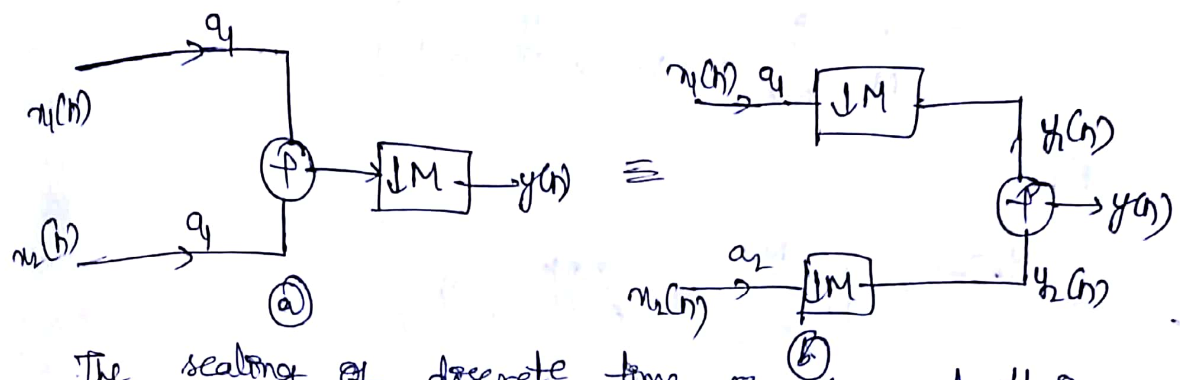


$F_y = \frac{F_x}{D_1}$ $F_y = \frac{F_x}{D_1 D_2}$ $F_y = \frac{F_x}{D_1 D_2 \cdots D_n}$

Interpolation

$F_y = I_1 F_x$ $F_y = I_1 I_2 F_x$ $F_y = I_1 I_2 \cdots I_n F_x$

Global identities:



The scaling of discrete time signals and their additions at the nodes are independent of sampling rate

(a) $\Rightarrow y(n) = a_1 x_1(n) + a_2 x_2(n)$

$$Y(e^{j\omega}) = \frac{a_1}{M}\sum_{k=0}^{M-1} X_1\left(e^{j\left(\frac{\omega-2\pi k}{M}\right)}\right) + \frac{a_2}{M}\sum_{k=0}^{M-1} X_2\left(e^{j\left(\frac{\omega-2\pi k}{M}\right)}\right)$$

(b) $\Rightarrow a_1 x_1 n$

$$Y_1(e^{j\omega}) = \frac{a_1}{M}\cdot\sum_{k=0}^{M-1} X_1\left(e^{j\left(\frac{\omega-2\pi k}{M}\right)}\right),\ \cancel{Y_2(e^{j\omega}) = \frac{a_2}{M}}$$

$$Y_2(e^{j\omega}) = \frac{a_2}{M}\sum_{k=0}^{M-1} X_2\left(e^{j\left(\frac{\omega-2\pi k}{M}\right)}\right)$$

$$\therefore Y(e^{j\omega}) = Y_1(e^{j\omega}) + Y_2(e^{j\omega}) \Rightarrow (a) = (b)$$

2) A delay of ~~'m'~~ ~~M~~ sample delays is the same as a delay of one sample period after the down sampler.

$$x(n) \to \boxed{z^{-M}} \xrightarrow{x_1(n)} \boxed{\downarrow M} \xrightarrow{y(n)} \;\equiv\; x(n) \to \boxed{\downarrow M} \xrightarrow{y_1(n)} \boxed{z^{-1}} \to$$

$$x_1(z) = z^{-M} x(z)$$

$$Y(z) = \frac{1}{M} \sum_{k=0}^{M-1} X_1\left(e^{j\left(\frac{\omega - 2\pi k}{m}\right)}\right)$$

$$= \frac{1}{M} \sum_{k=0}^{m-1} z^{-m/M} x\left(e^{-j\frac{2\pi k}{m}} z^{1/M}\right)$$

$$= \frac{1}{M} \sum_{k=0}^{m-1} z^{-1} x\left(e^{-j\frac{2\pi k}{m}} z^{1/m}\right)$$

RHS

$$Y_1(z) = \frac{1}{M} \sum_{k=0}^{M-1} x\left(e^{-j\frac{2\pi k}{M}} z^{1/M}\right)$$

$$Y(z) = z^{-1} Y_1(z)$$

$$Y(z) = z^{-1} \frac{1}{M} \sum_{k=0}^{M-1} x\left(e^{-j\frac{2\pi k}{M}} z^{1/M}\right)$$

LHS = RHS

3) A filter is having T.F $H(z^M)$ before the downsampler is equal to the filter T.F $H(z)$ after the down sampler

$$X(z) \to \boxed{H(z^M)} \xrightarrow{Y_1(z)} \boxed{\downarrow M} \xrightarrow{Y(z)} \;\equiv\; X(z) \to \boxed{\downarrow M} \xrightarrow{Y_1(z)} \boxed{H(z)} \to$$

$$Y_1(z) = H(z^M) X(z)$$

$$Y(z) = \frac{1}{M}\sum_{k=0}^{M-1} Y_1\left(e^{\frac{-j2\pi k}{M}} z^{1/M}\right)$$

$$= \frac{1}{M}\sum_{k=0}^{M-1} H(z^{M/M})\, X\left(e^{\frac{-j2\pi k}{M}} z^{1/M}\right)$$

$$= H(z) \cdot \frac{1}{M}\sum_{k=0}^{M-1} X\left(e^{\frac{-j2\pi k}{M}} z^{1/M}\right)$$

RHS

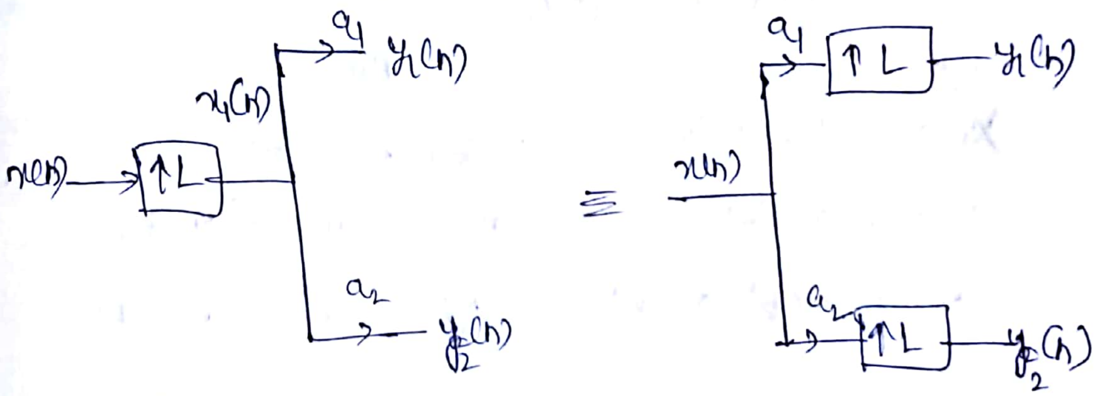
$$X_1(z) = \frac{1}{M}\sum_{k=0}^{M-1} X\left(e^{\frac{-j2\pi k}{M}} z^{1/M}\right)$$

$$Y(z) = X_1(z) H(z)$$

LHS = RHS.

4) The scaling of discrete time signals & their additions at the nodes are independent of the sampling rate.

(1, 2, 3 → Decimation
4, 5, 6 → Interpolation)



$X_1(z) = X(z^L)$ $\qquad$ $Y_1(z) = a_1 X(z^L)$

$Y_1(z) = a_1 X(z^L)$ $\qquad$ $Y_2(z) = a_2 X(z^L)$

$Y_2(z) = a_2 X(z^L)$

5) A delay of one sample before upsampling is equal to delay of 'L' samples after

up sampling.

$x(n) \rightarrow \boxed{z^{-1}} \xrightarrow{x_1(n)} \boxed{\uparrow L} \rightarrow y(n) \equiv x(n) \rightarrow \boxed{\uparrow L} \xrightarrow{y_1(n)} \boxed{z^{-L}}$

$X_1(z) = z^{-1} X(z)$

$Y(z) = X_1(z^L)$

$Y(z) = z^{-L} X(z^L)$

$Y_1(z) = X(z^L)$

$Y(z) = z^{-L} Y_1(z)$

$= z^{-L} X(z^L)$

6) Transfer function ($H(z)$) before upsampler is equal to T.F ($H(z^L)$) after the upsampler.

$X(z) \rightarrow \boxed{H(z)} \xrightarrow{X_1(z)} \boxed{\uparrow L} \rightarrow Y(z)$

$\equiv$

$X(z) \rightarrow \boxed{\uparrow L} \rightarrow \boxed{H(z^L)} \rightarrow Y(z)$

LHS

$X_1(z) = H(z) X(z)$

$Y(z) = X_1(z^L)$

$Y(z) = H(z^L) X(z^L)$

RHS

$Y_1(z) = X(z^L)$

$Y(z) = H(z^L) Y_1(z)$

$Y(z) = H(z^L) X(z^L)$

Filter design and implementation of sampling rate conversion:

$x(n) \rightarrow \underset{h(n)}{\boxed{\text{Filter}}} \xrightarrow{v(n)} \boxed{\downarrow M} \rightarrow y(n)$

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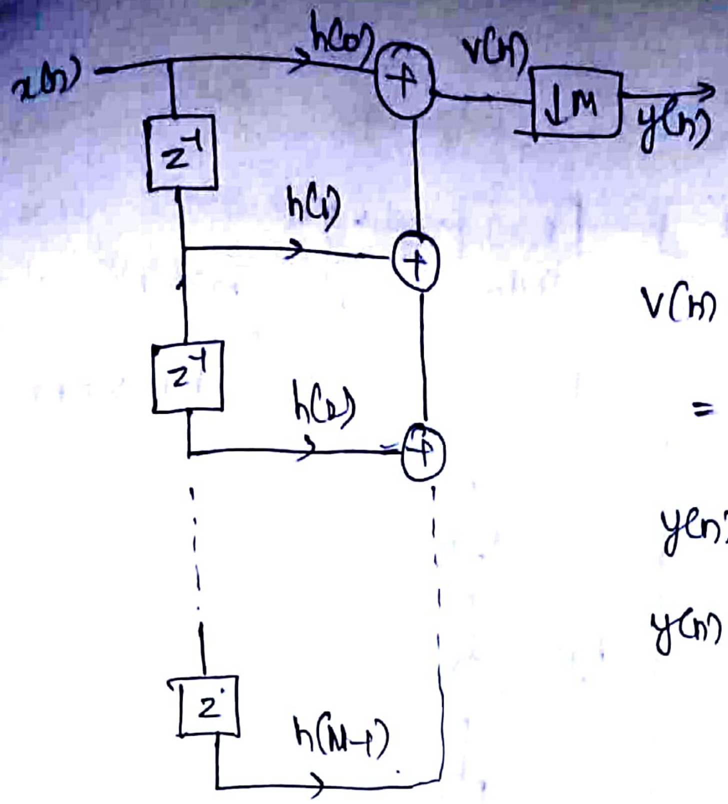


$$v(n) = x(n) * h(n)$$

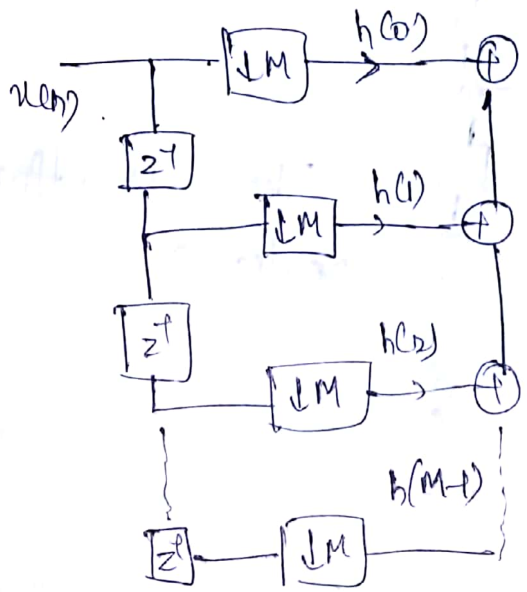
$$= \sum_{k=0}^{N-1} h(k)\, x(n-k)$$

$$y(n) = v(nM)$$

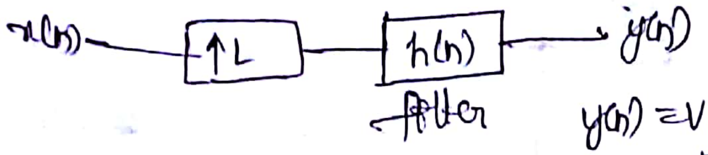
$$y(n) = \sum_{k=0}^{N-1} h(k)\, x(nM-k)$$

This realisation is inefficient as $x(1-k), x(3-k) \ldots$ are not used but still calculated.

$\therefore$ It is ~~eff~~ modified as

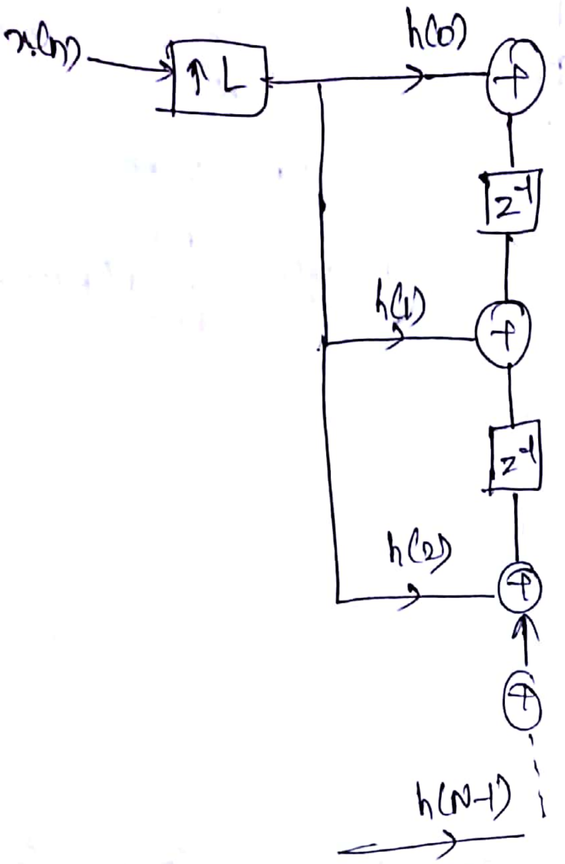


For interpolator:



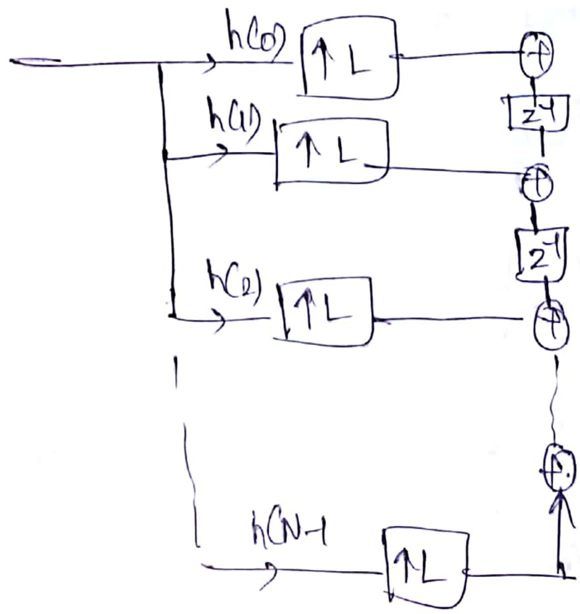
$y(n) = v(n) * h(n)$

$= \sum_{k=0}^{M-1} h(k)\, v(n-k)$



This is inefficient realisation

$\equiv$



This is efficient realisation.

Show that upsampler and downsampler are ~~time~~ variant systems.

Down sampler

Sol: $y(n) = x(nM)$

$y(n,k) = x(nM-k)$

$y(n-k) = x(M(n-k))$

$y(n,k) \neq y(n-k)$

$\therefore$ Time variant

Up sampler,

$y(n) = x\left(\frac{n}{L}\right)$

$y(n,k) = x\left(\frac{n}{L}-k\right)$

$y(n-k) = x\left(\frac{n-k}{L}\right)$

$y(n,k) \neq y(n-k)$

$\therefore$ Time Variant

UNIT-IV

FIR filters:

→ FIR filters can be obtained by multiplying IIR " with ~~the~~ window function.

Advantages of FIR over IIR:

→ FIR filters are stable as the I.R (impulse response) of finite duration. $\left(\because \sum_{n=-\infty}^{\infty} |h(n)| < \infty\right)$

→ It produces linear phase

→ FIR filters are non-recursive

→ No. of designing methods are available

Disadvantages of FIR over IIR:

→ Executive time is high

→ Design methods of FIR are costly. (more no. of adders and multipliers)

Realisation of FIR filters:

1) Direct form (or transversal structure

2). Cascade

3) Linear phase realisation.

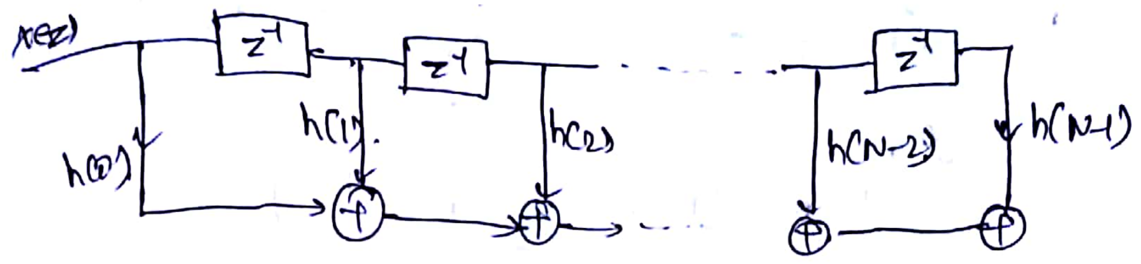
$\cancel{H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}}$ $H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$

($\because$ It is finite and N^{th} order)

$$H(z) = h(0) + h(1) z^{-1} + h(2) z^{-2} + \dots + h(N-1) z^{-(N-1)}$$

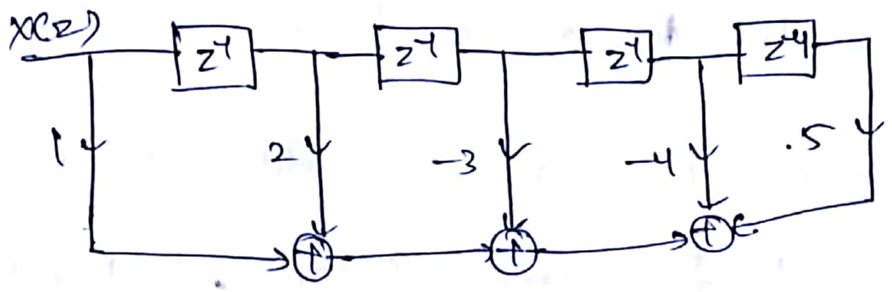
$$Y(z) = h(0) X(z) + h(1) X(z) z^{-1} + \dots + h(N-1) X(z) z^{-(N-1)}$$

1) **Direct form: (Transversal form)**



Q) Determine direct form realisation of system function

$$H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$$



2) **Cascade realisation:**

The given T.F is divided into no. of T.F's

$$H(z) = H_1(z) H_2(z) \dots$$

and each T.F is realised using T.F direct form.

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + \dots + h(N-1) z^{-(N-1)}$$

Based on N (odd or even), type of realisation of the filter is determined.

When N is odd, N-1 is even and H(z) will

$$H(z) = \prod_{k=0}^{\frac{N-1}{2}} \left(b_{k_0} + b_{k_1} z^{-1} + b_{k_2} z^{-2}\right)$$

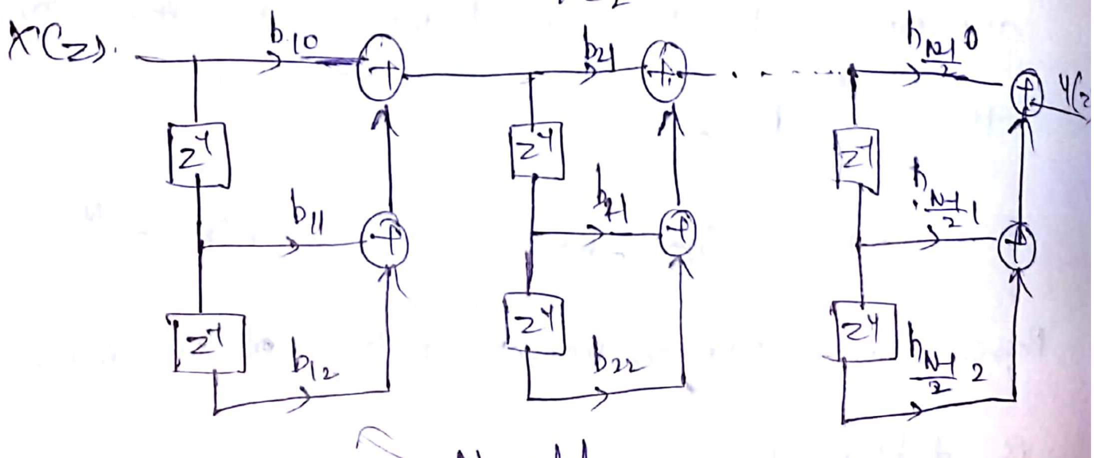
$$H(z) = (b_{10} + b_{11} z^{-1} + b_{12} z^{-2})(b_{20} + b_{21} z^{-1} + b_{22} z^{-2}) \cdot$$

$$\cdot \left(b_{\frac{N-1}{2}0} + b_{\frac{N-1}{2}1} z^{-1} + b_{\frac{N-1}{2}2} \cdot z^{-2}\right)$$

'$\frac{N-1}{2}$' second order functions and each 2nd order factor can be realised in Direct form and is cascaded to realize H(z)

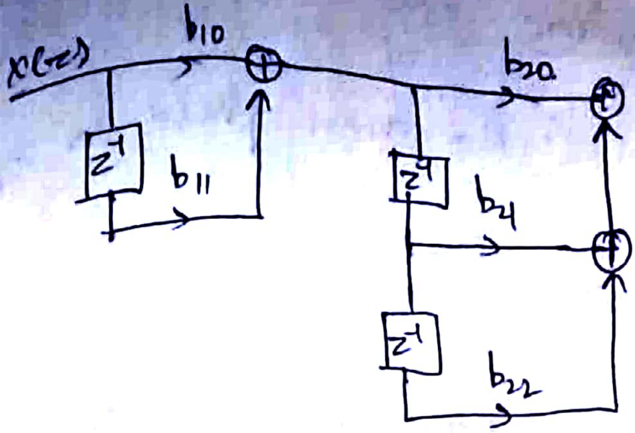
→ If N is even, N-1 will be odd and H(z) will have 1 first order factor and '$\frac{N-2}{2}$' second order factors. Each factor can be realised in direct form-I & cascaded.

$$H(z) = (b_{10} + b_{11} z^{-1}) \prod_{k=2}^{N/2} (b_{k_0} + b_{k_1} z^{-1} + b_{k_2} z^{-2})$$



N-odd

N-even



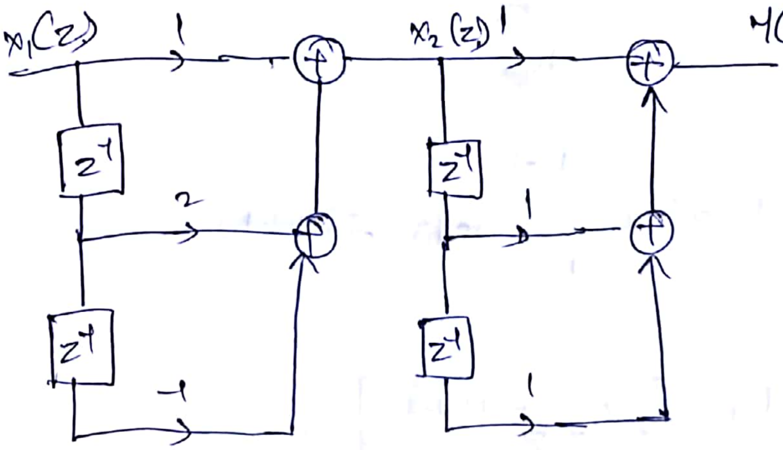
Q) obtain the cascade realisation of the system functⁿ

$$H(z) = (1+2z^{-1}-z^{-2})(1+z^{-1}-z^{-2})$$

sol: $H_1(z) = 1+2z^{-1}-z^{-2}$, $H_2(z) = 1+z^{-1}-z^{-2}$

$$Y_1(z) = X_1(z) + 2z^{-1}X(z) - z^{-2}X(z)$$

$$Y_2(z) = X_2(z) + z^{-1}X_2(z) - z^{-2}X_2(z)$$



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9/3/18

Linear phase realization:

$h(n) = h(N-1-n)$ (∵ No linear phase for

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

Depending upon 'N', FIR filter can be realized a

N – even: (Symmetrical i.e. 0 to $\frac{N}{2}-1$ and $\frac{N}{2}$ to

$$H(z) = \sum_{n=0}^{\frac{N-2}{2}} h(n)\, z^{-n} + \sum_{n=N/2}^{N-1} h(n)\, z^{-n}$$

put $n = N-1-n$

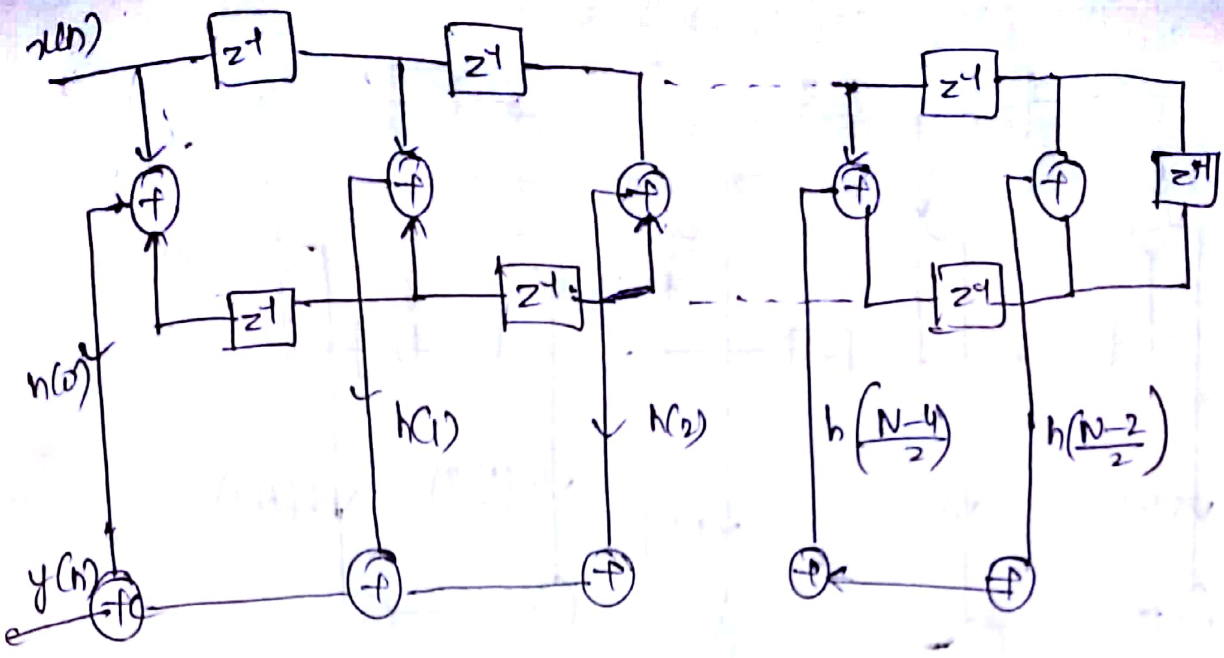
$$= \sum_{n=0}^{\frac{N-2}{2}} h(n)\, z^{-n} + \sum_{\cancel{n=\frac{N}{2}}\ n=0}^{(N-2)/2} h(N-1-n)\, z^{-(N-1-n)}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n)\, z^{-n} + \sum_{n=0}^{\frac{N-2}{2}} h(n)\, z^{-(N-1-n)}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} h(n) \left[z^{-n} + z^{-(N-1-n)} \right]$$

Ex: Let $N = 6$.

$$\Rightarrow \sum_{n=0}^{2} h(n) \left[z^{-n} + z^{-(5-n)} \right]$$



N odd

(For N = 7 $\quad 0 \rightarrow 6 \rightarrow$ centre of symmetry

$0 \rightarrow 2 \quad 3 \quad 4 \rightarrow 6$)

symmetrical

$$H(z) = \sum_{n=0}^{\frac{N-3}{2}} h(n)\, z^{-n} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n)\, z^{-n} + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)}$$

$$N-1-\frac{N+1}{2}$$

$$= \frac{N-2-N-1}{2}$$

$$\frac{N-3}{2}$$

put $n = N-1-n$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n)\, z^{-n} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n)\, z^{-(N-1-n)} + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)}$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) \left[z^{-n} + z^{-(N-1-n)} \right] + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)}$$

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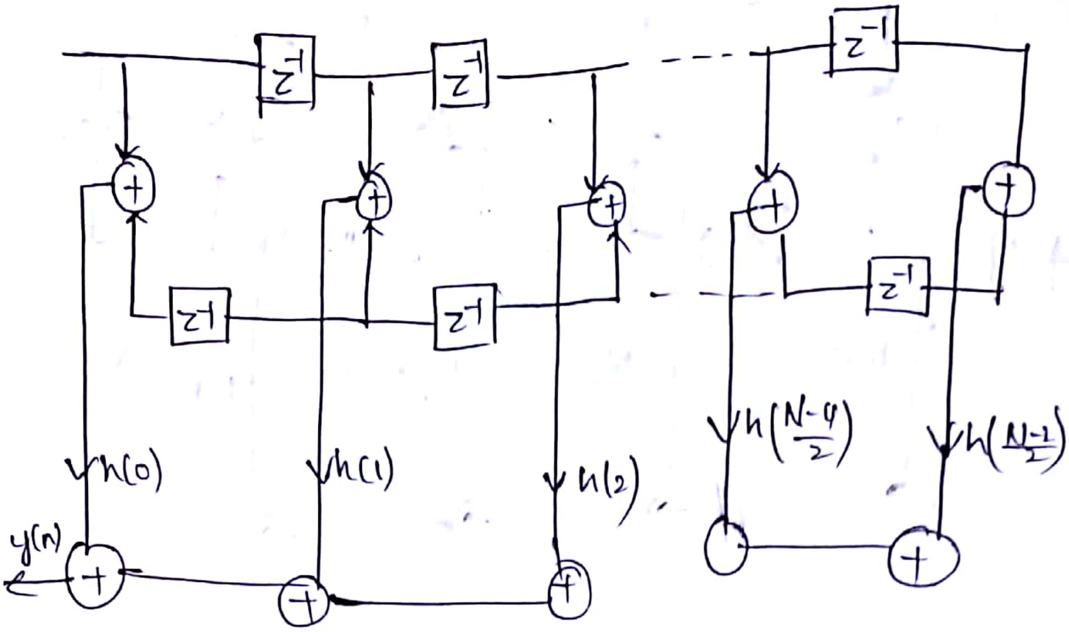


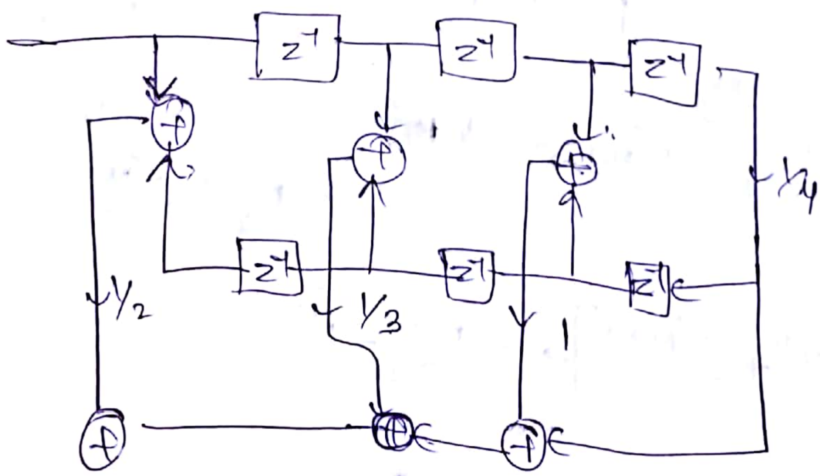
Q) Realise the system function

$$H(z) = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + \frac{1}{3}z^{-5} + \frac{1}{2}z^{-6}$$

Sol:

$h(n) = h(N-1-n)$, $N=7$ odd

$h(0) = (6)$ . . . linear phase



cascade.

Q) $H(z) = \underbrace{\left(\frac{1}{2} + z^{-1} + \frac{1}{2} z^{-2}\right)\left(1 + \frac{1}{3} z^{-1} + z^{-2}\right)}_{\text{cascade}}$

→ linear phase → linear phase

Realise using cascade realization using ~~linear phase~~ minimum no. of multipliers.

Linear phase FIR filters:

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

h(n) is may be symmetric or antisymmetric

($\because h(n) = -h(N-1-n)$)

Phase delay (for 1 frequency)

$$\tau_p = \frac{-\theta(\omega)}{\omega}$$

Group delay (for more than 1 frequencies)

$$\tau_g = \frac{-d\theta(\omega)}{d\omega}$$

$$z = e^{j\omega}$$

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n}$$

$$H(e^{j\omega}) = \pm |H(e^{j\omega})|\, e^{j\theta(\omega)}$$

$$\tau_p = \frac{-\theta(\omega)}{\omega}, \quad \tau_g = \frac{-d\theta(\omega)}{d\omega}$$

For FIR filters with linear phase,

$$\theta(\omega) = -\alpha\omega \qquad -\pi \leq \omega \leq \pi$$

↳ constant phase delay

$$\tau_p = \tau_g = -\alpha$$

↳ independent of freq

$$\sum_{n=0}^{N-1} h(n)\, e^{-j\omega n} = \pm |H(e^{j\omega})|\, e^{j\theta(\omega)}$$

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$$\sum_{n=0}^{N-1} h(n)\cos\omega n = \pm|H(e^{j\omega})| \cos\theta(\omega) \quad \text{--- (1)}$$

$$-\sum_{n=0}^{N-1} h(n)\sin\omega n = \pm|H(e^{j\omega})| \sin\theta(\omega) \quad \text{--- (2)}$$

Take the ratio $\frac{(2)}{(1)}$

$$\frac{\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n} = \frac{\sin\alpha\omega}{\cos\alpha\omega}$$

$$\sum_{n=0}^{N-1} h(n)\sin(\alpha-n)\omega = 0$$

$$h(n) = h(N-1-n)$$

$$\alpha = \frac{N-1}{2}$$

→ FIR filters have constant τ_p and τ_g when $h(n)$ is symmetric about $\alpha = \frac{N-1}{2}$ and also maintains a linear phase

→ If N - odd, $\frac{N-1}{2}$ is centre of symmetry
N - even, " " " "

(3)

0 1 2 3

→ If only constant ~~phase~~ group delay is required and not " ~~group~~ phase " , $\theta(\omega) = \beta - \alpha\omega$.

0, 1, 2, 3

→ If only constant phase delay is required and not " group "

$$\sum_{n=0}^{N-1} h(n)\, e^{-j\omega n} = \pm\, H(e^{j\omega})\, e^{j\theta(\omega)}$$

$$\frac{-\sum_{n=0}^{N-1} h(n) \sin \omega n}{\sum_{n=0}^{N-1} h(n) \cos \omega n} = \frac{\sin(\beta - \alpha\omega)}{\cos(\beta - \alpha\omega)}$$

$$\sum_{n=0}^{N-1} h(n) \sin(\beta - (\alpha - n)\omega) = 0.$$

$$\cancel{\beta = \pi/2} \quad \beta = \pi/2$$

$$\sum_{n=0}^{N-1} h(n) . \cos((\alpha - n)\omega) = 0$$

$$h(n) = -(h(N-1-n))$$

$$\alpha = \frac{N-1}{2}$$

FIR filters have constant group and not constant phase delay when impulse response is antisymmetrical about α, $\alpha = \frac{N-1}{2}$

Frequency response of a linear phase FIR ~~sys~~ filters:

Case (i) symmetric $h(n)$ and 'N' is odd

$n = N-1-n$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}$$

0 to $\frac{N-1}{2} - 1$, $\frac{N-1}{2}$, $\frac{N-1}{2} + 1$ to $N-1$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n)\, e^{-j\omega(N-1-n)}$$

$$h(n) = h(N-1-n)$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) \cdot e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{j\omega\left(\frac{N-1}{2}-n\right)} + h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j\omega\left(\frac{N-1}{2}-n\right)} \right]$$

$$\frac{N-1}{2} - n = p$$

$$e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{p=1}^{\frac{N-1}{2}} h\left(\frac{N-1}{2}-p\right) e^{j\omega p} + h\left(\frac{N-1}{2}\right) + \sum_{p=1}^{\frac{N-1}{2}} h\left(\frac{N-1}{2}-p\right) e^{-j\omega p} \right]$$

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{p=1}^{N-1/2} 2h\left(\frac{N-1}{2}-p\right) \cos \omega p + h\left(\frac{N-1}{2}\right) \right]$$

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right), \quad a(n) = 2h\left(\frac{N-1}{2}-n\right)$$

$$H(e^{j\omega}) = \bar{H}(e^{j\omega})\, e^{j\theta(\omega)}$$

$$\theta(\omega) = -\alpha\omega = -\left(\frac{N-1}{2}\right)\omega$$

$$h(n) = h(N-1-n) \Rightarrow \boxed{\alpha = \frac{N-1}{2}}$$

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

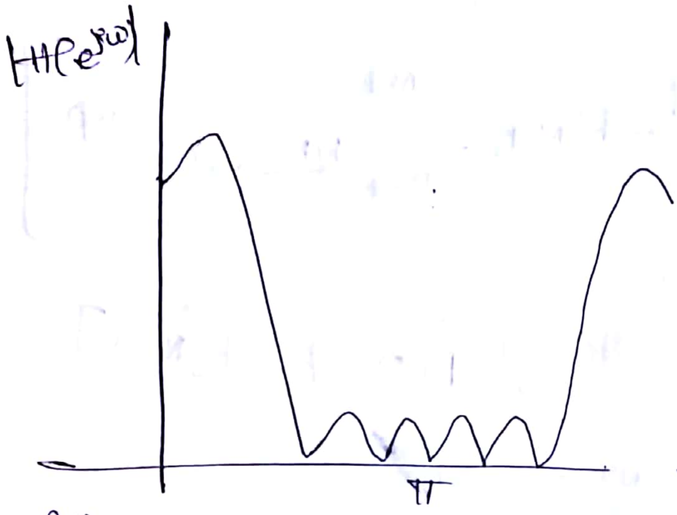
→ $\bar{H}(e^{j\omega})$ is real and even function of 'ω'.

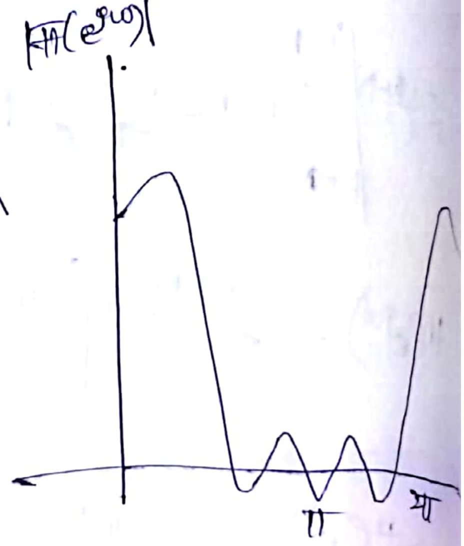
magnitude and phase of $H(e^{j\omega})$ are

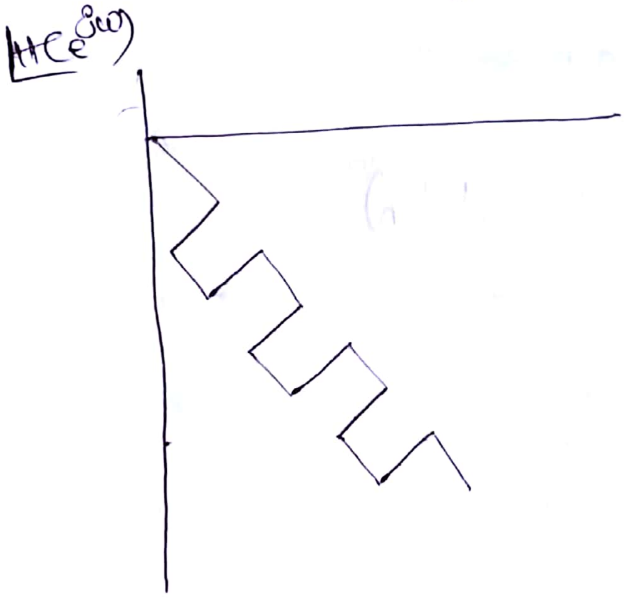
$$|H(e^{j\omega})| = |\bar{H}(e^{j\omega})|$$

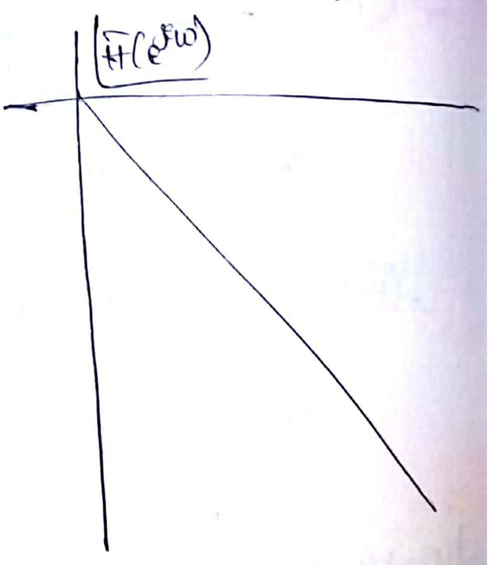
$$\theta(\omega) = -\alpha\omega \quad , \quad \alpha = \frac{N-1}{2}$$

→ $\bar{H}(e^{j\omega})$ is zero phase frequency response to distinguish it from the magnitude response









Symmetric & Even

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=1}^{N/2} b(n) \cdot \cos\left(\omega\left(n-\frac{1}{2}\right)\right)$$

Fourier series method of designing FIR filter:

T.F of FIR filter

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)\, e^{-j\omega n}$$ (Non causal with infinite duration)

$H(e^{j\omega})$ $\downarrow$ Desired I.R

$h_d(n)$ $\hookrightarrow$ coefficients (F.S)

$$\underbrace{h_d(n)}_{-\infty \text{ to } +\infty} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$h_d(n)$ should be truncated to get finite causal signal.

$$h(n) = h_d(n) \quad |n| \le \frac{N-1}{2} \qquad -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N-1}{2}\right)$$

$$0$$

$$H(z) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} h(n)\, z^{-n}$$

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \ldots + h(1) z^{-1} + h(0)$$

$$+ h(-1) z + h(-2) z^{2} + \ldots + h\left(-\left(\frac{N-1}{2}\right)\right) z^{\frac{N-1}{2}}$$

$$= h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n)\, z^{-n} + h(-n)\, z^{n}\right]$$

$h(n)$ is symmetric $\Rightarrow h(n) = h(-n)$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{-n} + z^{n}\right]$$

$$\boxed{H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)}$$

Q) Design an ideal LPF with freq. Response

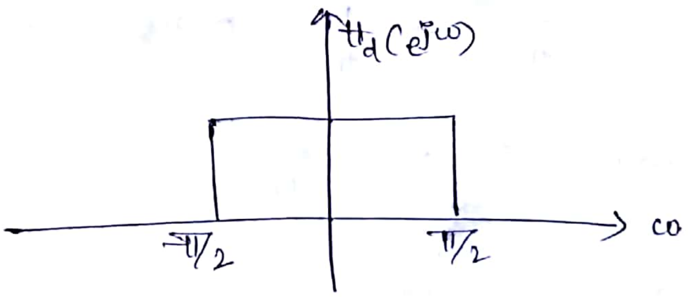
$H_d(e^{j\omega}) = 1$ for $\frac{-\pi}{2}$ ~~to~~ $\le |\omega| \le \frac{\pi}{2}$

$= 0 \qquad \frac{\pi}{2} \le \omega \le \pi$

Find the values of $h(n)$ for $N=11$ and find $H(z)$ and also plot the Magnitude Response

Sol: $H_d(e^{j\omega}) = 1 \qquad \frac{-\pi}{2} \le |\omega| \le \frac{\pi}{2}$

$= 0 \qquad \frac{\pi}{2} \le \omega \le \pi$



$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi j n} e^{j\omega n} \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2\pi j n} \left[e^{j\frac{\pi}{2}n} - e^{j\frac{\pi}{2}n} \right]$$

$$h_d(n) = \frac{1}{\pi n} \sin\frac{\pi n}{2} \qquad -\infty \le n \le \infty$$

$$h(n) = \frac{\sin \pi n/2}{\pi n} \qquad |n| \le 5 \quad \text{—} (1)$$

$$h(0) = h_d(n) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j\omega(0)}\, d\omega = \frac{1}{2\pi}\, \omega \Big|_{-\pi/2}^{\pi/2} = \frac{1}{2}$$

($\because h(n)$ is (1) indeterminant.

$$h(0) = \lim_{n\to 0} \frac{\sin\frac{\pi n}{2}}{\pi n} = \frac{1}{2}\lim_{n\to 0}\frac{\sin\left(\frac{n\pi}{2}\right)}{\pi n/2} = \frac{1}{2}$$

$$h(1) = h(-1) = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi} = 0.3183$$

$$h(2) = h(-2) = \frac{\sin \pi}{2\pi} = 0$$

$$h(3) = h(-3) = \frac{\sin\frac{3\pi}{2}}{3\pi} = \frac{-1}{3\pi} = -0.106$$

$$h(4) = h(-4) = \frac{\sin 2\pi}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin 5\pi/2}{5\pi} = \frac{1}{5\pi} = 0.063$$

$$H(z) = h(0) + \sum_{n=1}^{5} h(n)\left[z^{n} + z^{-n}\right]$$

$$= \frac{1}{2} + 0.318\left[z^{1} + z^{-1}\right] - 0.106\left[z^{3} + z^{-3}\right]$$
$$+ 0.063\left[z^{5} + z^{-5}\right]$$

$H(z) = \cancel{0.063 z^5 + 0 + -0.106 z^3 + 0.318 z^1 + \frac{1}{2} + 0.318 z^{-1}} - 0.106 z^{-3} + 0.063 z^{-5}$

$H(z) = z^{-5}\left[\frac{1}{2} + 0.318\left[z^1 + z^{-1}\right] - 0.106\left[z^3 + z^{-3}\right] + 0.063\left[z^5 + z^{-5}\right]\right]$

$= 0.063 - 0.106 z^{-2} + 0.318 z^{-4} + 0.318 z^{-8} - 0.106 z^{-8} + 0.063 z^{-10}$

$h(0) = h(10) = 0.063$, $h(1) = h(9) = 0$

$h(1) = h(8) = -0.106$ $h(4) = h(6) = 0.318$

$h(3) = h(7) = 0$, $h(5) = 0.5$

It is symmetric with N-odd.

$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$

$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.5$

$a(n) = 2h\left(\frac{N-1}{2} - n\right)$

$a(1) = 2h(5-1) = 2h(4) = 0.636$

$a(2) = 2h(5-2) = 2h(3) = 0$

$a(3) = 2h(2) = -0.212$

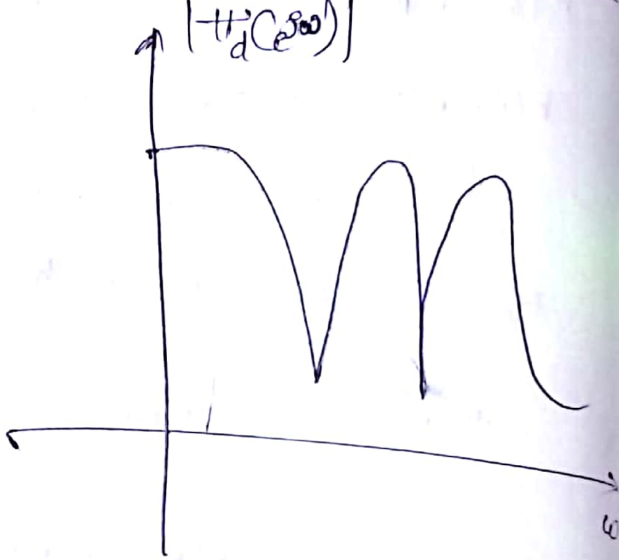
$a(4) = 2h(1) = 0$

$a(5) = 2h(0) = 0.126$

$$H(e^{j\omega}) = 0.5 + 0.0636 \cos\omega - 0.212 \cos 3\omega + 0.126 \cos 5\omega)$$

~~ω=0~~

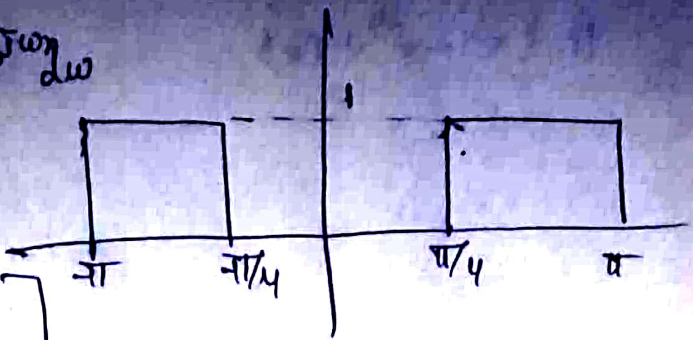
ω	0°	30	60°	90°	120°	150	180
$\lvert H(e^{j\omega}) \rvert$	1.05	0.941	1.093	0.5	+0.093	0.058	1.0[illegible]
dB	0.42	-0.52	0.772	-0.6.02			



② Design an ideal HP filter with freq response $H_d(e^{j\omega}) = 1 \quad \frac{\pi}{4} \le \omega \le \pi$

$= 0 \quad \omega \le \frac{\pi}{4}$ Find the values of $h(n)$ for $N = 11$ and find the magnitude response

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$



$$= \frac{1}{2\pi}\left[\int_{\pi/4}^{-\pi} e^{j\omega n} d\omega + \int_{-\pi}^{-\pi/4} e^{j\omega n} d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{\left(e^{j\omega n}\right)_{\pi/4}^{\pi}}{jn} + \frac{\left(e^{j\omega n}\right)_{-\pi}^{-\pi/4}}{jn}\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\pi n} - e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi n}{4}} - e^{-j\pi n}}{jn}\right]$$

$$= \frac{1}{\pi n}\left[\sin \pi n - \sin\frac{\pi n}{4}\right]$$

$$h(n) = h_d(n) \quad |n| \leq 5 \qquad \left(\because \frac{N-1}{2} = \frac{11-1}{2} = 5\right)$$

$n=0$

$$h_d(n) = \frac{1}{2\pi}\left[\int_{\pi/4}^{\pi} 1.d\omega + \int_{-\pi}^{-\pi/4} 1 d\omega\right] = \frac{1}{2\pi}\left[\pi - \frac{\pi}{4} - \frac{\pi}{4} + \pi\right]$$

$$h(0) = \frac{3}{4}$$

$h(1) = h(-1) = -0.225$

$h(2) = h(-2) = -0.159$

$h(3) = h(-3) = -0.075$

$h(4) = h(-4) = 0$

$h(5) = h(-5) = 0.045$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]$$

$$= \frac{3}{4} - 0.225(z^1 + z^{-1}) - 0.159(z^2 + z^{-2}) - 0.075(z^3 + z^{-3}) + 0.045(z^5 + z^{-5})$$

$H'(z) = z^{-5} H(z)$

$H(0) = H(10) = 0.045$

$H(1) = H(9) = 0$

$H(2) = H(8) = -0.075$

$H(3) = H(7) = 0.159$

$H(4) = H(6) = -0.225$

$H(5) = \frac{3}{4} = 0.75$

N-odd & symmetric

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

$a_0 = h\left(\frac{N-1}{2}\right) \qquad a_n = 2h\left(\frac{N-1}{2} - n\right)$

Q) For the above problem, find the magnitude response using hanning window

Sol:

$$\omega_{Hn} = 0.5 + 0.5 \cos\left(\frac{2\pi n}{N-1}\right) \quad |n| < 5 \quad \left(\because -\left(\frac{N-1}{2}\right) < n < \frac{N-1}{2}\right)$$

$\omega(n) = \omega(-n)$

$\omega_{Hn}(0) = 1$

$\omega_{Hn}(1) = \omega_{Hn}(-1) = 0.904$

$\omega_{Hn}(2) = \omega_{Hn}(-2) = 0.6554$

$\omega_{Hn}(3) = \omega_{Hn}(-3) = 0.3454$

$\omega_{Hn}(4) = \omega_{Hn}(-4) = 0.09549$

$\omega_{Hn}(5) = \omega_{Hn}(-5) = 0.$

$h(0) = h_d(0)\, w_{Hn}(0)$

$= (0.75)(1) = 0.75$

$h(1) = h_d(1)\, w_{Hn}(1) = -0.2034$

$h(2) = 0.104$

$h(3) = -0.026$

$h(4) = 0$

$h(5) = 0$

1) $h_d(n)$

2) $w_T(n)$ . . .

$w_{hn}(n)$ . . .

$w_R(n)$ . . .

Multiply with $h_d(n)$

3) $h(n) = h_d(n)\, w_p(n)$

$p \to$ Re, hamming

4) $H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left(z^{-n} + z^{n}\right)$

5) $H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$

$h(n)$

$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$

$a_0 = h\left(\frac{N-1}{2}\right) \quad a(n) = 2h\left(\frac{N-1}{2} - n\right)$

FIR characteristics

Symmetric → odd

→ even

F.S method of

window techniques

Adv & Dis of FIR

Comp. of FIR & IIR filters

2/7

Design of FIR filters using windows:

Infinite duration → finite duration } convergence

$H_d(e^{j\omega})$

↓

$h_d(e^{j\omega}) \longrightarrow \infty$ duration.

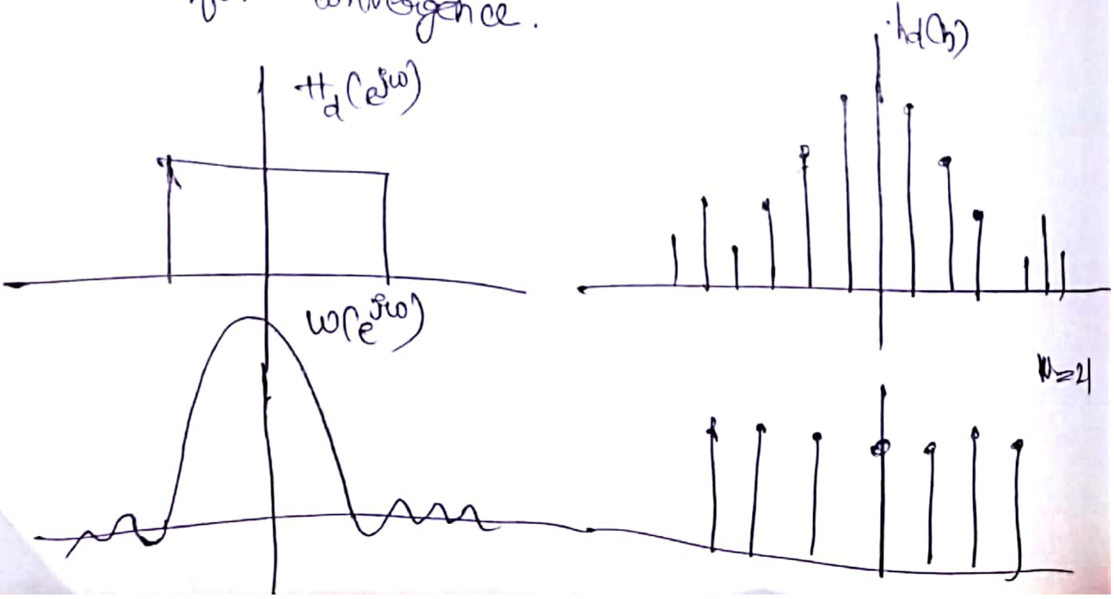
is ~~truncated~~ truncated to $h_d(n)$ $|n| \leq \frac{N-1}{2}$

~~Slower~~ (slow convergence of fourier series leads to oscillations in SB & PB.)

~~Ab~~

Abrupt conversion of infinite Impulse Response to finite I.R leads to oscillations in SB & PB which is called Gibb's phenomenon.

To reduce oscillations, window functions are used for convergence.



Characteristics of window

1) The main lobe consists of max. energy.

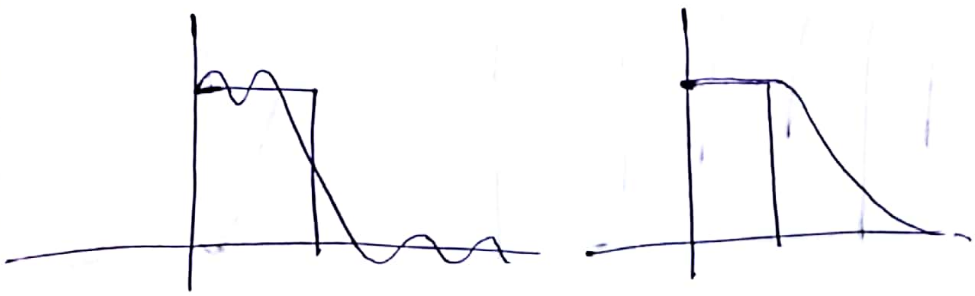
2) ~~Amplitude of~~ Highest side lobes level must be minimum (small)

3) Amplitude of side lobes decreases with ω

4) Main lobe width should be less and ~~height~~ amplitude should be high (narrow)

~~$H(e^{j\omega})$~~

$$H(e^{j\omega}) = H_d(e^{j\omega}) * W(e^{j\omega})$$ (∵ Multiplication in t-domain ⇒ conv in freq. domain)



i) Rectangular window:

$$W_R(n) = 1 \qquad |n| < \frac{N-1}{2} \rightarrow -\left(\frac{N-1}{2}\right) \leq n \leq \frac{N-1}{2}$$

$$= 0 \quad \text{O.W.}$$

$$h(n) = h_d(n) \cdot w_R(n)$$

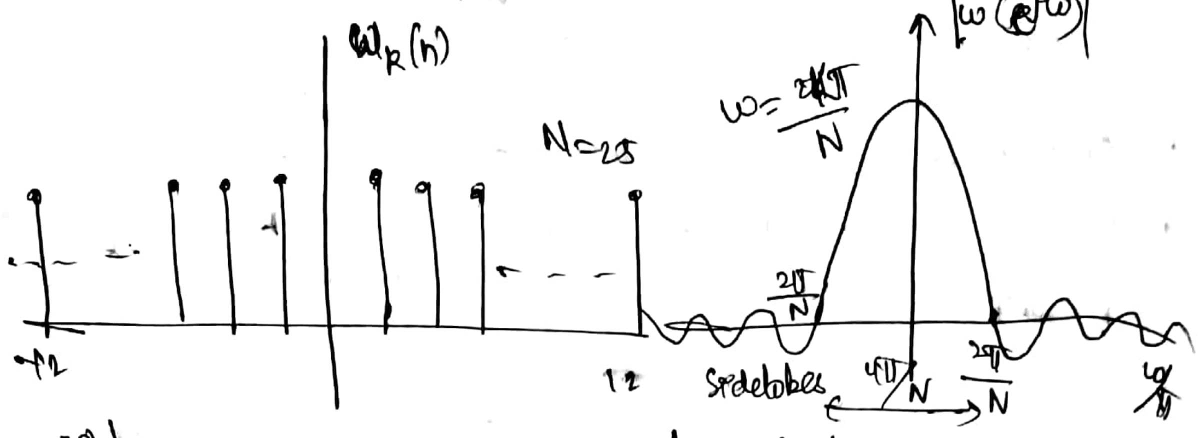
$$h(n) = h_d(n) \qquad |n| < \frac{N-1}{2}$$

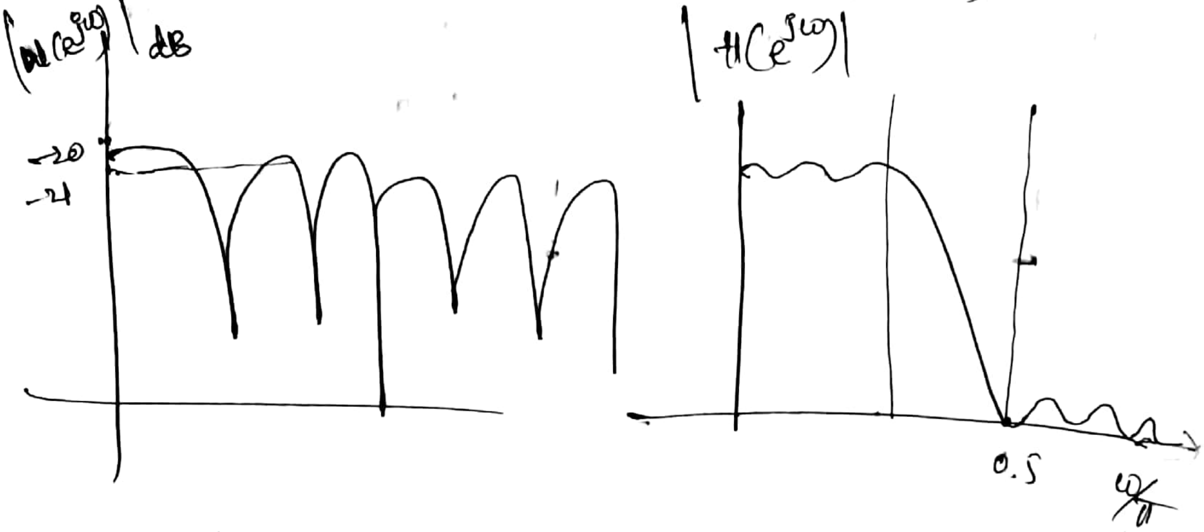
$$W_R(e^{j\omega}) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} e^{-j\omega n}$$

$$= e^{j\omega\left(\frac{N-1}{2}\right)} + \ldots + e^{j\omega} + 1 + e^{-j\omega} + \ldots + e^{-j\omega\left(\frac{N-1}{2}\right)}$$

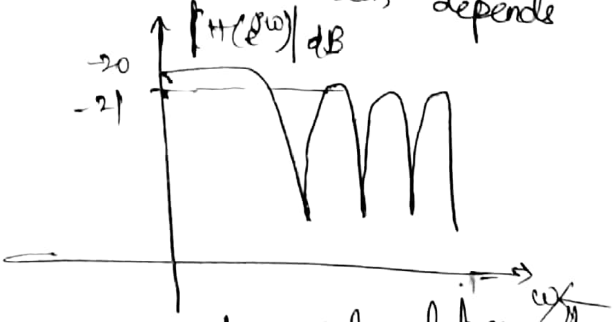
$$= e^{j\omega\left(\frac{N-1}{2}\right)} \left[1 + e^{-j\omega} + \ldots + e^{-j\omega(N-1)}\right]$$

$$= \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin \omega/2}$$





→ ~~transition zone~~ can be reduced by increasing N. $\left(\because \frac{4\pi}{N}\right)$ i.e. main lobe width depends on N.

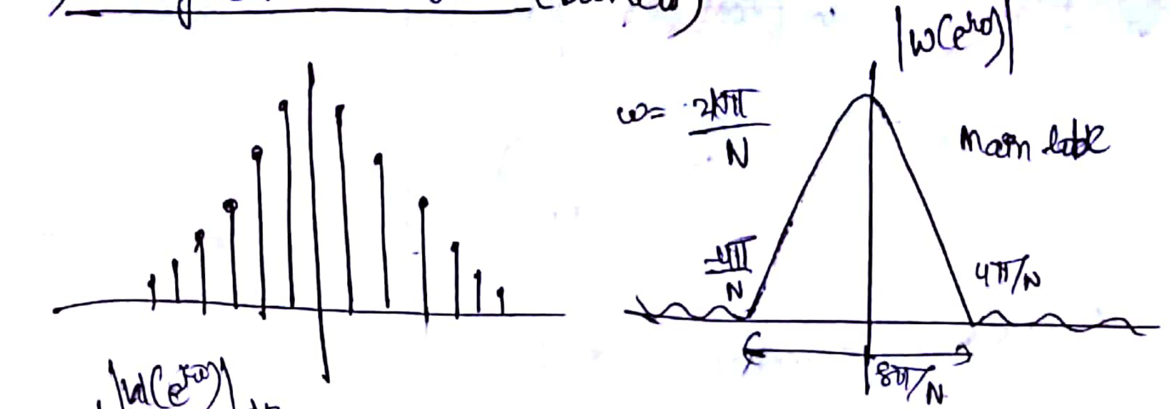


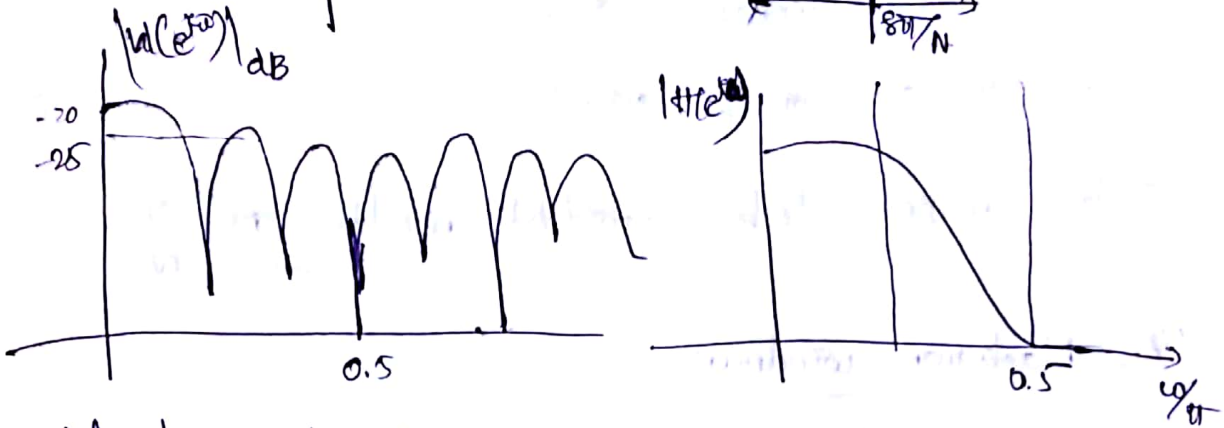
Disadvantages:

1) Ripples in PB & SB due to side lobes (first side lobe contain 22% of energy) due

2) to ~~ad~~ abrupt change.

1) Triangular window: (Barlett)





~~Advantages:~~ Disadvantages:

1) stop band attenuation is less

2) width of main lobe is more

~~Advantages:~~

~~1)~~ Amplitude of side lobes is comparitively less

$\Rightarrow$ less no. of ripples.

2) Raised cosine window:

$$w_\alpha(n) = \alpha + (1-\alpha)\frac{\cos 2\pi n}{N-1} \quad |n| \le \frac{N-1}{2}$$

$$= 0 \quad \text{OW}$$

Apply F.T $\qquad \cos \to \frac{e^{j\omega}+e^{-j\omega}}{2}$

Refer T.B

$$\to W_\alpha(e^{j\omega}) = \alpha \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin(\omega/2)} + \frac{1-\alpha}{2}\left[\frac{\sin\left(\frac{\omega N}{2} - \frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2} + \frac{\pi}{N-1}\right)}\right]$$

$$+ \frac{1-\alpha}{2}\left[\frac{\sin\left(\frac{\omega N}{2}+\frac{\pi N}{N-1}\right)}{\sin\left(\frac{\omega}{2}+\frac{\pi}{N-1}\right)}\right]$$

$\alpha = 0.5 \Rightarrow$ Hanning window

$\alpha = 0.54 \Rightarrow$ Hamming window.

→ The main lobe ~~width~~ width is $\frac{8\pi}{N}$

4. <u>Blackman window:</u>

$$W_\alpha(e^{j\omega}) = e$$

$$\omega_B(n) = 0.42 + 0.5 + 0.5\cos\frac{2\pi N}{N-1} + 0.08\cos\frac{4\pi n}{N-1}$$

$$|n| \leq \frac{N-1}{2}$$

~~Window~~

Scanned by CamScanner

Window	Peak side lobe (dB)	Main lobe width	Min. SB attenuation.
Rectangular	−13	$\frac{4\pi}{N}$	−21
Triangular	−25	$\frac{8\pi}{N}$	−25
Hanning	−31	$\frac{8\pi}{N}$	−44
Hamming	−41	$\frac{8\pi}{N}$	−53
Blackman	−57	$\frac{12\pi}{N}$	−74

Hamming window is preferred among all.