Q1. check the whether the following system $y(n) = x(n) + \frac{1}{x(n-1)}$ is i] Linear ii] Time invariant iii] casual iv] stable.

Sol:

given

$$y(n) = x(n) + \frac{1}{x(n-1)}$$

i] Linear system (or) non-linear system:

$$y_1(n) = x_1(n) + \frac{1}{x_1(n-1)}$$

$$y_2(n) = x_2(n) + \frac{1}{x_2(n-1)}$$

$$a y_1(n) + b y_2(n) = a\left[x_1(n) + \frac{1}{x_1(n-1)}\right] + b\left[x_2(n) + \frac{1}{x_2(n-1)}\right] \quad —①$$

$$y_3(n) = T[a x_1(n) + b x_2(n)]$$

$$= a x_1(n) + b x_2(n) + \frac{1}{a x_1(n-1) + b x_2(n-1)} \quad —②$$

$\therefore y_3(n) \neq a y_1(n) + b y_2(n)$

$\therefore$ the system is non-linear.

ii] Time variant & Invariant system:

condition for time invariant is $y(n,K) = y(n-K)$

$$y(n,K) = x(n-K) + \frac{1}{x(n-K-1)} \quad —①$$

$$y(n-K) = x(n-K) + \frac{1}{x(n-K-1)} \quad —②$$

$\therefore y(n,K) = y(n-K)$

$\therefore$ The system is time invariant

iii] casual

$$y(n) = x(n) + \frac{1}{x(n-1)}$$

if $n = -1 \Rightarrow y(-1) = x(-1) + \frac{1}{x(-1-1)} = \underbrace{x(-1)}_{\text{present}} + \frac{1}{\underbrace{x(-2)}_{\text{past}}}$

$n=0 \Rightarrow y(0) = x(0) + \frac{1}{x(0-1)} = \underset{\text{present}}{x(0)} + \underset{\text{past}}{\frac{1}{x(-1)}}$

$n=1, \Rightarrow y(1) = x(1) + \frac{1}{x(1-1)} = \underset{\text{present}}{x(1)} + \underset{\text{past}}{\frac{1}{x(0)}}$

$\therefore$ The system o/p depends on present & past values of the i/p then it is called casual system.

iv] stable

if $x(n) = \delta(n)$ then $y(n) = h(n)$

$\therefore h(n) = \delta(n) + \frac{1}{\delta(n-1)}$

we have $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

$\Rightarrow \sum_{n=-\infty}^{\infty} \delta(n) + \frac{1}{\delta(n-1)}$

$\Rightarrow \sum_{n=0}^{\infty} \delta(n) + \frac{1}{\delta(n-1)}$

$= \delta(0) + \frac{1}{\delta(-1)} + \delta(1) + \frac{1}{\delta(0)} + \delta(2) + \frac{1}{\delta(1)} \cdots$

$= 1 + \frac{1}{0} + 0 + \frac{1}{1} + 0 + \frac{1}{0}$

$= \infty$

$\therefore$ The system is unstable.

Q2. Determine the frequency response ~~of an LTI system~~ & sketch the magnitude & phase response of the system $y(n) = \frac{1}{2}[x(n) + x(n-2)]$

Sol: given

$y(n) = \frac{1}{2}[x(n) + x(n-2)]$

Apply DTFT on b.s.

$Y(e^{j\omega}) = \frac{1}{2}[X(e^{j\omega}) + e^{-j2\omega} X(e^{j\omega})]$

$= Xe^{j\omega}[0.5 + 0.5e^{-j2\omega}]$

$= Xe^{j\omega}[0.5 + 0.5(\cos 2\omega - j\sin 2\omega)]$

$= Xe^{j\omega}[0.5 + 0.5\cos 2\omega - 0.5j\sin 2\omega]$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = [0.5 + 0.5\cos 2\omega - 0.5 j \sin 2\omega]$$

$$H(e^{j\omega}) = 0.5[1+\cos 2\omega] - j0.5\sin 2\omega$$

Magnitude :

$$|H(e^{j\omega})| = \sqrt{(0.5)^2(1+\cos 2\omega)^2 + (0.5)^2(\sin 2\omega)^2}$$

$$= 0.5\sqrt{1+\cos^2 2\omega + 2\cos 2\omega + \sin^2 2\omega}$$

$$= 0.5\sqrt{2(1+\cos 2\omega)}$$

$$= 0.5\sqrt{2(2\cos^2\omega)}$$

$$= 0.5\sqrt{4\cos^2\omega}$$

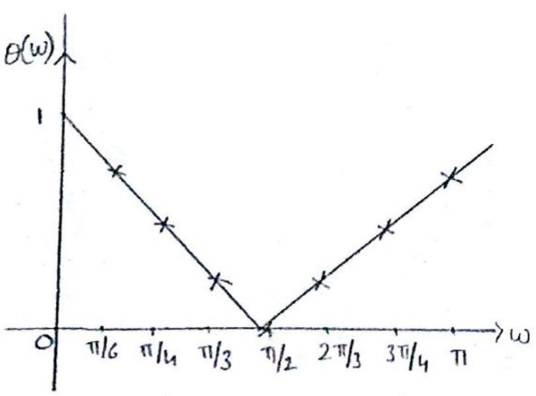
$$= \cos\omega$$

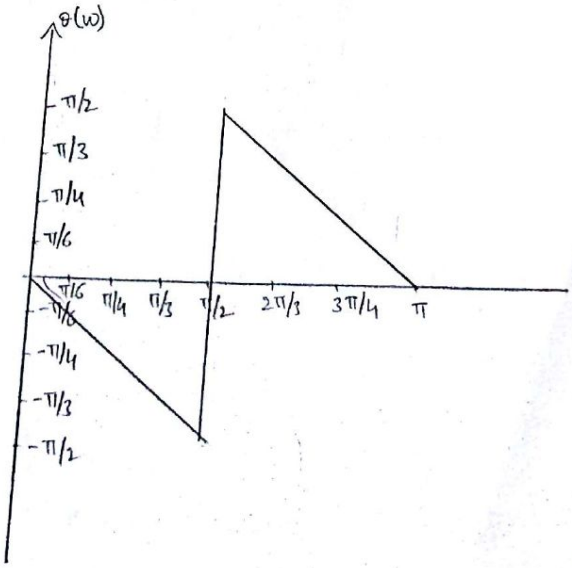
$$\underline{|\theta(\omega)} = \tan^{-1}\left[\frac{-0.5\sin 2\omega}{0.5(1+\cos 2\omega)}\right] = \tan^{-1}\left[\frac{-2\sin\omega\cos\omega}{2\cos^2\omega}\right] = \tan^{-1}(-\tan\omega) = -\omega$$

$$\underline{|H(e^{j\omega})} = -\omega \text{ for } H(e^{j\omega}) > 0$$

$$= -\omega + \pi \quad \text{for } H(e^{j\omega}) < 0$$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\omega$ : | $0^\circ$ | $30^\circ$ | $45^\circ$ | $60^\circ$ | $90^\circ$ | $120^\circ$ | $150^\circ$ | $180^\circ$ |
| $\lvert H(e^{j\omega})\rvert$ : | 1 | 0.862 | 0.707 | 0.5 | 0 | 0.5 | 0.707 | 1 |
| $\theta(\omega)$ : | $0^\circ$ | $-\pi/6$ | $-\pi/4$ | $-\pi/6$ | $-\pi/2$ | $\pi/3$ | $\pi/4$ | 0 |



Q3. obtain the circular convolution b/w the sequences using concentric circle method

a. $x_1(n) = \{1, 1, 2, 1\}$ and $x_2(n) = \{2, 3, 1, 1\}$

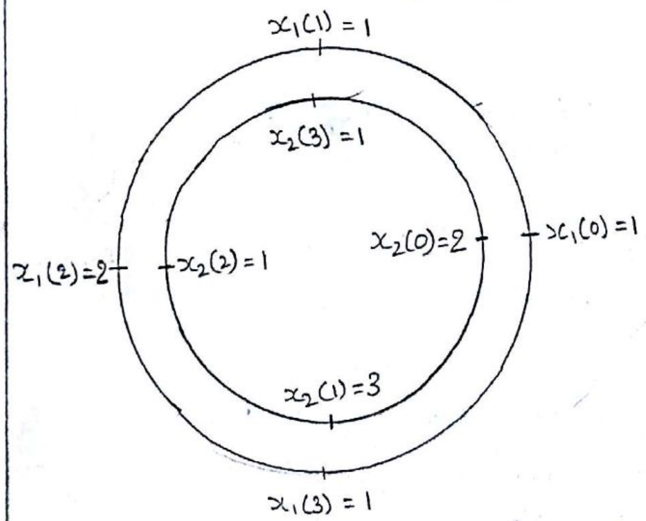
b. $x_1(n) = \{3, -2, 1, 4, 1\}$ and $x_2(n) = \{2, 5, 3\}$

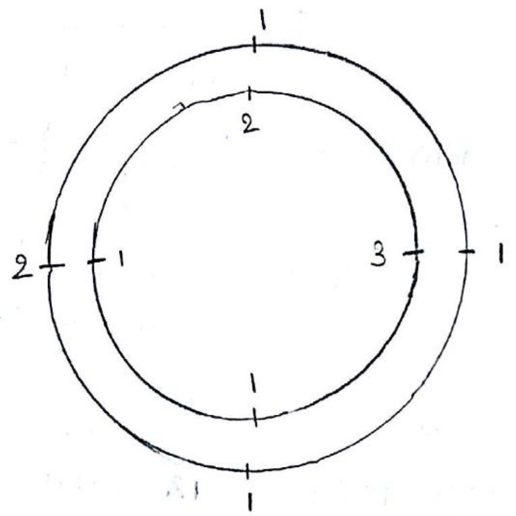
Sol: given

a. $x_1(n) = \{1, 1, 2, 1\}$ & $x_2(n) = \{2, 3, 1, 1\}$

$x_1(n)$ has 4 samples & $x_2(n)$ has 4 samples

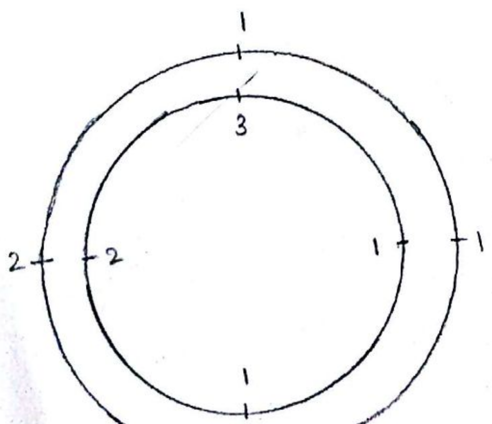
$\therefore\ x_1(n) = \{\underset{x_1(0)}{1}, \underset{x_1(1)}{1}, \underset{x_1(2)}{2}, \underset{x_1(3)}{1}\}$ $\quad x_2(n) = \{\underset{x_2(0)}{2}, \underset{x_2(1)}{3}, \underset{x_2(2)}{1}, \underset{x_2(3)}{1}\}$

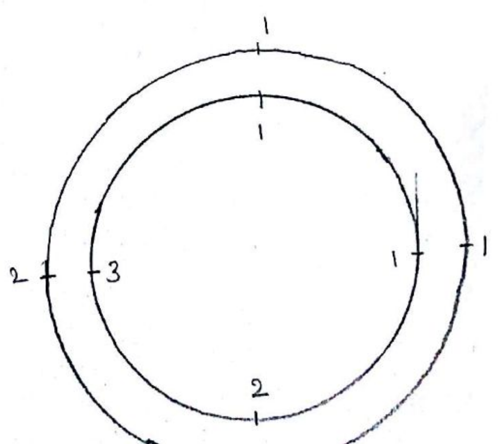



$x_3(0) = 1(2) + 1(1) + 1(2) + 1(3)$
$= 2 + 1 + 2 + 3 = 8$

$x_3(1) = 1(3) + 1(2) + 2(1) + 1(1)$
$= 3 + 2 + 2 + 1 = 8$




$x_3(2) = 1(1) + 3(1) + 2(2) + 1(1) = 9$

$x_3(3) = 1(1) + 1(1) + 2(3) + 2(1) = 10$

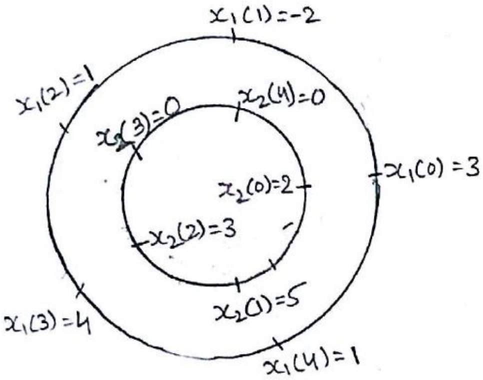
$\therefore\ x_3(n) = \{8, 8, 9, 10\}$

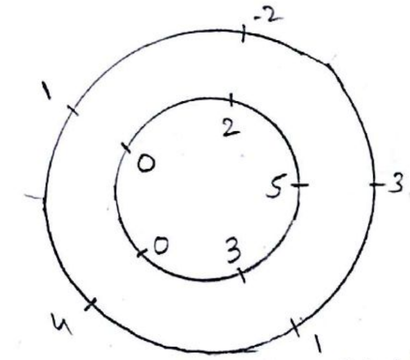
b. $x_1(n) = \{3, -2, 1, 4, 1\}$ & $x_2(n) = \{2, 5, 3\}$

$x_1(n)$ has 5 samples & $x_2(n)$ has 3 samples the result in sequence $x_3(n)$ contains $\max(L, M) = \text{Max}(5, 3) = 5$

To perform circular convolution we must add $5-3=2$ we must add zeros to the second sequence.

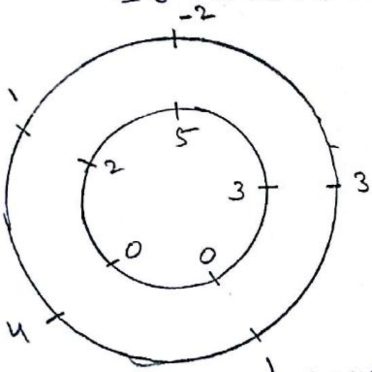
$x_1(n) = \{\underset{x_1(0)}{3}, \underset{x_1(1)}{-2}, \underset{x_1(2)}{1}, \underset{x_1(3)}{4}, \underset{x_1(4)}{1}\}$ $\quad x_2(n) = \{\underset{x_2(0)}{2}, \underset{x_2(1)}{5}, \underset{x_2(2)}{3}, \underset{x_2(3)}{0}, \underset{x_2(4)}{0}\}$

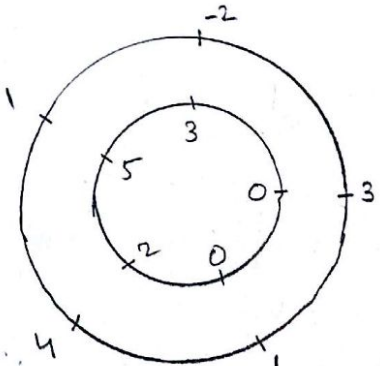



$x_3(0) = 3(2) + 0(-2) + 0(1) + 4(3) + 1(5)$
$= 6 + 0 + 0 + 12 + 5 = 23$

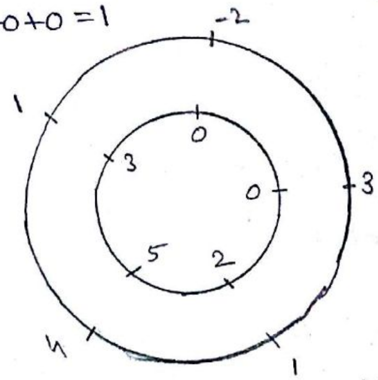
$x_3(1) = 3(5) + 2(-2) + 0(1) + 4(0) + 1(3)$
$= 15 - 4 + 0 + 0 + \cancel{3} = 14$




$x_3(2) = 3(3) + 5(-2) + 2(1) + 0(4) + 1(0)$
$= 9 - 10 + 2 + 0 + 0 = 1$

$x_3(3) = 3(0) + 3(-2) + 5(1) + 2(4) + 1(0)$
$= 0 - 6 + 5 + 8 + 0 = 7$




$x_3(4) = 0(3) + 0(-2) + 3(1) + 5(4) + 2(1) = 0 + 0 + 3 + 20 + 2 = 25$

$\therefore x_3(n) = \{23, 14, 1, 7, 25\}$

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Q4. Find the DFT of the sequence $x(n) = 1$ for $0 \le n \le 2$
$= 0$ otherwise

for i] N=4 & ii] N=8. plot magnitude & phase responses & comment on the result.

Sol:

$$x(0) = x(1) = x(2) = 1$$
$$x(3) = x(4) = \ldots\ldots = 0$$

$N = 4$

$$x(n) = \{1, 1, 1, 0\}$$

$$X(K) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi}{4}nk}$$

for K=0:

$$X(0) = \sum_{0}^{3} x(n)\, e^{0}$$
$$= x(0)e^{0} + x(1)e^{0} + x(2)e^{0} + x(3)e^{0}$$
$$= 1(1) + 1(1) + 1(1) + 0(1)$$
$$= 3$$

$\therefore |X(0)| = 3$, $\angle H(0) = \theta = 0^\circ$

for K=1

$$X(1) = \sum_{0}^{3} x(n)\, e^{-j\frac{\pi}{2}n}$$
$$= x(0) + x(1)e^{-j\pi/2} + x(2)e^{-j\pi} + x(3)e^{-j3\pi/2}$$
$$= 1 + \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} + \cos\pi - j\sin\pi + 0$$
$$= \cancel{1} - j \cancel{-1} = -j$$

$\therefore |X(1)| = 1$, $\angle X(1) = -\pi/2$

for K=2

$$X(2) = \sum_{0}^{3} x(n)\, e^{-j\pi n}$$
$$= x(0) + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$$
$$= 1 + \cos\pi - j\sin\pi + \cos 2\pi - j\sin 2\pi + 0$$
$$= \cancel{1} \cancel{-1} + 1 = 1$$

$\therefore |X(2)| = 1$, $\angle X(2) = 0^\circ$

for K=3

$$X(3) = \sum_{0}^{3} x(n)\, e^{-j3\pi n/2}$$
$$= x(0) + x(1)e^{-j3\pi/2} + x(2)e^{-j3\pi} + x(3)e^{-j9\pi/2}$$

$X(3) = 1 + \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} + \cos 3\pi - j\sin 3\pi + 0$

$= \cancel{1} + j - \cancel{1} = j$

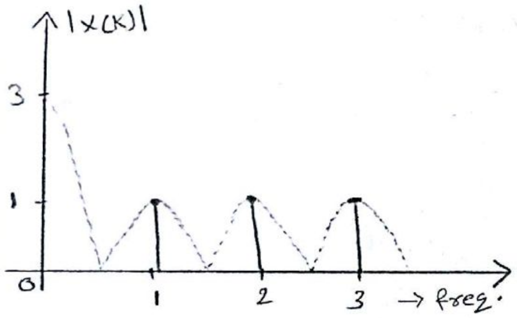
$\therefore\ |X(3)| = 1 \quad , \quad \angle X(3) = \pi/2$

$\therefore\ X(K) = \{ 3, -j, 1, j \}$

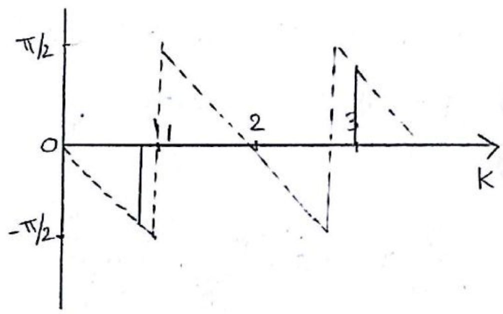
$|X(K)| = \{ 3, 1, 1, 1 \}$

$\angle X(K) = \{ 0^\circ, -\pi/2, 0^\circ, \pi/2 \}$

Magnitude spectrum:




phase spectrum:




<u>N=8</u>

$L = 3$ ~~zeros should be added~~ $N-L$ zeros i.e. $8-3 = 5$ zeros

$\therefore\ x(n) = \{ 1, 1, 1, 0, 0, 0, 0, 0 \}$

$$X(K) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi}{8}nK} = \sum_{n=0}^{7} x(n)\, e^{-j\frac{\pi}{4}nK}$$

for $K=0$

$$X(0) = \sum_{n=0}^{7} x(n) = 1+1+1+0+0+0+0+0 = 3$$

$\therefore\ |X(0)| = 3 \quad , \quad \angle X(0) = 0^\circ$

for $K=1$

$$X(1) = \sum_{n=0}^{7} x(n)\, e^{-j\pi n/4}$$

$= x(0) + x(1)\, e^{-j\pi/4} + x(2)\, e^{-j\pi/2}$

$= 1 + 0.707 - j0.707 + 0 - j$

$= 1.707 - j1.707$

$\therefore\ |X(1)| = 2.414 \quad , \quad \angle X(1) = -\pi/4$

for $K=2$

$$X(2) = \sum_{n=0}^{7} x(n) e^{-j\pi n/2} = x(0) + x(1) e^{-j\pi/2} + x(2) e^{-j\pi}$$

$$= 1 + \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} + \cos\pi - j\sin\pi = \cancel{1} - j - \cancel{1} = -j$$

$$\therefore |X(2)| = 1 \ , \ \angle X(2) = -\pi/2$$

for $K=3$

$$X(3) = \sum_{n=0}^{7} x(n) e^{-j3\pi n/4} = x(0) + x(1) e^{-j3\pi/4} + x(2) e^{-j3\pi/2}$$

$$= 1 + \cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4} + \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}$$

$$= 1 - 0.707 - j0.707 + j = 0.293 + j0.293$$

$$\therefore |X(3)| = 0.414 \ , \ \angle X(3) = \frac{\pi}{4}$$

for $K=4$

$$X(4) = \sum_{n=0}^{7} x(n) e^{-j\pi n} = x(0) + x(1) e^{-j\pi} + x(2) e^{-j2\pi}$$

$$= 1 + \cos\pi - j\sin\pi + \cos 2\pi - j\sin 2\pi$$

$$= 1 - 1 + 1 = 1$$

$$\therefore |X(4)| = 1 \ , \ \angle X(4) = 0$$

for $K=5$

$$X(5) = \sum_{n=0}^{7} x(n) e^{-j5\pi n/4}$$

$$= x(0) + x(1) e^{-j5\pi/4} + x(2) e^{-j5\pi/2}$$

$$= 1 + \cos\frac{5\pi}{4} - j\sin\frac{5\pi}{4} + \cos\frac{5\pi}{2} - j\sin\frac{5\pi}{2}$$

$$= 1 - 0.707 + j0.707 - j = 0.293 - j0.293$$

$$\therefore |X(5)| = 0.414 \ , \ \angle X(5) = -\frac{\pi}{4}$$

for $K=6$

$$X(6) = \sum_{n=0}^{7} x(n) e^{-j3\pi n/2} = x(0) + x(1) e^{-j3\pi/2} + x(2) e^{-j3\pi}$$

$$= 1 + \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} + \cos 3\pi - j\sin 3\pi$$

$$= 1 + j - 1 = j$$

$$\therefore |X(6)| = 1 \quad \angle X(6) = \frac{\pi}{2}$$

for K=7

$$X(7) = \sum_{n=0}^{7} x(n)\, e^{-j7n\pi/4}$$

$$= 1 + e^{-j7\pi/4} + e^{-j7\pi/2}$$

$$= 1 + \cos\frac{7\pi}{4} - j\sin\frac{7\pi}{4} + \cos\frac{7\pi}{2} - j\sin\frac{7\pi}{2}$$

$$= 1 + 0.707 + j0.707 + j$$

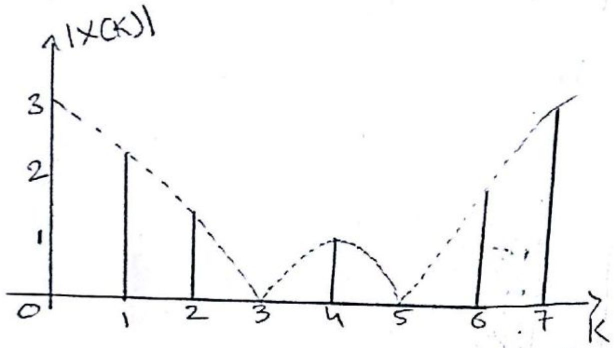
$$= 1.707 + j1.707$$

$$\therefore |X(7)| = 2.414, \quad \angle X(7) = \frac{\pi}{4}$$

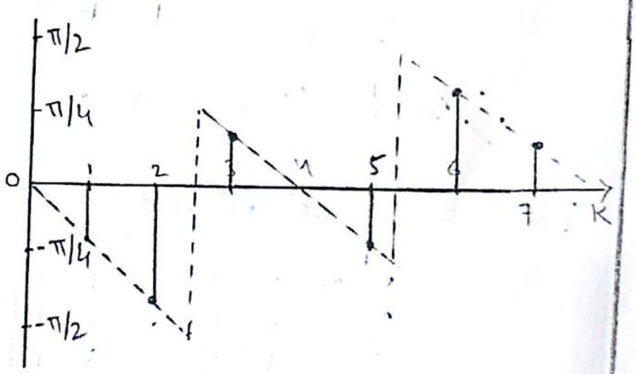
$$|X(K)| = \{3, 2.414, 1, 0.414, 1, 0.414, 1, 2.414\}$$

$$\angle X(K) = \left\{0, -\frac{\pi}{4}, -\frac{\pi}{2}, \frac{\pi}{4}, 0, -\frac{\pi}{4}, \frac{\pi}{2}, \frac{\pi}{4}\right\}$$

Magnitude spectrum:




Phase spectrum:




from N=4 point DFT the separation b/w successive samples is high & it results in poor resolution.

for N=8 the separation b/w samples is less & it increases the freq. resolution & gives the better displayed version of the plot because the spectrum has high density.

Q5. perform linear convolution of the two sequences $x(n) = \{1, 3, -2, 1, 3, 6, 4, 3, 2, 1, 1, 5\}$ and $h(n) = \{1, 2, 1, 1\}$ using over-lap save method.

sol:

$x(n) \longrightarrow L_S = 12$

$h(n) \longrightarrow M = 4$

$y(n) \longrightarrow L_S + M - 1 = 12 + 4 - 1 = 15$

let us assume no. of data points $L = 4$

each block length $N = L+M-1 = 4+4-1 = 7$

$x_1(n) = \{0, 0, 0, 1, 3, -2, 1\}$

$x_2(n) = \{3, -2, 1, 3, 6, 4, 3\}$

$x_3(n) = \{6, 4, 3, 2, 1, 1, 5\}$

To perform circular convolution we must add N-M zeros [i.e. 7-4 = 3 zeros]

$y_1(n) = x_1(n) \; (N) \; h(n)$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 1 \\ 5 \\ 5 \\ 1 \end{bmatrix}$$

$\therefore y_1(n) = \{3, -1, 1, 1, 5, 5, 1\}$

$y_2(n) = x_2(n) \; (N) \; h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \\ 3 \\ 6 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 19 \\ 11 \\ 3 \\ 6 \\ 11 \\ 20 \\ 20 \end{bmatrix}$$

$\therefore y_2(n) = \{19, 11, 3, 6, 11, 20, 20\}$

$y_3(n) = x_3(n) \; (N) \; h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 3 \\ 2 \\ 1 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 18 \\ 22 \\ 22 \\ 18 \\ 12 \\ 8 \\ 10 \end{bmatrix}$$

$\therefore y_3(n) = \{18, 22, 22, 18, 12, 8, 10\}$

discard M-1 samples [i.e 4-1=3] from each block & write remaining samples.

$\therefore y(n) = \{1, 5, 5, 1, 6, 11, 20, 20, 18, 12, 8, 10\}$

Q6. Develop an algorithm for the implementation of Radix-2 DIT FFT algorithm.

Sol: Let $x(n)$ is N point sequence where N is a power of 2. decimate (or) break the sequence into two sub sequences of length $\frac{N}{2}$. one has even indexed values of $x(n)$ & the other sequences has odd indexed values.

$$x_e(n) = x(2n) \qquad 0 \le n \le \frac{N}{2}-1$$

$$x_o(n) = x(2n+1) \qquad 0 \le n \le \frac{N}{2}-1$$

consider the DFT of the given sequence $X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nK}$

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK} \qquad \left[\because W_N = e^{-j\frac{2\pi}{N}}\right]$$

$$= \sum_{n=0\,(\text{even})}^{N-1} x(n) W_N^{nK} + \sum_{n=0\,(\text{odd})}^{N/2-1} x(n) W_N^{nK}$$

$$= \sum_{n=0}^{N/2-1} x(2n) W_N^{2nK} + \sum_{n=0}^{N/2-1} x(2n+1) W_N^{(2n+1)K}$$

$$\left[\because W_N^{2nK} = e^{-j\frac{2\pi}{N}nK(2)} = e^{-j\frac{2\pi}{N/2}nK} = W_{N/2}^{nK}\right.,$$

$$\left. W_N^{(2n+1)K} = W_N^{2nK} \cdot W_N^{K} = W_{N/2}^{nK} W_N^{K}\right]$$

$$X(K) = \underbrace{\sum_{n=0}^{N/2-1} x_e(n) W_{N/2}^{nK}}_{X_e(K)} + \underbrace{\sum_{n=0}^{N/2-1} x_o(n) W_{N/2}^{nK} W_N^{K}}_{W_N^K X_o(K)}$$

$$X(K) = X_e(K) + W_N^K X_o(K) \qquad 0 \le K \le \frac{N}{2}-1 \quad \text{—— (1)}$$

$X_e(K)$ → DFT of even indexed sequence

$X_o(K)$ → DFT of odd indexed sequence

using periodicity property

$$X_e(K) = X_e(K - N/2) \text{ —— (2)} \qquad X_o(K) = X_o(K - N/2) \text{ —— (3)}$$

using symmetry property of twiddle factor

$$W_N^K = -W_N^{(K-N/2)}$$

Sub ② & ③ in ①

$$X(K) = X_e(K-N/2) - W_N^{(K-N/2)} X_o(K-N/2) \quad ; \quad \frac{N}{2} \le K \le N-1$$

Let us take $N=8$

$$X(K) = X_e(K) + W_8^K X_o(K) \quad ; \quad 0 \le K \le 3$$

$$X(K) = X_e(K-4) - W_8^{(K-4)} X_o(K-4) \quad ; \quad 4 \le K \le 7$$

$$X(0) = X_e(0) + W_8^0 X_o(0)$$

$$X(1) = X_e(1) + W_8^1 X_o(1)$$

$$X(2) = X_e(2) + W_8^2 X_o(2)$$

$$X(3) = X_e(3) + W_8^3 X_o(3)$$

for $N=4$

$$X(4) = X_e(0) - W_8^0 X_o(0)$$

$$X(5) = X_e(1) - W_8^1 X_o(1)$$

$$X(6) = X_e(2) - W_8^2 X_o(2)$$

$$X(7) = X_e(3) - W_8^3 X_o(3)$$

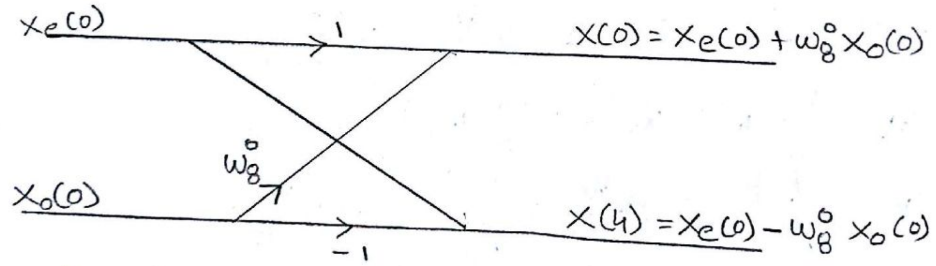
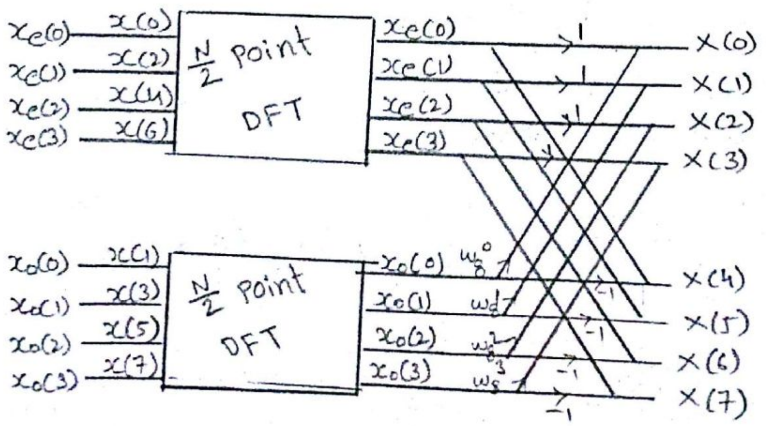



fig: flow graph of butterfly diagram.



for $\frac{N}{2}$ point DFT

$$X(K) = X_e(K) + W_N^K X_o(K) \qquad 0 \leq K \leq \frac{N}{2} - 1$$

$$= X_e(K-\frac{N}{2}) - W_N^{(K-\frac{N}{2})} X_o(K-\frac{N}{2}) \qquad \frac{N}{2} \leq K \leq N-1$$

$\frac{N}{4}$ point DFT of even indexed sequence

$$X_e(K) = X_{ee}(K) + W_{N/2}^K X_{eo}(K) \qquad 0 \leq K \leq \frac{N}{4} - 1$$

we have $W_{N/2}^K = W_N^{2K}$

$$X_e(K) = X_{ee}(K) + W_N^{2K} X_{eo}(K) \qquad 0 \leq K \leq \frac{N}{4} - 1$$

symmetry & periodicity property

$$X_e(K) = X_{ee}(K - N/4) - W_N^{2(K-N/4)} X_{eo}(K-\frac{N}{4}) \qquad \frac{N}{4} \leq K \leq \frac{N}{2} - 1$$

$\frac{N}{4}$ point DFT of odd indexed sequence

$$X_o(K) = X_{oe}(K) + W_{N/2}^K X_{oo}(K) \qquad 0 \leq K \leq \frac{N}{4} - 1$$

$$X_o(K) = X_{oe}(K) + W_N^{2K} X_{oo}(K) \qquad 0 \leq K \leq \frac{N}{4} - 1$$

periodicity & symmetry property

$$X_o(K) = X_{oe}(K-\frac{N}{4}) - W_N^{2(K-\frac{N}{4})} X_{oo}(K-\frac{N}{4}) \qquad \frac{N}{4} \leq K \leq \frac{N}{2} - 1$$

for $N=8$

$$X_e(K) = X_{ee}(K) + W_8^{2K} X_{eo}(K) \qquad 0 \leq K \leq 1$$

$K=0 \Rightarrow X_e(0) = X_{ee}(0) + W_8^0 X_{eo}(0)$

$K=1 \Rightarrow X_e(1) = X_{ee}(1) + W_8^2 X_{eo}(1)$

$$X_e(K) = X_{ee}(K-\frac{N}{4}) - W_N^{2(K-N/4)} X_{eo}(K-\frac{N}{4}) \qquad \frac{N}{4} \leq K \leq \frac{N}{2} - 1$$

$$2 \leq K \leq 3$$

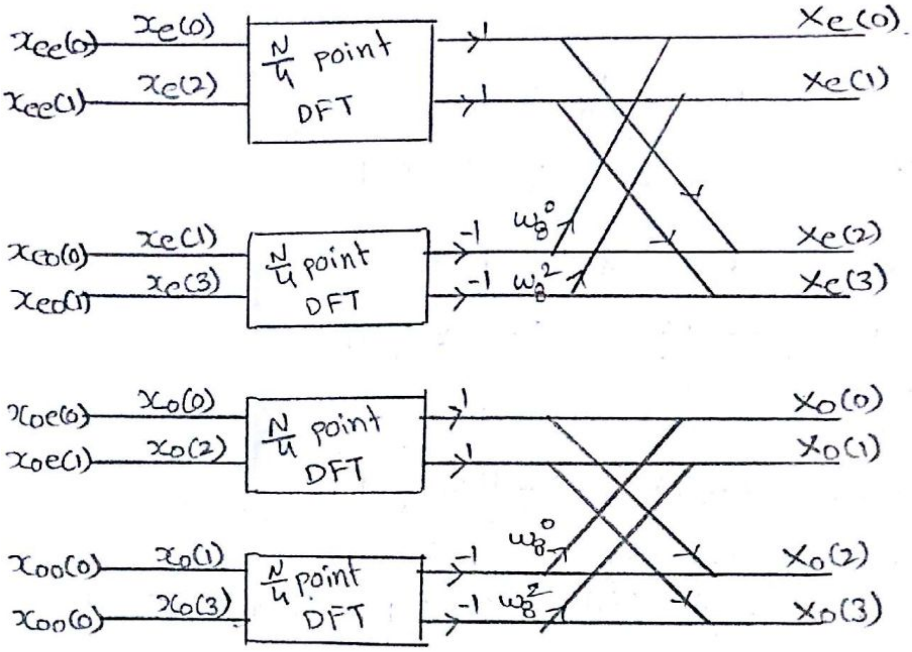
$X_e(2) = X_{ee}(0) - W_8^0 X_{eo}(0)$

$X_e(3) = X_{ee}(1) - W_8^2 X_{eo}(1)$

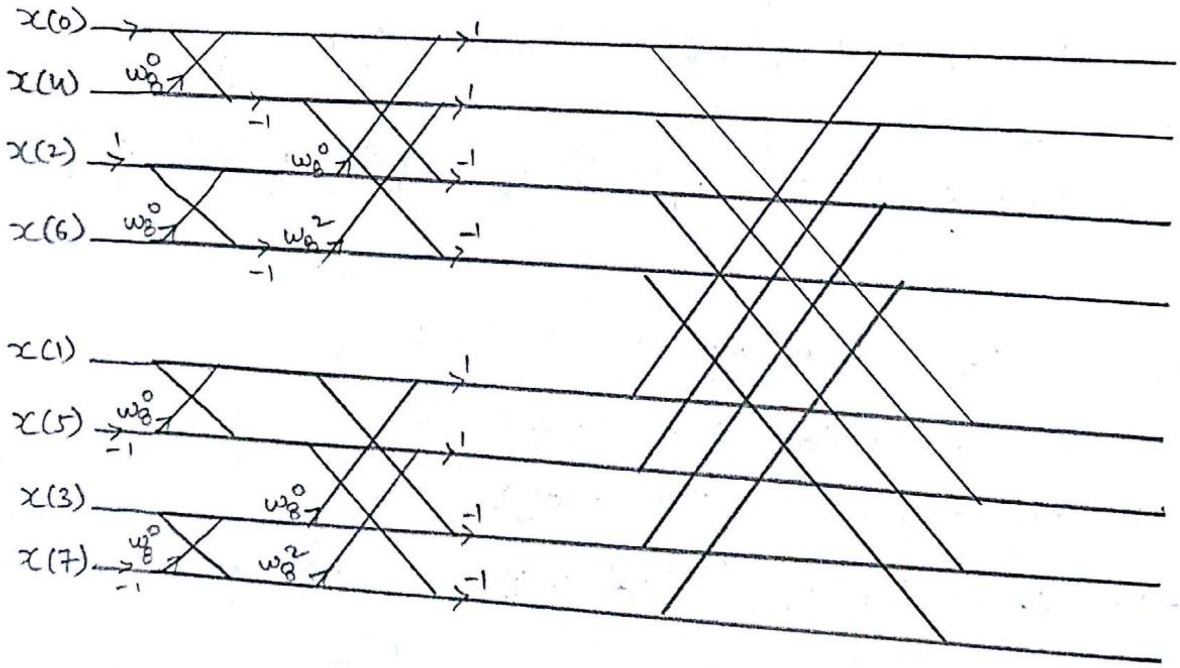
odd indexed

$$\left.\begin{aligned} X_o(0) &= X_{oe}(0) + W_8^0 X_{oo}(0) \\ X_o(1) &= X_{oe}(1) + W_8^2 X_{oo}(1) \end{aligned}\right\} \quad 0 \leq K \leq 1$$

$$\left.\begin{aligned} X_o(2) &= X_{oe}(0) - W_8^0 X_{oo}(0) \\ X_o(3) &= X_{oe}(1) - W_8^2 X_{oo}(0) \end{aligned}\right\} \quad 2 \leq K \leq 3$$



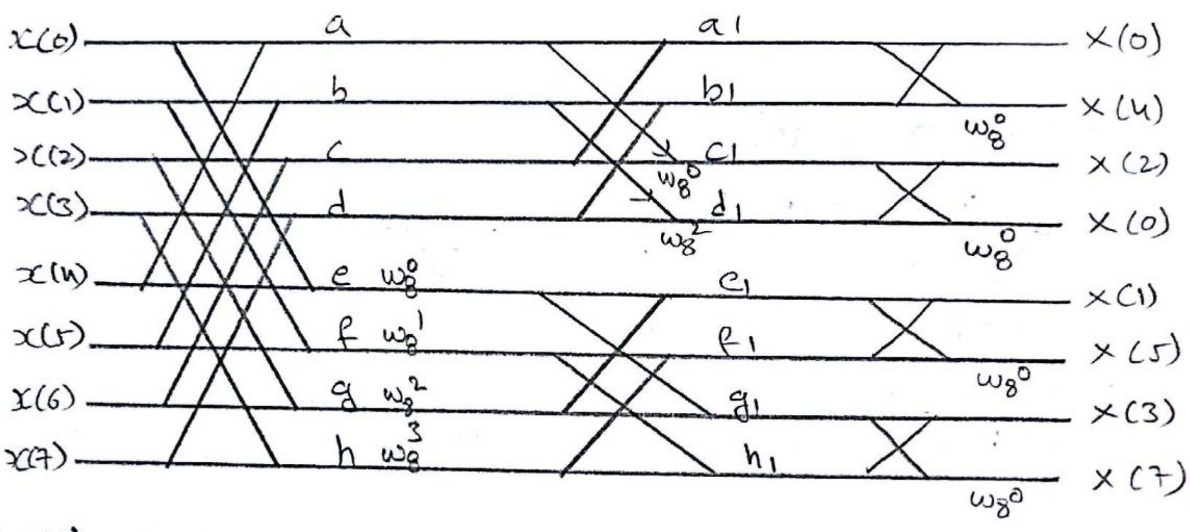

final flow graph



Q7 Find the DFT of the sequence $x(n) = \{2,1,2,1,2,1,2,1\}$ using Radix-2 DIF FFT algorithm.

Sol:

$$x(n) = \{\underset{x(0)}{2}, \underset{x(1)}{1}, \underset{x(2)}{2}, \underset{x(3)}{1}, \underset{x(4)}{2}, \underset{x(5)}{1}, \underset{x(6)}{2}, \underset{x(7)}{1}\}$$




Twiddle factor

$$W_8^0 = e^{-j\frac{2\pi}{8}(0)} = e^0 = 1$$

$$W_8^1 = 0.707 - j0.707$$

$$W_8^2 = -j$$

$$W_8^3 = -0.707 - j0.707$$

stage-I

$a = x(0) + x(4) = 2 + 2 = 4$

$b = x(1) + x(5) = 1 + 1 = 2$

$c = x(2) + x(6) = 2 + 2 = 4$

$d = x(3) + x(7) = 1 + 1 = 2$

$e = x(0) - x(4) = 2 - 2 = 0$

$f = x(1) - x(5) = 1 - 1 = 0$

$g = x(2) - x(6) = 2 - 2 = 0$

$h = x(3) - x(7) = 1 - 1 = 0$

stag-II

$a_1 = a + c = 4 + 4 = 8$

$b_1 = b + d = 2 + 2 = 4$

$c_1 = (a - c) W_8^0 = (4 - 4) 1 = 0$

$d_1 = (b - d) W_8^2 = (2 - 2)(-j) = 0$

$e_1 = (e-g)\,\omega_8^2 = (0-0)(-j) = 0$

$f_1 = 0$

$g_1 = 0$

$h_1 = 0$

Stage-III

$x(0) = a_1 + b_1 = 8 + 4 = 12$

$x(4) = (a_1 - b_1)\,\omega_8^0 = (8-4)1 = 4$

$x(2) = c_1 + d_1 = 0$

$x(6) = (c_1 - d_1)\,\omega_8^0 = 0$

$x(1) = e_1 + f_1 = 0$

$x(5) = (e_1 - f_1)\,\omega_8^0 = 0$

$x(3) = g_1 + h_1 = 0$

$x(7) = (g_1 - h_1)\,\omega_8^0 = 0$

$\therefore\ x(K) = \{12, 4, 0, 0, 0, 0, 0, 0\}$