Representation of discrete signals ($x[n]$)

It is denoted by $x[n]$

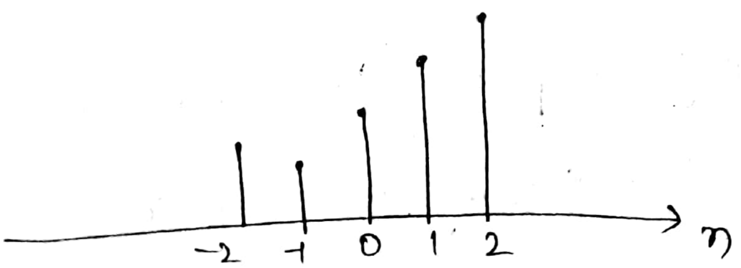
$n \rightarrow$ sample index or time index

It can be represented by

1) graphical representation
2) Functional "
3) Tabular "
4) Sequence "

1) Graphical Representation:

$x(-2) = 2$, $x(-1) = 1$, $x(0) = 3$, $x(1) = 4$, $x(2) = 5$



2) Function representation:

$$x[n] = \begin{cases} 2 & n = -2 \\ 1 & n = -1 \\ 3 & n = 0 \\ 4 & n = 1 \\ 5 & n = 2 \end{cases}$$

3) Tabular Representation:

n	-2	-1	0	1	2
$x[n]$	2	1	3	4	5

ii) Sequence Representation.

$$x[n] = \{2, 1, \underset{\underset{n=0}{\uparrow}}{3}, 4, 5\}$$

Classification of discrete time signals:

1) Energy & power signals.
2) Even & Odd signals
3) Causal & Noncausal signals
4) Periodic and Aperiodic signals

Energy and Power signals:

→ Energy signal: Energy is finite, avg. power is zero.

→ Power signal: Avg. power is finite, energy is infinite

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \underset{N \to \infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$N \to$ sequence length.

Eg: $x[n] = \{1, 2, 3, 4\}$

$N = 4$

$N = 0$ to $N-1$

E = finite, avg. power is zero → energy signal

$E \to \infty$, P - finite → power signal.

① $x[n] = \left(\frac{1}{3}\right)^n u(n)$

Sol: $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{2n}$$

$$= 1 + \left(\frac{1}{9}\right) + \left(\frac{1}{9}\right)^2 + \cdots$$

$$= \frac{1}{1-\frac{1}{9}} = \frac{9}{8}$$

E → finite ⇒ Energy signal.

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \ \& \ \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \text{ (finite value)}$$

$$= 0$$

E → finite, $P_{avg} = 0$ → Energy signals.

(or)

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{3}\right)^n$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \frac{1-\left(\frac{1}{9}\right)^{N+1}}{1-\frac{1}{9}}$$

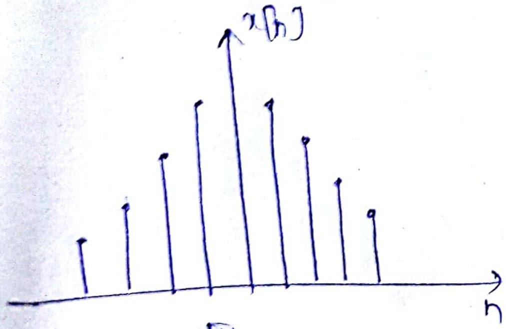
② $x[n] = e^{2n} u[n]$

2) Even and odd signals:

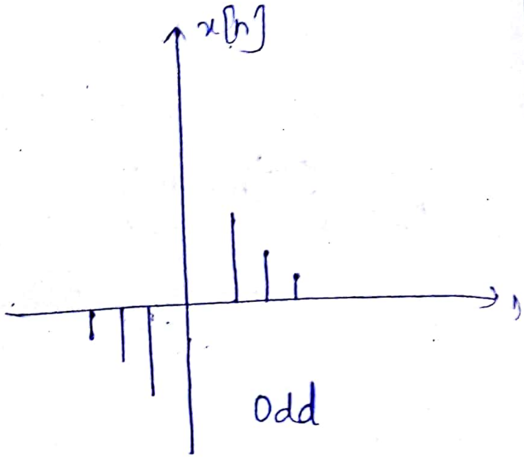
If $x[n]$ is a signal

→ If $x(-n) = x(n)$ → even → $\cos(\omega n), t^2$

$x(-n) = -x(n)$ → odd → $\sin(\omega n), t^3$



Even.



Odd

Even part of signal $x_e(n) = \dfrac{x(n) + x(-n)}{2}$

Odd part of signal $x_o(n) = \dfrac{x(n) - x(-n)}{2}$

3) Causal & Non-causal signal:

If $x(n) = 0$ for $n < 0$ → causal

otherwise it is noncausal signal.

Eg. Causal

$x(n) = a^n u(n)$

$x(n) = \{-1, 1, 2, 3, 4\}$

Noncausal.

$x(n) = a^n u(n+1)$

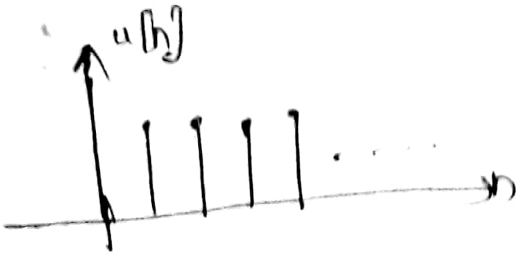
$x(n) = \{-1, 1, \underset{\uparrow}{2}, 3, 4\}$

Basic signals:

Unit step

$u(n) = 1 \quad n > 0$

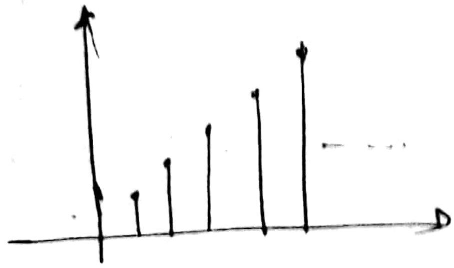
$= 0 \quad n < 0$



Ramp:

$r(n) = n \quad n \geq 0$

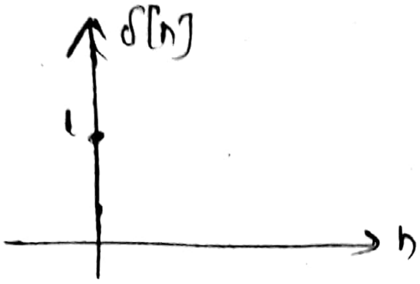
$= 0 \quad n < 0$



Impulse:

$\delta(n) = 1 \quad n = 0$

$= 0 \quad n \neq 0$



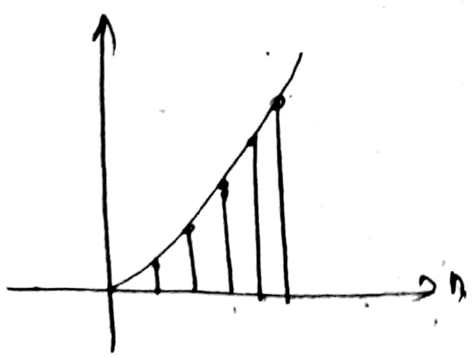
Exponential:

$x[n] = a^n u[n]$

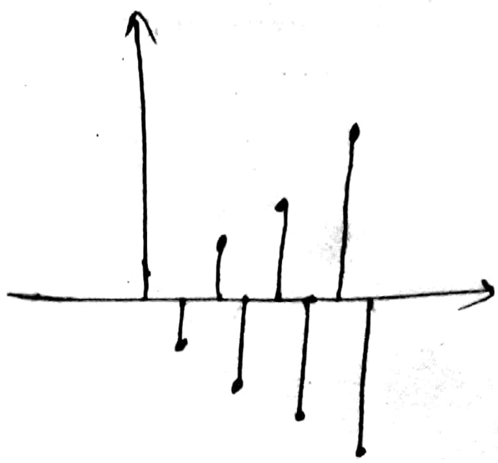
$a > 0 \qquad 0 < a < 1$

$a < 1 \qquad -1 < a < 0$

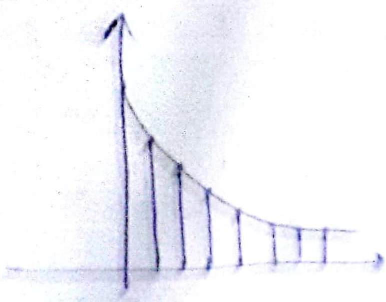
if $a > 1 \rightarrow$ increasing



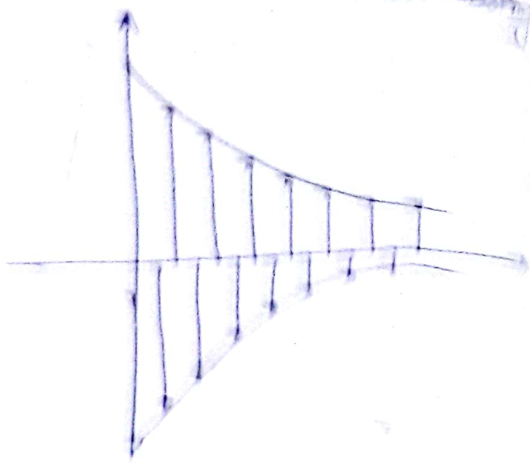
if $a < 1 \rightarrow$ both sides $\uparrow$



If $0<a<1$

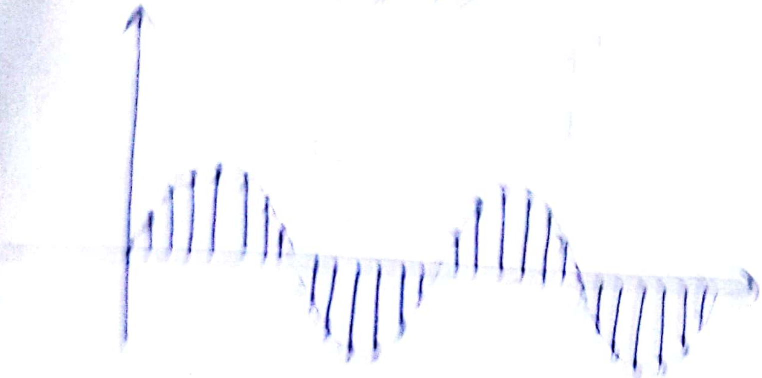


$-1<a<0$ → both sides decreasing



Sinusoidal signal:

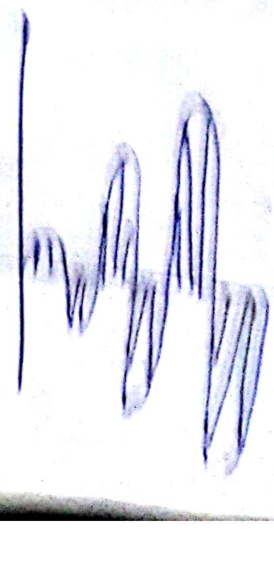
$x[n] = A \sin(\omega_0 n + \phi)$

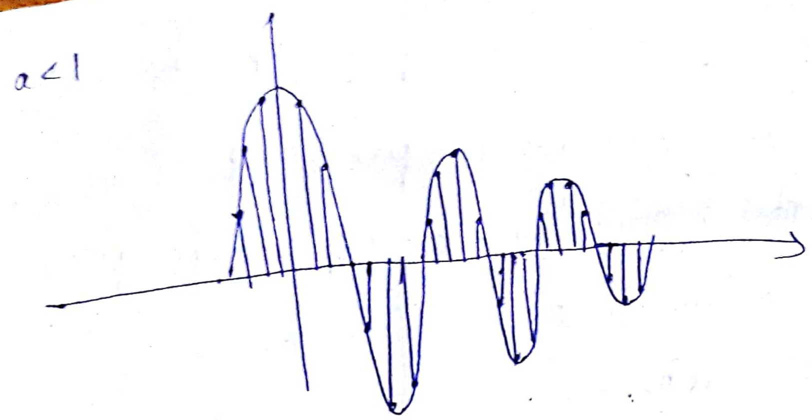


Complex exponentials:

$x[n] = A^n e^{j(\omega_0 n + \phi)}$

$A = 1$ sinusoidal





4) Periodic & Aperiodic signals (sequences)

Signals which is having definite pattern and repeats for every 'N' are called periodic signals.

For continuous $x(t+T) = x(t)$

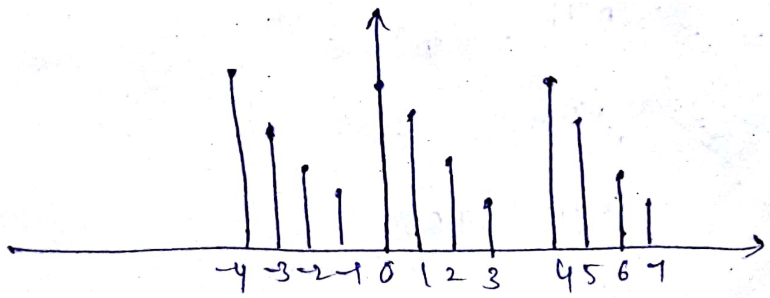
For discrete $x(n+N) = x(n)$

$N \rightarrow$ sequence length.

The smallest value of 'N' at which the signal repeats is called fundamental period (which satisfies above condition)

→ A discrete time signal $x[n]$ is said to be periodic with period 'N' if $x(n+N) = x(n)$

Otherwise, it is aperiodic



→ If $x_1(n)$ is periodic with period 'N_1'

and $x_2(n)$ is periodic with period 'N_2' then their linear combination need not be periodic but all discrete time sinusoidal signals may or may not be periodic

Ex: $x[n] = A \sin(\omega_0 n + \phi)$

$x(n+N) = x(n)$

$A \sin((\omega_0(n+N)) + \phi) = A \sin(\omega_0 n + \phi)$

$A \sin(\omega_0 n + \omega_0 N + \phi) = A \sin(\omega_0 n + \phi)$

It is periodic when $\omega_0 N$ is an integral multiple of '2π'

$\omega_0 N = 2\pi m$

$N = \frac{2\pi m}{\omega_0}$

$$\boxed{\omega_0 = 2\pi\left(\frac{m}{N}\right)}$$

From the above equation, the discrete time signal to be periodic, the fundamental period ω_0 must be rational multiple of '2π'

① $x(n) = e^{j\omega_0 n}$

Sol: $x(n+N) = e^{j\omega_0(n+N)} = x(n) = e^{j\omega_0 n}$

$\cancel{e^{j\omega_0 n}} e^{j\omega_0 N} = \cancel{e^{j\omega_0 n}}$

$\omega_0 N = 2\pi k$

$$\boxed{N = \frac{2\pi k}{\omega_0}}$$

② $x(n)$

Sol:

③ x

Sol:

'ω_0

④

Sol:

(2) $x(n) = e^{j6\pi n}$

Sol: $e^{j6\pi(n+N)} = e^{j6\pi n}$

$e^{j6\pi N} = 1$

$6\pi N = 2\pi k$

$N = \frac{k}{3}$

$N = \frac{2\pi m}{6\pi} = \frac{1\times m}{3}$

$N=1$ for $m=3$

(3) $x(n) = e^{j\frac{3}{5}\left(n+\frac{1}{2}\right)}$

Sol: $x(n+N) = x(n)$

$e^{j\frac{3}{5}\left(n+N+\frac{1}{2}\right)} = e^{j\frac{3}{5}\left(n+\frac{1}{2}\right)}$

$e^{j\frac{3}{5}N} = 1$

$\frac{3}{5}N = 2\pi k$

$N = \frac{2\pi m}{3}\times 5$

N is not an integer

'ω_0' is not an integral multiple of '2π'

∴ Aperiodic

(4) $x(n) = \cos\frac{\pi}{3}n - \cos\frac{3\pi}{4}n$

Sol: $N_1 = 2\cancel{\pi}\cdot\frac{m}{\frac{\cancel{\pi}}{3}} = 6m$, $N_2 = 2\cancel{\pi}\cdot\frac{m}{\frac{3\cancel{\pi}}{4}} = \frac{8m}{3}$

$N_2 = 8$ for $m=3$

$\frac{N_1}{N_2} = \frac{\overset{3}{\cancel{6m}}}{\frac{{}_4\cancel{8m}}{\cancel{3}}} = \frac{3}{4}$

$N = 4N_1 = 3N_2 = 24$

① $x(n) = \{-2, 5, \underset{\uparrow}{1}, -3\}$

Sol: $x(-2) = -2$

$x(-1) = 5$

$x(0) = 1$

$x(1) = -3$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$x(n) = \{-2, 5, \underset{\uparrow}{1}, -3\}$

$x(-n) = \{-3, \underset{\uparrow}{1}, -2, 5\}$

$x_e(n) = \{-1, 1, \underset{\uparrow}{1}, 1, -1\}$

$x_o(n) = \{-1, 4, \underset{\uparrow}{0}, -4, 1\}$

② $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$

Sol: $x(-n) = \{5, 4, 3, 2, \underset{\uparrow}{1}\}$

$$x_e(n) = \frac{x(n) + x(-n)}{2} = \{2.5, 3\ 1.5, 1, 1, 1.5, 2, 2.5\}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2} = \{-2.5, -2, -1.5, -1, 0, 1, 1.5, 2, 2.5\}$$

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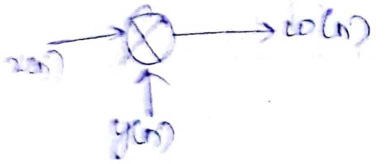
Operation on signals:

1) **Signal multiplication:**

$w(n)$ is product of 2 signals $x(n) \cdot y(n)$

$w(n) = x(n) \cdot y(n)$



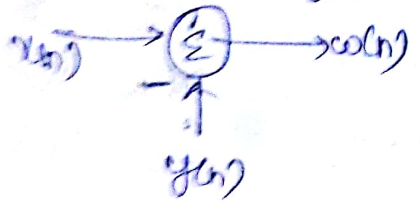
~~The~~ corresponding samples are multiplied ($1^{st} \times 1^{st}$)

~~It~~ is used in FIR (Finite Impulse Response) ~~filter~~ in DSP.

~~An~~ infinite length sequence is multiplied with window ~~function~~ (finite sequence) resulting in finite length sequence.

2) **Signal addition:**

$w(n) = x(n) + y(n)$



3) **Scalar multiplication:**

4) **Time shifting:**

~~If~~ $x(n)$ is a sequence, $x(n-k)$ is shifted version of $x(n)$

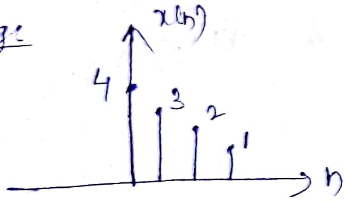
$x(n-k)$

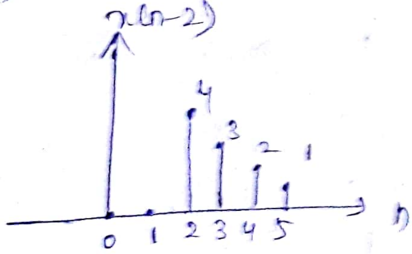
k is +ve ⇒ eg. k = 2

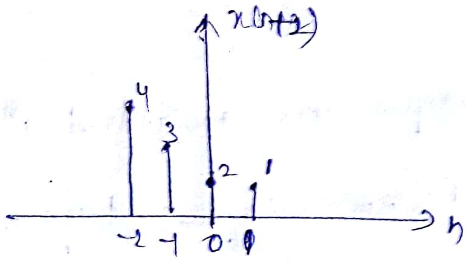
It is right shifted by 2 units

k is -ve ⇒ $y(n) = x(n+2)$ ⇒ ~~to~~ left shift by 2 units

Eg:

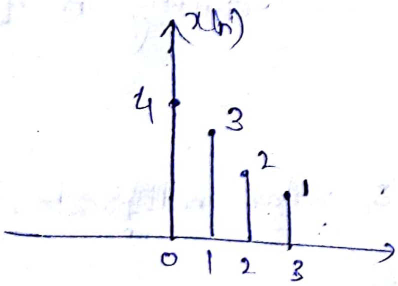


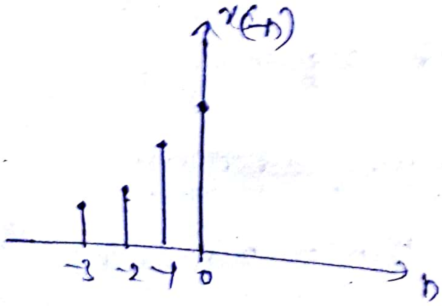


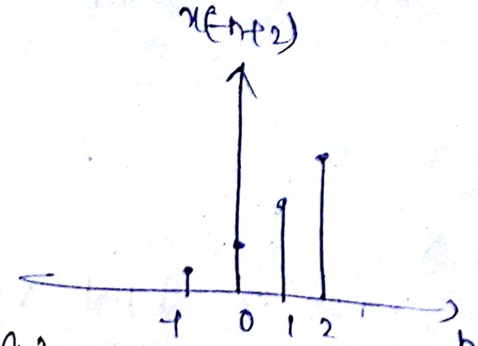


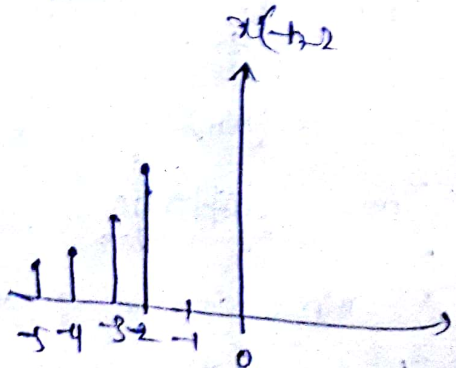
5) Time Reversal:

It is folded / flip over sequence of $x(n)$ i.e. $x(-n)$

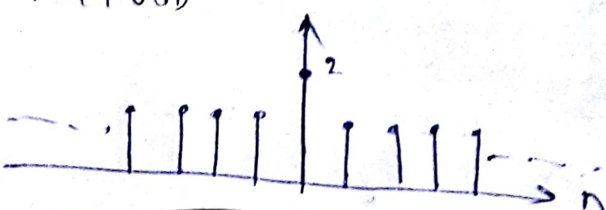






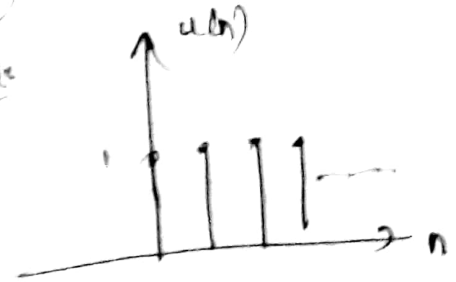


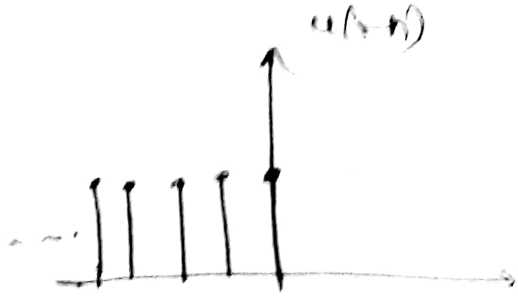
*** (i) $u(n) + u(-n) = 1 + \delta(n)$

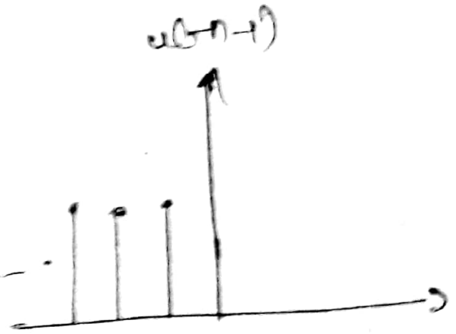


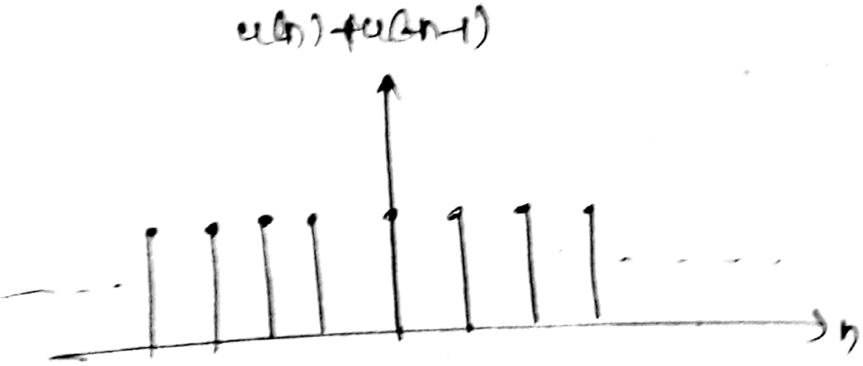
(c) $u(n) + u(-n-1) = 1 \quad \forall n$

Sol:









6) <u>Time scaling:</u>

$x(n) \longrightarrow x(nk)$ or $x\left(\frac{n}{k}\right)$

Time shifting can be represented as

$x(n) \longrightarrow \boxed{z^{-1}} \longrightarrow x(n-1)$

$x(n) \longrightarrow \boxed{z^{-1}} \longrightarrow \boxed{z^{-1}} \longrightarrow x(n-2)$

For advanced

$x(n) \longrightarrow \boxed{z} \longrightarrow x(n+1)$

$x(n) \longrightarrow \boxed{z} \longrightarrow \boxed{z} \longrightarrow x(n+2)$

Classification of discrete-time systems:

1) Linear or Nonlinear systems:

- A system is said to be linear if it satisfies homogenity and superposition principle.

Homogenity:

If i/p is multiplied by a factor, o/p should also be multiplied by the same factor.

$$x(n) \rightarrow y(n)$$
$$ax(n) \rightarrow ay(n)$$

Superposition:

Sum of weighted individual i/p results in " " " " o/p's

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$
$$y_1(n) = T[x_1(n)]$$

① $y(n) = n\,x(n)$

Sol:
$$y_1(n) = n\,x_1(n)$$
$$y_2(n) = n\,x_2(n)$$
$$y_3(n) = n\,[a_1 x_1(n) + a_2 x_2(n)]$$
$$= n a_1 x_1(n) + n a_2 x_2(n)$$
$$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 n x_1(n) + a_2 n x_2(n)$$

$\therefore$ The system is linear.

(2) $y(n) = x(n) + \frac{1}{x(n-1)}$

$$y_1(n) = x_1(n) + \frac{1}{x_1(n-1)}$$

$$y_2(n) = x_2(n) + \frac{1}{x_2(n-1)}$$

$$y_3(n) = a_1 x_1(n) + a_2 x_2(n) + \frac{1}{a_1 x_1(n-1) + a_2 x_2(n-1)}$$

$$a_1 y_1(n) + a_2 y_2(n) = a_1 x_1(n) + \frac{a_1}{x_1(n-1)} + a_2 x_2(n) + \frac{a_2}{x_2(n-1)}$$

$\therefore$ It is nonlinear.

(3) $y(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(n-k)$

Sol:
$$y_1(n) = \frac{1}{N} \sum_{k=0}^{N-1} x_1(n-k)$$

$$y_2(n) = \frac{1}{N} \sum_{k=0}^{N-1} x_2(n-k)$$

$$y_3(n) = \frac{1}{N} \sum_{k=0}^{N-1} a_1 x_1(n-k) + a_2 x_2(n-k)$$

$$= a_1 \frac{1}{N} \sum_{k=0}^{N-1} x_1(n-k) + \frac{a_2}{N} \sum_{k=0}^{N-1} x_2(n-k)$$

$$= a_1 y_1(n) + a_2 y_2(n)$$

$\therefore$ It is linear.

(4) $y(n) = n x^2(n)$

Sol:
$$y_1(n) = n x_1^2(n)$$

$$y_2(n) = n x_2^2(n)$$

$$y_3(n) = n\left(a_1 x_1(n) + a_2 x_2(n)\right)^2$$

$$= n\, a_1^2 x_1^2(n) + a_2^2 x_2^2(n) + 2a_1 a_2 x_1(n) x_2(n)$$

$$a_1 y_1(n) + a_2 y_2(n) = a_1 n x_1^2(n) + a_2 n x_2^2(n)$$
$$\neq y_3(n)$$

$\therefore$ It is non linear.

(5) $y(n) = x(n) + \frac{1}{2x(n-2)}$

$$y_1(n) = x_1(n) + \frac{1}{2x_1(n-2)}$$

$$y_2(n) = x_2(n) + \frac{1}{2x_2(n-2)}$$

$$y_3(n) = a_1 x_1(n) + a_2 x_2(n) + \frac{1}{2(a_1 x_1(n-2) + a_2 x_2(n-2))}$$

$$a_1 y_1(n) + a_2 y_2(n) = a_1 x_1(n) + \frac{1}{2a_1 x_1(n-2)} + a_2 x_2(n) + \frac{1}{2a_2 x_2(n-2)}$$

$$= a_1 x_1(n) + a_2 x_2(n) + \frac{1}{2a_1 x_1(n-2)} + \frac{1}{2a_2(x_2(n-2))}$$
$$\neq y_3(n)$$

$\therefore$ It is non-linear.

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Static and Dynamic system:

Static system: (Memoryless system)

→ It is a memoryless system.

→ output depends only on the present i/p

Eg: $y(n) = x(n)$ → static

→ A system is said to be dynamic system or system with memory, if the o/p depends on past or future values and present or past o/p's

Eg: $y(n) = x(n) + x(n-2)$

$y(n) = x(n) + x(n-2)$ → Dynamic

$y(n) + 2y(n-1) = x(n) + 3x(n-2)$

$y(n) + 2y(n-1) = x(n) + 2(x(n-2))$

It depends on past i/p's and past o/p's.

Causal and Non causal systems

→ A system is said to be causal if the o/p depends on past and present i/p's.

→ Causal systems are real systems (or) causal systems are physically realisable.

Eg: $y(n) = x(n) + x(2n)$ → Non-causal system

$n=1$

$y(1) = x(1) + x(2)$
↓
future i/p

$y(n) = x(n) + x(n-1)$ → causal system.

→ A system is said to be non causal, if the o/p depends on future i/p's

→ A system is said to be causal, if $h(n) = 0$, $n < 0$ (in terms of impulse response)

① $y(n) = x(-n)$

Soln: $y(-1) = x(1)$

Noncausal.

② $y(n) = x(2n)$

$y(1) = x(2)$

Noncausal.

③ $y(n) = [\sin(x(n))]$

Causal.

<u>Time invariant system & Time variant system:</u>

→ A system is said to be time invariant system if the I/p-o/p relationship does not change with time (or shifting

→ Eg: $x(n) \rightarrow y(n)$

$x(n-k) \rightarrow y(n-k)$

→ It is said to be time invariant if the o/p characteristics does not change with time.

$y(n,k) = y(n-k)$ i.e. delay in I/p leads to delay in o/p

Eg: $x(n) \rightarrow y(n)$

$x(n-k) \rightarrow y(n,k) = T[x(n-k)] = y(n)\Big|_{x(n)=x(n-k)}$

$y(n-k) = y(n)\Big|_{n=n-k}$

① $y(n) = x(n+1)$

Sol: $y(n,k) = T[x(n-k)] = y(n)\big|_{x(n) = x(n-k)}$

$y(n-k) = y(n)\big|_{n = n-k}$

$y(n-k) = \cancel{y}\ x(n-k+1)$

$y(n,k) = y(n-k)$

TIV

② $y(n) = x\left(\frac{n}{2}\right)$

Sol: $y(n,k) = x\left(\frac{n}{2} - k\right)$

$y(n-\cancel{k}) = x\left(\frac{n-k}{2}\right)$

$y(n,k) \neq y(n-k)$

$\therefore$ T.V

③ $y(n) = x(-n)$

$y(n,k) = x(-n-k)$

~~$y(n,k) = x(-n-k)$~~

$y(n-k) = x(-(n-k))$

$= x(-n+k)$

$\therefore$ T.V

LTI systems

→ If a system satisfies linearity and time invariant property, then it is said to be LTI system.

→ All discrete-time systems are described by difference equations

→ All " " " " " " differential equations

If the coefficients of difference (or) differential eqn are constants, then it is time invariant.

Eg: $y(n) = x(n) + x(n-1) + 3x(n-2) \longrightarrow$ T.IV

$y(n) = x(n) + n\,x(n-1) + 3x(n-2) \longrightarrow$ T.V

Difference equations consists of past or present I/p's only. i.e $x(n), x(n-1), x(n-2)$ and past o/p's

Eg: $y(n)+3n\,y(n-1)=x(n)+8x(n-1)+2x(n-2) \rightarrow$ T.V

① $y(n)=x(n)+n\,x(n-2)$

T.V

② $y(n)=x^2(n-2)$

T.I.V.

$y(n,k)=x^2(n-k-2)$

$y(n-k)=x^2(n-k-2)$

Stable and Unstable systems:

→ A system is said to be stable if bounded I/p produces bounded o/p (BIBO)

Bounde I/p : $|x(n)| \leq M \rightarrow$ (+ve real finite value)

$|y(n)| \leq M_y$

→ If I/p is bounded, o/p is infinite → Unstable

The necessary and sufficient condition for stability is $\boxed{\sum_{n=-\infty}^{\infty} |h(n)| < \infty}$

i.e if impulse response of the system has finite value, then it is stable

→ Stability of system indicates usefulness of system.

If i/p $x(n) = \delta(n)$

$y(n) = h(n)$

$y(n) = x(n) * h(n)$

Proof: $y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$ $\qquad y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$|y(n)| = \sum_{k=-\infty}^{\infty} |x(k)\, h(n-k)|$

$\leq \sum_{k=-\infty}^{\infty} |x(k)|\,|h(n-k)|$

$|y(n)| \leq M_x \sum_{k=-\infty}^{\infty} h(n-k)$ $\quad$ ($\because$ bounded i/p $\Rightarrow |x(n)| = M_x$)

Replace $n-k = m$

The o/p is bounded if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

① $h(n) = \left(\frac{1}{2}\right)^n u(n)$. The impulse response of the system is $h(n) = \left(\frac{1}{2}\right)^n u(n)$. Check whether it is stable or unstable

Sol: $\sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u(n)$ $\quad \left(\because \sum_{n=-\infty}^{\infty} h(n)\right)$

$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$

$= 1 + \left(\frac{1}{2}\right) + \ldots \infty \Rightarrow \frac{1}{1-\frac{1}{2}}$ $\quad \left(\because \frac{a}{1-r}\right)$

$= 2 < \infty$

$\therefore$ It is stable

(2) $y(n) = a\,x(n-7)$

Sol: Let $x(n) = \delta(n) \Rightarrow y(n) = h(n)$

$h(n) = a\,\delta(n-7) \quad = a \text{ for } n = 7$

$= 0 \text{ for } n \neq 7$

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{\infty} a\,\delta(n-7)$$

$$= a\,\delta(7-7)$$

$$= a$$

$\therefore$ It is stable

(3) $y(n) = a\,x(n+1) + b\,(x\,n-1)$

Sol: $h(n) = a\,\delta(n+1) + b\,\delta(n-1)$

$$\sum_{n=-\infty}^{\infty} h(n) = \ldots + a\,\delta(0) + b\,\delta(-2) + a\,\delta(1) + b(-1) + a\,\delta(2) + b\,\delta(0)$$

$$= a+b < \infty$$

$\therefore$ It is stable

(4) $h(n) = u(n)$

Sol: $$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{\infty} u(n)$$

$$= 1+1+\ldots+1 < \infty$$

$\therefore$ It is unstable.

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FIR & IIR systems:

Any LTI discrete time system can be classified according to the type of impulse response.

If impulse response h(n) is finite duration system is finite impulse Response system (FIR)

If impulse response h(n) is infinite duration, system is infinite impulse Response System (IIR)

Impulse Response and convolution sum:

Arbitary sequence: A sequence of any extent or any order.

Any arbitary sequence x(n) can be expressed (or) represented by delayed and weighted impulse response δ(n)

Eg: $x(0).\delta(n) \rightarrow x(0) \quad n=0$

$0 \quad n \neq 0$

$x(-1)\,\delta(n+1) \rightarrow x(-1) \quad n=-1$

$0 \quad n \neq -1$

$x(n) \rightarrow -2 \leq n \leq 2$

$$x(n) = x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0).\delta(n) + x(1)\delta(n-1) + x(2).\delta(n-2)$$

For an infinite sequence $x(n)$ can be represented as

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$$

If the i/p $x(n) = \delta(n)$

$$y(n) = T[x(n)] = T[\delta(n)] = h(n)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$$

$$y(n) = T[x(n)] = \sum_{k=-\infty}^{\infty} T[x(k) . \delta(n-k)]$$

$$= \sum_{k=-\infty}^{\infty} x(k) T[\delta(n-k)]$$

(if system is linear)

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

(i.e if system is time invariant)

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k) . h(n-k)} \longrightarrow \text{convolution sum}$$

(or)

$$\boxed{y(n) = x(n) * h(n)}$$

and convolution sum is possible only when the system is LTI system.

*** → If the input sequence $x(n)$ of length 'N_1', and the impulse response $h(n)$ of length 'N_2', then the convolution of two sequences produce the o/p of length N_1+N_2-1

→ The limits of the convolution sum can be modified according to the type of the sequence and system.

For causal system,

$$h(n)=0 \quad n<0$$

For causal i/p sequence, $x(n)=0 \quad n<0$.

Case (i) For non-causal system excited by a noncausal i/p

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \text{ — non causal i/p}$$

$$= \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k) \rightarrow \text{ " system.}$$

Case (ii) For non-causal system, causal i/p sequence
($x(n)=0$; $n<0$)

$$y(n) = \sum_{k=0}^{\infty} x(k)\, h(n-k).$$

$$= \sum_{k=-\infty}^{n} h(k) \cdot x(n-k)$$

Case (iii) For causal system, non-causal i/p

$$y(n) = \sum_{k=0}^{\infty} h(k) \cdot x(n-k) = \sum_{k=-\infty}^{n} x(k)\, h(n-k)$$

Case (iv) For causal system, causal i/p

$$y(n) = \sum_{k=0}^{n} h(k)\, x(n-k) = \sum_{k=0}^{n} x(k)\, h(n-k)$$

$h(k) \rightarrow 0$ to ∞, $x(n-k) \rightarrow -\infty$ to n

① Determine the convolution sum of 2 sequences.

$$x(n) = \{\underset{\uparrow}{3}, 2, 1, 2\}, \quad h(n) = \{1, \underset{\uparrow}{2}, 1, 2\}$$

Sol: (i) Using Tabular method:

$x(n)$

$h(n)$ \ $x(n)$	3	2	1	2
1	3	2	1	2
2	6	4	2	4
1	3	2	1	2
2	6	4	2	4

$$y(n) = \{3, \underset{\uparrow}{8}, 8, 12, 9, 4, 4\}$$

$N_1 + N_2 - 1 = 4 + 4 - 1 = 7$

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Graphical Method :-

Steps to do convolution sum using graphical method:

Step 1:- Chooce the initial value of 'n' that will give the starting time for evaluating the o/p sequence $y(n)$. If $x(n)$ is starts at $n = n_1$ and $h(n)$ starts at $n = n_2$. Then $n = n_1 + n_2$ is a good choice.

Step 2:- Express the both sequences in terms of index scale.

Step 3: fold $h(k)$ about $k=0$ to obtain $h(-k)$ & shift by n to the right if n is +ve & shift left if n is -ve. to obtain $h(n-k)$.

Step 4: Multiply the 2 sequences $x(k)$ & $h(n-k)$ element by element & sum of the products to get $y(n)$.

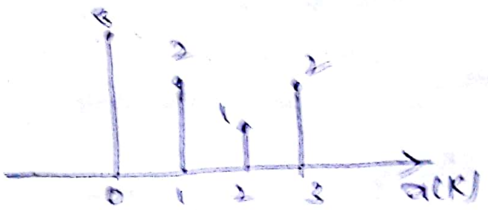
Step 5: ~~loop~~ Increment index n ~~by~~ shift the sequence $h(n-k)$ to the right by one sample & do step '4'.

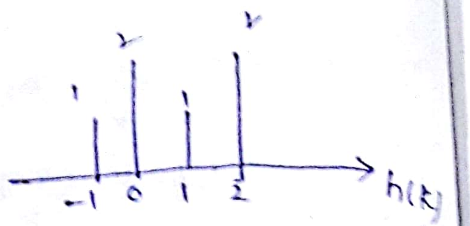
Step 6: Repeat step 5 until the sum of products is zero for remaining values of 'n'

* $x(n) = \{3, 2, 1, 2\}$, $h(n) = \{1, \underset{\uparrow}{2}, 1, 2\}$

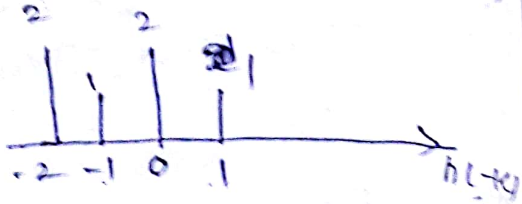
$n_1 = 0$, $n_2 = -1$

$n = n_1 + n_2 = 0 - 1 = -1$. $y(-1)$

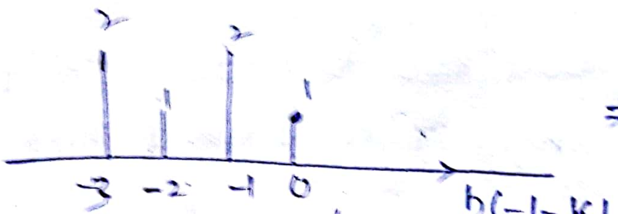




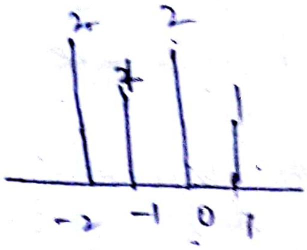
$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$



$n = -1$, shift by left -1 $-1-k$ $-1($



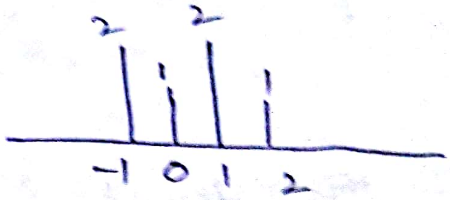
$\Rightarrow y(-1) = 3$ $h(-1-k)$



$y(0) = 6 + 2 = 8$

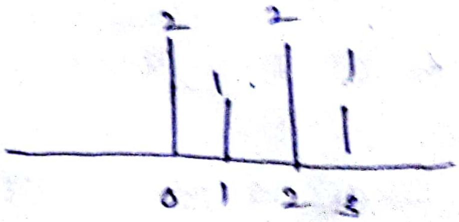
$y(1) = \cancel{6+2+2=10}$

$3 + 4 + 1 = 8$ $h(-1$

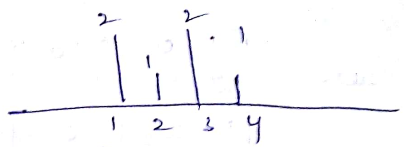


$y(2) = 6 + 2 + 2 + 2$

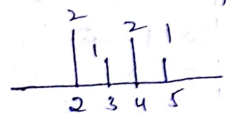
$= 12$

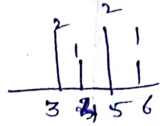


$y(3) = 4 + 1 + 4 = 9$



$y(4) = 2 + 2 = 4$





$y(5) = 4$

$y(n) = \{3, \underset{\uparrow}{8}, 8, 12, 9, 4, 4\}$

<u>Matrix Method:-</u>

$H \quad X$

$$H = \begin{bmatrix} h(0) & 0 & 0 & \cdots \\ h(1) & h(0) & 0 & \cdots \\ h(2) & h(1) & h(0) & \\ \vdots & & & \\ h(N_2-1) & h(N_2-2) & \cdots & \\ 0 & h(N_2-1) & h(N_2-2) & \cdots \\ \vdots & & & \\ 0 & 0 & \cdots & h(N_2-1) \end{bmatrix}_{(N_1+N_2-1)\times N_1} \quad X = \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N_1-1) \end{bmatrix}_{N_1\times 1}$$

$$~~y(n)~~ \; y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 2 & 1 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 8 \\ 12 \\ 9 \\ 4 \\ 4 \end{bmatrix}$$

$= [$

→ **Tabular Array method:-**

	-3	-2	-1	0	1	2	3	4	5	6
x(k)				3	2	1	2			
h(k)			1	2	1	2				
h(-k)		2	1	2	1					
h(-1-k) = 3	2	1	2	1						
h(-0-k) = 8		2	1	2	1					
h(1-k) = 8			2	1	2	1				
h(2-k) = 12				2	1	2	1			
h(3-k) = 9					2	1	2	1		
h(4-k) = 4						2	1	2	1	
h(5-k) = 4							2	1	2	1

* $x(n) = \{4,$ …

→ $h(n) = a^n \cdot u(n)$

find the convolu…

i) $a = b$, ii)

Sol: $y(n) = \sum_{k=-\infty}^{\infty}$

$= \sum_{k=}^{\infty}$

$x(n) =$

impulse resp…

∴ $y(n) =$

if $a = b$

$y(n)$

* $x(n) = \{4, 2, 1, 3\}$, $h(n) = \{1, \underset{\uparrow}{2}, 2, 1\}$

→ $h(n) = a^n \cdot u(n)$ for $\forall n$, $x(n) = b^n u(n)$

find the convolution is of 2 finite duration sequences

i) $a = b$, ii) $a \neq b$.

$$x(n) = b^n u(n)$$
$$= b^n \; ; n \geq 0$$

Sol: $y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$= \sum_{k=-\infty}^{\infty} \cancel{b^k u(k) \cdot a}$

$x(n) = 0$, $h(n) = 0$ for $n < 0$, so, i/p sequence & impulse respones are causal.

$\therefore \; y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$.

$= \sum_{k=0}^{n} b^k \cdot a^{n-k}$

$= a^n \sum_{k=0}^{n} \left(\frac{b}{a}\right)^k$

$= a^n \left[1 + \frac{b}{a} + \left(\frac{b}{a}\right)^2 + \dots + (b/a)^n\right]$

if $a = b$

$y(n) = a^n [1 + 1 + \dots + n]$

$= (n+1)\, a^n$.

if $a \neq b$

$$y(n) = a^n \left[\frac{1-\left(\frac{b}{a}\right)^{n+1}}{1-b/a} \right]$$

$$= \frac{a^{n+1}\left(1-(b/a)^{n+1}\right)}{a-b}$$

→ $x(n) = 3^n u(n)$ & $h(n) = \left(\frac{1}{3}\right)^n u(n)$.

$x(n) * h(n)$

$$x(n) * h(n) = \sum_{k=0}^{n} 3^k \left(\frac{1}{3}\right)^{n-k}$$

$$= \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} 3^k \cdot 3^k$$

$$= \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} 3^{2k} = \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} 9^n$$

$$= \cancel{\left(\frac{1}{3}\right)^n \frac{1-3^{2n+1}}{1-9}} = \left(\frac{1}{3}\right)^n \frac{9^{n+1}-1}{9-1}$$

$$= \left(\frac{1}{3}\right)^n \frac{9^{n+1}-1}{8}$$

→ $x(n) = n+3$ for $0<n<2$, $h(n) = a^n u(n)\ \forall n$ find convo.

$x(n)$ & $h(n)$ are causal

$$y(n) = \sum_{k=0}^{n} (k+3)\, a^{n-k}.$$

Inverse System & de-convolution:

A system is said to be invertible if the i/p to the system can be recovered from o/p.

$$x(n) \longrightarrow \boxed{\begin{array}{c}\text{system}\\ h_1(n)\end{array}} \xrightarrow{y(n)} \boxed{\begin{array}{c}\text{system}\\ h_2(n)\end{array}} \longrightarrow x(n)$$

$$h_1(n) * h_2(n) = \delta(n)$$

$$y(n) = x(n) * h_1(n)$$

$$x(n) = y(n) * h_2(n)$$

$$= [x(n) * h_1(n)] * h_2(n)$$

$= \cancel{x(n)}$ then $h_1(n) * h_2(n) = \delta(n)$.

Deconvolution:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

for One sided sequence

$$y(n) = \sum_{k=0}^{\infty} x(k)\, h(n-k)$$

$$y(0) = x(0)\, h(0) + x(0)\, h(-1) + \ldots$$

zero because one sided sequence

$$y(0) = x(0)\, h(0).$$

$$x(0) = \frac{y(0)}{h(0)}$$

$$y(1) = x(0)h(1) + x(1)h(0)$$

$$x(1) = \frac{y(1) - x(0)h(1)}{h(0)}$$

$$y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0)$$

$$x(2) = \frac{y(2) - x(0)h(2) - x(1)h(1)}{h(0)}$$

$$\boxed{x(n) = \frac{y(n) - \sum_{k=0}^{n-1} x(k)\, h(n-k)}{h(0)}}$$

→ $h(n) = \{1, 2, 2, 1\}$, $y(n) = \{4, 10, 13, 13, 10, 7, 3\}$.

$$x(0) = \frac{y(0)}{h(0)} = \frac{4}{1} = 4$$

$$7 = N_1 + N_2 - 1$$

$$7 = N_1 + 4 - 1$$

$$7 = N_1 + 3$$

$$N = 4$$

$$x(1) = \frac{y(1) - x(0)h(1)}{h(0)}$$

$$= \frac{10 - 4(2)}{1}$$

$$= 2$$

$$x(2) = \frac{y(2) - x(0)h(2) - x(1)h(1)}{h(0)}$$

$$= \frac{13 - 4(2) - 2(2)}{1} = 1$$

$$x(3) = \frac{y(3) - \sum_{K=0}^{2} x(K)\, h(3-K)}{h(0)}$$

$$= \frac{13 - 4(1) - 2(2) - 1(2)}{1}$$

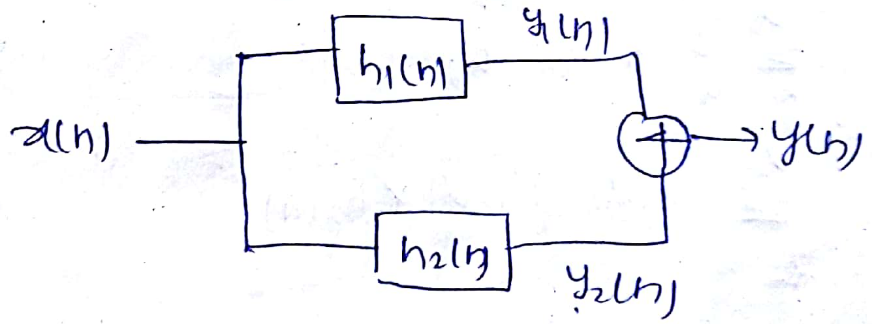
$$= 13 - 4 - 4 - 2$$

$$= 3$$

Inter connection of LTI System:-

i) Parallel Connection

The 2 systems are connected in parallel with impulse responses $x_1(n)$ & $x_2(n)$ then the overall impulse response is equal to the sum of 2 impulse responses.



$$y_1(n) = x(n) * h_1(n)$$

$$y_2(n) = x(n) * h_2(n)$$

$$y(n) = y_1(n) + y_2(n)$$

$$= \sum_{K=-\infty}^{\infty} x(K)\, h_1(n-K) + \sum_{K=-\infty}^{\infty} x(K)\, h_2(n-K)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\left[h_1(n-k) + h_2(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

where $h(n) = h_1(n) + h_2(n)$.

2) cascade connection:-

$$x(n) \longrightarrow \boxed{h_1(n)} \xrightarrow{y_1(n)} \boxed{h_2(n)} \longrightarrow y(n)$$

$$y_1(n) = x(n) * h_1(n)$$

$$y(n) = y_1(n) * h_2(n)$$

$$y_1(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h_1(n-k) * h_2(n)$$

$$= \sum_{k=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} x(v)\, h_1(k-v) * h_2(n-k)$$

$$k - v = p$$

$$= \sum_{v=-\infty}^{\infty} x(v) \underbrace{\sum_{p=-\infty}^{\infty} h_1(p) \cdot h_2(n-p-v)}_{h_1(n) * h_2(n)}$$

$$= \sum_{k=-\infty}^{\infty} x(n)\, h(n-k)$$

$\therefore h(n) = h_1(n) * h_2(n)$ $\quad$ $(h_1(k)\, h(n-k))$

$h(n-k) = h_1(k)\, h_2(n-k))$

Time response analysis of discrete-time LTI systems:

The o/p of LTI can also be determined by using time response analysis (one method is convolution sum)

$$y(n) = -\sum_{k=0}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

Total response = Zerostate response + zero i/p response

Zerostate response $\downarrow$ initial conditions are zero. depends on i/p

Zero i/p response $\downarrow$ $x(n)=0$ depends on initial conditions.

Order = max(N, M)

The system satisfies both LTI (ie coefficients are constants) and causality (i.e depends on past i/p's and o/p's)

Eg: $y(n) = a\, y(n-1) + x(n)$

Let $x(n)=0$ for $n<0$ and initial condition of $y(n)$ for $n=-1$ exists ie. $y(-1) \neq 0$

Sol: $n=0$

$y(0) = ay(-1) + x(0)$

$n=1$

$y(1) = a\,y(0) + x(1)$

$= a(ay(-1) + x(0)) + x(1)$

$= a^2 y(-1) + ax(0) + x(1)$

$y(2) = a[a^2y(-1) + ax(0) + x(1)] + x(2)$

$= a^3 y(-1) + a^2(x(0)) + ax(1) + x(2)$

$$y(n) = a^{n+1} y(-1) + \sum_{k=0}^{n} a^k x(n-k) \quad n \geq 0$$

$y(n) = a^{n+1} y(-1)$. when $x(n) = 0 \rightarrow$ Zero i/p

$y(n) = \sum_{k=0}^{n} a^k x(n-k)$ when initial conditions are zero. It depends only on i/p

The zero i/p response is called as natural response is obtained by substituting i/p signal is zero. It depends on the nature of the system and the initial states only. It is also called natural response.

Zero state response is also called as forced response and here the o/p depends only on the applied i/p and all the initial contions are zero.

$y(n) = y_n(n) + y_p(n)$

natural response - zero I/p response (or) Homogeneous Sol^n

forced response → zero state "

<u>Natural Response:</u>

The natural response $y_n(n)$ is the sol^n of the D.E with I/p is equal to zero.

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \underbrace{\sum_{k=0}^{M} b_k\, x(n-k)}_{\text{zero}}$$

$$y(n) + \sum_{k=1}^{N} a_k\, y(n-k) = 0$$

$$\sum_{k=0}^{k} a_k\, y(n-k) = 0 \qquad \text{Let } a_0 = 1$$

$$y_n(n) = \lambda^n$$

$$\sum_{k=0}^{N} a_k\, \lambda^{n-k} = 0, \quad a_0 = 1$$

$$\lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \ldots + a_N \lambda^{n-N} = 0$$

$$\lambda^{n-N}\left[\lambda^N + a_1 \lambda^{N-1} + \ldots + a_N\right] = 0$$

$$\lambda^N + a_1 \lambda^{N-1} + \ldots + a_N = 0$$

It is called as characteristic equation of the system

Roots decide the nature of the system.

Let the roots be $\lambda_1, \lambda_2, \ldots \lambda_N$

$$(\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\lambda_N) = 0$$

(i) If the roots are distinct, then it has 'N' solutions. i.e. $c_1\lambda^n \quad c_2\lambda^n \quad c_3\lambda^n \ldots c_N \lambda^N$

$$y_h(n) = c_1\lambda_1^n + c_2\lambda_2^n + c_3\lambda_3^n + \dots + c_N\lambda_N^n$$

let $\lambda_1 = 3, \lambda_2 = 4$

$$y_h(n) = c_1 3^n + c_2 4^n$$

(ii) If the roots λ_1 is repeated 'm' times, and the remaining $(N-m)$ roots are distinct then the characteristic eqn is

$$(\lambda-\lambda_1)^m(\lambda-\lambda_{m+1}) \dots (\lambda-\lambda_N) = 0$$

~~$y_h(n) = c_1\lambda$~~

$$y_h(n) = (c_1 + c_2 n + c_3 n^2 + \dots + c_m n^{m-1})\lambda_1^n + c_{m+1}\lambda_{m+1}^n + \dots + c_N\lambda_{N-m}^n$$

Eg: $\lambda_1 = -1,\ \lambda_2 = -1,\ \lambda_3 = -1,\ \lambda_2 = 3$

$$y_h(n) = (c_1 + c_2 n + c_3 n^2)(-1)^n + c_4(3)^n$$

(iii) If the roots are complex, solution is

$\lambda_1 = a + jb, \quad \lambda_2 = a - jb$

$$y_h(n) = r^n(A_1\cos n\theta + A_2\sin n\theta)$$

$r = \sqrt{a^2+b^2}$, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

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→ find the natural response of an LTI system described by difference equation

$$y(n) + 2y(n-1) + y(n-2) = x(n) + x(n-1).$$

with the initial conditions $y(-1) = y(-2) = 1$,

Sol: Zero i/p $x(n) = 0$

$$y(n) + 2y(n-1) + y(n-2) = 0$$

Assume $y_h(n) = \lambda^n$

$$\lambda^n + 2\lambda^{n-1} + \lambda^{n-2} = 0$$

$$\lambda^{n-2}[\lambda^2 + 2\lambda + 1] = 0$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$(\lambda+1)^2 = 0$$

$$\lambda_1 = -1, \quad \lambda_2 = -1.$$

$$y_h(n) = (C_1 + C_2 n)(-1)^n$$

$n = 0$, $y(0) = C_1$

$n = 1$, $y(1) = -C_1 + C_2$

from the diff eqn

$n = 0$,

$$y(0) + 2y(-1) + y(-2) = 0$$

$$y(0) + 3 = 0$$

$$y(0) = -3$$

$y(1) + 2y(0) + y(-1) = 0$

$y(1) - 6 + 1 = 0$

$y(1) = 5$

from ①

$y(0) = c_1 \Rightarrow c_1 = -3$

$y(1) = -c_1 - c_2$

$5 = 3 - c_2$

$c_2 = -2$

Homogeneous soln.

$y_h(n) = \cancel{0\,(-3-2n)(-1)^n}$

$y_h(n) = -3(-1)^n u(n) - 2n(-1)^n u(n)$

→ ~~forced response~~

→ find the NR $\quad y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

with the initial conditions

$y(-1) = y(-2) = 1$

Forced Response:- ($y_p(n)$)

Zero state Response (Forced response) is the response where the initial conditions are zero and o/p depends on the i/p only. It consists of homogenous solution and particular solution

$$y_h(n) + y_p(n)$$

can be obtained by finding the roots of the characteristic eqn

$x(n) \to n \geq 0$

$x(n) = 0, n < 0$

→ Particular sol^n depends on i/p only

i/p $x(n)$	$y_p(n)$
unit step $u(n)$	$K\, u(n)$
$A M^n$	$K M^n$
$A n^M$	$K_0 n^M + K_1 n^{M-1} + \dots + K_M$
$A \sin \omega n$, $A \cos \omega n$	$C_1 \cos(\omega n) + C_2 \sin(\omega n)$
$\delta(n)$	0

The constants can be obtained by substituting these sol^{n}s $y_p(n)$ & $x(n)$ in difference eqn.

The homogenous sol^n is obtained without initial conditions in case of forced response.

→ find the forced response of the LTI system ... the diff equation

$y(n)+2y(n-1)+y(n-2)=x(n)+x(n-1)$

with i/p $x(n)=(1/2)^n u(n)$.

$y_f(n)=y_h(n)+y_p(n)$

$y_h(n)=(C_1+C_2 n)(-1)^n$.

$x(n)=(1/2)^n u(n)$

$y_p(n)=K(1/2)^n u(n)$

substitute in diff eq.

$K(\frac{1}{2})^n+2K(\frac{1}{2})^{n-1}+(1/2)^{n-2}=(1/2)^n+(1/2)^{n-1}$

$n=2$

$\frac{K}{4}+\frac{K}{2}+K=\frac{1}{4}+\frac{1}{2}$

~~K+2K+4K~~ $=1+2$

~~5K = -1~~

~~K = -1/3~~

$\frac{K+8K}{4}=\frac{1+2}{4}$

$9K=3$

$K=1/3$

$y_p(n)=1/3(1/2)^n u(n)$

$y_f(n) = y_h(n) + y_p(n)$

$= (C_1 + C_2 n)(-1)^n u(n) + \frac{1}{3}(\frac{1}{2})^n u(n)$

C_1, C_2 can be obtained with the initial condition

i.e. $y(-1) = y(-2) = 0$.

$n=0$: $y(0) = C_1 + \frac{1}{3}$

$n=1$: $y(1) = -C_1 - C_2 + \frac{1}{6}$

from the diff eqn

$n=0$
$y(0) + 2y(-1) + y(-2) = x(0) + x(-1)$

$y(0) + 0 + 0 = 1 + 0$

$y(0) = 1$ ——— (1)

$n=1$ $y(1) + 2y(0) + y(-1) = x(1) + x(0)$

$y(1) + 2 = \frac{1}{2} + 1$

$y(1) = -\frac{1}{2}$ — (2)

$y(0) = 1$

$C_1 + \frac{1}{3} = 1$

$C_1 = 2/3$

$y(1) = -\frac{1}{2}$

$-C_1 - C_2 + 1/6$

$C_2 = 0$

$y_f(n) = (\frac{2}{3} + 0)(-1)^n u(n) + (\frac{1}{3})(\frac{1}{2})^n u(n)$

$y_f(n) = (\frac{2}{3})(-1)^n u(n) + (\frac{1}{3})(\frac{1}{2})^n u(n)$.

→ Find the natural & forced response of LTI system described by the difference equation

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

when the i/p $x(n)$ is $(-1)^n u(n)$ & the initial conditions are $y(-1) = y(-2) = 1$.

Sol: Natural response

when $x(n) = 0$

$$y(n) - 4y(n-1) + 4y(n-2) = 0$$

$$y(n) = \lambda^n$$

$$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 4\lambda + 4) = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = \frac{4 \pm \sqrt{16-4}}{2} \qquad (\lambda-2)^2 = 0$$

$$\cancel{= \frac{4 \pm \sqrt{12}}{2}} \qquad \lambda = 2, 2$$

$$\cancel{= 2 \pm \sqrt{3}} \qquad \lambda_1 = 2, \lambda_2 = 2$$

$$y_h(n) = (C_1 + C_2 n)\lambda^n$$

$$y_h(n) = (C_1 + C_2 n)\, 2^n$$

$y(-1) = y(-2) = 1$

$y(0) = c_1$

$y(1) = (c_1 + c_2)\,2$

from the homogeneous diff eqn.

$y(0) - 4y(-1) + 4y(-2) = 0$

$y(0) - 4(1) + 4(1) = 0$

$y(0) = 0$

$y(1) - 4y(0) + 4y(-1) = 0$

$y(1) = -4$

$c_1 = 0$

$(c_1 + c_2)\,2 = -4$

$c_2 = -2$

$y_h(n) = y_n(n) = -2n \cdot 2^n\, u(n).$

↓

Because we find it for causal sequence.

$y_f = y_h(n) + y_p(n)$

$y_h(n) = (c_1 + c_2 n)2^n$

We take without taking initial conditions because it is zero state soln.

$x(n) = (-1)^n u(n)$

$y_p(n) = K(-1)^n u(n)$

substitute in diff eqn.

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

$n = 2$

$$K - 4K(-1) + 4K = 1 - (-1)$$

$$9K = 2$$

$$K = 2/9$$

$$y_p(n) = \frac{2}{9}(-1)^n u(n).$$

$$y(n) = (C_1 + C_2 n)\, 2^n + \frac{2}{9}(-1)^n u(n)$$

$$y(-1) = y(-2) = 0$$

$$\cancel{y(-1) = (C_1 - C_2)\frac{1}{2} + \frac{2}{9}(-1)(0)}$$

$$\cancel{0 = \frac{C_1 - C_2}{2}}$$

$$y(0) = C_1 + 2/9$$

$$y(1) = 2C_1 + 2C_2 + 2/9$$

$$y(0) - 4y(-1) + 4y(-2) = x(0) - x(-1) \rightarrow \text{It takes previous value}$$

$$y(0) - 4(0) + 4(0) = 1$$

$$y(0) = 1$$

$y(1) - 4y(0) + 4y(-1) = x(1) - x(0)$

$y(1) - 4 = -1-1 \Rightarrow y(1) = 2$

$1 = C_1 + 2/9$

$C_1 = 1 - 2/9$

$= 7/9$

$y(1) = \frac{14}{9} + 2C_2 \cancel{+} \frac{2}{9}$

$2 = \frac{12}{9} + 2C_2$

$2C_2 = 2 - \frac{12}{9}$

$= 6/9$

$C_2 = 3/9 = 1/3$

$$\boxed{y_f(n) = \left(\frac{7}{9} + 1/3\, n\right) 2^n u(n) + \frac{2}{9}(-1)^n u(n)}$$

$y(n) = y_h(n) + y_f(n)$

$= -2n \cdot 2^n u(n) + \frac{7}{9} 2^n u(n) + 1/3\, n\, 2^n u(n) + \frac{2}{9}(-1)^n u(n)$

$y(n) = \left(\frac{7}{9} - \frac{5}{3}n\right) 2^n u(n) + \frac{2}{9}(-1)^n u(n)$

(or) $y(n) = (C_1 + C_2 n)\, 2^n + 2/9\, (-1)^n u(n)$

If we substitute initial conditions as zero we get forced response

If we substitute initial conditions we get total response

→ $y(n) = y(n-1) + 0.5\, y(n-2) + x(n) + x(n-1)$. find natural, force, total response with i/p $x(n) = 2^n u(n)$
initial condition $y(-1) = y(-2) = 1$.

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Impulse Response:-

$$y(n) = y_h(n) + y_p(n), \quad \text{if } x(n) = \delta(n)$$
then $y_p(n)$
$$= y_h(n) + y_p(n)$$
$\therefore y(n) = y_h(n)$

→ find the impulse response of LTI system described by the diff equation $y(n) = 0.6\, y(n-1) - 0.08\, y(n-2) + x(n)$

Sol:-

if i/p is impulse the particular soln $y_p(n) = 0$
Hence, the $h(n)$ can be computed from the homogeneous soln with the initial conditions are zero.

$$y(n) = 0.6\, y(n-1) - 0.08\, y(n-2)$$

$$y(n) = \lambda^n$$

$$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$$

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$\lambda^2 - 0.4\lambda - 0.2\lambda + 0.08 = 0$$

$$\lambda(\lambda - 0.4) - 0.2(\lambda - 0.4) = 0$$

$$\lambda = 0.4,\ 0.2$$

$\lambda_1 = 0.4, \quad \lambda_2 = 0.2$

$y(n) = c_1 \lambda_1^n + c_2 \lambda_2^n$

$y(n) = c_1 (0.4)^n + c_2 (0.2)^n, \quad n \geq 0.$

$\cancel{c_1 (0.4)^n + c_2}$

$n = 0, \quad y(0) = c_1 + c_2$

$n = 1, \quad y(1) = 0.4 c_1 + 0.2 c_2$

from the diff eq.

$n = 0, \quad y(0) = 0.6\, y(-1) - 0.08\, y(-2) + x(0)$

All the initial conditions are zero

$c_1 + c_2 = 0 - 0 + 1 \quad y(0) = 0$

$c_1 \cancel{=c_2} \quad c_1 + c_2 = 1$

$n = 1, \quad y(1) = 0.6\, y(0) - 0.08\, y(-1) + x(1)$

$0.4 c_1 + 0.2 c_2 = 0.6 - 0 + 0$

Here we consider i/p because we are considering Particular Sol'n so, we have to consider x(n)

$0.4 c_1 + 0.2 c_2 = 0.6$

$0.2 c_1 + 0.2 c_2 = 0.2$

$0.2 c_1 = 0.4$

$c_1 = 2, \quad c_2 = -1$

$y_h(n) = 2 (0.4)^n - (0.2)^n \; ; \; n \geq 0$

$= 2 (0.4)^n u(n) - (0.2)^n u(n)$

Step response:-

→ find impulse & step response of DTLTI system, whose diff eqn given by $y(n) = y(n-1) + 0.5\, y(n-2) + x(n)$ $+ x(n$

Sol:

$$y(n) = y_h(n) + y_p(n)$$

$$y_p(n) = K\, u(n)$$

$$y_h(n) = y(n-1) + 0.5\, y(n-2)$$

$$y(n) - y(n-1) - 0.5\, y(n-2) = 0$$

$$\lambda^n - \lambda^{n-1} - 0.5\lambda^{n-2} = 0$$

$$\lambda^{n-2}[\lambda^2 - \lambda - 0.5] = 0$$

$$\lambda^2 - \lambda - 0.5 = 0$$

$$\cancel{\lambda^2 - \lambda + 0.5\lambda - 0.5 = 0}$$ $\lambda = \dfrac{1 \pm \sqrt{1+2}}{2}$

$$\cancel{\lambda(\lambda-1) + 0.5(\lambda-1) = 0}$$ $= \dfrac{1 \pm \sqrt{3}}{2}$

$$\cancel{\lambda = 1,\ \lambda = -0.5}$$

$$\cancel{\lambda_1 = 1,\ \lambda_2} = -0.5$$ $\lambda_1 = 1.366,$

$\lambda_2 = -0.366.$

$$\cancel{y_h(n) = C_1(1)^n + C_2(-0.5)^n}$$

$$\cancel{= C_1 + C_2(-0.5)^n}$$

By substi $\cancel{y_h(n) = \left(\frac{1}{2}\right)^n\left[C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x\right]}$

$$y_h(n) = C_1\left(\frac{1+\sqrt{3}}{2}\right)^n + C_2\left(\frac{1-\sqrt{3}}{2}\right)^n$$

$y_p(n) = K u(n)$ sub. in diff eqn

$$K u(n) = K u(n-1) + 0.5K u(n-2) + u(n) + u(n-1)$$

$$K = K + 0.5K + 1 + 1$$

$$-2 = 0.5K \Rightarrow K = -4.$$

$$y_p(n) = -4u(n)$$

$$y_h(n) = C_1(1.366)^n + C_2(-0.366)^n.$$

$$y(n) = C_1(1.366)^n + C_2(-0.366)^n - 4u(n)$$

$$y(0) = C_1 + C_2 - 4$$

$$y(1) = 1.366\,C_1 - 0.366\,C_2 - 4$$

$$\cancel{y(n) = C_1(1.366)^{n-1} + C_2(-0.366)^{n-2}}$$

from diff eqn

$$y(n) = y(n-1) + 0.5\,y(n-2) + x(n) + x(n-1)$$

$$y(0) = y(-1) + 0.5\,y(-2) + u(0) + u(-1)$$

$$y(0) = 1$$

$$y(1) = y(0) + 0.5\,y(-1) + x(1) + u(0)$$

$$y(1) = 2$$

$$C_1 + C_2 - 4 = 1 \Rightarrow C_1 + C_2 = 5$$

$$1.366C_1 - 0.366C_2 - 4 = 2$$

$$1.366C_1 - 0.366C_2 = 6$$

MSC

$\cancel{0\;1.366}$

$0.366c_1 + 0.366c_2 = 1.830$

$1.366c_1 - 0.366c_2 = 6$

$1.732c_1 = 7.830$

$c_1 = 5.09$

$c_2 = -0.9$

$y(n) = 5.09(1.366)^n - 0.09(-0.366)^n - 4u(n)$

Impulse

$y(n) = c_1(1.366)^n + c_2(-0.366)^n$

$y(0) = c_1 + c_2$

$y(1) = 1.366c_1 - 0.366c_2$

By differen

$y(0) = y(-1) + 0.5y(-2) + \delta(0) + \delta(-1)$

$y(0) = 1$

$y(1) = y(0) + 0.5y(-1) + \delta(1) + \delta(0)$

$y(1) = 1 + 1$

$y(1) = 2$

$c_1 + c_2 = 1$

$1.366c_1 - 0.366c_2 = 2$

$c_1 = 1.366, \quad c_2 = -0.366$

19/12/17

Frequency Domain Representation of discrete time signals & systems:-

$$X(\omega) = \int_{-\infty}^{\infty} x(t).e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega).e^{j\omega t} d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n).e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{0}^{2\pi} X(e^{j\omega}).e^{j\omega n} d\omega.$$

$\delta(n) = 0$ if $x(n) = u(n)$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} 1.e^{-j\omega n}$$

$$= \frac{1}{1-e^{-j\omega}}.$$

$a^n u(n) = x(n)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n).e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (a.e^{-j\omega})^n$$

$$= \frac{1}{1-a.e^{-j\omega}}. \quad a<1$$

$X(e^{j\omega}) = X_R(e^{j\omega}) + j X_I(e^{j\omega})$

$|X(e^{j\omega})| \cdot \underline{/\theta(\omega)}$

~~Rlem~~ Real part → even function.

Imaginary part → odd function.

magnitude → even function.

angle → odd function.

Existence of DTFT

→ ~~It~~ exists for all periodic signals, but aperiodic signals should satisfy certain conditions.

The sufficient condition for existence of DTFT for any ~~aperi~~ aperiod sequence $x(nT)$ is

$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty \quad \text{(Absolutely summable)}$$

should be finite.

→ If the sequence is not satisfied the above condition also the fourier transform exist

Ex: Unit step, $\cos\omega_0 n$, $\sin\omega_0 n$, $e^{j\omega_0}$

$-a^{n} \cdot e^{-j\omega n}$

$(a^{-1} \cdot e^{+j\omega})^{n}$

$(a^{-1} e^{j\omega})^{n}$

$\sum (a^{-1} \cdot e^{j\omega})^{n} = \frac{-a^{-1} e^{j\omega}}{1 - a^{-1} e^{j\omega}}$

$a\, e^{j\omega} + a\, (e^{j\omega})^{2} + \ldots$

$\frac{\frac{1}{a} e^{j\omega}}{1 - a\, e^{j\omega}}$

$\frac{(a\, e^{j\omega})^{x}}{(a\, e^{j\omega}) - 1}$

$\frac{1}{a - e^{j\omega}}$

→ find the DTFT of the input $\delta(n+3) - \delta(n-3)$ –

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n) \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{+\infty} \delta(n+3) - \delta(n-3)$$

$$= e^{j3\omega} - e^{-j3\omega}$$

$x(n) = \delta(n)$

$X(z) = 1$

$\delta(n+3) = z^{3} X(z)$

$\delta(n-3) = z^{-3} X(z)$

$= z^{3} + z^{-3}$

$= e^{j3\omega} + e^{-j3\omega}$

→ $x(n) = \{1, 2, 3, 4\}$

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\omega n}$$

$$= 1 + 2 e^{-j\omega} + 3 \cdot e^{-j2\omega} + 4 \cdot e^{-j3\omega}$$

$u(-n-1)$

$u(n-1)$

$u(-n-1)$

$\sum_{n=-\infty}^{-n+1} a^{n} (1) e^{-j\omega n}$

$(a \cdot e^{j\omega})^{-n-1}$

$a + (a e^{-j\omega})^{-1}$

$+ (a e^{-j\omega})^{-1}$

→ $x(n) = -a^{n} \cdot u(-n-1)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{-1} -a^{n} \cdot e^{-j\omega n}$$

$$= -\sum_{n=-\infty}^{-1} (a^{-1} \cdot e^{+j\omega})^{-n}$$

$$= -\sum_{n=\infty}^{1} (a^{-1} e^{j\omega})^{n}$$

$$= -\sum_{n=1}^{\infty} (a^{-1} \cdot e^{j\omega})^{n} = \frac{-a^{-1} e^{j\omega}}{1 - a^{-1} e^{j\omega}}$$

$$= \frac{1}{1 - a e^{-j\omega}}$$

$a e^{j\omega} + a e^{j\omega} +$

$\frac{a^{-1} e^{j\omega}}{1 - a e^{j\omega}}$

$\frac{(a e^{j\omega})^{-1}}{(a e^{j\omega})^{-1} - 1}$

$\frac{1}{a e^{j\omega}}$

$1+2+3$

$3+2+1$

$1+1+1+3$

$3+2+1$

$\rightarrow x(n) = (1/4)^n u(n+1)$

$x(n) = (0.5)^n u(n) - 2^n u(-n-1)$

$$\sum_{n=-\infty}^{\infty} (1/4)^n e^{-j\omega n} = \cancel{(1/4)^{-1} + 1 + (1/4)^1 + \dots}$$

$$= \cancel{1 \cdot (1/4)^{-1} [1 + (1/4) + (1/4)^2 + \dots}$$

$$= \cancel{4 \left[\frac{1}{1-1/4}\right]}$$

$$= \cancel{4 \left[\frac{1}{3/4}\right]}$$

$$= \cancel{16/3}$$

$$= \sum_{n=-1}^{\infty} (1/4 \, e^{-j\omega})^n$$

$$= (1/4 \cdot e^{-j\omega})^{-1} + (1/4 \, e^{-j\omega})^0 + (1/4 \, e^{-j\omega})^1 + \dots$$

$$= (1/4 \, e^{-j\omega})^{-1} [1 + (1/4) e^{-j\omega} + (1/4 \, e^{-j\omega})^2 + \dots$$

$$= \frac{4 \cdot e^{j\omega}}{1 - 1/4 \, e^{-j\omega}}$$

(3) $x(n) = (0.5)^n u(n) - 2^n u(-n-1)$

Sol: $X(e^{j\omega}) = \sum_{n=0}^{\infty} (0.5)^n e^{-j\omega n} - \sum_{n=-\infty}^{-1} (2)^n e^{-j\omega n}$

$$= \frac{1}{1-0.5 e^{-j\omega}} - \sum_{n=1}^{\infty} (2^{-1} e^{j\omega})^n$$

$$= \frac{1}{1-0.5 e^{-j\omega}} - \frac{2^{-1} e^{j\omega}}{1-2^{-1} e^{j\omega}} \quad (\because a < 1)$$

$$= \frac{1}{1-0.5 e^{-j\omega}} + \frac{1}{1-2e^{-j\omega}}$$

Properties of DTFT:-

1) Linearity

$F[x_1(n)] = X_1(e^{j\omega})$; $F[x_2(n)] = X_2(e^{j\omega})$.

$$F[a x_1(n) + b x_2(n)] = a X_1(e^{j\omega}) + b.X_2(e^{j\omega})$$

2) Periodicity.

$F[x(n)] = X(e^{j\omega})$

if $X(e^{j\omega})$ is periodic in ω. with period 2π

$$X(e^{j\omega}) = X(e^{j(\omega \pm 2\pi k)})$$

3) Time shifting

$F[x(n)] = X(e^{j\omega})$.

$$F[x(n-k)] = e^{-j\omega k}.X[e^{j\omega}]$$

$$= \sum_{n=-\infty}^{\infty} x(n-k).e^{-j\omega n}$$

$n-k = p$

$$= \sum_{p=-\infty}^{\infty} x(p).e^{-j\omega p}.e^{-j\omega k}$$

$$= e^{-j\omega k}.X(e^{j\omega})$$

4) frequency shifting property

$F[x(n)] = X(e^{j\omega})$

$$F[x(n).e^{j\omega_0 n}] = X[e^{j(\omega-\omega_0)}]$$

$$= \sum_{n=-\infty}^{\infty} x(n).e^{j\omega_0 n}.e^{-j\omega n}.$$

$$= \sum_{n=-\infty}^{\infty} x(n).\ e^{-j(\omega-\omega_0)n}$$

$$= X[e^{+j(\omega-\omega_0)}]$$

Time reversal property

$$F[x(n)] = X(e^{j\omega})$$

$$F[x(-n)] = X(e^{-j\omega})$$

$$F[x(-n)] = \sum_{n=-\infty}^{\infty} x(-n)\ e^{-j\omega n}$$

~~x(n)~~ k(-n) $\quad$ fu $\quad e^{-j\omega n}$

$$= \sum_{n=-\infty}^{\infty} x(n).\ e^{j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(n).\ e^{-j(-\omega)n}$$

$$= X(e^{-j\omega})$$

Differentiation in frequency

$$F[x(n)] = X(e^{j\omega})$$

$$F[n\,x(n)] = j.\frac{d}{d\omega}X(e^{j\omega})$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n).\ e^{-j\omega n}$$

$$\frac{d}{d\omega}X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n).\ e^{-j\omega n}\ (-jn)$$

$$\frac{-1}{j}\frac{d}{d\omega}X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} n\,x(n).\ e^{-j\omega n}$$

$$j\frac{d}{d\omega}X(e^{j\omega}) = F[n\,x(n)]$$

Time convolution:-

$$F[x_1(n)] = X_1(e^{j\omega})$$

$$F[x_2(n)] = X_2(e^{j\omega})$$

$$F[x_1(n) * x_2(n)] = X_1(e^{j\omega}) \cdot X_2(e^{j\omega})$$

$$x_1(n) * x_2(n) = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$$

$$F[x_1(n) * x_2(n)] = \sum_{n=-\infty}^{\infty} x_1(n) * x_2(n) \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)\, e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x_1(k) \sum_{n=-\infty}^{\infty} x_2(n-k) \cdot e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x_1(k) \cdot X_2(e^{j\omega}) \cdot e^{-j\omega k}$$

$$= X_2(e^{j\omega}) \cdot X_1(e^{j\omega})$$

Frequency convolution:-

$$F[x_1(n)] = X_1(e^{j\omega})$$

$$F[x_2(n)] = X_2(e^{j\omega})$$

$$F[x_1(n) \cdot x_2(n)] = \frac{1}{2\pi}\int_0^{2\pi} X_1(e^{j\theta}) * X_2(e^{j(\omega-\theta)})\, d\theta$$

Correlation Property:-

if $F[x_1(n)] = X_1(e^{j\omega})$

$F[x_2(n)] = X_2(e^{j\omega})$.

$F[r_{x_1x_2}(l)] = \Gamma_{x_1x_2}(e^{j\omega}) = X_1(e^{j\omega}) \cdot X_2(e^{-j\omega})$

Modulation property:-

$F[x(n)\cos\omega_0 n] = \frac{1}{2}[X[e^{j(\omega+\omega_0)}] + X[e^{j(\omega-\omega_0)}]]$

Symmetry Property:-

$X(e^{j\omega}) = X_R(e^{j\omega}) + jX_I(e^{j\omega})$.

$X(e^{j\omega}) = |X(e^{j\omega})| e^{j\theta(\omega)}$

$X(e^{j\omega}) = \sqrt{X_R^2(e^{j\omega}) + X_I^2(e^{j\omega})}$

$|X(e^{-j\omega})| = \sqrt{X_R(e^{-j\omega})^2 + X_I(e^{-j\omega})^2}$

$= |X(e^{j\omega})|$

$\theta(\omega) = \tan^{-1}\left(\frac{X_I(\omega)}{X_R(\omega)}\right)$

$\theta(\omega)$ for $X(e^{-j\omega}) = \tan^{-1}\left(\frac{X_I(-\omega)}{X_R(\omega)}\right)$

$= \tan^{-1}\left(\frac{-X_I(\omega)}{X_R(\omega)}\right)$

$= -\theta(\omega)$

① $y(n) + y(n-1) = \cos 2n$. Find the total response of the system.

② Find the DTFT for the following I/p's

a) $x(n) = 2^{n} u(-n)$

b) $x(n) = e^{-2n}$ for $n = 0, 1, 2$

e^{3n} for $n = -1, -2, -3$

c) $x(n) = e^{-2n}$

③ $x(n) = \left(\frac{3}{-4}\right)^{n} u(n)$ $h(n) = \left(\frac{-1}{2}\right)^{n} u(n)$ Find the DTFT of the o/p response $(Y(e^{j\omega}))$

④ $x(n) = (-1)^{n} u(n)$ $h(n) = \left(\frac{1}{2}\right)^{n} u(n)$ Find the DTFT of the o/p response.

⑤ $y(n) + y(n-1) - 2y(n-2) = x(n-1) + 2x(n-2)$. Find the impulse response of the system.

($\because N = M = 2$ so $y_h(n)$ includes an impulse function $A\delta(n)$)

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Frequency Response analysis of discrete time systems:-

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

(or)

$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k)\, h(k)$$

Let $x(n) = e^{j\omega n}$

$$y(n) = \sum_{k=-\infty}^{\infty} e^{j\omega(n-k)} \cdot h(k)$$

$$y(n) = e^{j\omega n} \sum_{k=-\infty}^{\infty} e^{-j\omega k} \cdot h(k)$$

$$y(n) = e^{j\omega n} \cdot H(e^{j\omega})$$

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h(k) \cdot e^{-j\omega k}$$

$H(e^{j\omega})$ period is 2π

$$H(e^{j\omega}) = |H(e^{j\omega})| \cdot e^{j\theta(\omega)}$$

$$\theta(\omega) = \angle H(e^{j\omega})$$

$y(n) - a y(n-1) = x(n)$

Apply DTFT

$$Y(e^{j\omega}) - a e^{-j\omega} \cdot Y(e^{j\omega}) = X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 - a \cdot e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{1}{1-a\cos\omega - j\sin\omega}$$

$$= \frac{1}{1-a\cos\omega + ja\sin\omega}$$

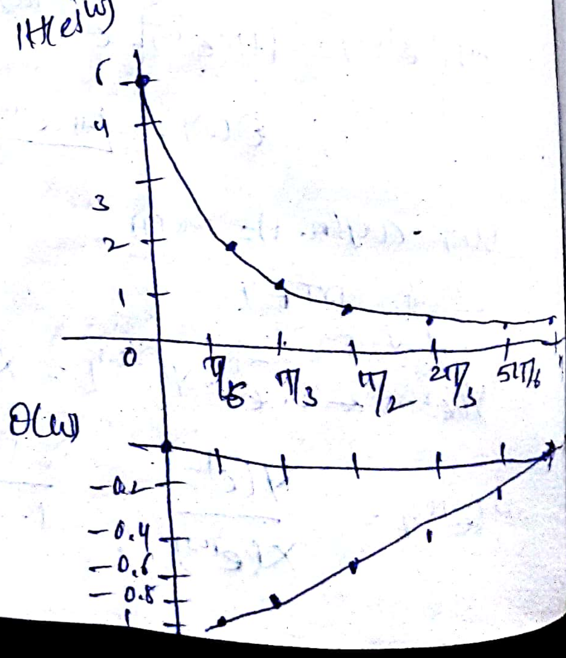
$$|H(e^{j\omega})| = \frac{1}{\sqrt{(1-a\cos\omega)^2 + (a\sin\omega)^2}}$$

$$= \frac{1}{\sqrt{1+a^2-2a\cos\omega}}$$

Let $a = 0.8$.

$$\theta(\omega) = -\tan^{-1}\left(\frac{a\sin\omega}{1-a\cos\omega}\right)$$

$\omega =$	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
$\lvert H(e^{j\omega})\rvert$	5	1.98	1.09	0.78	0.64	0.57	0.5[illegible]
$\angle H(e^{j\omega})$	0	−0.9	−0.8	−0.6	−0.45	−0.23	0



→ find the fr

$y(n) =$

Sol: $Y(e^{j\omega}) =$

$H(e^{j\omega}$

$H($

$|H$

find the frequency response of

$$y(n) = \frac{1}{2}x(n) + \frac{1}{2}x(n-2)$$

ol: $Y(e^{j\omega}) = \frac{1}{2}X(e^{j\omega}) + \frac{1}{2}e^{-j2\omega}.X(e^{j\omega})$

$$H(e^{j\omega}) = \frac{1}{2} + \frac{1}{2}.e^{-j2\omega}$$

$$= \frac{1+e^{-j2\omega}}{2}$$

$\frac{\pi}{2} - \tan^{-1}(\theta)$

$\tan^{-1}(-\theta)$

$= \frac{\pi}{2} - \tan^{-1}(\theta)$

$$H(e^{j\omega}) = \frac{1+\cos 2\omega - j\sin 2\omega}{2}$$

$$|H(e^{j\omega})| = \frac{\sqrt{(1+\cos 2\omega)^2 + (\sin 2\omega)^2}}{2}$$

$$= \frac{1}{2}\sqrt{1+\cos^2 2\omega + 2\cos 2\omega + \sin^2 2\omega}$$

$$= \frac{1}{2}\sqrt{2(1+\cos 2\omega)}$$

$$= \frac{1}{2}\sqrt{2.2\cos^2\omega}$$

$$= \frac{\cancel{2}}{\cancel{2}}\cos\omega$$

$$= \cos\omega$$

$\tan(-\theta) = -\tan(\theta)$

$\tan^{-1}(\tan(-\omega))$

1.046

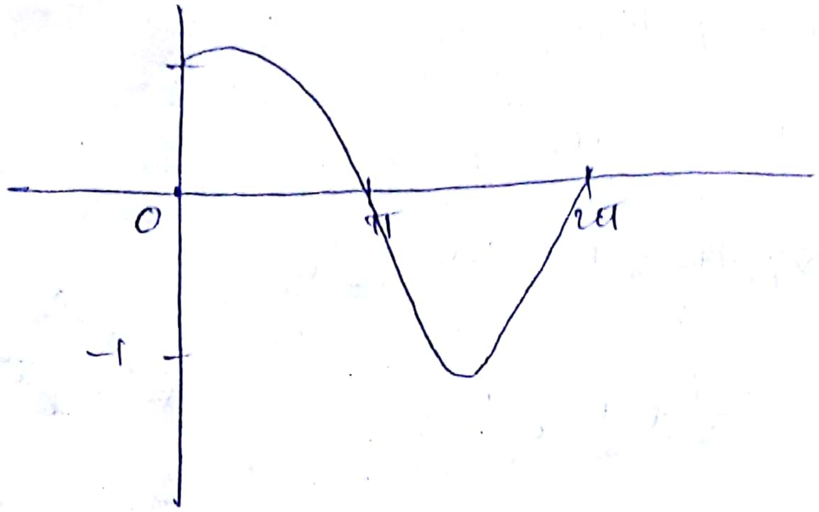
3.14

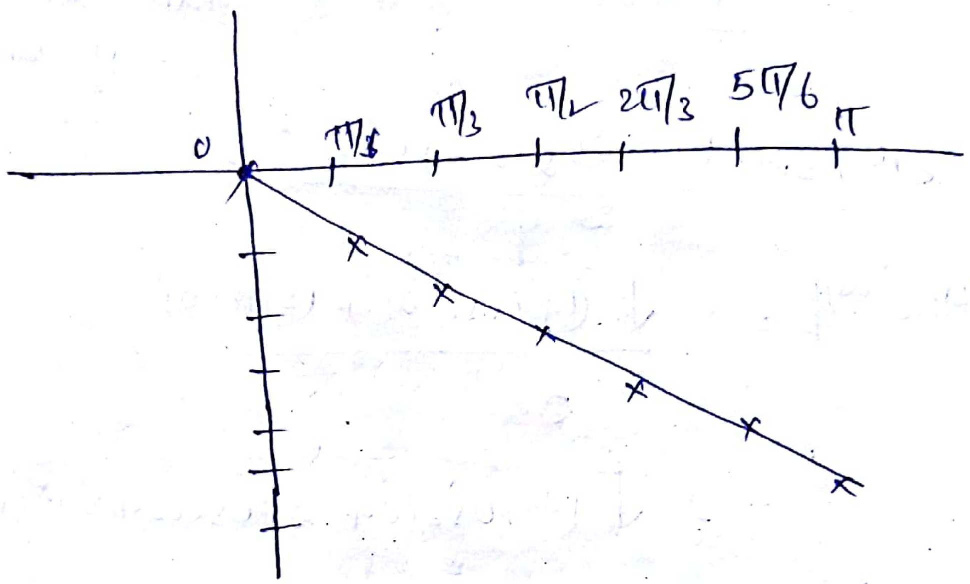
3

$$\theta(\omega) = \tan^{-1}\left(\frac{-\sin 2\omega}{1+\cos 2\omega}\right)$$

$$= \tan^{-1}\left(\frac{-\cancel{2}\sin\omega\cos\omega}{\cancel{2}\cos^2\omega}\right)$$

$$= \tan^{-1}(-\tan\omega) = -\omega$$





$\frac{z}{z-a}$

$\frac{1}{a^{-1}-z^{-1}}$

→ $h(n) = \frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)$

→ $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) - x(n-1)$ $\frac{y}{x}$

→ $h(n) = \frac{1}{2}\cdot\left(\left(\frac{1}{2}\right)^n + \left(\frac{-1}{4}\right)^n\right)u(n)$

→ $h(n) = \left(\frac{-1}{4}\right)\delta(n+1) + \frac{1}{2}\delta(n) - \frac{1}{2}\delta(n-1)$

a) Is it stable b) causal or not

c) freq response

$\rightarrow$ $h(n) = \frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)$

$H(e^{j\omega}) = \frac{1}{2} + e^{-j\omega} + \frac{1}{2}e^{-j2\omega}$

$= \frac{1}{2} + \cos\omega - j\sin\omega + \frac{1}{2}(\cos 2\omega - j\sin 2\omega)$

$= \frac{1}{2} + \cos\omega + \frac{1}{2}\cos 2\omega - j(\sin\omega + \frac{1}{2}\sin 2\omega)$

$= \frac{1}{2}[1+\cos 2\omega] + \cos\omega - j(\sin\omega + \frac{1}{\cancel{2}}\cancel{2}\sin\omega\cos\omega)$

$= \frac{1}{\cancel{2}}(\cancel{2}\cos^2\omega) + \cos\omega - j(\sin\omega + \sin\omega\cos\omega)$

$= \cos\omega(\cos\omega + 1) - j\sin\omega(1+\cos\omega)$

$= \cos\omega \cdot \cos^2\omega/_2 - j\sin\omega\cos^2\omega/_2$

$= \cos^2\omega/_2(\cos\omega - j\sin\omega)$ $\quad$ $a+jb$ $\quad$ $\sqrt{a^2+b^2}$

$|H(e^{j\omega})| = \cos^2\omega/_2 \cdot \sqrt{\cos^2\omega + \sin^2\omega}$

$= \cos^2\omega/_2$

$\angle\theta(e^{j\omega}) = \tan^{-1}\left(\frac{-\sin\omega}{\cos\omega}\right)$ $\quad$ $\omega = 0$ $\quad$ $\omega =$

$\omega = 0$, $\omega = \pi/4$, $\pi/2$, $3\pi/4$, π

$= -\underline{\underline{\omega}}$

ω	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
$\lvert H(e^{j\omega})\rvert$	2	1.86	1.5	1	0.5	0.13	0
$\angle\theta(e^{j\omega})$	0	0.52	1.04	0	-1.04	-0.52	0

$\sum_{n=0} h(n)$

*

$$y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n) - x(n-1)$$

$$Y(e^{j\omega}) - \frac{3}{4} e^{-j\omega} Y(e^{j\omega}) + \frac{1}{8} e^{-j2\omega} Y(e^{j\omega}) = X(e^{j\omega}) - e^{-j\omega}$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - e^{-j\omega}}{1 - \frac{3}{4} e^{-j\omega} + \frac{1}{8} e^{-j2\omega}}$$

let $e^{j\omega} = z$

$$H(z) = \frac{1 - z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$H(z) = \frac{z(z-1)}{z^2 - \frac{3}{4} z + \frac{1}{8}}$$

$$= \frac{8z(z-1)}{8z^2 - 6z + 1}$$

$$= \frac{8z(z-1)}{8z^2 - 4z - 2z + 1}$$

$$= \frac{8z(z-1)}{4z(2z-1) - 1(2z-1)}$$

$$H(z) = \frac{8z(z-1)}{(2z-1)(4z-1)}$$

$$\frac{H(z)}{z} = \frac{8(z-1)}{(2z-1)(4z-1)}$$

$$= \frac{z-1}{(z-\frac{1}{2})(z-1/4)}$$

$\angle a \cdot \angle a_2$

$\angle a + a_2$

$\frac{\angle a}{\angle a_2}$

$\angle a_1 - a$

$$\frac{H(z)}{z} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-1/4}$$

$$z-1 = A(z-1/4) + B(z-1/2)$$

$z = 1/4 \Rightarrow -3/4 = B(-1/4) \Rightarrow B = 3$

$z = 1/2,\ -1/2 = \cancel{A(1/4)} \Rightarrow A \Rightarrow -2$

$$\frac{H(z)}{z} = \frac{-2}{z-1/2} + \frac{3}{z-1/4}$$

$$H(z) = \frac{-2}{1-1/2 z^{-1}} + \frac{3}{1-1/4 z^{-1}}$$

$$H(e^{jw}) = \frac{-2}{1-1/2\, e^{-jw}} + \frac{1}{1-\frac{1}{4} e^{-jw}}$$

$|H(e^{jw})| =$ $\phi_1 - \phi_2$

$$-\tan^{-1}\frac{+1/2 \sin w}{1+1/2 \cos w} - \tan^{-1}\frac{+1/4 \sin w}{1+1/4 \cos w}.$$

$$|H(e^{jw})| = \frac{2}{\sqrt{(1-\frac{1}{2}\cos w)^2 + \sin^2 w}} + \frac{1}{\sqrt{(1-\frac{1}{4}\cos w)^2 + \sin^2 w}}$$

$$\angle \theta(e^{jw}) = -\tan^{-1}\frac{1/2 \sin w}{1-1/2 \cos w} - \tan^{-1}\frac{1/4 \sin w}{1-1/4 \cos w}.$$

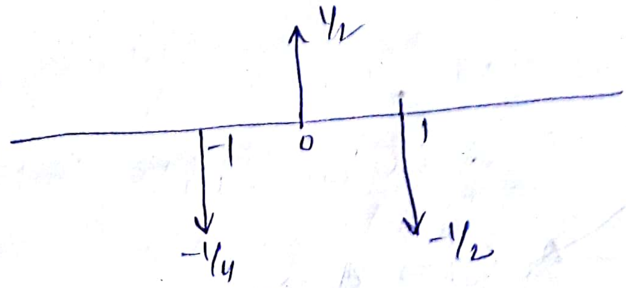
plotted with in matlab

$\rightarrow h(n) = -\frac{1}{4}\delta(n+1) + \frac{1}{2}\delta(n) - \frac{1}{2}\delta(n-1)$

$\sum_{n=-\infty}^{\infty} h(n) = -\frac{1}{4} + \frac{1}{2} - \frac{1}{2} = -\frac{1}{4}$

Absolutely summable. so, stable.

b)



$h(n) \neq 0$ for $n<0$ so, non causal

$H(e^{j\omega}) = -\frac{1}{4} e^{j\omega} + \frac{1}{2} - \frac{1}{2} e^{-j\omega}$

$= \frac{1}{2} - \frac{1}{2}\cos\omega + \frac{1}{2} j \sin\omega - \frac{1}{4}\cos\omega + j\frac{1}{2}\sin\omega$

$\frac{1}{2} + \frac{1}{4} \quad \frac{2+1}{4}$

$= \cancel{\frac{1}{2}(1 - \cos\omega}$

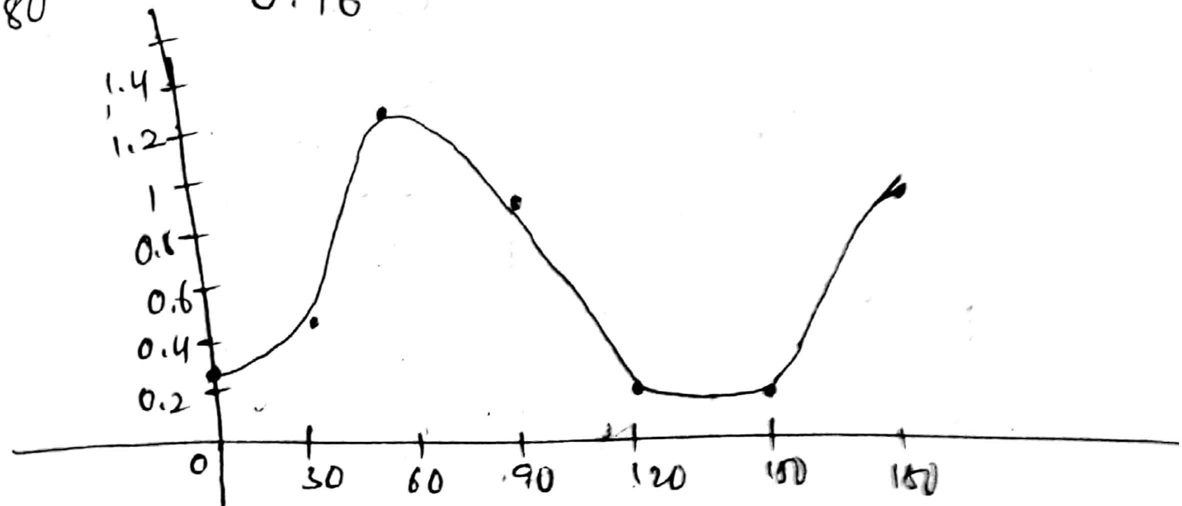
$= \left(\frac{1}{2} - \frac{1}{2}\cos\omega - \frac{1}{4}\cos\omega\right) + j\left(\frac{1}{2}\sin\omega - \frac{1}{4}\sin\omega\right)$

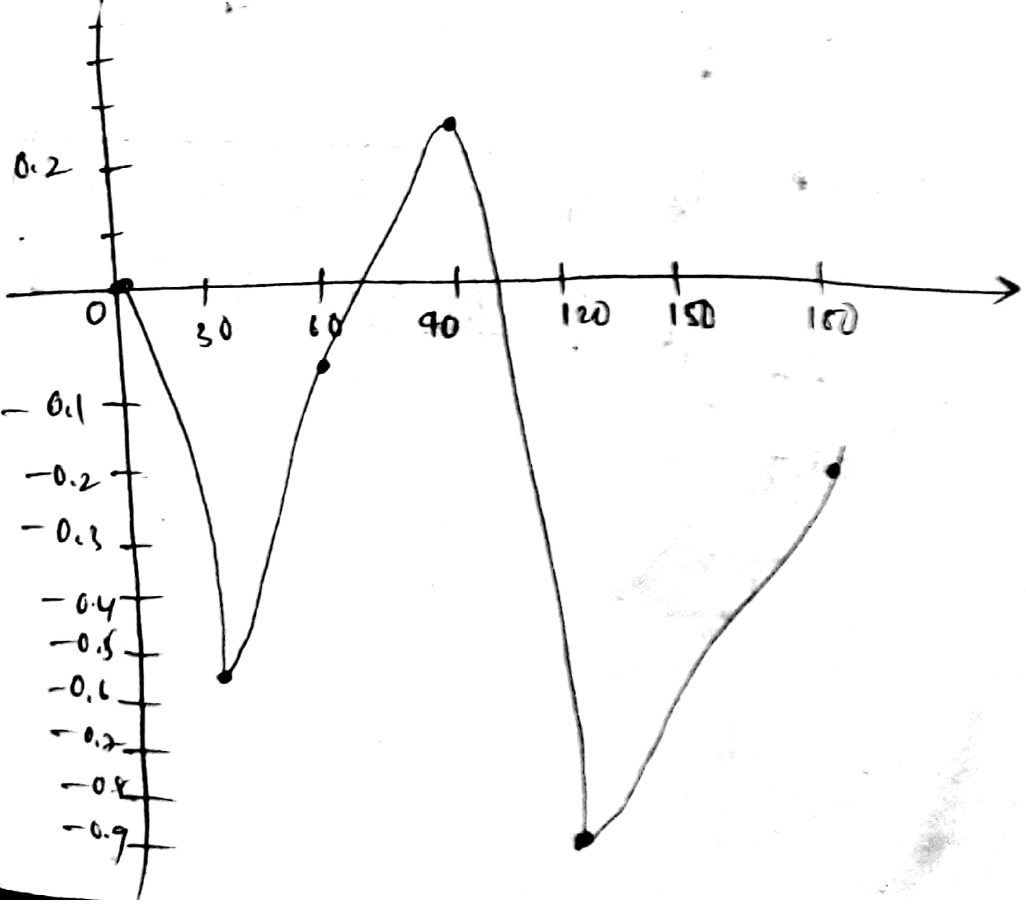
$= \left(\frac{1}{2} - \frac{3}{4}\cos\omega\right) + j\left(+\frac{1}{4}\sin\omega\right)$

$|H(e^{j\omega})| = \sqrt{\left(\frac{1}{2} - \frac{3}{4}\cos\omega\right)^2 + \left(\frac{1}{4}\sin\omega\right)^2}$

$\angle\theta(e^{j\omega}) = \tan^{-1}\left(\frac{+\frac{1}{4}\sin\omega}{\frac{1}{2} - \frac{3}{4}\cos\omega}\right)$

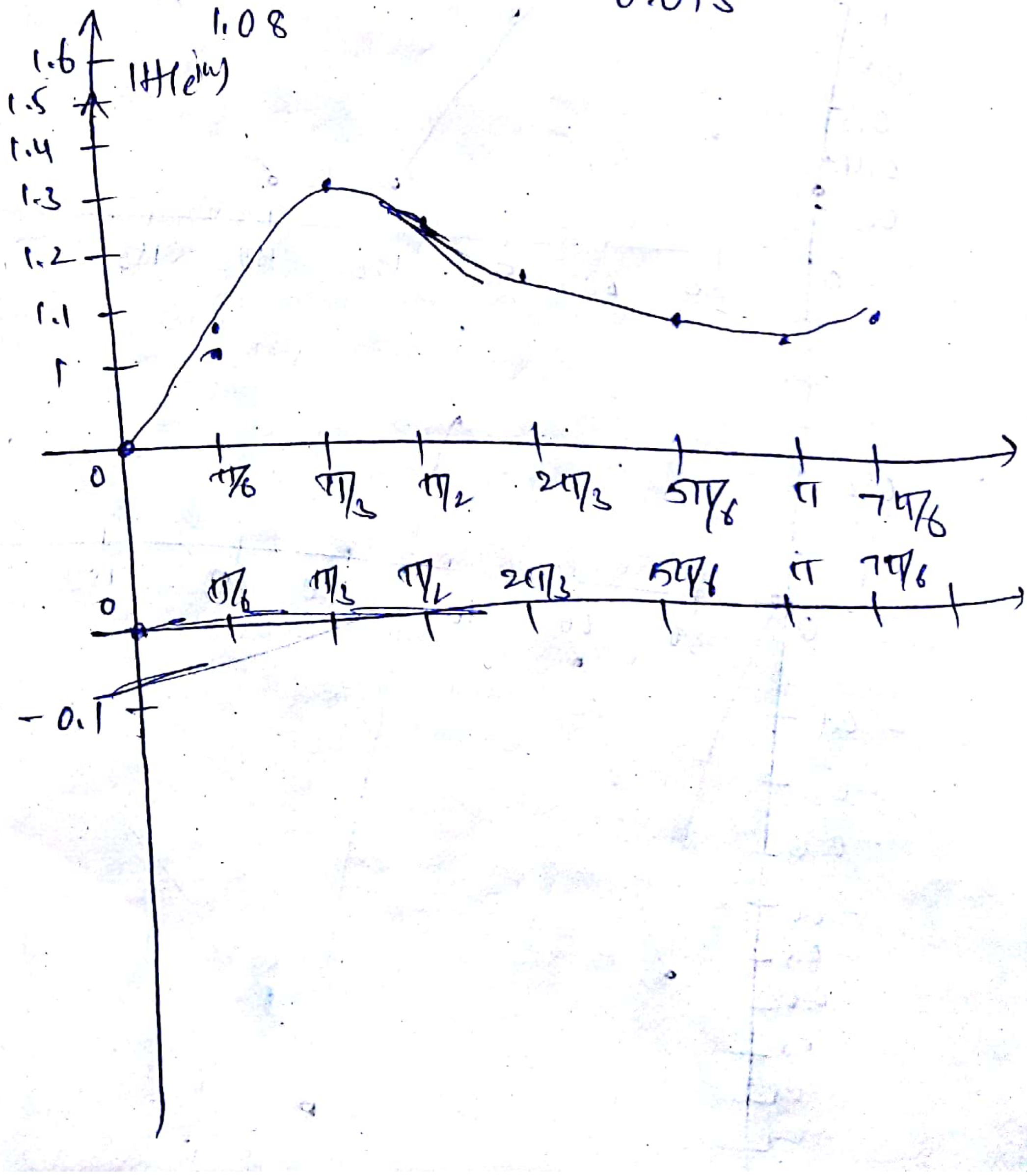
ω	$\lvert H(e^{j\omega})\rvert$	$\angle \theta(e^{j\omega})$
0°	$1/4$	0°
30	0.4568	-0.57
60	1.216	-0.0606
90	0.865	0.26
120	0.1825	-0.919
150	0.180	1.43
180	0.96	-0.208

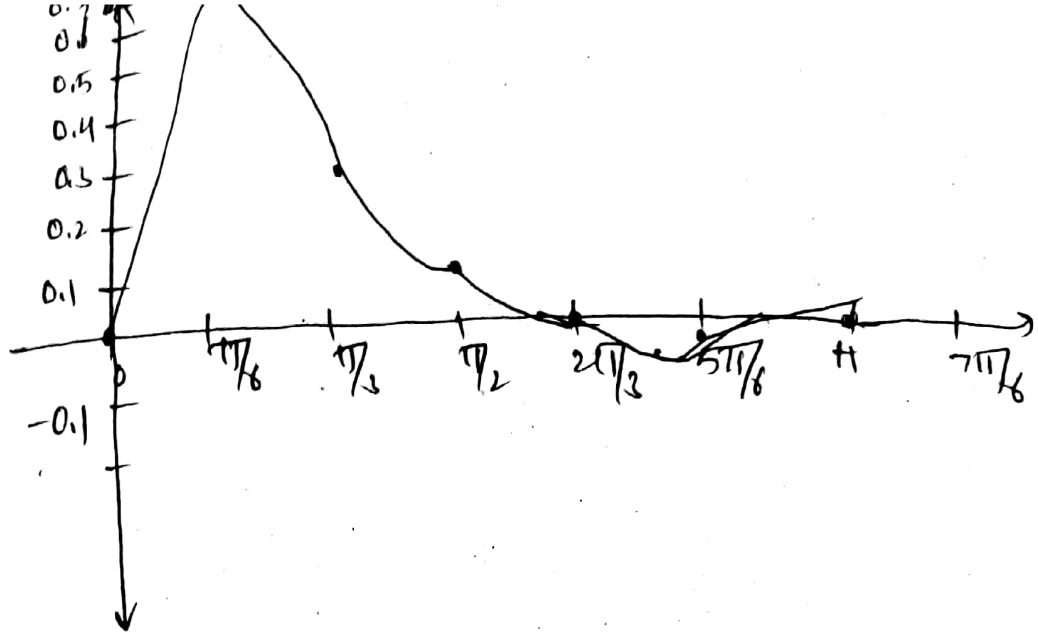




* $y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n) - x(n-1)$

ω	$\lvert H(e^{j\omega})\rvert$	$\angle \theta(e^{j\omega})$
0	0	0
$\pi/6$	1.05	0.73
$\pi/3$	1.28	0.21
$\pi/2$	1.22	0.076
$2\pi/3$	1.14	0
$5\pi/6$	1.08	-0.013
π	1.06	0
$7\pi/6$	1.08	0.013





$$h(n) = \frac{1}{2}\left(\left(\frac{1}{2}\right)^n + \left(-\frac{1}{4}\right)^n\right)u(n)$$

$$= \frac{1}{2}\left(\left(\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{4}\right)^n u(n)\right)$$

$$H(e^{j\omega}) = \frac{1}{2}\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}} + \frac{1}{1+\frac{1}{4}e^{-j\omega}}\right)$$

$$= \frac{1}{2}\left[\frac{1+\frac{1}{4}e^{-j\omega}+1-\frac{1}{2}e^{-j\omega}}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1+\frac{1}{4}e^{-j\omega}\right)}\right]$$

$$= \frac{1}{2}\left[\frac{2-\frac{1}{4}e^{-j\omega}}{1+\frac{1}{4}e^{-j\omega}-\frac{1}{2}e^{-j\omega}-\frac{1}{8}e^{-j2\omega}}\right]$$

$$= \frac{\cancel{2}}{2}\left[\right.$$

$$|H(e^{j\omega})| = \frac{\sqrt{\left(1-\frac{1}{4}\cos\omega\right)^2+\left(\frac{1}{4}\sin\omega\right)^2}}{\sqrt{\left(1-\frac{1}{2}\cos\omega-\frac{1}{8}\cos 2\omega\right)^2+\left(\frac{1}{2}\sin\omega+\frac{1}{8}\sin\omega\right)^2}}$$

$$\theta = \tan^{-1}\left(\frac{1/4 \sin w}{1 - 1/4 \cos w}\right) - \tan^{-1}\left(\frac{\frac{1}{2}\sin w + 1/8 \sin 2w}{1 - 1/2 \cos w - 1/8 \cos 2w}\right)$$

w	$\lvert H(e^{jw}) \rvert$	θ
		0
0	1.20	−0.45
$\pi/6$	0.98	−0.34
$\pi/3$	0.83	−0.17
$\pi/2$	0.83	−0.052
$2\pi/3$	0.92	-6.6×10^{-4}
$5\pi/6$	1.05	
π	1.11	0

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Steady state & Transient response of a system:-

$x(n) = e^{jwn}$ (Eternal ~~syst~~ signals are those which extends from $-\infty$ to ∞)

$y(n) = e^{jwn} H(e^{jwn})$

$= \sum_{n=-\infty}^{\infty} h(k)\, x(n-k)$

$y(n) = a\, y(n-1) + x(n)$ $\quad n \geq 0$

$y(n) = a^{n+1} y(-1) + \sum_{k=0}^{n} a^k x(n-k)$

$x(n) = e^{jwn}$

$y(n) = a^{n+1} y(-1) + \sum_{k=0}^{n} a^k e^{jw(n-k)}$

$y(n) = a^{n+1} y(-1) + e^{jwn} \sum_{k=0}^{n} (a e^{-jw})^k$

$= a^{n+1} y(-1) + \frac{e^{jwn}\left(1 - a^{n+1} e^{-jw(n+1)}\right)}{1 - a e^{-jw}}$

$$y(n) = \underbrace{a^{n+1} y(-1) - e^{j\omega n}\left(\frac{a^{n+1} e^{-j\omega(n+1)}}{1-ae^{-j\omega}}\right)}_{\text{Transient response}} + \underbrace{\frac{e^{j\omega n}}{1-ae^{-j\omega}}}_{\underbrace{e^{j\omega n} H(e^{j\omega})}_{\text{steady state}}}$$

$$\left(\because Y(e^{j\omega}) = ae^{-j\omega} Y(e^{j\omega}) + X(e^{j\omega}),\ H(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}\right)$$

Phase delay, Group delay

When I/p is sinusoidal, $x(n) = A \sin(\omega n) \quad -\infty < n < \infty$

$$x(n) = \frac{A}{2j}\left[e^{j\omega n} - e^{-j\omega n}\right]$$

$$= \frac{1}{2j}\left[A e^{j\omega n}|H(e^{j\omega})| e^{j\theta(\omega)} - A e^{-j\omega n}|H(e^{-j\omega})| e^{j\theta(-\omega)}\right]$$

$$= \frac{1}{2j} A |H(e^{j\omega})| \left[e^{j\omega n} e^{j\theta(\omega)} - e^{-j\omega n} e^{-j\theta(\omega)}\right]$$

$$= \frac{A |H(e^{j\omega})|}{2j}\left[e^{j(\omega n + \theta(\omega))} - e^{-j(\omega n + \theta(\omega))}\right]$$

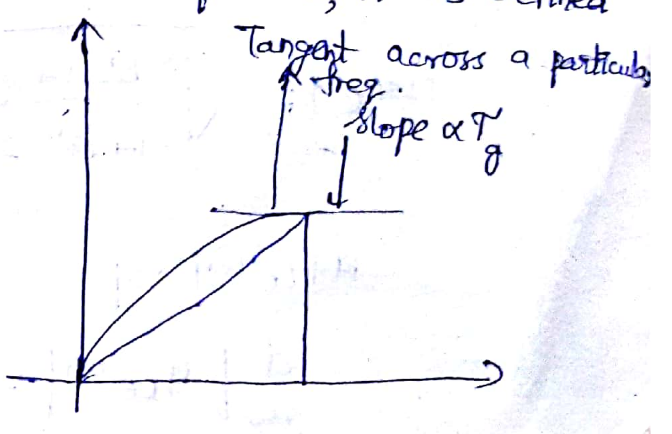
$$y(n) = A|H(e^{j\omega})| \sin(\omega n + \theta(\omega))$$

$$y(n) = A|H(e^{j\omega})| \sin\left(\omega\left(n + \frac{\theta(\omega)}{\omega}\right)\right)$$

When I/p is sinusoidal signal, o/p is also sinusoidal signal multiplied with a factor of magnitude $|H(e^{j\omega})|$ & delay is defined as phase delay $\left(\frac{\theta(\omega)}{\omega}\right)$. This delay plays an important role when it is passed through filter, so it is referred as phase delay. When the i/p consists of more than one harmonics of 'sine', it is defined as group delay

$$\boxed{\tau_p = \frac{-\theta(\omega)}{\omega}}$$

$$\boxed{\tau_g = \frac{-d\theta(\omega)}{d\omega}}$$



Problem

→ The causal LTI system is described by D.E where a is real & $y(n) - a\,y(n-1) = bx(n) + x(n-1)$ find the value of b condition $a \neq b$ in magnitude. find the value of b such that freq. response satisfies the condition

$$|H(e^{j\omega})| = 1, \ \forall \omega.$$

Sol

$$y(n) - a\,y(n-1) = bx(n) + x(n-1)$$

$$Y(e^{j\omega})\,[1 - a e^{-j\omega}] = X(e^{j\omega})\,[b + e^{-j\omega}]$$

$$H(e^{j\omega}) = \frac{b + e^{-j\omega}}{1 - a e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{(b + \cos\omega) - j\sin\omega}{1 - a\cos\omega + j\sin\omega}$$

$$(b+\cos\omega)^2 + \cancel{\sin^2\omega} = (1 - a\cos\omega)^2 + \sin^2\omega$$

$$b + \cos\omega = 1 - a\cos\omega$$

$$(1+a)\cos\omega = 1 - b$$

$$\cos\omega = \frac{1-b}{1+a}$$

$$|H(e^{j\omega})| = \sqrt{\frac{b^2 + 1 + 2b\cos\omega}{1 + a^2 - 2a\cos\omega}}$$

$$|H(e^{j\omega})| = 1$$

$$\frac{d}{d\omega}|H(e^{j\omega})| = 0$$

$$\frac{d}{d\omega}|H(e^{j\omega})|^2 = 0$$

$$\frac{d}{d\omega}\left(\frac{b^2+1+2b\cos\omega}{1+a^2-2a\cos\omega}\right) = 0$$

$$\frac{-2a(-\sin\omega) -}{(1+a^2-2a\cos\omega)^2}$$

$$\frac{(1+a^2-2a\cos\omega)[-2b\sin\omega] - (1+b^2+2b\cos\omega)(2a\sin\omega)}{(1+a^2-2a\cos\omega)^2} = 0$$

$$-2b\sin\omega - 2a^2b\sin\omega + 4ab\cancel{\cos\omega}\sin\omega - 2a\sin\omega - 2ab^2\sin\omega - 4ab\sin\omega\cancel{\cos\omega} = 0$$

$$-2ab\sin\omega(a+b) - 2\sin\omega(a+b) = 0$$

$$(a+b)(2ab\sin\omega + 2\sin\omega) = 0$$

$$2\sin\omega(a+b)(ab+1) = 0$$

$$\boxed{b = -1/a}$$

→ The freq. domain representation of i/p $X(e^{j\omega}) = j \; (-\pi \le \omega \le 0)$

$-j \; (0 \le \omega \le \pi)$

find $x(n)$.

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) \cdot e^{j\omega n} \cdot d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} j \, e^{j\omega n} d\omega + \int_{0}^{\pi} -j \, e^{j\omega n} d\omega\right]$$

$$= \frac{1}{2\pi}\left[j \left(\frac{e^{j\omega n}}{jn}\right)_{-\pi}^{0} - j\left(\frac{e^{j\omega n}}{jn}\right)_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{1-(-1)^{n}}{n}\right) - \left(\frac{(-1)^{n}-1}{n}\right)\right]$$

$$= \frac{1}{2\pi}\left[\frac{1-e^{-j\pi n}}{n} - \frac{e^{j\pi n}-1}{n}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1-e^{-j\pi n} - e^{j\pi n}+1}{n}\right]$$

$$= \frac{1}{2\pi}\left[\frac{2-2\cos\pi n}{n}\right]$$

$$= \frac{1-\cos\pi n}{\pi n} = \frac{2\sin^2 \pi n/2}{\pi n}$$

$$= \frac{\sin^2 \pi n/2}{\pi n/2}$$

→ find the res

$y(n) = \frac{1}{2}$

$x(n) = 3$

Sol: $y(n) =$

$y(n) =$

$y(e^{j}$

the

→ find the response of the system

$y(n) = \frac{1}{2}[x(n) - x(n-2)]$ to the I/p ~~x(n) = 3~~

$x(n) = 3 + 4\cos(\frac{\pi}{3}n + 60)$ in $-\infty \leq n \leq \infty$.

Sol: $y(n) = A \cdot |H(e^{j\omega})| \sin(\omega n + \theta(\omega))$

$y(n) = \frac{1}{2}[x(n) - x(n-2)]$

$Y(e^{j\omega}) = \frac{1}{2}[X(e^{j\omega}) - e^{-j2\omega} \cdot X(e^{j\omega})]$

$H(e^{j\omega}) = \frac{1}{2}[1 - e^{-j2\omega}]$

$= \frac{1}{2}[1 - \cos 2\omega + j\sin 2\omega]$

$= \sin^2\omega + j\sin\omega\cos\omega$

$H(e^{j\omega}) = \sin\omega(\sin\omega + j\cos\omega)$

$|H(e^{j\omega})| = \sin\omega$ $\qquad \tan(\frac{\pi}{2} - \theta) = \cot\theta$

$\angle\theta(\omega) = \cancel{\tan^{-1}(\ldots)}\ \tan^{-1}\left(\frac{\sin\omega}{1-\cos\omega}\right)$ $\qquad \frac{\pi}{2} - \theta = \tan^{-1}(\cot\theta)$

$= \frac{\pi}{2} \cancel{\omega} = 0$ $\qquad 1.866$, $.732$, 2

$|H(e^{j\omega})|_{\omega=0} = 0$, $\angle\theta(\omega) = \pi/2$

$|H(e^{j\omega})|_{\omega=\pi/3} = 0.866$

$= \tan^{-1}\left(\frac{\sin 2\pi/3}{1 - \cos 2\pi/3}\right)$

$= 68.9°$.

$y(n) = 4 \times 0.866 \cdot \cos(\frac{\pi}{3} n + 60^\circ + 89^\circ)$

→ A LTI system is described by the D.E

$y(n) = a\, y(n-1) + x(n)$ where $a = 0.9$

determine the O/p of the system to the i/p

$x(n) = 5 + \sin(\frac{\pi}{2} n) + 20 \cos(\pi n + \pi/4)$

2/1/18

UNIT-II

DISCRETE FOURIER TRANSFORM

DTFT → ~~by~~ after performing transform on discrete signal, the result is a continuous function. DTFT is suitable for infinite length sequence & finite length also.

DFT → DFT can be performed on only finite length sequence and both i/p & o/p are of discrete time.

→ When DTFT is sampled into N finite components from 0 to 2π, DFT is obtained.

→ The o/p of DFT is finite, discrete & periodic.

If 'ω' is replaced by $\frac{2\pi}{N}k$ in DTFT, it is DFT

k = Integral multiple of fundamental frequency

N – No. of samples.

Discrete Fourier Series:

Fourier Series is for periodic sequences.

→ let the i/p sequence is $x_p(n)$ which is periodic with period of 'N' samples.

$$x_p(n) = x_p(n+N)$$

→ If any signal/sequence is periodic, it

30/12/17

~~UNIT-II~~

~~Discrete fourier Transform.~~

~~Discrete~~ fourier series:-

can be represented by weighted sum of complex exponentials.

$n \to 0$ to $N-1$;

$k \to 0$ to $N-1$;

$$\text{DFS of } x_p(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_p(k)\, e^{j\frac{2\pi k n}{N}}$$

$X_p(k) \to$ Discrete Fourier series coefficient & $k = 0$ to $N-1$.

To compute $x_p(k)$ multiply the whole eqn with $e^{-j\frac{2\pi}{N}mn}$ and take the summation over n is values from 0 to N-1

$$\Rightarrow \sum_{n=0}^{N-1} x_p(n).\, e^{-j\frac{2\pi}{N}mn} = \frac{1}{N} \sum_{n=0}^{N-1}\sum_{k=0}^{N-1} X_p(k).\, e^{j\frac{2\pi}{N}nk}.\, e^{-j\frac{2\pi}{N}mn}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X_p(k) \sum_{n=0}^{N-1} .\, e^{j\frac{2\pi}{N}(k-m)n}$$

from above eqn

$$\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k-m)n} = N \quad k-m = 0, \pm N, \pm 2N$$
$$\qquad\qquad 0 \quad \text{ow}$$

$$\sum_{n=0}^{N-1} x_p(n) \cdot e^{-j\frac{2\pi}{N}mn} = \frac{1}{\cancel{N}} \sum_{k=0}^{N-1} \cdot x_p(k), \cancel{N}$$

$$\boxed{X_p(k) = \sum_{n=0}^{N-1} \cdot x_p(n) \cdot e^{-j\frac{2\pi}{N}nk}}$$ from $k=0$ to $N-1$

$$x_p(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_p(k) \cdot e^{j\frac{2\pi}{N}nk}.$$

Properties:-

1) Linearity:-

$$DFS[x_{1p}(n)] = X_{1p}(k)$$

$$DFS[x_{2p}(n)] = X_{2p}(k)$$

$$a_1 x_{1p}(n) + a_2 x_{2p}(n) = a_1 X_{1p}(k) + a_2 X_{2p}(k).$$

2) Time shifting

if $DFS[x_p(n)] = X_p(k)$

$$DFS[x_p(n-m)] = e^{-j\frac{2\pi}{N}mk} \cdot X_p(k)$$

Frequency shifting

if $DFS[x_p(n)] = X_p(k)$

$$e^{j\frac{2\pi}{N}lk} x_p(n) \longleftrightarrow X_p(k-l).$$

Convolution:

if $DFS[x_{1p}(n)] = X_{1p}(k)$

$DFS[x_{2p}(n)] = X_{2p}(k)$

$$x_{1p}(n) * x_{2p}(n) \longleftrightarrow X_{1p}(k) \cdot X_{2p}(k).$$

$$DFS\left[\sum_{n=0}^{N-1} x_{1p}(n) \cdot x_{2p}(m-n)\right] = X_{1p}(k) \cdot X_{2p}(k).$$

03/01/18

~~Time aliasing due to frequency sampling:~~

Discrete Fourier Transform:

$x(10)$

In DFT, both $x(n)$ & its DFT $X(k)$ $x(2)$

are discrete and of finite length. $e^{j\frac{2\pi}{N}k}$

When $x(n)$ is discrete (0 to N-1 → finite length)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

↓

continuous

If the DTFT $X(e^{j\omega})$ is sampled at 'N' distinct

points equally spaced, DFT is obtained.

* (Frequency domain sampling)

$X(e^{j\omega_k})$ is DFT $k \to 0$ to $N-1$

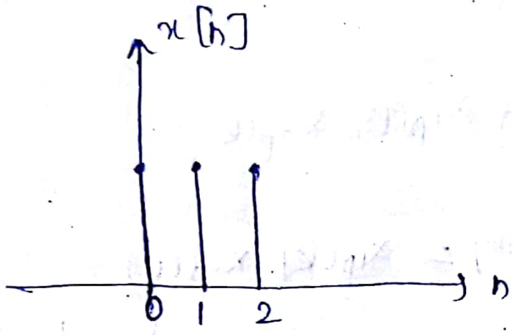
$$\omega_k = \frac{2\pi}{N}$$

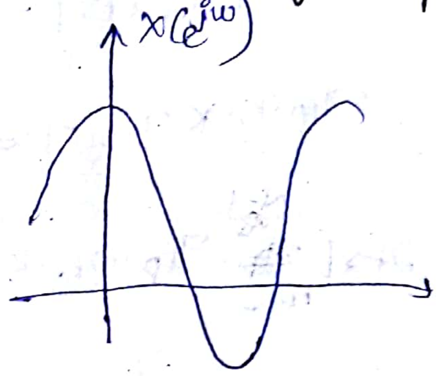
(∵ sampled into N samples)

→ Time domain sampling leads to frequency domain aliasing

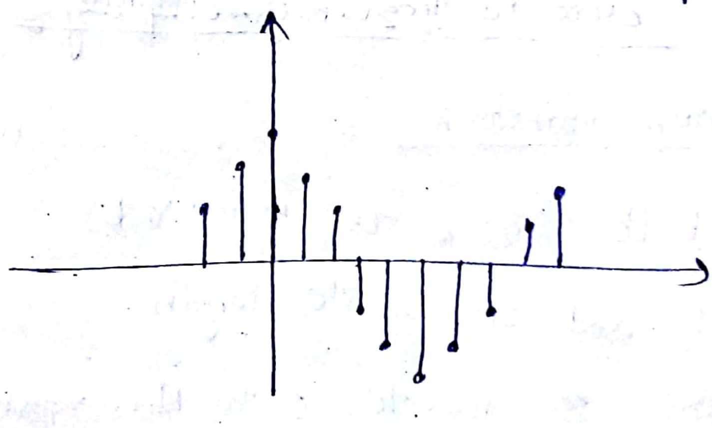
→ Frequency " " " time domain aliasing

* <u>Time domain aliasing due to frequency sampling:</u>

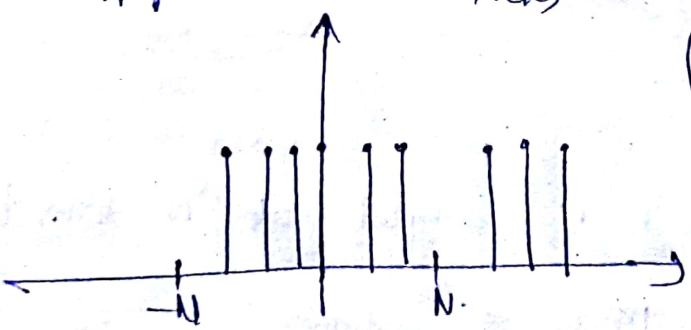




$X(e^{j\omega})$ after frequency domain sampling



Apply IDFT to X(k)



(∵ If X(k) is discrete periodic in freq. domain $x_p(n)$ is also discrete and periodic)

$N > L$ → condition for no aliasing

N → no. of samples/length of DFT

L → I/p sequence length

Ex: if $N=9$, $L=10$ (as above) ($N \geq L$)

then there will not any overlap

if $L=9$, $N=7$, samples will be overlapped.

→ To perform DFT (or) IDFT, $N \geq L$ should be satisfied

→ If $N=10$, $L=7$ then $(N-L)$ no. of zeros are appended to perform DFT

$$x(n) = \begin{cases} x_p(n) & ; 0 \leq n \leq N-1 \\ 0 & ; o.w \end{cases}$$

$$x(n) = x_p(n) \cdot w_R(n)$$

$$w_R(n) = \text{Rectangular funct}^n = \begin{cases} 1 & ; 0 \leq n \leq N-1 \\ 0 & ; o.w \end{cases}$$

DFT → sampling in frequency domain

$$X(e^{j\omega_k}) = \sum_{n=0}^{N-1} x(n) e^{-j\omega_k n}$$

$$\omega_k = \frac{2\pi}{N} k$$

$$X(e^{j\frac{2\pi}{N}k}) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk}$$

$$\downarrow$$

$X(k)$ $k \to 0$ to $N-1$

$$\boxed{\text{DFT of } x(n) \Rightarrow X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk} \quad k \to 0 \text{ to } N-1}$$

→ $y(n)$ is periodic with period 'N'. Any periodic sequence can be represented with $1 +$ (finite terms) unique

then IDFT → $y(n) = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\, e^{j\frac{2\pi}{N}nk}$

$n = 0$ to $N-1$

$$x(n) = \begin{cases} y(n) & ; 0 \le n \le N-1 \\ 0 & ; \text{otherwise} \end{cases}$$

$$X(k) = Y(k)\, W_R(k)$$

then,

$$\boxed{\text{IDFT of } X(k) = x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}nk} \quad n = 0 \text{ to } N-1}$$

$$\boxed{\begin{aligned} DFT[x(n)] &= X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}nk} \\ IDFT[X(k)] &= x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}nk} \end{aligned}} \rightarrow N\text{-point DFT}$$

– When $N > L$, zeros are added which is called zero padding.

<u>Advantages:</u>

→ Frequency resolution is increased

→ To perform DFT

<u>Twiddle factor:</u>

Denoted by W_N

$$\boxed{W_N = e^{-j\frac{2\pi}{N}}}$$

→ $x_p(n)$ is periodic with period ...

sequence can be represented with F.S (Fourier series)

then IDFT ⇒ $x_p(n) = \frac{1}{N} \sum_{n=0}^{N-1} X_p(k) e^{j\frac{2\pi}{N}nk}$

$n \to 0$ to $N-1$

$$X(k) = \begin{cases} X_p(k) \; ; & 0 \le n \le N-1 \\ 0 \; ; & OW \end{cases}$$

$$X(k) = X_p(k) \cdot W_R(k)$$

then,

$$\boxed{\text{IDFT of } X(k) \Rightarrow x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}nk} \quad n \to 0 \text{ to } N-1}$$

$$\boxed{\begin{aligned} DFT[x(n)] = X(k) &= \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}nk} \\ IDFT[X(k)] = x(n) &= \frac{1}{N} \sum_{n=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}nk} \end{aligned}}$$ → N-point DFT

→ When $N > L$, zeros are added which is called zero padding.

Advantages:

→ Frequency resolution is increased

→ To perform DFT

Twiddle factor:

Denoted by W_N

$$\boxed{W_N = e^{-j\frac{2\pi}{N}}}$$

$$|W_N| = \left|e^{-j2\pi/N}\right| = \left|\cos\frac{2\pi}{N} - j\sin\frac{2\pi}{N}\right| = 1$$

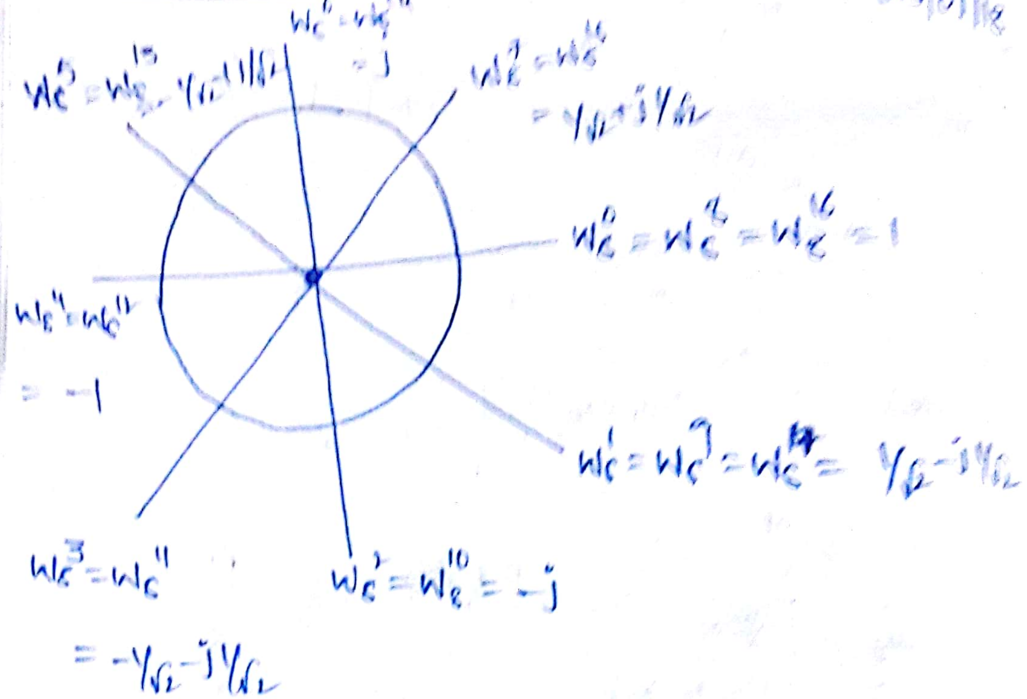
$$\angle W_N^{nk} = \frac{-2\pi}{N}(nk)$$

$$\Rightarrow \boxed{X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}} \rightarrow \text{DFT}$$

$$\boxed{x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cdot W_N^{-nk}} \rightarrow \text{IDFT}$$

$kn = r$	W_N^{nk}	magnitude	Phase
0	$W_8^0 = e^{-j\frac{2\pi}{8}(0)}$	1	0
1	$W_8^1 = e^{-j\frac{2\pi}{8}(1)} = \frac{1-j}{\sqrt{2}}$	1	$-\pi/4$
2	$W_8^2 = e^{-j\frac{2\pi}{8}\cdot 2} = e^{-j\pi/2} = -j$	1	$-\pi/2$
3.	$W_8^3 = e^{-j\frac{2\pi}{8}(3)} = \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$	1	$-3\pi/4$
4	$W_8^4 = e^{-j\frac{2\pi}{8}(4)}$	+1	$-\pi$

08/01/18



$$e^{-j\frac{2\pi}{N}nk} = W_N^{nk}$$

Twiddle factor is periodic with period N.

$$W_N^r = W_N^{(r\pm N)} = W_N^{(r\pm 2N)} = \dots$$

Twiddle is symmetric

$$W_N^r = W_N^{(r\pm N/2)}$$

* To overcome time aliasing effect $N \geq L$ so, that $N-L$ is zeros to the i/p it is called zero padding.

Find the DFT of a sequence $x(n) = \{1,1,0,0\}$

& IDFT of $X(K) = \{1,0,1,0\}$.

Proceed only with ~~diff~~ in lab

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \\ W_N^0 & W_N^4 & W_N^8 & W_N^{12} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$W_N^0 = W_N^8 = W_N^{16} = 1$

$W_N^1 =$

$$= \begin{bmatrix} 1 & 1/\sqrt{2} - j1/\sqrt{2} & -j & -1/\sqrt{2} - j1/\sqrt{2} \\ 1 & -j & -1 & j \\ 1 & -1/\sqrt{2} - j1/\sqrt{2} & j & 1/\sqrt{2} - j1/\sqrt{2} \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$= \cancel{1 + 1/\sqrt{2} - j1/\sqrt{2}}$

$$X(K) = \sum_{n=0}^{N-1} x(n) . e^{-j\frac{2\pi}{N}nk}$$

$$X(0) = \sum_{n=0}^{3} x(n) = 1+1+0+0 = 2$$

$$X(1) = \sum_{n=0}^{3} x(n) . e^{-j\frac{2\pi}{4}n}$$

$= 1 + 1 . 1/\sqrt{2} - j1/\sqrt{2} =$

$$W_4^0 = e^{-j\frac{2\pi}{4}\cdot(0)} = 1$$

$$W_4^1 = e^{-j\frac{2\pi}{4}} = -j$$

$$W_4^2 = e^{-j\frac{2\pi}{4}\cdot 2} = -1$$

$$W_4^3 = e^{-j\frac{2\pi}{4}3} = e^{-j3\pi/2} = j.$$

$N=4$

$$\begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \quad 1+1 = 2 \qquad \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$a+b+c+d=1$

$a-bj-c+dj=0$

$(a-c)-j(b-d)=0$

$a=c, \quad b=d$

$2a+2b=1$

$a+b=1/2$

$a+b=0.5$

$a-b+c-d=1$

$a-b=0.5$

$2a=1$

$a=0.5$

$b=0$

*find the DFT of a sequence $x(n)=1 \quad 0\le n\le 2$

$= 0 \quad$ OW

i) $N=4$, ii) $N=8$

plot $|X(K)|$ & $\angle X(K)$ & comment on result

W_N^{nk}

W_N^0

$$\begin{array}{c} \begin{array}{cccc} (0,0) & (0,1) & (0,2) & (0,3) \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ (1,0)\ 1 & (1,1)\ -j & (1,2)\ -1 & (1,3)\ j \\ (2,0)\ 1 & (2,1)\ -1 & (2,2)\ 1 & (2,3)\ -1 \\ (3,0)\ 1 & (3,1)\ j & (3,2)\ -1 & (3,3)\ -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \end{array}$$

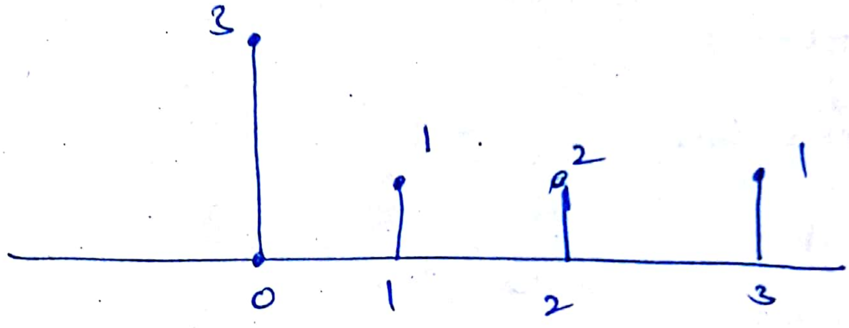
=

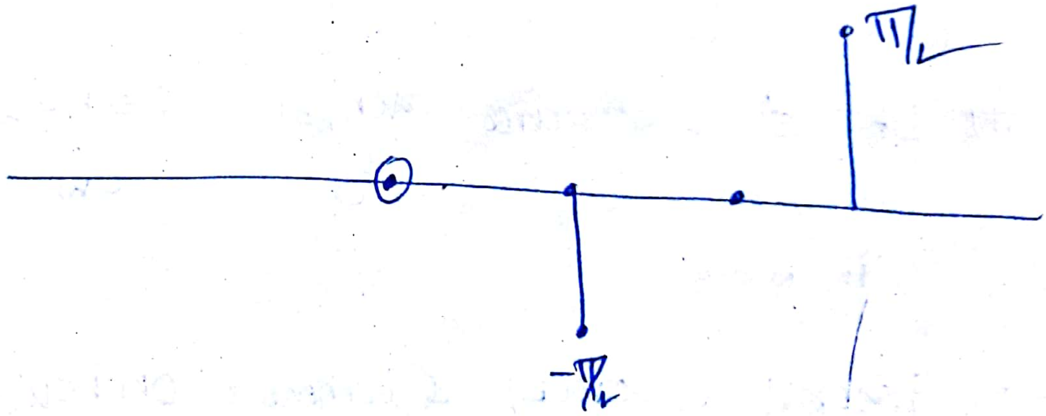
$$= \begin{bmatrix} 3 \\ -j \\ 1 \\ j \end{bmatrix}$$

$X(0) = 3, \quad X(1) = -j \quad X(2) = 1, \quad X(3) = j$

$|X(0)| = 3 \quad |X(1)| = 1 \quad |X(2)| = 1 \quad |X(3)| = 1$

$\angle X(0) = 0 \quad \angle X(1) = -\pi/2 \quad \angle X(2) = 0 \quad \angle X(3) = \pi/2$





$$\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^1 & W_8^2 & W_8^4 & W_8^3 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} \end{bmatrix}$$

$$\begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^2 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1/\sqrt{2}-j1/\sqrt{2} & -j & & & & & & \\ 1 & & & & & & & & \\ 1 & & & & & & & & \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$1 + 1/\sqrt{2} - 1/\sqrt{2} \quad -j$

06/01/17. Find the D.F.T of

$$x(n) = \cos \frac{2\pi r n}{N} \qquad 0 \le r \le N-1.$$

Sol:

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N} nk}$$

$$X(K) = \sum_{n=0}^{N-1} \cancel{\cos \frac{2\pi r n}{N}} \cdot \frac{\left(e^{j\frac{2\pi r n}{N}} + e^{-j\frac{2\pi r n}{N}}\right)}{2} \cdot e^{-j\frac{2\pi}{N}nk}$$

$$X(K) = \sum_{n=0}^{N-1} \frac{e^{-j\frac{2\pi}{N}(k-r)n} + e^{-j\frac{2\pi}{N}(r+k)n}}{2}$$

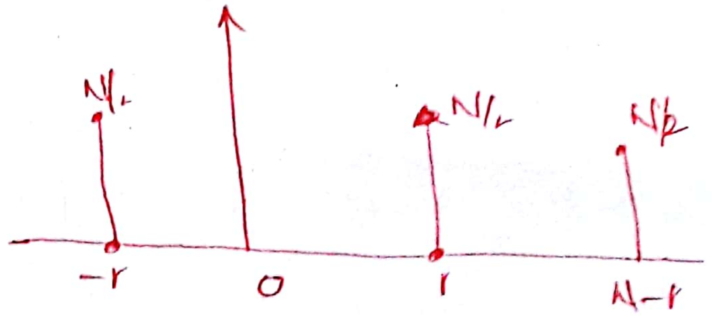
$$X(K) = \sum_{n=0}^{N-1} \frac{1}{2}\left[W_N^{-rn} + W_N^{rn}\right] W_N^{nk}$$

$$= \sum_{n=0}^{N-1} \frac{1}{2}\left[\underset{\downarrow}{W_N^{(k-r)n}} + W_N^{(r+k)n}\right]$$

N if k = r   N if k = −r (−r means N−r)

0 OW   0 OW

$$X(K) = \begin{cases} N/2 & \text{if } K = r \\ N/2 & \text{if } K = -r \\ 0 & \text{OW} \end{cases}$$



Realation ship of DFT to other Transforms:-

i) $X(K) = X(e^{j\omega})\Big|_{\text{at } \omega = \frac{2\pi}{N}k}$

ii) Relation b/w DFT & z transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{N} \sum_{k=0}^{N-1} X(K)\, e^{j\frac{2\pi}{N}nk} \cdot z^{-n}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X(K) \sum_{n=-\infty}^{\infty} \left(e^{j\frac{2\pi}{N}k} \cdot z^{-1}\right)^n$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot \frac{1 - z^{-N}}{1 - e^{j\frac{2\pi}{N}k} z^{-1}}$$

$$= \left(\frac{1 - z^{-N}}{1 - e^{j\frac{2\pi}{N}k} \cdot z^{-1}}\right) \frac{1}{N} \cdot \sum_{k=0}^{N-1} X(K)$$

$$X(z) = \frac{1 - z^{-N}}{N} \sum_{K=0}^{N-1} \frac{X(K)}{1 - e^{j\frac{2\pi}{N}k} \cdot z^{-1}}$$

* Find the z transform of the sequence $x(n) = u(n) - u(n-8)$ & sampling at 6 points on the unit circle using the relation

$$X(K) = X(z)\Big|_{at\ z = e^{j\frac{2\pi k}{3}}}$$

$x(n) = \{1, 1, 1, 1, 1, 1, 1, 1\}$.

$$X(z) = \frac{1}{1-z^{-1}} - \cancel{e^{-j\omega 8}} \cdot \frac{1}{1-z^{-1}} z^{-8}$$

$$X(K) = \frac{1}{1-e^{-j\frac{\pi}{3}k}} - \cancel{e^{-j\omega 8}} \cdot \frac{1}{1-e^{-j\frac{\pi}{3}k}} \cdot e^{-j\frac{\pi}{3}8k}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot \cancel{e} z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + \dots + z^{-7}$$

$$X(K) = X(z)\Big|_{z = e^{j\pi k/3}}$$ where $K \to 0$ to 5.

$$X(K) = 1 + e^{-j\pi k/3} + e^{-j2\pi k/3} + e^{-j3\pi k/3} + e^{-j4\pi k/3} + e^{-j5\pi k/3} + e^{-j6\pi k/3} + e^{-j7\pi k/3}$$

$$\underset{(1)}{\downarrow} \qquad \underset{e^{-j\pi k/3}}{\downarrow}$$

$$X(K) = 2 + 2e^{-j\pi K/3} + e^{-j2\pi K/3} + e^{-j\pi K} + e^{-j4\pi K/3} + e^{-j5\pi K/3}.$$

$x(n) = \{2, 2, 1, 1, 1, 1\}$

Properties of DFT:-

Periodicity :-

if $DFT[x(n)] = X(K)$

$x(n+N) = x(n)$

$X(K+N) = X(K)$

linearity:-

$x_1(n)$ N_1 $\quad$ $x_2(n)$ N_2

if $D[\underset{N_1}{x_1(n)}] = X_1(K)$

$D[\underset{N_2}{x_2(n)}] = X_2(K)$

then $D[a x_1(n) + b x_2(n)] = a X_1(K) + b X_2(K)$

$N_3 = \max[N_1, N_2]$

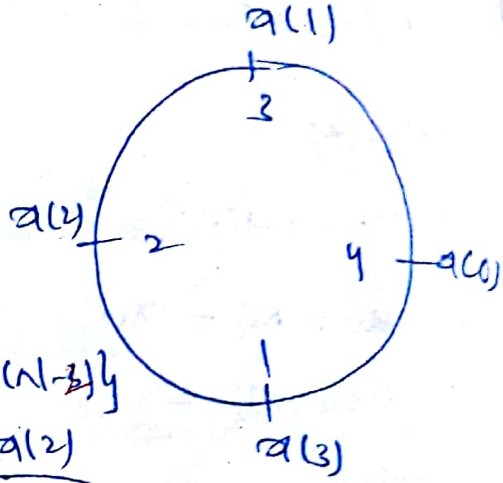
for linear combination both should have same lengths.

if $N_2 < N_1$ then $(N_1 - N_2)$ no. of zeros are added to $x_2(n)$ & vice versa

Circular ~~linear~~ shift of a sequence:-

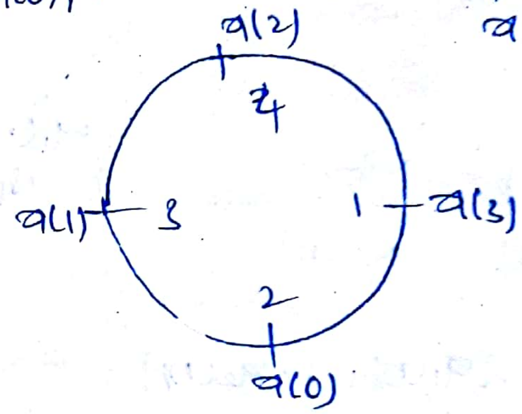
$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$

$$x((n))_N = x(n \text{ modulo } N)$$



$$x(n) = \{x(0), x(1), \ldots x(N-1)\}$$

$$x((n-1))_N = \{x(N-1), x(0), \ldots x(N-2)\}$$



$$x((n-2))_N = \{x(N-2), x(N-1), \ldots x(N-3)\}$$

$\vdots$

$$x((n-k))_N = \{x(n-k), x(n-k+1), \ldots x(n-k)\}$$

$\vdots$

$$x((n-N))_N = \{x(0), x(1), \ldots x(N-1)\}$$

$$= x(n)$$

Circular time shift property:-

$$x((n-m))_N = x(n-m+N)$$

if $DFT[x(n)] = X(K)$

$$x((n-m))_N \longrightarrow e^{-j\frac{2\pi}{N}mK} X(K)$$

$$DFT[x((n-m))_N] = \sum_{n=0}^{N-1} x((n-m))_N \cdot e^{-j\frac{2\pi}{N}nK}$$

0 7

0 3 4 7

$$= \sum_{n=0}^{m-1} x((n-m))_N \cdot e^{-j\frac{2\pi}{N}nK} + \sum_{n=m}^{N-1} x((n-m))_N \cdot e^{-j\frac{2\pi}{N}nK}$$

$$\cancel{\sum_{n=0}^{m-1} x((n-m))_N \cdot e^{-j\frac{2\pi}{N}nK}}$$

$$= \sum_{n=0}^{m-1} x(n-m+N) \cdot e^{-j\frac{2\pi}{N}nK} + \sum_{n=m}^{N-1} x(n-m+N) \cdot e^{-j\frac{2\pi}{N}nK}$$

$$n-m+N = l$$

$$l = N-m$$

$$m-1-m+N = l \Rightarrow l = N-1$$

$$= \sum_{N-m}^{N-1} x(l) \cdot e^{-j\frac{2\pi}{N}(l+m-N)K} + \sum_{l=N}^{2N-m-1} x(l) \cdot e^{-j\frac{2\pi}{N}(l+m-N)K}$$

$$= \sum_{l=N-m}^{m-1} x(l) \cdot e^{-j\frac{2\pi}{N}(l+m)K} + \sum_{l=0}^{N-m-1} x(l) \cdot e^{-j\frac{2\pi}{N}(l+m)K}$$

(4)

$x(n-m)$

$e^{-j\frac{2\pi}{N}10}$

$N=4$

$x(n+1)$

$e^{-j\frac{2\pi}{N}10}$

$x(n+1)$

$x(n-4)$

$x(n)$

$x(n-4)$

$x(4-4)$

$$= \sum_{l=0}^{N-1} x(l) \cdot e^{-j\frac{2\pi}{N}(l+m)k}$$

$$= e^{-j\frac{2\pi}{N}mk} \cdot \sum_{l=0}^{N-1} x(l) \cdot e^{-j\frac{2\pi}{N}lk}$$

$$= e^{-j\frac{2\pi}{N}mk} \cdot X(K)$$

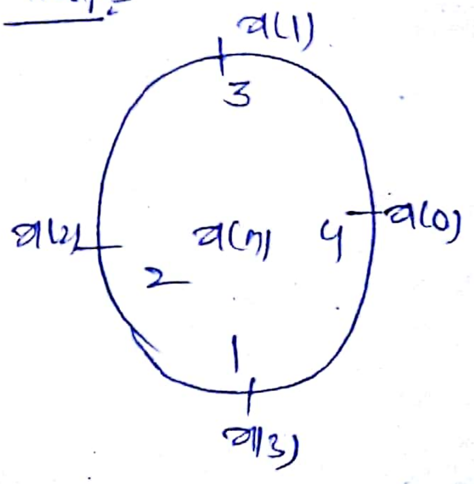
<u>Time Reversal property:-</u>

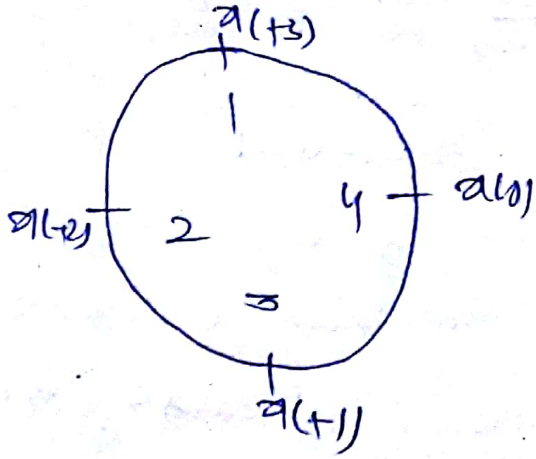
The time reversal of an end point sequence $x(n)$ is obtained by wrapping the sequence $x(n)$ around a circle in clock wise direction.

$$DFT[x((-n))_N] = X((-K))_N$$

$$x(N-n) \qquad x(N-K)$$

<u>Proof:-</u>





$$DFT[x(n)] = \sum_{n=0}^{N-1} (x(n))_N e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} x(N-n)\, e^{-j\frac{2\pi}{N}nk}$$

$N-n=m$

$$= \sum_{m=0}^{N-1} x(m).\, e^{-j\frac{2\pi}{N}(N-m)k}$$

$$= \sum_{m=0}^{N-1} x(m).\, e^{j\frac{2\pi}{N}mk}$$

$$= \sum_{m=0}^{N-1} x(m).\, e^{-j\frac{2\pi}{N}(N-k)m}$$

$$= X(N-k)$$

$$= X((-k))_N.$$

Circular frequency shifting

if $DFT[x(n)] = X(k)$ then

$$DFT[e^{+j\frac{2\pi}{N}ln}\, x(n)] = X((k-l))_N.$$

$$DFT[x(n)] = \sum_{n=0}^{N-1} x(n).\, e^{-j\frac{2\pi}{N}kn}$$

$$DFT[e^{j\frac{2\pi}{N}ln}\, x(n)] = \sum_{n=0}^{N-1} x(n).\, e^{j\frac{2\pi}{N}ln}.\, e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} x(n).\, e^{-j\frac{2\pi}{N}(k-l)n}$$

$$= x((k-l))_N.$$

Complex Conjugate property:-

if $DFT[a(n)] = X(K)$

$$DFT[a^*(n)] = X^*(N-K) = X^*((-K))_N$$

$$DFT[a(n)] = \sum_{n=0}^{N-1} a(n).\, e^{-j\frac{2\pi}{N}nk}$$

$$DFT(a^*(n)) = \sum_{n=0}^{N-1} a^*(n).\, e^{-j\frac{2\pi}{N}nk}$$

$$= \left[\sum_{n=0}^{N-1} a(n).\, e^{j\frac{2\pi}{N}nk}\right]^*$$

$$= \left[\sum_{n=0}^{N-1} a(n).\, e^{-j\frac{2\pi}{N}(-k)n}\right]^*$$

$$= \left[X^*((-k))_N\right]$$

$$= X^*(N-K)$$

Circular Convolution:-

$$DFT[a_1(n) \circledN a_2(n)] = X_1(K).\, X_2(K)$$

$$X_1(K) = \sum_{n=0}^{N-1} a_1(n).\, e^{-j\frac{2\pi}{N}kn}$$

$$X_2(K) = \sum_{n=0}^{N-1} a_2(n).\, e^{-j\frac{2\pi}{N}kn}$$

$$x_{3p}(n) = \sum_{m=0}^{N-1} x_{1p}(m) \cdot x_{2p}(n-m)$$

$$x_3((n))_N = \sum_{m=0}^{N-1} x_1((m))_N \; x_2((n-m))_N$$

$$x_3(n) = \sum_{m=0}^{N-1} x_1(n) \cdot x_2((n-m))_N$$

$$x_3(n) = x_1(n) \, N \, x_2(n)$$

$$x_3(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{j\frac{2\pi}{N}Kn}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X_1(K) \cdot X_2(K) \cdot e^{j\frac{2\pi}{N}Kn}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} \left[\sum_{m=0}^{N-1} x_1(m) e^{-j\frac{2\pi}{N}mn} \sum_{l=0}^{N-1} x_2(l) \cdot e^{-j\frac{2\pi}{N}lK} \right] e^{j\frac{2\pi}{N}K}$$

$$\cancel{= \frac{1}{N} \sum_{K=0}^{N-1} x_1(m) \sum_{l=0}^{N-1} x_2(l) \cdot e^{-j\frac{2\pi}{N}(l+m-n)K}}$$

$N =$ if $l = n-m$

0 ow

$$\cancel{= \frac{1}{N} \sum}$$

$$= \frac{1}{N} \sum_{m=0}^{N-1} x_1(m) \cdot \sum_{l=0}^{N-1} x_2(l) \cdot \sum_{K=0}^{N-1} e^{-j\frac{2\pi}{N}(l+m-n)K}$$

$N \quad l = n-m$

$0 \quad l \neq n-m$

$$= \frac{1}{\cancel{N}} \sum_{m=0}^{N-1} x_1(m) \cdot x_2((n-m))_N \cdot \cancel{N}$$

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m) \cdot x_2((n-m))_N$$

* The 5 point DFT of a sequence $x(n)$ denoted by $X(K)$. plot sequence whose DFT is $Y(K) = e^{\frac{-j4\pi K}{5}} . X(K)$

sol:- Where $x(n) = \{1, 2, 2, 1\}$

$$DFT[x(n-m)] = e^{-j\frac{2\pi}{N}mK} . X(K).$$

$x(n)$ - 4

$$e^{-j\frac{2\pi}{5}(2K)}$$

$$y(n) = (x(n-2))_5$$

$$x(n) = \{1, 2, 2, 1, 0\}$$

0 to 4

$n=0$, $y(0) = x((-2))_5 = x(3) = 1$

$n=1$, $y(1) = x((-1))_5 = x(4) = 0$

$x(0)$

$n=2$, $y(2) = x((0))_5 = 1$

$n=3$, $y(3) = x((1))_5 = 2$

$n=4$, $y(4) = x(2)_5 = 2$

$$y(n) = \{1, 0, 1, 2, 2\}$$

$x(n) = \{1, 3, 1, 1, 2, 1\}$ 6

* $$\boxed{Y(K) = e^{-j\pi K} X(K)}$$

$$= e^{-j\frac{2\pi}{6}(3)K} X(K)$$

$$= x((n-3))_6$$

$$= \{1, 2, 1, 1, 3, 1\}$$

* Consider the length $N=8$ & sequence

$x(n) = \{1, 2, -3, 0, 1, -1, 4, 2\}$. Evaluate the following functions of $X(K)$ without computing the DFT

A) $X(0)$ B) $X(4)$ C) $\sum_{K=0}^{7} X(K)$

D) $\sum_{K=0}^{7} e^{-j\frac{3\pi K}{4}} . X(K)$ E) $\sum_{K=0}^{7} |X(K)|^2$

Sol: $X(0) = \sum_{n=0}^{7} x(n) . e^{-j\frac{2\pi}{N} nk} \quad (k=0)$

$= \sum_{n=0}^{7} x(n)$

$= 6$

$X(4) = \sum_{n=0}^{7} x(n) . e^{-j\frac{2\pi}{8} . 4n}$

$= \sum_{n=0}^{7} (-1)^n x(n)$

$=$ ~~1+2~~.

$=$ ~~1(1)+(-1)2+(-3)~~

$= +(1)-(2)+(-3)-(0)+(1)-(-1)+(4)-2$.

$= 7-7 = 0$

$\sum_{K=0}^{7} X(K) = N x(0)$ $\qquad x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) . e^{j\frac{2\pi}{N} nk}$

$= 8(1)$ $\qquad x(0) . N = \sum_{K=0}^{N-1} X(K)$

$= 8$

d) $\sum_{k=0}^{7} e^{-j\frac{3\pi}{4}k} X(k) = \sum_{k=0}^{7} e^{-j\frac{2\pi}{8}(3)k} \cdot X(k)$

$$= N\, x((0-3))_8$$

$$= N\, x(5)$$

$$= -8$$

$$\frac{1}{N}\sum_{k=0}^{7}|X(k)|^2 = \sum_{n=0}^{N-1}(x(n))^2$$

$$= N\left(\sum_{n=0}^{N-1} x(n)\right)^2$$

$$= 8\left[\sum_{n=0}^{7}|x(n)|^2\right]$$

$$= 8[1+4+9+0+1+1+16+4]$$

$$= 8\times 36$$

$$= 288$$

Circular Convolution

If $x_1(n)$ length is N_1, & $x_2(n)$ length is N_2.

$x_3(n) = x_1(n)$ Ⓝ $x_2(n)$, $L_3 = \mathrm{Max}(N_1, N_2)$

To perform circular convolution, the sequence length must be same

So, if $N_1 < N_2$ then append $(N_2 - N_1)$ no. of zeros to $x_1(n)$

if $N_2 < N_1$ then append $(N_1 - N_2)$ no of zeros to $x_2(n)$

* Methods f
sequences:
1> Concentric
2> Matrix

Concentric

$x_1(n$

$x_2(n$

Procedure:
1> ~~Take~~
clockwise
1> Graph
around a
2> Start
of $x_2(n)$
in clockw
3> Multip
sum of th
4> Rotate
wise di
next va
5> Repeat
first samp

If $x(n) \rightarrow$ of length L ; $h(n) \rightarrow$ length M $\Big\}$ $Max(L, M) = A$

$x(n) * h(n) \rightarrow$ length $\rightarrow L+M-1 = B$

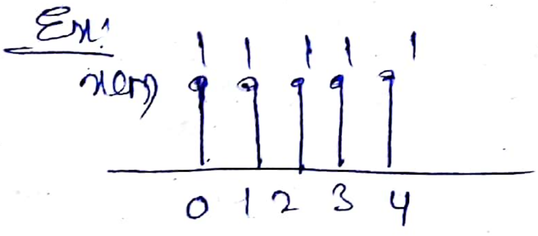
if $A = L$; then $M-1$ points are extra in B (linear) (circular)

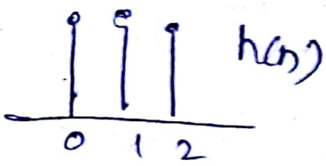
if $A = M$ then $L-1$ pts are extra in B.

→ To perform linear convolution from circular convolution add '$(M-1)$' zeros to $x(n)$ & '$(L-1)$' zeros to $h(n)$

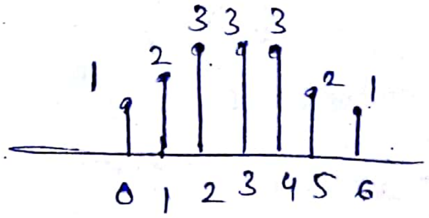
→ There is a time aliasing effect that occurs in circular convolution as '$(M-1)$' points overlap first samples in circular convolution.

Ex:





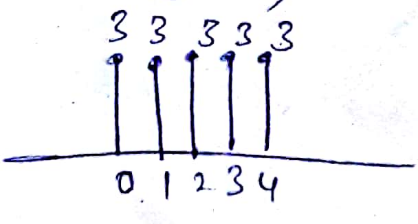
$x(n) * h(n)$



$y(0) + y(5)$

$y(1) + y(6)$

$x(n)$ (N) $h(n)$



* Determine the output response $y(n)$ if $h(n) = \{1, 1, 1\}$ and $x(n) = \{1, 2, 3, 1\}$ using

a) linear convolution b) circular convolution

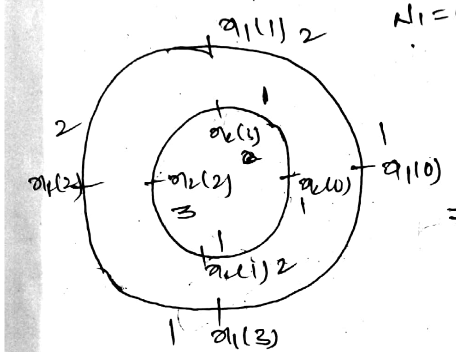
c) circular convolution with zero padding. linear con from circular.

* $h(n) = \{1, 2, 2, 1\}$, $x(n) = \{1, -1, 1, -1\}$.

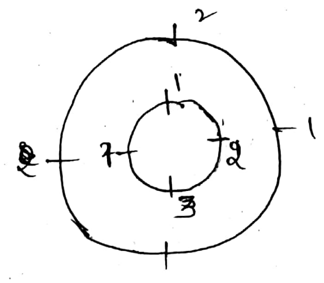
$\triangleright$ $x_1(n) = \{1, 2, 2, 1\}$, $x_2(n) = \{1, 2, 3, 1\}$

$N_1 = 4$, $N_2 = 4$

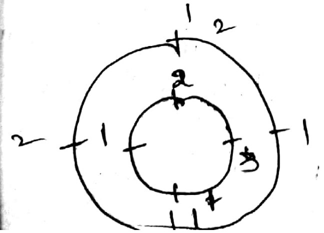
$x_3(n) = x_1(n) \circledast x_2(n)$



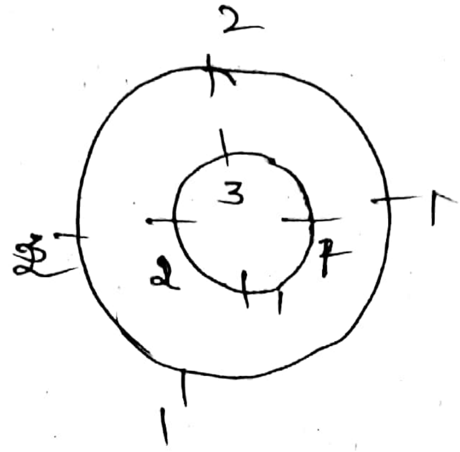
$\Rightarrow x_3(0) = 1 + 2 + 6 + 2$
$= 11$



$\Rightarrow x_3(1) = 2 + 2 + 2 + 3$
$= 9$



$\Rightarrow x_3(2) = 2 + 4 + 2 + 2$
$= 10$



$\Rightarrow a_3(3) = 1 + 6 + 4 + 1 = 12$

$a_3(n) = \{11, 09, 10, 12\}$

2) $a_1(n) = \{1, 2, 3, 1\}$, $a_2(n) = \{4, 3, 2, 2\}$

$$X_1(K) = \sum_{n=0}^{3} a_1(n) \cdot e^{-j\frac{2\pi}{N}nK}$$

$K = 0$,

$$X_1(0) = \sum_{n=0}^{3} a_1(n) \cdot e^{-j\frac{2\pi}{N}n(0)}$$

$$= \sum_{n=0}^{3} a_1(n) = 1 + 2 + 3 + 1 = 7$$

$K = 1$

$$X_1(1) = \sum_{n=0}^{3} a_1(n) \cdot e^{-j\frac{2\pi}{4} \cdot n(1)}$$

$$= \sum_{n=0}^{3} a_1(n) \cdot e^{-jn\pi/2}$$

$\cos n\theta - j\sin n\theta$

$$= a_1(0) + a_1(1) \cdot e^{-j\pi/2} + a_1(2) \cdot e^{-j\pi} + a_1(3) \cdot e^{-j3\pi/2}$$

$$= 1 + 2(-j) + 3(-1) + 1(j)$$

$$= 1 - 2j - 3 + j = -2 - j$$

$k=2$

$$X_1(2)=\sum_{n=0}^{3} x_1(n).\, e^{-j\frac{2\pi}{4}.n(2)}$$

$$=\sum_{n=0}^{3} x_1(n).\, e^{-jn\pi}$$

$$= x_1(0)+x_1(1)(-1)+x_2(2)+x_2(1)(-1)$$

$$=1-2+3-1=1$$

$k=3$

$$X_1(3)=\sum_{n=0}^{3} x_1(n).\, e^{-j\frac{2\pi}{4}.n(3)} = \sum_{n=0}^{3} x_1(n).\, e^{-j\frac{3\pi n}{2}}$$

$$= x_1(0)+x_1(1).\, e^{-j3\pi/2}+x_2(2).\, e^{-j3\pi}+x_2(3).\, e^{-j\frac{9\pi}{2}}$$

$4\pi+\pi/2$

$$=1+2.(j)+3(-1)+1(-j)$$

$$=1+2j-3-j=-2+j.$$

$$X_2(0)=\sum_{n=0}^{3} x_2(n) = 4+3+2+2=11$$

$n=-1$

$e-1$

$$X_2(1)=\sum_{n=0}^{3} x_2(n).\, e^{-j\frac{2\pi}{4}n}$$

$$=4+3(-j)+2(-1)+2(j)$$

$$=4-3j-2+2j = 2-j$$

$$X_2(2)=\sum_{n=0}^{3} x_2(n).\, e^{-j\frac{2\pi}{4}.2n}$$

$$=4+3(-1)+2(1)+2(-1)$$

$$=4-3+\cancel{2}-\cancel{2}=1$$

$q+1$

$-1\ 2\ y$

$1\ \cos\ \ldots j\sin\ldots$

$x_3(3).\, e^{-j3\pi/2}$

$$X_2(3) = 4 + 3(j) + 2(-1) + 2(-j)$$

$$= 4 + 3j - 2 - 2j = 2 + j$$

$$X_3(K) = \{X_1(K) \cdot X_2(K)\}$$ $$-4 + 2j - 2j - 1$$

$$= \{7, -2-j, 1, -2+j\}\ \{11, 2-j, 1, 2+j\}$$

$$= \{77, (-4 + \cancel{2j} - \cancel{2j} - 1), 1, (-1) - 4\}$$

$$= \{77, -5, 1, -5\}$$

IDFT

$$x_3(n) = \frac{1}{N}\sum_{K=0}^{3} X_3(K) \cdot e^{j\frac{2\pi}{N}nK}$$

$$x_3(0) = \frac{1}{4}\sum_{K=0}^{3} X_3(K) = \frac{1}{4}[77 - 5 + 1 - 5] = 2.17$$

$$x_3(1) = \frac{1}{4}\sum_{K=0}^{3} X_3(K) \cdot e^{j\frac{2\pi}{4}(1)(K)}$$

$$= \frac{1}{4}\{X_2(0) + X_2(1) \cdot e^{j\pi/2} + X_2(2) \cdot e^{j\pi} + X_2(3) \cdot e^{j3\pi/2}$$

$$= \frac{1}{4}[77 - 5(j) + 1(-1) + (-5)(-j)$$

$$= \frac{1}{4}\ 11 - \cancel{5j} - 1 + \cancel{5j} = 10$$

$$= \frac{1}{4}[77 - \cancel{5j} - 1 + \cancel{5j}] = \frac{1}{4}(76)$$

$$= 19$$

$$x_2(2) = \frac{1}{4}\sum_{k=0}^{3} X_2(k).\, e^{j\frac{2\pi}{4}.n.2}$$

$$= \frac{1}{4}\left[X_2(0) + X_2(1).\, e^{j\pi} + X_2(2).\, e^{j2\pi} + X_2(3).\, e^{j3\pi}\right]$$

$$= \frac{1}{4}\left[77 - 5(-1) + 1(1) - 5(-1)\right]$$

$$= \frac{1}{4}\left[77 + 5 + 1 + 5\right] = \cancel{22}\ 22$$

$$x_2(3) = \sum_{k=0}^{3} X_2(k).\, e^{j\frac{2\pi}{4}.n.(3)}$$

$$= \frac{1}{4}\left[X_2(0) + X_2(1).\, e^{j\frac{3\pi}{2}} + X_2(2).\, e^{j3\pi} + X_2(3).\, e^{j\frac{9\pi}{2}}\right]$$

$$= \frac{1}{4}\left[77 - 5(-j) + 1(-1) - 5(j)\right]$$

$$= \frac{1}{4}\left[77 + \cancel{5j} - 1 \cancel{+5j}\right] = \frac{76}{4} = 19$$

$$= \{17, 19, 22, 19\}$$

$\rightarrow$ $x_1(n) = \{1, 1, -1, 2\}$, $\quad x_2(n) = \{2, 0, 1, 1\}$

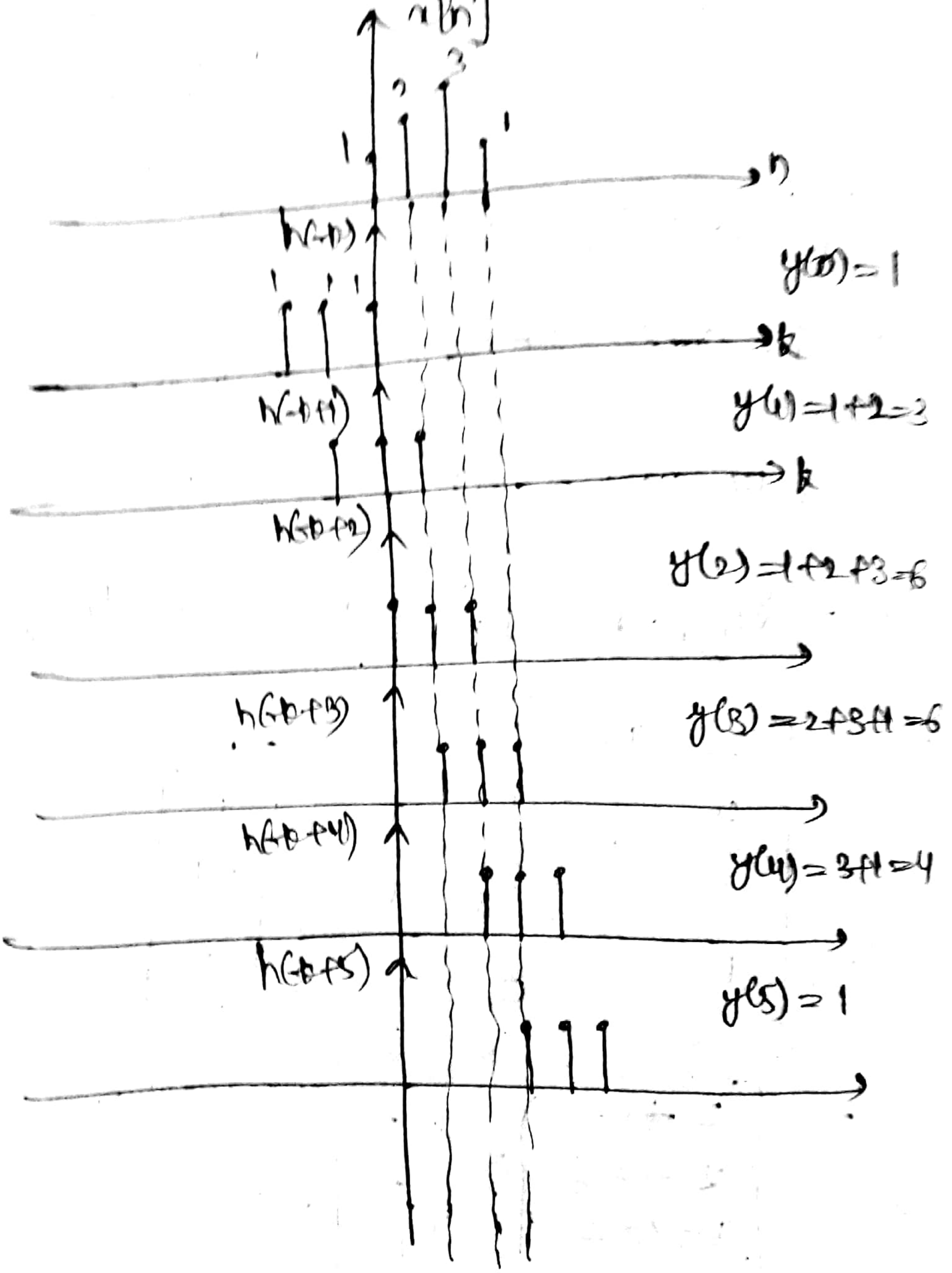
$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1-1 \\ 2-1+2 \\ 1-\cancel{1}+\cancel{2} \\ 1+1+4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 6 \end{bmatrix}$$

~~x(n):~~

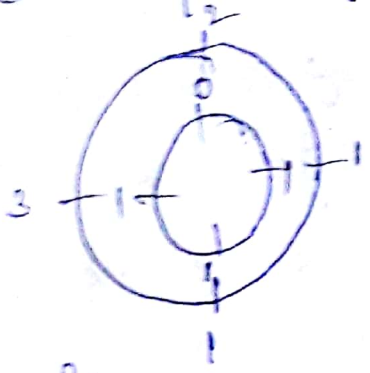
(i) Determine $y(n)$; $h(n) = \{1, 1, 1\}$, $x(n) = \{1, 2, 3, 1\}$ using (a) linear convolution (b) circular convolution (c) circular convolution with zero padding.

Sol. $h(n) = \{1, 1, 1\}$, $x(n) = \{1, 2, 3, 1\}$

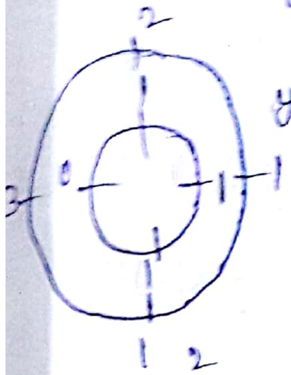


$y(n) = \{1, 3, 6, 6, 4, 1\}$

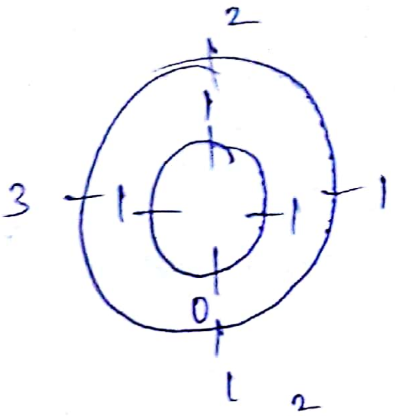
(1) $x(n) = \{1, 2, 3, 1\}$, $h(n) = \{1, 1, 1, 0\}$



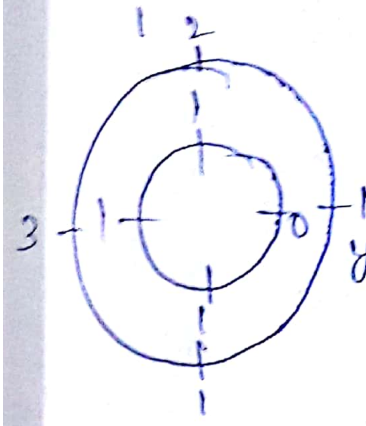
$y(0) = 1+3+1 = 5$



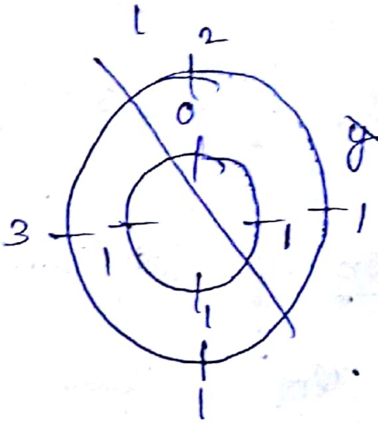
$y(1) = 1+2+1$
$= 4$



$y(2) = 1+2+3$
$= 6$



$y(3) = 2+3+1$
$= 6$



~~$y(4) = 1+3+1 = 5$~~

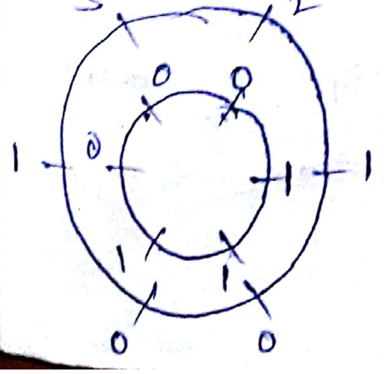
$y(n) = \{5, 4, 6, 6\}$

(2) $x(n) = \{1, 2, 3, 1\}$, $h(n) = \{1, 1, 1\}$

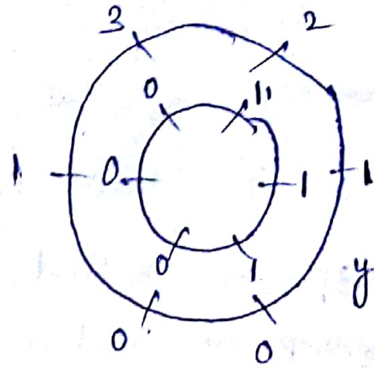
$N = 4$, $L = 3$

$N+L-1 = 4+3-1 = 6$

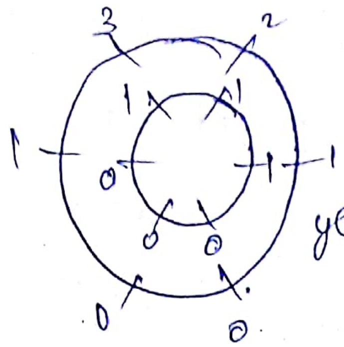
$x(n) = \{1, 2, 3, 1, 0, 0\}$, $h(n) = \{1, 1, 1, 0, 0, 0\}$



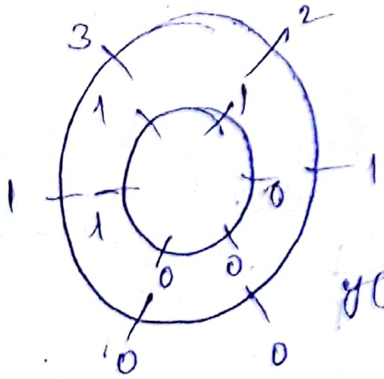
$y(0) = 1$



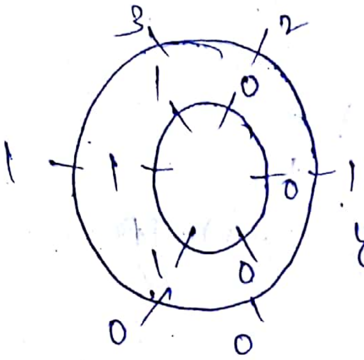
$y(1) = 3$



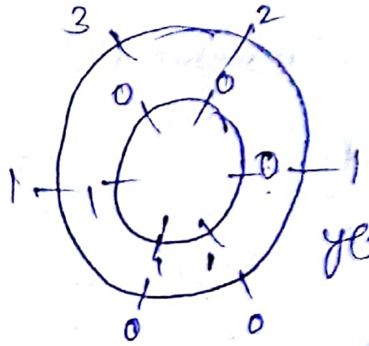
$y(2) = 6$



$y(3) = 6$



$y(4) = 3+1 = 4$

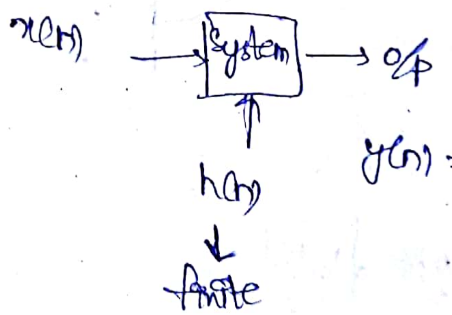


$y(5) = 1$

$y(n) = \{1, 3, 6, 6, 4, 1\}$

16/1/18

Filtering ~~of~~ long duration sequences:



$y(n) = x(n) * h(n)$

We cannot store long duration signal $x(n)$ before convolution (Circular convolution)

∴ I/p sequence $x(n)$ is divided into blocks and it is processed and the o/p's are combined

It can be done in two methods.

1) Overlap save method

2) Overlap add method.

Overlap save method:

$x(n) \rightarrow$ length $= L_s$

$h(n) \rightarrow$ length $= M$

length of each block $= N = L+M-1$

$L \rightarrow$ data points

$M-1$ data points from previous data

$x_1(n) = \{0, 0, x(0), x(1) \ldots\ldots x(L-1)\}$

$x_2(n) = \{x(L-M+1), \ldots\ldots x(L-1), x(L), x(L+1) \ldots x(2L-1)\}$

$x_3(n) = \{x(2L-M+1), \ldots\ldots x(2L-1), x(2L), x(2L+1) \ldots x(3L-1)\}$

$h(n) \rightarrow$ length $= M$, $L-1$ zeros are appended i.e. $L+M-1$

$y_1(n) = x_1(n) \; (N) \; h(n) = \{y_1(0), y_1(1) \ldots\}$

$y_2(n) = x_2(n) \; (N) \; h(n)$

$\vdots$

$y_5(n) = x_5(n) \; (N) \; h(n)$

Eg: $L_s = 15$

$M = 3 \qquad M-1 = 2$

$N = 5 \qquad L = 3$

$x_1(n) = \{0, 0, x(0), x(1), x(2)\}$

$x_2(n) = \{x(1), x(2), x(3), x(4), x(5)\}$

$x_3(n) = \{x(4), x(5), x(6), x(7), x(8)\}$

$x_4(n) = \{x(7), x(8), x(9), x(10), x(11)\}$

$x_5(n) = \{x(10), x(11), x(12), x(13), x(14)\}$

$y_1(n) = x_1(n) \, (N) \, h_1(n) = \{y_1(0), y_1(1), y_1(2), y_1(3), y_1(4)\}$

$y_2(n) = x_2(n) \, (N) \, h(n) = \{y_2(0), y_2(1), \ldots y_2(4)\}$

$\{y_3(0)\; y_3(1) \ldots y_3(4)\}$

$\vdots$

First $M-1$ points are discarded, as the $M-1$ points i/p's are repeated.

$y(n) = \{y_1(2), y_1(3), y_1(4), y_2(2), y_2(3), y_2(4), y_3(2), y_3(3), y_3(4),$

$y_4(3), y_4(4), y_5(2), y_5(3), y_5(4)\}$

<u>Overlap add method:</u>

$x(n) \rightarrow L_s, \; h(n) \rightarrow M, \; N = L+M-1$

Here 'L' data points and $M-1$ zeros are ~~taken~~ as i/p

$N = 5, = L+M-1, \; M = 3 \Rightarrow L = 3$

$x_1(n) = \{x(0), x(1), \ldots x(L-1); 0, 0, \ldots 0\}$

$x_2(n) = \{x(L), x(L+1) \ldots x(2L-1), 0, 0, \ldots 0\}$

~~$x_3(n) =$~~

$\therefore x_1(n) = \{x(0), x(1), x(2), 0, 0\}$

$x_2(n) = \{x(3), x(4), x(5), 0, 0\}$

$\vdots$

$x_5(n) = \{x(12), x(13), x(14), 0, 0\}$

For $h(n)$, $L-1$ zeros are appended and convolution is performed.

$y_1(n) = \{y_1(0), y_1(1), \ldots y_1(4)\}$

$y_2(n) = \{y_2(0), y_2(1) \ldots y_2(4)\}$

$\vdots$

$y_5(n) = \{y_5(0); y_5(1), \ldots y_5(4)\}$

$y(n) = \{y_1(0), y_1(1), y_1(2), y_1(3)+y_2(0), y_1(4)+y_2(1), y_2(2), y_2(3)+y_3(0),$

$y_2(4)+y_3(1), y_3(2), y_3(3)+y_4(0), y_3(4)+y_4(1), y_4(2), y_4(3)+y_5(0), y_4(4)+y_5(1), y_5$

$x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$

$h(n) = \{1, 1, 1\}$. $L = 10$

M = 3

Overlap save method:

$N = L_s + M - 1$

$L = 10, M = 3$

$N = L + M - 1$

$N = L + 2$

$N = 5, L = 3$

$x_1(n) = \{(M-1)$ padded zeros, L data points$\}$

$x_2(n) = \{(M-1)$ previous data points, L new data points$\}$

$x_1(n) = \{0, 0, 3, -1, 0\}$

$x_2(n) = \{-1, 0, 1, 3, 2\}$

$x_3(n) = \{3, 2, 0, 1, 2\}$

$x_4(n) = \{1, 2, 1, 0, 0\}$

$y_1(n) = x_1(n) \; (N) \; h(n)$

$h(n) = \{1, 1, 1, 0, 0\}$

$y_5(1), y_5(2)\}$

$$y_1(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ -1 \\ 0 \end{bmatrix}$$

$$y_1(n) = \{-1, 0, 3, 2, 2\}$$

$$y_2(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

$$y_2(n) = \{4, 1, 0, 4, 6\}$$

$$y_3(n) = \{6, 7, 5, 3, 3\}$$

$$y_4(n) = \{1, 3, 4, 3, 1\}$$

$$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$$

Overlap add method

$x_1(n) = \{L$ datapoints $(M-1)$ zeros padded$\}$

$$x_1(n) = \{3, -1, 0, 0, 0\}$$

$$x_2(n) = \{1, 3, 2, 0, 0\}$$

$$x_3(n) = \{0, 1, 2, 0, 0\}$$

$$x_4(n) = \{1, 0, 0, 0, 0\}$$

$$y_1(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 4 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$y_1(n) = \{3, 2, 2, -1, 0\}$

$y_2(n) = \{1, 4, 6, 5, 2\}$

$y_3(n) = \{0, 1, 3, 3, 2\}$

$y_4(n) = \{1, 1, 1, 0, 0\}$

3	2	2	-1	0										
			1	4	6	5	2							
						0	1	3	3	2				
									1	1	1	0	0	

$g(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$

DITFFT

Radix 2-pt DITFFT

We perform FFT only for 2-power number of points. If the sequence length is power 2.

Append zeros.

$N = 2^M \rightarrow M \rightarrow$ Integer

$x(n)$ is divi

$x_e(n) = x(2n)$

$x_o(n) = x(2n+1)$

$W_N^{nK+N} = W_N^{nK}$

$W_N^{nK+N/2} = -W_N^{nK}$

$$X(K) = \sum_{n=0}^{N-1} x(n) . W_N^{nK}$$

$$= \sum_{\substack{n=0 \\ \text{even}}}^{N-1} x(n) W_N^{nK} + \sum_{\substack{n=0 \\ \text{odd}}}^{N-1} x(n) . W_N^{nK}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x_e(n) . W_N^{2nK} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) . W_N^{(2n+1)K}$$

$$= \sum_{n=0}^{N/2-1} x(2n) . W_N^{2nK} + W_N^{K} \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) . W_N^{2nK}$$

$$W_N^{r} = \left(e^{-j\frac{2\pi}{N}}\right)^{r} = W_{N/2}$$

$$\sum_{n=0}^{\frac{N}{2}-1} x_e(n).\, W_{N/2}^{nk} + W_N^{k} \sum_{n=0}^{\frac{N}{2}-1} x_o(n).\, W_{N/2}^{nk}$$

$$X(K) = X_e(K) + W_N^{K} \cdot X_o(K) \qquad K = 0 \text{ to } \frac{N}{2}-1$$

$$X(K) = X_e(K-N/2) + W_N^{(K-N/2)}\, X_o(K-N/2)$$

$N=8$

$x_e(n) = x(2n)$ $\qquad$ $x_o(n) = x(2n+1)$

$x_e(0) = x(0)$ $\qquad$ $x_o(0) = x(1)$

$x_e(1) = x(2)$ $\qquad$ $x_o(1) = x(3)$

$x_e(2) = x(4)$ $\qquad$ $x_o(2) = x(5)$

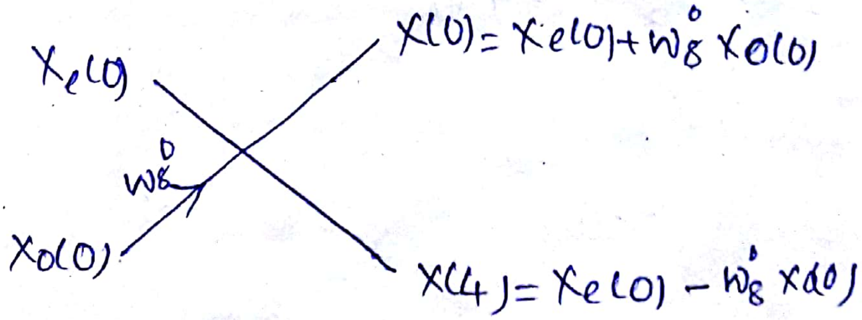
$x_e(3) = x(6)$ $\qquad$ $x_o(3) = x(7)$

$X(0) = X_e(0) + W_8^0\, X_o(0)$ $\qquad$ $X(4) = X_e(0) - W_8^0\, X_o(0)$

$X(1) = X_e(1) + W_8^1\, X_o(1)$ $\qquad$ $X(5) = X_e(1) - W_8^1\, X_o(1)$

$X(2) = X_e(2) + W_8^2\, X_o(2)$ $\qquad$ $X(6) = X_e(2) - W_8^2\, X_o(2)$

$X(3) = X_e(3) + W_8^3\, X_o(3)$ $\qquad$ $X(7) = X_e(3) - W_8^3\, X_o(3)$



Steps to perform radix 2 DIT Algorithm

* → The no. of i/p samples $N = 2^M$ where M is a integer
→ The i/p sequence is shuffled through bit reversal
→ The no. of stages in the flow graph $M = \log_2^N$
→ Each stage consists of $N/2$ butterflies
→ i/p's or O/p's for each butterfly are separated by 2^{m-1} samples. where m represents stage index. for example, for I^{st} stage $m=1$, 2^{nd} stage $m=2$, etc.
→ The no. of complex multiplication & Additions $\frac{N}{2}\log_2^N$ & $N\log_2^N$ respectively.
→ The twiddle factor exponents are function of stage index m. and is given by $K = \frac{Nt}{2^m}$

$t \rightarrow 0, 1, 2, 3, \ldots 2^{(m-1)} - 1$

$m=1$

$$K = \frac{Nt}{2^m}, \quad t=0$$

$$K = \frac{8 \cdot 0}{2^1} = 0 \quad , W_8^0$$

$m=2$

$$K = \frac{Nt}{2^m}, \quad t=0,1$$

$$K=0, \quad K = \frac{8 \cdot 1}{2^2} = 2$$

$$W_8^0, W_8^2$$

→ The no. of sets or sections of butterflies in each stage is given by the formula 2^{M-m}.

2^{3-1} 2^2 2^{3-3} 2^{3-2} 2^1

* The exponent repeat factor which the no. of times the exponent sequence associated with m is repeated is given by 2^{M-m}.