AE 502: Homework Project 2

Due Date: March 19, 2023, 11:59pm CT

Problem

So-called "Molniya orbits" were discovered by Soviet scientists in the 1960s as an alternative to geostationary orbits, which, when launched from high latitudes, require large launch energies to achieve a high perigee and to change inclination in order to orbit over the equator. A long duration stay over a target communications area could be achieved using highly elliptical orbits with an apogee over the desired territory, see Figure 1. The orbit's name refers to the "lightning" speed with which the satellite passes through perigee. In order to minimize station keeping fuel expenditure Molniya orbits were designed to be "frozen" under Earth's J_2 perturbation, i.e.

$$\dot{\omega} = \frac{3}{4} \cdot n \cdot J_2 \cdot \left(\frac{R}{a}\right)^2 \frac{5\cos^2 i - 1}{(1-e^2)^2} = 0 \tag{1}$$

and keep the nodal precession small,

$$\dot{\Omega} = -\frac{3}{2} \cdot n \cdot J_2 \cdot \left(\frac{R}{a}\right)^2 \frac{\cos i}{(1-e^2)^2} \ll 1. \tag{2}$$

Here, n is the satellite's mean motion, for Earth $J_2 = 0.00108$ and $R = 6370\,\mathrm{km}$, a is the semimajor axis and e the orbital eccentricity. Furthermore, i is the inclination and ω and Ω denote the argument of perigee and the argument of the ascending node, respectively. Assume the Earth's $GM = 3.986 \times 10^5\,\mathrm{km}^3/\mathrm{s}^2$.

1. (25 points) What orbital elements a, e, i would you choose for your Molniya orbit if the perigee altitude cannot be lower than 600km to keep drag at bay, and you want your satellite to orbit the Earth three times per day? What is your lowest $\dot{\Omega}$ drift rate?

2. (25 points) What orbital elements a, e, i would you choose to optimize your Molniya orbit around Mars, if the perigee altitude cannot be lower than 400km ($J_2 = 0.00196$, $R = 3390\,\mathrm{km}$, $GM = 4.282 \times 10^4\,\mathrm{km}^3/\mathrm{s}^2$)? Assume the orbital period of your satellite is one Martian Day (24 hours, 39 minutes and 35 seconds). What is your lowest $\dot{\Omega}$ drift rate?

3. (50 points) Code up your favorite (non-averaged) perturbed equations of motion [e.g. Curtis, 2014, Ch.12], either Planetary Equations or special perturbations equations, and solve them numerically to study the actual effect of Earth's J_2 perturbation on the following Molniya orbit: $a = 26600\,\mathrm{km}, i = 1.10654\,\mathrm{rad}, e = 0.74, \omega = 5\,\mathrm{deg}, \Omega = 90\,\mathrm{deg}, M_0 = 10\,\mathrm{deg}$. Plot the time evolution of all orbital elements except the mean anomaly (M) for 100 days. Describe your results!

4. (BONUS QUESTION - no partial credit - 100 points) One of the undesirable properties of the Planetary Equations in Keplerian orbital elements that were presented in the lecture are their singularities. Several of Lagrange's Planetary equations, for instance, become singular for co-planar orbits ($i = 0\,\mathrm{deg}$). This issue can be solved by using an alternative set of orbital elements, for instance, equinoctial elements[1].

$$\begin{aligned}
a &= a \\
h &= e \cdot \sin(\omega + \Omega) \\
k &= e \cdot \cos(\omega + \Omega) \\
p &= \tan(i/2) \sin\Omega \\
q &= \tan(i/2) \cos\Omega \\
\lambda_0 &= M_0 + \omega + \Omega
\end{aligned}$$

   Use the method of Lagrange/Poisson brackets to construct Gauss Planetary Equations in those elements! Equations (13) - (19) in Broucke and Cefola [1972] could be helpful. Present the resulting equations! Use the resulting Gauss planetary equations to solve the following J_2 perturbed two body problem around the Earth:
   $a = 7000\mathrm{km}, e = 0.05, i = 0\,\mathrm{rad}, h = p = q = 0, k = e, \lambda_0 = 0.$

Good luck!

1 Deliverables

Please write a short report and upload your code to your own GitHub repository! No extensions! Late submissions will not be graded!

References

Howard Curtis. *Orbital mechanics for engineering students.* Butterworth-Heinemann, 2014.

[1] Not to be confused with modified equinoctial elements!

R. A. Broucke and P. J. Cefola. On the Equinoctial Orbit Elements. *Celestial Mechanics*, 5(3):303–310, May 1972. doi: 10.1007/BF01228432. URL https://ui.adsabs.harvard.edu/abs/1972CeMec...5..303B.

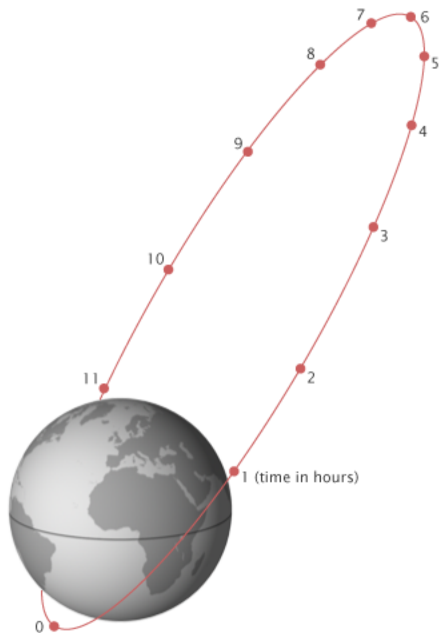

Figure 1: Sketch of a Molniya orbit, source: NASA.