Assignment-11

Sample (i)	x_i^a	y_i^a
1	0.2	3.4
2	0.4	3.8

Do manual calculations for 2 iterations with first 2 samples using Nesterov Accelerated Gradient (NAG) optimizer.

Step.1: Read dataset, $\eta = 0.1$, epochs $= 2$, $m = 1$, $c = 1$, $\gamma = 0.9$, $v_m = 0$ and $v_c = 0$

Step.2: set iter $= 1$

Step 3: set sample $= 1$

Step 4: $y_i = mx_i + c$

$y_1 = 1(0.2) - 1$

$= -0.8$

Step 5: $E = \frac{1}{2}(3.4 - (-0.8))^2$

$= 8.82$

Step 6: $\frac{\partial E}{\partial m} = -(y_i^a - (m + \gamma * v_m^{t-1}) * x_i^a - c - \gamma * v_c^{t-1}) * x_i^a$

$= -(3.4 - (1 + 0.9 * 0)(0.2) + 1 - 0.9 * 0)(0.2)$

$= -0.84$

$\frac{\partial E}{\partial c} = -(y_i^a - (m + \gamma * v_m^{t-1})(x_i^a) - c - \gamma * v_c^{t-1}$

$= -(3.4 - (1 + 0.9 * 0)(0.2) + 1 - 0.9 * 0)$

$= -4.2$

Step. 7: $v_m = \gamma * v_m^{t-1} - \eta\left(\frac{dE}{dm}\right)$

$v_m = 0.9 * 0 + 0.1 * (0.84)$

$= 0.084$

$v_c = \gamma * v_c^{t-1} - \eta \frac{dE}{dc}$

$= 0.9 * 0 + 0.1 * (4.2)$

$= 0.42$

Step. 8: $m = m + v_m^t$ $\quad c = c + v_c^t$

$= 1 + 0.084$ $\quad = -1 + 0.42$

$= 1.084$ $\quad = -0.58$

Step.9: Sample = Sample + 1 = 2 < $n_s = 2$

Step.10: $y_i = mx_i + c$

$y_a = (1.084)(0.4) - 0.58$

$= -0.1464$

Step.11: $E = \frac{1}{2}(y_i - mx_i - c)^2$

$= \frac{1}{2}(3.8 - (-0.1464))^2$

$= 7.79$

Step.12: $\frac{\partial E}{\partial m} = -(y_i^a - (m + \gamma * v_m^{t-1})(x_i^a) - c - \gamma * v_c^{t-1}$

(x_i^a)

$= -(3.8 - (1.084 + 0.9 * 0.084)(0.4) -$

$(-0.58) - 0.9 * 0.42)(0.$

$= -1.74$

$\frac{\partial E}{\partial c} = -(y_i^a - (m + \gamma * v_m^{t-1})(x_i^a) - c - \gamma * v_c^{t-1})$

$= -(3.8 - (1.084 + 0.9 * 0.084)(0.4) -$

$(-0.58) - 0.9 * 0.42)$

$= -4.36$

Step.13: $v_m = \gamma * v_m^{t-1} - \eta \frac{\partial e}{\partial m}$

$= 0.9(0.084) - 0.1(1.74)$

$= 0.25$

$v_c = \gamma * v_c^{t-1} - \eta \frac{\partial e}{\partial c}$

$= 0.9(0.42) + 0.1(4.36)$

$= 0.814$

Step.14: $m = m + v_m$

$= 1.084 + 0.25$

$= 1.334$

$c = c + v_c$

$= -0.58 + 0.814$

$= 0.234$

Step.15: $sample = sample + 1 = 2 \nleq n_s = 2$

Step.16: $iter = iter + 1 = 2 \leq epochs = 2$

Step.17: set $sample = 1$

Step.18: $y_1 = (1.334)(0.2) + 0.234$

$= 0.5008$

Step.19 = $E = \frac{1}{2}(3.4 - 0.5008)^2$

$= 4.20268032$

Step.20 = $\frac{\partial E}{\partial m} = -(y_i^a - (m + \gamma * v_m^{t-1})(x_i^a) - c - \gamma * v_c^{t-1})(x_i^a)$

$= -(3.4 - (1.334 + 0.9 * 0.25)(0.2)$

$- 0.234 - 0.9 * 0.814).(0.2)$

$= (-2.1216)(0.2)$

$= -0.42432$

$\frac{\partial E}{\partial c} = -(y_i^a - (m + \gamma * v_m^{t-1})(x_i^a) - c - \gamma * v_c^{t-1})$

$= -2.1216$

Step.21: $v_m = \gamma * v_m^{t-1} - \eta \frac{dE}{\partial m}$

$= 0.9 * 0.25 - 0.1(-0.42432)$

$= 0.267432$

$$V_c = \gamma * V_c^{t-1} - \eta \frac{\partial E}{\partial c}$$

$$= 0.9 * 0.834 - 0.1(-2.1816)$$

$$= 0.94476$$

Step.22: $m = m + V_m$ $\quad c = c + V_c$

$= 1.334 + 0.267432$ $\quad = 0.234 + 0.94476$

$= 1.17876$

$= 1.601432$

Step.23: $Sample = Sample + 1 = 2 \leq n_s$

Step 24: $y_2 = mx_2 + c$

$= (1.601432)(0.4) + 1.17876$

$= 1.8193328$

Step.25: $E = \frac{1}{2}(y_2^a - y_2)^2$

$= \frac{1}{2}(3.8 - 1.8193328)^2$

$= 1.96152128$

Step. 26:

$$\frac{\partial e}{\partial m} = -(3.8 - (1.601432 + 0.9 * 0.267432)(0.4) - 1.17876 - 0.9 * 0.94476)(0.4)$$

$$= -(1.034107768)(0.4)$$

$$= -0.413643072$$

$$\frac{\partial G}{\partial c} = -1.034107768$$

Step. 27: $v_m = \gamma * v_m^{t-1} - \eta \frac{\partial G}{\partial m}$

$$= 0.9 * 0.267432 - 0.1(-0.413643072)$$

$$= 0.282053107$$

$$v_c = \gamma * v_c^{t-1} - \eta \frac{\partial G}{\partial c}$$

$$= 0.9 * 0.94476 - 0.1(-1.034107)$$

$$= 0.953694768$$

Step: 28: $m = m + V_m$ $\quad c = c + V_c$

$= 1.601432 + 0.282053107$ $\quad = 1.17826 + 0.953694768$

$= 1.88348511$ $\quad = 2.13245427$

Step. 29: Sample = Sample + 1 = 3 $\neq n_s = 2$

Step. 30: iter = iter + 1 = 3 $\neq$ epochs

stop.

$$MSE = \frac{1}{2}\left\{\left(3.4 - (1.88348511)(0.2) - 2.13245427\right)^2 + \left(3.8 - (1.88348511)(0.4) - 2.13245427\right)^2\right\}$$

$$= \frac{1}{2}\left(0.804336708 + 0.835672391\right)$$

$$= 0.820004549$$