

Assignment - 9

Sample (i)	x_i^a	y_i^a
1	0.2	3.4
2	0.4	3.8

Do manual calculations for two iterations for first 2 samples using momentum optimizer.

Step.1: Read dataset, $\eta = 0.1$, epochs = 2, $m = 1$, $c = -1$, $\gamma = 0.9$, $v_m = 0$ and $v_c = 0$

Step.2: Set iter = 1

Step.3: set sample = 1

Step.4: $y = mx_i + c$

$$\begin{aligned} y &= (1)(0.2) - 1 \\ &= -0.8 \end{aligned}$$

$$\begin{aligned} \text{Step.5: } e &= \frac{1}{2} (3.4 - (-0.8))^2 \\ &= 8.82 \end{aligned}$$

$$\text{Step. 6} \quad \frac{\partial e}{\partial m} = -(y_i^a - mx_i^a - c)x_i^a$$

$$= -(3.4 + 0.8)(0.2)$$

$$= -0.84$$

$$\frac{\partial e}{\partial c} = -(y_i^a - mx_i^a - c)$$

$$= -(3.4 + 0.8)$$

$$= -4.2$$

$$\text{Step. 7:} \quad v_m^t = \gamma v_m^{t-1} - \eta \frac{\partial e}{\partial m}$$

$$= 0.9 \times 0 - 0.1(-0.84)$$

$$= 0.084$$

$$v_c^t = \gamma v_c^{t-1} - \eta \frac{\partial e}{\partial c}$$

$$= 0.9 \times 0 + 0.1(-4.2)$$

$$= -0.42$$

$$\text{Step. 8:} \quad m = m + v_m^t$$

$$c = c + v_c^t$$

$$m = 0 + 0.084 = 0.084$$

$$c = -1 + (-0.42) = -1.42$$

Step. 9: Sample = Sample + 1 = 2 and $2 < n_s = 4$

Step. 10:

$$y_i = mx_i + c$$

$$y_2 = (1.084)(0.4) - 0.58$$

$$= -0.1464$$

Step. 11:

$$e = \frac{1}{2} (3.8 + 0.1464)^2$$

$$= 7.79$$

Step. 12:

$$\frac{\partial e}{\partial m} = -(3.8 + 0.1464)(0.4)$$

$$= -1.58$$

$$\frac{\partial e}{\partial c} = -(3.8 + 0.1464)$$

$$= -3.94$$

Step. 13:

$$v_m = 0.9 * 0.084 + 0.1 * (1.58)$$

$$= 0.2336$$

$$v_c = 0.9 * 0.42 + 0.1 * (3.94)$$

$$= 0.772$$

Step. 14:

$$m = 1.084 + 0.2336 \quad c = -0.58 + 0.772$$

$$= 1.3176$$

$$= 0.192$$

Step. 15: $\text{Sample} = \text{Sample} + 1 = 2$ and $2 \text{ not } < n_s = 2$

Step. 16: $\text{iter} = \text{iter} + 1 = 2$ and $\text{iter} < \text{epochs} = 2$

Step 17: Set $\text{sample} = 1$

Step. 18: $y = mx_i + c$

$$y_1 = (1.3176)(0.2) + 0.192$$
$$= 0.45552$$

Step. 19: $e = \frac{1}{2} (3.4 - 0.45552)^2$

$$= 4.7224$$

~~4.7224~~ 4.33498124

Step. 20: $\frac{\partial e}{\partial m} = -(3.4 - 0.45552)(0.2)$

$$= -0.588896$$

$$\frac{\partial e}{\partial c} = -(3.4 - 0.45552)$$

$$= -2.94448$$

Step. 21: $v_m = 0.9 * 0.2336 + 0.1 * (0.588896)$

$$= 0.2691296$$

$$v_c = 0.9 * 0.772 + 0.1 * (2.94448)$$
$$= 0.989248$$

$$\begin{aligned} \text{Step 22: } m &= m + V_m^t & c &= c + V_c^t \\ &= 1.3176 + 0.2691296 & &= 0.196 + 0.989248 \\ &= 1.586726 & &= 1.181248 \end{aligned}$$

$$\text{Step 23: Sample} = \text{Sample} + 1 = 2 \leq n_s = 2$$

$$\begin{aligned} \text{Step 24: } y_2 &= mx_2 + c \\ &= (1.586726)(0.4) + 1.181248 \\ &= 1.8159384 \end{aligned}$$

$$\begin{aligned} \text{Step 25: } e &= \frac{1}{2} (3.8 - 1.8159384)^2 \\ &= \frac{1}{2} (1.9840616)^2 \\ &= 1.96825022 \end{aligned}$$

$$\begin{aligned} \text{Step 26: } \frac{\partial e}{\partial m} &= (3.8 - 1.8159384)(0.4) \\ &= 0.79362464 \\ &= \cancel{0.787300088} \\ \frac{\partial e}{\partial c} &= -(3.8 - 1.8159384) \\ &= \cancel{1.9840616} \\ &= -1.9840616 \end{aligned}$$

$$\text{Step. 27: } v_m = 0.9 * 0.2691296 + 0.1 * (0.7936246) \\ = 0.956478816 \\ 0.321579104$$

$$v_c = 0.9 * 0.989248 + 0.1 * (1.9840616) \\ = 1.08872936$$

$$\text{Step. 28: } m = m + v_m t \quad C = C + v_c t \\ = 1.586226 \quad = 1.181248 + \\ + 0.321579104 \quad 1.08872936 \\ = 1.9083051 \quad = 2.26997736$$

$$\text{Step. 29: } \text{Sample} = \text{Sample} + 1 = 3 \neq n_s = 2$$

$$\text{Step. 30: } \text{iter} = \text{iter} + 1 = 3 \neq \text{epochs}$$

$$\text{MSE} = \frac{1}{2} \sum (y_i - mx_i - c)^2 \\ = \frac{1}{2} \left((3.4 - (1.9083051)(0.2) - 2.26997736)^2 + \right. \\ \left. (3.8 - (1.9083051)(0.4) - 2.26997736)^2 \right) \\ = \frac{1}{2} (0.560045114 + 0.58782981) \\ = 0.57393746$$