**Department Master Syllabus**

**Camden County College**

**Blackwood, New Jersey**

**Course Title:** Explorations in Mathematical Thought

**Course Number**: MTH-117

**Department/Program Affiliation:** Mathematics

**Date of Review: 11/2018**

(This Department Master Syllabus has been examined by the program/department faculty members and it is decided that no revision is necessary at this time.)

**Date of Last Revision**: **11/2018**

(This Department Master Syllabus has been examined by the program/department faculty members and it is decided a change requiring a revision is necessary at this time.)

**N.B.** A change to the course materials alone (textbooks and/or supplementary materials) may not constitute a revision. Any other change to the items listed below on this form is considered a revision and requires approval by the program faculty at a Program/Department Meeting and by the division at a Chairs and Coordinator Meeting.

**Credits:** 3

**Contact Hours: Lecture\_\_**3**\_\_ Lab\_\_**0**\_\_ Other \_**0**\_**

**Prerequisites:** MTH-100(Algebraic Concepts) **AND** ENG-013 (Reading Skills III) **OR** proper placement exam score**.**

**Co-requisites:** None

**Course Description:**

This is a general education mathematics course in which students are exposed to basic concepts and principles in the philosophy of mathematics and mathematical logic; including set theory; axiomatic systems and algebraic structures; the concept of infinity; number theory; and proof; among other topics. **This course is for the student majoring in liberal arts; it is not intended for students majoring in mathematics/science.**

**Course Student Learning Outcomes: (**Cognitive, Psychomotor, Affective Domains)

Upon completion of this course, students will be able to:

* construct correct logical arguments, as assessed by tests, quizzes, homework, or projects.
* understand the evolution of the notions of number and infinity, as assessed by tests, quizzes, homework, or projects.
* understand the role and significance of a mathematical structure, as assessed by tests, quizzes, homework, or projects.
* understand the arguments and perspectives of various philosophies of mathematics, as assessed by tests, quizzes, homework, or projects.
* demonstrate an understanding of contemporary mathematical problems and challenges, as assessed by tests, quizzes, homework, or projects.
* develop their own processes, concepts and techniques for solving problems, as assessed by tests, quizzes, homework, or projects.

**General Education Student Learning Outcomes (if applicable)**

Students will apply appropriate mathematical and statistical concepts and operations to interpret data and to solve problems.

**Course Outline:**

**Several topics will be selected from the following, depending upon student and faculty interests:**

* Laws of logic and syllogisms

1. Simple and compound statements
2. Truth tables
3. Logical equivalence
4. Tautologies and contradictions
5. Conditional and bi-conditional statements
6. Valid and invalid arguments; rules of inference
7. Multi-valued and fuzzy logic
8. Introduction to the predicate calculus; quantifiers
9. Statements with multiple quantifiers and their negations
10. Arguments with quantified statements

* Proof in mathematics (done in conjunction with Introduction to Number Theory)

1. What is proof in life, in science and in mathematics?
2. Disproof by counterexample
3. Proof by exhaustion
4. Direct proof
5. Proof by division into cases
6. Proof by contradiction

* The evolution of the notion of number

1. The natural numbers
2. The integers
3. The rational numbers
4. The real numbers
5. The complex numbers

* Introduction to Number Theory

1. Divisibility
2. Parity
3. Primes and composites
4. Congruence and modulo arithmetic
5. The unique factorization theorem
6. The quotient-remainder theorem
7. Proof of the irrationality of 
8. Proof of the existence of infinitely many primes
9. Fermat’s Last Theorem and Goldbach’s Conjecture
10. Open problems in Number Theory

* Number bases

1. Binary representation of numbers
2. Binary addition
3. Other bases

* Set theory

1. Sets, subsets and power sets
2. Venn diagrams and operations with sets: union, intersection, set difference and complementation
3. Set identities

* Boolean algebras

1. The axiomatic approach in mathematics
2. The axioms defining a Boolean algebra
3. Examples of Boolean algebras
4. The Principle of Duality
5. Proving new theorems in a Boolean algebra

* Cardinality and transfinite numbers

1. The cardinality of a set
2. The finite, the countable and the uncountable
3. The cardinality of the set of rational numbers
4. The cardinality of the set of real numbers: Cantor’s diagonal process
5. A hierarchy of infinities
6. The Continuum Hypothesis and its resolution

* The concept of chaos in mathematics

1. The discrete logistic equation and chaos
2. Other examples of chaos
3. Implications

* Philosophy of mathematics

1. Mathematics: discovered or invented?
2. Platonism
3. Logicism
4. Intuitionism
5. G. H. Hardy’s views on mathematics

* Gödel’s Theorems

1. Consistency and completeness in mathematics
2. Gödel’s theorems and their implications
3. Gödel’s proof – central ideas

* Paradoxes in mathematics and logic

1. Self-reference and the liar’s paradox
2. Russell’s paradox
3. The raven’s paradox
4. Cantor’s set
5. Koch’s snowflake

* Contemporary problems in mathematics

1. Hilbert’s tenth problem
2. The four color theorem
3. The traveling salesman problem
4. The 3n+1 problem
5. The P vs. NP problem and its implications
6. Examples of NP-complete problems
7. The Riemann Hypothesis and its implications

* Great moments and figures in the history of mathematics

1. Euclid and the architecture of mathematics
2. Pythagoras: beyond the rational numbers
3. Hamilton and the liberation of algebra
4. Polyai , Lobachevski and Gauss and the liberation of geometry
5. Cantor: an infinitude of infinities
6. Gödel and the inherent constraints in mathematics
7. Andrew Wiles: proving Fermat’s Last Theorem

* Mathematics and the arts and humanities

1. M. C. Escher: Tilings and Tessellations
2. J. L. Borges: “The Aleph”
3. Mathematics and literature: Oulipo
4. “A Beautiful Mind”
5. Mathematics and music

Instructors may cover other appropriate additional topics not listed above with the permission of the course coordinator. Other topics may be assigned as projects for an in-depth exploration.

**Course Activities:**

The classroom activities will include formal and informal lectures where presentation, explanation and illustration of new material and discussion of homework problems take place. Active student participation should be encouraged.

**Assessment of Student Learning Outcomes:**

The student will be evaluated on the degree to which student learning outcomes are achieved. A variety of methods may be used such as tests (a minimum of two), class participation, projects, homework assignments, journals, etc.

**Grading:**

A: 90 to 100

B: 80 to 89

C: 70 to 79

D: 60 to 69

F: below 60

I: Incomplete (given only in case of an extreme emergency)

NA: Not attending

XA: Never attended

W: Withdrawals (student must submit an official withdrawal form by the deadline)

**Course Materials:**

**Textbook(s):**

1. *Discrete Mathematics with Applications* (Current Ed.), by Epp
2. *A Transition to Advanced Mathematics* (Current Ed.), by Smith, Eggen and St. Andre
3. *Doing Mathematics* (Current Ed.), by Galovich
4. *A Mathematician’s Apology*, by Hardy

**Supplemental Materials:**

**Books**

1. *Foundations and Fundamental Concepts of Mathematics* (Current Ed.), by Eves
2. *Conjecture and Proof: An Introduction to Mathematical Thinking*, by Schwartz
3. *The Mathematical Experience*, by Davis and Hersh
4. *Invitation to Mathematics*, by Jacobs
5. *Mathematics: A Discrete Introduction*, by Scheinerman
6. *Concepts of Modern Mathematics*, by Stewart
7. *Mathematics: The New Golden Age*, by Devlin
8. *Math through the Ages*, by Berlinghoff and Gouvea
9. *An Episodic History of Mathematics*, by Krantz
10. *Essentials of Mathematics: Introduction to Theory, Proof, and the Professional Culture*, by Hale
11. *Engines of Logic*, by Martin
12. *Fermat’s Enigma*, by Singh
13. *Incompleteness: The Proof and Paradox of Kurt Gödel*, by Goldstein
14. *Mindtools: The Five Levels of Mathematical Reality*, by Rucker
15. *Calculus Gems,* by Simmons

**Film**

1. PBS documentary: *The Proof*
2. Documentary: *Julia Robinson and Hilbert’s Tenth Problem*
3. Documentary: *Hard Problems: The Road to the World’s Toughest Math Contest*
4. *Fermat’s Room*

**Fiction**

Short story: *The Aleph*, by J. L. Borges

**Web**

Course relevant; instructor recommended websites.