Euchre Rotation Charts

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Introduction

Euchre is a playing-card game involving 4 individuals in teams of 2. Each round involves a dealer initiating the round and the players play through a series of tricks. After 5 tricks points are resolved and the next player is dealer.

When more than 4 people play Euchre, you could make a tournament of multiple games to resolve points across many people. The list of rounds, which individuals are partnered together, and which pairs play against other pairs for each game, is called a rotation chart.

Definitions

A *player* denoted p_i represents an individual involved in the tournament. We will denote the first player p_0 and the n^{th} player p_{n-1}.

All games played across players in the tournament at a given time is called a *round*. The list of rounds needed to play the entire tournament is called the *rotation table*, or the description of each player's partner and opponents across each round.

A pair of individuals playing together in a Euchre is called a partnership. It is the set containing both players. Two players p_i, p_j in a partnership $\{p_i, p_j\}$ will notated as t_{ij}. We say that p_i is *partners with* p_j.

A game of Euchre is played between two partnerships. We can define the game as the set of two partnerships $\{t_{ij}, t_{mn}\}$. We device an alternative simple notation for this: (ij, mn) or $(i.j, m.n)$ if ambiguous.

A round is pair of games and byes of distinct players. *Byes* is a set of players not part of a game in that round. We say a round $R = (G, B)$.

A tournament of n players is determined by a *rotation chart* $T_n = \{R_i\}$ which is a set of rounds R_i.

A set of games, or a set of sets of parterships, can be cumbersome to notate. We devise alternative notation.

Rotation Chart Criteria

We call a rotation chart T *optimal* if it meets the following criteria.

1) In the union of all partnerships of all games of all rounds of T each player p_i is partners with each other player exactly once.
2) In the list construct from all byes across all rounds has each player the same number of times.
3) Rounds contain $\lfloor \frac{n}{4} \rfloor$ games.
4) Each player p_i plays 2 games as the opponent of each other player.

Trivial Example

Suppose we have a rotation chart T_4 as follows.

Round	Games
R_0	(12, 34)
R_1	(13, 24)
R_2	(14, 23)

We show that this is optimal.

1) Each player p_i is partners with every other $p_j \neq p_i$.
2) Each player has the same number of byes 0.
3) Rounds contain $\lfloor \frac{4}{4} \rfloor = 1$ games.
4) Each player plays 2 games as the opponent of each other player.

Other Optimal Rotation Chart

We can use a constructive argument to get us to an optimal rotation chart. Let's start with an induction argument that shows that given an optimal rotation chart T_{2^n} we can construct an optimal rotation chart of $T_{2^{n+1}}$.

Hmm?

Round	Partnerships
R_0	12, 34, 56, 78
R_1	13, 24, 57, 68
R_2	14, 23, 58, 67
R_3	15, 26, 37, 48
R_4	16, 27, 38, 45
R_5	17, 28, 35, 46
R_6	18, 25, 36, 47

Round	Games
R_0	(12, 34) (56, 78)
R_1	(13, 57) (24, 68)
R_2	(14, 67) (23, 58)
R_3	15, 26, 37, 48
R_4	16, 27, 38, 45
R_5	17, 28, 35, 46
R_6	18, 25, 36, 47

How many Rounds?

If T_n is optimal, then there is at least $n-1$ games, since each player must partner with each other $n-1$ players. Because each round must contain $\lfloor \frac{n}{4} \rfloor$ games, there will be $n \bmod 4 = r$ byes for a round of n players. If T_n is optimal, each player has the same number of byes, which means a player has $n-1$ rounds playing with a partner, and r rounds as a bye. This means there must be precisely $n-1+r$ rounds, since a player is either a bye or a partner.

T_n has $n-1+(n \bmod 4)$ rounds.

Observations

The rotation chart T_4 is unique. So is T_5?

Rotation Chart T_4

Round	Games
R_0	(12, 34)
R_1	(13, 24)
R_2	(14, 23)

Rotation Chart T_5

Round	Games	Byes
R_0	(12, 35)	4
R_1	(13, 45)	2
R_2	(14, 23)	5
R_3	(15, 24)	3
R_4	(25, 34)	1

Pretty sure this is unique (up to transpositions). Is it clear how I would construct this from T_4?