```
* simplex_alg.hpp
   * Simplex algorithm.

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3
4
    * 15.02.2009
^6_7
8 #ifndef NUMERIC SIMPLEX ALG HPP
9
  #define NUMERIC_SIMPLEX_ALG_HPP
10
11 #include <iterator>
12 #include <algorithm>
13 #include < numeric>
14 #include <functional>
15 #include < vector >
16
17 #include <boost/numeric/ublas/matrix.hpp>
18 #include <boost/numeric/ublas/vector.hpp>
19 #include <boost/numeric/ublas/storage.hpp>
20 #include <boost/numeric/ublas/matrix_proxy.hpp>
21 #include <boost/numeric/ublas/functional.hpp>
  #include <boost/bind.hpp>
23
  #include <boost/optional.hpp>
24
25
  #include "numeric_common.hpp"
26
27 #include "li_vectors.hpp"
28 #include "iterator.hpp"
  #include "submatrix.hpp"
29
30 #include "subvector.hpp"
31 #include "invert_matrix.hpp"
  #include "combination.hpp'
33
34
  namespace numeric
35
  {
36
  {\bf name space} \ {\rm simplex}
37
38
        TODO: Move implementation lower.
     // TODO: Code may be overgeneralized.
39
40
     // TODO: Rename 'value_type' by 'scalar_type'.
     // TODO: Replace 'basis' by 'basic'.
41
42
43
     // Types of linear programming solving results.
44
     enum simplex_result_type
45
46
       srt\_min\_found \,=\, 0\,,
                                                Function has minimum and it was founded.
47
       {\tt srt\_not\_limited} \;,
                                                Function is not limited from below.
48
                                                Set of admissible points is empty.
       \operatorname{srt}\_\operatorname{none} ,
                                             ^{\prime\prime}/ Loop in changing basis detected.
       srt_loop,
49
50
51
     //\ Types\ of\ searching\ first\ basic\ vector\ results\,.
52
53
     enum first_basic_vector_result_type
54
                                             // Found first basic vector.
55
       fbrt\_found = 0,
56
       fbrt none,
                                             // Set of admissible points is empty.
57
     };
58
59
     // Types of searching next basic vector results.
60
     enum next_basic_vector_result_type
61
62
       nbrt\_next\_basic\_vector\_found = 0, // Found next basic vector.
63
       nbrt\_min\_found,
                                             // Current basic vector is solution of problem.
                                             // Function is not limited from below.
64
       nbrt not limited,
65
       nbrt_none,
                                             // Set of admissible points is empty.
                                             // Loop in changing basis detected.
66
       nbrt_loop,
67
68
69
     namespace
70
     {
```

```
template < class MatrixType, class VectorType >
 71
         bool assert basic vector (MatrixType const &A, VectorType const &b, VectorType const
 72
              &x )
 73
 74
            // TODO: Assert that value types in all input is compatible, different types for
                 different vectors.
 75
           BOOST\_CONCEPT\_ASSERT((\,u\,b\,l\,as::MatrixExpressionConcept< MatrixType>))\,;
 76
           BOOST CONCEPT ASSERT((ublas::VectorExpressionConcept<VectorType>));
 77
                                                                        value\_type\,;
 78
            typedef typename MatrixType::value_type
            typedef vector < value_type>
 79
                                                               vector_type;
            {\bf typedef} \ \ {\rm matrix}{<} {\rm value\_type}{>}
                                                               matrix_type;
 80
            typedef basic_range<size_t, long>
 81
                                                               {\tt range\_type}\,;
           typedef std::vector<size t>
 82
                                                                        range container type;
 83
            typedef linear_independent_vectors<vector_type> li_vectors_type;
 84
 85
           range type const N(0, A. size2()), M(0, A. size1());
 86
 87
           \begin{aligned} & \text{BOOST\_ASSERT(N.size()} > 0); \\ & \text{BOOST\_ASSERT(M.size()} > 0); \end{aligned}
 88
 89
 90
             // \ TODO: \\ //BOOST\_ASSERT(M.\ size() < N.\ size()); 
 91
 92
            //BOOST\_ASSERT(is\_linear\_independent(matrix\_rows\_begin(A), matrix\_rows\_end(A)));
 93
 94
           BOOST ASSERT(x.size() == N.size());
 95
 96
           BOOST_ASSERT(b. size() == M. size());
 97
           \begin{aligned} & \texttt{BOOST\_ASSERT(std::find\_if(x.begin(), x.end(), boost::bind<} \\ & \texttt{bool}>(\texttt{std::less}< \\ & \texttt{value\_type}>(), \ \_1, \ 0.)) \\ & = \texttt{x.end())}; \end{aligned}
 98
 99
100
           range_container_type Nkp;
            {\tt copy\_if}(N.\,begin\,(\,)\;,\;N.\,end\,(\,)\;,\;std::back\_inserter\,(Nkp)\;,
101
                 boost::bind<bool>(std::logical not-bool>(), boost::bind<bool>(eq zero functor<
102
            \begin{array}{c} value\_type>(0.)\;,\;\;boost::bind<value\_type>(x\;,\;\;\_1)\;)\;)\;;\\ BOOST\_ASSERT(Nkp.size\;()\;>\;0)\;;\\ \end{array} 
103
           {\tt BOOST\_ASSERT(Nkp.size())} <= {\tt M.size())};
104
105
106
            li_vectors_type basicVectorLICols;
           BOOST_ASSERT(is_linear_independent(matrix_columns_begin(submatrix(A, M. begin(), M.
107
                 end(), Nkp.begin(), Nkp.end()),
108
                                                        matrix columns end (submatrix (A, M. begin (), M.
                                                            end(), Nkp.begin(), Nkp.end())));
109
110
            // Asserting that basic vector lies in set of admissible points.
            for (size_t r = 0; r < M. size(); ++r)
111
112
            {
              value\_type \  \, \textbf{const} \  \, result = std::inner\_product(row(A, \ r).begin() \, , \, row(A, \ r).end() \, ,
113
                   x.begin(), 0.);
              BOOST\_ASSERT(eq\_zero(result - b[r]));
114
115
116
117
           return true;
118
119
       120
       /\!/ Finds next basic vector, that closer to goal of linear programming problem.
121
       template < class MatrixType, class VectorType >
122
123
       inline
124
       first_basic_vector_result_type
         find_first_basic_vector( MatrixType const &A, VectorType const &b, VectorType const &
125
126
                                         VectorType &basicV )
127
         /\!/ TODO: Assert that value types in all input is compatible, different types for
128
              different vectors.
         BOOST_CONCEPT_ASSERT((ublas::MatrixExpressionConcept<MatrixType>));
BOOST_CONCEPT_ASSERT((ublas::VectorExpressionConcept<VectorType>));
129
130
131
132
         typedef typename VectorType::value_type
                                                               value type;
```

```
typedef ublas::vector<value_type>
133
                                                           vector_type;
134
         typedef ublas::matrix<value type>
                                                           matrix_type;
         typedef ublas::scalar_vector<value_type>
135
                                                           scalar vector type;
136
         typedef ublas::basic_range<size_t, long>
                                                           range_type;
137
         typedef ublas::identity_matrix<value_type> identity_matrix_type;
         typedef ublas::matrix_row<matrix_type>
138
                                                           matrix row type;
139
140
         range type const N(0, A.size2()), M(0, A.size1());
141
142
          / TODO
        \begin{array}{l} \mbox{BOOST\_ASSERT(N.\,size()>0);} \\ \mbox{BOOST\_ASSERT(M.\,size()>0);} \end{array}
143
144
145
        BOOST\_ASSERT(M.\,size\,()\,<\,N.\,size\,()\,)\,;\\ BOOST\_ASSERT(\,is\_linear\_independent\,(\,matrix\_rows\_begin\,(A)\,,\,\,matrix\_rows\_end\,(A)\,)\,)\,;\\
146
147
        BOOST ASSERT (basic V . size ()
148
                                             = N. size ();
        BOOST_ASSERT(c.size()
BOOST_ASSERT(b.size()
149
                                             == N. size());
150
                                             == M. size());
151
152
         vector\_type \ newC(N.\,size\,() \ + \ M.\,size\,()\,)\,, \ newB(M.\,size\,()\,)\,, \ newBasicV(N.\,size\,() \ + \ M.\,size\,()\,)
              , \text{ newResultV}(N. \text{size}() + M. \text{size}());
         matrix type newA(M. size(), N. size() + M. size());
153
154
155
         // Filling new 'c'.
         ublas::project (newC, ublas::range (0, N. size ())) = scalar\_vector\_type (N. size (), 0);
156
157
         ublas :: project(newC, ublas :: range(N. size(), N. size() + M. size())) =
             scalar vector type (M. size (), 1);
158
159
          ^{\prime}/ Filling new 'A' and new 'b'.
160
         for (size_t r = 0; r < M.size(); ++r)
161
162
           value type const factor = (b[r] >= 0 ? 1 : -1);
163
            / TODO:
164
           //ublas::project(matrix\_row\_type(ublas::row(newA, r)), ublas::range(0, N. size())) =
165
                 factor * ublas :: row(A, r);
166
           matrix_row_type row(newA, r);
           ublas::vector_range<matrix_row_type>(row, ublas::range(0, N.size())) = factor *
167
                ublas::row(A, r);
168
169
           newB[r] = factor * b[r];
170
171
         project (newA, ublas::range(0, M.size()), ublas::range(N.size(), N.size() + M.size()))
              = identity_matrix_type(M. size());
172
173
         // Filling new basic vector.
174
         ublas::project(newBasicV, ublas::range(0, N.size())) = scalar_vector_type(N.size(),
             0.);
         ublas::project (newBasicV \,, \ ublas::range (N.\,size \,() \,, \ N.\,size \,() \,+\, M.\,size \,() \,) \,) \,=\, newB;
175
        BOOST_ASSERT(assert_basic_vector(newA, newB, newBasicV));
176
177
178
         // Solving auxiliary problem.
179
        simplex_result_type const result = solve_augment_with_basic_vector(newA, newB, newC,
             newBasicV, newResultV);
180
        {	t BOOST\_ASSERT(result = srt\_min\_found);} // it always has solution
181
         if (eq_zero(ublas::vector_norm_inf<vector_type>::apply(ublas::project(newResultV,
182
              ublas::range(N.size(), N.size() + M.size()))))
183
184
            // Found basic vector.
           basicV = ublas::project(newResultV, ublas::range(0, N.size()));
185
186
           assert basic vector(A, b, basicV);
187
           return fbrt found;
188
189
         else
190
           // Set of admissable points is empty.
191
192
           return fbrt none;
193
194
      }
195
```

```
//\ Finds\ next\ basic\ vector\,,\ that\ closer\ to\ goal\ of\ linear\ programming\ problem\,.
196
197
          template < class MatrixType, class VectorType >
198
          inline
199
          next\_basic\_vector\_result\_type
200
              find next basic vector (MatrixType const &A, VectorType const &b, VectorType const &c
201
                                                             VectorType const &basicV, VectorType &nextBasicV)
202
           {
               // TODO: Assert that value types in all input is compatible, different types for
203
                       different\ vectors .
204
              BOOST CONCEPT ASSERT((ublas::MatrixExpressionConcept<MatrixType>));
              BOOST_CONCEPT_ASSERT((ublas::VectorExpressionConcept<VectorType>));
205
206
207
              typedef typename MatrixType::value type
                                                                                                            value\_type\,;
208
              {\bf typedef}\ {\tt vector}{<} {\tt value\_type}{>}
                                                                                                            vector_type;
              typedef matrix<value_type>
                                                                                                            matrix_type;
209
210
              {\bf typedef} \ \ {\bf typename} \ \ {\tt vector\_type::size\_type}
                                                                                                            size type;
               typedef basic_range<size_t, long>
211
                                                                                                            range_type;
212
              typedef std::vector<size type>
                                                                                                            range_container_type;
213
               typedef linear_independent_vectors<vector_type> li_vectors_type;
214
               typedef identity_matrix<value_type>
                                                                                                            identity_matrix_type;
215
              range_type const N(0, A.size2()), M(0, A.size1());
216
217
                // TODO
218
              BOOST\_ASSERT(N. size() > 0);
219
220
              BOOST ASSERT(M. size() > 0);
221
222
              BOOST_ASSERT(M. size() < N. size());
              \begin{array}{lll} BOOST\_ASSERT(is\_linear\_independent(matrix\_rows\_begin(A)\,,\;matrix\_rows\_end(A)))\,;\\ BOOST\_ASSERT(basicV.size() &== N.size())\,; \end{array}
223
224
225
              BOOST ASSERT (nextBasicV. size() = N. size());
              BOOST_ASSERT(c.size()
BOOST_ASSERT(b.size()
                                                                          == N. size());
226
227
                                                                          == M. size());
228
229
              BOOST_ASSERT(assert_basic_vector(A, b, basicV));
230
231
              range container type Nkp, Nk;
232
233
                  / Filling 'Nkp'.
               // Using check with precision.
234
               copy_if(N.begin(), N.end(), std::back_inserter(Nkp),
235
236
                       boost::bind<bool>(std::logical not<bool>(), boost::bind<bool>(eq zero functor<
                              value\_type>()\;,\;\;boost::bind<value\_type>(basicV\;,\;\;\_1))));
              BOOST_ASSERT(Nkp.size() > 0);
237
              238
239
                        \overline{N}kp.end());
              BOOST\_ASSERT (is\_linear\_independent (matrix\_columns\_begin (submatrix (A, M. begin (), M. end, M. begin (), M. end, M
240
                      (), Nkp.begin(), Nkp.end())),
                                                                                   matrix columns end (submatrix (A, M. begin (), M. end
241
                                                                                          (), Nkp.begin(), Nkp.end())));
242
                // Iterating through basises till find suitable (Nk).
243
244
               bool foundBasis (false);
              combination:: first\_combination < size\_type > (std::back\_inserter(Nk)\;,\;M.\;size\;()\;)\;;
245
246
              do
247
248
                  BOOST ASSERT(std::adjacent find(Nk.begin(), Nk.end(), std::greater<size type>()) ==
                           \overline{N}k. end());
                  BOOST ASSERT(Nk. size() == M. size());
249
                  if (std::includes(Nk.begin(), Nk.end(), Nkp.begin(), Nkp.end()))
250
251
252
                       bool const is LI = is _ linear _ independent (
                               \begin{array}{ll} matrix\_columns\_begin(submatrix(A,\ M.\ begin()\ ,\ M.\ end()\ ,\ Nk.\ begin()\ ,\ Nk.\ end())),\\ matrix\_columns\_end & (submatrix(A,\ M.\ begin()\ ,\ M.\ end()\ ,\ Nk.\ begin()\ ,\ Nk.\ end()))) \end{array} 
253
254
255
                       if (isLI)
256
257
                          // Basis was found.
258
```

```
259
                           foundBasis = true:
260
261
                           range container type Nkz, Lk;
262
263
                            // Filling 'Nkz'.
                           std::set_difference(Nk.begin(), Nk.end(), Nkp.begin(), Nkp.end(), std::
264
                                   back_inserter(Nkz));
265
                           BOOST ASSERT(std::adjacent find(Nkz.begin(), Nkz.end(), std::greater<size type
                                   >()) = Nkz.end());
266
267
                            // Filling 'Lk'.
                           \mathtt{std} :: \mathtt{set\_difference} \, (\mathtt{N.\,begin} \, () \, \, , \, \, \, \mathtt{Nk.\,begin} \, () \, \, , \, \, \, \mathtt{Nk.\,end} \, () \, \, , \, \, \, \mathtt{std} :: \\
268
                                   back_inserter(Lk));
269
                           BOOST_ASSERT(Nk. size() == M. size());
270
271
                           BOOST ASSERT(Nkz.size() + Nkp.size() == M.size());
                           BOOST\_ASSERT(Lk.\,size\,()\,=\!\!-N.\,size\,()\,-M.\,size\,()\,)\,;
272
273
274
                           // \ \ Calculating \ \ 'A' \ \ submatrix \ \ inverse \, .
275
                           matrix_type BNk(M.size(), M.size());
BOOST_VERIFY(invert_matrix(submatrix(A, M.begin(), M.end(), Nk.begin(), Nk.end
276
277
                                   ()), BNk));
278
                           BOOST\_ASSERT(eq\_zero(ublas::matrix\_norm\_inf < matrix\_type > ::apply(ublas::prod(matrix\_type) > ::apply(ublas::prod(matr
                                   submatrix (A, M. begin (), M. end (), Nk. begin (), Nk. end ()), BNk) -
                                   identity_matrix_type(M. size(), M. size()))));
279
                           // \ \ Calculating \ \ 'd' \ \ vector \, .
280
                           vector_type d(M.size());
281
282
                           d = c - ublas::prod(ublas::trans(A), vector type(ublas::prod(ublas::trans(BNk),
                                     subvector(c, Nk.begin(), Nk.end())));
283
284
                           .begin(), Nk.end())));
285
286
                           vector\_subvector< vector\_type>\ dLk(subvector(d,\ Lk.begin(),\ Lk.end()));
287
                           typename vector_subvector<vector_type>::const_iterator jkIt = std::find_if(
288
                                   dLk.begin(), dLk.end(),
289
                                   boost::bind < bool > (sl\_functor < value\_type > () , \_1, ~0.)); ~//~ Check~with
                                            precision. If vector satisfies this, than it will satisfy optimal point
                                              criteria.
290
291
                           if (jkIt = dLk.end())
292
293
                                // d[Lk] >= 0, current basic vector is optimal.
                               nextBasicV = basicV;
294
295
                               return nbrt_min_found;
296
                           }
297
                           else
298
                               // Searhcing next basic vector.
299
300
301
                                size_type const jk = Lk[jkIt.index()];
302
                               BOOST ASSERT(sl(d(jk), 0.) && !eq zero(d(jk)));
303
304
                                vector_type u(scalar_vector<value_type>(N. size(), 0.));
                               subvector(u, Nk.begin(), Nk.end()) = ublas::prod(BNk, ublas::column(A, jk));
305
306
                               \mathbf{u}[\mathbf{j}\mathbf{k}] = -1;
307
                               vector_subvector<vector_type> uNk(subvector(u, Nk.begin(), Nk.end()));
308
309
                               typename vector_subvector<vector_type>::const_iterator iuIt = std::find_if(
310
                                       uNk.begin(), uNk.end(),
                                       boost::bind<\!\!bool>\!\!(sg\_functor<\!\!value\_type>\!\!()\;,\;\;\_1,\;\;0.)\;)\;;\;\;//\;\;\mathit{Check}\;\;\mathit{with}
311
                                                precision. Some errors may occur due to this.
312
313
                               if (iuIt = uNk.end())
314
315
                                    // u <= 0, goal function is not limited from below.
316
                                   return nbrt_not_limited;
317
                               else
318
```

```
319
                   // Found u[iu] > 0.
320
                  BOOST ASSERT((*iuIt > 0.) && sg(*iuIt, 0));
321
322
323
                  bool canCalculateNextBasicV(false);
324
                  if (Nkp.size() == Nk.size())
325
326
                    canCalculateNextBasicV = true;
327
328
                  if (!canCalculateNextBasicV)
329
                    vector_subvector<vector_type> uNkz(subvector(u, Nkz.begin(), Nkz.end()));
330
331
                    if (std::find_if(uNkz.begin(), uNkz.end(), boost::bind<bool>(sg_functor
                        value\_type>(), _1, _0.)) = uNkz.end())
332
                      canCalculateNextBasicV = true;
333
334
335
                  if (canCalculateNextBasicV)
336
337
                       Basic\ vector\ is\ not\ singular\ or\ u[Nkz] <=\ 0.
                    // Now we need to find 'theta' so that one coordinate of new basis vector
338
                         will become zero,
                    //\ and\ one\ coordinate\ to\ `theta'.
339
340
                    boost::optional < std::pair < size\_t \;, \; value\_type > > \; minTheta;
341
342
                    343
                      size_t const r = Nk[ri];
344
                      if (sg(u[r], 0)) // not strict check
345
346
                        static value_type const maxTheta = infinity <value_type >();
347
348
349
                        value type const theta = basicV(r) / u(r);
350
                         if (theta < maxTheta && (!minTheta || theta < minTheta->second))
351
                          minTheta = std::make_pair(r, theta);
352
353
354
                      else if (\mathbf{u}[\mathbf{r}] > 0 \&\& eq zero(\mathbf{u}[\mathbf{r}])
355
                         // Adjusting u[r] to zero, needed for cases when basic vector has
356
                            near zero components.
357
                        u[r] = adjust(u[r]);
358
                      }
359
360
361
                      / Finally constructing next basic vector.
                    BOOST VERIFY(minTheta);
362
363
                    nextBasicV = basicV - minTheta->second * u;
364
                    BOOST\_ASSERT(eq\_zero(nextBasicV[minTheta->first]));
365
                    // Adjusting new basic vector.
                    nextBasicV = apply to all<functor::adjust<value type> >(nextBasicV);
366
367
368
                      // Debug: Checking new basis vector 'Nkp'.
369
370
371
                      range_container_type Nkp1;
372
373
                      \verb"copy_if" (N. begin" () , N. end" () , std :: back_inserter" (Nkp1) ,
                           boost::bind<bool>(std::logical_not<bool>(), boost::bind<bool>(
374
                              nextBasicV, _1)));
375
                       //\ Nkp1 = Nkp - \{minTheta->first\} + \{jk\}
376
377
                      BOOST_ASSERT(std::find(Nkp.begin(), Nkp.end(), jk)
                          Nkp. end()):
                      BOOST_ASSERT(std::find(Nkp.begin(), Nkp.end(), minTheta->first)!=
378
                          Nkp.end());
379
                      BOOST ASSERT(std::find(Nkp1.begin(), Nkp1.end(), jk)
                          Nkp1.end());
380
                      BOOST ASSERT(std::find(Nkp1.begin(), Nkp1.end(), minTheta->first) ==
                          Nkp1.end());
```

```
381
382
                       range_container_type diff;
                       std::set symmetric difference(Nkp.begin(), Nkp.end(), Nkp1.begin(),
383
                           Nkp1.end(), std::back_inserter(diff));
384
385
                       BOOST ASSERT(diff.size() >= 2);
386
                       // End of debug.
387
388
389
                     BOOST\_ASSERT(basicV.size() = nextBasicV.size() && basicV.size() = c.
                         \mathtt{size}\,(\,)\,)\,;\ //\ debug
390
                         Asserting that next basic vector not increases goal function.
391
                     BOOST_ASSERT(std::inner_product(c.begin(), c.end(), basicV.begin(), 0.)
                                   std::inner_product(c.begin(), c.end(), nextBasicV.begin(),
392
                     BOOST\_ASSERT(\,assert\_\,basic\_\,vector\,(A,\ b\,,\ nextBasicV\,)\,)\,;
393
394
395
                     return nbrt next basic vector found;
396
                   }
397
                   else
398
                     // Continuing and changing basis.
399
400
                }
401
402
              }
            }
403
404
405
        } while (combination::next_combination(Nk.begin(), N.size(), M.size()));
406
407
         // Basis not found: loop detected.
408
        return nbrt loop;
409
410
411
         Solves linear programming problem described in augment form:
412
           min\ (c^T * x),\ where\ x:\ x>=0,\ A*\ x=b,
       / using provided first basic vector.
413
414
      template < class MatrixType, class VectorType >
415
      inline
416
      simplex_result_type
        solve_augment_with_basic_vector( MatrixType const &A, VectorType const &b, VectorType
417
             const &c.
418
                                            VectorType const &basicV, VectorType &resultV)
419
420
        // TODO: Assert that value types in all input is compatible, different types for
            different vectors.
        {\tt BOOST\ CONCEPT\ ASSERT((ublas::MatrixExpressionConcept<MatrixType>));}
421
422
        BOOST_CONCEPT_ASSERT((ublas::VectorExpressionConcept<VectorType>));
423
424
        typedef typename MatrixType::value_type value_type;
425
        typedef ublas::vector<value type>
                                                   vector type;
426
        vector_type curBasicV = basicV;
427
        BOOST_ASSERT(assert_basic_vector(A, b, curBasicV));
428
429
430
        while (true)
431
432
          vector_type nextBasicV(basicV.size());
          next basic vector result type const result = find next basic vector (A, b, c,
433
              curBasicV, nextBasicV);
434
          switch (result)
435
          case nbrt_next_basic_vector_found:
436
437
            BOOST_ASSERT(assert_basic_vector(A, b, nextBasicV));
438
            curBasicV = nextBasicV;
439
            break;
440
441
          case nbrt min found:
442
            BOOST ASSERT(curBasicV == nextBasicV);
443
            result\overline{V} = curBasicV;
            BOOST\_ASSERT(\,assert\_\,basic\_\,vector\,(A,\ b\,,\ result\,V\,)\,)\,;
444
```

```
445
              return srt min found;
446
              break;
447
448
           case nbrt_not_limited:
449
              return srt_not_limited;
450
              break:
451
452
           case nbrt none:
453
             return srt_none;
454
              break;
455
456
           case nbrt_loop:
457
              return srt_loop;
458
              break:
459
           }
460
461
          / Impossible case.
462
463
         BOOST ASSERT(0);
464
         return srt_none;
465
466
          Solves linear programming problem described in augment form:
467
468
            min\ (c^T * x), where x: x >= 0, A * x = b
      \mathbf{template} < \mathbf{\ class\ } \mathrm{MatrixType} \;, \; \mathbf{\ class\ } \mathrm{VectorType} \; > \;
469
470
      inline
       simplex_result_type solve_augment( MatrixType const &A, VectorType const &b, VectorType
471
            const &c.
                                                 VectorType &resultV )
472
473
         // TODO: Assert that value types in all input is compatible, different types for
474
              different\ vectors .
         BOOST_CONCEPT_ASSERT((ublas::MatrixExpressionConcept<MatrixType>));
BOOST_CONCEPT_ASSERT((ublas::VectorExpressionConcept<VectorType>));
475
476
477
         {\bf typedef\ typename\ } {\rm MatrixType::value\_type}
478
                                                                    value\_type \, ;
479
         typedef ublas::vector<value_type>
                                                                    vector_type;
         typedef ublas::matrix<value type>
                                                                    matrix type;
480
         typedef ublas::basic_range<size_t , long>
481
                                                                    range_type;
482
         typedef std::vector<size_t>
                                                                    range_container_type;
         typedef linear_independent_vectors<vector_type> li_vectors_type;
483
484
485
         range type const N(0, A.size2()), M(0, A.size1());
486
487
          / TODO
         \begin{array}{l} {\rm BOOST\_ASSERT(N.\,size\,()\,>\,0)\,;} \\ {\rm BOOST\_ASSERT(M.\,size\,()\,>\,0)\,;} \end{array}
488
489
490
491
         // Removing linear dependent constraints.
         matrix\_type\ newA(M.\,size(),\ N.\,size());
492
493
         vector type newb (M. size ());
         size_t^n extAddingRow = 0;
494
495
496
         li vectors type liARows;
497
498
         for (size_t r = 0; r < M.size(); ++r)
499
500
           matrix_row<MatrixType const> ARow(A, r);
501
           value type const bval = b(r);
502
503
           if (eq_zero(norm_2(ARow)))
504
                Omitting zero rows.
505
506
             BOOST\_ASSERT(eq\_zero(b(r))); \ /\!/\ \textit{TODO: Handle as incorrect input return state}.
507
              continue;
508
509
510
           if (liARows.is independent(ARow))
511
512
              // Adding linear independent constraint to result matrix.
              row(newA, nextAddingRow) = ARow;
513
```

```
newb(nextAddingRow) = bval;
514
515
516
            liARows.insert(ARow);
517
518
            ++nextAddingRow;
519
          }
520
          else
521
          {
522
               Omitting linear dependent constraints.
            // FIXME: Must be checked is absolute terms is correspondent!
523
524
          }
525
526
        BOOST_ASSERT(nextAddingRow <= A.size2());
527
        newA.resize(nextAddingRow, N.size(), true);
528
529
        newb.resize(nextAddingRow, true);
530
531
        if (newA.size1() = newA.size2())
532
           // Linear program problem is well defined system of linear equations.
533
534
          size_t const size = newA.size1();
535
          matrix_type invNewA(size, size);
BOOST_VERIFY(invert_matrix(newA, invNewA)); // TODO: Handle zero determinant case.
536
537
538
539
          resultV = prod(invNewA, newb);
          BOOST_ASSERT(assert_basic_vector(newA, newb, resultV));
BOOST_ASSERT(assert_basic_vector(A, b, resultV));
540
541
542
543
          return srt min found;
        }
544
545
        else
546
547
          BOOST_ASSERT(newA.size1() < newA.size2());
          return solve li augment (newA, newb, c, result V);
548
549
        }
550
      }
551
        Solves linear programming problem described in augment form:
552
553
           min\ (c^T*x), where x: x>=0, A*x=b and rank(A) is equal to number of
          columns.
      template < class MatrixType, class VectorType >
554
555
      simplex_result_type solve_li_augment( MatrixType const &A, VectorType const &b,
556
          VectorType const &c,
                                                VectorType &resultV )
557
558
559
        // TODO: Assert that value types in all input is compatible, different types for
        different vectors.

BOOST_CONCEPT_ASSERT((ublas::MatrixExpressionConcept<MatrixType>));
560
561
        BOOST CONCEPT ASSERT((ublas:: VectorExpressionConcept<VectorType>));
562
563
        typedef typename MatrixType::value_type
                                                             value_type;
        typedef ublas::vector<value_type>
                                                             vector_type;
564
        typedef ublas::matrix<value_type>
565
                                                             matrix_type;
566
        typedef ublas::basic_range<size_t, long>
                                                             range_type;
        typedef std::vector<size t>
567
                                                             range container_type;
568
        typedef linear_independent_vectors<vector_type> li_vectors_type;
569
570
        range_type const N(0, A.size2()), M(0, A.size1());
571
572
         // TODO
        BOOST_ASSERT(N. size() > 0);
573
574
        BOOST_ASSERT(M. size() > 0);
575
        BOOST_ASSERT(M. size() < N. size());
576
        BOOST ASSERT(is linear independent(matrix rows begin(A), matrix rows end(A)));
577
578
        BOOST ASSERT(c.size() == N.size());
579
580
        BOOST ASSERT(b. size() == M. size();
581
```

```
//\ Searching\ first\ basic\ vector\ using\ artificial\ basis. vector\_type\ firstBasicV\left(N.\ size\left(\right)\right);
582
583
          first_basic_vector_result_type const result = find_first_basic_vector(A, b, c,
584
               firstBasicV);
585
586
          if (result == fbrt\_found)
587
            BOOST\_ASSERT(assert\_basic\_vector(A, b, firstBasicV));
588
            // Solving linear programming problem starting from founded basic vector.
589
590
            return solve_augment_with_basic_vector(A, b, c, firstBasicV, resultV);
591
          }
          else
592
593
            \begin{array}{ll} {\rm BOOST\_ASSERT(\,result == \,fbrt\_none)}\,;\\ //\,\,Set\,\,of\,\,admissible\,\,points\,\,is\,\,empty\,. \end{array}
594
595
596
            return srt none;
597
         }
598
      // End of namespace 'simplex'.
599
      // End of namespace 'numeric'.
600
601
602 #endif // NUMERIC_SIMPLEX_ALG_HPP
```