Project Report

CS 211: High Performance Computing Project 2
High Performance Sequential Codes for Solving Large Linear Systems

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PART #1

1. **GEPP**
   a. **Execution steps**

   The code for Project 2 part #1 is in the C file named *'part1nonblock.c'*. Since Project 2 uses LAPACK library functions, I've run the code on TARDIS cluster as follows:

   Jobfile:

```
#PBS -l nodes=1:ppn=1,walltime=05:00:00
JOB_PATH=/home/rgura001/HPC/Project_2
module load gcc-4.7.2
cd $JOB_PATH
./lapack1
```

   Compilation command:

```
gcc -o lapack1 part1nonblock.c -I/opt/lapack/include
/opt/lapack/lib/liblapacke.a /opt/lapack/lib/liblapack.a
/opt/lapack/lib/librefblas.a -lgfortran -lm -lrt
```

   b. **Analysis**

   The LAPACK library functions are naturally more efficient than my implementations of LU factorization and Forward/Backward Substitution. The example outputs above show that LAPACK functions do significantly more floating point operations per second than my functions.

   c. **Example Outputs**

Run No. 1

```
------------------------------------------------------------
Number of Equations: 1000
Part1.1: Library Functions

 Solved LU Factorization --- Exec. time = 0.0000003894165233

 Solved LU Factorization --- Gflops = 1.7119629671983707


Part1.2: My GEPP Functions

 Solved LU Factorization --- Exec. time = 0.0000037660348937

 Solved LU Factorization --- Gflops = 0.1770208416771166
Error = 0.000000
Error < 1e-3
------------------------------------------------------------
Number of Equations: 2000
Part1.1: Library Functions

 Solved LU Factorization --- Exec. time = 0.0000033101344295

 Solved LU Factorization --- Gflops = 1.6112135162328569
```

```
Part1.2: My GEPP Functions

 Solved LU Factorization --- Exec. time = 0.0000316135113016

 Solved LU Factorization --- Gflops = 0.1687042379587639
Error = 0.000000
Error < 1e-3
------------------------------------------------------------
Number of Equations: 3000
Part1.1: Library Functions

 Solved LU Factorization --- Exec. time = 0.0000111924121454

 Solved LU Factorization --- Gflops = 1.6082324137197073


Part1.2: My GEPP Functions

 Solved LU Factorization --- Exec. time = 0.0001084534342550

 Solved LU Factorization --- Gflops = 0.1659698480149737
Error = 0.000000
Error < 1e-3
------------------------------------------------------------
Number of Equations: 4000
Part1.1: Library Functions

 Solved LU Factorization --- Exec. time = 0.0000270183779486

 Solved LU Factorization --- Gflops = 1.5791720268297129


Part1.2: My GEPP Functions

 Solved LU Factorization --- Exec. time = 0.0002613807465434

 Solved LU Factorization --- Gflops = 0.1632356905812916
Error = 0.000000
Error < 1e-3
------------------------------------------------------------
Number of Equations: 5000
Part1.1: Library Functions

 Solved LU Factorization --- Exec. time = 0.0000595707626194

 Solved LU Factorization --- Gflops = 1.3988965336198007


Part1.2: My GEPP Functions

 Solved LU Factorization --- Exec. time = 0.0005012123220414

 Solved LU Factorization --- Gflops = 0.1662635367660640
Error = 0.000000
Error < 1e-3
```

Run No.2

```
------------------------------------------------------------
Number of Equations: 1000
Part1.1: Library Functions

 Solved LU Factorization --- Exec. time = 0.0000003923552297

 Solved LU Factorization --- Gflops = 1.6991405139850970


Part1.2: My GEPP Functions
```

```
 Solved LU Factorization --- Exec. time = 0.0000039388338067

 Solved LU Factorization --- Gflops = 0.1692548351592785
Error = 0.000000
Error < 1e-3
------------------------------------------------------------
Number of Equations: 2000
Part1.1: Library Functions

 Solved LU Factorization --- Exec. time = 0.0000033143234737

 Solved LU Factorization --- Gflops = 1.6091770690518601


Part1.2: My GEPP Functions

 Solved LU Factorization --- Exec. time = 0.0000321935111769

 Solved LU Factorization --- Gflops = 0.1656648541389220
Error = 0.000000
Error < 1e-3
------------------------------------------------------------
Number of Equations: 3000
Part1.1: Library Functions

 Solved LU Factorization --- Exec. time = 0.0000112643597759

 Solved LU Factorization --- Gflops = 1.5979603242486871


Part1.2: My GEPP Functions

 Solved LU Factorization --- Exec. time = 0.0001116101413257

 Solved LU Factorization --- Gflops = 0.1612756671229879
Error = 0.000000
Error < 1e-3
------------------------------------------------------------
Number of Equations: 4000
Part1.1: Library Functions

 Solved LU Factorization --- Exec. time = 0.0000270698764510

 Solved LU Factorization --- Gflops = 1.5761677650737538


Part1.2: My GEPP Functions

 Solved LU Factorization --- Exec. time = 0.0002588004170768

 Solved LU Factorization --- Gflops = 0.1648632067467101
Error = 0.000000
Error < 1e-3
------------------------------------------------------------
Number of Equations: 5000
Part1.1: Library Functions

 Solved LU Factorization --- Exec. time = 0.0000596116018556

 Solved LU Factorization --- Gflops = 1.3979381653790723


Part1.2: My GEPP Functions

 Solved LU Factorization --- Exec. time = 0.0005114343875311

 Solved LU Factorization --- Gflops = 0.1629404188787770
Error = 0.000000
Error < 1e-3
```

PART #2

2. Blocked GEPP

a. Execution steps

The code for Project 2 part #1 is in the C file named *'part2blocked.c'*.
Jobfile:

```
#PBS -l nodes=1:ppn=1,walltime=05:00:00
JOB_PATH=/home/rgura001/HPC/Project_2
module load gcc-4.7.2
cd $JOB_PATH
./lapack2
```

Compilation command:

```
gcc -o lapack2 part2blocked.c -I/opt/lapack/include
/opt/lapack/lib/liblapacke.a /opt/lapack/lib/liblapack.a
/opt/lapack/lib/librefblas.a -lgfortran -lm -lrt
```

b. Analysis

Block size 30 to 50 gives higher performance compared to other block sizes. There is a significant decrease in gflops beyond block size 200. There is a noticeable decline in gflops for block size 500.

As can be seen from the example output, the blocked version does more floating point operations per second than the naïve version.

c. Example Outputs

Run 1 – Block Sizes {10,20,30,40,50}

```
Part1.2: My GEPP Functions
Matrix size = 1000
 Solved LU Factorization --- Exec. time = 0.0000041410158314

 Solved LU Factorization --- Gflops = 0.1609910934448963

Part2: Blocked GEPP Functions
Matrix size = 1000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0000032964533530

 Solved LU Factorization --- Gflops = 0.2022375551155307

Error = 0.000000
Error < 1e-3


 Block size = 20
 Solved LU Factorization --- Exec. time = 0.0000033795701005

 Solved LU Factorization --- Gflops = 0.1972637485982671

Error = 0.000000
Error < 1e-3
```

```
 Block size = 30

 Solved LU Factorization --- Exec. time = 0.0000032932450920

 Solved LU Factorization --- Gflops = 0.2024345738126785

Error = 0.000000


 Block size = 40
 Solved LU Factorization --- Exec. time = 0.0000037907790355

 Solved LU Factorization --- Gflops = 0.1758653459926542

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0000036958615668

 Solved LU Factorization --- Gflops = 0.1803819365566120

Error = 0.000000
Error < 1e-3

------------------------------------------------------------

Part1.2: My GEPP Functions
Matrix size = 2000
 Solved LU Factorization --- Exec. time = 0.0000354180666357

 Solved LU Factorization --- Gflops = 0.1505822829965921

Part2: Blocked GEPP Functions
Matrix size = 2000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0000234674942642

 Solved LU Factorization --- Gflops = 0.2272647123412394

Error = 0.000000
Error < 1e-3


 Block size = 20
 Solved LU Factorization --- Exec. time = 0.0000235160131715

 Solved LU Factorization --- Gflops = 0.2267958133222860

Error = 0.000000
Error < 1e-3


 Block size = 30

 Solved LU Factorization --- Exec. time = 0.0000228573722132

 Solved LU Factorization --- Gflops = 0.2333309920142026

Error = 0.000000


 Block size = 40
 Solved LU Factorization --- Exec. time = 0.0000242457809299

 Solved LU Factorization --- Gflops = 0.2199695422782267

Error = 0.000000
Error < 1e-3
```

```
 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0000258827308863

 Solved LU Factorization --- Gflops = 0.2060575971196321

Error = 0.000000
Error < 1e-3


------------------------------------------------------------
Part1.2: My GEPP Functions
Matrix size = 3000
 Solved LU Factorization --- Exec. time = 0.0001135641650483

 Solved LU Factorization --- Gflops = 0.1585007030373025

Part2: Blocked GEPP Functions
Matrix size = 3000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0000815565537289

 Solved LU Factorization --- Gflops = 0.2207057456086003

Error = 0.000000
Error < 1e-3


 Block size = 20
 Solved LU Factorization --- Exec. time = 0.0000845188132487

 Solved LU Factorization --- Gflops = 0.2129703353385858

Error = 0.000000
Error < 1e-3


 Block size = 30
 Solved LU Factorization --- Exec. time = 0.0000784029305875

 Solved LU Factorization --- Gflops = 0.2295832549257141

Error = 0.000000
Error < 1e-3


 Block size = 40
 Solved LU Factorization --- Exec. time = 0.0000853285541870

 Solved LU Factorization --- Gflops = 0.2109493143474435

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0000858233728372

 Solved LU Factorization --- Gflops = 0.2097330762583028

Error = 0.000000
Error < 1e-3
```

```
Part1.2: My GEPP Functions
Matrix size = 4000
 Solved LU Factorization --- Exec. time = 0.0002720447400995

 Solved LU Factorization --- Gflops = 0.1568369476692241
```

```
Part2: Blocked GEPP Functions
Matrix size = 4000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0002105619732663

 Solved LU Factorization --- Gflops = 0.2026323462152634

Error = 0.000000
Error < 1e-3


 Block size = 20
 Solved LU Factorization --- Exec. time = 0.0002089372326806

 Solved LU Factorization --- Gflops = 0.2042080586560026

Error = 0.000000
Error < 1e-3


 Block size = 30

 Solved LU Factorization --- Exec. time = 0.0001942295851447

 Solved LU Factorization --- Gflops = 0.2196713061755606

Error = 0.000000


 Block size = 40
 Solved LU Factorization --- Exec. time = 0.0001791464382559

 Solved LU Factorization --- Gflops = 0.2381664245298498

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0001708812155165

 Solved LU Factorization --- Gflops = 0.2496861140512684

Error = 0.000000
Error < 1e-3


------------------------------------------------------------
Part1.2: My GEPP Functions
Matrix size = 5000
 Solved LU Factorization --- Exec. time = 0.0004333255921304

 Solved LU Factorization --- Gflops = 0.1923111278141440

Part2: Blocked GEPP Functions
Matrix size = 5000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0003414579936564

 Solved LU Factorization --- Gflops = 0.2440514935409308

Error = 0.000000
Error < 1e-3


 Block size = 20
 Solved LU Factorization --- Exec. time = 0.0003437308673933

 Solved LU Factorization --- Gflops = 0.2424377361430825

Error = 0.000000
Error < 1e-3
```

```
 Block size = 30

 Solved LU Factorization --- Exec. time = 0.0003329964487776

 Solved LU Factorization --- Gflops = 0.2502529190303335

Error = 0.000000
Error < 1e-3


 Block size = 40
 Solved LU Factorization --- Exec. time = 0.0003373596759960

 Solved LU Factorization --- Gflops = 0.2470162833993237

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0003455371281430

 Solved LU Factorization --- Gflops = 0.2411704171449933

Error = 0.000000
Error < 1e-3

================================================
```

Run 2 – Block Sizes {10,50,100,200,500}

```
Part1.2: My GEPP Functions
Matrix size = 1000
 Solved LU Factorization --- Exec. time = 0.0000047668307424

 Solved LU Factorization --- Gflops = 0.1398553258336849

Part2: Blocked GEPP Functions
Matrix size = 1000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0000031488668807

 Solved LU Factorization --- Gflops = 0.2117163703413279

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0000034354973622

 Solved LU Factorization --- Gflops = 0.1940524460907239

Error = 0.000000
Error < 1e-3


Block size = 100
 Solved LU Factorization --- Exec. time = 0.0000032820639983

 Solved LU Factorization --- Gflops = 0.2031242130000034

Error = 0.000000
Error < 1e-3
```

```
 Block size = 200
 Solved LU Factorization --- Exec. time = 0.0000035108194463

 Solved LU Factorization --- Gflops = 0.1898891916444496

Error = 0.000000
Error < 1e-3


 Block size = 500
 Solved LU Factorization --- Exec. time = 0.0000041402825639

 Solved LU Factorization --- Gflops = 0.1610196058803040

Error = 0.000000
Error < 1e-3

------------------------------------------------------------

Part1.2: My GEPP Functions
Matrix size = 2000
 Solved LU Factorization --- Exec. time = 0.0000369823738821

 Solved LU Factorization --- Gflops = 0.1442128444847315

Part2: Blocked GEPP Functions
Matrix size = 2000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0000235809158050

 Solved LU Factorization --- Gflops = 0.2261715947526196

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0000231707037985

 Solved LU Factorization --- Gflops = 0.2301757158395473

Error = 0.000000
Error < 1e-3


 Block size = 100
 Solved LU Factorization --- Exec. time = 0.0000237571330331

 Solved LU Factorization --- Gflops = 0.2244939793828074

Error = 0.000000
Error < 1e-3


 Block size = 200
 Solved LU Factorization --- Exec. time = 0.0000282706164159

 Solved LU Factorization --- Gflops = 0.1886528844958613

Error = 0.000000
Error < 1e-3


 Block size = 500
 Solved LU Factorization --- Exec. time = 0.0000313920594342

 Solved LU Factorization --- Gflops = 0.1698943436480006

Error = 0.000000
Error < 1e-3
```

```
------------------------------------------------------------

Part1.2: My GEPP Functions
Matrix size = 3000
 Solved LU Factorization --- Exec. time = 0.0001232911300063

 Solved LU Factorization --- Gflops = 0.1459959041585415

Part2: Blocked GEPP Functions
Matrix size = 3000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0000792964498885

 Solved LU Factorization --- Gflops = 0.2269962908214760

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0000773561161496

 Solved LU Factorization --- Gflops = 0.2326900689429068

Error = 0.000000
Error < 1e-3


 Block size = 100
 Solved LU Factorization --- Exec. time = 0.0000793391195498

 Solved LU Factorization --- Gflops = 0.2268742091181442

Error = 0.000000
Error < 1e-3


 Block size = 200
 Solved LU Factorization --- Exec. time = 0.0000847795412876

 Solved LU Factorization --- Gflops = 0.2123153738110497

Error = 0.000000
Error < 1e-3


 Block size = 500
 Solved LU Factorization --- Exec. time = 0.0001089410540201

 Solved LU Factorization --- Gflops = 0.1652269675735168

Error = 0.000000
Error < 1e-3

------------------------------------------------------------

Part1.2: My GEPP Functions
Matrix size = 4000
 Solved LU Factorization --- Exec. time = 0.0002903228175044

 Solved LU Factorization --- Gflops = 0.1469628430635465

Part2: Blocked GEPP Functions
Matrix size = 4000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0001877195706666

 Solved LU Factorization --- Gflops = 0.2272893897805464

Error = 0.000000
Error < 1e-3
```

```
 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0001853292789571

 Solved LU Factorization --- Gflops = 0.2302208636798476

Error = 0.000000
Error < 1e-3


 Block size = 100
 Solved LU Factorization --- Exec. time = 0.0002088309134208

 Solved LU Factorization --- Gflops = 0.2043120243443028

Error = 0.000000
Error < 1e-3


 Block size = 200
 Solved LU Factorization --- Exec. time = 0.0002156536431313

 Solved LU Factorization --- Gflops = 0.1978481144447808

Error = 0.000000
Error < 1e-3


 Block size = 500
 Solved LU Factorization --- Exec. time = 0.0002685800160430

 Solved LU Factorization --- Gflops = 0.1588601687320987

Error = 0.000000
Error < 1e-3

------------------------------------------------------------

Part1.2: My GEPP Functions
Matrix size = 5000
 Solved LU Factorization --- Exec. time = 0.0005952932884730

 Solved LU Factorization --- Gflops = 0.1399870197547295

Part2: Blocked GEPP Functions
Matrix size = 5000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0003708342774957

 Solved LU Factorization --- Gflops = 0.2247185289776515

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0003608969174884

 Solved LU Factorization --- Gflops = 0.2309061931403331

Error = 0.000000
Error < 1e-3


 Block size = 100
 Solved LU Factorization --- Exec. time = 0.0003693711327203

 Solved LU Factorization --- Gflops = 0.2256086790530128

Error = 0.000000
Error < 1e-3
```

```
 Block size = 200
 Solved LU Factorization --- Exec. time = 0.0003952681380808

 Solved LU Factorization --- Gflops = 0.2108273480831157

Error = 0.000000
Error < 1e-3


 Block size = 500
 Solved LU Factorization --- Exec. time = 0.0005356080785468

 Solved LU Factorization --- Gflops = 0.1555864010853385

Error = 0.000000
Error < 1e-3
```

Optimizations:

Using gcc Flag O1-

Analysis: The performance increases slightly with increase in block size till block size 200 and then decreases. Compiling with optimization flag O1, O2 and O3, the example outputs show that the number of floating point operations done per second increase. There a significant improvement from O1 to O2 and a slight improvement from O2 to O3.

```
Part1.2: My GEPP Functions
Matrix size = 1000
 Solved LU Factorization --- Exec. time = 0.0000007723627128

 Solved LU Factorization --- Gflops = 0.8631523190382018

Part2: Blocked GEPP Functions
Matrix size = 1000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0000006791573875

 Solved LU Factorization --- Gflops = 0.9816085033762352

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0000009164135903

 Solved LU Factorization --- Gflops = 0.7274735705738410

Error = 0.000000
Error < 1e-3


 Block size = 100
 Solved LU Factorization --- Exec. time = 0.0000008213442191

 Solved LU Factorization --- Gflops = 0.8116775538495746

Error = 0.000000
Error < 1e-3


 Block size = 200
```

```
 Solved LU Factorization --- Exec. time = 0.0000007554778792

 Solved LU Factorization --- Gflops = 0.8824436625143726

Error = 0.000000
Error < 1e-3


 Block size = 500
 Solved LU Factorization --- Exec. time = 0.0000010447665527

 Solved LU Factorization --- Gflops = 0.6381010809680423

Error = 0.000000
Error < 1e-3
```

Using gcc Flag O2-

```
Part1.2: My GEPP Functions
Matrix size = 1000
 Solved LU Factorization --- Exec. time = 0.0000006342407987

 Solved LU Factorization --- Gflops = 1.0511254843324538

Part2: Blocked GEPP Functions
Matrix size = 1000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0000005522854477

 Solved LU Factorization --- Gflops = 1.2071052558470161

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0000010304187164

 Solved LU Factorization --- Gflops = 0.6469861776332451

Error = 0.000000
Error < 1e-3


 Block size = 100
 Solved LU Factorization --- Exec. time = 0.0000009414436296

 Solved LU Factorization --- Gflops = 0.7081323254332533

Error = 0.000000
Error < 1e-3


 Block size = 200
 Solved LU Factorization --- Exec. time = 0.0000008145365715

 Solved LU Factorization --- Gflops = 0.8184613066995589

Error = 0.000000
Error < 1e-3


 Block size = 500
 Solved LU Factorization --- Exec. time = 0.0000010343998969

 Solved LU Factorization --- Gflops = 0.6444960683873860

Error = 0.000000
Error < 1e-3
```

Using gcc Flag O3-

```
Part1.2: My GEPP Functions
Matrix size = 1000
 Solved LU Factorization --- Exec. time = 0.0000006785511151

 Solved LU Factorization --- Gflops = 0.9824855516930746

Part2: Blocked GEPP Functions
Matrix size = 1000
 Block size = 10
 Solved LU Factorization --- Exec. time = 0.0000005523277409

 Solved LU Factorization --- Gflops = 1.2070128245507399

Error = 0.000000
Error < 1e-3


 Block size = 50
 Solved LU Factorization --- Exec. time = 0.0000011415050402

 Solved LU Factorization --- Gflops = 0.5840242865097599

Error = 0.000000
Error < 1e-3


 Block size = 100
 Solved LU Factorization --- Exec. time = 0.0000010234119333

 Solved LU Factorization --- Gflops = 0.6514157642228240

Error = 0.000000
Error < 1e-3


 Block size = 200
 Solved LU Factorization --- Exec. time = 0.0000009619409740

 Solved LU Factorization --- Gflops = 0.6930432164631604

Error = 0.000000
Error < 1e-3


 Block size = 500
 Solved LU Factorization --- Exec. time = 0.0000011128575839

 Solved LU Factorization --- Gflops = 0.5990583847433671

Error = 0.000000
Error < 1e-3
```