GROUP 30

Option Pricing Project

20 April 2024

Option importance:

Option pricing is crucial in financial markets as it enables investors to manage risk exposure, and make informed investment decisions. By accurately pricing options, market participants can understand price fluctuations, speculate on future movements, and optimize portfolio performance.

Objective:

This project aims to delve into the intricacies of option pricing by employing both the Binomial and Black-Scholes models. We will explore the pricing dynamics of options, focusing on a comprehensive analysis of call and put options.

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Data Collection

The data utilized was gathered from Yahoo Finance. It encompassed both historical stock prices and option prices.

The stock chosen for analysis was AAPL (Apple Inc.), a widely-traded company in the technology sector. The data spanned a one-year period from January 1, 2023, to January 1, 2024, allowing for a thorough examination of its performance and option pricing dynamics over that timeframe.

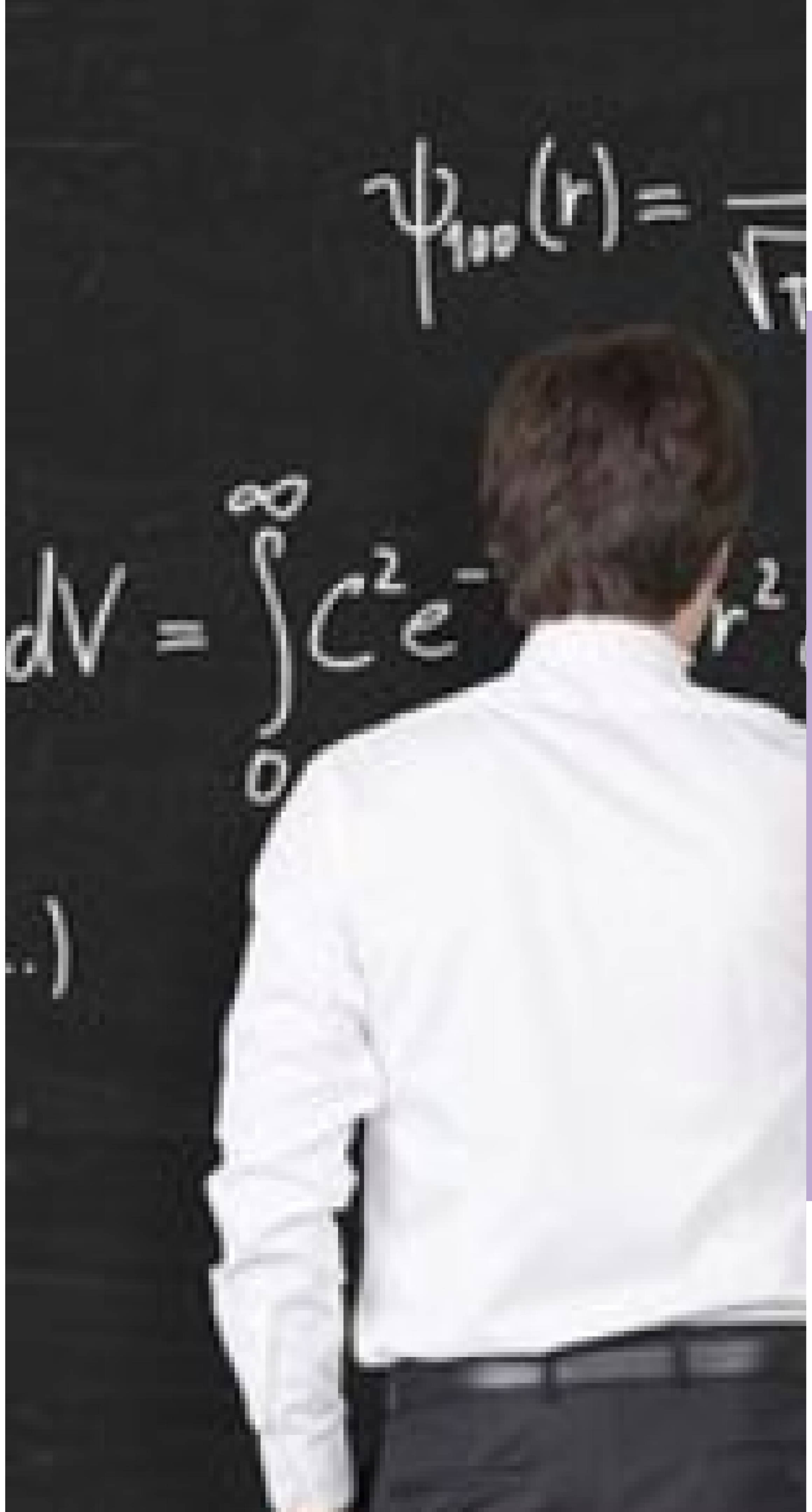
Estimating Annual Volatility:

- Using historical stock prices for one-year period.
- Volatility was calculated as the standard deviation of the logarithmic returns of the stock prices.
- This method captures the degree of fluctuation or variability in the stock's price over the specified timeframe.

- Volatility is a critical component in option pricing models like Black-Scholes and Binomial Model.
- It reflects the market's expectations of future price movements and directly impacts option prices.
- Higher volatility implies greater potential price swings, leading to higher option premiums, and vice versa.

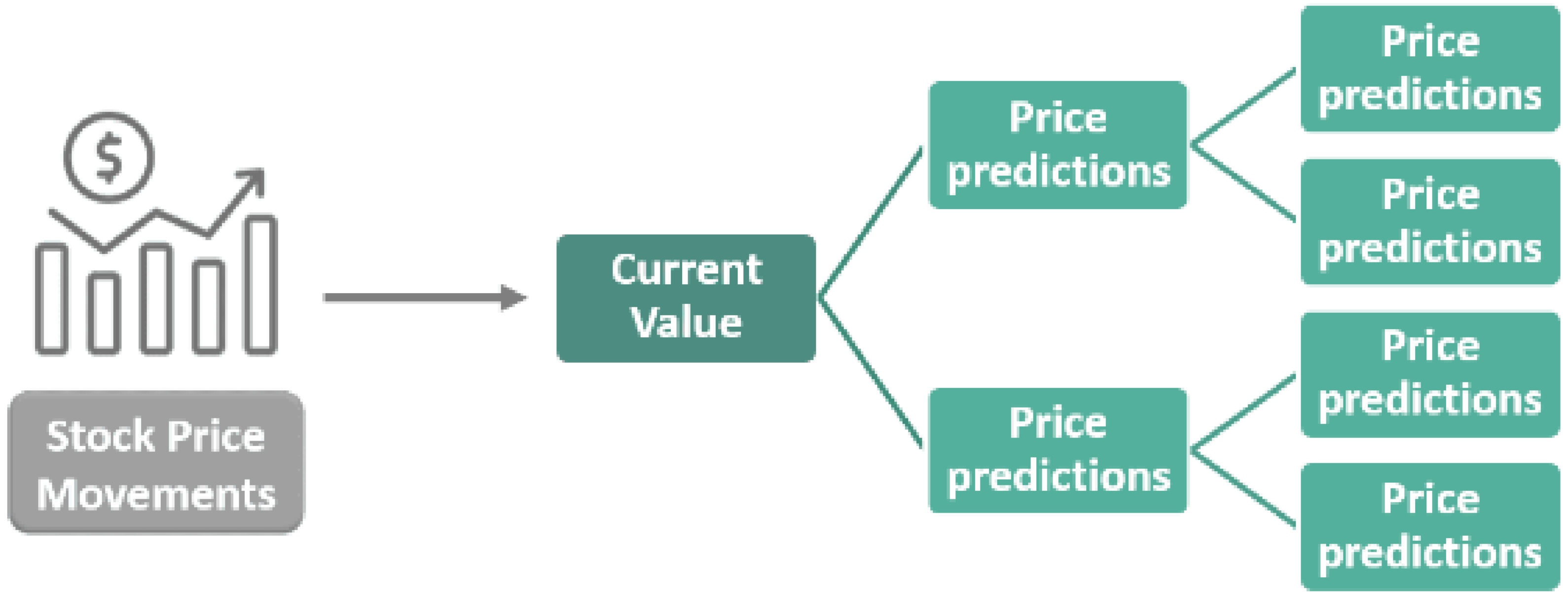
Binomial Model

- The Binomial model is a discrete-time model that calculates option prices by simulating the possible future price movements of the underlying asset over multiple time steps.
- This model considers the risk-neutral probabilities of the asset's price going up or down at each step, along with factors like the strike price, time to maturity, risk-free rate, and volatility.



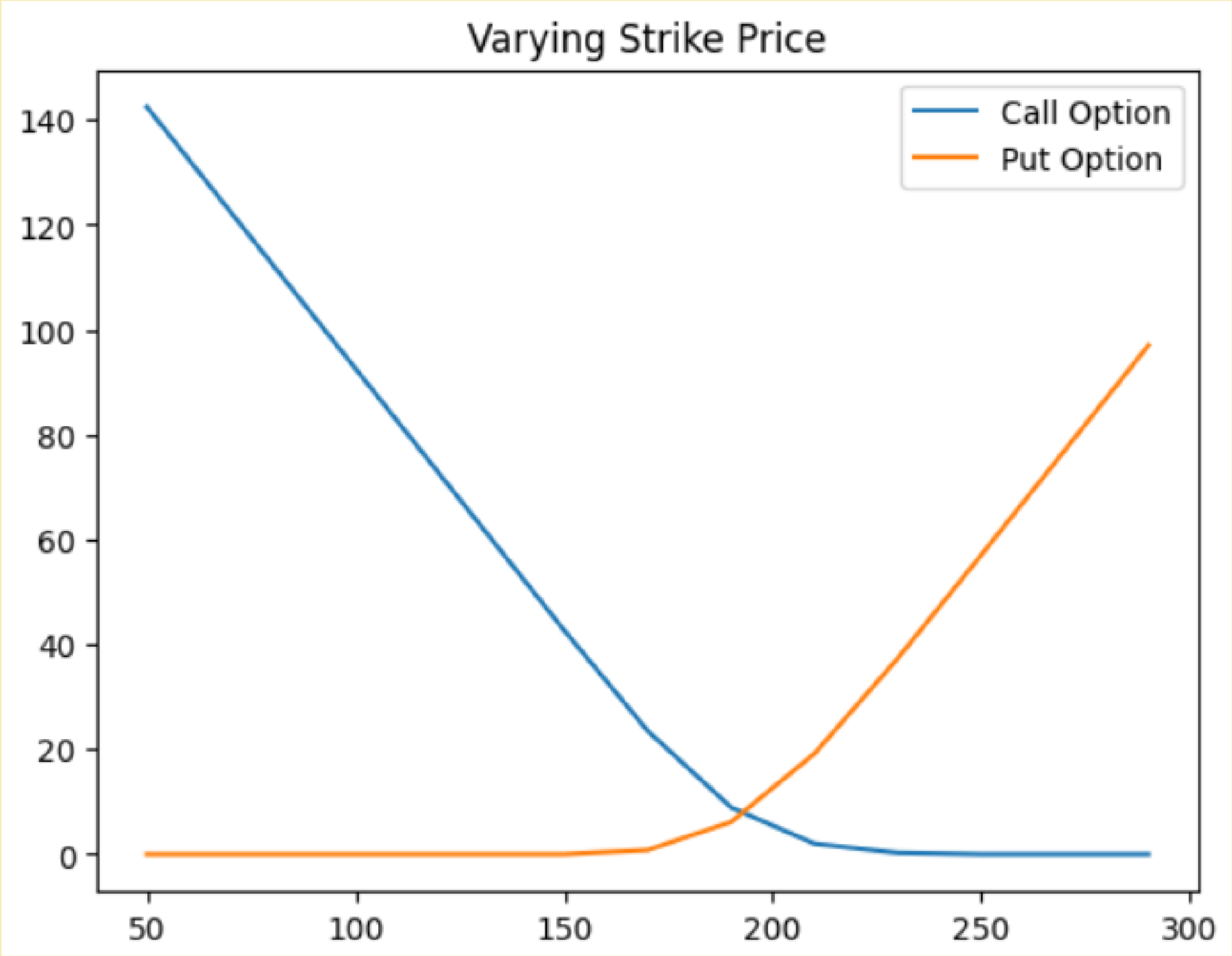
Binomial Model

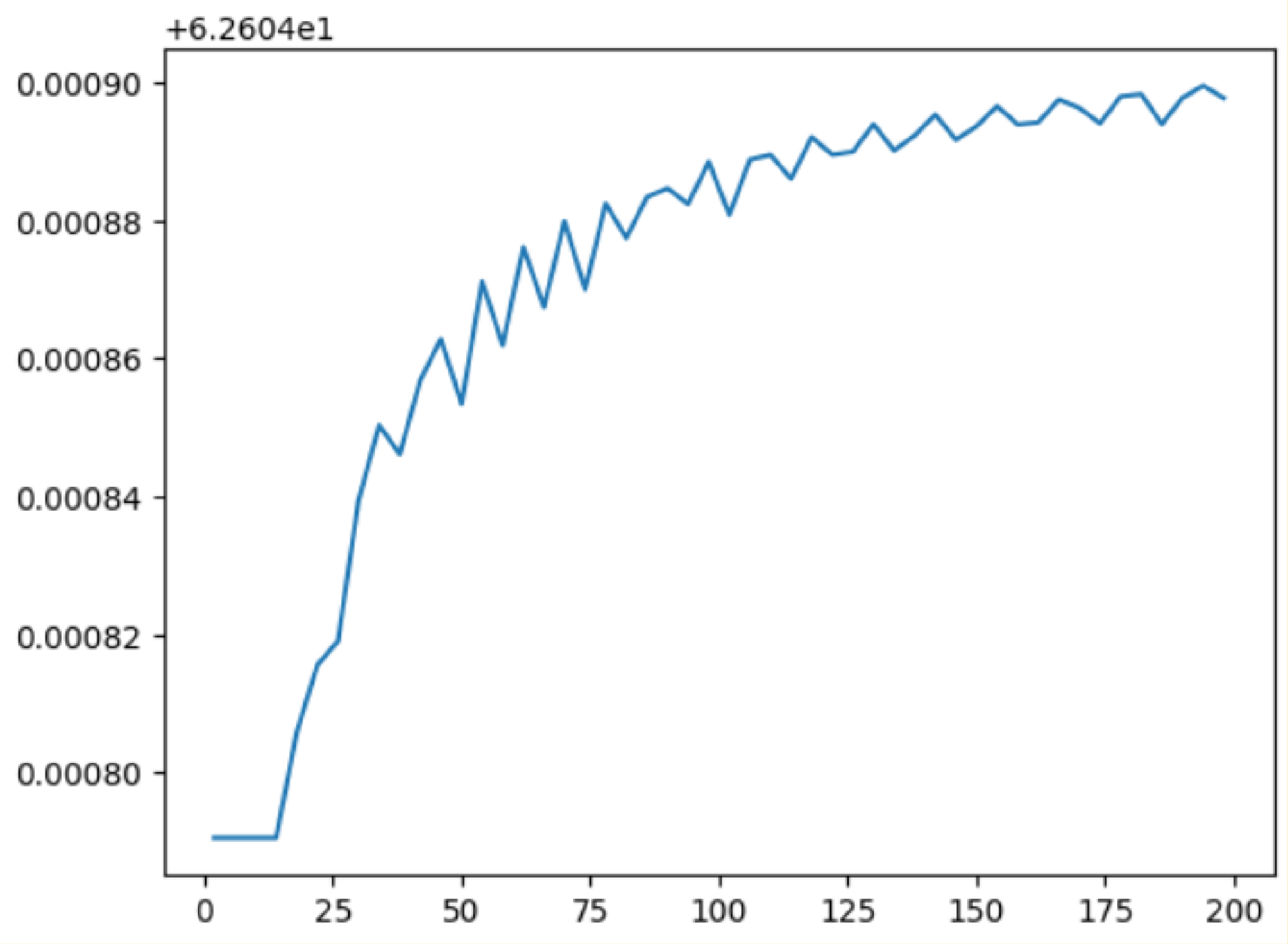
- The function **binomial_option_price** to calculate the option price (both call and put) using the Binomial model.
- The parameters for this model include the current stock price, strike price, time to maturity, risk-free interest rate (10 years' US Treasury rate), annual volatility, and the number of steps in the Binomial model.
- It then calculates the call option price for a specified strike price, time to maturity, and number of steps.



Plotting Option Prices

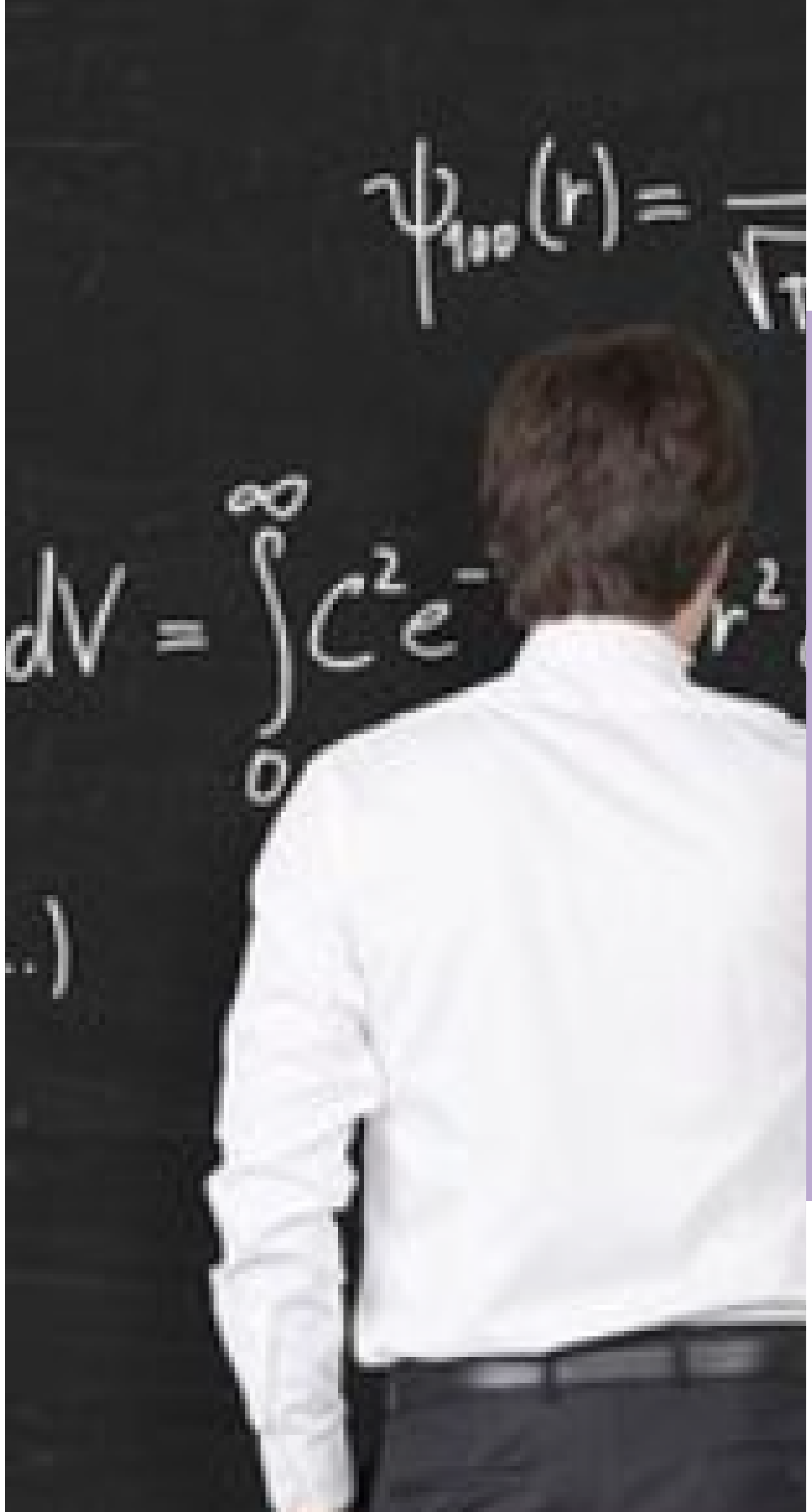
- We generated plots to visualize how option prices vary with changing strike prices and with increasing steps in the Binomial model.
- The first plot shows the call and put option prices for different strike prices, illustrating how option prices change with varying strike prices.
- The second plot focuses on the call option price for a fixed strike price of $130, showing how the option price converges as the number of steps in the Binomial model increases.





Black-Scholes Model

- The Black-Scholes model is a continuous-time mathematical model used to calculate option prices. It assumes that asset prices follow a lognormal distribution and that markets are efficient and frictionless.
- This model takes into account factors such as the current stock price, strike price, time to maturity, risk-free interest rate, and volatility to determine the theoretical value of options.



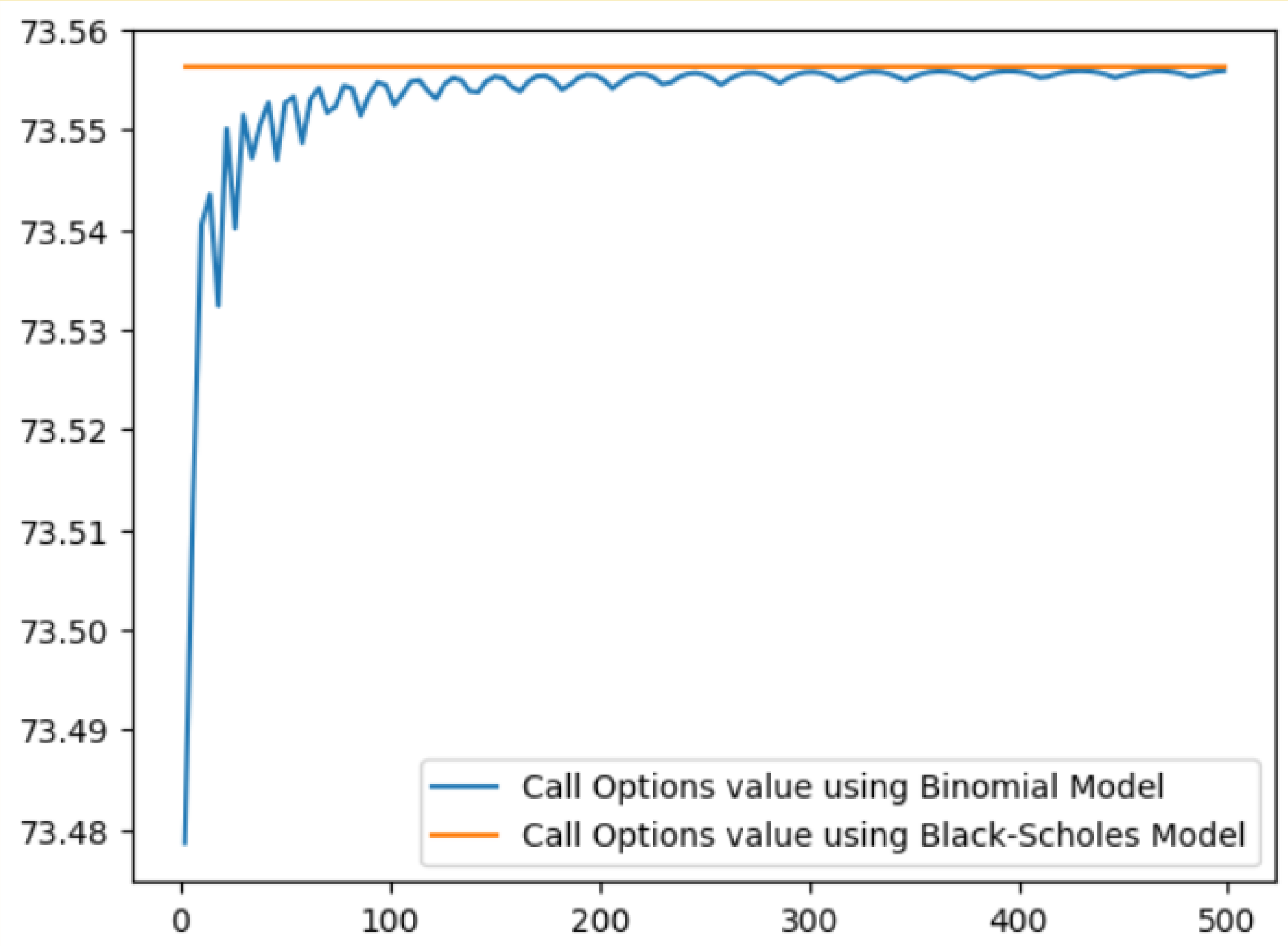
Black-Scholes Model

- Two functions are defined: **black_scholes_call** to calculate the call option price and **black_scholes_put** to calculate the put option price using the Black-Scholes formula.
- The Black-Scholes formula calculates option prices based on the stock price (S), strike price (X), time to maturity (T), risk-free interest rate (r), and annual volatility (sigma).

$$\overbrace{C(S,t)}^{\text{Call Price}} = \overbrace{N(d_1)}^{\text{Risk-adj. prob. that option won't expire ITM}}\underbrace{S}_{\text{Stock Price}} - \overbrace{N(d_2)}^{\text{Risk-adj. prob. that option will expire ITM}}\underbrace{K}_{\text{Strike Price}}\overbrace{e^{-rT}}^{\text{Discounts strike price to present time}}$$

Plotting Option Prices

- The plot compares call option prices calculated using the Binomial model and Black-Scholes model for a range of steps in the Binomial model.
- It helps visualize how the option prices converge as the number of steps in the Binomial model increases and compares them with the Black-Scholes model.



Delta Neutral Portfolio

A delta-neutral portfolio is a portfolio of financial securities or derivatives where the overall delta, or sensitivity to changes in the price of the underlying asset, is close to zero. In options trading, delta measures the rate of change of the option price with respect to changes in the price of the underlying asset.

When a portfolio is delta-neutral, it means that the overall value of the portfolio is not significantly affected by small changes in the price of the underlying asset. Delta-neutral portfolios are often used by traders to hedge against directional movements in the market and to profit from changes in volatility.

To create a delta-neutral portfolio, a trader typically takes offsetting positions in options or other derivatives that have opposite deltas. For example, if a trader holds a long position in call options (positive delta), they might also take a short position in put options (negative delta) to offset the positive delta and achieve delta neutrality.

$$\mathrm{call\_quantity} = -\frac{\mathrm{put\_d} \times \mathrm{call\_price}}{\mathrm{call\_d} \times \mathrm{put\_price}}$$

And similarly, we can derive the expression for $\mathrm{put\_quantity}$:

$$\mathrm{put\_quantity} = -\frac{\mathrm{call\_d} \times \mathrm{put\_price}}{\mathrm{put\_d} \times \mathrm{call\_price}}$$

These equations determine the quantities of call and put options neede

delta-neutral.

Derivative

- call_d: Delta of the call option
- put_d: Delta of the put option
- call_price: Price of the call option
- put_price: Price of the put option

To construct a delta-neutral portfolio, we want the sum of the deltas of the call options to be equal in magnitude but opposite in sign to the sum of the deltas of the put options.

Let call_quantity be the quantity of call options and put_quantity be the quantity of put options required to achieve this balance.

The equation for the portfolio delta (Portfolio Delta) can be written as:

$$\text{Portfolio Delta} = \text{call\_d} \times \text{call\_quantity} + \text{put\_d} \times \text{put\_quantity}$$

For a delta-neutral portfolio, Portfolio Delta should be close to zero.

Now, let's set Portfolio Delta equal to zero and solve for call_quantity and put_quantity:

$$0 = \text{call\_d} \times \text{call\_quantity} + \text{put\_d} \times \text{put\_quantity}$$

$$\text{put\_quantity} = -\frac{\text{call\_d}}{\text{put\_d}} \times \text{call\_quantity}$$

Given the prices of the options, we can also use the equation for the value of the portfolio (Portfolio Value):

$$\text{Portfolio Value} = \text{call\_price} \times \text{call\_quantity} + \text{put\_price} \times \text{put\_quantity}$$

For simplicity, let's assume that the value of the portfolio is zero initially.

Substituting the expression for put_quantity into the equation for Portfolio Value, we get:

$$0 = \text{call\_price} \times \text{call\_quantity} + \text{put\_price} \times \left(-\frac{\text{call\_d}}{\text{put\_d}} \times \text{call\_quantity}\right)$$

$$\text{call\_quantity} = -\frac{\text{put\_d} \times \text{call\_price}}{\text{call\_d} \times \text{put\_price}}$$

How to get implied Volatility

To calculate the imlpied volatility we want that the estimated price using the black scholes model should be close to market price. So we need to find roots of the function:

$f(\sigma)=C(S,K,r,T,\sigma)-\text{Market Price}$

Where:

$f(\sigma)$ is the difference between the theoretical price of the call option calculated using the Black-Scholes formula and the observed market price of the option.

σ is the implied volatility, which we want to solve for.

$C(S,K,r,T,\sigma)$ is the theoretical price of the call option calculated using the Black-Scholes formula.

Market Price is the observed market price of the call option.

Then we have used the Newton-Raphson method which iteratively improves the initial guess for the implied volatility until it finds a solution where $f(\sigma)$ is close to zero.

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Thank You