MARKOWITZ PORTFOLIO OPTIMIZATION

GROUP 30

INTRODUCTION

The primary goal is to construct an optimal investment portfolio using the principles of Markowitz's modern portfolio theory. By applying mean-variance optimization, we aim to find the perfect balance between risk and return. This analysis will provide valuable insights for investors seeking to maximize portfolio performance.

Through this project, we explore the dynamics of diverse assets, considering stocks, bonds, and ETFs, and employ financial analytics to guide portfolio construction.

CHOSEN ASSETS

NVDA (NVIDIA CORPORATION)

Industry: Technology - Semiconductors

Brief Description: A leading graphics processing unit (GPU) manufacturer, known for its innovations in gaming, AI, and data centers.

MSFT (MICROSOFT CORPORATION)

Industry: Technology - Software

Brief Description: A global technology giant, widely recognized for its software products, including Windows, Office, and cloud services.

IBM (INTERNATIONAL BUSINESS MACHINES CORPORATION)

IIndustry: Technology - IT Services

Brief Description: A multinational technology and consulting company, offering a range of IT services, software, and hardware solutions.

AMZN (AMAZON.COM INC.)

Industry: E-commerce and Cloud Computing

Brief Description: A multinational technology and e-commerce giant, known for its online retail platform, cloud services, and digital streaming.

GOOGL (ALPHABET INC.)

Industry: Technology - Internet Services

Brief Description: The parent company of Google, involved in online advertising, search engines, and various internet-related services.

BLK (BLACKROCK INC.)

Industry: Financial Services - Asset Management

Brief Description: One of the world's largest investment management firms, offering a diverse range of financial products and services.

JPM (JPMORGAN CHASE & CO.)

Industry: Financial Services - Banking

Brief Description: A multinational investment bank and financial services company, providing a wide range of banking and financial solutions.

V (VISA INC.)

IIndustry: Financial Services - Payment Technology

Brief Description: A global payments technology company, facilitating electronic funds transfers worldwide.

DIS (THE WALT DISNEY COMPANY)

Industry: Entertainment and Media

Brief Description: A diversified multinational mass media and entertainment conglomerate, known for its film studio, theme parks, and media networks.

NFLX (NETFLIX INC.)

Industry: Technology - Streaming Services

Brief Description: A leading subscription-based streaming service, offering a wide variety of TV shows, movies, and original content.

SIMPLE RETURN

Simple return, also known as arithmetic return or price return, is a measure used to calculate the rate of return on an investment over a single period of time. It is expressed as a percentage and represents the change in the value of an investment from the beginning of the period to the end, without considering any intermediate cash flows such as dividends or interest payments.

The result is typically expressed as a percentage to indicate the relative change in value.

FORMULAS

① Obj : $\text{Max } \mu^T w$

s.t

$$O^T w = 1$$

$$w^T C w = \text{Risk}$$

② obj : $\text{Min } w^T C w$

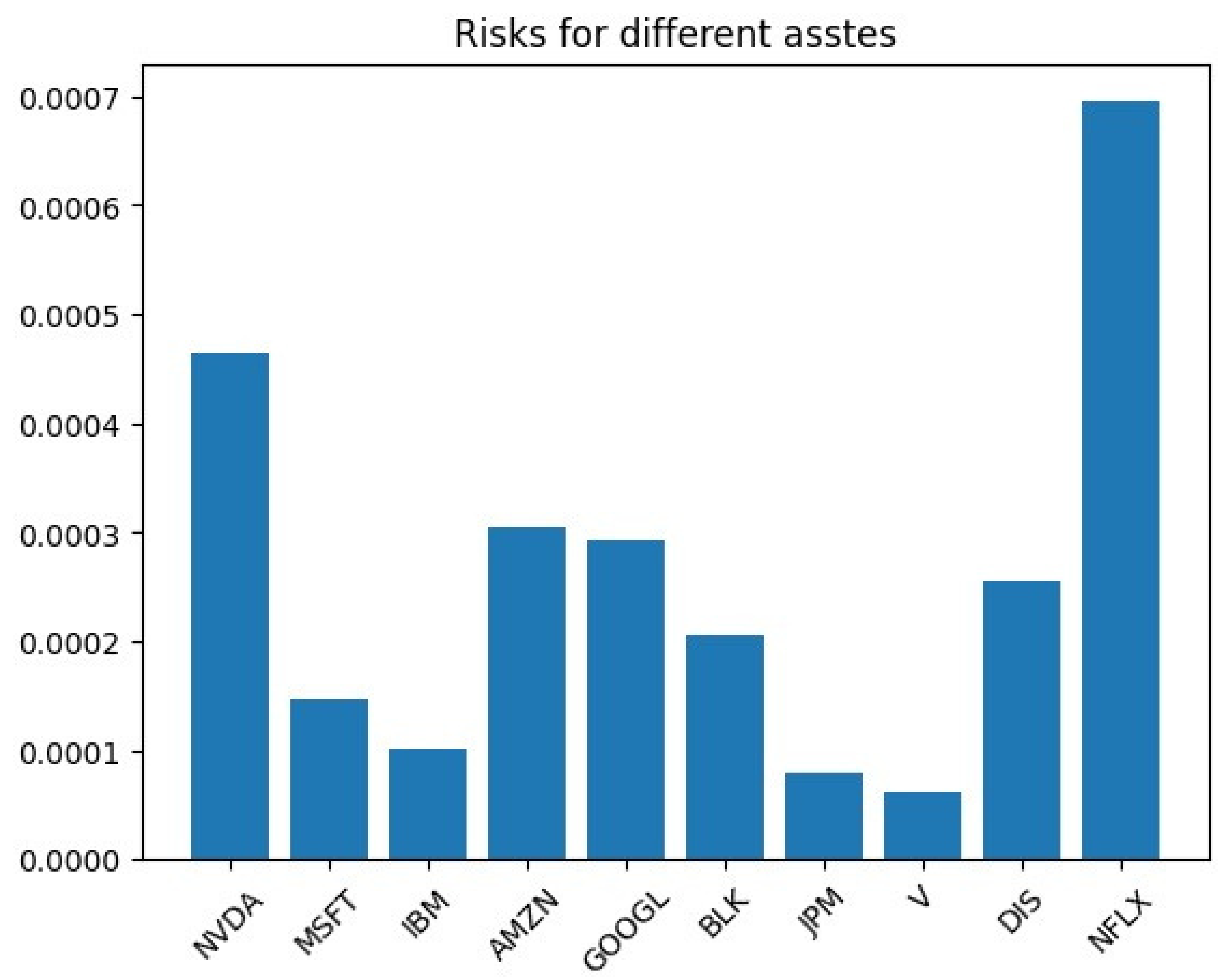
$$O^T w = 1$$

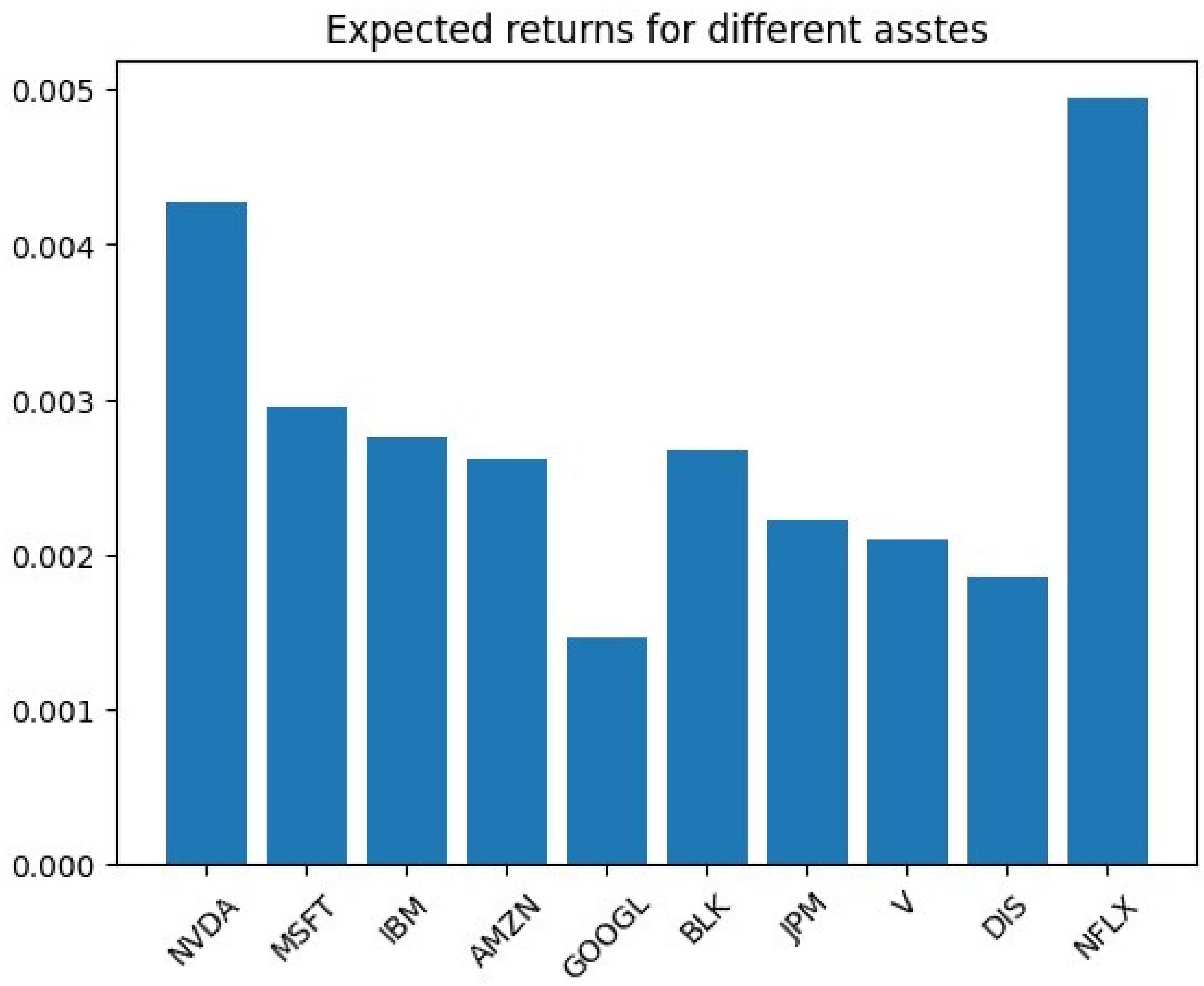
$$\mu^T w = \text{Return}$$

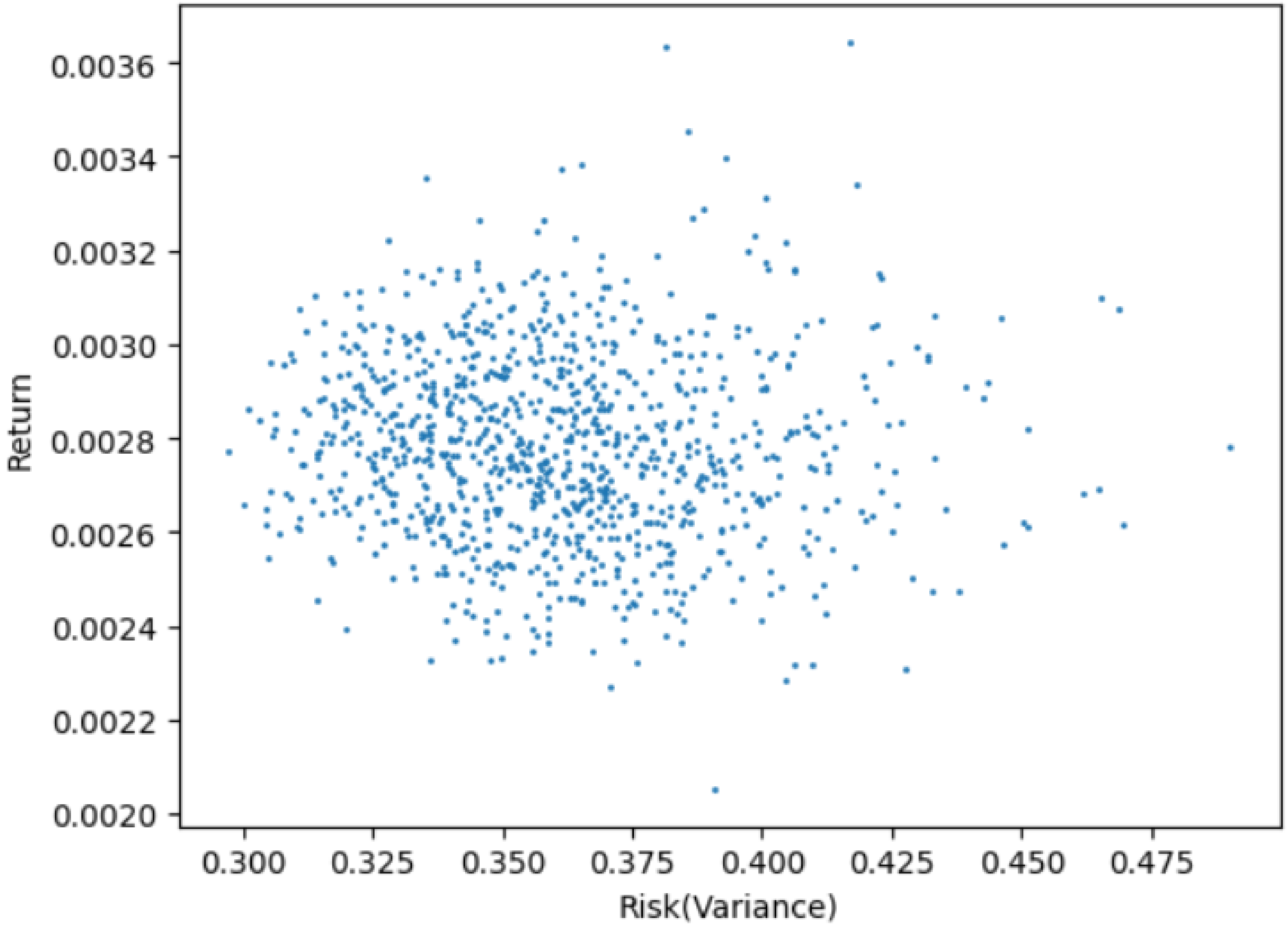
$$\text{Rate of Return Formula} = \frac{\text{Current Value} - \text{Original Value}}{\text{Original Value}} \times 100$$

Why not log return?

- **Interpretability**: Simple returns are more intuitive and easier to interpret for many investors. They directly represent the percentage change in the value of an investment over a specific period.
- **Ease of Calculation**: Simple returns are easier to calculate and understand, especially for individuals who are not familiar with the concept of logarithms.
- **Historical Convention**: In many contexts, including historical financial data and financial reporting, simple returns have been traditionally used, making them more familiar and widely accepted.


Risks for different asstes
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
NVDA
MSFT
IBM
AMZN
GOOGL
BLK
JPM
V
DIS
NFLX


Expected returns for different asstes
0.005
0.004
0.003
0.002
0.001
0.000
NVDA
MSFT
IBM
AMZN
GOOGL
BLK
JPM
V
DIS
NFLX



SCATTER PLOT

- The plot illustrates the trade-off between risk and return for different portfolio compositions. Investors can use this plot to visualize how changing the asset allocation affects the risk-return profile of the portfolio.

MARKOWITZ'S MEAN VARIANCE OPTIMIZATION

Markowitz's mean-variance optimization is a method to build an optimal portfolio by balancing expected return and risk. It considers assets' expected returns, their correlation, and seeks to find the best mix of assets to maximize returns for a given level of risk, or minimize risk for a given level of return. This optimization process results in a set of portfolios on the efficient frontier, which represent the best risk-return trade-offs. It's a fundamental concept in modern finance, guiding portfolio construction by diversifying across assets with different risk-return profiles.

GLOBAL MINIMUM RISK

The global minimum risk portfolio refers to the portfolio that offers the lowest possible level of risk among all possible portfolios in the investment universe. In the context of Markowitz's mean-variance optimization, it represents the portfolio with the smallest variance or standard deviation of returns, regardless of its expected return.

The global minimum risk portfolio can be obtained by setting the target risk level to the lowest possible value (e.g., 0) in the **optimal_portfolio()** function. This will find the portfolio allocation that minimizes risk without considering expected return. Alternatively, one can iteratively search for the portfolio with the smallest risk among a range of risk levels.

DERIVATIVE

Obj : Max $\mu^T w$

s.t

$$w^T C w = R \quad \text{— (1)}$$

$$O^T w = 1$$

$$L(w, \lambda) = \mu^T w - \lambda_1 (w^T C w - R) - \lambda_2 (O^T w - 1)$$

$$\frac{\partial L}{\partial w} = \mu - C w \lambda_1 - \lambda_2 O = 0 \quad \text{— (2)}$$

$$w^T C w = R \quad \text{— (3)}$$

$$O^T w = 1 \quad \text{— (4)}$$

$$w = \frac{C^{-1}(\mu - \lambda_2 O)}{\lambda_1} \quad \text{— (5)}$$

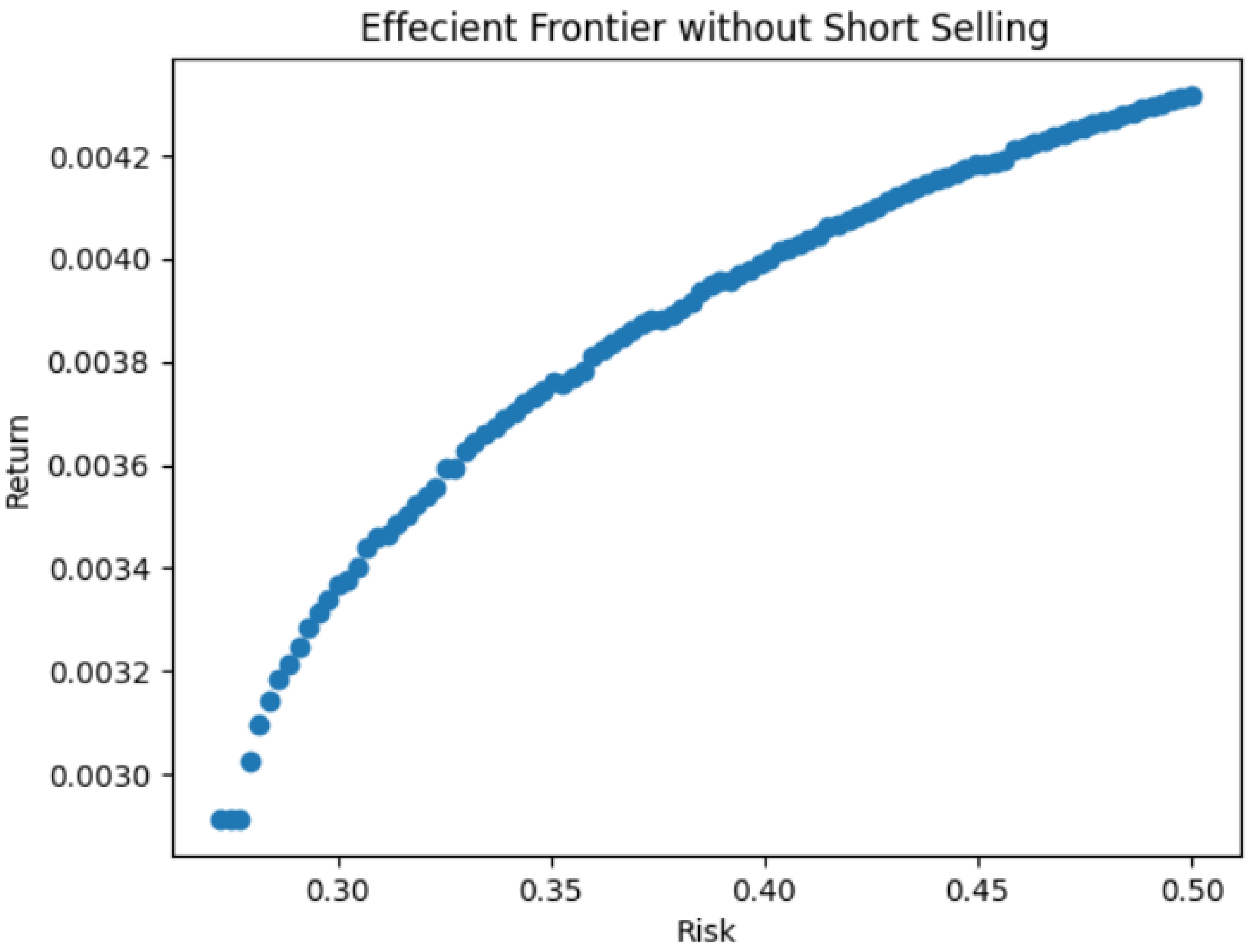
from eq^n (4) and (5)

$$\boxed{\begin{gathered} O^T C^{-1} (\mu - \lambda_2 O) = \lambda_1 \\ \left[\frac{C^{-1}(\mu - \lambda_2 O)}{\lambda_1}\right]^T C \left[\frac{C^{-1}(\mu - \lambda_2 O)}{\lambda_1}\right] = R \end{gathered}}$$

EFFICIENT FRONTIER

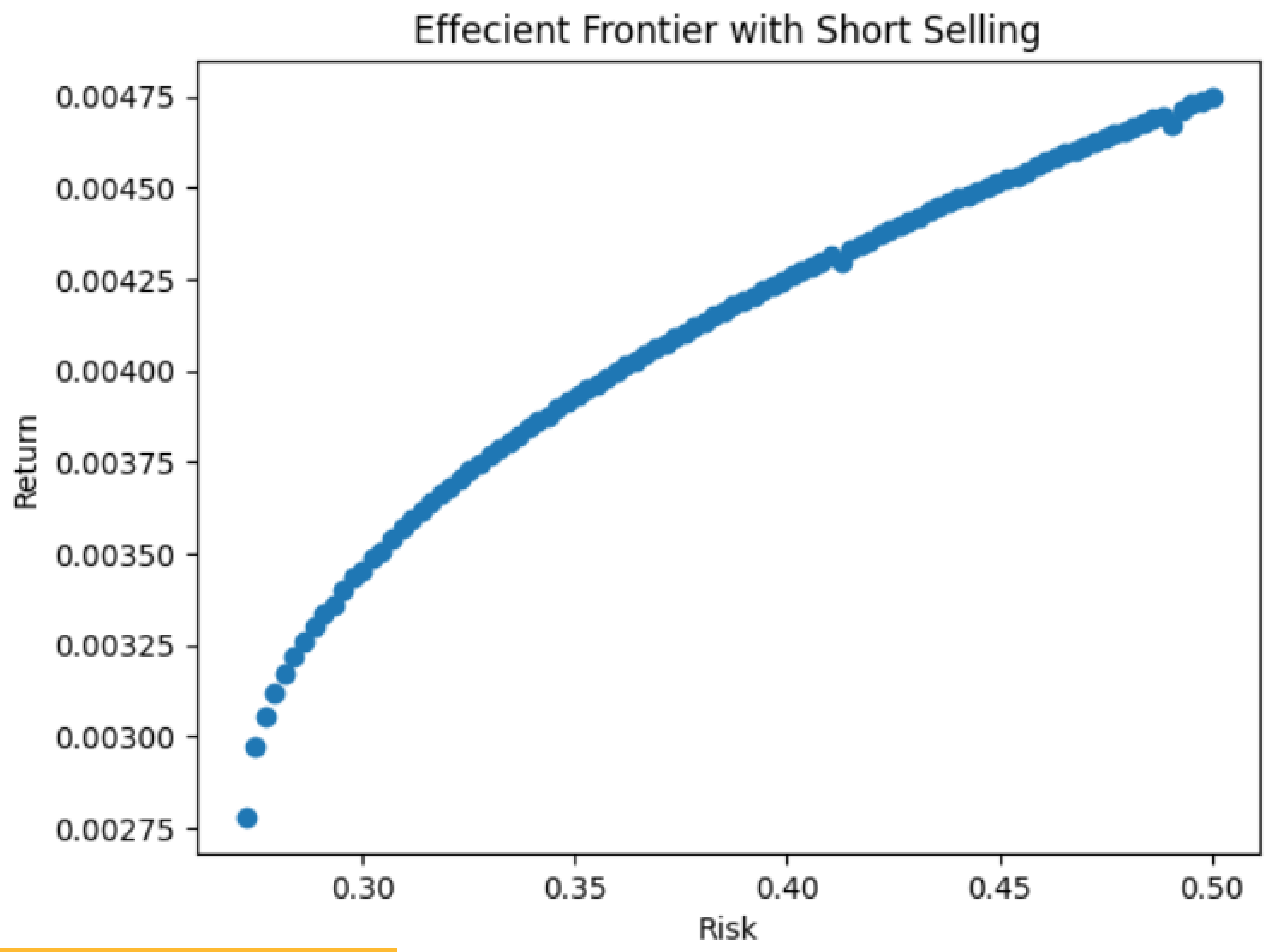
The efficient frontier is a concept in portfolio theory that represents a set of optimal portfolios that offer the highest expected return for a given level of risk or the lowest risk for a given level of expected return. Portfolios that lie on the efficient frontier are considered efficient because they provide the best risk-return trade-offs.

In the context of Markowitz's mean-variance optimization, the efficient frontier is determined by systematically varying the allocation of assets in the portfolio and calculating the expected return and risk (variance) of each portfolio.



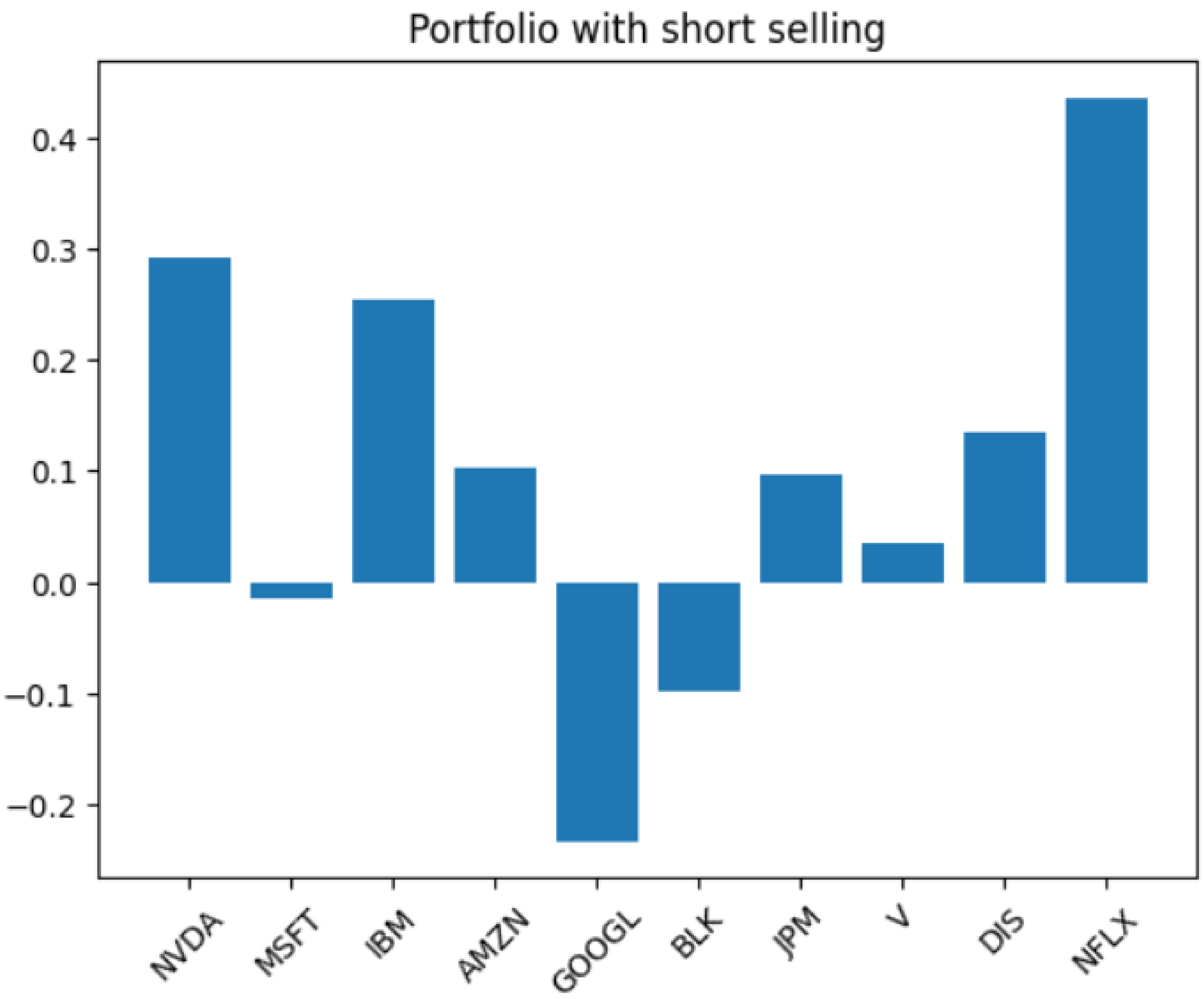
EFFECIENT FRONTIER WITHOUT SHORT SELLING

- This plot is generated using the results of the mean-variance optimization function.
- It represents the set of optimal portfolios that maximize returns for a given level of risk (variance) without allowing short selling.
- The curve depicts the efficient frontier, showing the highest possible returns for any given level of risk. Portfolios lying on this curve are considered efficient because they offer the best risk-return trade-offs without short selling.



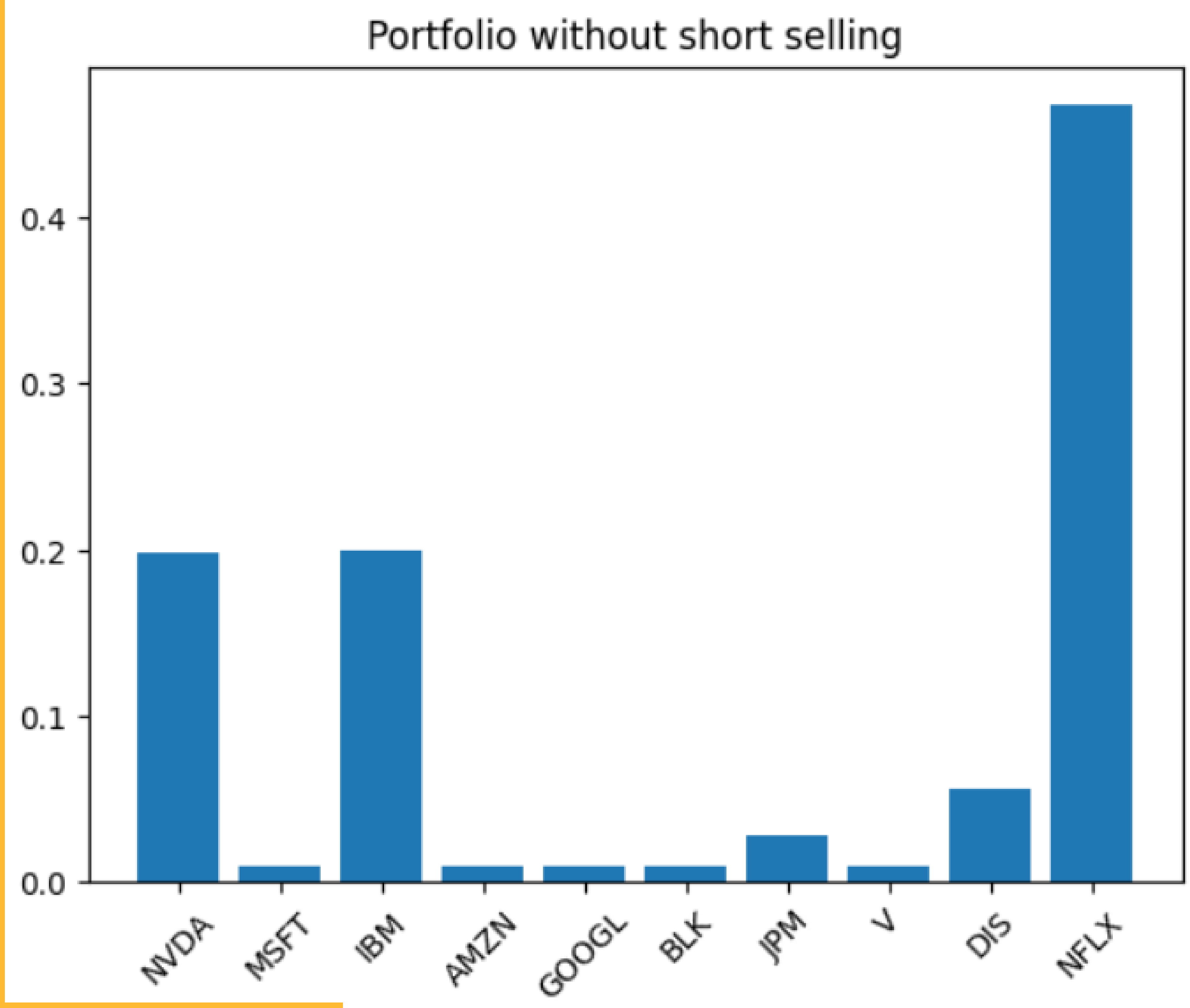
EFFECIENT FRONTIER WITH SHORT SELLING

- Similar to the previous plot, this one represents the efficient frontier but allows for short selling.
- Short selling refers to the practice of borrowing assets and selling them with the intention of buying them back at a lower price, profiting from the price decrease.
- Allowing short selling expands the feasible set of portfolios, potentially leading to higher returns or lower risk compared to portfolios without short selling.



PORTIFOLIO WITH SHORT SELLING

- This plot displays the optimal portfolio weights for a specified level of risk when short selling is allowed.
- Each bar represents the weight of a particular asset in the optimized portfolio.
- Investors can use this plot to understand the allocation of their investment across different assets in the portfolio, considering the impact of short selling on asset weights.



PORTFOLIO WITHOUT SHORT SELLING

- Similar to the previous plot, this one shows the optimal portfolio weights, but without allowing short selling.
- It provides insight into the allocation of assets in the portfolio when short selling is restricted, potentially leading to different weight distributions compared to portfolios with short selling.

A BRIEF DISCUSSION OF THE TRADE-OFF BETWEEN RISK AND RETURN

In portfolio choices guided by Markowitz's mean-variance optimization, there exists a trade-off between risk and return. This trade-off is illustrated by the efficient frontier, where portfolios with higher expected returns typically come with higher levels of risk, and portfolios with lower risk tend to offer lower returns. Investors must balance their risk tolerance and investment objectives when selecting portfolios, as opting for higher returns often requires accepting greater levels of risk, while prioritizing capital preservation may entail settling for lower returns. Markowitz's framework provides a systematic approach to navigate this trade-off, enabling investors to construct portfolios that align with their risk preferences and financial goals.

THE LIMITATIONS OF MARKOWITZ OPTIMIZATION AND ITS REAL-WORLD APPLICATIONS.

- **Markowitz portfolio optimization**, while a powerful tool for portfolio construction, is based on several simplifying assumptions that may not fully capture the complexities of real-world financial markets. Here are three drawbacks of Markowitz's risk-return theory:

- **Normal Distribution Assumption**: Markowitz assumes that asset returns follow a normal distribution, which implies that returns are symmetrically distributed around the mean. In reality, asset returns often exhibit non-normal distributions, such as skewness. This can lead to underestimation or overestimation of risk and may result in suboptimal portfolio allocations.

- **Constant Expected Returns and Volatility**: Markowitz assumes that expected returns and volatilities of assets are constant over time. However, in practice, asset returns and volatilities can vary significantly due to changes in market conditions, economic factors, and investor sentiment. Ignoring these fluctuations can lead to inaccurate estimates of expected returns and risk, resulting in suboptimal portfolio decisions.

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--> We can't include the risk-free assets in our portfolio as the variance will be zero, which will result in a non-invertible covariance matrix.

--> We are not able to define a exact relation between the optimal return and risk of the portfolio, we need simulations to make the efficient frontier

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THANK YOU