Personalized Driver/Vehicle Lane Change Models for ADAS

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***Abstract*—Lane changes are stressful maneuvers for drivers, particularly during high-speed traffic flows. Advanced driver-assistance systems (ADASs) aim to assist drivers during lane change maneuvers. A system that is developed for an average driver or all drivers will have to be conservative for safety reasons to cover all driver/vehicle types. Such a conservative system may not be acceptable to aggressive drivers and could be perceived as too aggressive by the more passive drivers. An ADAS that takes into account the dynamics and characteristics of each individual vehicle/driver system during lane change maneuvers will be more effective and more acceptable to drivers without sacrificing safety. In this paper, we develop a methodology that learns the characteristics of an individual driver/vehicle response before and during lane changes and under different driving environments. These characteristics are captured by a set of models whose parameters are adjusted online to fit the individual vehicle/driver response during lane changes. We develop a two-layer model to describe the maneuver kinematics. The lower layer describes lane change as a kinematic model. The higher layer model establishes the kinematic model parameter values for the particular driver and represents their dependence on the configuration of the surrounding vehicles. The proposed modeling framework can be used as a kernel component of ADAS to provide more personalized recommendations to the driver, increasing the potential for more widespread acceptance and use of ADAS. We evaluated the proposed methodology using an actual vehicle and three different drivers. We demonstrated that the method is effective in modeling individual driver/vehicle responses during lane change by showing consistency of matching between the model outputs and raw data.**

***Index Terms*—Adaptation models, intelligent vehicles, machine learning, man machine systems.**

I. Introduction

DRIVING through the traffic network of a modern city can be stressful and often challenging due to high variations of vehicle speeds, congestion, and accidents. The latest trend to improve this urban driving experience is equipping cars with advanced driver-assistance systems (ADASs). These systems aim to make driving safer and more comfortable [1]. One particular area of interest for such systems is the lane change maneuver. This is the maneuver that is stressful for drivers as it involves changes in the longitudinal speed and involves lateral speed and movement in the presence of other vehicles that are also moving. The National Highway Traffic Safety Administration (NHTSA) estimates that lane changing and merging collisions made up about 5% of all police-reported collisions in 2007 and accounted for about 0.5% of all fatal cases [2]. These particular numbers do not indicate the significance of the problem. However, studies suggest that risky lane changes result in unstable traffic flow [3] and lane changing/merging accidents contribute to approximately 10% of all crash-caused traffic delays [4].

The major contributing factor to lane change accidents is failure to detect the other vehicle. It was found to be the main contributing factor in approximately 75% of lane change crashes [5]. NHTSA reports that 78% of lane change accidents occur at speeds smaller than 25 km/h [6], and on average, it takes 1.5 s to cross into the adjacent lane from the time of lane change initiation [7]. Other studies show that the auditory and visual reaction time of an average person lies in the range of 180–200 ms [8]. These facts indicate that there is enough time for ADAS to notify the driver about potentially dangerous situations.

The assistance system can be effective only if it captures personal driving styles and the dynamics of the driver/vehicle system. Most ADASs are designed with the average driver characteristics in mind and are difficult to manually tune to fit the characteristics of a particular driver. An ADAS designed for an average driver may be found to be too conservative and annoying to more aggressive drivers and too aggressive to more passive drivers. This in turn may inhibit most drivers from using an ADAS and taking advantage of its benefits. To provide automatic tuning, ADASs must incorporate a model that includes variables of interest and can automatically adapt to the characteristics of a particular vehicle and driver. When tuned, the model can be effectively used by ADASs to provide the driver with personalized assistance.

In this paper, we are not proposing a new ADAS, but we are developing a modeling framework that takes into account the driver/vehicle response before and during the maneuver that can be utilized by ADAS to provide personalized assistance to drivers. We consider the way the driver prefers to adjust his/her longitudinal position with respect to the surrounding vehicles to find a gap to perform the lane change, the kinematic characteristics of the maneuver, and the driver's acceptable gaps to initiate the maneuver (see Fig. 1). The acceptable gaps

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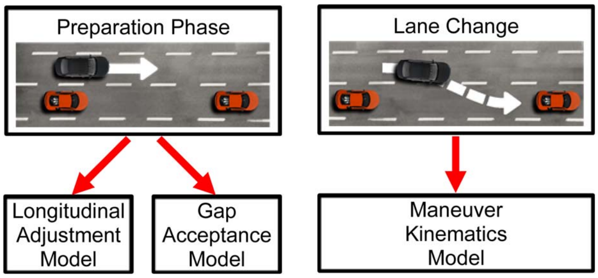

Fig. 1. Models associated with the lane change maneuver.

describe the distances between the subject vehicle and the surrounding vehicles for which the driver finds performing a lane change maneuver acceptable [9]–[11]. When the driver finds the gaps acceptable, he/she initiates the maneuver.

There are two common approaches to the trajectory-modeling problem, i.e., stochastic and kinematic. Stochastic models are able to establish a relationship between hazards imposed to the subject vehicle and the lane change trajectory shape. Possible modeling techniques are Stochastic Switched AutoRegressive eXogenous model (SS-ARX) [12], [13]; hidden Markov model [14]–[16]; neural network [13], [17]–[19]; fuzzy system [19]–[21]; Bayesian network [22]–[24]; support vector machine [25]; and Gaussian mixture model (GMM) [18], [26]. If properly trained, such models are accurate, flexible, and adjustable for different personalizing aspects of driving. However, when stochastic models are directly used to model a highly nonlinear function (such as driver's behavior or vehicle trajectory), these methods might lack physical meaning for their model parameters. On the contrary, kinematic models describe the lane change maneuver in a form of equations with physical meaning, e.g., lateral acceleration versus time. The two most widely used kinematic models are polynomial [27]–[29] and sinusoidal [30], [31]. These models provide meaningful kinematic information (trajectory shape, acceleration curves, etc.) about the maneuver. However, their rigid structure does not take into account all aspects of driving behavior, such as dependence of trajectory shape on speed and surrounding traffic configuration. In addition, a system that depends on a human response is nondeterministic by nature. Therefore, an attempt to describe it solely with a kinematic model could not be effective.

In this paper, we propose a new maneuver trajectory model that combines the advantages of both modeling methods. The proposed model consists of two layers: stochastic (GMM) and kinematic (sinusoidal lane change model). We use driving performance measurements along with information about the surrounding vehicles as model inputs. We focus our effort solely on developing models that can be used as a kernel of the ADAS to provide personalized recommendations to the driver based on his/her lane changing characteristics. We evaluated the proposed methodology using an actual vehicle and real-time experiments involving a considerable number of real-time data from different lane change maneuvers. The experiments are extended to three different drivers to demonstrate the difference in lane change characteristics between different people. We demonstrated that the method is effective in modeling individual driver/vehicle responses during lane change by showing consistency of matching between the model outputs and raw data. In our work, we assume that the system clearly identifies the driver behind the wheel. This can be implemented by asking the driver to identify himself before he starts the trip. This feature is already available in most new cars and allows different drivers (usually up to three) to specify themselves so that the car seat and side mirrors are adjusted automatically. Other more sophisticated methods include face recognition [32] or performing an analysis of the driver's style [33]–[35] along the course of the trip.

This paper is organized as follows. Section II presents our approach of how to describe the driver's characteristics for a premaneuver longitudinal adjustment and gap acceptance to initiate the maneuver. Section III describes the technique used for lane change trajectory modeling. The results of the experiments are given in Section IV, and conclusions are presented in Section V.

II. Longitudinal Adjustment and Acceptable Gaps

Before we describe our approach, we establish the formal definitions associated with a lane change maneuver.

A. *Definitions*

The lane change maneuver belongs to the category of tactical level driving tasks along with vehicle following, turning, overtaking, and other short-term tasks [36]. Alternatively, the other two task levels are strategic (i.e., destination and route selection) and operational (i.e., pedals and steering wheel use). The NHTSA defines lane change as "a driving maneuver that moves a vehicle from one lane to another where both lanes have the same direction of travel."

The maneuver can be classified as mandatory lane change (MLC) when the driver must leave the current lane or discretionary lane change (DLC) when the driver performs lane change to improve driving conditions [10]. The type of lane change affects the driver's behavior. For instance, the driver may tolerate smaller gaps between vehicles in the destination lane for an MLC than for a DLC maneuver due to the imperative nature of MLCs.

Merging maneuvers occur when entering into the main lane from a ramp or when the two lanes merge into one. It is a representative case of an MLC [11]. The merging scenario can be detected from the GPS data, e.g., when the vehicle is approaching a ramp. Similarly, other lane change situations such as an MLC before the intersection to perform a turn can be anticipated and classified. In this paper, we focus on describing a general class of lane change maneuvers. Particular scenarios can be derived by placing conditions on the model variables and introducing other modeling parameters.

We establish definitions for the start and endpoints of lane changing. Correct detection of the lane change initiation point is crucial because it defines when a driver makes the final decision to start the maneuver based on his judgment of the road situation. NHTSA proposes several indicators of the lane change initiation [7]. We adopt the definition which states that lane change initiation is the point where the vehicle begins lateral movement toward the destination lane. The lane change initiation point, endpoint, and trajectory shape can be effectively determined from yaw rate, lateral acceleration, and steering angle only on a straight flat road. This is a substantial limitation on the effectiveness of the lane change assist system. In contrast, data of lane markings can provide effective maneuver detection in all cases for the following reasons:

- Missing lane-marking cases are extremely rare on the urban roads and highways in the United States.
- Modern video processing algorithms allow effective lane-marking recognition, even in the case of partially missing or degraded markings [37]–[39].

In addition, if lane markings are absent on a multiple-lane road, then the car's alignment in the lanes, and therefore the lane change maneuver, is *ad hoc*, and no effective lane change assistance can be provided in this case.

The accuracy of the initiation point detection can be improved if lane-marking data are augmented with visual data on the driver's gaze status. In this case, the initiation point could be marked when the driver returns his/her gaze to the forward view upon glancing at the mirrors or side windows before initiating the maneuver. The endpoint of the maneuver is not as important as the initiation point in terms of safety analysis, but when wrongly detected, it may distort the information regarding the completion time and the shape of the maneuver. By analogy to the initiation point, we say that the lane change ends when the lateral vehicle position in the adjacent lane settles with respect to the lane.

We utilize the data of surrounding vehicles as a factor that affects a driver's behavior. We use longitudinal projection of the relative distance between the subject and another vehicle of interest (we denote it as D_X) to characterize the relative positions of the vehicles. In addition, we use the longitudinal projection of the relative speed between the subject and the vehicle of interest V_R.

B. Longitudinal Adjustment

If the gap in the destination lane is not acceptable for the driver to start the maneuver, then he/she might perform longitudinal adjustment. There are two strategies to adjust the position, i.e., slow down with respect to the vehicles in the destination lane to move in behind them or speed up with respect to these vehicles to find a spot ahead of them. To analyze longitudinal adjustment, other studies [40] assume that surrounding vehicles are at a constant speed. This assumption is not appropriate for a system that will provide recommendations to the driver in real-world situations. Absolute values of longitudinal speeds are not as important as their relation to the speed of the vehicles in the destination lane. We characterize the way the driver performs longitudinal adjustment with three parameters. The first two describe a portion of the adjustments and their direction with respect to the overall number of lane changes

$$\alpha = \frac{n^+ + n^-}{N}, \qquad \beta = \frac{n^+ - n^-}{n^+ + n^-} \tag{1}$$

where n^+ is the number of positive longitudinal adjustments before the lane change maneuver; n^- is the number of negative longitudinal adjustments before the lane change maneuver; and N is the total number of lane change maneuvers. Coefficient α describes how often a driver performs longitudinal adjustment before the maneuver, whereas β shows which direction of adjustment is more preferable to the driver. The third parameter is the relative speed with respect to the vehicle in the destination lane, with respect to which the driver performs longitudinal adjustment. We store the peak value of the relative speed for every longitudinal adjustment before the maneuver. From the collected speed values, we construct distribution curves (see Fig. 2) for a particular driver that, along with coefficients in (1), can be used to describe the individual driver's longitudinal adjustment characteristics before the lane change maneuver.

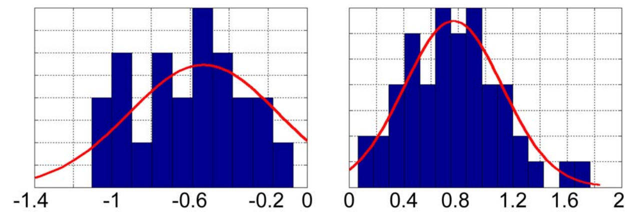

Fig. 2. Example of distribution curves of the peak relative speed during longitudinal adjustments. (a) Negative adjustment. (b) Positive adjustment.

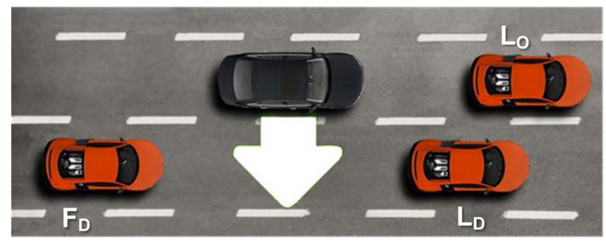

Fig. 3. Pre-lane change traffic configuration.

C. Gap Acceptance

The driver initiates the maneuver at the moment when he/she feels the positions of the surrounding vehicles are acceptable for his/her comfort and safety standards. The gap acceptance depends on personal preferences, current speed, urgency for changing lanes, direction of the maneuver (left or right), and other factors. To model the gap acceptance for an individual driver, we take into account three surrounding vehicles that affect a driver's decision on adjusting his/her longitudinal position and initiating lane change (see Fig. 3): the leading vehicle in the origin lane (L_O), the leading vehicle in the destination lane (L_D), and the following vehicle in the destination lane (F_D). The latter two gaps are critical variables that affect the driver's decision to initiate lane change [10], whereas information on the gap acceptance with respect to the leading vehicle in the origin lane is important for providing personalized longitudinal adjustment recommendation to find the gap in the destination lane. The critical gaps are assumed to have a log-normal distribution [10], whereas the gap to the leading vehicle in the origin lane has a normal distribution as a noncritical variable. These assumptions are based on analysis of real-time data not included in this paper.

To initiate the maneuver, the driver must evaluate the surrounding traffic environment. Judgment is made based on factors such as relative distance and relative speed with respect to the surrounding vehicles [11]. In our model, we use the longitudinal projection of the relative distance D_X between the subject and one of the three vehicles of interest as a dependent variable. That is, we aim to model the distance that is accepted by the driver depending on independent variables, i.e., factors that might affect the driver's decision. One of such factors is the relative speed between the subject and the vehicles of interest. A driver might prefer to keep a larger gap between his/her vehicle and another if the other vehicle is closing the gap between them. Similar to the relative distance, we use the longitudinal projection of the relative speed V_R between the subject and one of the three vehicles of interest. Subject speed V also affects the gap acceptance: Drivers usually accept smaller gaps when the traffic density is high and the speeds are low [9]. Direction of lane change (left or right) is another parameter that we take into account in the model. A driver might demonstrate different gap acceptance behaviors depending on whether he/she is changing lanes to the left or to the right lane [41]. Another important factor that affects gap acceptance is whether the lane change is mandatory or discretionary. Drivers are ready to tolerate lower gaps for MLCs than for DLCs [42].

It is hard to recognize in advance which type of lane change a driver is about to perform. We propose an idea that the blinker status might be considered as a reliable indicator of lane change type. Its usage in advance is correlated with the urgency of the lane change maneuver [41]. That is, when the driver switches on the blinker in advance before the lane change, there is a high chance that he/she has a strong need to perform the maneuver. NHTSA reports that blinkers are used in no more than 45% of lane changes [43]. Moreover, some drivers have a habit of switching the blinker on when the maneuver has been already initiated [41]. Therefore, in majority of cases, if the blinker is activated in advance, it shows forethought demonstrating a stronger desire to change lanes. The lane change assist system can address this need by relaxing the threshold conditions associated with a driver's comfort zone for the lane change maneuver to find an empty spot in the destination lane as soon as possible.

We take into account the previously described factors to develop the appropriate models for each driver/vehicle system. We view the acceptance gap separately for the three surrounding vehicles of interest and model them in the form of a linear regression with independent variables of the subject speed V, the relative speed V_R, and two additional dichotomous (categorical) variables: blinker status b (0: off; 1: on) and the direction of the lane change d (0: left; 1: right). The critical gaps for the leading and the following vehicles in the destination lane are assumed to have a log-normal distribution [42]. We establish the regression model for the vehicles in the destination lane as follows:

$$\ln(D_X) = \gamma + \delta V + \varepsilon \max(0, V_R) - \zeta \min(0, V_R) - \eta b - \theta d. \quad (2)$$

Parameter γ represents the y-intercept term: It is the gap acceptance value when all explanatory variables are set to be zero. The values of η and θ are the differential intercept coefficients that represent the effect of blinker status and the direction of the lane change on the gap acceptance. The value of δ characterizes the gap acceptance dependence on the subject speed. Coefficients ε and ζ represent the slopes corresponding to the gap acceptance dependence on positive and negative relative speeds V_R, respectively. We split the relative speed term into positive and negative intervals because the magnitude of its effect on the gap acceptance is different depending on the sign. For instance, other studies indicate that, for the lead vehicle in the destination lane, the influence of the relative speed on the gap acceptance is strongest when the vehicle is faster than the subject vehicle [10]. The necessary assumptions for applying the linear regression such as homoscedasticity (constant variance) and small correlation between the predictions are satisfied for all three gaps. Equation (2) can be written in vector form as $\ln(D_X) = x\psi$, where $x = (1, V, \max(0, V_R), -\min(0, V_R), -b, -d)$ is a vector of independent variables, and $\psi = (\gamma, \delta, \varepsilon, \zeta, \eta, \theta)^T$ is the parameter vector. If n data samples are available, we can combine them into a matrix $X = (x_1^T, x_2^T, \ldots, x_n^T)^T$. The corresponding measurement vector of gap acceptance is represented as $Y = (\ln(D_{X1}), \ln(D_{X2}), \ldots, \ln(D_{Xn}))$. We use the least squares method to determine the coefficients of the regression model as $\widehat{\Psi} = (X^T X)^{-1} X^T Y$.

To establish a similar regression model for the lead vehicle in the origin lane, we take into account the fact that the corresponding gap is a noncritical variable [42]. Therefore, we assume its distribution to be normal and omit the direction variable in the model to obtain

$$D_X = \gamma + \delta V + \varepsilon \max(0, V_R) - \zeta \min(0, V_R) - \eta b. \quad (3)$$

We use the estimated mean of acceptable gaps for the particular driver along with the estimation of the variance to assess if the actual gaps are acceptable for the driver to change lanes. The acceptance probability of a gap is calculated as a probability that the actual distance D^{actual} to the vehicle is greater than the preferred gap $D^{\text{preferred}}$. We view gap acceptance separately for the vehicles of interest. Consequently, the probability that the driver will find the gaps to the vehicles in the destination lane acceptable to initiate the maneuver is given as a product of acceptance probabilities for the separate vehicles [10]

$$P(change) = P(accept\ L_D) \cdot P(accept\ F_D) = P[D_{XL_D}^{\text{actual}} > D_{XL_D}^{\text{preferred}}] \cdot P[D_{XF_D}^{\text{actual}} > D_{XF_D}^{\text{preferred}}]. \quad (4)$$

During the course of a drive, the system collects data on the gaps at the initiation point of lane change maneuvers to estimate the parameters of the linear regression. Until the system collects enough data to establish the corresponding coefficients, we use the mean and variance of the gaps, disregarding their dependence on speed, relative speed, blinker status, and lane change direction. This approach allows for a quick initial style evaluation, with accuracy improvement when the linear regression results are generated.

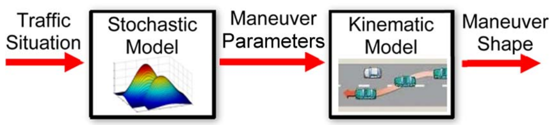

Fig. 4. Two-layer model structure.

The knowledge of a driver's acceptance gaps and the way he/she performs longitudinal adjustment before the maneuver can be used to provide personalized recommendations. For instance, an optimization problem can be formulated to determine an appropriate longitudinal adjustment to find an opening in the destination lane [44].

III. Modeling of Lane Change Kinematics

In addition to the way a driver performs longitudinal adjustment before the lane change maneuver and his/her gap acceptance at the maneuver initiation point, it is important to know the way a driver performs the maneuver itself. We propose a two-layer model structure that combines both stochastic and kinematic modeling techniques (see Fig. 4). The lower layer describes the lane change maneuver in the form of a kinematic model. The higher layer reflects the kinematic model parameter's dependence on a particular driver's style and the surrounding traffic. Such a structure provides model adaptation and, at the same time, contains physically meaningful parameters.

We choose GMM and sinusoidal lane change model as the stochastic and kinematic counterparts, respectively. The GMM is chosen because its effectiveness has been demonstrated in modeling other driving tasks [45], [46]. The sinusoidal lane change model is selected because of its simplicity (only two parameters) and yet adequate representation of the lane change. The small number of model parameters results in a low computational cost, which is important when implementing the system in real time.

A. Sinusoidal Lane Change Model

The sinusoidal lane change kinematic model establishes a relationship between time and lateral acceleration during the maneuver

$$a_{\mathrm{lat}}(t) = \frac{2\pi H}{t_{\mathrm{lat}}^2} \sin\left(\frac{2\pi}{t_{\mathrm{lat}}} t\right) \tag{5}$$

where a_{lat} is the lateral acceleration, t is the time from the beginning of the maneuver, H is the final lateral displacement, and t_{lat} is the time needed to complete the lane change maneuver. Parameter t_{lat} characterizes the driver's style. The lateral acceleration in (5) is associated only with the lane change maneuver. That is, if performed on a curvy road, the total lateral acceleration of the car consists of two components: lateral acceleration associated with the lane change maneuver and the centrifugal acceleration. In addition, uneven road surfaces might introduce another component of acceleration. The profile of the lateral acceleration associated with the lane change is estimated using the vehicle position relative to the lane markings. We note that the lane change behavior of the driver in a straight-line lane may be different than that in a curvy lane. For simplicity, in our work, we do not model the effect of the road curvature on the trajectory shape. We believe, however, that the same methodology can be applied to obtain personalized models that take into account the curvature of the road.

We chose the sinusoidal model rather than the polynomial model because data of the initiation and endpoints of the maneuver are sufficient for calculating the parameters of the sinusoidal model. Knowledge of the initiation and endpoints allows us to calculate the duration and total lateral displacement. That is, we avoid relying on the car's lateral acceleration curve, which might be corrupted by signals not related to the lane change maneuver. Lateral acceleration due solely to the lane change maneuver can be obtained as the second derivative of the lateral position with respect to lanes. However, a second derivative is not a desirable estimation either. Hence, we prefer the sinusoidal model that does not require the knowledge of the acceleration curve to determine its parameters. When the model coefficients are calculated, one can estimate the curves of the lateral acceleration, speed, and position trajectory associated with a lane change. This information can be used to evaluate whether the conditions are safe and appropriate for the driver's style, and this, in return, can be used to provide recommendations to the driver.

We calculate the parameters t_{lat} and H for each particular instance of lane change. They are stored in a data vector along with the subject vehicle speed and the distances to the leading vehicle in the origin lane, the leading vehicle in the destination lane, and the following vehicle in the destination lane. These values are captured at the time point of the lane change initiation. We later use a set of such vectors to train the GMM.

B. GMM

We use GMM to adjust the kinematic model parameters in (5) to a particular driver's style and to establish their dependence on the surrounding traffic environment. We define the traffic situation vector of 4×1 size that contains the parameters that affect the duration of the lane change maneuver as $s = [V, H, D_{XL_D}, D_{XF_D}]^T$, where V is the subject vehicle speed, H is the total lateral displacement during the lane change maneuver, and D_X is the longitudinal projection of the relative distance between the subject and another vehicle of interest. The combined model layout and its signals are shown in Fig. 5. We do not consider the distance to the leading vehicle in the origin lane. Data analysis indicates that this parameter is noncritical as it does not affect the duration of the lane change maneuver. This shows that, when initiating the maneuver, the driver's attention switches solely to the vehicles in the destination lane. Another parameter that might affect the duration of the lane change is the road curvature. For instance, on a left-curved road, it might take the driver more time to perform the lane change to the left due to centrifugal acceleration already applied to the car. If the measurement of the road curvature is available, it should be added to the model. For the sake of simplicity, we did not include it in this paper.

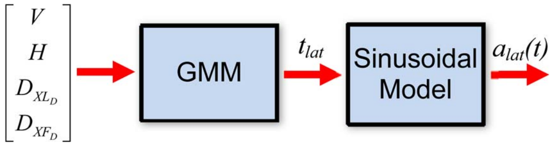

Fig. 5. Lane change kinematic model layout.

We define an augmented vector of 5×1 size to also contain the time required for the driver to complete the lane change maneuver as $r = [s, t_{\mathrm{lat}}]^T$. We represent the joint probability density function between the prediction vector and the duration of the maneuver t_{lat} in the form of the multivariate Gaussian distribution functions' weighted sum

$$\begin{aligned} p(d_t|\omega_i, \mu_i, \Sigma_i) &= \sum_{i=1}^{M} \omega_i N_i(d_t|\mu_i, \Sigma_i) \\ &= \sum_{i=1}^{M} \omega_i \frac{1}{(2\pi)^{D/2} |\Sigma_i|^{1/2}} \\ &\quad \times \exp\left(-\frac{1}{2}(d_t - \mu_i)^T (\Sigma_i)^{-1}(d_t - \mu_i)\right) \end{aligned} \tag{6}$$

where $N_i(r_t, \mu_i, \Sigma_i)$ is the multivariate Gaussian distribution of dimension D (in our case, $D = 5$), with mean vector μ_i of size $D \times 1$ and covariance matrix of Σ_i size $D \times D$; M is the number of Gaussian components; and ω_i represents the mixture weights whose sum is equal to 1. We refer to (6) as the GMM [47]. Let us denote the GMM parameters as $\theta = \{\omega_i, \mu_i, \Sigma_i\}$, $i = 1, \ldots, M$. Then the GMM in (6) is denoted as $p(r_t|\theta)$. We can estimate the model parameters given a data set for multiple-lane-change maneuvers. To perform accurate GMM training, we use sample entries where the leading and the following vehicles in the destination lane ($\mathrm{L_D}$ and $\mathrm{F_D}$) are present when the maneuver is initiated.

Given a data set for a particular driver/vehicle, the GMM parameters θ can be estimated using the maximum-likelihood (ML) method. Given the model configuration and training data set $R = \{r_1, r_2, \ldots, r_N\}$, which consists of data vectors with time intervals t denoted by natural numbers, the goal of the ML estimator is to find the set of model parameters θ that maximizes the likelihood of the GMM function. We assume that the data vectors are independent random variables, and then, the log-likelihood function has the following form:

$$l(\theta) = \sum_{t=1}^{N} \log\left(p(r_t; \theta)\right). \tag{7}$$

Due to the function's nonlinearity with respect to the set of parameters θ, direct maximization of the log-likelihood function is impossible. Therefore, we use the iterative version of the expectation–maximization (EM) algorithm [49], which guarantees a monotonic increase in the model's likelihood value at each step of iteration with the objective of finding the set of parameters θ that maximizes (7).

Let the EM estimate of $\theta = \{\omega_i, \mu_i, \Sigma_i\}$ at instant k be $\widehat{\theta}^k$. The update of $\widehat{\theta}$ is generated as follows. At every iteration, we calculate the *a posteriori* probability for each component i by using the GMM parameter set from the previous iteration $\widehat{\theta}^k$

$$\Pr_i^{l+1}(r_t) = \frac{\hat{\omega}_i^k N_i(r_t; \widehat{\mu}_i^k, \widehat{\Sigma}_i^k)}{\sum_{j=1}^{M} \widehat{\omega}_j^k N_j(r_t; \widehat{\mu}_j^k, \widehat{\Sigma}_j^k)}. \tag{8}$$

Then, the distribution parameters are calculated as follows:

$$\begin{aligned} \widehat{\omega}_i^{k+1} &= \frac{1}{N} \sum_{t=1}^{N} \Pr_i^{k+1}(r_t) \\ \widehat{\mu}_i^{k+1} &= \frac{\sum_{t=1}^{N} \Pr_i^{k+1}(r_t) r_t}{\sum_{t=1}^{N} \Pr_i^{k+1}(r_t)} \\ \widehat{\Sigma}_i^{k+1} &= \frac{\sum_{t=1}^{N} \Pr_i^{k+1}(r_t) \left(r_t - \widehat{\mu}_i^{k+1}\right) \left(r_t - \widehat{\mu}_i^{k+1}\right)^T}{\sum_{t=1}^{N} \Pr_i^{k+1}(r_t)}. \end{aligned} \tag{9}$$

At the end of each iteration, we update the value of the log-likelihood function $l(\widehat{\theta}^{k+1})$

$$l\left(\widehat{\theta}^{k+1}\right) = \sum_{t=1}^{N} \log\left(p(r_t; \widehat{\theta}^{k+1})\right). \tag{10}$$

The iteration (8)–(10) is repeated until the increase in the log-likelihood function becomes smaller than a specified value, i.e., $l(\widehat{\theta}^{k+1}) - l(\widehat{\theta}^k) < \alpha$. In this paper, we set the value of the threshold α to be equal to 10^{-10}. We found this number to be appropriate for the task. A higher value would reduce the accuracy fit of the model, whereas a lower value would increase the computation cost without any significant improvement in accuracy.

C. Maneuver Shape Prediction

After the model is trained, it can be used to predict lane change duration t_{lat}, given the traffic situation vector s. We approximate the total lateral displacement parameter H as the width of the lane in which the subject vehicle is currently. If the width cannot be estimated because of a lack of right or left lane markings, then we assign it to be 3.7 m, which is a standardized lane width according to the U.S. Interstate Highway System. We perform an initial maneuver feasibility check by assessing the longitudinal distance D_X to the leading and the following vehicles in the destination lane to determine if there is enough space to fit the subject vehicle. If the check is passed, we determine the expected lane change duration as the one that maximizes the GMM probability density function

$$t_{\mathrm{lat}} = \arg\max_{t_{\mathrm{lat}}} p(s, t_{\mathrm{lat}}; \widehat{\theta}) \tag{11}$$

where $\widehat{\theta}$ is a vector of the estimated model parameters. If one or more of the two surrounding vehicles ($\mathrm{L_D}$ and $\mathrm{F_D}$) are absent, then we substitute their corresponding entries with fictitious far-distanced cars. In this case, we assign the relative distance D_X to be a large number (100 m).

The GMM establishes dependence between lane change trajectory parameters and traffic environment for a particular driver. However, it requires a large number of samples for its training. Until there are enough data to train the GMM, we establish probability density functions of the kinematic parameters associated with lane changing without assuming any dependence between them. This allows rough but quick insight into a driver's style; hence, the corresponding ADAS can be supported with some individual driver characteristics as soon as possible. When available, the GMM will replace the rough estimates with more precise estimates to provide a more accurate and personalized description of a driver's style. The expected maneuver duration is then substituted to (5) to predict the lane change acceleration curve, which, in turn, can be integrated to obtain lateral velocity and trajectory with respect to the lanes.

The gap acceptance, longitudinal adjustment, and maneuver kinematic models can be used as a kernel of a personalized ADAS. The system should evaluate the surrounding traffic environment in real time and provide the driver with recommendations if it is reasonable to initiate the maneuver. If the gaps are not acceptable, the system should provide longitudinal adjustment recommendations to the driver to find the gap in the destination lane. In addition to driver's acceptance, the system can perform supplementary safety evaluation of the maneuver. The duration and shape of the maneuver can be assessed by ADAS with respect to their feasibility and safety to provide appropriate recommendations to the driver. For instance, when the expected duration of the lane change is abnormally small or the peak lateral acceleration value is too large for a particular driver, the maneuver may be considered not desirable. In addition, the gaps in the destination lane can be also assessed from a safety perspective [40]. The design of ADAS, however, is outside the scope of this paper and is part of future work.

IV. Experiments

We used a customized vehicle to run experiments and collect on-road data. The vehicle is pre-equipped from the factory with sensors to measure the values of speed, lane markings, and blinker status. The data are accessed through a controller area network bus for real-time data processing and storage. In addition, the vehicle is equipped with side-facing radars and front- and rear-facing lidars (i.e., sensors that measure distance by illuminating objects with a laser). The sensors provide 360° coverage of object detection in a vicinity of up to 50 m and allow measurement of the relative distance, speed, and angle to the surrounding vehicles. The presented modeling framework is implemented as a real-time application on the vehicle built-in PC.

Three drivers participated in the experiments. They were of similar age (27, 28, and 31 years old), were the same gender (male), and had similar driving experience (4, 5, and 6 years). Prior to data collection, they had an opportunity to perform casual daily trips for several months without any restrictions or requirements to the trips to adapt to the vehicle and to feel comfortable demonstrating their natural driving behavior. The drivers performed 97 trips in total with the average duration of a trip lasting 51 min in the areas of Los Angeles and Palo Alto, CA, USA. The drivers were not restricted to a particular route and were given no restrictions, including the duration of the journeys. During driving, the onboard PC recorded collected data with a frequency of 30 Hz. The process of data acquisition and recording was hidden from the drivers; thus, the system did not intrude on the driving in any way or affect natural driving behavior. We should note that the use of three drivers was just to demonstrate that our method can easily distinguish between different driving styles. The number three is not used for any statistical inference or analysis. For each driver, however, we collected a large number of data to train and validate the models.

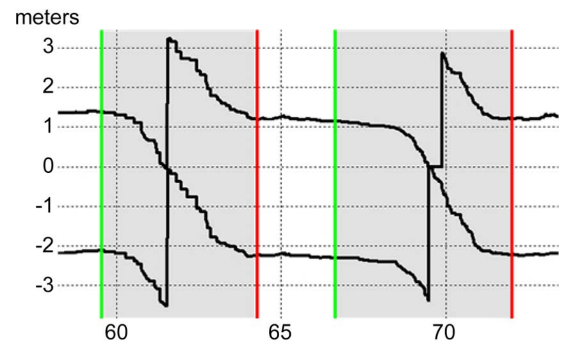

Fig. 6. Vehicle's lateral position with respect to lanes and lane change detection.

A. Data Processing

We used the collected lane change data to determine the lane change initiation and endpoints. By assessing the video signal from a camera, we established that the overall portion of lane changes performed without detected lane markings was only 12%. Part of this missing data was reconstructed by performing lane signal interpolation for the cases where data were missing for less than 3 s. We detected lane change maneuvers in a set of data by first searching points where the vehicle crossed a lane (see Fig. 6). A jump in the vehicle lateral position with respect to one of the lanes at the moment when the vehicle crosses a lane can be explained by switching in what is considered to be the neighboring lanes of the vehicle. When the crossing point was detected, in its proximity, we looked for the two extremum points of the lateral position with respect to the lanes: The one that occurs before the lane crossing specifies the initiation point, whereas the one after indicates the end of the maneuver. We used additional filtering techniques to avoid classifying changing more than one lane at a time or interpreting turning as a lane change.

To distinguish the L_O, L_D, and F_D vehicles from all the vehicles detected by radar (see Fig. 7), we used their relative position and their location with respect to the lane markings. By doing so, we detected the closest vehicles in the current and neighboring lanes, which corresponded to the position of the L_O, L_D, and F_D vehicles. After a lane change maneuver was performed and the endpoint was detected, we referred to the traffic configuration at the initiation point. We stored the relative speeds and distances from the vehicles of interest.

The parameters associated with gap acceptance regression and GMM were estimated in quasi-real time. Chunks of driving data were processed every fixed period of time after which we

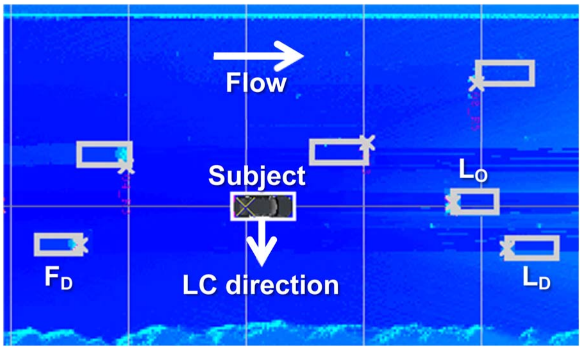

Fig. 7. Object detection from the radar data.

TABLE I
CONTINGENCY TABLE OF THE DATA—NUMBER OF SAMPLES

Direction	Blinker	Driver 1	Driver 2	Driver 3	All
Left	On	15	2	20	37
	Off	183	67	42	292
Right	On	18	6	17	41
	Off	252	62	33	347
All		468	137	112	717

updated the model parameters. We performed GMM training when there were enough lane change sample points. We set the minimum number of data to be 100. We find this number to be reasonable for establishing a well-trained model and avoiding ill-conditioning. We perform lane change shape prediction and gap acceptance estimation in real time. That is, when the associated models are trained, at every moment in time, we calculate the gap acceptable to the driver and the expected lane change kinematic parameters of the potential maneuvers to the left and right. The system is implemented as a C++ application and is able to provide online prediction with 30-Hz frequency.

B. General Data Analysis

Table I gives a summary of the collected data. Overall, we collected 717 lane changes for the three drivers. There are slightly more lane changes to the right than the left, i.e., 388 versus 329. We classify the driver indicating the desire to change lanes in advance when he switches the blinker on at least 3 s before the maneuver. The amount of samples when the blinker was used in advanced is small, i.e., only 78. The overall portion of such lane changes is 10.8% for the three drivers. On average, it took 11.6 h to collect 100 lane changes, i.e., the number we specified to be the minimum for GMM training after which the system can start providing reliable personalized lane change duration prediction. Before this number is reached, the system can provide personalized assistance based on the duration probability density function that can be constructed with a smaller number of samples.

C. Longitudinal Adjustment and Gap Acceptance

Table II presents the parameters described in (1) that characterize the drivers' habits of adjusting their longitudinal position before the maneuver. The data indicate that the majority of the lane changes were not preceded by longitudinal adjustment. If there was an adjustment, then it was in the positive direction.

TABLE II
LONGITUDINAL ADJUSTMENT OF THE DRIVERS

Parameter	Driver 1	Driver 2	Driver 3	All
Number of positive adjustments (n^+)	51	9	2	62
Number of negative adjustments (n^-)	19	6	7	32
Total number of lane changes (N)	468	137	112	717
α	0.15	0.11	0.08	0.13
β	0.46	0.2	-0.56	0.32
Mean of relative speed peak, km/h	7.2	5.3	4.6	5.7

TABLE III
MEAN VALUES OF LANE CHANGE PARAMETERS FOR ALL AVAILABLE DATA

		D_{XL_O}, m	D_{XL_D}, m	D_{XF_D}, m	V_{RL_O}, m/s	V_{RL_D}, m/s	V_{RF_D}, m/s
Left	Mean	20.1	16.1	18.0	0.63	0.52	-0.23
	Std. Dev.	7.94	12.24	10.12	1.65	3.32	3.04
Right	Mean	22.7	15.1	17.4	0.65	0.67	0.12
	Std. Dev.	7.72	10.34	9.67	1.70	2.98	2.46
All	Mean	21.4	15.8	17.7	0.64	0.60	-0.05
	Std. Dev.	7.90	11.56	9.96	1.68	2.15	2.70

TABLE IV
LINEAR REGRESSION COEFFICIENTS AND THEIR SIGNIFICANCE

		γ	δ	ε	ζ	η	θ
L_O	Value	9.38	0.11	1.68	0.9	1.16	-
	Signif.	1.00	1.00	1.00	0.95	0.90	-
L_D	Value	2.11	0.03	0.42	0.11	0.45	0.50
	Signif.	1.00	0.96	0.98	1.00	0.92	0.91
F_D	Value	2.23	0.02	0.35	0.06	0.67	0.73
	Signif.	1.00	0.95	0.93	0.91	0.97	0.96

Table III provides information on the mean values of lane change parameters for all of the collected data. The distributions of all the parameters presented in the table are normal, except for the relative distance to the lead and following vehicles in the destination lane, which had a log-normal distribution. This indicates that the relative distances to the following and the preceding vehicles in the destination lane are the two critical parameters that affect a driver's decision to initiate a lane change.

Table IV presents the values and significance (i.e., probability that an effect is not due to just chance alone) of the linear regression models' coefficients for Driver 1. Coefficients related to the lead vehicle in the destination lane (L_D) and the following vehicle in the destination lane (F_D) correspond to (2). The coefficients related to the lead vehicle in the origin lane correspond to (3). All associated significance values are greater than 0.9, which indicates that their respective terms are important predictors.

The sign of the coefficients indicates that Driver 1 needs larger gaps when driving at higher speeds, when the relative speed between his vehicle and the surrounding vehicles is high, and when he performs maneuvers to the right, as opposed to the

TABLE V
LINEAR REGRESSION MODEL CHARACTERISTICS

	R^2	Significance	Std. error
L_O	0.33	1.00	3.25
L_D	0.48	1.00	0.64
F_D	0.52	1.00	0.54

TABLE VI
FIT ACCURACY VERSUS NUMBER OF GMM COMPONENTS

Number of GMM components	Mean difference between actual and expected lane change duration, s	
	Training set	Raw set
2	0.75	1.03
3	0.56	0.91
4	0.50	0.84
5	0.48	0.82
7	0.46	0.81
10	0.45	0.81

left. In general, Driver 1 accepts smaller gaps for maneuvers when he switches on the blinker signal in advance. A small standard error, significance values of zero, and substantial R^2 values (see Table V) support the effectiveness of the constructed model. The regression coefficients for the other two drivers indicate that they have different gap acceptance preferences. For instance, Driver 3 prefers larger gaps, and there is no strong dependence between blinker usage and gap acceptance.

D. Lane Change Kinematics

We trained the GMM by using only samples that contained the leading and the following vehicles in the destination lane. The number of sample data collected for Driver 1 is 45% of the overall number (210 samples). We used 80% of this data as a training set and 20% as a raw data set for model validation. We measured the accuracy of the model by assessing the difference between the actual and expected lane change duration periods. An increase in the number of the GMM components M improves the accuracy of the model (see Table VI). However, an increase in components also increases the computational cost. We set this number to be 4 to balance the accuracy–computation cost for the real-time application. The GMM outperforms the baseline model, which uses the average maneuver duration as a prediction: The four-component GMM mean prediction error for the raw set is 0.84 s, whereas the baseline model's error is 1.44 s on the same data set. Therefore, we can conclude that the proposed framework is effective. The baseline model can be used up to the point when there are enough data to train the GMM, which gives a more accurate prediction.

Another indicator of the effectiveness of the model is its ability to capture a driver's personal style. If the model is trained on the set of one driver and used to predict lane change duration on a raw data set belonging to another driver, then we observe deterioration in the accuracy of the predictions: The mean prediction error increases from 0.84 to 1.82 s when the Driver 1 model was applied to the Driver 3 data set. The results indicate that the model is able to distinguish between the drivers of very similar backgrounds.

The sinusoidal model adequately describes the lane change lateral acceleration with respect to the lanes. The mean error between the lateral position obtained by integrating (5) with parameters established by the GMM and the actual lateral displacement in time for the maneuvers that were performed on a straight road is 0.35 m for Driver 1. This indicates that the kinematic model represents lateral displacement reasonably well.

V. CONCLUSION AND FUTURE WORK

In this paper, we have developed a methodology that models how a driver changes lanes with a particular vehicle. The methodology combines advantages of stochastic and kinematic modeling techniques to come up with models that can run in real time with the capability of adjusting their parameters as new data are received for better accuracy. The models capture the behavior that involves adjusting longitudinal position and speed before the lane change maneuver, gap acceptance to initiate the maneuver, and the kinematics of the maneuver. This personalized modeling framework can be used to develop a lane change driver-assistance system as part of ADAS to provide appropriate and desirable recommendations to the driver that are personalized to his/her driving characteristics and dynamics. We used experimental data to train the proposed models and validate them. The experimental results demonstrate the effectiveness of the approach.

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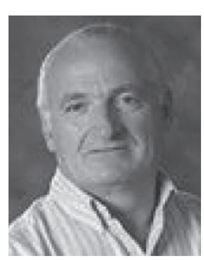

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