1

a.

A	C	P(A,C)
T	T	0.014 + 0.012 = 0.026
T	F	0.126 + 0.048 = 0.174
F	T	0.392 + 0.144 = 0.536
F	F	0.168 + 0.096 = 0.264

b.

C	P(C)
T	0.014 + 0.012 + 0.392 + 0.144 = 0.562
F	0.126 + 0.048 + 0.168 + 0.096 = 0.438

C.

A	C	P(A/C)
T	T	0.026 / 0.562 = 0.0462
T	F	0.174 / 0.438 = 0.3972
F	T	0.536 / 0.562 = 0.9537
F	F	0.264 / 0.438 = 0.6027

d.

A	B	C	P(A, B/C)	
T	T	T	0.014 / 0.562	= 0.0249
T	F	T	0.012 / 0.562	= 0.0213
F	T	T	0.392 / 0.562	= 0.6975
F	F	T	0.144 / 0.562	= 0.2562
T	T	F	0.126 / 0.438	= 0.2876
T	F	F	0.048 / 0.438	= 0.1095
F	T	F	0.168 / 0.438	= 0.3835
F	F	F	0.096 / 0.438	= 0.2191

$\boxed{2}$

a. Bayesian Network factorization of the joint $P(x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) =$

$P(x_2)$
$P(x_3)$
$P(x_4)$
$P(x_5 | x_2, x_3)$
$P(x_6 | x_3, x_4)$
$P(x_7 | x_5)$
$P(x_8 | x_3, x_5, x_6)$
$P(x_9 | x_5, x_7, x_8)$

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b. (i)

x_2 is binary, x_3 can take on 3 possible values ..... x_9 can take 9 possible values,

So, the number of independent parameters for $p(x_2, x_3 \ldots x_9)$, without using Bayesian factorization,

$2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 - 1$
$= 362879$

(i) Find the number of independent parameters for $p(x_2, x_3 \ldots x_9)$ using Bayesian factorization,

-> independent parameter for each factor,

$P(x_2) = 2 - 1 = 1$
$P(x_3) = 3 - 1 = 2$
$P(x_4) = 4 - 1 = 3$
$P(x_5 | x_2, x_3) = (5-1) \times 2 \times 3 = 24$
$P(x_6 | x_3, x_4) = (6-1) \times 3 \times 4 = 60$
$P(x_7 | x_5) = (7-1) \times 5 = 30$
$P(x_8 | x_5, x_3, x_6) = (8-1) \times 5 \times 3 \times 6 = 630$
$p(x_9 | x_7, x_5, x_8) = (9-1) \times 7 \times 5 \times 8 = 2240$

So, Total independent parameters are
$= 1 + 2 + 3 + 24 + 60 + 30 + 630 + 2240$
$= 2990$

$\boxed{3}$

$\rightarrow$ $P(A) = \langle 0.4, 0.6 \rangle$

$P(B \mid A = T) = \langle 0.1, 0.9 \rangle$

$P(B \mid A = F) = \langle 0.8, 0.2 \rangle$

$P(C \mid B = T) = \langle 0.7, 0.3 \rangle$

$P(C \mid B = F) = \langle 0.4, 0.6 \rangle$

$P(D \mid C = T) = \langle 0.82, 0.18 \rangle$

$P(D \mid C = F) = \langle 0.37, 0.63 \rangle$

$$A \rightarrow B \rightarrow C \rightarrow D$$

Joints. $P(A)\,P(B \mid A)\,P(C \mid B)\,P(D \mid C)$

(i) $\boxed{\text{Compute } P(B)}$

$\rightarrow$ $Q = B$, $E = \phi$, $W = A, C, D$

$\rightarrow$ Elimination order : A, D, C

$\rightarrow$ <u>Eliminate A</u>

$\psi(B, A) = P(A)\,P(B \mid A)$

B	A	P(A) P(B|A)
T	T	$0.4 \times 0.1 = 0.04$
T	F	$0.6 \times 0.8 = 0.48$
F	T	$0.4 \times 0.9 = 0.36$
F	F	$0.6 \times 0.2 = 0.12$

Sum of A, $\tau_1(B) = \sum_A \psi(B, A)$

B	$\tau_1(B)$
T	0.52
F	0.48

- New Fectorization

$T_1(B)\ P(C|D)\ P(D|C)$

→ **Eliminate D**

Multiply fectors of D,

$\psi(D,C) = P(D|C)$

D	C	P(D|C)
T	T	0.82
T	F	0.37
F	T	0.18
F	F	0.63

Sum of D $T_2(C) = \sum_{D} \psi(D|C)$

C	$T_2(C)$
T	1
F	1

- New Fectorization

$T_1(B)\ P(C|B)\ T_2(C)$

→ **Eliminate C**

Multiply fectors of C,

$\psi(C,B) = P(C|B)\ T_2(C)$

C	B	$P(C\|B)\,\tau_2(C)$
T	T	$0.7 \times 1 = 0.7$
T	F	$0.4 \times 1 = 0.4$
F	T	$0.3 \times 1 = 0.3$
F	F	$0.6 \times 1 = 0.6$

Sum of C, $\tau_3(B) = \sum_{C}' \psi(C, B)$

B	$\tau_3(B)$
T	1
F	1

New Factorization $\tau_1(B)\,\tau_3(B)$

- Multiplication and Normalization

B	$\tau_1(B)\,\tau_2(B)$	Normalization
T	$0.52 \times 1 = 0.52$	$0.52/0.52+0.48 = 0.52$
F	$0.48 \times 1 = 0.48$	$0.48/0.48+0.52 = 0.48$

(ii) $\boxed{\text{Compute } P(C\|A=T)}$

→ $Q = C$, $E = A = T$, $W = B, D$

→ $P(A=T)\,P(B|A=T)\,P(C|B)\,P(D|C)$

→ Elimination order : D, B

→ Eliminate D

Multiply all the factors of D

$\psi(D, C) = P(D|C)$

D	C	$P(D\|C)$
T	T	0.82
T	F	0.37
F	T	0.18
F	F	0.63

Sum of D, $\tau_1(C) = \sum_D \psi(D, C)$

C	$\tau_1(C)$
T	1
F	1

New Factorization

$P(A=T)\,P(B|A=T)\,P(C|B)\,\tau_1(C)$

→ Eliminate B

Multiply factors of B,

$\psi(C, B) = P(B|A=T)\,P(C|B)$

C	B	$P(B\|A=T)\,P(C\|B)$
T	T	$0.1 \times 0.7 = 0.07$
T	F	$0.9 \times 0.4 = 0.36$
F	T	$0.1 \times 0.3 = 0.03$
F	F	$0.9 \times 0.6 = 0.54$

Sum of B, $\tau_2(C) = \sum_B \psi(C, B)$

C	$\tau_2(C)$
T	0.43
F	0.57

New factorization

$P(A=T)\,\tau_2(c)\,\tau_1(c)$

→ Multiply and Normalization

C	$P(A=T)\,\tau_2(c)\,\tau_1(c)$	Normalization
T	$0.4 \times 0.43 \times 1 = 0.172$	$0.172/0.4 = 0.43$
F	$0.4 \times 0.57 \times 1 = 0.228$	$0.228/0.4 = 0.57$

(iii) $\boxed{\text{Compute } P(A, B \mid C=T, D=F)}$

$Q = A, B$, $E = C=T, D=F$, $W = \phi$

→ $P(A)\,P(B|A)\,P(C=T|B)\,P(D=F|C=T)$

A	B	$P(A)\,P(B/A)\,P(C=T/B)\,P(D=F/C=T)$
T	T	$0.4 \times 0.1 \times 0.7 \times 0.18 = 0.00504$
T	F	$0.4 \times 0.9 \times 0.4 \times 0.18 = 0.02592$
F	T	$0.6 \times 0.8 \times 0.7 \times 0.18 = 0.06048$
F	F	$0.6 \times 0.2 \times 0.4 \times 0.18 = 0.00864$

a.

$$EU(action) = \sum_{x,y} P(Y|x)\, U(Y, action)$$

X	Y	action	$P(Y|x)\, U(Y, action)$
T	T	T	$0.2 \times 800 = 160$
T	T	F	$0.2 \times 400 = 80$
T	F	T	$0.8 \times 200 = 160$
T	F	F	$0.8 \times 1000 = 800$
F	T	T	$0.7 \times 800 = 560$
F	T	F	$0.7 \times 400 = 280$
F	F	T	$0.3 \times 200 = 60$
F	F	F	$0.3 \times 1000 = 300$

→ For action a

$EU(a) = 160 + 160 + 560 + 60 = 940$

→ For action ¬a

$EU(\neg a) = 80 + 800 + 280 + 300 = 1460$

→ action ¬a has a higher utility. So, we should take ¬a.

Ans: ¬a action should take.

5

$P(Rain) = p$
$P(\neg Rain) = 1 - p$

$$\begin{aligned}
EU(take) &= 0 \times (1-p) \times 100 \\
&+ 0 \times p \times 0 \\
&+ 1 \times (1-p) \times 20 \\
&+ 1 \times p \times 70 \\
&= 0 + 0 + 20 - 20p + 70p \\
&= 50p + 20
\end{aligned}$$

$$\begin{aligned}
EU(\neg take) &= 1 \times (1-p) \times 100 \\
&+ 0 \times p \times 0 \\
&+ 0 \times (1-p) \times 20 \\
&+ 0 \times p \times 70 \\
&= 100 - 100p + 0 + 0 + 0
\end{aligned}$$

Exp. Utility (take) = Exp. Utility ($\neg$take)

$$\begin{aligned}
20 + 50p &= 100 - 100p \\
150p &= 80 \\
p &= 80/150 \\
&= 0.53
\end{aligned}$$

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