Machine Learning HW5: CIS 519

Rutuja Moharil

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Taking 1 late day remaining from 1 late day
Collaborators : Sagar , Sai ,Gauri,Tien

1 Problem Set

1.1 K means

1. Here we have been given the data after performing one iteration. We can summarise this as follows
   Data points $= [4, 1, 9, 12, 6, 10, 2, 3, 9]$
   Clustered data :

Data point	Cluster
1	C1
2	C1
3	C1
4	C2
9	C2
12	C2
6	C2
10	C2
9	C2

   In the next iteration we need to find the mean of the data points that are in the cluster 1 and 2 respectively .Using these new data points then we can update the euclidean distance between the datapoints and the new centroids . This can be solved as

   (a) ***Iteration1***

$$Centroid1 = \frac{1+2+3}{3} = 2$$
$$Centroid2 = \frac{4+9+12+6+10+9}{6} = 8.33$$

Data point	Dist 1	Dist 2	Cluster
1	1	7.33	C1
2	0	6.33	C1
3	1	5.33	C1
4	2	4.33	C1
9	7	1.33	C2
6	4	2.33	C2
10	8	2.33	C2
9	7	1.33	C2
12	10	4.33	C2

(b) So the new clusters are as follows
C1=[1,2,3,4]
C2=[9,6,10,9,12]

(c) ***Iteration2***
Here again we follow the same steps as before and lets calculate the new centroids and assign clusters accordingly.

$$Centroid1 = \frac{1+2+3+4}{4} = 2.5$$
$$Centroid2 = \frac{9+12+6+10+9}{5} = 9.2$$

Data point	Dist 1	Dist 2	Cluster
1	1.5	8.2	C1
2	0.5	7.2	C1
3	1.5	6.2	C1
4	2.5	5.2	C1
9	6.5	0.2	C2
6	3.5	3.2	C2
10	7.5	0.8	C2
9	6.5	0.2	C2
12	9.5	2.8	C2

(d) Clusters formed here are as follows
Cluster 1 = C1= [1,2,3,4]

Cluster 2 = C2= [9,6,10,9,12]

2. As we can see in both the iterations the clusters contain the same data points .This basically means that Cluster 1 or C1 in both iterations has same datapoints. The same logic goes for Cluster 2 or C2 .
It means that none of the data points has moved to other cluster. Also the means/centroid of these clusters is constant. So this becomes the stopping condition for our algorithm. Which is basically the convergence condition. So the K means algorithm has converged in this case.

1.2 K means and Variance

1. On increasing the value of K we can see the number of partitions decreases and this is because of the fact that incrementing K leads to decrease in the intra-cluster variance.

2. If K is equal to the dataset instance then the variance becomes zero or vanishes. Since in this every point of the dataset would be a cluster and hence it would be a case of zero error as there is a single point in a cluster now . So this case can be thought of as a maximum accuracy case and without any compression whatsover.

1.3 Reinforement learning I

So here we have been given the reward equation as

$$R_t = r_(t+1) + r_(t+2) + r_(t+3).....r_\tau$$
$$\tau \text{ is the final time step}$$

After running the learning agent we find that there is no improvement is escaping the maze . Now this is a case where agent is unable to find an exit in the first iteration and so it doesn't about the +1 reward since it has never experienced it.
One method is to assign the non-goal state to -1. So when an agent visits those states a lot, the states scores get worse and worse. So one solution is to formulate the reward equation as :

$$R_t = r_1 + r_2 + r_3.....r_\tau$$
$$\text{Let's say that } r_\tau = -1, R_t = -(N-1)$$
$$\text{Once it reaches the goal the value of } R_t = 0$$

1.4 Reinforcement learning II

Both the signs of the rewards and the intervals are important. Lets consider the case where we want to avoid running into the edges. One good way to avoid the edges is to assign a negative sign rather a negative cost to it. So the negative cost makes sure that the reward will only decrease for the edge case in that episode and hence that particular step will not be taken.

$$v_\pi(s) = E_\pi\left\{G_t|s_t = s\right\} \text{Using the discounted reward equation we have} \tag{1}$$

$$v_\pi(s) = E_\pi\left\{\sum_{k=0}^{\infty}\gamma^k R_{t+k+1}|s_t = s\right\} \text{Adding a constant C gives us the follows} \tag{2}$$

$$v'_\pi(s) = E_\pi\left\{\sum_{k=0}^{\infty}\gamma^k\left(R_{t+k+1} + C\right)|s_t = s\right\} \tag{3}$$

$$= E_\pi\left\{\sum_{k=0}^{\infty}\gamma^k R_{t+k+1} + C\sum_{k=0}^{\infty}\gamma^k|s_t = s\right\} \tag{4}$$

$$K = \sum_{k=0}^{\infty}\gamma^k c \tag{5}$$

We see that adding a constant C to all rewards does not affect the relative values of any states under any policies.

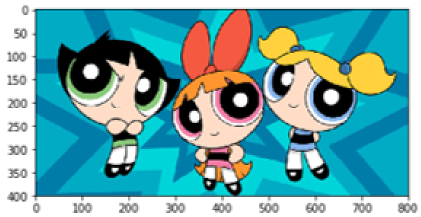
2 Programming part

2.1 K means

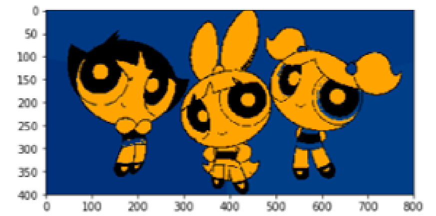
K-Means finds the best centroids by alternating between (1) assigning data points to clusters based on the current centroids (2) choosing centroids (points which are the center of a cluster) based on the current assignment of data points to clusters.

Here are the 3 images chosen randomly with a different set of K values to show how a difference in K affects the reconstruction and segmentation as well .

Example 1

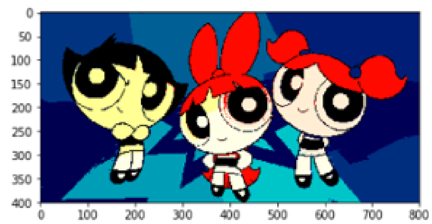


Original Image



K = 5 segmented

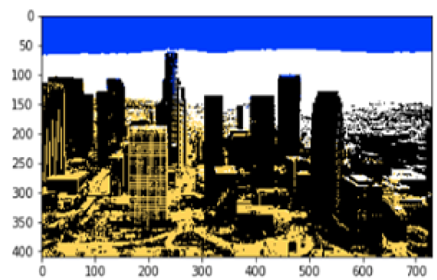
K = 8 segmented



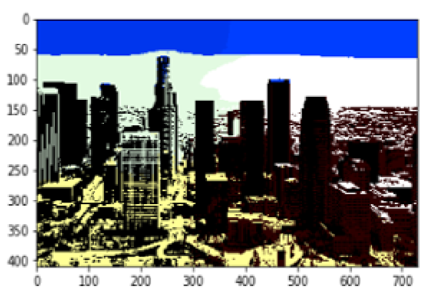
K = 14 segmented

Example 2

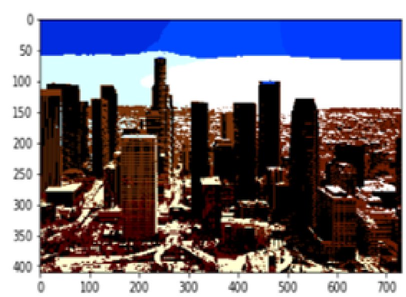
Original Image



K= 5 segmented

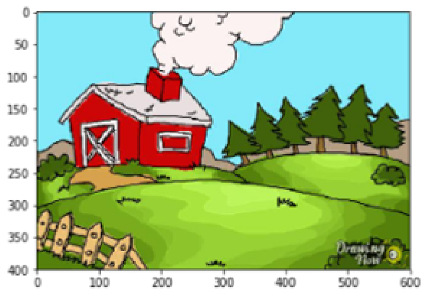


K = 8 segmented

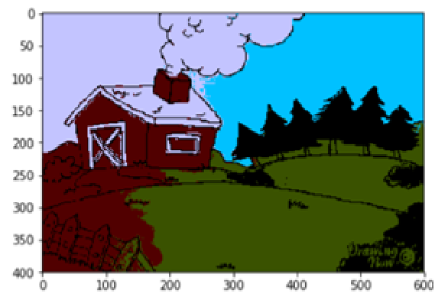


K = 14 segmented

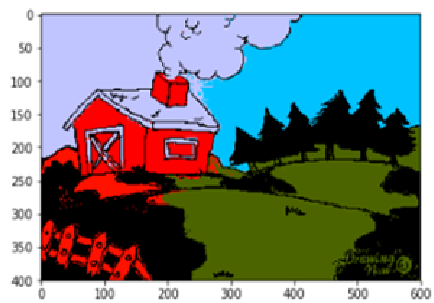
Example 3



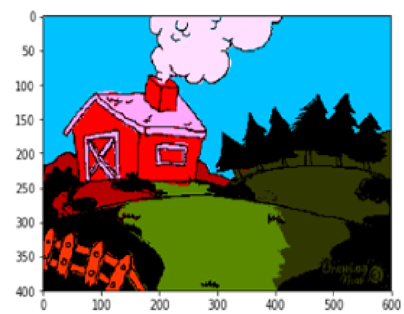
Original Image



K = 5 segmented



K = 8 segmented



K = 14 segmented