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Control of semi-autonomous teleoperation system with time delays[☆]

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ABSTRACT

In teleoperation systems operating in complex environments, due to the cognitive limitations of the human operator and lack of complete information about the remote environment, safety and performance of such systems can potentially be comprised. In order to ensure the safety and enhance the efficiency of complex teleoperation systems operating in cluttered environments, in this paper we investigate a semi-autonomous control framework for bilateral teleoperation. The semi-autonomous teleoperation system is composed of heterogeneous master and slave robots, where the slave robot is assumed to be a redundant manipulator. Considering robots with different configurations, and in the presence of dynamic uncertainties and asymmetric communication delays, we first develop a control algorithm to ensure position and velocity tracking in the task space. Additionally in the absence of dynamic uncertainty, and in the presence of human operator and environmental forces, all signals of the proposed teleoperation system are proven to be ultimately bounded. The redundancy of the slave robot is then utilized for achieving autonomous sub-task control, such as singularity avoidance, joint limits, and collision avoidance. The control algorithms for the proposed semi-autonomous teleoperation system are validated through numerical simulations on a non-redundant master and a redundant slave robot.

1. Introduction

Teleoperated robotic systems have emerged as a useful tool to accomplish tasks in remote or hazardous environments, as was witnessed during the recent Fukushima Daiichi nuclear disaster. A bilateral teleoperation system is composed of master and slave robots, where the signals are exchanged between the two robots via a communication channel. On being manipulated by a human operator, the controlled coupling between the master and slave robots is utilized by the slave robot for carrying out tasks remotely. However, due to the fact that the master and slave robots may be separated by a considerable distance, the human operator is not able to access complete information about the environment. This lack of information, coupled with the cognitive limitations of the human operator, limits the capabilities of the teleoperation system. Hence, this limitation necessitates the study of semi-autonomous robotic systems where there is shared autonomy between the human operator and the remote slave robotic system. The idea of semi-autonomous robotic systems has been utilized for health care (Ettelt, Furtwängler, Hanebeck, & Schmidt, 1998), search and rescue (Doroodgar, Ficocelli, Mobedi, & Nejat, 2010), and under water vehicles (Li, Jun, Lee, & Hong, 2005). In this paper, we study a semi-autonomous control framework for task-space bilateral teleoperation system, where the slave robot is able to accomplish additional tasks autonomously.

Control of teleoperation system has been studied in Chopra, Spong, and Lozano (2008), Lee and Spong (2006) and Nuño, Ortega, Barabanov, and Basañez (2008); however, the problem was solved in the joint space with the assumption that, the master and slave robots are kinematically identical. Due to the practical importance of heterogeneous manipulators, several researchers have recently studied teleoperation systems where the master and slave robots have different configurations. Building on the work (Chopra et al., 2008), scaled synchronization has been proposed for bilateral teleoperators with different configurations (Kawada, Yoshida, & Namerikawa, 2007), but the master and slave robots system were assumed to be kinematically identical and non-redundant manipulators. Teleoperation of redundant manipulators was studied in Nath, Tatlicioglu, and Dawson (2009),

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where the robots are assumed to track a desired trajectory in task space. However, the teleoperation system was developed without considering communication delays and the master and slave robots were required having the same degrees of freedom. Synchronization of heterogeneous robotic manipulators following a desired trajectory in the task space was recently presented in Liu and Chopra (2012). Even though heterogeneity of the robotic manipulators and communication delays were considered, the controller required all agents to have knowledge of a common trajectory, which may not be practical for all teleoperation systems.

The study of teleoperation using nonidentical robots has been recently addressed (Liu & Chopra, 2011; Malysz & Sirouspour, 2011). Task-space teleoperation with a redundant slave robot has been studied in the presence of constant delays (Liu & Chopra, 2011). A control theoretic framework was proposed to guarantee the position and velocity tracking between the master and slave robots. However, external (human and environmental) forces were not considered and the performance of the force reflection was not studied. An interesting teleoperation system has been developed in Malysz and Sirouspour (2011), where the system utilizes dual master robots to control different frames assigned on the slave robot. Even though the master and slave robots were considered to be nonidentical, the slave robot required complete control from the human operator. Moreover, the issue of communication delays, a significant issue in the study of teleoperation systems, was not considered.

It is well known that, the presence of time delays in a closed-loop system affects the stability of the teleoperation system (Anderson & Spong, 1989; Richard, 2003). The problem of constant time delays in bilateral teleoperation systems was addressed using the scattering or the wave-variable formulation (Anderson & Spong, 1989; Niemeyer & Slotine, 1991). Even though the stability problem is solved by the scattering transformation, position drifts resulting from the offset of initial conditions is a well-known problem in such systems (Chopra, Spong, Ortega, & Barabanov, 2006). Without relying on the use of scattering transformation, passive control for nonlinear robotic teleoperation was studied Lee and Spong (2006) with constant time delays under the assumption that the system dynamics are known. The result has been further studied in Nuño et al. (2008) by demonstrating that, it is possible to control a teleoperation system with a simple PD controller. Recently, without using the scattering transformation, passivity-based synchronization Chopra et al. (2008) has been utilized to synchronize the state of master and slave robot in the presence of dynamic uncertainties. To overcome a drawback of the adaptive gravity compensation algorithm addressed in Chopra et al. (2008), a new adaptive controller was proposed (Nuño, Ortega, & Basañez, 2010) to overcome the problem.

As introducing autonomy for various sub-tasks and ensuring stability of the teleoperation system in the presence of time delays (Hokayem & Spong, 2006) are important goals for teleoperating in complex environments, in this paper, we propose a semi-autonomous control system for task space teleoperation. Considering both time delays and dynamic uncertainties, the objective of this paper is to develop a teleoperation system where the slave robot can autonomously achieve an additional task while tracking the position and velocity of the master robot. Hence, the human operator only focuses on controlling the position of the end-effectors by manipulating the master robot while the slave robot, in addition to tracking the master position in the task space, is able to accomplish several tasks autonomously.

The proposed teleoperation system is constituted by a master robot, which could be a non-redundant or redundant manipulator, with a redundant slave robot. Since the degrees of freedom in a redundant manipulator is more than the dimension of the task space, the motion of the redundant robot in the null space of the Jacobian matrix will not influence the task-space motion. Therefore, this property is utilized for achieving several sub-tasks, such as singularity avoidance, joint limits, and collision avoidance, to enhance the overall performance of the teleoperation system. In addition, an obstacle avoidance algorithm, which is adapted from a previously proposed collision avoidance scheme for multi-agent system, is proposed to avoid obstacles in the remote slave environment.

The contributions of this paper can be summarized as follows:

- In contrast to Chopra et al. (2008, 2006), Lee and Spong (2006) and Nuño et al. (2008) assuming kinematic similarity and Malysz and Sirouspour (2011) and Nath et al. (2009) assuming perfect communication channel, the teleoperation system with heterogeneous robots is studied in this paper under asymmetric and unknown communication delays.
- By utilizing the redundancy of the slave robot, the semi-autonomous behavior can be achieved with the use of only one master robot, while Malysz and Sirouspour (2011) requires dual master robots to control the slave robot.
- In free motion, the tracking errors can be guaranteed to approach the origin independent of constant time delays, even if there is initial mismatch between the master and slave robots (Theorem 3.1).
- The proposed teleoperation system is able to achieve tracking even when the human operator exerts a damping force (Theorem 3.2). Moreover, in hard contact, the signals of the system are shown to be ultimately bounded with force reflection (Theorem 3.3).
- Based on the proposed semi-autonomous teleoperation framework, three sub-task controls are discussed in this paper. In addition, a previously developed collision avoidance control, which was originally designed for multi-agent system, is modified and applied in this paper for obstacle avoidance for the redundant slave robots (Section 4).

The paper is organized as follows. The control problem is formulated in Section 2, and the theoretical results for task-space teleoperation system with dynamic uncertainties and communication delays are presented in Section 3. Subsequently, the semi-autonomous control framework for the redundant slave robot is discussed in Section 4. The numerical examples for semi-autonomous teleoperation with communication delays are discussed in Section 5. Finally, Section 6 summarizes the results and discusses future research directions.

2. Problem formulation

With the assumption that manipulators in the teleoperation system are modeled by Lagrangian systems (see Appendix) and driven by actuated revolute joints, the dynamics of the master and slave robots are given as

$$\begin{cases} M_1(q_1)\ddot{q}_1 + C_1(q_1,\dot{q}_1)\dot{q}_1 + g_1(q_1) = J_1^T(q_1)F_1 + \tau_1 \\ M_2(q_2)\ddot{q}_2 + C_2(q_2,\dot{q}_2)\dot{q}_2 + g_2(q_2) = -J_2^T(q_2)F_2 + \tau_2 \end{cases} \quad (1)$$

where the subscripts $\{1,\ 2\}$ denote the master robot and slave robot, $q_1(t) \in R^n$, $q_2(t) \in R^m$, $M_1(q_1) \in R^{n\times n}$, $M_2(q_2) \in R^{m\times m}$, $C_1(q_1,\dot{q}_1) \in R^{n\times n}$, $C_2(q_2,\dot{q}_2) \in R^{m\times m}$, $g_1(q_1) \in R^n$, $g_2(q_2) \in R^m$, $\tau_1(t) \in R^n$ and $\tau_2(t) \in R^m$ are the vectors of applied torques, $J_1(q_1) \in R^{n\times n}$ and $J_2(q_2) \in R^{n\times m}$ are the Jacobian matrices, and $F_1(t), F_2(t) \in R^{n\times 1}$ are the forces exerted by the human operator and the environment on the end-effectors of the master and slave robot, respectively. In order to achieve semi-autonomous teleoperation, the slave robot in this paper is assumed to be a redundant manipulator. For the sake of simplicity, the master robot in the system is assumed to be a non-redundant manipulator;

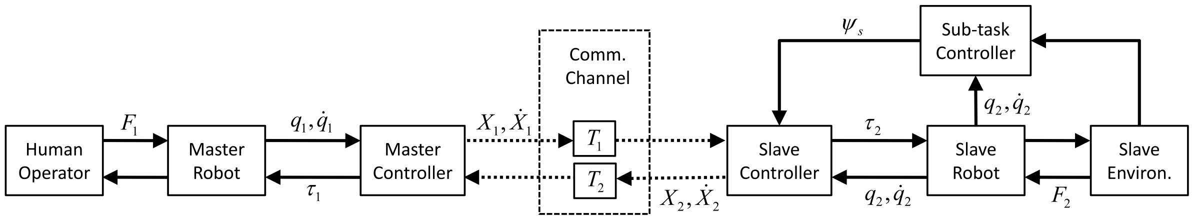


Fig. 1. Framework of the proposed semi-autonomous teleoperation system.

however, a redundant master robot can also be easily incorporated in the proposed teleoperation framework.

Let $X_1(t), X_2(t) \in R^n$ represent the position of the end-effector in the task space. It is related to the joint space vector as

$$X_1 = h_1(q_1), \qquad \dot{X}_1 = J_1(q_1)\dot{q}_1$$
$$X_2 = h_2(q_2), \qquad \dot{X}_2 = J_2(q_2)\dot{q}_2$$

where $h_1(\cdot) \in R^{n\times n}, h_2(\cdot) \in R^{m\times n}$ denote the mapping between the joint space and the task space, and $J_1(q_1) = \partial h_1(q_1)/\partial q_1, J_2(q_2) = \partial h_2(q_2)/\partial q_2$ are the Jacobian matrices that are assumed to be known.

In general, lack of complete information (such as obstacles, slave joint limits) about the remote environment, can make teleoperation a tedious task for the human operator. To address this issue, and to ensure that the teleoperation system is not restricted by the cognitive limitation of the human operator, a semi-autonomous teleoperation framework is studied in this paper. As seen in Fig. 1, the position and velocity signals are transmitted between the master and slave controller via a communication channel, which is subjected to constant delays. In the proposed framework, a teleoperation controller is developed so that the end-effector of the slave robot tracks the corresponding position of the master robot. Additionally, a sub-task controller is also developed that exploits the redundancy of the slave robot, to ensure autonomous compliance with other goals, such as obstacle avoidance, etc., in teleoperation mission. The theoretical formulations proposed in the paper ensure that the interaction of the sub-task controller and the teleoperation controller results in a stable closed-loop system. Moreover, the feedback signals from the slave robot provide the human operator with a perception of the remote environment. Hence, the human operator only focuses on manipulating the end-effector of the slave robot, and the redundant slave robot is able to achieve an additional sub-task autonomously while tracking the master robot.

We define the tracking errors as

$$\begin{cases} e_1(t) = X_2(t - T_2) - X_1(t) \\ e_2(t) = X_1(t - T_1) - X_2(t) \end{cases} \tag{2}$$

where T_1 and T_2 are the constant time delays in the communication channel. In the rest of this paper, for the sake of simplicity, the argument of time-dependent signals are omitted, for example $e_1 \equiv e_1(t)$, unless otherwise required for the sake of clarity. To develop the aforementioned semi-autonomous teleoperation system, the following problems are studied in this paper:

P1 In the presence of asymmetric communication delays and dynamic uncertainties, design a synchronization controller for the heterogeneous master and slave robots in free motion to accomplish the position and velocity tracking (Theorem 3.1) such that

$$\begin{cases} \lim_{t\to\infty} e_1(t) = \lim_{t\to\infty} e_2(t) = 0 \\ \lim_{t\to\infty} \dot{e}_1(t) = \lim_{t\to\infty} \dot{e}_2(t) = 0. \end{cases} \tag{3}$$

P2 If the human operator provides a damping force and the slave robot is allowed to move freely (Theorem 3.2), demonstrate that the position and velocity of the master and the slave robots converge asymptotically (3).

P3 On hard contact of the slave robot with the remote environment, and when the human operator exerts non-passive force, ensure the boundedness of the position tracking e_1, e_2 (2), and force reflection errors (Theorem 3.3).

P4 Based on the proposed teleoperation framework, study the semi-autonomous behavior for P1–P3 by utilizing the redundancy of the slave robot (Section 4).

The task-space teleoperation system between heterogeneous robotic systems is first studied in Section 3, and the semi-autonomous control is presented in Section 4.

3. Task space teleoperation

Let the control input τ_i (1), $i = \{1, 2\}$ be given as

$$\begin{aligned} \tau_i &= \hat{M}_i a_i + \hat{C}_i v_i + \hat{g}_i - K_i s_i - J_i^T \bar{\tau}_i \\ &= Y_i \hat{\Theta}_i - K_i s_i - J_i^T \bar{\tau}_i, \end{aligned} \tag{4}$$

where $\hat{M}_i(q_i)$, $\hat{C}_i(q_i, \dot{q}_i)$, and $\hat{g}_i(q_i)$ denote the estimate of M_i, C_i, and g_i, which may include unknown parameters of the manipulator. Additionally, the formulation $Y_i(q_i, \dot{q}_i, v_i, a_i)\hat{\Theta}_i = \hat{M}_i a_i + \hat{C}_i v_i + \hat{g}_i$ is due to Property 1 (see Appendix) for Lagrangian systems, K_i is a positive-definite diagonal matrix, and $\bar{\tau}_i$ is the coordinating control that will be subsequently defined.

The signal a_i, v_i, and s_i in (4) are defined as

$$\begin{cases} s_1 = -J_1^{-1}\lambda e_1 + \dot{q}_1 \\ v_1 = \dot{q}_1 - s_1 = J_1^{-1}\lambda e_1 \\ a_1 = \ddot{q}_1 - \dot{s}_1 = \dot{J}_1^{-1}\lambda e_1 + J_1^{-1}\lambda \dot{e}_1 \end{cases} \tag{5}$$

$$\begin{cases} s_2 = -J_2^{+}\lambda e_2 + \dot{q}_2 - (\mathscr{I}_m - J_2^{+}J_2)\psi_s \\ v_2 = \dot{q}_2 - s_2 = J_2^{+}\lambda e_2 + (\mathscr{I}_m - J_2^{+}J_2)\psi_s \\ a_2 = \ddot{q}_2 - \dot{s}_2 = \dot{J}_2^{+}\lambda e_2 + J_2^{+}\lambda \dot{e}_2 + \dfrac{d}{dt}[(\mathscr{I}_m - J_2^{+}J_2)\psi_s] \end{cases} \tag{6}$$

where λ is a positive control constant, $\psi_s \in R^m$ is the negative gradient of an appropriately defined function (for sub-task control in Fig. 1), $\mathscr{I}_m$ is a $m \times m$ identity matrix, $J_1^{-1} \in R^{n\times n}$ is the inverse of J_1, and $J_2^{+} \in R^{m\times n}$ is the pseudo-inverse of J_2, which is defined by $J_2^{+} = J_2^T(J_2J_2^T)^{-1}$ and satisfies $J_2J_2^{+} = \mathscr{I}_n$.

By defining $r_i = J_i s_i$ and substituting s_i into r_i, we obtain that

$$r_i = J_i s_i = -\lambda e_i + J_i \dot{q}_i = -\lambda e_i + \dot{X}_i, \tag{7}$$

where the property of the pseudo-inverse matrix J_2^{+} that $J_2(\mathscr{I}_m - J_2^{+}J_2) = 0$ (Zergeroglu, Dawson, Walker, & Behal, 2000) is utilized.

Substituting the controller (4) in the robot dynamics (1), the closed-loop system for the master and slave robots can be written as

$$\begin{cases} M_1\dot{s}_1 + C_1 s_1 + K_1 s_1 = Y_1\tilde{\Theta}_1 - J_1^T\bar{\tau}_1 + J_1^T F_1 \\ M_2\dot{s}_2 + C_2 s_2 + K_2 s_2 = Y_2\tilde{\Theta}_2 - J_2^T\bar{\tau}_2 - J_2^T F_2 \end{cases} \tag{8}$$

where $\tilde{\Theta}_i = \hat{\Theta}_i - \Theta_i$ is the estimation error of unknown parameters.

Define the coordinating control $\bar{\tau}_1$ and $\bar{\tau}_2$ as

$$\bar{\tau}_i = k_r r_i - K_J \dot{e}_i, \tag{9}$$

where k_r is a positive constant gain, and K_J is a positive-definite constant matrix. Let the time-varying estimates of the uncertain parameters evolve as

$$\dot{\hat{\Theta}}_i = -\Gamma_i Y_i^T s_i, \tag{10}$$

where Γ_i is a positive definite matrix.

Denote by $\mathscr{C} = \mathscr{C}([-T_i, 0], R^n)$, the Banach space of continuous functions mapping the interval $[-T_i, 0]$ into R^n, with the topology of uniform convergence. Let $z = [s_1\ s_2\ e_1\ e_2\ \tilde{\Theta}_1\ \tilde{\Theta}_2]$ and define $z_t = z(t+\phi) \in \mathscr{C}$, $-T_i \le \phi \le 0$ as the state of the system (Hale & Verduyn, 1993). We assume in this paper that, $z(\phi) = \eta(\phi)$, $\eta \in \mathscr{C}$ and all signals belong to $\mathscr{L}_{2e}$, the extended $\mathscr{L}_2$ space. Based on the aforementioned formulation, the following result provides a solution to the problem (P1).

Theorem 3.1. *Consider the closed-loop teleoperation system described by (8), (9) and the update law (10). Assume that, the Jacobian matrix of the non-redundant master manipulator is full rank. Then in free motion ($F_1 = F_2 = 0$) the task-space position error (e_i) and the velocity error ($\dot{e}_i$) asymptotically approach the origin independent of the constant communication delays.*

Proof. Consider a positive-definite storage functional V for the system as

$$V(z_t) = \frac{1}{2}\sum_{i=\{1,2\}}\left(s_i^T M_i s_i + \tilde{\Theta}_i^T\Gamma_i^{-1}\tilde{\Theta}_i + \lambda e_i^T K_J e_i + \int_{t-T_i}^{t}\dot{X}_i^T(\sigma)K_J\dot{X}_i(\sigma)d\sigma\right).$$

Taking the time derivative of the storage function, we get

$$\dot{V}(z_t) = \sum_{i=\{1,2\}}\left(s_i^T(-C_i s_i - K_i s_i - J_i^T\bar{\tau}_i + Y_i\tilde{\Theta}_i) + \frac{1}{2}s_i\dot{M}_i s_i + \tilde{\Theta}_i^T\Gamma_i^{-1}(-\Gamma_i Y_i^T s_i) + \lambda e_i^T K_J\dot{e}_i + \frac{1}{2}\dot{X}_i^T K_J\dot{X}_i - \frac{1}{2}\dot{X}_i^T(t-T_i)K_J\dot{X}_i(t-T_i)\right).$$

Using Property 2 and substituting the coordinating control (9), the derivative becomes

$$\dot{V}(z_t) = \sum_{i=\{1,2\}}\left(-s_i^T J_i^T(k_r r_i - K_J\dot{e}_i) - s_i^T K_i s_i + \lambda e_i^T K_J\dot{e}_i + \frac{1}{2}\dot{X}_i^T K_J\dot{X}_i - \frac{1}{2}\dot{X}_i(t-T_i)K_J\dot{X}_i(t-T_i)\right).$$

As $r_i = J_i s_i$ and $r_i = -\lambda e_i + \dot{X}_i$, the derivative can be rewritten as

$$\dot{V}(z_t) = \sum_{i=\{1,2\}}\left(-k_r r_i^T r_i + (-\lambda e_i + \dot{X}_i)^T K_J\dot{e}_i - s_i^T K_i s_i + \lambda e_i^T \times K_J\dot{e}_i + \frac{1}{2}\dot{X}_i^T K_J\dot{X}_i - \frac{1}{2}\dot{X}_i(t-T_i)K_J\dot{X}_i(t-T_i)\right).$$

Substituting $\dot{e}_1 = \dot{X}_2(t-T_2) - \dot{X}_1$ and $\dot{e}_2 = \dot{X}_1(t-T_1) - \dot{X}_2$ yields

$$\begin{aligned}\dot{V}(z_t) &= -k_r r_1^T r_1 - k_r r_2^T r_2 - s_1^T K_1 s_1 - s_2^T K_2 s_2 - \frac{1}{2}\dot{X}_1^T K_J\dot{X}_1 \\ &\quad + \dot{X}_1^T K_J\dot{X}_2(t-T_2) - \frac{1}{2}\dot{X}_2^T(t-T_2)K_J\dot{X}_2(t-T_2) \\ &\quad - \frac{1}{2}\dot{X}_2^T K_J\dot{X}_2 + \dot{X}_2^T K_J\dot{X}_1(t-T_1) \\ &\quad - \frac{1}{2}\dot{X}_1^T(t-T_1)K_J\dot{X}_1(t-T_1) \\ &= -\sum_{i=\{1,2\}}\left(k_r r_i^T r_i + s_i^T K_i s_i + \frac{1}{2}\dot{e}_i^T K_J\dot{e}_i\right) \le 0.\end{aligned}$$

As V is positive-definite and $\dot{V}$ is negative semi-definite, $\lim_{t\to\infty} V$ exists and is finite. Therefore, $r_i, s_i, \dot{e}_i \in \mathscr{L}_2$, and $s_i, \tilde{\Theta}_i, e_i \in \mathscr{L}_\infty$. From (9), we obtain that $\bar{\tau}_i \in \mathscr{L}_\infty$, hence we have that $\dot{s}_i \in \mathscr{L}_\infty$ from (8) by utilizing Properties 3 and 4. As $s_i \in \mathscr{L}_2$, and $\dot{s}_i \in \mathscr{L}_\infty$, it can show that $\lim_{t\to\infty} s_i(t) = 0$. Since $s_i, \dot{s}_i \in \mathscr{L}_\infty$, the derivative of $r_i = J_i s_i$, which is $\dot{r}_i = \dot{J}_i s_i + J_i\dot{s}_i$, results in $\dot{r}_i \in \mathscr{L}_\infty$. As $r_i \in \mathscr{L}_2$ and $\dot{r}_i \in \mathscr{L}_\infty$, we have that $\lim_{t\to\infty} r_i(t) = 0$. Taking the derivative of $r_i = -\lambda e_i + \dot{X}_i$, we get $\dot{r}_i = -\lambda\dot{e}_i + \ddot{X}_i$, then $\ddot{X}_i \in \mathscr{L}_\infty$, which implies $\ddot{e}_i \in \mathscr{L}_\infty$. Noting that, $\dot{e}_i \in \mathscr{L}_2$ and $\ddot{e}_i \in \mathscr{L}_\infty$, $\lim_{t\to\infty}\dot{e}_i(t) = 0$.

From the definition of r_1 and r_2 in (7), we have that

$$r_1 = -\lambda e_1 + \dot{X}_1 \tag{11}$$

$$r_2 = -\lambda e_2 + \dot{X}_2. \tag{12}$$

Delaying (11) by T_1 and subtracting from (12) yields

$$\begin{aligned} r_1(t-T_1) - r_2 &= -\lambda(e_1(t-T_1) - e_2) + (\dot{X}_1(t-T_1) - \dot{X}_2) \\ &= -\lambda(X_2(t-T_1-T_2) + X_2 \\ &\quad - 2X_1(t-T_1)) + \dot{e}_2. \end{aligned} \tag{13}$$

Since $\lim_{t\to\infty} r_i(t) = \lim_{t\to\infty}\dot{e}_i(t) = 0$, taking limit of (13) yields that

$$-2\lim_{t\to\infty}(X_1(t-T_1) - X_2(t)) = \lim_{t\to\infty}(X_2(t) - X_2(t-T_1-T_2)).$$

By noting that, $X_2(t) - X_2(t-T_1-T_2) = \int_{t-T_1-T_2}^{t}\dot{X}_2(\sigma)d\sigma$, the above equation can be rewritten as

$$-2\lim_{t\to\infty} e_2(t) = \lim_{t\to\infty}\int_{t-T_1-T_2}^{t}\dot{X}_2(\sigma)d\sigma. \tag{14}$$

Observing that $\lim_{t\to\infty} r_2(t) = 0$, taking limit of (12) results in $\lambda\lim_{t\to\infty} e_2(t) = \lim_{t\to\infty}\dot{X}_2(t)$. Hence, (14) becomes

$$-\frac{2}{\lambda}\lim_{t\to\infty}\dot{X}_2(t) = \lim_{t\to\infty}\int_{t-T_1-T_2}^{t}\dot{X}_2(\sigma)d\sigma. \tag{15}$$

Since $\lim_{t\to\infty}\dot{e}_1(t) = \lim_{t\to\infty}\dot{e}_2(t) = 0$, $\lim_{t\to\infty}(\dot{e}_2(t) + \dot{e}_1(t-T_1)) = 0$ gives that

$$\lim_{t\to\infty}(\dot{X}_2(t-T_1-T_2) - \dot{X}_2(t)) = 0.$$

From the above equation, we have that $\lim_{t\to\infty}\dot{X}_2(t)$ is either a constant or a periodic signal with period $T_1 + T_2$. By assuming first that, $\lim_{t\to\infty}\dot{X}_2(t)$ is a periodic signal, we have $\lim_{t\to\infty}\int_{t-T_1-T_2}^{t}\dot{X}_2(\sigma)d\sigma = \text{constant}$. Thus, from (15) we get

$$-\frac{2}{\lambda}\lim_{t\to\infty}\dot{X}_2(t) = \lim_{t\to\infty}\int_{t-T_1-T_2}^{t}\dot{X}_2(\sigma)d\sigma = \text{constant},$$

which contradicts the assumption that $\lim_{t\to\infty}\dot{X}_2(t)$ is a periodic signal. Accordingly, $\lim_{t\to\infty}\dot{X}_2$ can only be a constant. Denoting

$\lim_{t\to\infty}\dot{X}_2(t)=\bar{X}_{c2}$, where $\bar{X}_{c2}$ is a constant, (15) can be rewritten as

$$-\frac{2}{\lambda}\bar{X}_{c2}=\lim_{t\to\infty}\int_{t-T_1-T_2}^{t}\dot{X}_2(\sigma)d\sigma=(T_1+T_2)\bar{X}_{c2},$$

where the second equality results from the mean value theorem. Therefore, we have $(T_1+T_2)\bar{X}_{c2}+\frac{2}{\lambda}\bar{X}_{c2}=0$. Since T_1+T_2 and λ are both positive constants, the only solution is $\bar{X}_{c2}=0$, which leads us to $\lim_{t\to\infty}\dot{X}_2(t)=0$. Following similar arguments, we have that $\lim_{t\to\infty}\dot{X}_1(t)=0$. As $\lim_{t\to\infty}r_i(t)=\lim_{t\to\infty}\dot{X}_i(t)=0$, from (7) we get $\lim_{t\to\infty}e_i(t)=0$. Consequently, the position and velocity tracking errors of the closed-loop teleoperation system are stable and approach the origin independent of the constant communication delays. □

Remark 1. The convergence of position and velocity errors between the master and slave robots in the teleoperation system can be guaranteed if $\lim_{t\to\infty}e_i(t)=0$ and $\lim_{t\to\infty}\dot{e}_i(t)=0$. In Theorem 3.1, we have also shown that, $\lim_{t\to\infty}s_i(t)=0$ as the convergence of s_i to the origin is necessary (see Section 4) for utilizing the null space of the redundant manipulator to accomplish semi-autonomous control. If the sub-task control is not required, for example in non-redundant robots, the term K_is_i in the control input (4) could be eliminated. Following the proof of Theorem 3.1 for $K_1=0$, we have that $\lim_{t\to\infty}e_i(t)=\lim_{t\to\infty}\dot{e}_i(t)=0$ and $\lim_{t\to\infty}s_2(t)=0$. Hence, the proposed task-space teleoperation system can still guarantee the convergence of position and velocity errors in free motion.

Our next result addresses the case when the human operator provides a damping force, while there is no contact force between the slave robot and the remote environment (P2). The external force for the teleoperation system is given as $F_1=-k_d\dot{X}_1$ and $F_2=0$, where k_d is a positive constant. Denoting $\bar{T}=T_1+T_2$ and $\Lambda=\frac{1}{4k_r\bar{T}^2}\left(\sqrt{k_d^2+16k_r^2\bar{T}^2+16k_rk_d\bar{T}^2}-k_d\right)>0$, we claim that

Theorem 3.2. *Consider the closed-loop teleoperation system described by (8), (9) and the update law (10). If the human operator provides a damping force, and the Jacobian matrix of the master manipulator is full rank, then for the range of gains satisfying* $\Lambda>\lambda>\frac{k_d}{2k_r}$, $e_i\to 0$ *and* $\dot{e}_i\to 0$ *as* $t\to\infty$. *Therefore, the teleoperation system achieves position and velocity tracking in the task space in the presence of constant communication delays.*

Proof. Consider a positive-definite storage functional for the system as

$$V(z_t)=\frac{1}{2}\sum_{i=\{1,2\}}\Bigg(s_i^TM_is_i+\tilde{\Theta}_i^T\Gamma_i^{-1}\tilde{\Theta}_i+\lambda e_i^TK_Je_i+\int_{t-T_i}^{t}\dot{X}_i^T(\sigma)K_J\dot{X}_i(\sigma)d\sigma\Bigg)+\lambda k_r(X_1-X_2)^T(X_1-X_2).$$

Following the proof of Theorem 3.1 and using $F_1=-k_d\dot{X}_1$, the derivative of V is given as

$$\dot{V}(z_t)=-\sum_{i=\{1,2\}}\left(k_rr_i^Tr_i+s_i^TK_is_i+\frac{1}{2}\dot{e}_i^TK_J\dot{e}_i\right)-k_dr_1^T\dot{X}_1+2\lambda k_r(X_1-X_2)^T(\dot{X}_1-\dot{X}_2).$$

Substituting (7) to expand the term $k_rr_i^Tr_i$, the derivative of V becomes

$$\begin{aligned}\dot{V}(z_t)=&-\sum_{i=\{1,2\}}\Bigg(\lambda^2k_re_i^Te_i-2\lambda k_re_i^T\dot{X}_i+k_r\dot{X}_i^T\dot{X}_i+s_i^TK_is_i+\frac{1}{2}\dot{e}_i^TK_J\dot{e}_i\Bigg)+\lambda k_de_1^T\dot{X}_1-k_d\dot{X}_1^T\dot{X}_1\\&+2\lambda k_r(X_1-X_2)\dot{X}_1+2\lambda k_r(X_2-X_1)\dot{X}_2\\=&-\sum_{i=\{1,2\}}\left(\lambda^2k_re_i^Te_i+k_r\dot{X}_i^T\dot{X}_i+s_i^TK_is_i+\frac{1}{2}\dot{e}_i^TK_J\dot{e}_i\right)\\&+2\lambda k_r(X_2(t-T_2)-X_1)^T\dot{X}_1+2\lambda k_r(X_1-X_2)\dot{X}_1\\&+\lambda k_de_1^T\dot{X}_1+2\lambda k_r(X_1(t-T_1)-X_2)^T\dot{X}_2\\&+2\lambda k_r(X_2-X_1)\dot{X}_2-k_d\dot{X}_1^T\dot{X}_1.\end{aligned}$$

By noting that $X_i(t-T_i)-X_i(t)=\int_{-T_i}^{0}\dot{X}_i(t+\sigma)d\sigma$, the above equation becomes

$$\begin{aligned}\dot{V}(z_t)=&-\sum_{i=\{1,2\}}\left(\lambda^2k_re_i^Te_i+k_r\dot{X}_i^T\dot{X}_i+s_i^TK_is_i+\frac{1}{2}\dot{e}_i^TK_J\dot{e}_i\right)\\&-2\lambda k_r\dot{X}_1^T\int_{-T_2}^{0}\dot{X}_2(t+\sigma)d\sigma-2\lambda k_r\dot{X}_2^T\\&\times\int_{-T_1}^{0}\dot{X}_1(t+\sigma)d\sigma+\lambda k_de_1^T\dot{X}_1-k_d\dot{X}_1^T\dot{X}_1.\end{aligned}\tag{16}$$

By expanding the term $\lambda k_de_1^T\dot{X}_1\le\frac{1}{2}\lambda k_d(e_1^Te_1+\dot{X}_1^T\dot{X}_1)$, integrating (16) from 0 to t, and using Lemma 1 (see Appendix) we get

$$\begin{aligned}&V(t)-V(0)\\&\le-\sum_{i=\{1,2\}}\left(\lambda^2k_r\|e_i\|_2^2+k_r\|\dot{X}_i\|_2^2+K_i\|s_i\|_2^2+\frac{1}{2}K_J\|\dot{e}_i\|_2^2\right)\\&\quad+2\lambda k_r\left(\frac{\alpha_1}{2}\|\dot{X}_1\|_2^2+\frac{T_2^2}{2\alpha_1}\|\dot{X}_2\|_2^2+\frac{\alpha_2}{2}\|\dot{X}_2\|_2^2+\frac{T_1^2}{2\alpha_2}\|\dot{X}_1\|_2^2\right)\\&\quad-k_d\|\dot{X}_1\|_2^2+\frac{1}{2}\lambda k_d(\|e_1\|_2^2+\|\dot{X}_1\|_2^2).\end{aligned}$$

The coefficients of $\|\dot{X}_1\|_2^2$, $\|\dot{X}_2\|_2^2$, and $\|e_1\|_2^2$ have to be negative in order to guarantee that $V(t)-V(0)\le 0,\ \forall t>0$. Therefore, we have that from the coefficient of $\|e_1\|_2^2$

$$\lambda k_r>\frac{1}{2}k_d,\tag{17}$$

and from the coefficients of $\|\dot{X}_1\|_2^2$ and $\|\dot{X}_2\|_2^2$

$$\begin{cases}\left(k_r+k_d-\frac{1}{2}\lambda k_d\right)>\lambda k_r\left(\alpha_1+\frac{T_1^2}{\alpha_2}\right)\\ k_r>\lambda k_r\left(\alpha_2+\frac{T_2^2}{\alpha_1}\right).\end{cases}\tag{18}$$

The above Eq. (18) has positive solutions α_1 and α_2 if $k_r+k_d-\frac{1}{2}\lambda k_d>\lambda^2k_r(T_1+T_2)^2=\lambda^2k_r\bar{T}^2$, which can be rewritten as

$$\lambda^2k_r\bar{T}^2+\frac{1}{2}\lambda k_d-(k_r+k_d)<0.\tag{19}$$

Observing that λ has to satisfy the inequality $\lambda>\frac{k_d}{2k_r}>0$ from (17) and $\Lambda>\lambda>-\Lambda$ from (19), we obtain that $\Lambda>\lambda>\frac{k_d}{2k_r}$.

Consequently, if gains and time delays satisfy the conditions $\Lambda>\lambda>\frac{k_d}{2k_r}$, then $V(t)-V(0)\le 0,\ \forall t>0$, and hence the signals $s_i,e_i,\dot{e}_i,\dot{X}_i\in\mathcal{L}_2$. Moreover, $s_i,\tilde{\Theta}_i,e_i,X_1-X_2\in\mathcal{L}_\infty$ because V is bounded. From the definition of r_i in (7), we have that $r_i\in\mathcal{L}_2$. Following the argument in the proof of Theorem 3.1, $\lim_{t\to\infty}s_i=\lim_{t\to\infty}r_i=\lim_{t\to\infty}e_i=\lim_{t\to\infty}\dot{e}_i=\lim_{t\to\infty}\dot{X}_i=0$. Hence, the position and velocity errors between the master and slave robot converge to the origin in the presence of communication delay. □

Remark 2. For the teleoperation system in Theorem 3.2, the exact values of k_d and $\bar{T}$ do not have to be known a priori. The inequality $\Lambda > \frac{k_d}{2k_r}$ has to be satisfied for the existence of λ. Thus, we have

$$\frac{1}{4k_r\bar{T}^2}\left(\sqrt{k_d^2 + 16k_r^2\bar{T}^2 + 16k_rk_d\bar{T}^2} - k_d\right) > \frac{k_d}{2k_r}.$$

After rearranging the above equation, we get that

$$4\left(\frac{k_r}{k_d}\right)^2 + 4\left(\frac{k_r}{k_d}\right) - 1 > \bar{T}^2. \tag{20}$$

By selecting the maximum acceptable k_d and $\bar{T}$, the range of control gain k_r can be obtained from (20). Noting that the control gain λ exists if (20) is satisfied, the desired value of λ can be selected from $\Lambda > \lambda > \frac{k_d}{2k_r}$.

In the last part of this section, we study the stability of task-space teleoperation when the slave robot is in contact with the environment, which is assumed to be passive with respect to r_2, and the human operator exerts a non-passive force to the master robot (P3). The human and environmental forces are given as

$$F_1 = K_f - k_h r_1, \qquad F_2 = k_e r_2, \tag{21}$$

where K_f is a positive bounded vector in R^n, and k_h, k_e are bounded nonnegative constants. In this case, we assume that, there is no dynamic uncertainty, which implies that $\tilde{\Theta}_i \equiv 0$. Hence, the closed-loop dynamics of the teleoperation system can be written as

$$\begin{cases} M_1\dot{s}_1 + C_1s_1 + K_1s_1 = -J_1^T\bar{\tau}_1 + J_1^TF_1 \\ M_2\dot{s}_2 + C_2s_2 + K_2s_2 = -J_2^T\bar{\tau}_2 - J_2^TF_2. \end{cases} \tag{22}$$

Letting $z = [s_1\ s_2\ e_1\ e_2]$, the result for the hard contact case is addressed.

Theorem 3.3. *Consider the closed-loop teleoperation system described by (22) and (9). If the external forces are given as (21), and the Jacobian matrix of the non-redundant master manipulator is full rank, then for the range of gains $k_r > \frac{1}{2(1-\lambda\bar{T})} > \frac{1}{2}$, all signals in the system are ultimately bounded.*

Proof. Consider a positive-definite storage functional V for the system as

$$V(z_t) = \frac{1}{2}\sum_{i=\{1,2\}}\Bigg(s_i^TM_is_i + \lambda e_i^TK_Je_i + \int_{t-T_i}^{t}\dot{X}_i^T(\sigma)K_J\dot{X}_i(\sigma)d\sigma + 2\lambda k_r\int_{t-T_i}^{t}(\theta - t + T_i)\dot{X}_i^T(\theta)\dot{X}_i(\theta)d\theta\Bigg) + \lambda k_r(X_1 - X_2)^T(X_1 - X_2).$$

It is to be noted that $V(z_t) > 0,\ \forall z(t) \neq 0$. Taking the time derivative of the storage functional, we get

$$\dot{V}(z_t) = \sum_{i=\{1,2\}}\Bigg(-k_rr_i^Tr_i - s_i^TK_is_i - \frac{1}{2}\dot{e}_i^TK_J\dot{e}_i + \lambda k_rT_i\dot{X}_i^T\dot{X}_i - \lambda k_r\int_{t-T_i}^{t}\dot{X}_i^T(\theta)\dot{X}_i(\theta)d\theta\Bigg) + r_1^TF_1 - r_2^TF_2 + 2\lambda k_r(X_1 - X_2)^T(\dot{X}_1 - \dot{X}_2).$$

Expanding the term $k_rr_i^Tr_i$ and substituting F_1, F_2 from (21), the above equation becomes

$$\begin{aligned}\dot{V}(z_t) = &-\sum_{i=\{1,2\}}\left(\lambda^2k_re_i^Te_i + k_r\dot{X}_i^T\dot{X}_i + s_i^TK_is_i + \frac{1}{2}\dot{e}_i^TK_J\dot{e}_i\right)\\ &- 2\lambda k_r\dot{X}_1^T\int_{t-T_2}^{t}\dot{X}_2(\sigma)d\sigma - 2\lambda k_r\dot{X}_2^T\\ &\times\int_{t-T_1}^{t}\dot{X}_1(\sigma)d\sigma + \lambda k_rT_1\dot{X}_1^T\dot{X}_1\\ &- \lambda k_r\int_{t-T_1}^{t}\dot{X}_1^T(\theta)\dot{X}_1(\theta)d\theta + \lambda k_rT_2\dot{X}_2^T\dot{X}_2\\ &- \lambda k_r\int_{t-T_2}^{t}\dot{X}_2^T(\theta)\dot{X}_2(\theta)d\theta\\ &+ K_f^Tr_1 - k_hr_1^Tr_1 - k_er_2^Tr_2.\end{aligned} \tag{23}$$

Utilizing Lemma 2 (see Appendix) for the integral terms in (23) and expanding r_1 in $K_f^Tr_1$, we obtain that

$$\begin{aligned}\dot{V}(z_t) \le &-\sum_{i=\{1,2\}}\left(\lambda^2k_re_i^Te_i + k_r\dot{X}_i^T\dot{X}_i + s_i^TK_is_i + \frac{1}{2}\dot{e}_i^TK_J\dot{e}_i\right)\\ &+ \lambda k_rT_1\dot{X}_1^T\dot{X}_1 + \lambda k_rT_1\dot{X}_2^T\dot{X}_2 + \lambda k_rT_2\dot{X}_2^T\dot{X}_2\\ &+ \lambda k_rT_2\dot{X}_1^T\dot{X}_1 - \lambda K_f^Te_1 + K_f^T\dot{X}_1 - k_hr_1^Tr_1 - k_er_2^Tr_2\\ \le &-\sum_{i=\{1,2\}}\left(\lambda^2k_re_i^Te_i + k_r\dot{X}_i^T\dot{X}_i + s_i^TK_is_i + \frac{1}{2}\dot{e}_i^TK_J\dot{e}_i\right)\\ &+ \lambda k_r\bar{T}\dot{X}_1^T\dot{X}_1 + \lambda k_r\bar{T}\dot{X}_2^T\dot{X}_2 - k_hr_1^Tr_1\\ &- k_er_2^Tr_2 + K_f^TK_f + \frac{1}{2}\dot{X}_1^T\dot{X}_1 + \frac{1}{2}\lambda^2e_1^Te_1\\ \le &-\lambda^2\left(k_r - \frac{1}{2}\right)e_1^Te_1 - k_r\lambda^2e_2^Te_2 - s_1^TK_1s_1 - s_2^TK_2s_2\\ &- \frac{1}{2}\dot{e}_1^TK_J\dot{e}_1 - \frac{1}{2}\dot{e}_2^TK_J\dot{e}_2 - \left(k_r - \frac{1}{2} - \lambda k_r\bar{T}\right)\dot{X}_1^T\dot{X}_1\\ &- \left(k_r - \lambda k_r\bar{T}\right)\dot{X}_2^T\dot{X}_2 - k_hr_1^Tr_1 - k_er_2^Tr_2 + K_f^TK_f.\end{aligned}$$

From the coefficient of $\dot{X}_1^T\dot{X}_1$, the gains can be chosen by ensuring $k_r > \frac{1}{2(1-\lambda\bar{T})} > \frac{1}{2}$, which implies $\lambda\bar{T} < 1$ and $k_r > \frac{1}{2}$. Then, the derivative of V becomes

$$\begin{aligned}\dot{V}(z_t) \le &-\lambda^2\left(k_r - \frac{1}{2}\right)e_1^Te_1 - \lambda^2k_re_2^Te_2 - s_1^TK_1s_1\\ &- s_2^TK_2s_2 + K_f^TK_f.\end{aligned}$$

Define $0 < \eta < 1$ and denote $K_{\min} := min\left\{\lambda_{\min}(K_i), (\lambda^2(k_r - \frac{1}{2}))\right\}$, where $\lambda_{\min}(\cdot)$ is the smallest eigenvalue of the enclosed matrix, we get

$$\begin{aligned}\dot{V} &\le -K_{\min}(1-\eta)\|z\|^2 - K_{\min}\eta\|z\|^2 + K_f^TK_f\\ &\le -K_{\min}(1-\eta)\|z\|^2 \quad \forall\,\|z\| \ge \sqrt{\frac{K_f^TK_f}{K_{\min}\eta}}.\end{aligned}$$

Since $K_{\min}, \eta$ are bounded away from zero, and K_f is assumed to be bounded, $\dot{V}(z_t) < 0,\ \forall z(t) \neq 0$ for large values of the norm of $z(t)$. Therefore, the trajectories of the system are ultimately bounded. □

Remark 3. In the case of hard contact scenario, the teleoperation system is expected to achieve static force reflection. However, in comparison to the previous work in joint-space teleoperation (Chopra et al., 2006; Lee & Spong, 2006), where the force reflection ($F_1 \to F_2$) was accomplished, the force feedback error in the proposed semi-autonomous teleoperation system with hard contact can only be guaranteed to be bounded. To study the static force reflection, suppose that $\dot{q}_1, \dot{q}_2, \ddot{q}_1, \ddot{q}_2 \to 0$ (Chopra et al., 2006; Lee & Spong, 2006), then we have $\dot{s}_1, \dot{s}_2 \to 0$. From the closed-loop dynamics of the teleoperation system (22) with Property 4, we get that

$$\begin{cases} F_1 \to -k_r\lambda(X_2 - X_1) + \zeta_1 K_1 s_1 \\ F_2 \to k_r\lambda(X_1 - X_2) - \zeta_2 K_2 s_2 \end{cases} \tag{24}$$

where ζ_1 is the inverse of J_1^T, and ζ_2 is the pseudo-inverse of J_2^T. Therefore, (24) guarantees that the force feedback error is bounded. Since the term $K_1 s_1$ can be eliminated for the non-redundant master robot, as discussed in Remark 1, the force feedback from the slave robot to the master robot becomes $F_1 = -k_r\lambda(X_2 - X_1)$. Consequently, the human operator feels a force proportional to the difference between the position of master and slave robot in the task space.

Since the slave robot is assumed to be a redundant manipulator, the null space of the Jacobian matrix has a minimum dimension of $m - n$, and the task-space motion will not be influenced by the link velocity in the null space. Hence, we can utilize this property to achieve a desired sub-task control by appropriately designing the vector ψ_s for the slave robot. According to Hsu, Hauser, and Sastry (1989), the sub-task tracking error is defined as $e_{s_N} = (\mathscr{I}_m - J_2^+ J_2)(\dot{q}_2 - \psi_s)$ for the redundant slave robot. Pre-multiplying $s_2(t)$ in (6) by $(\mathscr{I}_m - J_2^+ J_2)$, we obtain the relation between the sub-task tracking error e_{s_N} and s_2 as

$$\begin{aligned} (\mathscr{I}_m - J_2^+ J_2)s_2 &= (\mathscr{I}_m - J_2^+ J_2)J_2^+ \lambda e_2 + (\mathscr{I}_m - J_2^+ J_2)\dot{q}_2 \\ &\quad - (\mathscr{I}_m - J_2^+ J_2)(\mathscr{I}_m - J_2^+ J_2)\psi_s \\ &= (\mathscr{I}_m - J_2^+ J_2)(\dot{q}_2 - \psi_s) = e_{s_N}, \end{aligned} \tag{25}$$

where the properties of pseudo-inverse J_2^+

$$(\mathscr{I}_m - J_2^+ J_2)J_2^+ = 0,$$
$$(\mathscr{I}_m - J_2^+ J_2)(\mathscr{I}_m - J_2^+ J_2) = \mathscr{I}_m - J_2^+ J_2$$

are utilized. Hence, if $\lim_{t\to\infty} s_2(t) = 0$ (Theorems 3.1 and 3.2), the sub-task tracking errors approach the origin. Moreover, if s_2 is only ultimately bounded (Theorem 3.3), the sub-task tracking error will also be bounded as $\mathscr{I}_m - J_2^+ J_2$ is bounded. This result can be utilized in several sub-task controls that enable the semi-autonomous characteristics of the teleoperation system. Details on the semi-autonomous control problem (P4) are provided in the next section.

4. Semi-autonomous control for the slave robot

Based on the framework discussed in Section 3, the redundancy of the slave robot can be used for achieving sub-task control for enhancing the performance of the teleoperation system. The gradient projection method (Siciliano, 1990) is utilized in this paper with the proposed teleoperation framework in order to achieve semi-autonomous behavior of the slave robot. The sub-task of the slave robot can be controlled by designing the auxiliary function ψ_s for various applications and demand. Any differentiable auxiliary function can be used for ψ_s as long as it can be expressed in terms of joint angles or end-effector position. While other sub-task control methods might suffer from severe computational requirements (Nakamura, 1991; Siciliano, 1990), the gradient projection method is more useful and suitable for application in teleoperation systems.

As the slave manipulator is redundant, the null space of the Jacobian matrix has a minimum dimension of $m - n$. Therefore, the task-space velocity of the redundant manipulator will not be affected by the link velocity in the null space. The function $(\mathscr{I}_m - J_2^+ J_2)\psi_s$ in (6) can be considered as the desired velocity in the null space of J_2. Hence, we can define a differentiable function $f(q_2)$ for which a lower value corresponds more desirable configurations. In this section, we use q_s, X_s, and J_s to denote the generalized configuration coordinates, the position of the end-effector, and the Jacobian matrix of a redundant manipulator under the sub-task control. Then, the auxiliary function is given by

$$\psi_s = -\frac{\partial}{\partial q_s} f(q_s), \tag{26}$$

which is utilized for achieving the sub-task for the redundant slave robot. In this paper, three sub-task control objectives, that is, singularity avoidance, joint limits, and collision avoidance, are discussed for demonstrating the applicability of the proposed semi-autonomous architecture.

4.1. Singularity avoidance

The first sub-task that we consider for semi-autonomous teleoperation is singularity avoidance for the slave robotic system. The goal is to regulate the configuration of slave robot for avoiding configurations that result in singularity. To this end, we aim for increasing the manipulability of the manipulator (Nakamura, 1991; Nath et al., 2009; Yoshikawa, 1984). Hence, the differentiable function for this sub-task can be defined as $f(q_s) = -\sqrt{\det(J_s J_s^T)}$. Then, the negative gradient of the convex function is given as

$$\psi_s = -\frac{\partial}{\partial q_s} f(q_s) = \frac{\partial}{\partial q_s}\sqrt{\det(J_s J_s^T)}.$$

Using this auxiliary function, the slave robot is able to regulate its configuration to increase the manipulability while tracking the position of master robot in the task space. Simulation results will be demonstrated in the next section to show the utility of this approach.

4.2. Joint angle limits

In order to enhance the teleoperation performance, the redundancy in the slave robot can be utilized for respecting joint angle constraints that may occur due to the mechanical constraints or may be induced by the operating environment. For example, to maintain joint limits the function can be defined as (Tatlicioglu, McIntyre, Dawson, & Walker, 2008)

$$f(q_s) = -\Pi_{j=1}^{m}\left(\left(1 - \frac{q_{sj}}{q_{sj}^{\max}}\right)\left(\frac{q_{sj}}{q_{sj}^{\min}} - 1\right)\right),$$

where q_{sj} is the jth joint angle of the redundant robot with $j = 1, \ldots, m$, $q_{sj}^{\max}$ denotes the maximum angle for the jth joint, and $q_{sj}^{\min}$ denotes the minimum angle for the jth joint. Then, the auxiliary function is given by (26). Moreover, the function $f(q_s)$ can also be replaced by other functions to maintain joint angle limits. For example, if the function is defined as $f(q_s) = (q_{si} - 1)^2 + (q_{sj} - 0.5)^2$, then using (26), the sub-task control will force the ith joint towards 1 rad and the jth joint towards 0.5 rad.

4.3. Collision avoidance

In the last and practically important case, the sub-task control is used for guaranteeing collision avoidance between links of the slave robot and obstacles in the operating environment. Utilizing redundancy of the manipulators to achieve collision avoidance has been studied in Galicki (2005), Mohri, Yang, and Yamamoto (1995) and Rimon and Koditschek (1992). As collision avoidance was treated as a path-planning problem, these previous methods are more effective for off-line path planning, but are not ideal for real-time obstacle avoidance. A real-time algorithm has been presented in Khatib (1986) by utilizing attractive function for the goal position and repulsive function for obstacles to avoid collision. However, in a teleoperation system, the desired position or trajectory is manipulated by the human operator, so there is no predefined trajectory available for the slave robot.

Hence, in this paper, we develop a collision avoidance algorithm which is adapted from the formulation originally proposed in Stipanović, Hokayem, Spong, and Šiljak (2007). The proposed collision avoidance method can be utilized for real-time control, and only the local distance between the designated collision-free points on the robot and the obstacle is required for implementing the control algorithm. Moreover, the redundant robot is unaffected by the collision avoidance control if the designated collision-free points are outside the sensing regions. In addition, if there exists collision-free configurations and paths, the position tracking in the end-effector of the redundant robot can be guaranteed.

Denote as X_{sk}, the point on the redundant slave manipulator that needs to be protected from collisions with the obstacles in the environment, and let X_o denote the location of obstacles. Consider the avoidance function between X_{sk} and X_o as

$$f_k(q_s) = \left(\min\left\{0, \frac{\|X_{sk}-X_o\|^2 - R^2}{\|X_{sk}-X_o\|^2 - r^2}\right\}\right)^2, \quad k \in \Omega$$

where $\|X_{sk} - X_o\|$ is the distance between X_{sk} and X_o, Ω is the set of points that are designed for avoiding collision, R denotes the avoidance distance, and r denotes the avoidance region which is the smallest safe distance of $\|X_{sk} - X_o\|$. When the distance between X_{sk} and X_o is less than R, the aim of the avoidance function is to change the configuration of the redundant manipulator for guaranteeing that the distance $\|X_{sk} - X_o\|$ will remain greater than the safe distance r.

We assume that, $\|X_{sk} - X_o\|$ is larger than R for the initial configuration. Denoting the distance between X_{sk} and X_o as $d_{ko} = \|X_{sk} - X_o\|$, the sub-task control for X_{sk} is given as the negative gradient of the potential function and can be written as

$$\psi_{sk} = -\left[\frac{\partial f_k(q_s)}{\partial q_{s1}} \quad \frac{\partial f_k(q_s)}{\partial q_{s2}} \quad \cdots \quad \frac{\partial f_k(q_s)}{\partial q_{sm}}\right]^T, \tag{27}$$

where $\frac{\partial f_k(q_s)}{\partial q_{sj}}$ for $j = 1, \ldots, m$ is given by

$$\frac{\partial f_k(q_s)}{\partial q_{sj}} = \frac{\partial f_k(q_s)}{\partial X_{sk}}\frac{\partial X_{sk}}{\partial q_{sj}} = \begin{cases} 0 \quad \text{if } d_{ko} \geq R \\ 4\left[\dfrac{(R^2-r^2)(d_{ko}^2-R^2)}{(d_{ko}^2-r^2)^3}\right](X_{sk}-X_o)^T\dfrac{\partial X_{sk}}{\partial q_{sj}} \\ \quad \text{if } r < d_{ko} < R \\ \text{not defined} \quad \text{if } d_{ko} = r \\ 0 \quad \text{if } d_{ko} < r. \end{cases} \tag{28}$$

Due to the assumption that the manipulators are composed of actuated revolute joints, $\frac{\partial X_{sk}}{\partial q_{sj}}$ in the above equation can be obtained from the column of the Jacobian matrix.

Additional details can be seen from the diagram of the collision avoidance method in Fig. 2, where the redundant manipulator has to avoid an obstacle located at X_o. The points chosen to avoid the obstacle can be either at a joint X_{s1} or on a link X_{s2}. Using (27) and (28), we can keep the distance d_{1o} and d_{2o} larger than r, which is the safe distance between the manipulator and the obstacle. Taking the collision-free point at a joint as an example, the term $\frac{\partial X_{s1}}{\partial q_{sj}}$ in (28) is equal to the jth column of the Jacobian matrix with $L_{s1} = L_{s1}$ and $L_{si} = 0$, where $i = 2, \ldots, m$. Here, L_{si} denotes the length of link of the redundant manipulator. If the collision-free point is designed to be on a link as X_{s2} in Fig. 2, the term $\frac{\partial X_{s2}}{\partial q_{sj}}$ in (28) is equal to the jth column of the Jacobian matrix with $L_{s1} = L_{s1}$, $L_{s2} = l_{s2}$, and $L_{si} = 0$, where $i = 3, \ldots, m$. Here, l_{si} denotes the length from the ith joint to the collision-free point on the ith link.

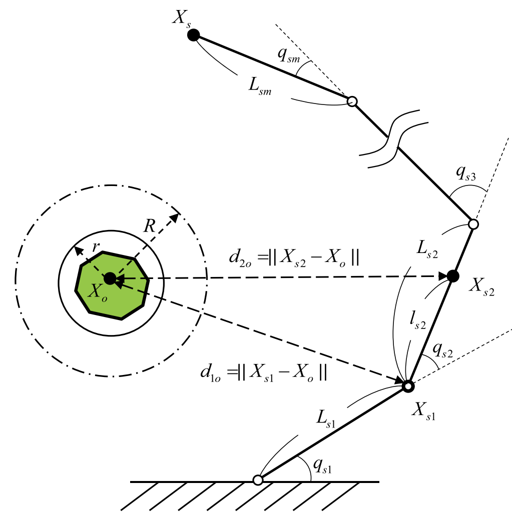


Fig. 2. Diagram of collision avoidance scenario.

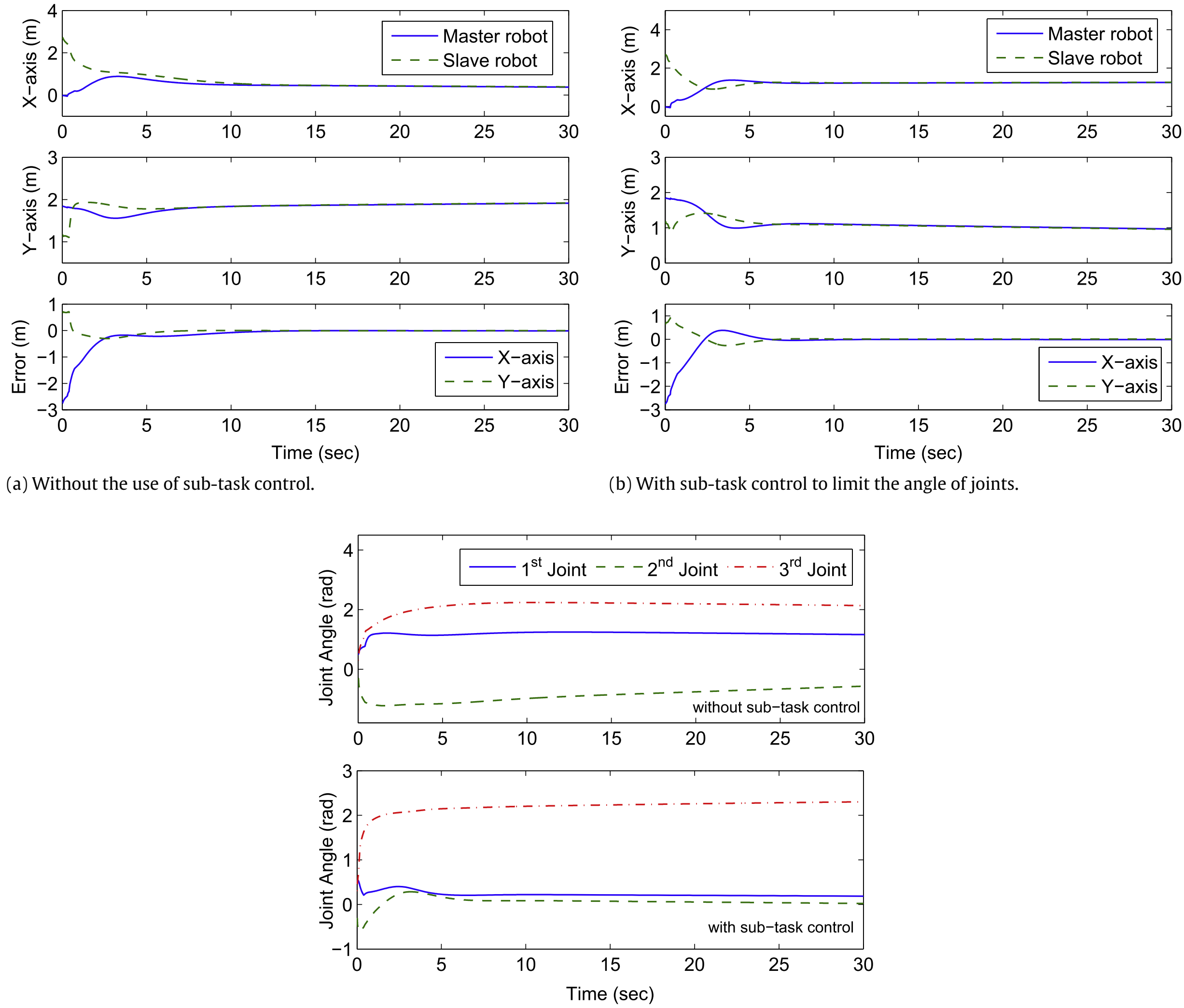
For different applications, there could be several such collision points, and in that case, the auxiliary function is the summation of these negative gradients ($\psi_s = \sum_{k\in\Omega} \psi_{sk}$) of the various avoidance functions. Moreover, this method can be extended to the case with multiple obstacles in the environment.

Remark 4. In the presence of hard contact (Theorem 3.3), the signals of the teleoperation system are ultimately bounded. Since the semi-autonomous control is based on the convergence of signal s_2, the fact that s_2 is only bounded under hard contact cannot guarantee the convergence of sub-task tracking errors, but can still ensure boundedness of the errors from (25). Based on the collision avoidance control proposed in this section, even though the sub-task tracking errors e_{sN} are not able to converge to the origin, the fact that s_2 is bounded still guarantees that ψ_s is bounded. Hence, from (27) and (28), the boundedness of ψ_s ensures that the designated collision-free points of the slave robot do not enter the regions of the safe distance r, provided existence of a collision-free configuration and trajectory is feasible. Consequently, collision avoidance control for the slave robot is guaranteed in the presence of human and/or environmental force.

5. Numerical examples

Numerical simulations are presented in this section to demonstrate the efficacy of the proposed semi-autonomous teleoperation system. In the simulation, we employ a 2-DOF planar master robot and a 3-DOF planar slave robot, which is a redundant manipulator. The reader is referred to Spong, Hutchinson, and Vidyasagar (2006) for the dynamics of the robots. The physical parameters of the manipulators are given as $m_1 = [3.14,\ 2.26]$ kg, $I_1 = [0.16,\ 0.07]$ kg m^2, $L_1 = [1.04,\ 0.96]$ m, $m_2 = [3.12,\ 1.85\ 1.02]$ kg, $I_2 = [0.12,\ 0.07,\ 0.04]$ kg m^2, $L_2 = [1.08,\ 0.98,\ 0.94]$ m, and $g = 9.8$. The control gains, which are assumed to be identical throughout this section, are given as $\lambda = 0.8$, $k_r = 8$, $K_1 = 2\mathscr{I}_2$, $K_2 = 2\mathscr{I}_3$, and $K_J = 5\mathscr{I}_2$. The communication time delays are given as $T_1 = 0.3$ s and $T_2 = 0.4$ s.

The first simulation illustrates the position tracking capabilities of the teleoperation system in the task space with and without uti-



(a) Without the use of sub-task control.

(b) With sub-task control to limit the angle of joints.

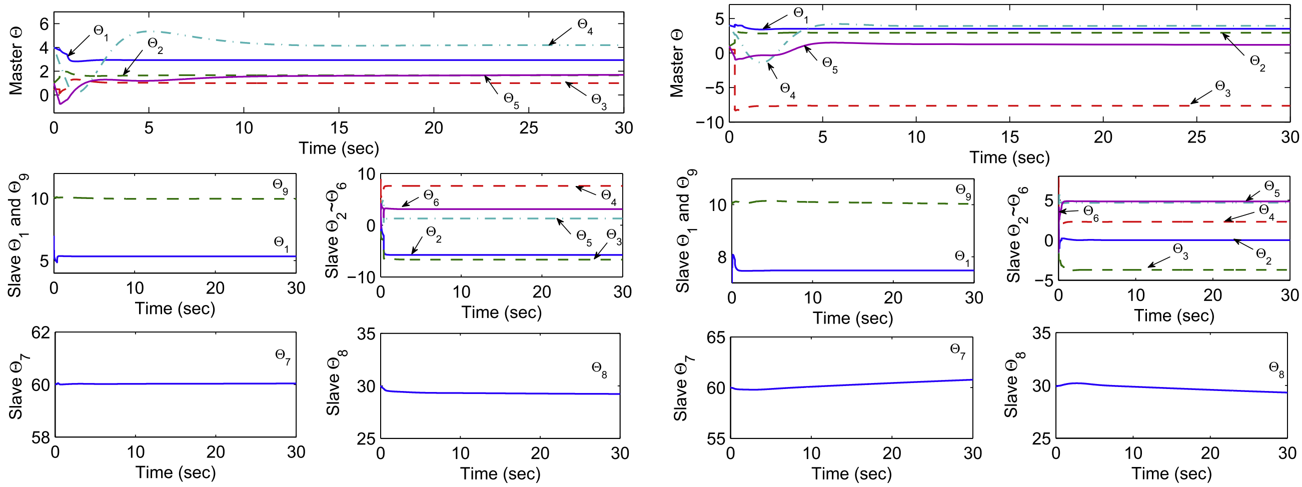
(c) Joint angles of the slave robot.

Fig. 3. Position configurations and joint angles of the master and slave robots with dynamic uncertainties, constant delays, and human damping force.

lizing joint limits sub-task control, and we consider the case where the human operator exerts a damping force on the master robot. The master and slave robots start from different initial positions, where the initial conditions are $q_1(0) = [1.2,\ 0.8]^T$ rad, $q_2(0) = [0.5, -0.3,\ 0.3]^T$ rad, and $\dot{q}_i(0) = \ddot{q}_i(0) = 0,\ i = \{1, 2\}$. Moreover, $\Gamma_1 = 0.75\mathscr{I}_5$, $\Gamma_2 = 0.75\mathscr{I}_9$, $\Theta_1(0) = [4,\ 1,\ 0.5,\ 4,\ 1]^T$, and $\Theta_2(0) = [7,\ 3,\ 1,\ 3,\ 1,\ 1,\ 60,\ 30,\ 10]^T$ are chosen for the adaptive control, and the damping gain is selected 12 N s/m. The results are shown in Figs. 3 and 4. In the absence of sub-task control for the slave robot under constant delays, Fig. 3(a) demonstrates that the position tracking errors between the master and slave robots converge to the origin.

With the use of sub-task control to limit the joint angles for $q_{21}^{\max} = 0.5$ rad, $q_{21}^{\min} = -0.5$ rad, $q_{22}^{\max} = 0.5$ rad, and $q_{22}^{\min} = -1$ rad, where q_{2i} denotes the ith joint of the slave robot, the semi-autonomous teleoperation system guarantees position tracking in the task space, and the tracking errors converge to the origin, as shown in Fig. 3(b). It is worth mentioning that, the final configuration of the teleoperation system in Fig. 3(b) is different from the result in Fig. 3(a) due to the influence of sub-task control in the slave robot, and as the human operator is assumed to only exert a damping force. Fig. 3(c) shows the joint angles of the slave robot with and without using sub-task control. It is evident that, the joint-limit sub-task control forces the joint angles to stay within the designed range without influencing the stability and position tracking capabilities of the system. The estimates of the uncertain dynamic parameters of the robotic systems are shown in Fig. 4.

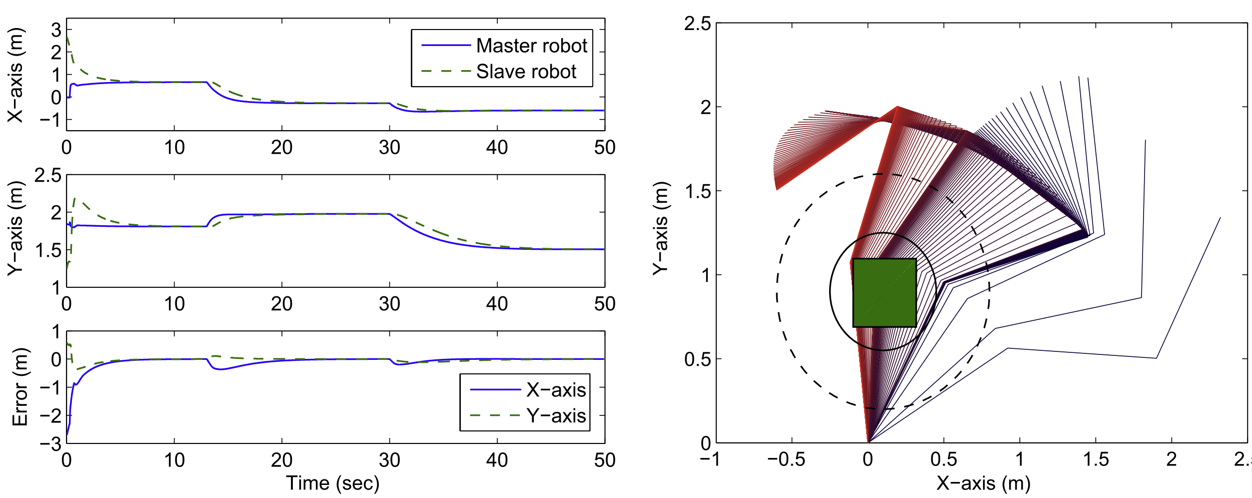
The second simulation illustrates the utilization of the (slave) sub-task controller for obstacle avoidance. The human operator exerts a force to manipulate the master robot from one set-point to another set-point, and there is no environmental force applied to the slave robot. Following Lee and Spong (2006), we assume that, the human operator exerts a spring–damper force, where the spring and damping gains are 80 N/m and 10 N s/m, for both the x and y directions. In the simulations, $F_1 = 0$ N at $t = 0$–13 s, $t = 23$–30 s, $t = 45$–50 s, and the human operator moves the master robot towards $X_1 = [-0.3,\ 2]^T$ m at $t = 13$–23 s and towards $X_1 = [-0.6,\ 1.5]^T$ m at $t = 30$–45 s. The obstacle that the slave robot needs to avoid is located at $X_o = [0.1,\ 0.9]^T$ m, and the collision distance and the safe distance are given as $R = 0.7$ m and $r = 0.35$ m, which are shown as the dashed circle and solid circle, respectively. By choosing the initial conditions as $q_1(0) = [1.2,\ 0.8]^T$ rad, $q_2(0) = [0.5, -0.3, 0.4]^T$ rad, and $\dot{q}_i(0) = \ddot{q}_i(0) = 0,\ i = \{1, 2\}$, the simulation results in the absence of sub-task control are shown in Fig. 5, and Fig. 6 demonstrates the results with the use of collision avoidance sub-task control. If no sub-task control is utilized, the links of the slave robot enter the region surrounding by the solid circle and collide with the obstacle as shown in Fig. 5(b). By utilizing the collision avoidance algorithm (27) and (28) for the



(a) Without the use of sub-task control.

(b) With the use of sub-task control for joint limits.

Fig. 4. Estimates of the dynamic uncertainty in the proposed semi-autonomous teleoperation system.



(a) Position configurations of the master and slave robot.

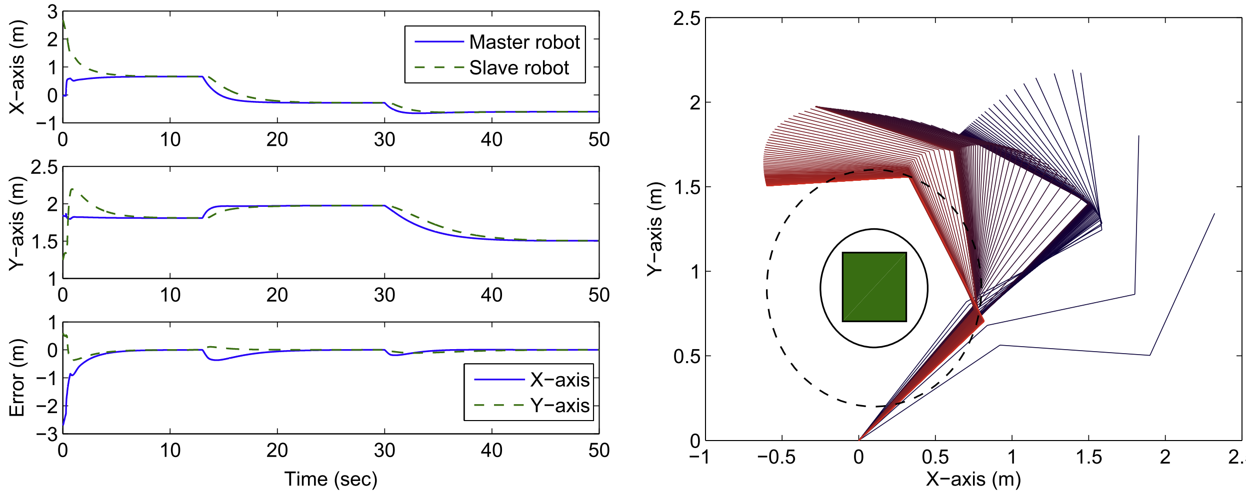
(b) Configurations of the slave robot without sub-task control.

Fig. 5. Configurations of slave robot in the presence of an obstacle in the environment without using collision avoidance control. The green box is assumed to be the obstacle in the remote environment. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

first two joints, the slave robot regulates its configuration to avoid colliding with the obstacle as seen in Fig. 6(b). Under the sub-task control for collision avoidance, the teleoperation system still achieves position tracking as shown in Fig. 6(a). Moreover, comparing the position configurations in Figs. 5(a) and 6(a), we can observe that the tracking performance is unaffected by the sub-task control, provided a collision-free configuration and trajectory exist.

Finally, we demonstrate the performance of the semi-autonomous teleoperation when the slave robot contacts the environment. In this simulation, we consider the case where the sub-task control ensures that the slave robot avoids singular configurations. The human operator is modeled as a spring–damper system whose spring and damping gains are 30 N/m and 15 N s/m for both the x and y directions. In the simulations, there is no human force before $t = 15$ s, and the human operator pushes the master to the position $X_1 = [-0.5,\ 1.5]^T$ m at $t = 15$–30 s and $X_1 = [0.5,\ 1]^T$ m after $t = 30$ s. In order to evaluate the stability in the presence of environmental force, we implement a wall in the remote environment at $x = 0$ m, which means that the slave robot will suffer an external force if its position in x direction is negative. The environmental force is modeled as a lightly damped spring–damper system, whose spring and damping gains are selected as 80 N/m and 0.1 N s/m. The simulation results are shown in Fig. 7(a) and (b) with sub-task control. When there is no human force before $t = 15$ s, the master and slave robots converge to each other as in the free motion case. After $t = 15$ s, the human operator exerts force to move the master robot towards the first set-point. Around $t = 19$ s, the slave robot contacts the wall in the remote environment, so the position errors in Fig. 7(a) do not approach the origin. As seen in Fig. 7(b), the human operator exerts a force to the master robot in order to push it moving towards $X_1 = [-0.5,\ 1.5]^T$ m. When the slave robot contacts the wall, the environmental force is reflected to the master robot, and hence the human operator is not able to push the master robot any further. When the human operator moves the master robot to another set-point after $t = 30$ s, the environmental force disappears, and the tracking errors of the teleoperation converge to the origin eventually. Moreover, the singularity avoidance sub-task control changes the configuration of the slave robot to increase the manipulability (Nakamura, 1991; Yoshikawa, 1984). The value of manipulability with and without using sub-task control are shown in Fig. 7(c). It is evident that, the sub-task control increases the manipulability as compared to the case when no sub-task control is utilized.

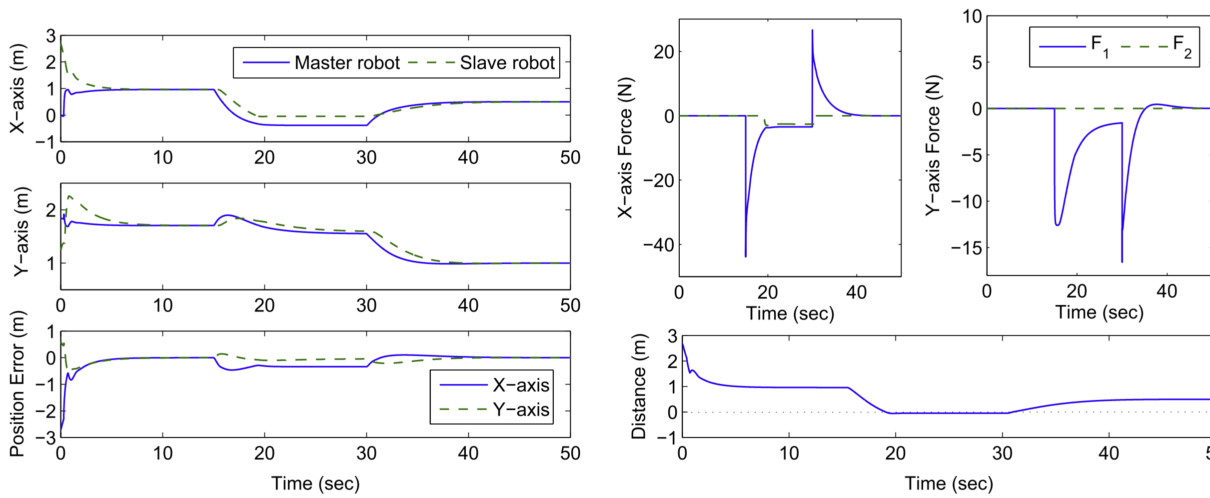
Remark 5. Three cases are illustrated in this section showing that by utilizing the proposed task-space teleoperation controller



(a) Position configurations of the master and slave robot.

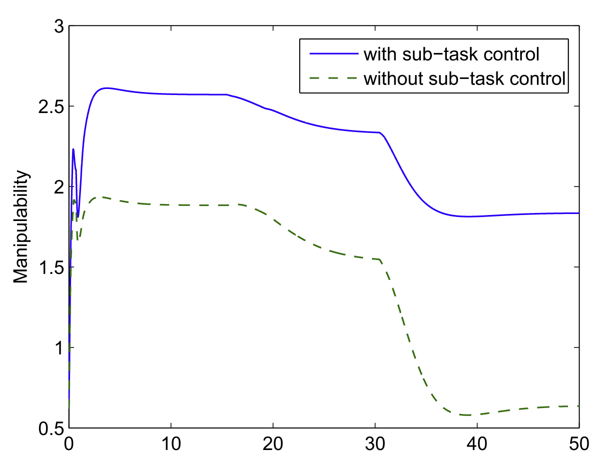
(b) Configurations of the slave robot with sub-task control.

Fig. 6. Configurations of slave robot with an obstacle in the environment and using collision avoidance control.



(a) Position configurations of the master and slave robots.

(b) External force and the distance between the end-effector of slave robot to the wall.



(c) Manipulability of the slave robot with and without using sub-task control.

Fig. 7. Simulation results of the proposed teleoperation system with hard contact and sub-task control to avoid configuration singularity.

(Section 3) and sub-task controller (Section 4), the system can achieve semi-autonomous teleoperation successfully. The human operator only needs to concern about manipulating the end-effector of the master robot, and the slave robot can regulate its configuration to follow the position and also achieve additional tasks, such as collision avoidance, joint limits, and increase of manipulability. Additionally, the case of slave robot contacting with the remote environment is also demonstrated that the external force can be reflected to the human operator.

6. Conclusions and future works

A semi-autonomous control framework was proposed in this paper to enable task space tracking and improve the overall performance of teleoperation systems. We demonstrated that the proposed control system in free motion can guarantee position and velocity tracking in the task space independent of the constant communication delays, and the initial position errors. The position and velocity tracking errors are guaranteed to converge to the origin even when the human operator exerts a damping force on the master robot. On hard contact with the environment, and when operated by the human operator, all signals of the teleoperation system were demonstrated to be ultimately bounded.

By exploiting the redundancy of the slave robot, the additional degrees of freedom were utilized to achieve several sub-tasks, such as singularity avoidance, joint limits, and collision avoidance. An obstacle avoidance algorithm, which is an adaptation of a previously studied collision avoidance scheme for multi-agent systems, was also proposed to ensure that the slave robot autonomously avoided obstacles in the remote environment. The efficacy of the proposed teleoperation system and the control algorithms was studied using numerical simulations with a 2-DOF master robot and a 3-DOF redundant slave robot. Future work encompasses studying the system performance under discrete time network effects (Chopra, Berestesky, & Spong, 2008; Huang & Lee, 2011), developing algorithms for the end-effector attitude tracking, and enhancing the quality of force reflection in the teleoperation system.

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Appendix

The master and slave robots in the semi-autonomous teleoperation are modeled as Lagrangian systems. Following Spong et al. (2006), in the absence of friction and disturbance, the dynamics of a n-link robotic system with revolute joints can be described as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u, \tag{29}$$

where $q \in R^n$ is the vector of generalized configuration coordinates, $u \in R^n$ is the vector of generalized forces acting on the system, $M(q) \in R^{n\times n}$ is the initial matrix, $C(q,\dot{q})\dot{q} \in R^n$ is the vector of Coriolis/Centrifugal forces and $g(q) = \frac{\partial H(q)}{\partial q} \in R^n$ is the gradient of the potential function $H(q)$. The above equations exhibit several fundamental properties due to their Lagrangian dynamic structure (Spong et al., 2006).

Property 1. *For any differentiable vector $\xi \in R^n$, the Lagrangian dynamics are linearly parameterizable which implies that*

$$M(q)\dot{\xi} + C(q,\dot{q})\xi + g(q) = Y(q,\dot{q},\xi,\dot{\xi})\Theta, \tag{30}$$

where Θ is a constant w-dimensional vector of unknown parameters, and $Y(q,\dot{q},\xi,\dot{\xi}) \in R^{n\times w}$ is the matrix of known functions of the generalized coordinates and their higher derivatives.

Property 2. *Under an appropriate definition of the matrix C, the matrix $\dot{M} - 2C$ is skew symmetric.*

Property 3. *The matrix $M(q)$ is symmetric positive definite, and there exist positive constants λ_m and λ_M such that*

$$\lambda_m \mathscr{I}_n \leq M(q) \leq \lambda_M \mathscr{I}_n \tag{31}$$

where $\mathscr{I}_n$ is a $n \times n$ identity matrix.

Property 4. *For $q, \dot{q}, \xi \in R^n$, there exists $k_c \in R^+$ such that the matrix of Coriolis/Centrifugal torques is bounded by*

$$|C(q,\dot{q})\xi| \leq k_c|\dot{q}|\,|\xi|. \tag{32}$$

We mention two lemmas that are utilized in this paper for the proof of stability.

Lemma 1 (Chopra et al., 2006)**.** *Given signals $x, y \in R^n$, $\forall T > 0$ there exists $\alpha > 0$ such that the following inequality holds*

$$-\int_0^t x^T(\sigma)\int_{-T}^0 y(\sigma+\theta)d\theta d\sigma \leq \frac{\alpha}{2}\|x\|_2^2 + \frac{T^2}{2\alpha}\|y\|_2^2, \tag{33}$$

where $\|\cdot\|_2$ denotes the $\mathscr{L}_2$ norm of the enclosed signal.

Lemma 2 (Hua & Liu, 2010)**.** *Given signals $x, y \in R^n$, $\forall T > 0$ and a positive definite matrix Υ such that the following inequality holds*

$$-2x^T(t)\int_{t-T}^t y(\sigma)d\sigma - \int_{t-T}^t y^T(\sigma)\Upsilon y(\sigma)d\sigma$$
$$\leq Tx^T(t)\Upsilon^{-1}x(t). \tag{34}$$

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