Model Based Control of Nonlinear Teleoperators

Submitted in partial fulfillment of the requirements
of the course project

AE 666 Adaptive and Learning Control Systems

by

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Chapter 1

Introduction

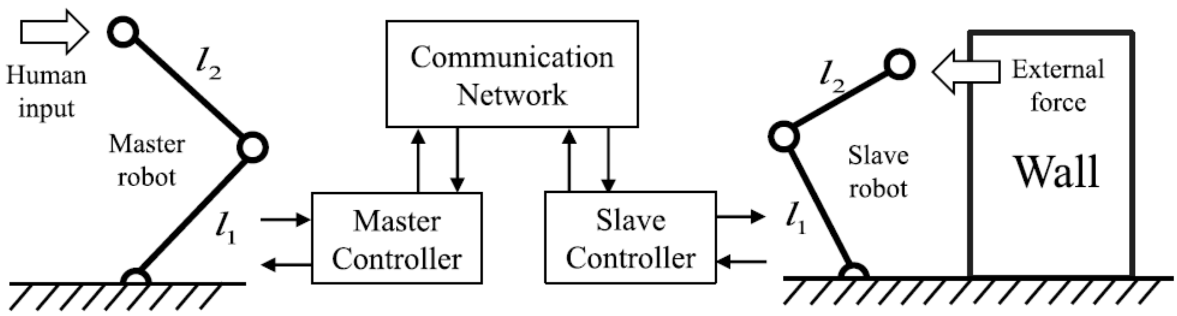

Figure 1.1: The 2 DOF master and slave teleoperation system

A simple teleoperation system typically consists of two robots - a master and a slave robot which communicate over network (wired or wireless, for example, LAN, WAN). A human operator interacts with the master robot with an aim to achieve a desired motion by the slave robot. The slave robot interacts with the environment around it and provides force feedback to the human operator through the master robot. The robots exchange the information about position, force A common example of such a system is a robotic surgery- where a haptic device such as Phantom Omni/Premium is used to control surgical robot like the Da Vinci.

In this work, we consider two planar 2 DOF non-identical robots as the master and the slave robots as in Figure 2.1. The primary control objective of the system is that the slave robot should track the master robot in absence of external force on the slave robot and the later should also reflect the environment force to the master robot (human operator). A force sensor can be used to achieve the force feedback objective, but due to high cost associated with industrial force sensors, this objective need to be achieved through the control system. The position sensor information of both the robots is available to each of the master and slave controller through network.

1.1 Single Robot Dynamics

The general dynamical equation of a robotic manipulator is given by-

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=\tau \tag{1.1}$$

where $q \in \mathbb{R}^n$ denotes the generalised configuration coordinates (joint angles in this case), $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix (a function of the joint angles) and $C(q,\dot{q})\dot{q} \in \mathbb{R}^{n\times n}$ matrix consist of the centrifugal and coriolis terms which is a function of joint angles as well as joint velocities, $g(q)=\frac{\partial H(q)}{\partial q} \in \mathbb{R}^n$ is the gradient of potential function $H(q)$ (generally the potential due to gravity), and $\tau \in \mathbb{R}^n$ is the input joint torque. Spong et al. (2006) We consider non redundant robotics manipulators and $X \in \mathbb{R}^n$ is the end-effector position of the robot. The relationship between the task space position X and joint space vector q is defined as-

$$X=h(q) \tag{1.2}$$

where $h(.) : \mathbb{R}^n \to \mathbb{R}^n$ is the transformation mapping from joint space to the task space. The task space velocity $\dot{X}$is related to the joint space velocity $\dot{q}$ by the following equation-

$$\dot{X} = J(q)\dot{q} \tag{1.3}$$

The dynamics is derived from Lagrangian method and has some special properties as follows-

1. For any differentiable vector $\eta \in \mathbb{R}^n$, the Lagrangian dynamics are linearly parameterizable that is

$$M(q)\dot{\xi} + C(q,\dot{q})\xi + g(q) = Y_d(q,\dot{q},\xi,\dot{\xi})\Theta_d \tag{1.4}$$

   where $\Theta_d \in \mathbb{R}^p$ is constant vector of unknown parameters and $Y_d(q,\dot{q},\xi,\dot{\xi}) \in \mathbb{R}^{n\times p}$ is the matrix of known functions of generalised coordinates and their derivatives.

2. The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric such that

$$\xi^T(\dot{M}(q) - 2C(q,\dot{q}))\xi = 0 \tag{1.5}$$

   where $\xi \in \mathbb{R}^n$

3. The matrix $M(q)$ is symmetric positive definite and there exists positive constants $\underline{m} <$ and $\overline{m}$ such that

$$\underline{m}I_n \leq M(q) \leq \overline{m}I_n \tag{1.6}$$

   where $I_n \in \mathbb{R}^{b\times n}$ is an identity matrix.

4. For $q, \dot{q}, \xi \in \mathbb{R}^n$, there exists a positive constant β_c such that the matrix of coriolis/centrifugal forces is bounded by

$$\|C(q,\dot{q})\xi\| \leq \beta_c\|\dot{q}\|\|\xi\| \tag{1.7}$$

where $\|.\|$ denotes the Euclidean norm of the enclosed signal.

5. For any differentiable vector $\xi \in \mathbb{R}^n$, the product of the Jacobian and the vector ξ can be linearly parameterized by-

$$J(q)\xi = Y_k(q,\xi)\Theta_k \tag{1.8}$$

where $\Theta_k \in \mathbb{R}^w$ is a constant vector of kinematic parameters and $Y_k(q,\xi)$ is the kinematic regressor matrix.

1.2 Teleoperation System Dynamics

The master and slave robots are assumed to have same degree of freedom. The teleoperation system dynamics is given by-

$$\begin{aligned} M_m(q_m)\ddot{q_m} + C_m(q_m\dot{q_m}), \dot{q_m} + g_m(q_m) &= \tau_m + J_m^T F_h \\ M_s(q_s)\ddot{q_s} + C_s(q_s\dot{q_s}), \dot{q_s} + g_s(q_s) &= \tau_s - J_s^T F_e \end{aligned} \tag{1.9}$$

where the subscripts m,s denote the master and slave robots respectively. $F_h, F_s \in \mathbb{R}^n$ are the human and environmental forces exerted on the master and slave robots respectively. These are applied on the end effectors and hence are transformed to the joint space by Jacobian $J_m(q_m), J_s(q_s) \in \mathbb{R}^{n\times n}$

The task space and joint space are related as-

$$\begin{aligned} X_m = h_m(q_m),\ \dot{X}_m = J(q_m)\dot{q}_m \\ X_s = h_s(q_s),\ \dot{X}_s = J(q_s)\dot{q}_s \end{aligned} \tag{1.10}$$

The task space positions X_m and X_s measurements are available to the controller through sensor like vision system, laser trackers, position sensors, etc. The tracking error between the master and slave robots are defined by-

$$e_m = X_s - X_m, \; e_s = X_m - X_s \tag{1.11}$$

The controller design objective to guarantee stability and task-space position tracking, that is, $lim_{t\to\infty} e_m(t) = 0, \; lim_{t\to\infty} e_s(t) = 0$

1.3 Robot Parameters Considered

The master and slave robots are considered as non identical 2-DOF planar robot manipulators. The physical paramters are choses as $m_m = [3.14, 2.26]kg, m_s = [2.54, 1.82]kg, I_m = [0.16, 0.07]kgm^2, I_s = [0.12, 0.06]kgm^2, l_m = [1.04, 0.96]m, l_s = [1.28, 0.84]m$ with $g = 0m/s^2$.

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Chapter 2

Stability Analysis of Uncontrolled System

The teleoperation system is meaningless if we do not have a controller which connects the two robots. Thus, an uncontrolled system is simply two independent robots. Since we consider the robots to be planar, the gravity torque terms $g(q) = 0$. The system dynamics is given by-

$$\begin{aligned} M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m &= J_m^T F_h \\ M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s &= -J_s^T F_e \end{aligned} \tag{2.1}$$

The dynamics of the master and slave are uncoupled. To investigate more about stability, the 2 DOF master robot is considered. In absence of any external force, the system remains in the initial state as $t \to \infty$. On the application of external force, the system will be disturbed from the initial position (excluding singularities) and will never settle to a steady. In case of singularities $(q_{2m} = 0 \ or \ \pi)$, the system will not move if the force applied is in the radial direction. The system resembles a 2-DOF mass-damper system with with mass and damping being a function of joint angles. For a 1-DOF mass damper system, the system will never return to initial position upon perturbation. The same behavior is expected in the 2-DOF system, it will not be guaranteed that the system returns to the initial position on perturbation for any initial configuration (except singularities with radial force input). The same is demonstrated through the simulations. A force input of 1N in both directions is given to the system for 0.1 seconds and it is observed that the system is set into oscillations.

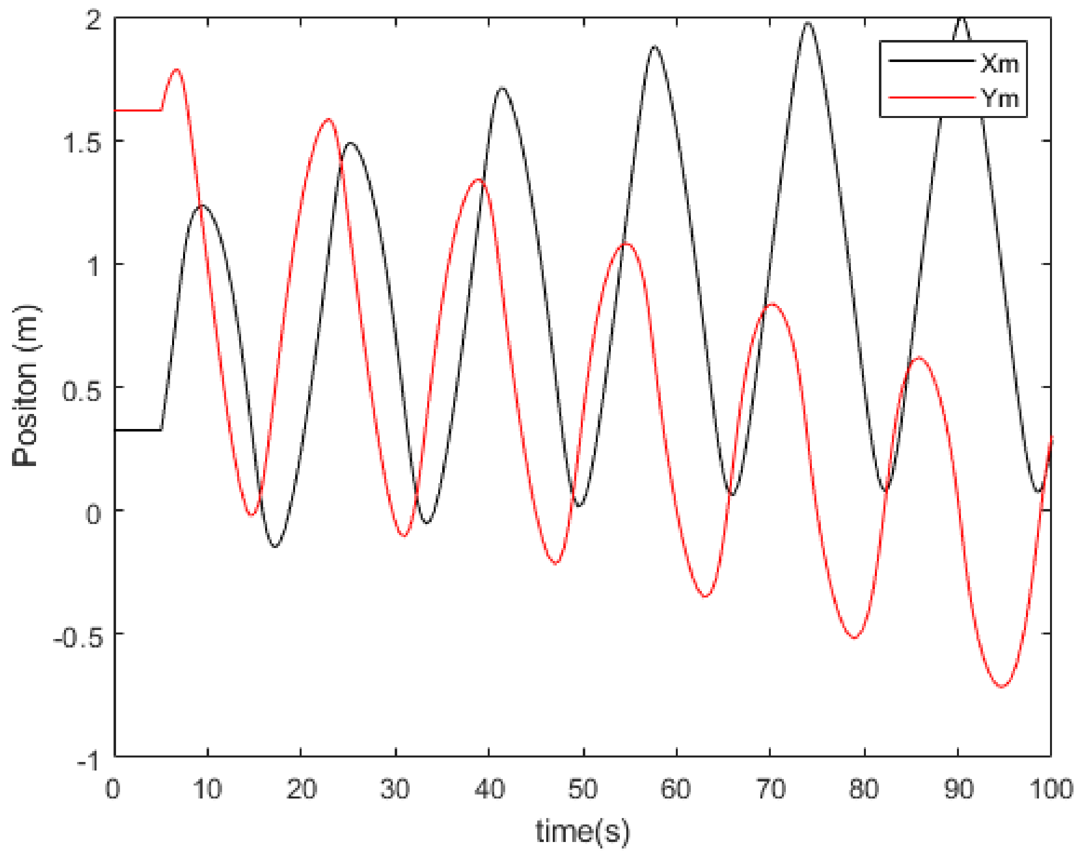

Figure 2.1: Response to Pulse Force Input

A single robot is typically controlled by using a PD or PID controller for set point tracking.

Chapter 3

Model Based Bilateral Teleoperation Control

3.1 Controller Design

The teleoperation system is highly non-linear as described in the previous sections. The model based control requires accurate modeling of the system for the tracking errors to converge to zero, which is practically difficult to obtain. For this work, it is assumed that accurate model is available. Hence, the availability or non-availability of the task space velocity makes no difference in absence of parameteric uncertainities. Liu and Khong (2015)

The subscript $i = m, s$ represents the master and slave robots respectively. The control input to the system is given by-

$$\begin{aligned}\tau_i &= M_i(q_i)\ddot{q}_{r,i} + C_i(q_i, \dot{q}_i)\dot{q}_{r,i} + g_i(q_i) - k_i s_i - k_r J_i^T r_i + k_d J_i^T \dot{e}_i \\ &= Y_{d,i}(q_i, \dot{q}_i, \dot{q}_{r,i}, \ddot{q}_{r,i})\Theta_{d,i} - k_i s_i - k_r J_i^T r_i + k_d J_i^T \dot{e}_i\end{aligned} \tag{3.1}$$

e_i is the task space velocity error, k_i, k_d, k_r are positive control gains. $\dot{q}_{r,i} \in \mathbb{R}^{\ltimes}$ in 3.1 is the joint space reference velocity given by

$$\dot{q}_{r,i} = J_i^{-1} \lambda e_i \tag{3.2}$$

where λ is positive control gain.

From the above definition, we get

$$\ddot{q}_{r,i} = \dot{J}_i^{-1} \lambda e_i + J_i^{-1} \lambda \dot{e}_i \tag{3.3}$$

where $\dot{J}_i^{-1}$ is the time derivative of the inverse of the Jacobian matrix and is obtained by $-J_i^{-1} \dot{J}_i J_i^{-1}$. The joint space sliding vector $s_i \in \mathbb{R}^{\ltimes}$ in equation 3.2 is given as

$$s_i = \dot{q}_i - \dot{q}_{r,i} \tag{3.4}$$

By defining $r_i = J_i s_i$ and substituting s_i into r_i, we get

$$r_i = J_i s_i = J_i \dot{q}_i - J_i^{-1} \lambda e_i) = J_i \dot{q}_i - \lambda e_i = \dot{X}_i - \lambda e_i \tag{3.5}$$

where $\dot{X}_i = J_i \dot{q}_i$ from equation 1.10 . Substituting the controller (3.1) into the robot dynamics (1.9), the closed loop teleoperation system is obtained as

$$
\begin{aligned}
M_m\dot{s}_m + C_m s_m + k_m s_m &= -k_r J_m^T r_m + k_d J_m^T \dot{e}_m + J_m^T F_h \\
M_s\dot{s}_s + C_s s_s + k_s s_s &= -k_r J_s^T r_s + k_d J_s^T \dot{e}_s - J_s^T F_e
\end{aligned}
\tag{3.6}
$$

The stability through Lyapunov method is proved further.

3.2 Stability Proofs

3.2.1 Tracking In Absence Of External Forces

It is assumed that there are no external forces, that is, F_h and F_e acting on the robots. The set-point tracking is desired in a free motion.

Theorem 3.1. *Consider the closed loop teleoperation system (3.6). If the Jacobian is non-singular, then in free motion with ($F_h = F_e = 0$), the task space position tracking error (e_i) and the velocity tracking error ($\dot{e}_i$) asymptotically approach the origin such that $X_s, \dot{X}_m - \dot{X}_s \to 0$ as $t \to \infty$. Additionally, $\dot{X}_m, \dot{X}_s \to 0$ as $t \to \infty$*

Proof. Consider the positive definite Lyapunov function candidate

$$
V = \frac{1}{2} \sum_{i=m,s} [s_i^T M_i s_i + \lambda k_d e_i^T e_i] \tag{3.7}
$$

The time derivative of the function V along the trajectories of the closed loop system (3.6) is given by

$$
\dot{V} = \sum_{i=m,s} [s_i^T (-C_i s_i - k_i s_i - k_r J_i^T r_i + k_d J_i^T \dot{e}_i) s_i + \lambda k_d e_i^T e_i] \tag{3.8}
$$

With the use of the skew-symmetry property (1.5), and $r_i = J_i^T s_i$. the time derivative of V

becomes

$$\dot{V} = \sum_{i=m,s} [-k_i s_i^T s_i - k_r r_i^T r_i + k_d r_i^T \dot{e}_i + \lambda k_d e_i^T \dot{e}_i] \tag{3.9}$$

Substituting $r_i = \dot{X}_i - \lambda e_i$ given in (3.2),the third term becomes $k_d r_i^T \dot{e}_i = k_d(\dot{X}_i - \lambda e_i)^T \dot{e}_i = k_d \dot{X}_i^T \dot{e}_i - \lambda k_d e_i^T \dot{e}_i$. After combining terms for both robots and simplification

$$\dot{V} = -k_m s_m^T s_m - k_s s_s^T s_s - k_r r_m^T r_m - k_r r_s^T r_s - k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \leq 0 \tag{3.10}$$

Thus, the zero solution of the teleoperation system is stable, and all signals are bounded such that $s_i, e_i \in L_\infty$. Integrating the above equation from $[0,t]$, we find that $s_i, r_i, \dot{X}_m - \dot{X}_s \in L_2$. Since the signals s_i, e_i are bounded and J_i is full rank, we have $\dot{q}_i$ bounded by observing (3.2) and (3.4). Hence, $\dot{X}_i$ is also bounded from (1.3) which leads to $\dot{e}_i \in L_\infty$. From the system dynamics (1.9) implies that the robot acceleration $\ddot{q}_i \in L_\infty$ as $\tau_i, \dot{q}_i$ are bounded from the properties (1.6) and (1.7). Since all signals are bounded, we have $\dot{s}_i \in L_\infty$ from the close loop system (3.6). As $s_i \in L_2$ and $\dot{s}_i \in L_\infty$, by invoking Barbalat's lemmawe conclude that $lim_{t\to\infty} s_i(t) = 0$ which leads to $lim_{t\to\infty} r_i(t) = 0$ since $r_i = J_i s_i$.

By differentiating (1.3) with $q_i, \dot{q}_i, \ddot{q}_i \in L_\infty$, we obtain that $\dot{J}_i \in L_\infty$. From the time derivative of r_i ($\dot{r}_i = \dot{J}_i s_i + J_i \dot{s}_i$), we get $\dot{r}_i \in L_\infty$ from $s_i, \dot{s}_i, J_i, \dot{J}_i$ being bounded. As $\dot{r}_i = \ddot{X}_i - \lambda \dot{e}_i$, $\ddot{X}_i$ is bounded. $q_i \in L_\infty$ implies that $X_i \in L_\infty$ from (1.2). Therefore, the velocity is uniformly continuous and $lim_{t\to\infty} \int_0^t \dot{X}_i(\sigma) d\sigma$ exists and is finite. By invoking Barbalat's lemma again, we conclude that $lim_{t\to\infty} \dot{X}_i(t) = 0$, which leads to $lim_{t\to\infty} \dot{q}_i(t) = 0$ from (1.10) as J_i is invertible.

As $lim_{t\to\infty} \dot{X}_i(t) = 0$ and $lim_{t\to\infty} r_i(t) = 0$ and from (3.5), the tracking error between the master and slave robots in teleoperation system converge to origin asymptotically such that $lim_{t\to\infty} e_i(t) = 0$. Further, differentiating $\dot{X}_i = J_i \dot{q}_i$, we obtain $\ddot{X}_i \in L_\infty$ as $\dot{q}_i, \ddot{q}_i, \dot{J}_i$ are bounded. Since $\dot{X}_m - \dot{X}_s \in L_2$ and $\ddot{X}_m - \ddot{X}_s \in L_\infty$, we conclude that $\dot{X}_m - \dot{X}_s \to 0$ as $t \to \infty$ such that $lim_{t\to\infty} \dot{e}_m(t) = lim_{t\to\infty} \dot{e}_s(t) = 0$.

□

3.2.2 Tracking In Presence Of External Forces

Theorem 3.2. *Consider the closed loop teleoperation system (3.6). If the Jacobian is non-singular*

1. *If $F_h, F_e \in L_\infty$ and $k_r > 1/2$, t then all signals of the bilateral teleoperation system are bounded.*

2. *If $F_h, F_e \in L_\infty, L_2$ and $k_r > 1/2$, then the task space position tracking error (e_i) and the velocity tracking error ($\dot{e}_i$) asymptotically approach the origin as $t \to \infty$.*

3. *If the gravitational term $g(q)$ of the robotic manipulators is zero or precompensated, when $(\dot{q}_m, \dot{q}_s, \ddot{q}_m, \ddot{q}_s) \to 0$, then the human and environmental forces are proportional to the tracking errors,i.e, $F_h \propto e_m$ and $F_e \propto e_s$.*

Proof. 1. Consider the Lyapunov function candidate (3.7). Taking the time derivative along the trajectories of the closed loop teleoperation system (3.6) and following the analysis in Theorem 3.1, we have

$$\dot{V} = s_m^T J_m^T F_h - s_s^T J_s^T F_e - k_m s_m^T s_m - k_s s_s^T s_s - k_r r_m^T r_m - k_r r_s^T r_s - k_d (\dot{X}_m - \dot{X}_s)^T (\dot{X}_m - \dot{X}_s) \tag{3.11}$$

Since $r_i = J_i s_i$, the first two terms on the right hand side become $r_m^T F_h$ and $r_s^T F_e$. By utilizing Young's inequality, we have that $r_m^T F_h \leq 1/2 r_m^T r_m + 1/2 F_h^T F_h$ and $r_s^T F_e \leq 1/2 r_s^T r_s + 1/2 F_e^T F_e$. For $k_r = 1/2 + \varepsilon$ with positive constant value ε, the derivative of V becomes

$$\dot{V} \leq 1/2 F_h^T F_h + 1/2 F_e^T F_e - k_m s_m^T s_m - k_s s_s^T s_s - \varepsilon r_m^T r_m - \varepsilon r_s^T r_s - k_d (\dot{X}_m - \dot{X}_s)^T (\dot{X}_m - \dot{X}_s) \tag{3.12}$$

Since the external forces F_h, F_e are bounded and J_m, J_s are non singular, the signals F_h and F_e are also bounded. The derivative of V in 3.12 can be considered for two different cases such that $\dot{V} \geq 0$ and $\dot{V} < 0$. If $\dot{V} \geq 0$, then $1/2 F_h^T F_h + 1/2 F_e^T F_e \geq k_m s_m^T s_m +$

$k_s s_s^T s_s + \varepsilon r_m^T r_m + \varepsilon r_s^T r_s + k_d(\dot{X}_m + \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \geq 0$. By considering the aforementioned inequality with V in (3.12), the states of teleoperation system will not grow unbounded while F_h and F_e are bounded. Therefore, s_i, r_i, and $(\dot{X}_m - \dot{X}_s)$ are all bounded. From the closed loop control system equation (3.6), all signals are bounded. Additionally, if $\dot{V} < 0$, then V is non-increasing function and $s_i, e_i \in L_\infty$. As s_i and e_i are bounded, (3.4) and (3.5) lead to $\dot{q}_i$ and r_i being bounded. From (1.3), we get $\dot{X}_i, \dot{e}_i \in L_\infty$. Consequently, all signals of teleoperation system are bounded.

2. By integrating the inequality (3.12) from 0 to t with $V(t)$ being positive definite, we have that $V(0) + 1/2\|F_h\|_2^2 + 1/2\|F_e\|_2^2 \geq k_m\|s_m\|_2^2 + k_s\|s_s\|_2^2 + \varepsilon\|r_m\|_2^2 + \varepsilon\|r_s\|_2^2 + k_d\|\dot{X}_m - \dot{X}_s\|_2^2$, where $\|.\|_2$ denotes the L_2 norm of the signal. Since F_h and F_e are square integrable, the aforementioned inequality results in $s_i, r_i, \dot{X}_m - \dot{X}_s \in L_2$. As F_h and F_e are bounded, all signals of the teleoperation system are bounded as discussed in the previous claim. Thus, following the proof in Theorem 3.1, the position and velocity tracking errors converge to the origin asymptotically such that $X_m - X_s, \dot{X}_m - \dot{X}_s \to 0$ as $t \to \infty$.

3. The last claim addresses the force reflection in the teleoperation system. In the closed loop system (3.6), if $(\dot{q}_m, \dot{q}_s, \ddot{q}_m, \ddot{q}_s) \to 0$, then $\dot{s}_i, \dot{q}_i \to 0$. Convergence of $\dot{q}_i$ to the origin with (1.3) and the property 1.7 implies that e_i and $C_i(q_i, \dot{q}_i)s_i$ approach zero. Consequently, from the closed loop system, we get

$$\begin{aligned} F_h &\to (J_m^T)^{-1}(k_m s_m + k_r J_m^T r_m) \\ F_e &\to (J_s^T)^{-1}(-k_s s_s - k_r J_s^T r_s) \end{aligned} \tag{3.13}$$

As $s_i \to -\lambda J_i^{-1} e_i$ and $r_i \to -e_i$ in steady state, the external forces can be written as

$$\begin{aligned} F_h &\to (-k_m \lambda (J_m J_m^T)^{-1} - k_r \lambda I_n) e_m \\ F_e &\to (k_s \lambda (J_s J_s^T)^{-1} + k_r \lambda I_n) e_s \end{aligned} \tag{3.14}$$

Since, J_i is bounded and invariant in steady state, we conclude that $F_h \propto e_m, F_e \propto e_s$ and the force feedback error is bounded.

□

Chapter 4

Simulation Results

The initial position and velocity are $q_m(0) = [0.8, 1.2]rad, q_s(0) = [1.0, 0.3]rad$, and $\dot{q}_m(0) = \dot{q}_s(0) = [0, 0]rad/s$. The control gains are selected as $\lambda = 0.8, k_m = k_s = 1, k_r = 4, k_d = 1, k_{pm} = k_{ps} = 0.5$. In this simulation, the teleoperation system is considered to move in free motion before $t = 15s$. After $t = 15s$, the human operator applies a spring damper force [Lee and Spong (2006), Liu and Chopra (2013)] on the end effector of the master robot to change the configuration. A virtual wall is implemented at in the remote environment at $x = 1m$ as seen in figure 2.1. The slave robot will be subject to an external force if its x position is larger than $1m$.

As shown in figures 4.1, 4.2, 4.3 the slave robot stably tracks the position of the master robot and the tracking errors converge to zero asymptotically in absence of environmental force. The position tracking is not achieved during $t = 30\ 45s$ because the slave robot is in contact with the remote environment. The external spring damper forces are shown in figures 4.4, 4.6 which demonstrates that the environmental force is reflected back to the human operator, although it is about $1/3$ times the magnitude (X-direction) which needs to be investigated further. The control forces are shown in figures 4.8, 4.10 which have quite high peaks, which can be controlled by tuning the input human force appropriately.

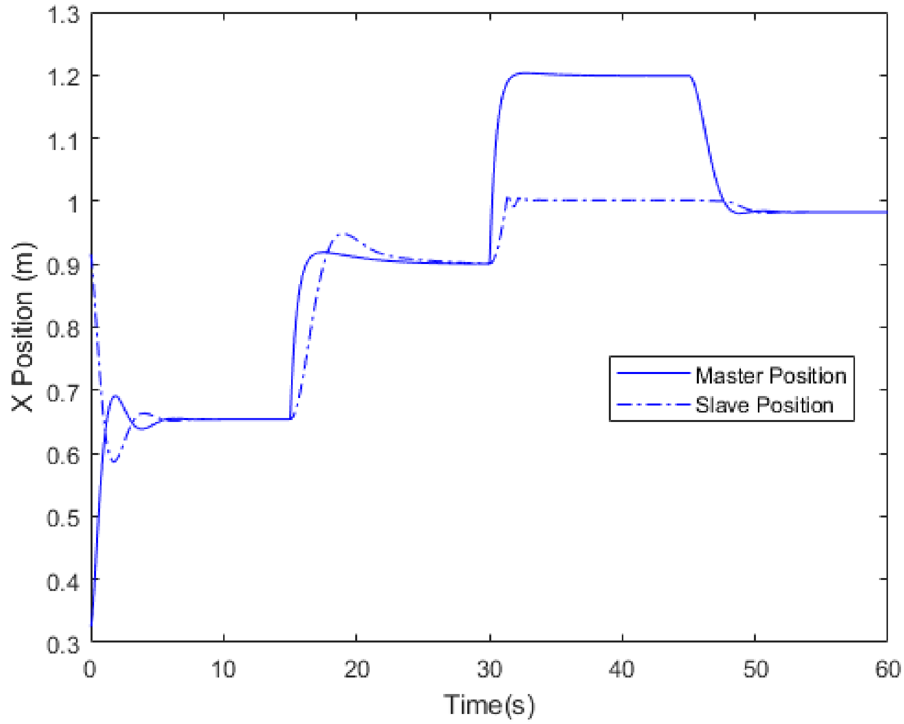

Figure 4.1: Simulation Results for X position tracking

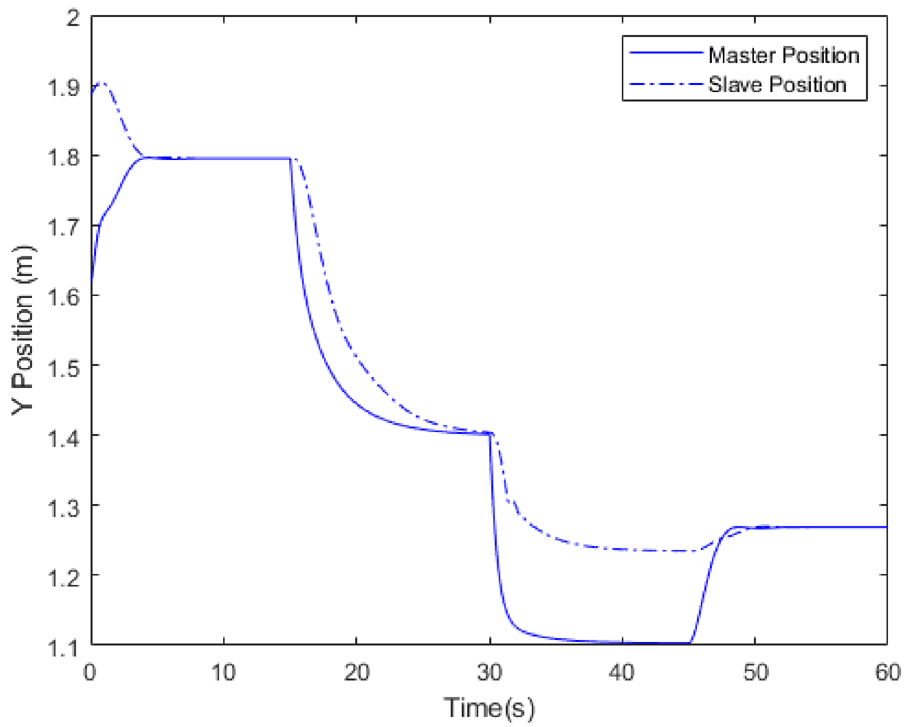

Figure 4.2: Simulation Results for Y position tracking

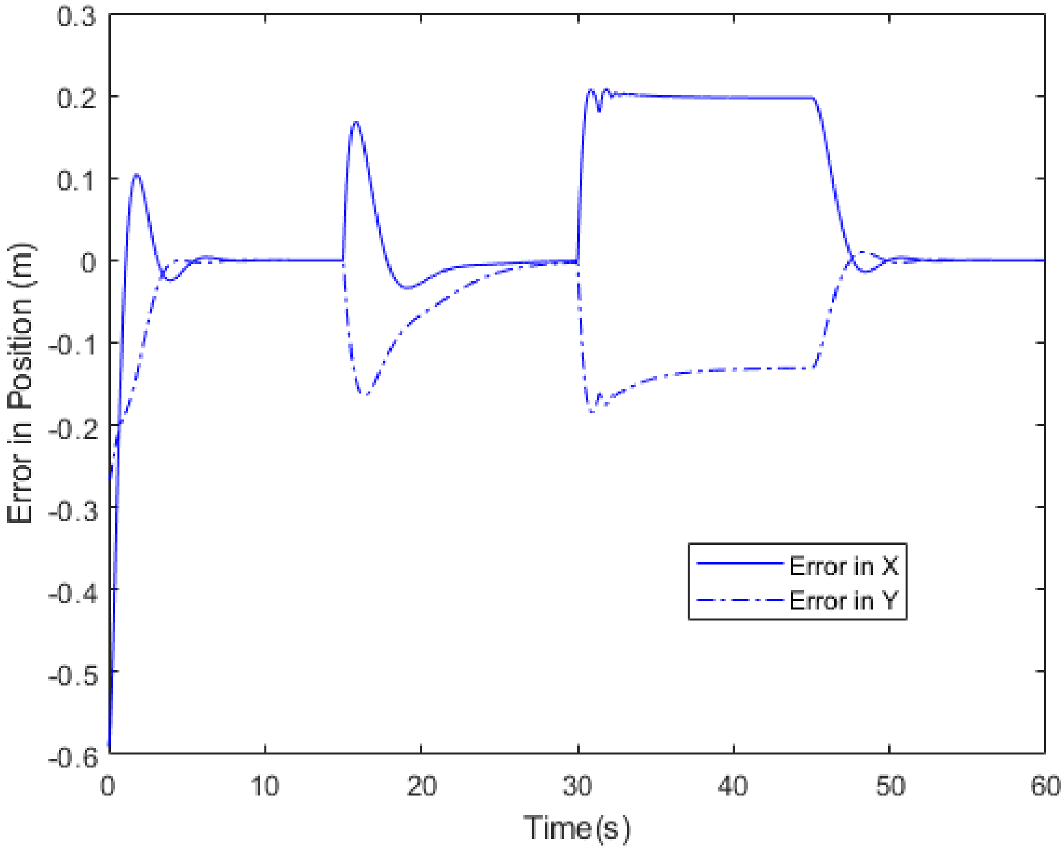

Figure 4.3: Simulation Results for error in position tracking

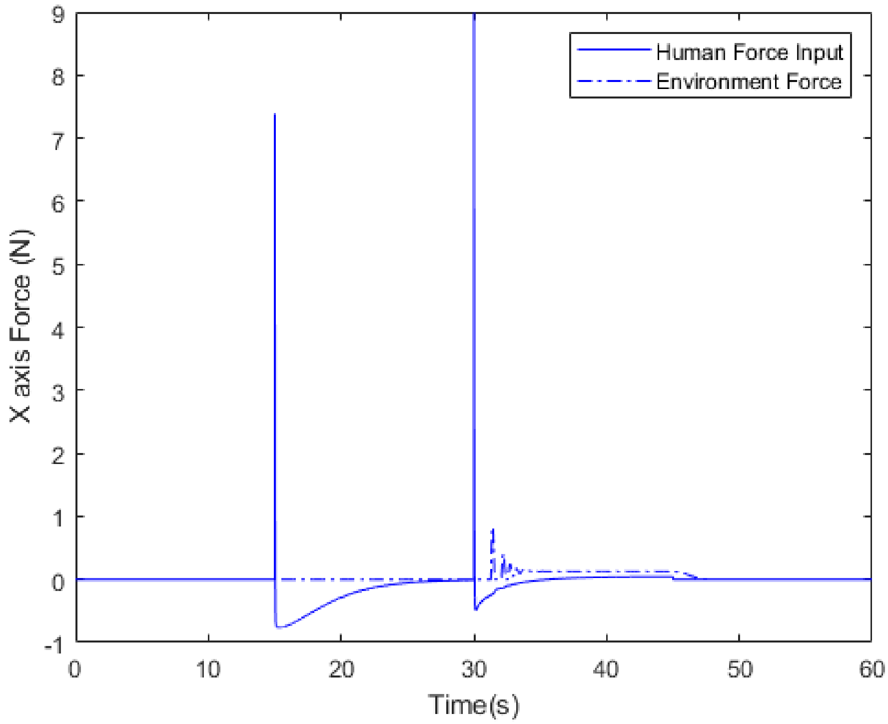

Figure 4.4: Simulation Results for X axis Force

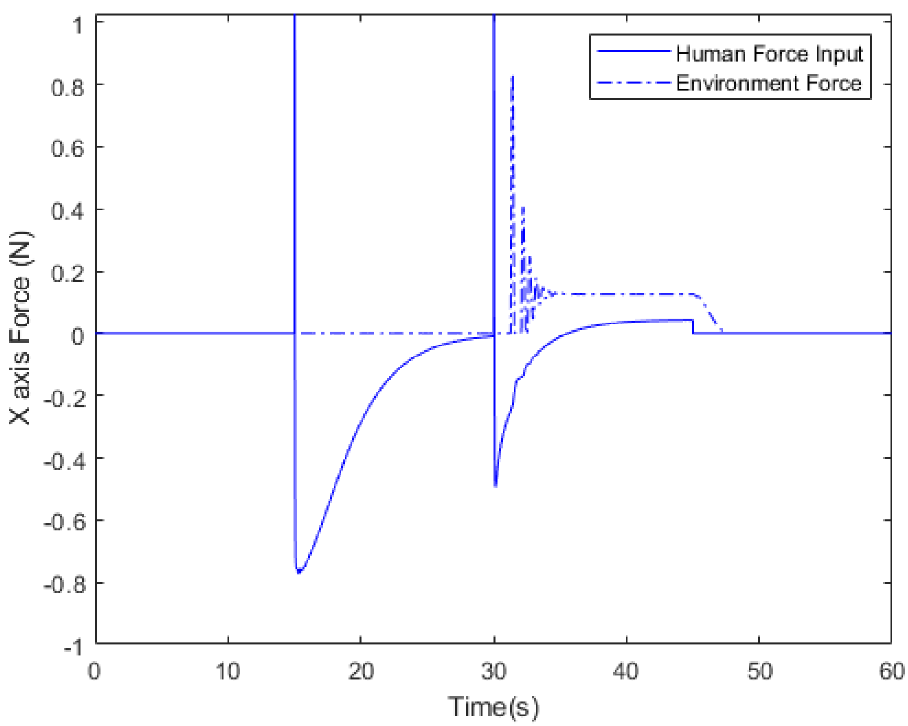

Figure 4.5: Zoomed view of Simulation Results for X axis Force

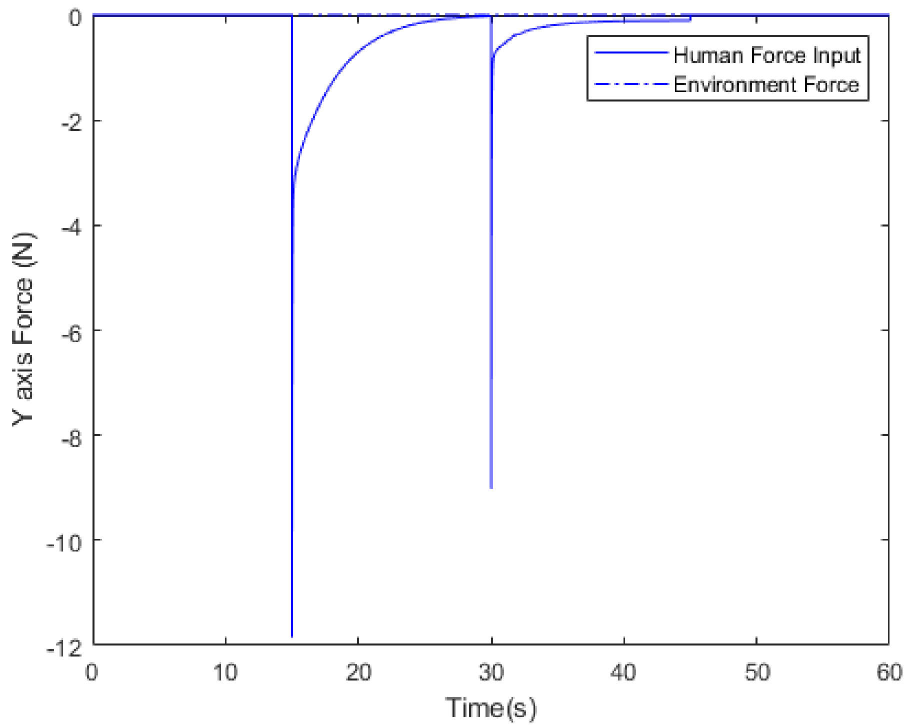

Figure 4.6: Simulation Results for Y axis Force

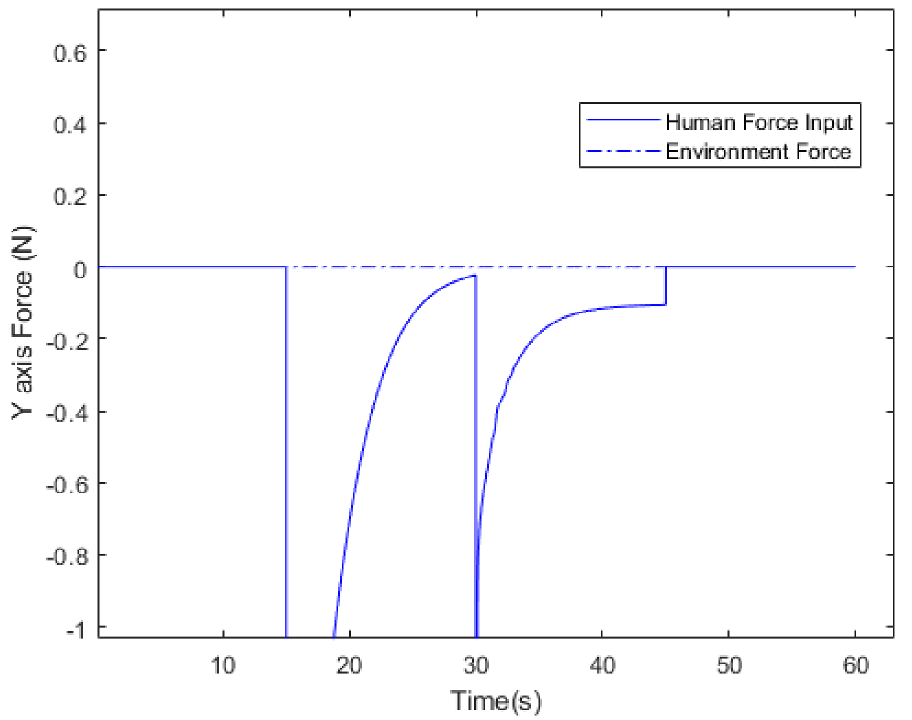

Figure 4.7: Zoomed view of Simulation Results for Y axis Force

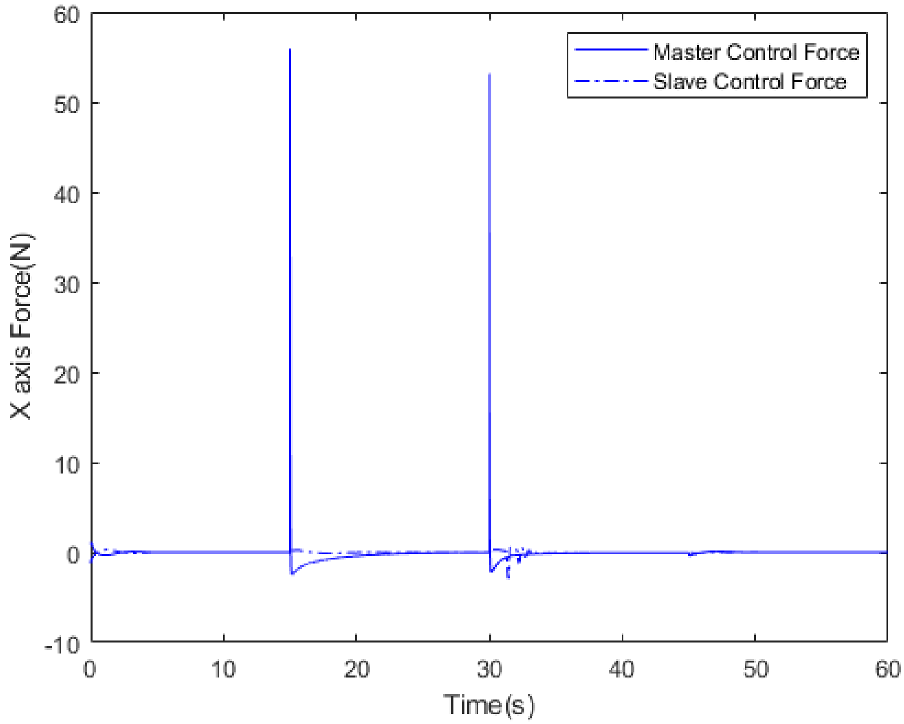

Figure 4.8: Simulation Results for X axis Control Force

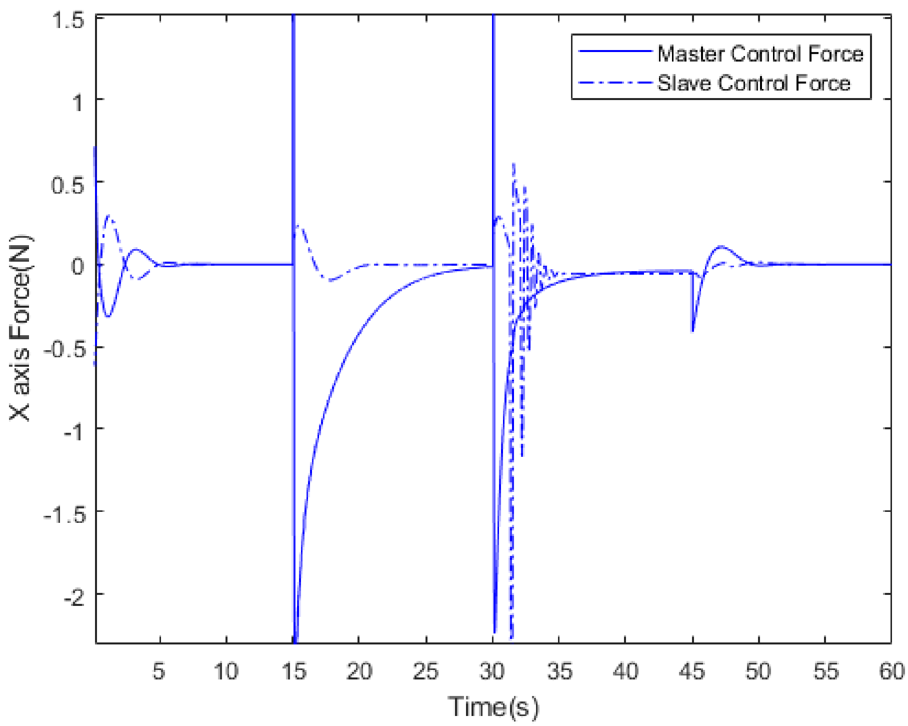

Figure 4.9: Zoomed view of Simulation Results for X axis Control Force

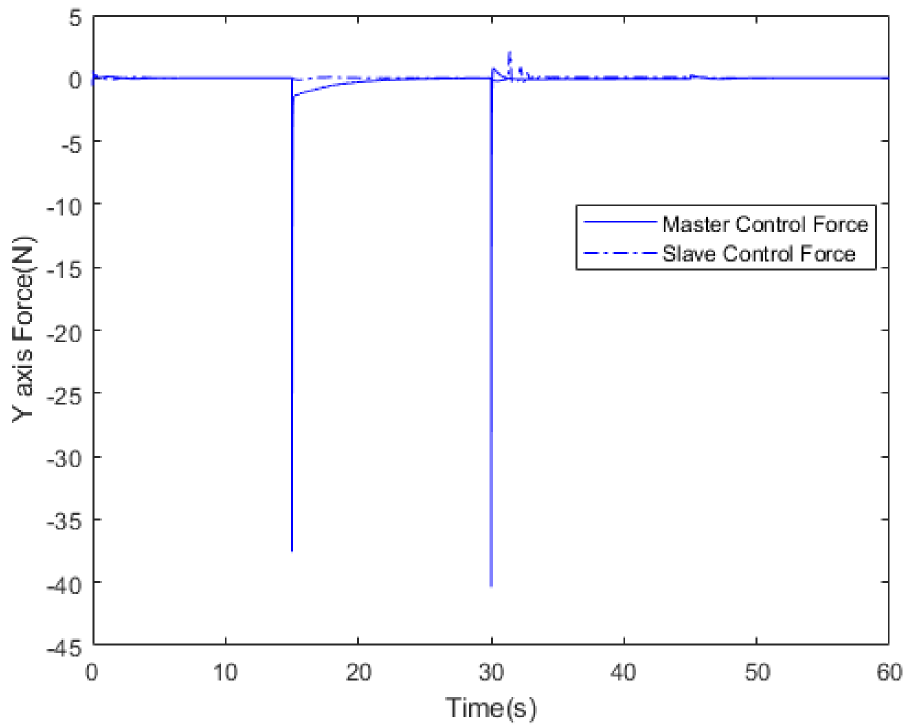

Figure 4.10: Simulation Results for Y axis Control Force

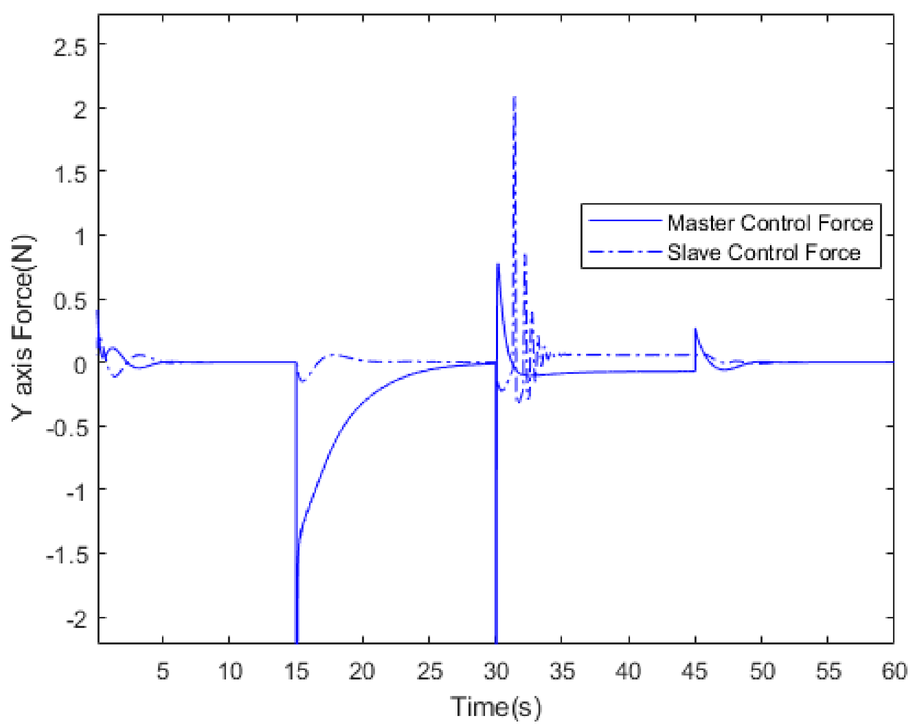

Figure 4.11: Zoomed view of Simulation Results for Y axis Control Force

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