Adaptive Control For Nonlinear Teleoperators With Uncertain Kinematics and Dynamics

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Introduction

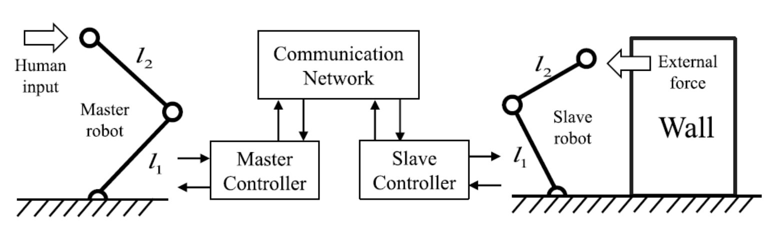


Figure: The 2 DOF master and slave teleoperation system

- A simple teleoperation system typically consists of two robots - a master and a slave robot which communicate over network
- A human operator interacts with the master robot with an aim to achieve a desired motion by the slave robot
- The slave robot interacts with the environment around it and provides force feedback to the human operator through the master robot
- The robots exchange the information about position, force

Single Robot Dynamics and Properties

- The general dynamical equation of a robotic manipulator is given by-

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{1}$$

  where $q \in \mathbb{R}^n$ denotes the generalised configuration coordinates (joint angles in this case), $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix (a function of the joint angles) and $C(q,\dot{q})\dot{q} \in \mathbb{R}^{n\times n}$ matrix consist of the centrifugal and coriolis terms which is a function of joint angles as well as joint velocities, $g(q) = \frac{\partial H(q)}{\partial q} \in \mathbb{R}^n$ is the gradient of potential function $H(q)$ (generally the potential due to gravity), and $\tau \in \mathbb{R}^n$ is the input joint torque.
- The relationship between the task space position X and joint space vector q is defined as-

$$X = h(q) \tag{2}$$

- The task space velocity $\dot{X}$is related to the joint space velocity $\dot{q}$ by the following equation-

$$\dot{X} = J(q)\dot{q} \tag{3}$$

- For teleoperation system with kinematic uncertainty, the parameters of the Jacobian matrix is unknown, so the estimated task space positions and velocities are expressed as-

$$\begin{aligned} \hat{X}_m = \hat{h}_m(q_m),\ \ \dot{\hat{X}}_m = \hat{J}_m(q_m)\dot{q}_m = Y_{k,m}(q_m,\dot{q_m})\hat{\Theta}_{k,m} \\ \hat{X}_s = \hat{h}_s(q_s),\ \ \dot{\hat{X}}_s = \hat{J}_s(q_s)\dot{q}_s = Y_{k,s}(q_s,\dot{q}_s)\hat{\Theta}_{k,s} \end{aligned} \tag{4}$$

Basic Properties

1. For any differentiable vector $\eta \in \mathbb{R}^n$, the Lagrangian dynamics are linearly parameterizable that is

$$M(q)\dot{\xi} + C(q,\dot{q})\xi + g(q) = Y_d(q,\dot{q},\xi,\dot{\xi})\Theta_d \tag{5}$$

   where $\Theta_d \in \mathbb{R}^p$ is constant vector of unknown parameters and $Y_d(q,\dot{q},\xi,\dot{\xi}) \in \mathbb{R}^{n\times p}$ is the matrix of known functions of generalised coordinates and their derivatives.

2. The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric such that

$$\xi^T(\dot{M}(q) - 2C(q,\dot{q}))\xi = 0 \tag{6}$$

   where $\xi \in \mathbb{R}^n$

3. The matrix $M(q)$ is symmetric positive definite and there exists positive constants $\underline{m} <$ and $\overline{m}$ such that

$$\underline{m}I_n \leq M(q) \leq \overline{m}I_n \tag{7}$$

   where $I_n \in \mathbb{R}^{b\times n}$ is an identity matrix.

4. For $q, \dot{q}, \xi \in \mathbb{R}^n$, there exists a positive constant β_c such that the matrix of coriolis/centrifugal forces is bounded by

$$\|C(q,\dot{q})\xi\| \leq \beta_c\|\dot{q}\|\|\xi\| \tag{8}$$

   where $\|.\|$ denotes the Euclidean norm of the enclosed signal.

5. For any differentiable vector $\xi \in \mathbb{R}^n$, the product of the Jacobian and the vector ξ can be linearly parameterized by-

$$J(q)\xi = Y_k(q,\xi)\Theta_k \tag{9}$$

   where $\Theta_k \in \mathbb{R}^w$ is a constant vector of kinematic parameters and $Y_k(q,\xi)$ is the kinematic regressor matrix. [Spong et al.(2006)Spong, Hutchinson, Vidyasagar, et al.]

Teleoperation System Dynamics

- The teleoperation system dynamics is given by-

$$\begin{aligned} M_m(q_m)\ddot{q_m} + C_m(q_m\dot{q_m}), \dot{q_m} + g_m(q_m) = \tau_m + J_m^T F_h \\ M_s(q_s)\ddot{q_s} + C_s(q_s\dot{q_s}), \dot{q_s} + g_s(q_s) = \tau_s - J_s^T F_e \end{aligned} \tag{10}$$

  where the subscripts m, s denote the master and slave robots respectively. $F_h, F_s \in \mathbb{R}^n$ are the human and environmental forces exerted on the master and slave robots respectively. These are applied on the end effectors and hence are transformed to the joint space by Jacobian $J_m(q_m), J_s(q_s) \in \mathbb{R}^{n\times n}$

- The task space and joint space are related as-

$$\begin{aligned} X_m = h_m(q_m),\ \ \dot{X}_m = J(q_m)\dot{q}_m \\ X_s = h_s(q_s),\ \ \dot{X}_s = J(q_s)\dot{q}_s \end{aligned} \tag{11}$$

- The tracking error between the master and slave robots are defined by-

$$e_m = X_s - X_m,\ \ e_s = X_m - X_s \tag{12}$$

  The controller design objective to guarantee stability and task-space position tracking, that is, $lim_{t\to\infty} e_m(t) = 0,\ \ lim_{t\to\infty} e_s(t) = 0$

Stability Analysis of Uncontrolled System

- An uncontrolled system is simply two independent robots. The dynamics of the master and slave are uncoupled.
- To investigate more about stability, the 2 DOF master robot is considered. In absence of any external force, the system remains in the initial state as $t \to \infty$. On the application of external force, the system will be disturbed from the initial position (excluding singularities) and will never settle to a steady state.
- A force input of 1N in both directions is given to the system for 0.1 seconds and it is observed that the system is set into oscillations.

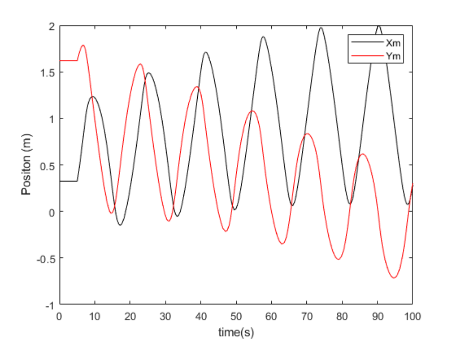


Figure: Response to Pulse Force Input

Model Based Bilateral Teleoperation Control

- The subscript $i = m, s$ represents the master and slave robots respectively. The control input to the system is given by-

$$\begin{aligned} \tau_i &= M_i(q_i)\ddot{q}_{r,i} + C_i(q_i, \dot{q}_i)\dot{q}_{r,i} + g_i(q_i) - k_i s_i - k_r J_i^T r_i + k_d J_i^T \dot{e}_i \\ &= Y_{d,i}(q_i, \dot{q}_i, \dot{q}_{r,i}, \ddot{q}_{r,i})\Theta_{d,i} - k_i s_i - k_r J_i^T r_i + k_d J_i^T \dot{e}_i \end{aligned} \tag{13}$$

  e_i is the task space velocity error, k_i, k_d, k_r are positive control gains. $\dot{q}_{r,i} \in \mathbb{R}^{\ltimes}$ in 27 is the joint space reference velocity given by

$$\dot{q}_{r,i} = J_i^{-1}\lambda e_i \tag{14}$$

  where λ is positive control gain.
- From the above definition, we get

$$\ddot{q}_{r,i} = \dot{J}_i^{-1}\lambda e_i + J_i^{-1}\lambda \dot{e}_i \tag{15}$$

  where $\dot{J}_i^{-1}$ is the time derivative of the inverse of the Jacobian matrix and is obtained by $-J_i^{-1}\dot{J}_i J_i^{-1}$.
- The joint space sliding vector $s_i \in \mathbb{R}^{\ltimes}$ in equation 28 is given as

$$s_i = \dot{q}_i - \dot{q}_{r,i} \tag{16}$$

- By defining $r_i = J_i s_i$ and substituting s_i into r_i, we get

$$r_i = J_i s_i = J_i \dot{q}_i - J_i^{-1}\lambda e_i) = J_i \dot{q}_i - \lambda e_i = \dot{X}_i - \lambda e_i \tag{17}$$

  where $\dot{X}_i = J_i \dot{q}_i$ from equation 11
- the closed loop teleoperation system is obtained as

$$\begin{aligned} M_m \dot{s}_m + C_m s_m + k_m s_m &= -k_r J_m^T r_m + k_d J_m^T \dot{e}_m + J_m^T F_h \\ M_s \dot{s}_s + C_s s_s + k_s s_s &= -k_r J_s^T r_s + k_d J_s^T \dot{e}_s - J_s^T F_e \end{aligned} \tag{18}$$

Stability Proof - In Absence Of External Forces

Theorem

Consider the closed loop teleoperation system (32). If the Jacobian is non-singular, then in free motion with $(F_h = F_e = 0)$*, the task space position tracking error* (e_i) *and the velocity tracking error* $(\dot{e}_i)$ *asymptotically approach the origin such that* $X_s, \dot{X}_m - \dot{X}_s \to 0$ *as* $t \to \infty$*. Additionally,* $\dot{X}_m, \dot{X}_s \to 0$ *as* $t \to \infty$

Proof.

Consider the positive definite Lyapunov function candidate

$$V = \frac{1}{2} \sum_{i=m,s} [s_i^T M_i s_i + \lambda k_d e_i^T e_i] \tag{19}$$

The time derivative of the function V along the trajectories of the closed loop system (32) is given by

$$\dot{V} = \sum_{i=m,s} [s_i^T (-C_i s_i - k_i s_i - k_r J_i^T r_i + k_d J_i^T \dot{e}_i) s_i + \lambda k_d e_i^T e_i] \tag{20}$$

With the use of the skew-symmetry property (6), and $r_i = J_i^T s_i$. the time derivative of V becomes

$$\dot{V} = \sum_{i=m,s} [-k_i s_i^T s_i - k_r r_i^T r_i + k_d r_i^T \dot{e}_i + \lambda k_d e_i^T \dot{e}_i] \tag{21}$$

□

Stability Proof - In Absence Of External Forces

Proof.

Substituting $r_i = \dot{X}_i - \lambda e_i$ given in (28),the third term becomes $k_d r_i^T \dot{e}_i = k_d(\dot{X}_i - \lambda e_i)^T \dot{e}_i = k_d \dot{X}_i^T \dot{e}_i - \lambda k_d e_i^T \dot{e}_i$. After combining terms for both robots and simplification

$$\dot{V} = -k_m s_m^T s_m - k_s s_s^T s_s - k_r r_m^T r_m - k_r r_s^T r_s - k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \leq 0 \tag{22}$$

Thus, the zero solution of the teleoperation system is stable, and all signals are bounded such that $s_i, e_i \in L_\infty$. Integrating the above equation from $[0, t]$, we find that $s_i, r_i, \dot{X}_m - \dot{X}_s \in L_2$. Since the signals s_i, e_i are bounded and J_i is full rank, we have $\dot{q}_i$ bounded by observing (28) and (30). Hence, $\dot{X}_i$ is also bounded from (3) which leads to $\dot{e}_i \in L_\infty$. From the system dynamics (10) implies that the robot acceleration $\ddot{q}_i \in L_\infty$ as $\tau_i, \dot{q}_i$ are bounded from the properties (7) and (8). Since all signals are bounded, we have $\dot{s}_i \in L_\infty$ from the close loop system (32). As $s_i \in L_2$ and $\dot{s}_i \in L_\infty$, by invoking Barbalat's lemma we conclude that $lim_{t\to\infty} s_i(t) = 0$ which leads to $lim_{t\to\infty} r_i(t) = 0$ since $r_i = J_i s_i$.
By differentiating (3) with $q_i, \dot{q}_i, \ddot{q}_i \in L_\infty$, we obtain that $\dot{J}_i \in L_\infty$. From the time derivative of r_i ($\dot{r}_i = \dot{J}_i s_i + J_i \dot{s}_i$), we get $\dot{r}_i \in L_\infty$ from $s_i, \dot{s}_i, J_i, \dot{J}_i$ being bounded. As $\dot{r}_i = \ddot{X}_i - \lambda \dot{e}_i$, $\ddot{X}_i$ is bounded. $q_i \in L_\infty$ implies that $X_i \in L_\infty$ from (2). Therefore, the velocity is uniformly continuous and $lim_{t\to\infty} \int_0^t \dot{X}_i(\sigma)d\sigma$ exists and is finite. By invoking Barbalat's lemma again, we conclude that $lim_{t\to\infty} \dot{X}_i(t) = 0$, which leads to $lim_{t\to\infty} \dot{q}_i(t) = 0$ from (11) as J_i is invertible.
As $lim_{t\to\infty} \dot{X}_i(t) = 0$ and $lim_{t\to\infty} r_i(t) = 0$ and from (31), the tracking error between the master and slave robots in teleoperation system converge to origin asymptotically such that $lim_{t\to\infty} e_i(t) = 0$. Further, differentiating $\dot{X}_i = J_i \dot{q}_i$, we obtain $\ddot{X}_i \in L_\infty$ as $\dot{q}_i, \ddot{q}_i, \dot{J}_i$ are bounded. Since $\dot{X}_m - \dot{X}_s \in L_2$ and $\ddot{X}_m - \ddot{X}_s \in L_\infty$, we conclude that $\dot{X}_m - \dot{X}_s \to 0$ as $t \to \infty$ such that $lim_{t\to\infty} \dot{e}_m(t) = lim_{t\to\infty} \dot{e}_s(t) = 0$.

□

Stability Proof - In Presence Of External Forces

Theorem

Consider the closed loop teleoperation system (32). If the Jacobian is non-singular

1. *If $F_h, F_e \in L_\infty$ and $k_r > 1/2$, t then all signals of the bilateral teleoperation system are bounded.*
2. *If $F_h, F_e \in L_\infty, L_2$ and $k_r > 1/2$, then the task space position tracking error (e_i) and the velocity tracking error ($\dot{e}_i$) asymptotically approach the origin as $t \to \infty$.*
3. *If the gravitational term $g(q)$ of the robotic manipulators is zero or precompensated, when $(\dot{q}_m, \dot{q}_s, \ddot{q}_m, \ddot{q}_s) \to 0$, then the human and environmental forces are proportional to the tracking errors,i.e, $F_h \propto e_m$ and $F_e \propto e_s$.*

Proof.

1. Consider the Lyapunov function candidate (35). Taking the time derivative along the trajectories of the closed loop teleoperation system (32) and following the analysis in Theorem 1, we have

$$\dot{V} = s_m^T J_m^T F_h - s_s^T J_s^T F_e - k_m s_m^T s_m - k_s s_s^T s_s - k_r r_m^T r_m - k_r r_s^T r_s - k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \tag{23}$$

Since $r_i = J_i s_i$, the first two terms on the right hand side become $r_m^T F_h$ and $r_s^T F_e$. By utilizing Young's inequality, we have that $r_m^T F_h \leq 1/2 r_m^T r_m + 1/2 F_h^T F_h$ and $r_s^T F_e \leq 1/2 r_s^T r_s + 1/2 F_e^T F_e$. For $k_r = 1/2 + \epsilon$ with positive constant value ϵ, the derivative of V becomes

$$\dot{V} \leq 1/2 F_h^T F_h + 1/2 F_e^T F_e - k_m s_m^T s_m - k_s s_s^T s_s - \epsilon r_m^T r_m - \epsilon r_s^T r_s - k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \tag{24}$$

Since the external forces F_h, F_e are bounded and J_m, J_s are non singular, the signals F_h and F_e are also bounded. The derivative of V in 47 can be considered for two different cases such that $\dot{V} \geq 0$ and $\dot{V} < 0$. If $\dot{V} \geq 0$, then $1/2 F_h^T F_h + 1/2 F_e^T F_e \geq k_m s_m^T s_m + k_s s_s^T s_s + \epsilon r_m^T r_m + \epsilon r_s^T r_s + k_d(\dot{X}_m + \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \geq 0$.

Stability Proof - In Presence Of External Forces

Proof.

1. By considering the aforementioned inequality with V in (47), the states of teleoperation system will not grow unbounded while F_h and F_e are bounded. Therefore, s_i, r_i, and $(\dot{X}_m - \dot{X}_s)$ are all bounded. From the closed loop control system equation (32), all signals are bounded. Additionally, if $\dot{V} < 0$, then V is non-increasing function and $s_i, e_i \in L_\infty$. As s_i and e_i are bounded, (30) and (31) lead to $\dot{q}_i$ and r_i being bounded. From (3), we get $\dot{X}_i, \dot{e}_i \in L_\infty$. Consequently, all signals of teleoperation system are bounded.
2. By integrating the inequality (47) from 0 to t with $V(t)$ being positive definite, we have that $V(0) + 1/2\|F_h\|_2^2 + 1/2\|F_e\|_2^2 \geq k_m\|s_m\|_2^2 + k_s\|s_s\|_2^2 + \epsilon\|r_m\|_2^2 + \epsilon\|r_s\|_2^2 + k_d\|\dot{X}_m - \dot{X}_s\|_2^2$, where $\|.\|_2$ denotes the L_2 norm of the signal. Since F_h and F_e are square integrable, the aforementioned inequality results in $s_i, r_i, \dot{X}_m - \dot{X}_s \in L_2$. Thus, following the proof in Theorem 1, the position and velocity tracking errors converge to the origin asymptotically such that $X_m - X_s, \dot{X}_m - \dot{X}_s \to 0$ as $t \to \infty$.
3. If $(\dot{q}_m, \dot{q}_s, \ddot{q}_m, \ddot{q}_s) \to 0$, then $\dot{s}_i, \dot{q}_i \to 0$. Convergence of $\dot{q}_i$ to the origin with (3) and the property 8 implies that e_i and $C_i(q_i, \dot{q}_i)s_i$ approach zero. Consequently, from the closed loop system, we get

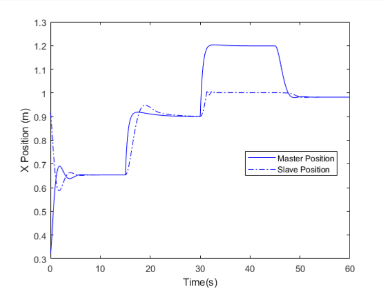
$$\begin{aligned} F_h &\to (J_m^T)^{-1}(k_m s_m + k_r J_m^T r_m) \\ F_e &\to (J_s^T)^{-1}(-k_s s_s - k_r J_s^T r_s) \end{aligned} \tag{25}$$

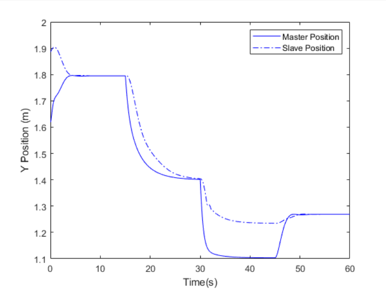
As $s_i \to -\lambda J_i^{-1} e_i$ and $r_i \to -e_i$ in steady state, the external forces can be written as

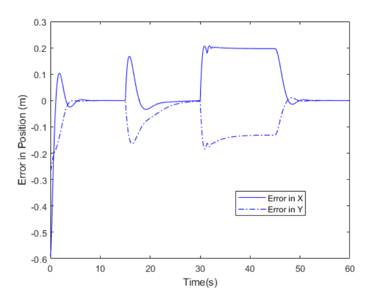
$$\begin{aligned} F_h &\to (-k_m \lambda (J_m J_m^T)^{-1} - k_r \lambda I_n) e_m \\ F_e &\to (k_s \lambda (J_s J_s^T)^{-1} + k_r \lambda I_n) e_s \end{aligned} \tag{26}$$

Since, J_i is bounded and invariant in steady state, we conclude that $F_h \propto e_m$, $F_e \propto e_s$ and the force feedback error is bounded.

Simulation Results-Position tracking

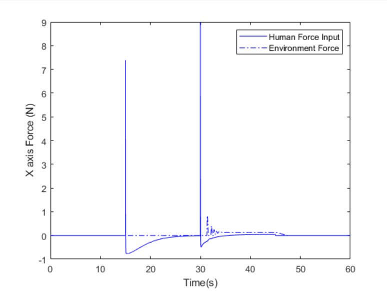


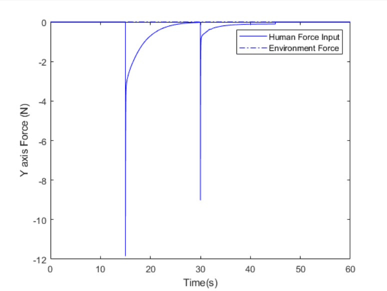


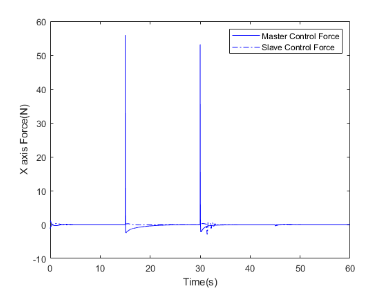


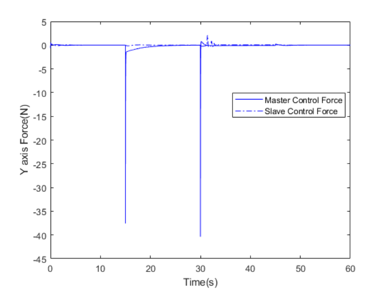
The master and slave robots are considered as non identical 2-DOF planar robot manipulators. The physical paramters are choses as $m_m = [3.14, 2.26]kg$, $m_s = [2.54, 1.82]kg$, $I_m = [0.16, 0.07]kgm^2$, $I_s = [0.12, 0.06]kgm^2$, $l_m = [1.04, 0.96]m$, $l_s = [1.28, 0.84]m$ with $g = 0m/s^2$. The initial position and velocity are
$q_m(0) = [0.8, 1.2]rad$, $q_s(0) = [1.0, 0.3]rad$, and $\dot{q}_m(0) = \dot{q}_s(0) = [0, 0]rad/s$. The control gains are selected as $\lambda = 0.8$, $k_m = k_s = 1$, $k_r = 4$, $k_d = 1$, $k_{pm} = k_{ps} = 0.5$.The slave robot stably tracks the position of the master robot and the tracking errors converge to zero asymptotically in absence of environmental force. The position tracking is not achieved during $t = 30\ 45s$ because the slave robot is in contact with the remote environment.

Simulation Results-Forces

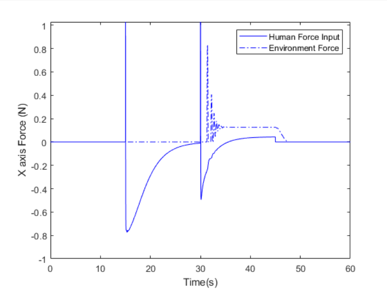


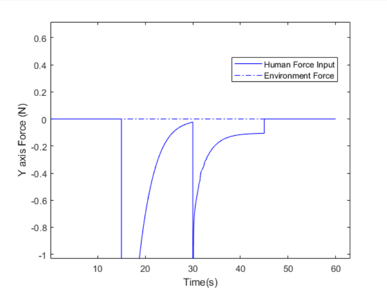


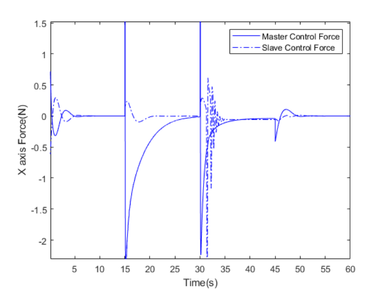


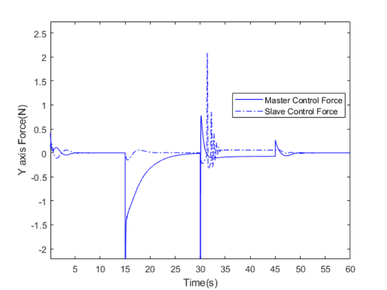


Simulation Results-Forces









Adaptive Model Based Control [Liu and Khong(2015)]

- The subscript $i = m, s$ represents the master and slave robots respectively. The control input to the system is given by-

$$\begin{aligned}\tau_i &= \hat{M}_i(q_i)\ddot{q}_{r,i} + \hat{C}_i(q_i,\dot{q}_i)\dot{q}_{r,i} + \hat{g}_i(q_i) - k_i s_i - k_r \hat{J}_i^{\,T} r_i + k_d \hat{J}_i^{\,T} \dot{e}_i \\ &= Y_{d,i}(q_i,\dot{q}_i,\dot{q}_{r,i},\ddot{q}_{r,i})\hat{\Theta}_{d,i} - k_i s_i - k_r \hat{J}_i^{\,T} r_i + k_d \hat{J}_i^{\,T} \dot{e}_i\end{aligned} \tag{27}$$

  All the quantities with hat symbol represent the estimate of the respective quantities. e_i is the task space velocity error, k_i, k_d, k_r are positive control gains.

- $\dot{q}_{r,i} \in \mathbb{R}^{\ltimes}$ in 27 is the joint space reference velocity given by

$$\dot{q}_{r,i} = \hat{J}_i^{\,-1}\lambda e_i \tag{28}$$

  where λ is positive control gain.

- From the above definition, we get

$$\ddot{q}_{r,i} = \dot{\hat{J}}_i^{-1}\lambda e_i + \hat{J}_i^{\,-1}\lambda \dot{e}_i \tag{29}$$

  where $\dot{\hat{J}}_i^{-1}$ is the time derivative of the inverse of the estimated Jacobian matrix and is obtained by $-\hat{J}_i^{\,-1}\dot{\hat{J}}_i\hat{J}_i^{\,-1}$.

- The joint space sliding vector $s_i \in \mathbb{R}^{\ltimes}$ in equation 28 is given as

$$s_i = \dot{q}_i - \dot{q}_{r,i} \tag{30}$$

- By defining $r_i = \hat{J}_i s_i$ and substituting s_i into r_i, we get

$$r_i = \hat{J}_i s_i = \hat{J}_i\dot{q}_i - \hat{J}_i^{\,-1}\lambda e_i) = \hat{J}_i\dot{q}_i - \lambda e_i = \dot{\hat{X}}_i - \lambda e_i \tag{31}$$

  where $\dot{\hat{X}}_i = \hat{J}_i\dot{q}_i$ from equation 11.

Adaptive Model Based Control

- The closed loop teleoperation system is obtained as

$$
\begin{aligned}
M_m\dot{s}_m + C_m s_m + k_m s_m &= Y_{d,m}\Delta\Theta_{d,m} - k_r \hat{J}_m{}^T r_m + k_d \hat{J}_m{}^T \dot{e}_m + J_m^T F_h \\
M_s\dot{s}_s + C_s s_s + k_s s_s &= Y_{d,s}\Delta\Theta_{d,s} - k_r \hat{J}_s{}^T r_s + k_d \hat{J}_s{}^T \dot{e}_s - J_s^T F_e
\end{aligned} \tag{32}
$$

  where $\Delta\Theta_{d,i}$ denotes the estimate error for the dynamic constant vector $\Theta_{d,i}$ such that $\Delta\Theta_{d,i} = \hat{\Theta}_{d,i} - \Theta_{d,i}$. Additionally, we define $\Delta\Theta_{k,i} = \hat{\Theta}_{k,i} - \Theta_{k,i}$ as the estimate error for the kinematic parameters.

- The uncertain parameters $\Delta\Theta_{d,i}$ and $\Delta\Theta_{k,i}$ are generated from the adaptive laws-

$$
\begin{aligned}
\dot{\hat{\Theta}}_{d,i} &= -\Gamma_{d,i} Y_{d,i}^T s_i \\
\dot{\hat{\Theta}}_{k,i} &= -\Gamma_{k,i}(k_{pi} Y_{k,i}^T \Delta X_i + k_d Y_{k,i}^T \dot{e}_i)
\end{aligned} \tag{33}
$$

  where $\Gamma_{d,i}$ and $\Gamma_{k,i}$ are positive-definite matrices, k_{pi} are positive control constants, and $\Delta X_i = \hat{X}_i - X_i$ denotes the estimate error between the actual end-effector position and the estimated position obtained from (4).

- In this work, a σ modification of the above mentioned adaptive laws is considered as follows-

$$
\begin{aligned}
\dot{\hat{\Theta}}_{d,i} &= -\Gamma_{d,i}(Y_{d,i}^T s_i + \sigma_{d,i}\hat{\Theta}_{d,i}) \\
\dot{\hat{\Theta}}_{k,i} &= -\Gamma_{k,i}(k_{pi} Y_{k,i}^T \Delta X_i + k_d Y_{k,i}^T \dot{e}_i + \sigma_{k,i}\hat{\Theta}_{k,i})
\end{aligned} \tag{34}
$$

  where $\sigma_{d,i}$ and $\sigma_{k,i}$ are positive-definite matrices.

- It is expected that the above adaptive laws ensure that the parameters $\hat{\Theta}_{d,i}$ and $\hat{\Theta}_{k,i}$ converge as $t \to \infty$.

Stability Proof: In Absence Of External Force

Theorem

Consider the closed loop teleoperation system (32). If the Jacobian is non-singular, then in free motion with ($F_h = F_e = 0$), the task space position tracking error (e_i) and the velocity tracking error ($\dot{e}_i$) asymptotically approach the origin such that $X_s, \dot{X}_m - \dot{X}_s \to 0$ as $t \to \infty$. Additionally, $\dot{X}_m, \dot{X}_s \to 0$ as $t \to \infty$

Proof.

Consider the positive definite Lyapunov function candidate

$$V = \frac{1}{2}\sum_{i=m,s}[s_i^T M_i s_i + \Delta\Theta_{d,i}^T \Gamma_{di}^{-1}\Delta\Theta_{d,i} + \Delta\Theta_{k,i}^T \Gamma_{ki}^{-1}\Delta\Theta_{k,i} + \lambda k_d e_i^T e_i + k_{pi}\Delta X_i^T \Delta X_i] \tag{35}$$

The time derivative of the function V along the trajectories of the closed loop system (32) is given by

$$\begin{aligned}\dot{V} = \sum_{i=m,s}[s_i^T(-C_i s_i - k_i s_i + Y_{d,i}\Delta\Theta_{d,i} - k_r \hat{J}_i^T r_i + k_d \hat{J}_i^T \dot{e}_i) + \frac{1}{2}s_i^T \dot{M}_i s_i + \\ \Delta\Theta_{d,i}^T \Gamma_{di}^{-1}\dot{\hat{\Theta}}_{d,i} + \Delta\Theta_{k,i}^T \Gamma_{ki}^{-1}\dot{\hat{\Theta}}_{k,i} + \lambda k_d e_i^T e_i + k_{pi}\Delta X_i^T \Delta\dot{X}_i]\end{aligned} \tag{36}$$

With the use of the skew-symmetry property (6), and $r_i = \hat{J}_i^T s_i$. the time derivative of V becomes

$$\dot{V} = \sum_{i=m,s}[-k_i s_i^T s_i - k_r r_i^T r_i + k_d r_i^T \dot{e}_i + \Delta\Theta_{k,i}^T \Gamma_{ki}^{-1}\dot{\hat{\Theta}}_{k,i} + \lambda k_d e_i^T \dot{e}_i + k_{pi}\Delta X_i^T \Delta\dot{X}_i] \tag{37}$$

Stability Proof: In Absence Of External Force

Proof.

Substituting $r_i = \dot{\hat{X}}_i - \lambda e_i$ given in (28),the third term becomes $k_d r_i^T \dot{e}_i = k_d(\dot{\hat{X}}_i - \lambda e_i)^T \dot{e}_i = k_d \dot{\hat{X}}_i^T \dot{e}_i - \lambda k_d e_i^T \dot{e}_i$. Since $\Delta \dot{X}_i = Y_{k,i} \Delta\Theta_{k,i}$, the last term in (37) can be written as $k_{pi} \Delta X_i^T \Delta \dot{X}_i = k_{pi} \Delta X_i^T Y_{k,i} \Delta\Theta_{k,i}$.

$$\dot{V} = \sum_{i=m,s} [-k_i s_i^T s_i - k_r r_i^T r_i + k_d \dot{\hat{X}}_i^T \dot{e}_i + \Delta\Theta_{k,i}^T \Gamma_{ki}^{-1} \dot{\hat{\Theta}}_{k,i} + k_{pi} \Delta\Theta_{k,i}{}^T Y_{k,i}^T \Delta X_i] \tag{38}$$

After combining terms the third term for $i = m, s$ in the summation of (38), we get
$k_d \dot{\hat{X}}_m^T \dot{e}_m + k_d \dot{\hat{X}}_s^T \dot{e}_s = -k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) + k_d \Delta\Theta_{k,m}{}^T Y_{k,m}^T \Delta X_m + k_d \Delta\Theta_{k,s}{}^T Y_{k,s}^T \Delta X_s$. From the above equation and utilizing the adaptive laws for uncertain kinematics (33), the derivative of Lyapunov function becomes-

$$\dot{V} = -k_m s_m^T s_m - k_s s_s^T s_s - k_r r_m^T r_m - k_r r_s^T r_s - k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \leq 0 \tag{39}$$

Thus, the zero solution of the teleoperation system is stable, and all signals are bounded such that $s_i, \hat{\Theta}_{d,i}, \hat{\Theta}_{k,i}, e_i, \Delta X_i \in L_\infty$. Integrating the above equation from $[0, t]$, we find that $s_i, r_i, \dot{X}_m - \dot{X}_s \in L_2$. Since the signals s_i, e_i are bounded and $\hat{J}_i$ is full rank, we have $\dot{q}_i$ bounded by observing (28) and (30). Hence, $\dot{X}_i$ is also bounded from (3) which leads to $\dot{e}_i \in L_\infty$. From the system dynamics (10) implies that the robot acceleration $\ddot{q}_i \in L_\infty$ as $\tau_i, \dot{q}_i$ are bounded from the properties (7) and (8). Since all signals are bounded, we have $\dot{s}_i \in L_\infty$ from the close loop system (32). As $s_i \in L_2$ and $\dot{s}_i \in L_\infty$, by invoking Barbalat's lemmawe conclude that $lim_{t\to\infty} s_i(t) = 0$ which leads to $lim_{t\to\infty} r_i(t) = 0$ since $r_i = \hat{J}_i s_i$.

□

Stability Proof: In Absence Of External Force

Proof.

By differentiating (3) with $q_i, \dot{q}_i, \ddot{q}_i \in L_\infty$, $\dot{\hat{\Theta}}_{di} \in L_\infty$ we obtain that $\dot{\hat{J}}_i \in L_\infty$. From the time derivative of r_i ($\dot{r}_i = \dot{\hat{J}}_i s_i + \hat{J}_i \dot{s}_i$), we get $\dot{r}_i \in L_\infty$ from $s_i, \dot{s}_i, \hat{J}_i, \dot{\hat{J}}_i$ being bounded. As $\dot{r}_i = \ddot{\hat{X}}_i - \lambda \dot{e}_i$, $\ddot{\hat{X}}_i$ is bounded. The boundedness of ΔX_i and X_i leads to $\hat{X}_i \in L_\infty$. Therefore, the approximate velocity is uniformly continuous and $lim_{t\to\infty} \int_0^t \dot{\hat{X}}_i(\sigma) d\sigma$ exists and is finite. By invoking Barbalat's lemma again, we conclude that $lim_{t\to\infty} \dot{\hat{X}}_i(t) = 0$, which leads to $lim_{t\to\infty} \dot{q}_i(t) = 0$ from (11) as $\hat{J}_i$ is invertible. Hence, we obtain $lim_{t\to\infty} \dot{X}_i(t) = 0$ from (11).

As $lim_{t\to\infty} \dot{\hat{X}}_i(t) = 0$ and $lim_{t\to\infty} r_i(t) = 0$ and from (31), the tracking error between the master and slave robots in teleoperation system converge to origin asymptotically such that $lim_{t\to\infty} e_i(t) = 0$. Further, differentiating $\dot{X}_i = J_i \dot{q}_i$, we obtain $\ddot{X}_i \in L_\infty$ as $\dot{q}_i, \ddot{q}_i, \dot{J}_i$ are bounded. Since $\dot{X}_m - \dot{X}_s \in L_2$ and $\ddot{X}_m - \ddot{X}_s \in L_\infty$, we conclude that $\dot{X}_m - \dot{X}_s \to 0$ as $t \to \infty$ such that $lim_{t\to\infty} \dot{e}_m(t) = lim_{t\to\infty} \dot{e}_s(t) = 0$. □

Stability Proof: In Absence Of External Force

Theorem

Theorem (3) with σ modification

Proof.

Following the steps in page (17) proof, the derivative of the Lyapunov function becomes-

$$\begin{aligned}\dot{V} = -k_m s_m^T s_m - k_s s_s^T s_s - k_r r_m^T r_m - k_r r_s^T r_s - k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \\ -\sigma_{d,i}\Delta\Theta_{d,i}^T\hat{\Theta}_{d,i} - \sigma_{k,i}\Delta\Theta_{k,i}^T\hat{\Theta}_{k,i}\end{aligned} \tag{40}$$

Comparing to (39) we have additional terms due to the σ modification. The term $-k_m s_m^T s_m - k_s s_s^T s_s - k_r r_m^T r_m - k_r r_s^T r_s - k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \leq 0$. For the Lyapunov derivative to be negative semi-definite, the additional terms must be $-\sigma_{d,i}\Delta\Theta_{d,i}^T\hat{\Theta}_{d,i} - \sigma_{k,i}\Delta\Theta_{k,i}^T\hat{\Theta}_{k,i} \leq 0$ This can be achieved if both $-\sigma_{d,i}\Delta\Theta_{d,i}^T\hat{\Theta}_{d,i} \leq 0$ and $-\sigma_{k,i}\Delta\Theta_{k,i}^T\hat{\Theta}_{k,i} \leq 0$. Hence,
$-\Delta\Theta_{d,i}^T\hat{\Theta}_{d,i} = -\Delta\Theta_{d,i}^T(\Delta\Theta_{d,i} + \Theta_{d,i}^*) < -\|\Delta\Theta_{d,i}\|_2^2 + \|\Delta\Theta_{d,i}\|_2\|\Theta_{d,i}^*\|_2$.
Now, $-\|\Delta\Theta_{d,i}\|_2^2 + \|\Delta\Theta_{d,i}\|_2\|\Theta_{d,i}^*\|_2 < 0 \iff \|\Delta\Theta_{d,i}\|_2 > \|\Theta_{d,i}^*\|_2 = k_1$ and
$-\|\Delta\Theta_{k,i}\|_2^2 + \|\Delta\Theta_{k,i}\|_2\|\Theta_{k,i}^*\|_2 < 0 \iff \|\Delta\Theta_{k,i}\|_2 > \|\Theta_{k,i}^*\|_2 = k_2$
Hence, for $\|\Delta\Theta_{d,i}\|_2 > k_1$ and $\|\Delta\Theta_{k,i}\|_2 > k_2$, the Lyapunov derivative is negative semi-definite and the rest of the proof follows identically with the proof of theorem (3).

□

Stability Proof: In Presence Of External Force

Theorem

Consider the closed loop teleoperation system (32). Assuming approximate Jacobian is non-singular

1. *If $F_h, F_e \in L_\infty$ and $k_r > 1/2$, t then all signals of the bilateral teleoperation system are bounded.*
2. *If $F_h, F_e \in L_\infty, L_2$ and $k_r > 1/2$, then the task space position tracking error (e_i) and the velocity tracking error $(\dot{e}_i)$ asymptotically approach the origin as $t \to \infty$.*
3. *If the gravitational term $g(q)$ of the robotic manipulators is zero or precompensated, when $(\dot{q}_m, \dot{q}_s, \ddot{q}_m, \ddot{q}_s) \to 0$, then the human and environmental forces are proportional to the tracking errors,i.e, $F_h \propto e_m$ and $F_e \propto e_s$.*

Proof.

1. Consider the Lyapunov function candidate (35). Taking the time derivative along the trajectories of the closed loop teleoperation system (32) and following the analysis in Theorem 1, we have

$$\dot{V} = s_m^T \hat{J}_m^T F_h - s_s^T \hat{J}_s^T F_e - k_m s_m^T s_m - k_s s_s^T s_s - k_r r_m^T r_m - k_r r_s^T r_s - k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \quad (41)$$

Defining $\bar{F}_h = (\hat{J}_m^T)^{-1} J_m^T F_h$ and $\bar{F}_e = (\hat{J}_s^T)^{-1} J_s^T F_e$ with $r_i = \hat{J}_i s_i$, the first two terms on the right hand side become $r_m^T \bar{F}_h$ and $r_s^T \bar{F}_e$. By utilizing Young's inequality, we have that $r_m^T \bar{F}_h \leq 1/2 r_m^T r_m + 1/2 \bar{F}_h^T \bar{F}_h$ and $r_s^T \bar{F}_e \leq 1/2 r_s^T r_s + 1/2 \bar{F}_e^T \bar{F}_e$. For $k_r = 1/2 + \epsilon$ with positive constant value ϵ, the derivative of V becomes

$$\dot{V} \leq 1/2 \bar{F}_h^T \bar{F}_h + 1/2 \bar{F}_e^T \bar{F}_e - k_m s_m^T s_m - k_s s_s^T s_s - \epsilon r_m^T r_m - \epsilon r_s^T r_s - k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \quad (42)$$

Since the external forces F_h, F_e are bounded and $\hat{J}_m, \hat{J}_s$ are non singular, the signals $\bar{F}_h$ and $\bar{F}_e$ are also bounded.

□

Stability Proof: In Presence Of External Force

Proof.

1. The derivative of V in 47 can be considered for two different cases such that $\dot{V} \geq 0$ and $\dot{V} < 0$. If $\dot{V} \geq 0$, then $1/2\bar{F}_h^T\bar{F}_h + 1/2\bar{F}_e^T\bar{F}_e \geq k_m s_m^T s_m + k_s s_s^T s_s + \epsilon r_m^T r_m + \epsilon r_s^T r_s + k_d(\dot{X}_m + \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) \geq 0$. By considering the aforementioned inequality with V in (47), the states of teleoperation system will not grow unbounded while $\bar{F}_h$ and $\bar{F}_e$ are bounded. Therefore, s_i, r_i, and $(\dot{X}_m - \dot{X}_s)$ are all bounded. From the closed loop control system equation (32), all signals are bounded. Additionally, if $\dot{V} < 0$, then V is non-increasing function and $s_i, \hat{\Theta}_{d,i}, \hat{\Theta}_{k,i}, e_i, \Delta X_i \in L_\infty$. As s_i and e_i are bounded, (30) and (31) lead to $\dot{q}_i$ and r_i being bounded. From (3), we get $\dot{X}_i, \dot{e}_i \in L_\infty$. Consequently, all signals of teleoperation system are bounded.
2. By integrating the inequality (47) from 0 to t with $V(t)$ being positive definite, we have that $V(0) + 1/2\|\bar{F}_h\|_2^2 + 1/2\|\bar{F}_e\|_2^2 \geq k_m\|s_m\|_2^2 + k_s\|s_s\|_2^2 + \epsilon\|r_m\|_2^2 + \epsilon\|r_s\|_2^2 + k_d\|\dot{X}_m - \dot{X}_s\|_2^2$, where $\|.\|_2$ denotes the L_2 norm of the signal. Since F_h and F_e are square integrable, the aforementioned inequality results in $s_i, r_i, \dot{X}_m - \dot{X}_s \in L_2$. As F_h and F_e are bounded, all signals of the teleoperation system are bounded as discussed in the previous claim. Thus, following the proof in Theorem 1, the position and velocity tracking errors converge to the origin asymptotically such that $X_m - X_s, \dot{X}_m - \dot{X}_s \to 0$ as $t \to \infty$.
3. The last claim addresses the force reflection in the teleoperation system. If the gravitational term $g(q)$ is zero or precompensated, then the control input becomes-

$$\begin{aligned} \tau_i &= \hat{M}_i(q_i)\ddot{q}_{r,i} + \hat{C}_i(q_i,\dot{q}_i)\dot{q}_{r,i} + \hat{g}_i(q_i) - k_i s_i - k_r \hat{J}_i^T r_i + k_d \hat{J}_i^T \dot{e}_i \\ &= \bar{Y}_{d,i}(q_i,\dot{q}_i,\dot{q}_{r,i},\ddot{q}_{r,i})\hat{\bar{\Theta}}_{d,i} - k_i s_i - k_r \hat{J}_i^T r_i + k_d \hat{J}_i^T \dot{e}_i \end{aligned} \tag{43}$$

□

Stability Proof: In Presence Of External Force

Proof.

where $\bar{Y}_{d,i}$ and $\hat{\bar{\Theta}}_{d,i}$ are corresponding to $Y_{d,i}$ and $\hat{\Theta}_{d,i}$ in property (5) without the gravitational term such that

$$M_i(q_i)\ddot{q}_{r,i} + C_i(q_i,\dot{q}_i)\dot{q}_{r,i} = \bar{Y}_{d,i}(q_i,\dot{q}_i,\dot{q}_{r,i},\ddot{q}_{r,i})\bar{\Theta}_{d,i} \tag{44}$$

In the closed loop system (32), if $(\dot{q}_m,\dot{q}_s,\ddot{q}_m,\ddot{q}_s) \to 0$, then $\dot{s}_i,\ddot{q}_{r,i} \to 0$. From (44) and property (5), $\bar{Y}_{d,i}(q_i,\dot{q}_i,\dot{q}_{r,i},\ddot{q}_{r,i}) \to 0$ in steady state as $\bar{\Theta}_{d,i}$ is a non-zero constant vector. Convergence of $\dot{q}_i$ to the origin with (3) and the property 8 implies that e_i and $C_i(q_i,\dot{q}_i)s_i$ approach zero. Consequently, from the closed loop system, we get

$$\begin{aligned} F_h &\to (J_m^T)^{-1}(k_m s_m + k_r \hat{J}_m^T r_m) \\ F_e &\to (J_s^T)^{-1}(-k_s s_s - k_r \hat{J}_s^T r_s) \end{aligned} \tag{45}$$

As $s_i \to \lambda \hat{J}_i^{-1} e_i$ and $r_i \to -e_i$ in steady state, the external forces can be written as

$$\begin{aligned} F_h &\to (-k_m\lambda(\hat{J}_m J_m^T)^{-1} - k_r\lambda I_n)e_m \\ F_e &\to (k_s\lambda(\hat{J}_s J_s^T)^{-1} + k_r\lambda I_n)e_s \end{aligned} \tag{46}$$

Since $Y_{k,i} \to 0_{(}n \times n)$ as $\dot{q}_i \to 0$, from (3) and (33) the actual and estimated Jacobian matrices $J_i, \hat{J}_i$ are bounded and invariant in steady state. Hence, we conclude that $F_h \propto e_m, F_e \propto e_s$ and the force feedback error is bounded.

□

Stability Proof: In Presence Of External Force

Theorem

Theorem (2) with σ modification.

Proof.

- Following the analysis in proof of Theorem 2, we have

$$\begin{aligned}\dot{V} \leq\ & 1/2\bar{F}_h^T\bar{F}_h + 1/2\bar{F}_e^T\bar{F}_e - k_m s_m^T s_m - k_s s_s^T s_s - \epsilon r_m^T r_m - \epsilon r_s^T r_s \\ & - k_d(\dot{X}_m - \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) - \sigma_{d,i}\Delta\Theta_{d,i}^T\hat{\Theta}_{d,i} - \sigma_{k,i}\Delta\Theta_{k,i}^T\hat{\Theta}_{k,i}\end{aligned} \tag{47}$$

  Since the external forces F_h, F_e are bounded and $\hat{J}_m$, $\hat{J}_s$ are non singular, the signals $\bar{F}_h$ and $\bar{F}_e$ are also bounded. The derivative of V in 47 can be considered for two different cases such that $\dot{V} \geq 0$ and $\dot{V} < 0$. If $\dot{V} \geq 0$, then $1/2\bar{F}_h^T\bar{F}_h + 1/2\bar{F}_e^T\bar{F}_e \geq$ $k_m s_m^T s_m + k_s s_s^T s_s + \epsilon r_m^T r_m + \epsilon r_s^T r_s + k_d(\dot{X}_m + \dot{X}_s)^T(\dot{X}_m - \dot{X}_s) + \sigma_{d,i}\Delta\Theta_{d,i}^T\hat{\Theta}_{d,i} + \sigma_{k,i}\Delta\Theta_{k,i}^T\hat{\Theta}_{k,i} \geq 0$ when $\|\Delta\Theta_{d,i}\|_2 > k_1$ and $\|\Delta\Theta_{k,i}\|_2 > k_2$. By considering the aforementioned inequality with V in (47), the states of teleoperation system will not grow unbounded while $\bar{F}_h$ and $\bar{F}_e$ are bounded. Therefore, s_i, r_i, and $(\dot{X}_m - \dot{X}_s)$ are all bounded. From the closed loop control system equation (32), all signals are bounded. Additionally, if $\dot{V} < 0$, then V is non-increasing function and $s_i, \hat{\Theta}_{d,i}, \hat{\Theta}_{k,i}, e_i, \Delta X_i \in L_\infty$. As s_i and e_i are bounded, (30) and (31) lead to $\dot{q}_i$ and r_i being bounded. From (3), we get $\dot{X}_i, \dot{e}_i \in L_\infty$. Consequently, all signals of teleoperation system are bounded.

Similarly, the proofs of the remaining claims follow from the proof of theorem (2) when $\|\Delta\Theta_{d,i}\|_2 > k_1$ and $\|\Delta\Theta_{k,i}\|_2 > k_2$.

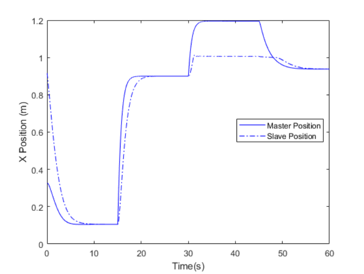
□

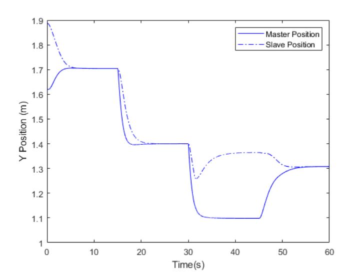
Simulations

- The master and slave robots are considered as non identical 2-DOF planar robot manipulators. The physical paramters are chosen as $m_m = [3.14, 2.26]kg$, $m_s = [2.54, 1.82]kg$, $I_m = [0.16, 0.07]kgm^2$, $I_s = [0.12, 0.06]kgm^2$, $l_m = [1.04, 0.96]m$, $l_s = [1.28, 0.84]m$ with $g = 0m/s^2$. The estimates of the physical parameters are chosen as $\hat{m}_m = [3.00, 2.00]kg$, $\hat{m}_s = [2.20, 2.00]kg$, $\hat{I}_m = [0.10, 0.10]kgm^2$, $\hat{I}_s = [0.2, 0.1]kgm^2$, $\hat{l}_m = [0.8, 1.2]m$, $\hat{l}_s = [1.1, 0.5]m$ with $g = 0m/s^2$
- The initial position and velocity are $q_m(0) = [0.8, 1.2]rad$, $q_s(0) = [1.0, 0.3]rad$, and $\dot{q}_m(0) = \dot{q}_s(0) = [0, 0]rad/s$. The control gains are selected as $\lambda = 0.8$, $k_m = k_s = 1$, $k_r = 4$, $k_d = 1$, $k_{pm} = k_{ps} = 0.5$, $\Gamma_{di} = I_{3\times3}$, $\Gamma_{ki} = I_{2\times2}$. For the σ modification, $\sigma_{di} = 0.01I_{3\times3}$ and $\sigma_{ki} = 0.01I_{2\times2}$.
- The parameters are given by $\Theta_{d,i} = \begin{bmatrix} m_{1i}(l_{1i}/2)^2 + m_{2i}(l_{1i}^2 + l_{2i}^2/4) + I_{1i} + I_{2i} \\ m_{2i}l_{1i}l_{2i}/2 \\ m_{2i}(l_{2i}/2)^2 + I_{2i} \end{bmatrix}$, $\Theta_{k,i} = \begin{bmatrix} l_{1i} \\ l_{2i} \end{bmatrix}$
- In this simulation, the teleoperation system is considered to move in free motion before $t = 15s$. After $t = 15s$, the human operator applies a spring damper force [[Lee and Spong(2006)], [Liu and Chopra(2013)]] on the end effector of the master robot to change the configuration. A virtual wall is implemented at in the remote environment at $x = 1m$ as seen in figure 2. The slave robot will be subject to an external force if its x position is larger than $1m$.
- We compare the adaptive control given in the paper and our proposed *sigma* modification. The plots of the adaptive control scheme and the *sigma* modification scheme are presented on a single page for easy comparison.

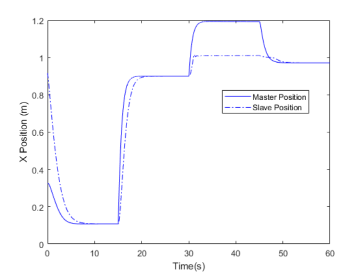
Simulation Results-Position Tracking

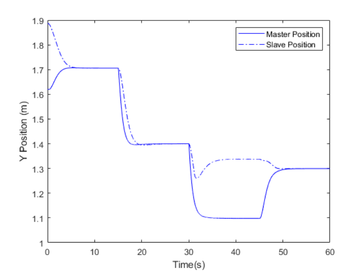
Literature





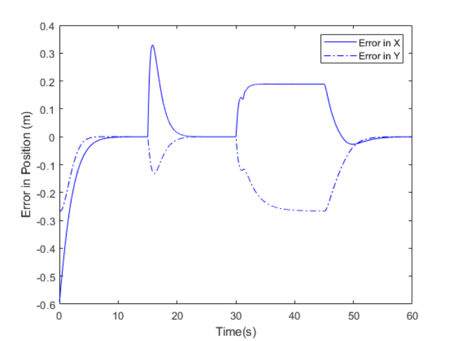
σ Modification



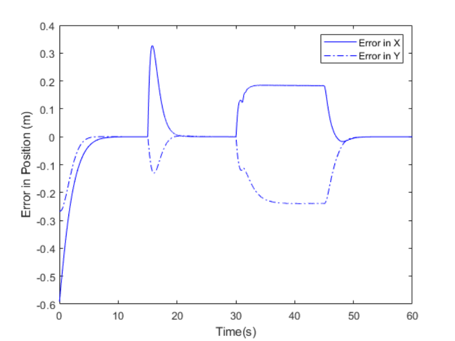


Simulation Results-Position Tracking

Literature



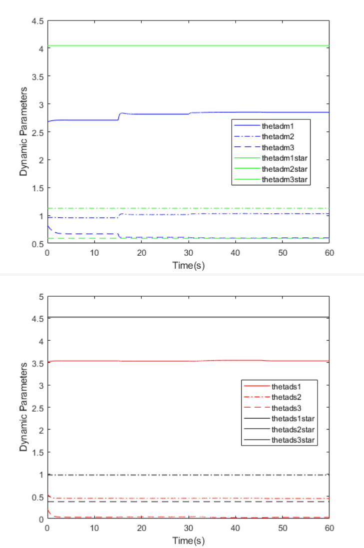
σ Modification



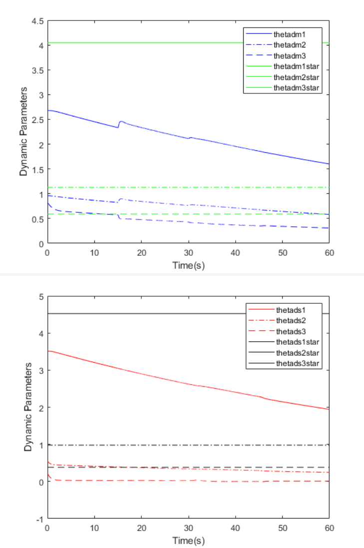
- The slave robot stably tracks the position of the master robot and the tracking errors converge to zero asymptotically in absence of environmental force. The position tracking is not achieved during $t = 30 - 45s$ because the slave robot is in contact with the remote environment. The adaptive and the σ modification controller have almost identical tracking behaviors for this case.
- The parameters are also compared to the ideal value of the parameters and it is evident that the parameters do not converge to the ideal values. It is clearly visible that the parameters converge to zero as $t \to \infty$ in case of the σ modification while the parameters settle to some value for the adaptive controller presented in paper. The simulations were run for longer time duration to confirm the convergence of kinematic and dynamic parameters to zero. Hence, the main aim of *sigma* modification is to ensure that the parameters do not drift in case of external disturbances and we confirm that the parameters converge to zero in this study.

Simulation Results-Dynamic Parameters

Literature

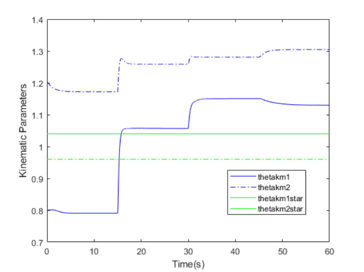


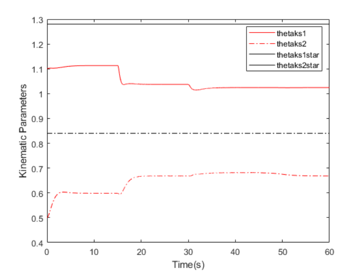
σ Modification



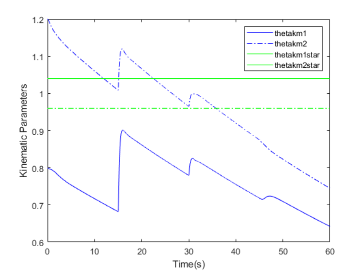
Simulation Results-Kinematic Parameters

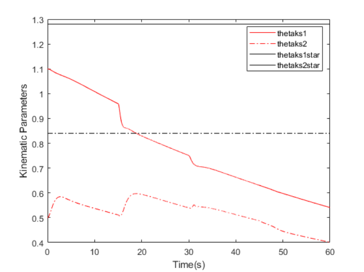
Literature





σ Modification





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