RUTVIJ DHOTEY : COMPUTER VISION HW1

1. Let us consider the training data to be

$$f(x_i, w_i) = \{(x_i, w_i), i = 1 \dots N\}$$

Here
x_i = Row
w_i = Height 'H'

Considering training Parameters to be ϕ_0, ϕ_1 we can use the maximum likelyhood estimation & considering further that the data pairs are independent & identically distributed, the equation is.

$$\boxed{P(w_i \mid x_i, \theta) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{-1}{2} \frac{(w_i - (\phi_0 + \phi_1 x_i))^2}{\sigma^2} \right\}}$$

Here the learning parameters $\theta = \{\phi_0, \phi_1, \sigma^2\}$

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2. To derive the values of $\{\phi_0, \phi_1, \sigma^2\}$

let

$$L = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{-1}{2} \frac{(w_i - (\phi_0 + \phi_1 x_i))^2}{\sigma^2} \right\}$$

as we need to consider derivatives, converting it to logarithmic terms would be easier.

$$\therefore \quad L = \operatorname{argmax}\left[\sum_{i=1}^{N} \left(\frac{-1}{2} \left[N(\log 2\pi) + N\log(\sigma^2) + \sum_{i=1}^{N} \frac{(w_i - (\phi_0 + \phi_1 x_i))^2}{\sigma^2} \right] \right) \right]$$

Hence taking the derivative of L wrt ϕ_0, ϕ_1 & σ^2 we will find the values.

$$L = \frac{-1}{2} N \log(2\pi) - \frac{1}{2} N(\log \sigma^2) - \frac{1}{2} \sum_{i=1}^{N} \frac{(w_i^2 - 2(\phi_0 + \phi_1 x_i) w_i + (\phi_0 + \phi_1 x_i)^2)}{\sigma^2}$$

$$= \frac{-1}{2} N \log(2\pi) - \frac{1}{2} N(\log \sigma^2) - \frac{1}{2} \frac{\sum(w_i^2 - 2\phi_0 w_i - 2\phi_1 x_i w_i + \phi_0^2 + 2\phi_0\phi_1 x_i + \phi_1^2 x_i^2)}{\sigma^2}$$

Now:

$$\frac{\partial L}{\partial \phi_0} = \frac{2w_i - 2\phi_0 - 2\phi_1 x_i}{2\sigma^2}$$

$$\frac{\partial L}{\partial \phi_1} = \frac{-2x_i w_i + 2\phi_0 x_i + 2\phi_1 x_i^2}{2\sigma^2}$$

$$\therefore \frac{dL}{\partial \phi_0} = 0 = w_i - \phi_0 - \phi_1 x_i$$

$$\frac{\partial L}{\partial \phi_1} = 0 = x_i w_i - \phi_0 x_i - \phi_1 x_i^2$$

$$\therefore \phi_0 + \phi_1 x_i = w_i \quad (1)$$

$$\phi_0 x_i + \phi_1 x_i^2 = x_i w_i \quad (2)$$

Converting it into a linear equation of matrices -

$$\left[\begin{array}{l} \sum \phi_0 + \sum \phi_1 x_i = w_i \\ \sum \phi_0 x_i + \sum \phi_1 x_i^2 = w_i x_i \end{array}\right.$$

$$\boxed{A = \underbrace{\begin{bmatrix} \sum_{i=1}^{N} 1 & \sum_{i=1}^{N} x_i \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 \end{bmatrix}}_{} \underbrace{\begin{bmatrix} \phi_0 \\ \phi_1 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} \sum_{i=1}^{N} w_i \\ \sum_{i=1}^{N} w_i \cdot x_i \end{bmatrix}}_{B}}$$

Solving for σ^2, consider $\sigma^2 = u$.

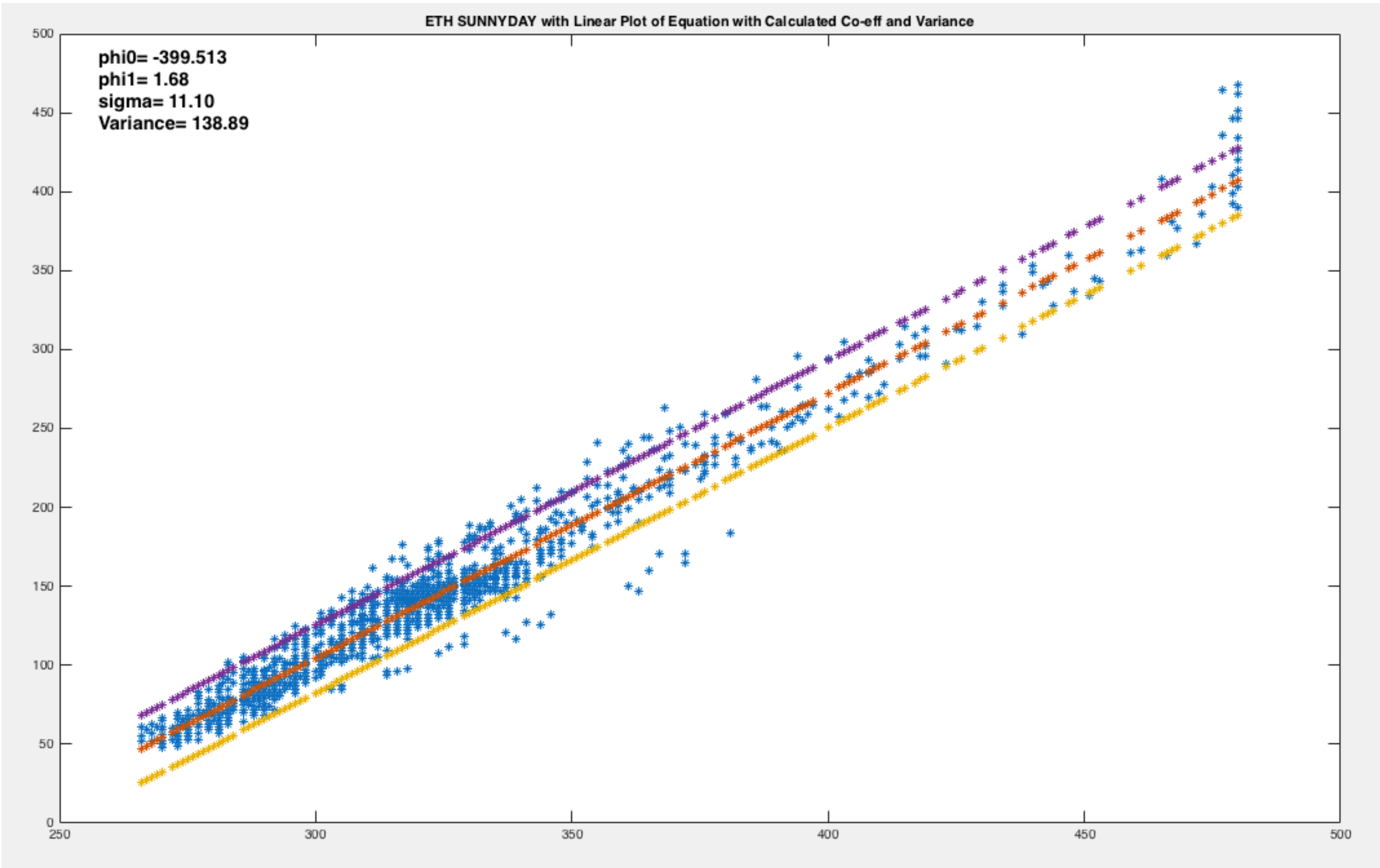
$$\therefore \frac{\partial L}{\partial u} = +\frac{1}{2}\frac{N}{u} - \frac{1}{2}\left(\frac{\sum_{i=1}^{N} (w_i - (\phi_0 + \phi_1 x_i))^2}{u^2}\right) = 0$$

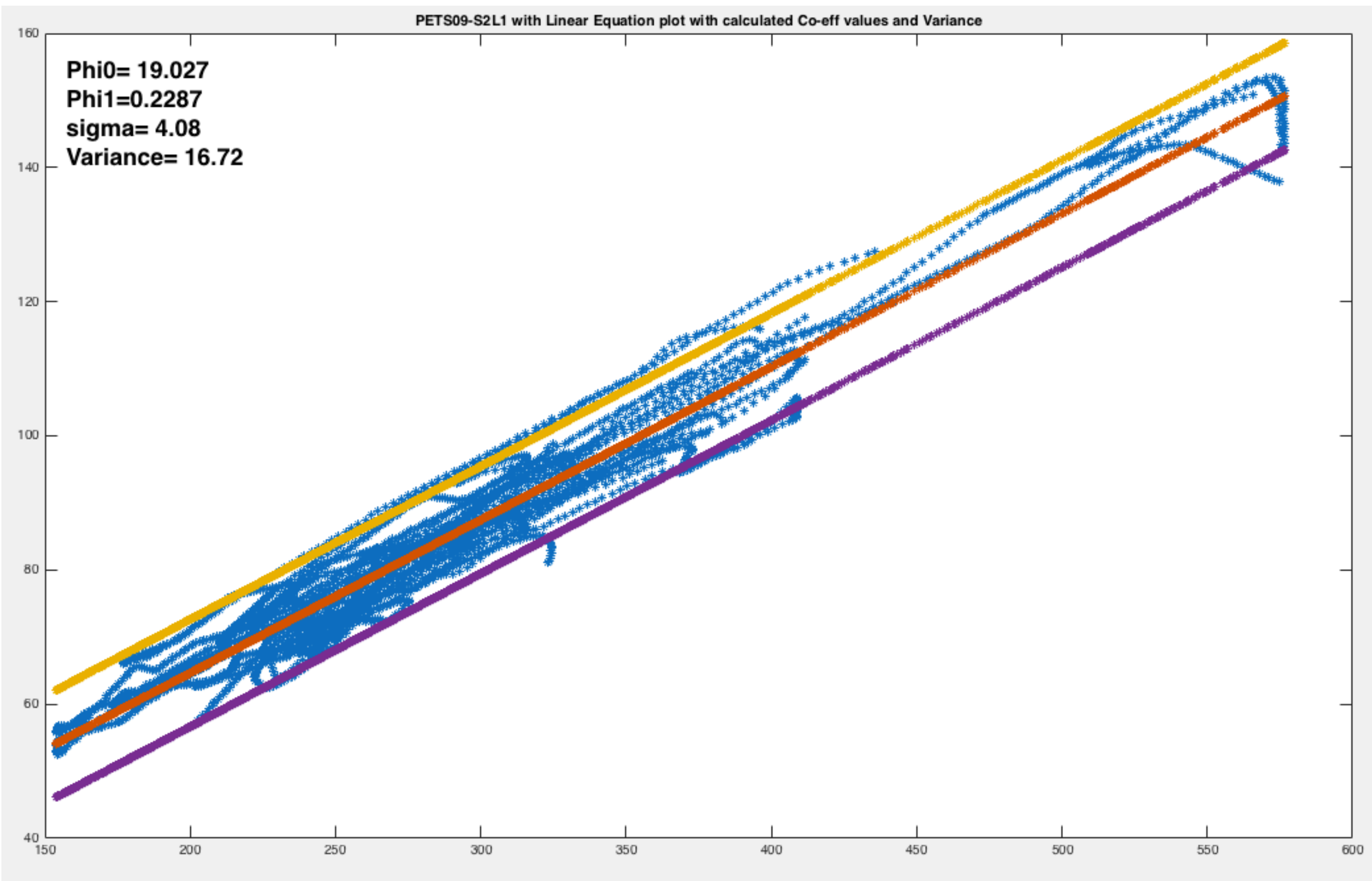
$$\therefore \frac{u^2}{u} = \sum_{i=1}^{N} \frac{(w_i - (\phi_0 + \phi_1 x_i))^2}{1}$$

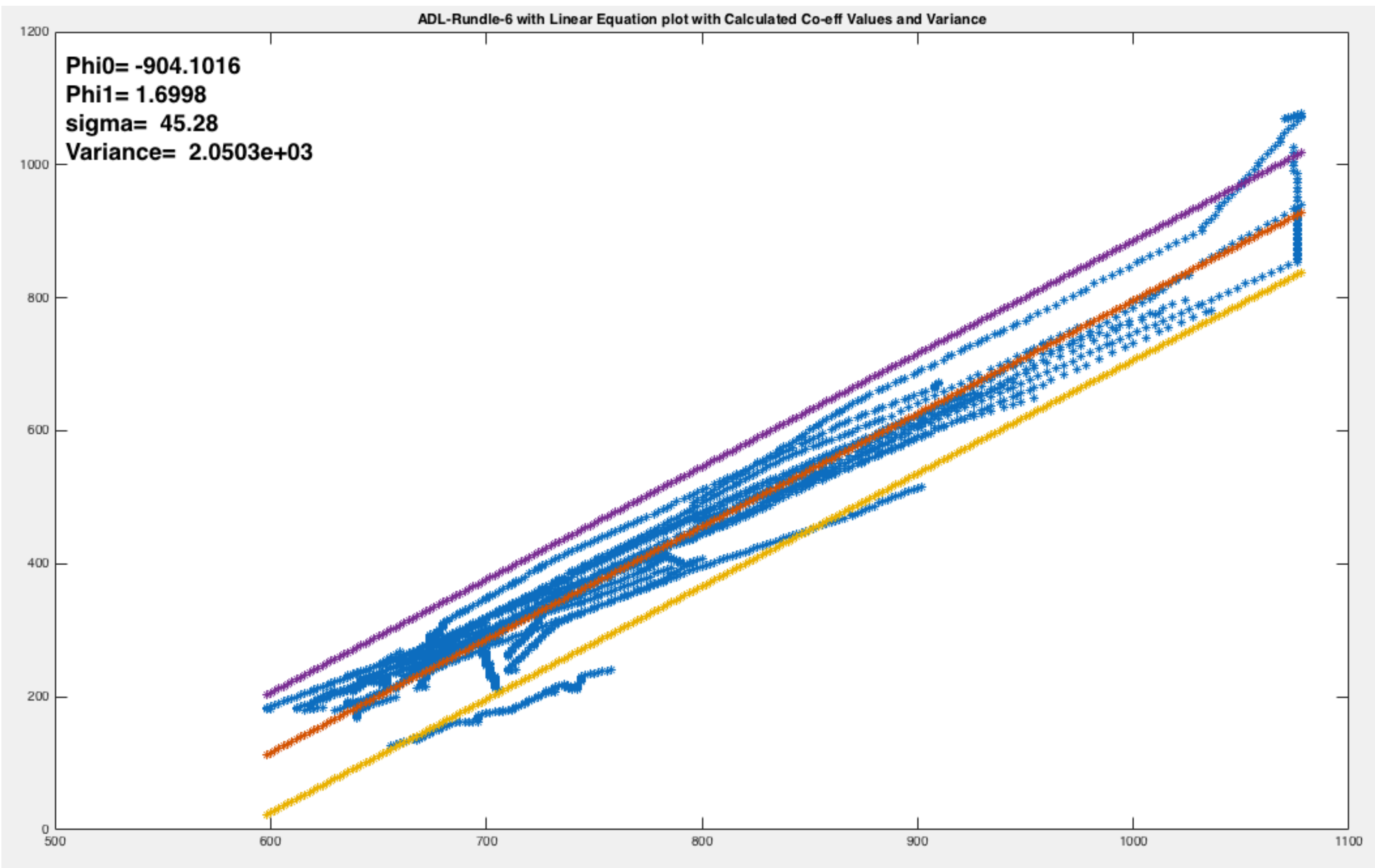
$$\boxed{\therefore u = \sigma^2 = \sum_{i=1}^{N} \frac{\left(w_i - (\phi_0 + \phi_1 x_i) \right)^2}{N}}$$

JUSTIFICATION OF MY VALUES

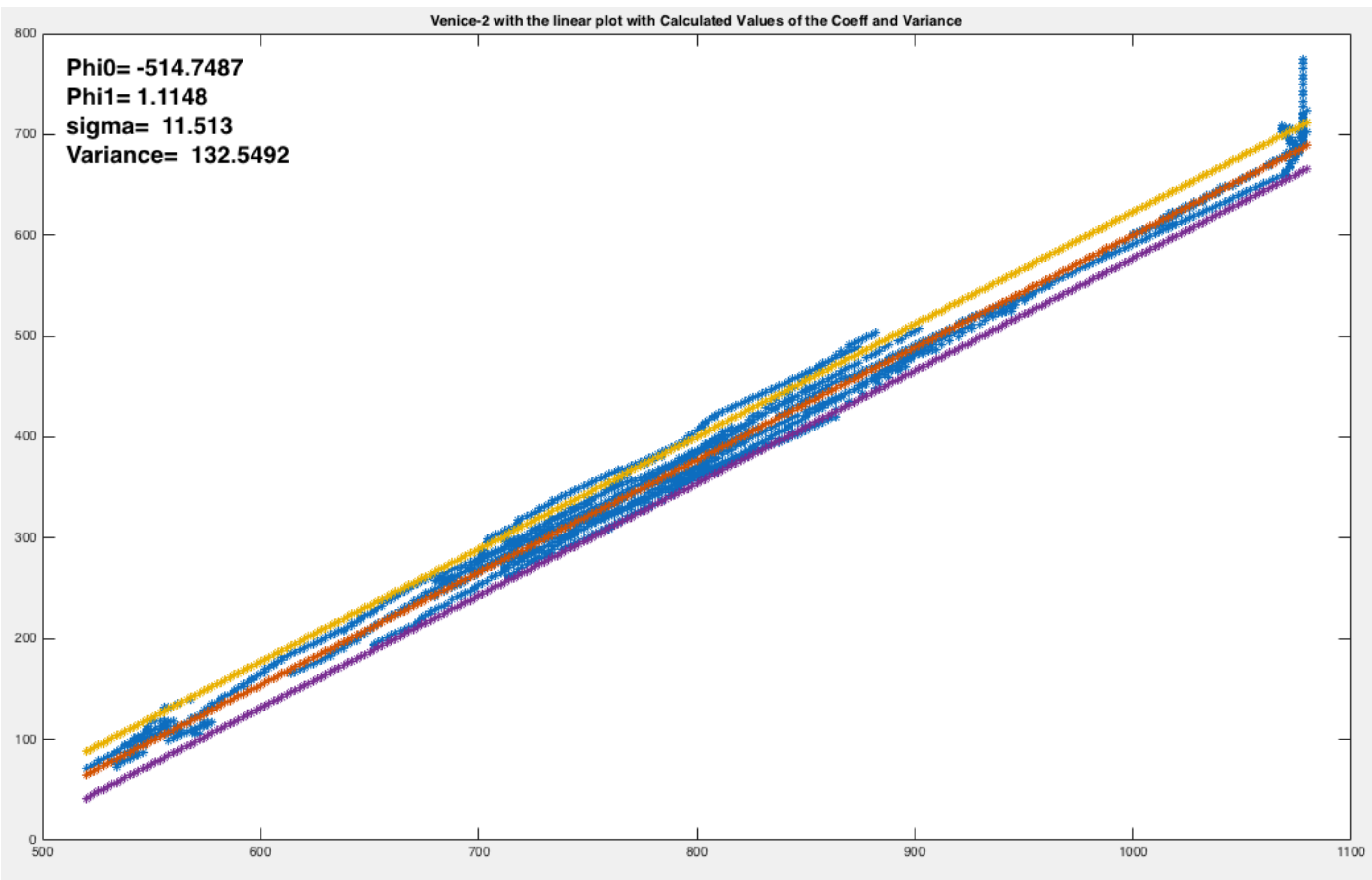
As shown in the next four plots we can clearly see that the calculated values(formula) works for the phi1 and phi0 calculated by the program. The distribution can also be seen by the two parallel lines to the ORANGE(Equation line) line drawn considering the sigma.




PETS09-S2L1 with Linear Equation plot with calculated Co-eff values and Variance
Phi0= 19.027
Phi1=0.2287
sigma= 4.08
Variance= 16.72
160
140
120
100
80
60
40
150
200
250
300
350
400
450
500
550
600



ADL-Rundle-6 with Linear Equation plot with Calculated Co-eff Values and Variance
Phi0= -904.1016
Phi1= 1.6998
sigma= 45.28
Variance= 2.0503e+03



Venice-2 with the linear plot with Calculated Values of the Coeff and Variance
Phi0= -514.7487
Phi1= 1.1148
sigma= 11.513
Variance= 132.5492
0
100
200
300
400
500
600
700
800
500
600
700
800
900
1000
1100

RESULTS FOR THE 2 DATA SETS NEEDED

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for the data set : PETS09-S2L1

Results

P0 = -399.513465746585
P1 = 1.68079799860489

Variance= 138.899394631459

for the data set :ETH-Sunnyday

Results

P0 = 19.0277416060443
P1 =  0.228239688273922

Variance=  16.723095077458
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