CS 2050 Worksheet 2

1.4 Quantifiers

1. Use quantifiers and predicates to represent the following statements:

   Let $x \in$ set of people → all ppl
   $P(x)$: x is a police officer
   $N(x)$: x is from New York
   $A(x)$: x has been to Australia

   a) "There is a police officer from New York who has been to Australia."

   $\exists x$

   $\neg(P(1) \vee P(2) \vee P(3))$

   $\neg P(1) \wedge \neg P(2)$

   $$\exists x\,(P(x) \wedge N(x) \wedge A(x))$$

   RHS

   b) "Being from New York is $\boxed{\text{necessary}}$ for someone to be a police officer."

   $\exists x$

   $$\forall x\,(P(x) \rightarrow N(x))$$

   c) "No person is both a police officer and someone who has been to Australia."

   $$\neg \exists x\,(P(x) \wedge A(x))$$

   d) Now negate your answer to part **b)** and simplify such that all negation symbols immediately precede predicates. In English, what does this negated statement now say?

   $\neg \forall x\,(P(x) \rightarrow N(x)) \rightarrow \exists x\, \neg(P(x) \rightarrow N(x))$ demorgan's
   $= \exists x\, \neg(\neg P(x) \vee N(x))$ Material imp.
   $= \exists x\,(\neg(\neg P(x)) \wedge \neg N(x))$ demorgan's
   $= \exists x\,(P(x) \wedge \neg N(x))$ double negation

   there exists an office who has not been to New York.

2. Decide whether the following statements are true or false:

   a) $\forall x(x + 1 > x)$, $x \in \mathbb{Z}$ True.

   b) $\exists x(x^2 < x)$, $x \in \mathbb{Z}$ False

   c) $\exists x(x^2 < x)$, $x \in \mathbb{R}$ True

   Ex: $.1^2 < .1$

   d) $\forall x(x^3 \neq 7)$, $x \in \mathbb{R}$

   True

1.5 Nested Quantifiers

1. Let $x \in$ set of students
   Let $y \in$ set of courses
   $U(y)$: y is an upper level course
   $M(y)$: y is an math level course
   $F(x)$: x is an first year student
   $P(x)$: x is an part time student
   $T(x, y)$: x student is taking course y

   Use quantifiers and the defined domains/predicates to represent the following statements:

   a) "Every student is taking at least one course." (From student to student, which course it is might be different)

   $\forall x \exists y (T(x,y))$

   b) "There is at least one specific course that every first year student is taking."

   $\exists y$ $\forall x$

   WRONG

   $\cancel{\exists y \forall x (F(x) \wedge T(x,y))}$ All students are first years and taking y course.

   $\exists y \forall x (F(x) \rightarrow T(x,y))$

   you need conditional b/c $\forall x$ is on inside.

   c) "At least one upper level course has a first year student taking it."

   $\exists y$ $\exists x$

   DONT FORGET

   $\exists x \exists y (U(y) \wedge F(x) \wedge T(x,y))$

2. Use quantifiers to represent the statement: "Every real number is less than another real number." (Be sure to provide a domain for any variable you create)

   $x, y \in \mathbb{R}$

   STUDY

   $\forall x \exists y (x < y)$

   Switch the quantifiers in your previous answer (so the first quantifier is now second and the second quantifier is now first). What does this new answer state, in English?

   $\exists y \forall x (x < y)$

   there exits one y where all the x values are values are less than y.

1.6 Rules of Inference

1. Use the given premises to conclude: $\neg r \rightarrow s$

$\neg p \rightarrow \neg r$

Step	Reason
1. $\neg p \wedge q$	Premise
2. $r \rightarrow p$	Premise
3. $\neg r \rightarrow s$	Premise
4. $s \rightarrow t$	Premise
5. $\neg p \rightarrow \neg r$	contrapositive (#2)
6. $\neg p \rightarrow s$	hypothetical syllogism (#5, #3)
7. $\neg p$	simplification (#1)

Teacher way

1. $\neg p \wedge q$
2. $r \rightarrow p$
3. $\neg r \rightarrow s$

Premise

8. s modus ponens (#6 #7)

9. t modus ponens (#8, #4)

IN Conclusion, given the premises, I have shown t by using logical inferences.

4. s -> t

5. ¬r -> t hypothetical (#3, #4)

6. ¬p -> ¬r contrapositive (#2)

7. ¬p -> t hypor syll. (5, 6)

8. ¬p simp. (#1)

9. t modus ponens (#7, #8)

2. For each rule, state it's name and fill-in whether c is "arbitrary" or "specific."

a) Rule: universal instantiation

$\forall x P(x)$

$P(c)$ for arbitrary c

b) Rule: Existential instantiation

$\exists x P(x)$

$P(c)$ for specific c

c) Rule: Existential generalization

$P(c)$ for specific c

$\exists x P(x)$

d) Rule: Universal generalization

$P(c)$ for arbitrary c

$\forall x P(x)$

$\exists x(Q(x))$

3) Given the following premises, use rules of inference to show: "There is an athlete who likes the cold."

$x \in$ athletes $P(x)$: x is a snowboarder $Q(x)$: x likes the cold Shaun White $\in$ athletes

"Shaun White is a snowboarder." -> P(Shaun white)

"If an athlete doesn't like the cold, then they aren't a snowboarder." ¬Q(x) -> ¬P(x)

① P(Shaun white) premise

② ∀x(¬Q(x) -> ¬P(x)) premise

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③ ¬Q(Shaun white) -> ¬P(Shaun white) universal instantiation (#2)

④ ¬(¬P(S.W)) -> ¬(¬Q(S.W)) Contrapositive (#3)

double negative (#4)

4. P(Shaun white) → Q(Shaun white)
5. Q(Shaun white) — modus ponens (#4, #1)
6. ∃x(Q(x)) — existential generalization (#5)

STUDY

Ex: everyone has at least one best friend

$$\forall x \exists y (F(x,y) \land (x \neq y))$$

at least one best friend.

x ∈ set of ppl

y ∈ set of programming language