

CS 2050 Homework 1

Q.1.

(a) Let p be " $1+1=3$ ".

Let q be "unicorns exist".

Note that p is false since $1+1=2 \neq 3$.

Since the statement is of form $p \rightarrow q$ and we found p to be false, automatically the statement is true.

(b) Let p be " $1+1=2$ ".

Let q be "horses can fly".

Note that p is true and q is false since $1+1=2$ and the horses don't fly.

Since the statement is of form $p \rightarrow q$ and we found p to be true and q to be false, the statement turns out to be false.

(c) Let p be " $1+2=3$ ".
Let q be " $2+2=4$ ".

Note that $1+2=3$ and $2+2=4$ and so p is true
and q is true.

Q

Since the statement is of form $p \rightarrow q$, and we
found p and q to be true respectively, the
statement is true.

Q.2.

- (a) If you want to pass Discrete Math, then ~~you~~ ~~do~~ you
~~need~~ to take exams.
↓ have
- (b) If there is thunder, then there is lightning.
- (c) If you can access Zybooks then you ~~will~~ have paid
subscription fees.

Q.3. Statement : If it rains tonight, then I will stay at home.

Let p be "It rains tonight".

Let q be "I will stay at home".

Now, Converse is $q \rightarrow p$

Contrapositive is $\neg q \rightarrow \neg p$

Inverse is $\neg p \rightarrow \neg q$

So, converse : If I stay at home, then it will rain tonight.

Contrapositive : If I don't stay at home, then it will not rain tonight.

Inverse : If it does not rain tonight, then I will not stay at home.

Q.4.

(a) Proposition $\rightarrow (p \wedge q \wedge r) \vee (\neg q \wedge t) \vee (\neg r \wedge \neg t)$

Number of rows = 2^n where n is the number of variables in proposition.

Note, there are 5 variables, which are p, q, r, s & t .
So, Number of rows = $2^5 = 32$

(b) Proposition $\equiv (p \wedge \neg p \vee p) \rightarrow (\neg q \vee q \wedge \neg q)$

Number of rows = 2^n , where n is the number of variables.

Note, there are just 2 variables p and q in this proposition.

So, Number of rows = $2^2 = \underline{4}$.

Q.5. Compound proposition is $(p \wedge q) \vee \neg r$

p	q	r	$(p \wedge q)$	$\neg r$	$(p \wedge q) \vee \neg r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

Q. 6.

(a)

"Access is granted whenever (the user has paid the subscription fee) and (enters a valid password)"

↑
operator \wedge

P

Signifies use of if, then form
q

So, the mathematical form is $(x \wedge p) \rightarrow q$.

(b) "Access is denied if (the user has not paid the subscription fee)." Signifies use of if, then

So, the mathematical form is $\neg x \rightarrow \neg q$

(c)

"If (the user has not entered a valid password) (but has paid the subscription fee), then (access is granted)." Signifies
 $\neg x \rightarrow q$

So, the mathematical form is $\neg p \wedge x \rightarrow q$.

Q.7.

(a) Person A says "The two of us are both knights" and Person B says "A is a knave."

~~Let p be "The two of us are both knights"~~
~~Let q be "A is a knave"~~

~~Let p be "A is a knight"~~
~~Let q be "B is a knight"~~

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$	$p \rightarrow (p \wedge q)$	$p \wedge q \leftrightarrow p$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	T
F	F	F	T	T	T

p	q	$\neg p$	$\neg p \rightarrow q$	$q \rightarrow \neg p$	$\neg p \leftrightarrow q$	$(p \wedge q \leftrightarrow p) \wedge (\neg p \leftrightarrow q)$
T	T	F	T	F	F	F
T	F	F	T	T	T	F
F	T	T	T	T	T	T
F	F	T	F	T	F	F

So, $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$ is true only when p is false & q is true.

So, [A is a knave and B is a knight].

(b) Both A and B say "I am a knight."

Let p be "A is a knight."

Let q be "B is a knight."

p	q	$p \rightarrow p$	$q \rightarrow q$	$p \leftrightarrow p$	$q \leftrightarrow q$	$(p \leftrightarrow p) \wedge (q \leftrightarrow q)$
T	T	T	T	T	T	T
T	F	T	FT	T	T	T
F	T	FT	T	T	T	T
F	F	FT	FT	T	T	T

Note that $(p \leftrightarrow p) \wedge (q \leftrightarrow q)$ is true ~~in~~ in all cases.

So, A can be a ~~not~~ knight or knave and B can be a knight or knave

Q.8. The conditional statement is $[\neg p \wedge (p \vee q)] \rightarrow q$

p	q	$\neg p$	$(p \vee q)$	$\neg p \wedge (p \vee q)$	$\neg p \wedge (p \vee q) \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Conclusion :-

Note that $\neg p \wedge (p \vee q) \rightarrow q$ is always true for any values of p and q . So, $\neg p \wedge (p \vee q) \rightarrow q$ is a tautology.

Q.9. $[\neg p \wedge (p \vee q)] \rightarrow q$

$$\equiv [\neg (\neg p \wedge (p \vee q))] \vee q \quad [\text{By material implication}]$$

$$\bullet \equiv [\neg (\neg p) \vee \neg (p \vee q)] \vee q \quad [\text{By DeMorgan's law}]$$

$$\equiv [p \vee \neg (p \vee q)] \vee q \quad [\text{By double negation law}]$$

$$\equiv [p \vee (\neg p \wedge \neg q)] \vee q \quad [\text{By DeMorgan's law}]$$

$$\equiv [(p \vee \neg p) \wedge (p \vee \neg q)] \vee q \quad [\text{By distributive law}]$$

$$\begin{aligned}
 &\equiv [T \wedge (p \vee \neg q)] \vee q \quad [\text{By negation law}] \\
 &\equiv [(p \vee \neg q) \wedge T] \vee q \quad [\text{By commutative law}] \\
 &\equiv [(p \vee \neg q) \vee q] \wedge [T \vee q] \quad [\text{By distributive law}] \\
 &\equiv [(p \vee \neg q) \vee q] \wedge [q \vee T] \quad [\text{By commutative law}] \\
 &\equiv [(p \vee \neg q) \vee q] \wedge T \quad [\text{Domination law}] \\
 &\equiv (p \vee \neg q) \vee q \quad [\text{By Identity law}] \\
 &\equiv p \vee (\neg q \vee q) \quad [\text{By associative law}] \\
 &\equiv p \vee (q \vee \neg q) \quad [\text{By commutative law}] \\
 &\equiv p \vee T \quad [\text{By negation law}] \\
 &\equiv T \quad [\text{By domination law}]
 \end{aligned}$$

Conclusion :-

Thus, $[G(p \wedge (p \vee q))] \rightarrow q \equiv T$ or ~~\perp~~
 $[\neg p \wedge (p \vee q)] \rightarrow q$ is a tautology as shown
 above.