# Recitation #1

$p \vee \neg(q \wedge r) \vee (r \wedge p)$

when making truth table

Left→Right go left to right in columns

| $p$ | $(q \wedge r)$ | $(r \wedge p)$ |
| --- | --- | --- |
| . | , | . |
| , | , | , |
| , | , | , |

...lling the truth, he's always... then he's telling the truth

If he's tell...
If he's a knight...

G: gawain is a knight
B: Bedivere is a knight

# CS 2050 Worksheet 1

## 1.2 Truth Table

a) You are on an island of knights and knaves, where knights always tell the truth and knaves always lie. You encounter two people Gawain and Bedivere. Gawain says "I am a knave or Bedivere is a knight" and Bedivere says nothing. Determine, if possible, what Gawain and Bedivere are.

$\neg G$ $\vee$ $B$

| $G$ | $B$ | $\neg G \vee B$ | $\neg G \vee B \to G$ | $G \to (\neg G \vee B)$ | $\neg G \vee B \leftrightarrow G$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | T | F | F |
| F | T | T | F | T | F |
| F | F | F | F | T | F |

So both are knights

b) Build a truth table to show that for some propositions p, q and r: $q \wedge \neg p \to r$ and $q \to (\neg p \to r)$ are equivalent.

order of precedence

| $p$ $q$ $r$ | $\neg p$ | $q \wedge \neg p$ | $q \wedge \neg p \to r$ | $\neg p \to \neg r$ | $q \to (\neg p \to \neg r)$ | $q \wedge \neg p \to r \equiv q \to (\neg p \to r)$ |
|---|---|---|---|---|---|---|
| T T T | F | F | T | T | T | T |
| T T F | F | F | T | T | T | T |
| T F T | F | F | T | T | T | T |
| T F F | F | F | T | T | T | T |
| F T T | T | T | T | T | T | T |
| F T F | T | T | F | F | F | T |
| F F T | T | F | T | T | T | T |
| F F F | T | F | T | F | T | T |

equal

Conclusion: since the proposition ... is true

## 1.3 Logical Equivalences

a) Use only logical equivalences and material implication to prove that $p \to q$ is equivalent to its contrapositive, $\neg q \to \neg p$.

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$\neg p \vee q$ material implication

1) $p \to q$ LHS
2) $\neg p \vee q$ material implication
3) $q \vee \neg p$ communitive
4) $\neg(\neg q) \vee \neg p$ double negative
5 $\neg q \to \neg p$ material implication

Conclusion: using logical equiv. I have shown that $p \to q \equiv \neg q \to \neg p$

$(\bar{p}+\bar{q})+p$

b) Use only logical equivalences and material implication to show whether or not $(p \wedge q) \to p$ is a tautology.

$\neg p \vee q$

$(p \wedge q) \to p$

$\neg(p \wedge q) \vee p$ material implication

$(\neg p \vee \neg q) \vee p$ deMorgan's

$(\neg p \vee p) \vee \neg q$ commutative

$T \vee \neg q$ negation

$\neg q \vee T$ commutative

$T$ domination

T must be on RHS

Conc: using logical equiv. I have shown this statement is a tautology.

c) Use only logical equivalences and material implication to show that $q \wedge \neg p \to r$ is equivalent to $q \to (\neg p \to r)$.

Prove $q \wedge \neg p \to r \equiv q \to (\neg p \to r)$

$q \wedge \neg p \to r$ LHS

$\bar{q}+p+r$

$\neg(q \wedge \neg p) \vee r$ material implication

$(\neg q \vee \neg(\neg p)) \vee r$ demorgan's

$(\neg q \vee p) \vee r$ double negation

$\neg q \vee (p \vee r)$ commutative

$\neg q \vee (\neg(\neg p) \vee r)$ double neg.

$\neg q \vee (\neg p \to r)$ material imp

Thus, Conc: by logical equiv., the proposition $q \wedge \neg p \to r$ is equiv. to the proposition $q \to (\neg p \to r)$.

$q \to (\neg p \to r)$ material imp

$A \land B$

A says "The two of us are both knights" and
B says "A is a knave"

$\neg A$

$(A \land B \to A)$ $(A \to A \land B)$

$(\neg A \to B)$ $(B \to \neg A)$

| $A$ | $B$ | $A \land B$ | $A \land B \to A$ | $A \to A \land B$ | $A \land B \leftrightarrow A$ | $\neg A$ | $\neg A \to B$ | $B \to \neg A$ | $\neg A \leftrightarrow B$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | F | T | F | F |
| T | F | F | T | F | F | F | T | T | T |
| F | T | F | T | T | T | T | T | T | T |
| F | F | F | T | T | T | T | F | T | F |

A is a knave

B is a knight

A B

Both A and B say "I am a knight."

$A \to A$

$B \to B$

| $A$ | $B$ | $A \to A$ | $B \to B$ | $A \leftrightarrow A$ | $B \leftrightarrow B$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | T | T | T | T |
| F | T | T | T | T | T |
| F | F | T | T | T | T |

$$\exists x\,(\forall y((x \neq y) \longrightarrow (\neg E(y,x) \vee \neg T(y,x))))$$

for all students in class, x uses snapchat or there exists y that uses snapchat and x and y are friends.

For all students in the class, they either use snapchat or someone else exists who uses snapchat and are friends with the original.

there exists a student x and student y for all classes they are distinct students and student x has taken all z classes if and only if student y has taken all z classes.

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