# CS 2050 Worksheet 9

## 6.1 The Basics of Counting

1. For the following problems, there is a running track with 4 individual lanes. Each of the 4 lanes are distinct and can only have one runner at a time.

(a) How many ways can you order 4 athletes in the lanes?

24 ways

$4 \cdot 3 \cdot 2 \cdot 1$

(b) If there are 8 athletes in total, how many ways can you order athletes in the lanes?

$8 \cdot 7 \cdot 6 \cdot 5 = 1680$

$8 \cdot 7 \cdot 6 \cdot 5$

(c) Suppose Usain Bolt is 1 of 8 athletes, and he always gets a lane. How many ways are there to order athletes now?

U $7 \cdot 6 \cdot 5$

$7$ U $6 \cdot 5$

$7 \; 6$ U $5$

$7 \; 6 \; 5$ U

$4(7 \cdot 6 \cdot 5)$

OR

# of ways to include Usain = # of ways for all 8 − # of ways that don't include Usain

$= 8 \cdot 7 \cdot 6 \cdot 5 - 7 \cdot 6 \cdot 5 \cdot 4$

$= 840$

2. How many integers from 1 to 100 are:

100

(a) divisible by 6?

$\left\lfloor \frac{100}{6} \right\rfloor = 16$

(b) divisible by 7?

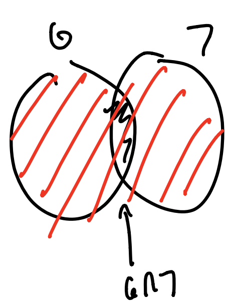
$\left\lfloor \frac{100}{7} \right\rfloor = 14$

do not share factors

(c) divisible by 6 and 7?

$\left\lfloor \frac{100}{42} \right\rfloor = 2$

(d) divisible by 6 or 7 (inclusive)?




we want red region

$d(6 \vee 7) = d(6) + d(7) - d(6 \wedge 7)$

$= 16 + 14 - 2$

$= 28$

3. How many integers from 1 to 100 are:

(a) divisible by 4?

$\left\lfloor \frac{100}{4} \right\rfloor = 28$

these share factors

(b) divisible by 4 and 6?

find LCM(4,6)

$4 = 2^2$

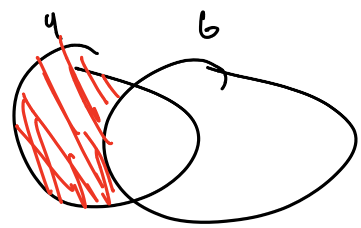
$6 = 2 \cdot 3$

$2^2 \cdot 3 = 12$

$\left\lfloor \frac{100}{12} \right\rfloor = 8$

(c) divisible by 4 but not by 6?

$n_{4 \text{ not } 6} = d(4) - d(4 \cap 6)$
$= 25 - 8$
$= 17$




4. DNA sequences can be represented with the characters A, G, C, and T.

(a) How many DNA sequences of length 8 start with "AT"?

$4^6$

A T
$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

(b) How many DNA sequences of length 8 don't contain "G"?

$3^8$

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

(c) How many DNA sequences of length 8 contain at least one "C"?

at least one "C" = All contain "C" − No contain "C"
$= 4^8 - 3^8$
$= 58975$

(d) How many DNA sequences of length 8 start with "AT," contain at least one "C," and don't contain "G"?

$3^6 - 2^6$
$= 665$

A T $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

5. How many palindromes are there of length 9 using these characters A, G, C, and T only?

$4^5$

$4 \cdot 4 \; 4 \; 4 \; 4 \;|\; 1 \; 1 \; 1 \; 1$

## 6.2 The Pigeonhole Principle

6. There are 16 CS2050 TAs and 8 different recitations. For the following problems, assume you are blindly selecting them one by one:

minimum #

(a) ~~How~~ many would you have to pick to guarantee a pair of two TAs from the same recitation are selected?

$n = 9$

$R_1$ $R_2$ $R_3$ $R_4$ $R_5$ $R_6$ $R_7$ $R_8$

9 picks $\left\lceil \frac{n}{8} \right\rceil = 2$

(b) Without knowing the schedules of the TAs how many would you have to pick to **guarantee** at least 3 that are currently taking CS 2200 with the amazing professor Umakishore Ramachandran or 3 that are not taking CS 2200 with the amazing professor Umakishore Ramachandran?

are taking Not taking

5 picks

$n = 5$

OR $\left\lceil \frac{n}{2} \right\rceil = 3$

7. There is a party with at least two people. Show that there must be at least two people with the same number of friends at the party. (It is assumed that if x is friends with y, then y is friends with x).

$n$ = # of people at party

$k$ = # of friend counts

$\left\lceil \frac{n}{k} \right\rceil = 2$

Case 1: everyone @ party have at least one friend.

Range = $\underbrace{[1, n-1]}_{\text{size} = n-1}$

Case 2: There is at least one person who has 0 friends.

Range of friends $= [0, n-1-1] = [0, n-2]$

size = n-1

b/c if someone has no friends, someone else would not be friends with them

Thus, $\left\lceil \frac{n}{n-1} \right\rceil = 2$

## 6.3 Permutations and Combinations

1. For the following problems, there are 30 students running for 5 student council positions:

(a) How many ways can winners be selected for the distinct roles of President, Vice President, Secretary, Treasury, and Activities Director?

$\underset{P}{30} \cdot \underset{VP}{29} \cdot \underset{S}{28} \cdot \underset{T}{27} \cdot \underset{AD}{26}$

$$P(30, 5) = \frac{30!}{(30-5)!} = \frac{30!}{25!}$$

(b) Now suppose the council is a committee with no distinction between roles. How many ways can 5 winners be selected for this committee?

__ __ __ __ __

$$C(30, 5) = \frac{30!}{5!\,25!} = \frac{30 \cdot 29 \cdot 28 \cdot 26 \cdot 25!}{5! \cdot 25!} = \frac{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

(c) Two candidates are lifelong rivals, and at MOST one of them can be in office. How many possible **committees** (see part b) are there now with this restriction?

# of commitees − # of committees w/ 2 rivals

A B

$= C(30, 5) - C(28, 3)$

2. How many bitstrings of length 20 are there that:

(a) have exactly 7 ones?

$\boxed{\underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}}$ ...... _ _ _ _ _

$C(20,7) = \frac{20!}{7!\,13!}$

choose 7 spots where the 1's will be

(b) have no more than 3 ones?

0, 1, 2, 3 number of 1's,

C(20,0) C(20,1) C(20,2) C(20,3)

$\boxed{C_{TOT} = C(20,0) + C(20,1) + C(20,2) + C(20,3)}$

3. (a) How many ways can you permute the string LUMBERJACK?

$\underline{10}\ \underline{9}\ \underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$ 10 unique characters

$= \boxed{10!}$

(b) How many permutations of LUMBERJACK contain the strings JACK and RUM?

$\boxed{\underline{J}\ \underline{A}\ \underline{C}\ \underline{K}}\ \boxed{\underline{R}\ \underline{U}\ \underline{M}}$ _ _ _

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

5!

(c) How many permutations of LUMBERJACK contain the strings BACK and LUMB?

$\boxed{\text{LUMBACK}}\ \boxed{E}\ \boxed{R}\ \boxed{J}$

$4 \cdot 3 \cdot 2 \cdot 1$

$\boxed{= 4!}$