CS 2050 Worksheet 5

you always take best choice at each step w/o consideration of the future.

3.1 Algorithms

1. Use the greedy algorithm to make change using quarters, dimes, nickels, and pennies for:

SHOW WORK

a) 57 cents

1) 1q → 32¢ 3) 1N → 2¢ 5) 1P → 0¢
2) 1q → 7¢ 4) 1P → 1¢

2q, 1N, 2P

57
− 25 quarter
32
25 quarter
7
5 nickel
2
− 2 2 pennies
0

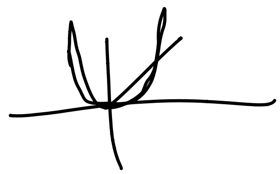
b) 24 cents

1) 1D → 14¢ 3) 1P → 3¢ 5) 1P → 1¢
2) 1D → 4¢ 4) 1P → 2¢ 6) 1P → 0¢

2D, 4P

2. Now after some tragic accident the nickel no longer exists as valid currency. Would any of the two situations above not be optimal with the greedy algorithm anymore? If so, show the new greedy algorithm results without nickels and a different but optimal result without nickels as evidence where optimal means fewer coins used.

The 57 cent problem would be affected, and it would not be as optimal.



57
− 25 quarter
32
− 25 quarter
7
− 7 7 pennies
0

9 coins

1a) uses 5 coins

3.2 The Growth of Functions

3. Determine the big-O of the following functions. Secondly, tell if the following function is $O(x^2)$:

a) $f(x) = 14x$ O(x) Yes → if it is less than, then yes

b) $f(x) = 2x^2 + 3x^2 * log x + 5$ $O(\log x \cdot x^2)$ NO

c) $f(x) = 213$ O(1) Yes

d) $f(x) = \frac{x^6}{2}$ $O(x^6)$ No

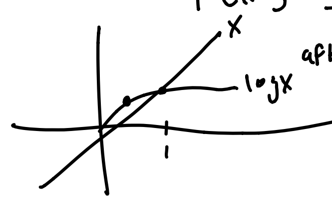
e) $f(x) = \frac{x^4+x^2+x}{x^3+3x^2}$ $O\left(\frac{x^4}{x^3}\right) = O(x)$

$\frac{O(x^4)}{O(x^3)} = O(x)$ Yes

$f(x) = x+3$

$f(x)$ is $O(g(x))$ if

$f(x) \le C \cdot g(x)$ after some x



after x > 1,
O(x) > O(log x)
thus O(log x) is O(x)

3.3 Complexity of Algorithms

4. Give a big-O estimate for the number of additions used in these algorithms:

```
count := 0
for i := 1 to 10
    count += i
```

O(1)

\# of steps is constant

```
count := 0
for i := 1 to n
    count += i
```

O(n)

```
count := 0
for i := 1 to n
    for j := 2 to n
        count += i + j
```

O(n·n) each for loop is n.

= O(n²)

```
count := 0
for i := 1 to n
    j := 2
    while j ≤ n
        count += i + j
        j = j * 3
```

O(n·log n)

normally if you have multiply you are cutting out half of numbers

j=1
j=3 1 2 3 4 5 6 7 8 9 10
j=9 *

all of these $x^2 + x$ can be brought up to cx^3 and it is still less

$2x^2 + 14x \le C \cdot x^3$ is $O(x^3)$

$2x^2 + 14x \le 2x^3 + 14x$ $x > 1$ $(x^2 \le x^3)$

$2x^3 + 14x \le 2x^3 + 14x^3$ $(x \le x^3)$ $x \ge 1$

$2x^3 + 14x^3 \le 16x^3$

$16x^3 \le Cx^3$