CS 2050 Worksheet 7

$18 = 2 \cdot 3^2$

4.3 Primes and Greatest Common Divisors

1. List all the integers $1 \leq n < 18$ that are relatively prime to 18.

1, 5, 7, 11, 13, 17

2. Use prime factorization to find the GCD and LCM of **147** and **42**.

$42 = 2 \cdot 21 = 2 \cdot 7 \cdot 3$

$147 = 3 \cdot 7^2$

$42 = 2 \cdot 3 \cdot 7$

$\gcd(147, 42) = 3 \cdot 7$

$\mathrm{LCM}(147, 42) = 2 \cdot 3 \cdot 7^2$

$3 \overline{)147}$ = 49; 12; 27

$147 = 3 \cdot 7 \cdot 7$

42 × 2 = 84

3. Use the **Euclidean Algorithm** to find GCD(93,42).

$\gcd(93, 42)$

$= \gcd(42, 9)$

$= \gcd(9, 6)$

$= \gcd(6, 3)$

$= \gcd(3, 0) = \boxed{3}$

93 mod 42

$42 \overline{)93}$ = 2; 84; 9

42 mod 9

$9 \overline{)42}$ = 4; 36; 6

$6 \overline{)9}$ = 1; 6; 3

5.2 Strong Induction

4. Use strong induction to prove that every amount of postage that is at least 12 cents can be made from 4-cent and 5-cent stamps.

$P(n)$: n amount of postage ($n \geq 12$) can be made from 4¢ and 5¢ stamps

Base Case:

$P(12)$ is true since 3×4¢ stamps yield 12¢ postage

$P(13)$ is true since 2×4¢ and 1×5¢ stamps ~~yi~~ yield 12¢ postage.

$P(14)$ is true since 1×4¢ and 2×5¢ stamps yield 12¢ postage.

$P(15)$ is true since 3×5¢ stamps yield 12¢ postage.

I have competed the base case.

Inductive Step:

I.H. We assume

$\forall j\ P(j)$ is true where $j \geq 12$

$12 \leq j \leq k$

where $k \geq 15$ ← biggest base case

smallest base case ↗

$P(k+1)$ is true since by our I.H. $p(k-3)$ is assumed true. I will use the $k-3$ postage, ~~and~~ if I add a 4¢ stamp to it will become a $k+1$ postage since $k-3+4 = k+1$. This will be the way to form a $k+1$ postage. This concludes the inductive step.

TA method: Assuming our I.H. we can make $k+1-4 = k-3$ postage since $k-3$ is in the bounds of j, so $P(k-3)$ is true. We can then add 1-4¢ stamp to the $k-3$¢ postage to give $k-3+4 = k+1$ postage. So $P(k+1)$ is true.

conc: $\forall j\, P(j) \rightarrow P(k+1)$

⇑ DO THIS

Conclusion: THE ONE ON NOTES

5. A jigsaw puzzle is put together by successively joining pieces that fit together into blocks. A move is made each time a piece is added to a block, or when two blocks are joined. Prove that no matter how the moves are carried out, exactly $n-1$ moves are required to assemble a puzzle with n pieces.

$P(n)$: A puzzle with n pieces will be assembled in $n-1$ moves, where $n \in \mathbb{Z}$, $n \geq 1$.

Base case:

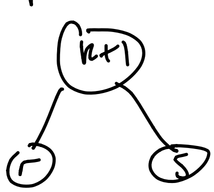
$P(1)$: A puzzle of 1 piece is already complete with 0 moves, and $n-1 = 1-1 = 0$.

$\therefore$ $P(1)$ is true. This concludes the basis step.

Inductive Hyp: $\forall j \; P(j)$ $1 \leq j \leq k$, $k \geq 1$ $j, k \in \mathbb{Z}$.

Inductive Step: proving $\forall j P(j) \rightarrow P(k+1)$

Let's say we have a puzzle with $k+1$ pieces, which can be broken into two piles of r-pieces and s-pieces, as shown.



$r, s \in \mathbb{Z}$
$1 \leq r \leq k$
$1 \leq s \leq k$

Since r and s are both in the bounds of j, by the inductive hypothesis we can say $P(r)$ and $P(s)$ are true.

Thus, an r-piece puzzle can be assembled in $r-1$ moves and a s piece puzzle can be assembled in $s-1$ moves.

The number of moves needed to assemble

$k+1$ puzzle = # of moves to assemble r + # of moves assemble s

+ the one more needed to combine two piles.

Thus, # of moves to assemble $k+1 = r-1+s-1+1$
$= (r+s)-1$

Since $r+s=k+1$, this equals $(k+1)-1$.

Thus, the # of moves needed to assemble

$k+1$ pieces $=(k+1)-1$

Note: this is $P(k+1)$

Conc: $\forall j\, P(j) \rightarrow P(k+1)$ by direct proof.

Conclusion: Since I have shown both the base case and inductive step to be true, $P(n)$ is true by strong induction, $\forall n\, P(n)$, $n \geq 1$.