CS 2050 Worksheet 4

2.1 Sets

1. Give the power sets (in list notation) of the following sets. Then state the cardinality of each power set.

A a) $\{a\}$ $\rightarrow 2^1 = 2$

$\mathbb{P}(\{a\}) = \{\emptyset, \{a\}\}$ $|\mathbb{P}(\{a\})| = 2$

B b) $\{a, b\}$ $2^2 = 4$

$\mathbb{P}(\{a,b\}) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$ $|\mathbb{P}(\{a,b\})| = 4$

C c) $\emptyset$ $2^0 = 1$

$P(\emptyset) = \{\emptyset\}$ $2^0 = 1$, thus $|\mathbb{P}(\emptyset)| = 1$

Now, let your power sets from a), b), and c) be called A, B, and C respectively.

Find $A \times B$: $A \times B = \{\langle\emptyset,\emptyset\rangle, \langle\emptyset,\{a\}\rangle, \langle\emptyset,\{b\}\rangle, \langle\emptyset,\{a,b\}\rangle, \langle\{a\},\emptyset\rangle, \langle\{a\},\{a\}\rangle, \langle\{a\},\{b\}\rangle, \langle\{a\},\{a,b\}\rangle\}$

Find $B - A$:

$B - A = \{\{b\}, \{a,b\}\}$ $\{\} \subset \{\{\}\}$ ✓

True/False: $\emptyset \subset C$ $\rightsquigarrow$ WHY

True/False: $\emptyset \in C$ $\{\} \in \{\{\}\}$

$\emptyset \in \{a,b\}$ False AV

2.2 Set Operators

$\emptyset \subseteq \{a,b\}$

2. Use set builder notation and logic to prove the distributive law that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

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$A \cup (B \cap C)$	given
$= \{x \mid x \in A \cup (B \cap C)\}$	def'n of set builder
$= \{x \mid x \in A \vee x \in (B \cap C)\}$	def'n of union
$= \{x \mid x \in A \vee (x \in B \wedge x \in C)\}$	def'n of intersection
$= \{x \mid (x \in A \vee x \in B) \wedge (x \in A \vee x \in C)\}$	distributive law for logic
$= \{x \mid (x \in A \cup B) \wedge (x \in A \vee x \in C)\}$	def'n of union
$= \{x \mid (x \in A \cup B) \wedge (x \in A \cup C)\}$	def'n of union
$= \{x \mid x \in (A \cup B) \cap (A \cup C)\}$	def'n of intersect.
$= (A \cup B) \cap (A \cup C)$	def'n of set builder

Concl: I've shown $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ using set builder notation and logic rules

3. Use set builder notation to represent the set $A = \{2, 3, 5, 9, 17, 33, ...\}$

$1 \quad 2 \quad 4 \quad 8 \quad 16$

$y = 2^x + 1$

$\{ 2^x + 1 \mid x \in \mathbb{Z}^{\geq 0} \}$

2.3 Functions

4. Given $f : \mathbb{Z}^{\geqslant 0} \to \mathbb{Z}^{\geqslant 0}$, is the function $f(x) = \lfloor \sqrt{x} \rfloor$ onto? One-to-one?

Not one-to-one, it is onto.

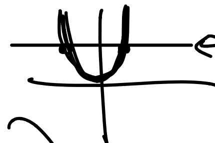
If the problem is changed so that $f : \mathbb{Z}^{\geqslant 0} \to \mathbb{Z}$, does it change whether f is onto or one-to-one?

Not onto, not one-to-one

5. Create your own function that is onto but not one-to-one. (Be sure to specify the domain and co-domain)

$f: \mathbb{R} \to \mathbb{R}^{\geq 0}$

$f(x) = x^2$



not onto-to-one b/c same out put

6. Create your own function that is one-to-one but not onto.

$f: \mathbb{R}^{\geq 0} \to \mathbb{R}$

$f(x) = x^2$

but what about these values

OR $f: \mathbb{R} \to \{1\}$

$f(x) = 1$

7a. Create a function that is both onto and one-to-one.

$f: \mathbb{R} \to \mathbb{R}$

$f(x) = x$

$f(x) = x^3$

7b. Find the inverse of the function you created in the previous question.

$f(x) = x^2$

swap

$f^{-1}(x) = \sqrt[3]{x}$

$x = (f^{-1}(x))^2$

$f^{-1}(x) = \sqrt{x}$

$y = x^2$

$x = y^2$

$y = \sqrt[2]{x}$

$f^{-1}(x) = \sqrt[2]{x}$

Order of operation

$(\ )$

$\overline{A}$

$\cap$

$\cup \quad -$

$\cancel{\times}$

Prove $A \cup (B \cap C)$

$A \cup (B \cap C)$ — given

$= \{x \mid x \in A \cup (B \cap C)\}$ — def'n of set builder

$= \{x \mid x \in A \vee x \in (B \cap C)\}$ — def'n of union

$= \{x \mid x \in A \vee (x \in B \wedge x \in C)\}$ — def'n of intersection

$= \{x \mid (x \in A \vee x \in B) \wedge (x \in A \vee x \in C)\}$ — distributive law for logic

$= \{x \mid (x \in A \cup B) \wedge (x \in A \vee x \in C)\}$ — def'n of union

$= \{x \mid (x \in A \cup B) \wedge (x \in A \cup C)\}$ — def'n of union

$= \{x \mid x \in (A \cup B) \cap (A \cup C)\}$ — def'n of intersect.

$= (A \cup B) \cap (A \cup C)$ def'n of set builder

Concl. I've shown $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ using set builder notation and logic rules