

Principles of cryptography Algorithms and Formulas

Find $\text{gcd}(a, b)$

Q	A	B	R
q_1	A_1	B_1	R_1
	B_1	R_1	\vdots
			\vdots

↓ ↓ ↓ ↓

Euclid's Algorithm
method-2 for gcd

$a > b$

Euclid-gcd (a, b) :

if $b = 0$
return a

else

return Euclid-gcd $(b, a \bmod b)$

Find $\text{gcd}(a, b)$

if $a > b$

$\text{gcd}(a, b) = \text{gcd}(b, a \bmod b)$

Relatively prime numbers:

They should have no prime factors in common.

only common factor is 1

$\text{gcd}(a, b) = 1$

a and b are relatively prime numbers

Euclid Totient Function

	criteria of 'n'	formula
$\phi(n)$	'n' is prime	$\phi(n) = (n-1)$
	$n = p \times q$ (p) and (q) are primes	$\phi(n) = (p-1)(q-1)$
	$n = a \times b$ either (a) or (b) is composite Both (a) & (b) are composite	$\phi(n) = n \times (1 - \frac{1}{p_1}) \times (1 - \frac{1}{p_2}) \dots$

Fermat's Little Theorem

(p) is prime number and a is the number integer not divisible by (p)

$$a^{p-1} = 1 \pmod{p}$$

Euler's Theorem

$$a^{\phi(n)} = 1 \pmod{n}$$

a, n are relatively prime

primitive Root

(x) is a primitive root of prime numbers p

if $a \bmod p, a^2 \bmod p, a^3 \bmod p \dots a^{p-1} \bmod p$ are distinct

$n, a = \{1, \dots, a\}$
 $a \bmod p, a^2 \bmod p$
 $\therefore a$ is P.R.

Multiplicative Inverse using Euclid's Algo

Q	A	B	R	T ₁	T ₂	T
				0	1	
1						1
	X	X	0	T ₁	X	X

$$T = T_1 - T_2 \times Q$$

T_1 is multiplicative inverse

At initial $T_1 = 0, T_2 = 1$

The Chinese Remainder Theorem

$$x = a_1 \pmod{m_1}$$

$$x = a_2 \pmod{m_2}$$

$$x = a_3 \pmod{m_3}$$

$$x = \sum_{i=0}^n (a_i M_i M_i^{-1})$$

Given	To find
a_1, a_2, a_3	M_1, M_2, M_3
m_1, m_2, m_3	$M_1^{-1}, M_2^{-1}, M_3^{-1}$

$$M = m_1 \times m_2 \times m_3$$

$$M_1 = \frac{M}{m_1}, M_2 = \frac{M}{m_2}$$

$$M_3 = \frac{M}{m_3}$$

finding inverse

$$M \times M^{-1} = 1 \pmod{m_i}$$

Trial & error.

Caesar cipher

$$C = E(P, K) \bmod 26$$

$$= (P + K) \bmod 26$$

$$P = D(C, K) \bmod 26$$

$$= (C - K) \bmod 26$$

here

$K = 3$

$$C = (P + 3) \bmod 26$$

$$D = (C - 3) \bmod 26$$

Shift cipher

A Caesar cipher is a special case of shift cipher



Play-Fair cipher

K	E	Y	W	O
R	D	A	B	C
F	G	H	I/J	L
M	N	P	Q	S
T	U	V	X	Z

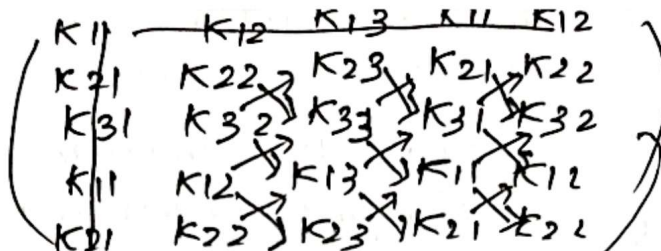
← 5x5 matrix →

Keyword is key word

same column | ↓ | wrap

same row → wrap

Rectangle
← →
swap



$$(P_1 P_2 P_3) \begin{pmatrix} R_1 & R_2 & R_3 \\ R_4 & R_5 & R_6 \\ R_7 & R_8 & R_9 \end{pmatrix} \text{mod } 26$$

-ve ↑
+ve ↓

poly alphabetic cipher

Encryption

$$C_i = (P_i + K_i \text{mod } m) \text{mod } 26$$

$$P_i = (C_i - K_i \text{mod } m) \text{mod } 26$$

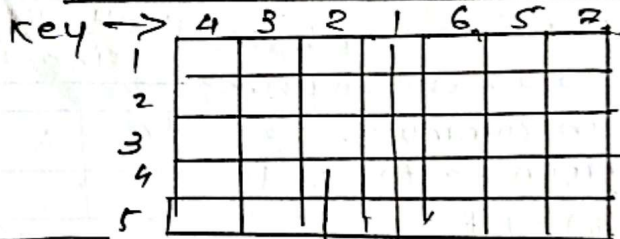
Transposition Techniques

Rail Fence Technique

Depth: D



Row-column Transposition



5x7 matrix
Cipher text ① ② ③ ... ⑥
this manner

Hill cipher * Imp

$$C = E(K, P) = P \times K \text{mod } 26$$

where K is a Key matrix

$$K = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{pmatrix} \text{mod } 26$$

For Decryption

$$P = D(K, C) = C \times K^{-1} \text{mod } 26$$

$$= P \times K \times K^{-1} \text{mod } 26$$

Key: $P_1 P_2 P_3$

plaintext $(P_1 P_2 P_3)$

ciphertext $(C_1 C_2 C_3)$

$$K^{-1} = \frac{1}{\det K} \times \text{adj } K$$

$$\therefore (C_1 C_2 C_3) = (P_1 P_2 P_3) \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{pmatrix} \text{mod } 26$$

poly alphabetic cipher

$$\begin{pmatrix} K_{11} & K_{12} & K_{13} & K_{11} & K_{12} \\ K_{21} & K_{22} & K_{23} & K_{21} & K_{22} \\ K_{31} & K_{32} & K_{33} & K_{31} & K_{32} \end{pmatrix}$$

