Boolean Algebra.

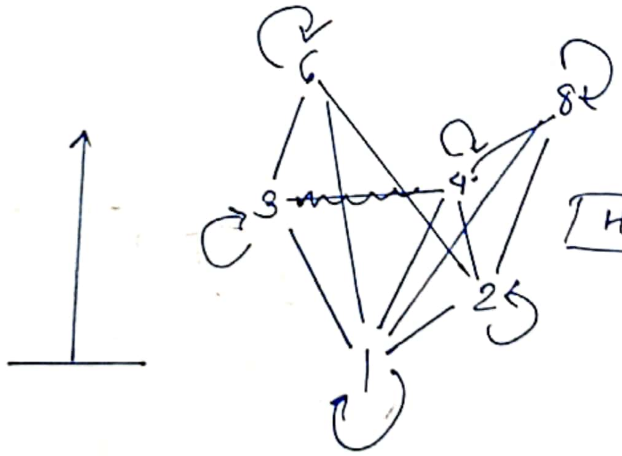
Cartesion product: product of two sets A and B denoted as $A \times B$ is the set of all ordered pairs of the form (a, b)

Binary Relation: A binary relation from A to B is a subset of $A \times B$

partially ordered set (poset) set A together with a partial ordering relation if is reflexive, antisymmetric and transitive

Hasse Diagram

$$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,8)$$
$$(2,2)\ (2,4), (2,6)\ (2,8)$$
$$(3,3)\ (3,6), (4,4), (4,8)$$
$$(6,6), (8,8)\}$$



Hasse Diagram

$S = \{a, b, c\}$

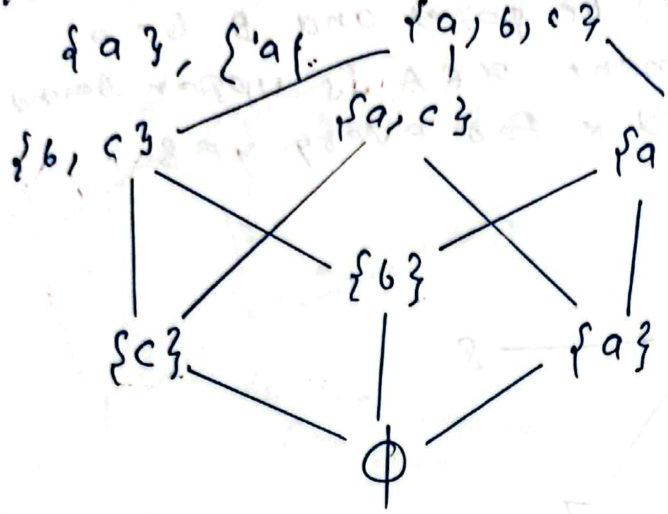
ϕ $\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}$

$\{a, b, c\}$ Draw Hasse Diagram

~~$R = \{\phi, \phi\}$,~~

$R = \{\phi, (\{\phi, \{a\}\}), \{\phi, \{b\}\}), \ldots$

$\{a\}, \{a$ ...



Hasse Diagram

<u>Maximal element</u> :- In a poset, if an element is not related to any other element

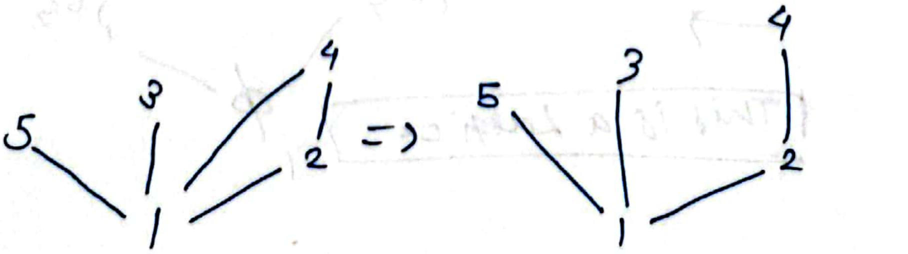
<u>minimal element</u> :- In a poset, if not element is related to an element

<u>Example</u> :- Let (P, R) be a poset

$P = \{1, 2, 3, 4, 5\}$ and R is ~~related~~ relation of divison

$P = \{1, 2, 3, 4, 5\} \Rightarrow \{1,1\}, 1,2, 1,3, 1,4, 1,5\}$, $(2,4)$,

~~$\{2, 4, 8$~~

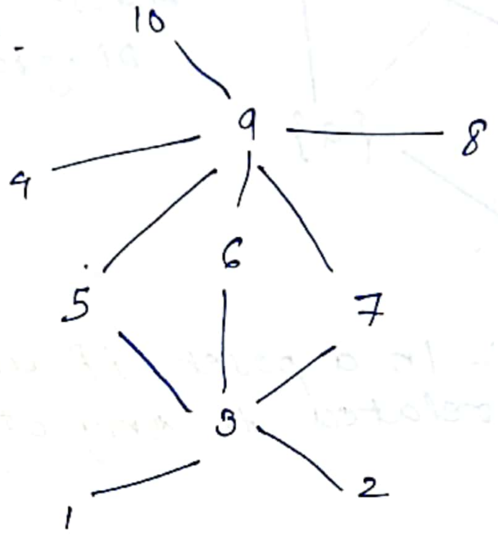


Theorem :- A finite non-empty poset (P, R) has atleast one maximal element and atleast one minimal element

LOWER BOUND AND UPPER BOUND

Upper Bound :- Let (P, R) be poset and B be a subset of P an element $x \in A$ is upper bound of B if y is related x for every $y \in B$

Example :-

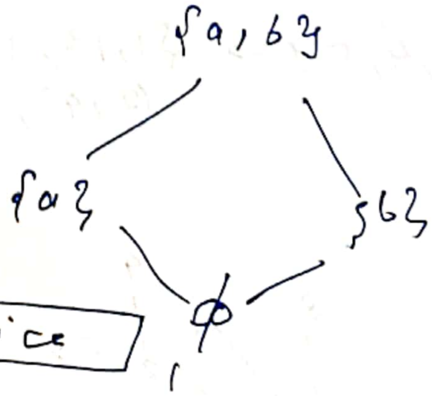


Lower Bound :- An element $x \in A$ is lower bound of B if x is related by y

Let $S = \{a, b\}$

$P(S) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

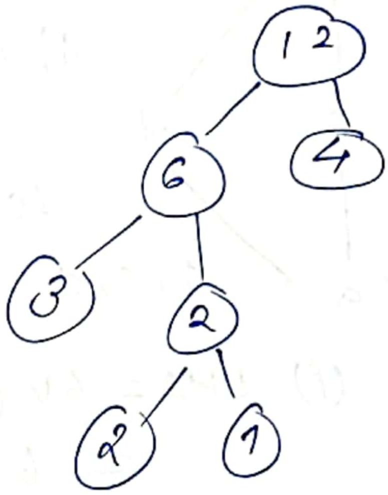
Hasse Diagram



$\longrightarrow$ This is a Lattice

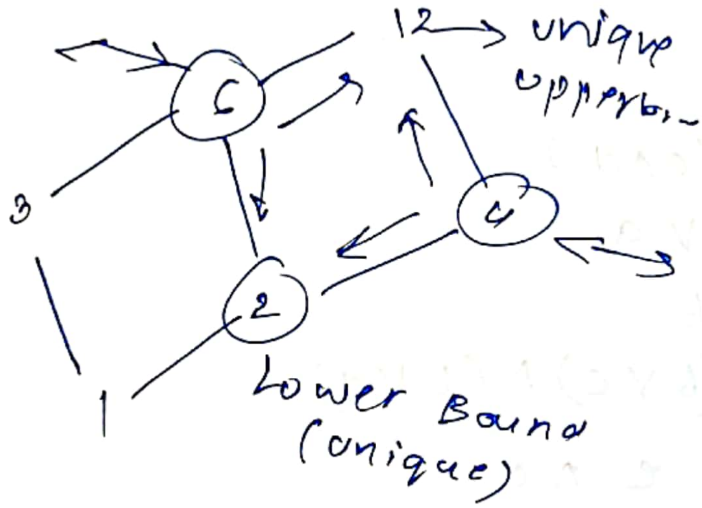
Let L be the set of all factors of 12 and let "|" divisibility relation on L. Show that (L "|") is a Lattice

Factor Tree



L = {1, 2, 3, 6, 12, 4)

L = {1, 2, 3, 4, 6, 12}

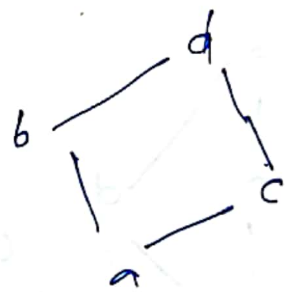


Properties of Lattice

→ 1) Idempotent Law

$a \vee a = a$

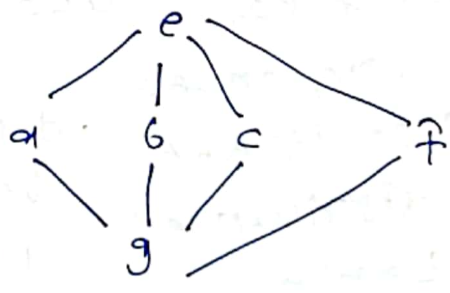
$a \wedge a = a$



→ 2) Associative Law

$(a \vee b) \vee c = a \vee (b \vee c)$

$(a \wedge b) \wedge c = a \wedge (b \wedge c)$

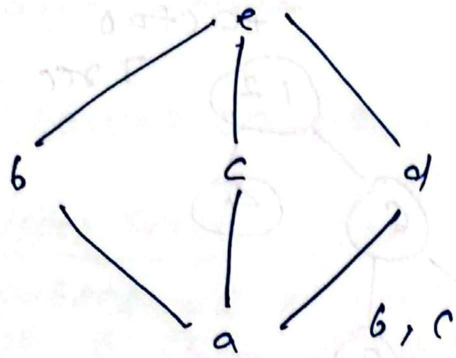


→ 3) Commutative - Law

$a \vee b = b \vee a$

$a \wedge b = b \wedge a$

Distributive-Lattice



i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Let us consider the elements b, c, d

(i) $LHS = b \vee (c \wedge d)$

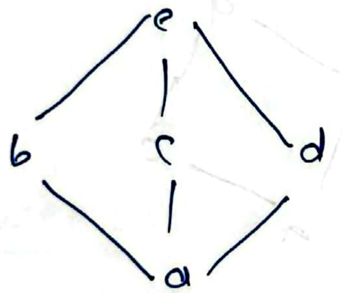
$= b \vee a$

$= b$

(ii) $RHS = (b \vee c) \wedge (b \vee d)$

$= e \wedge e$

$= e$



Least element : a

Greatest element : e

complement of b

LUB $(c, b) = e$

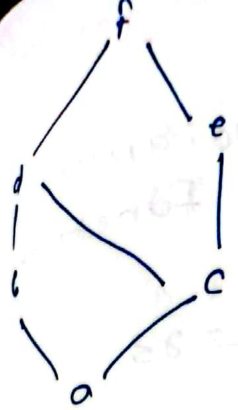
GLB $(c, b) = a$

Lattice has two complement hence its not a distributive Lattice

d is the complement of b because

LUB $(d, b) = e$

GLB $(d, b) = a$



Least element : a

Greatest element : f

$LUB(d,e) = f$

$GLB(d,e) = c \neq a$

(2) c is not complement of d

$LUB(c,d) = d \neq f$

$GLB(c,d) = c \neq a$

Q1A) Let A and B be two elements in a lattice $(A, \leq)$. Show that $a \wedge b = b$ if and only if $a \vee b = a$

$a \wedge b = b$ — Assume

$a \vee (a \wedge b) = a$ ...(absorption Law)

$a \vee b = a$ —(i)

let $a \vee b = a$

$b \wedge (a \vee b) = b$

$a \wedge b = b$ ..(absorption Law

Q2B) Let a, b, c elements in a Lattice $(A, \leq)$ show that :-

(i) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

(ii) $(a \wedge b) \vee (a \wedge c) \leq a \wedge (b \vee c)$

$a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

Generating functions

A numeric function is denoted by $(a_0, a_1, a_2, \ldots a_r \ldots)$ and a_r is rth term of function

Example :- $(1, 8, 27 \ldots \ldots, r^3 \ldots)$ $a_r = r^3$

Example :- $(0, 3, 6, 9, 12, 31, 63)$

then $a_r = \begin{cases} 3r \\ 2^r - 1 \end{cases}$

Generatic functions of Numeric f^n

let $(a_0, a_1, a_2, \ldots a_r \ldots)$

$(a_0, a_1 z, a_2 z^2 + a_3 z^3 + \ldots a_r z^r)$ is called generating functions.

$\boxed{S_\infty = \frac{a}{1-r}}$

Numeric Function	Generating f.
$a_r = 1$	$1 + z + z^2 + z^3 + \ldots = \frac{1}{1-z}$
$a_r = r$	$z + 2z^2 + 3z^3 = \frac{z}{(1-z)^2}$
$a_r = r^2$	$z + 4z^2 + 9z^3 = \frac{z(z+1)}{(1-z)^3}$
$a_r = {}^nC_r$	$= z + {}^nC_1 z + {}^nC_2 z^2 = (1+z)^n$
$a_r = \alpha^r$	$= 1 + \alpha z + \alpha^2 z^2 + \alpha^3 z^3 = \frac{1}{1-\alpha z}$

Example :- find generating fn 2, 4, 8, 16, 32

Generating function

$A(z) = 2 + 4z + 8z + 16z^3 + \cdots$

$2[1 + 2z + 4z + 4z^3 + \cdots]$

$= 2\left[\frac{z}{1-2z}\right]$; $|2z| < 1$

find generating function of sequence

$a_r = (r+2)(r+1)3^r$

put $r = 9$

$a_1 = (1+2)(1+1)3^{r=1}$

$(3)(2)(3)$

$= 18$

$a_2 = (2+2)(2+1)3^2$

$= (4)(3)(9)$

$= 12 \times 9 = 108$

$a_0 + a_1 z + a_2 z^2 + a_3 z^3 \cdots$

$(z) = 2 + 18z + 108z^2 + 540z^3 + \cdots$

$2[1 + 9z + 54z^2 + 270z^3 + \cdots]$

$2\left[1 + (-3)(-3z) + \frac{(-3)(-3-1)}{2}(-3z)^2 + \cdots\right]$ —(1)

$2(1-3z)^{-3} = \frac{2}{(1-3z)}$

$x \quad (\cancel{x6}) \quad \boxed{(x^5)^2 = x^{5 \times 2} = x^{10}}$

$\underline{x^{16}}$ in $(x^2 + x^3 + \dots)^{x5}$ find coeff of x^{16}

$x^{10} \cdot x^6 \qquad (x^2)^5 [1 + x + x^2 + x^3 \dots]^5$

$= x^{(10+6)}$

$= x^{16} \quad \underline{x^6} \rightarrow x^{10} [1 + x + \underline{x^2} + x^3 \dots]^5$

$(x^2)^{5} = x^{10} \qquad \cancel{1x} [1 + x + x^2 + \dots x^n] = (1-x)^{-n}$

$\underline{x^6} \qquad = (1-x)^{-5}$

$\underline{(1-x)^{-5}} = 1 + {}^5C_1 x + {}^6C_2 x^2 + \dots \underline{{}^{5+6}C_6 x^6}.$

coeff of x^{16} is ${}^{5+6-1}C_6$

$\underline{{}^{10}C_6}$

How many ways are there to select 25 toys from seven types of toys with between two and ~~six~~ six of each type?

$G.F = (x^2 + x^3 + x^4 + x^5 + x^6)^7 \cancel{+ x^7)}$

We want the coefficient x^{25} in

$x^{14} (1 + x^2 + x^3 + x^4)^7$

$x^{14} (1 + x^2 + x^3 + x^4)^7$

$1 + x + x^2 + \dots + x^m = \dfrac{1 - x^{m+1}}{1 - x}$

$\dfrac{\cancel{1 - x^5}}{}$

$= x^{14} (1 - x^5)^7 (1 - x)^{-7}$

${}^{17}C_{11} - 7 \cdot {}^{12}C_6 + 21 \cdot {}^7C_1$

$(1+x+x^2+x^3+x^4) = (1-x^5)^3(1-x)^{-3}$

coefficient of x^6 in (1) $= {}^{3+6-1}C_6 (-3) \cdot {}^3C_1 = {}^8C_6 - 3.3 = 19$

30 marks to 8 questions such that each question recieves at least 2 marks

coeff of

$\cancel{(1-x)}$

$(1-x^5)^7 = (1 + {}^7C_1 x^5 + {}^7C_2 (x^5)^2 + {}^7C_3 (x^5)^3$
${}^7C_4 (x^5)^4 \ldots)$

$= \boxed{(1 + {}^7C_1 x^5 + {}^7C_2 \cancel{(10} x^{10} + {}^7C_3 (x)^{15} + {}^7C_4 (x)^{20} + \ldots}$

$(1-x)^7 (1 + \cancel{{}^7C_1 x^5 + {}^7C_2 x^{10}} \ldots)$
$(\cancel{1+x+}$

$\boxed{(1-x)^{-7} = 1+x+x^2+\ldots)}$

$(1 + {}^7C_1 x^5 + {}^7C_2 x^{10} + {}^7C_3 x^{15} + {}^7C_4 x^{20})$
$(1+x+x^2+x^3+x^4 \ldots)$

$= 1 + {}^7C_1 x^5 + 1 + {}^7C_1 x^{6} + 1 + {}^7C_1 x^7 +$
$1 + {}^7C_1 x^8 + 1 + {}^7C_1 x^9$

$\cancel{{}^7C_2 x}$ ${}^7C_2 x^{11}$

$(1-x)^{-7} = 1 + {}^{7}C_1 x + {}^{8}C_2 x^2 + {}^{9}C_3 x^3 +$

${}^{10}C_3 x^4 + + {}^{11}C_4 x^5 + {}^{12}C_5 x^6 +$

$\cancel{{}^{13}C_5}$ $\quad {}^{13}C_6 x^7$

$C_5 x^6$

${}^{7}C_1 x^5 \times + {}^{12}C_5 x^6$

${}^{7}C_1 . {}^{12}C_6 x^{11}$

How many ways are there to get sum of 25 when 10 distinct dice are rolled?

A dice has 6 possiblities

dice $= \{1, 2, 3, 4, 5, 6\}$

$G.F = (1 + x^2 + \dots x^6)^{10}$

$= x^{10}(1-x^6)^{10}(1-x)^{-10}$

$(1-x^6)^{10} = \cancel{1 - {}^{10}C_1 x^6 + {}^{10}C_2}$

$(1-x^m)^n = 1 - \boxed{{}^{10}C_1 x^6 \; (2)} + \boxed{{}^{10}C_2 x^{12} \; (1)} + {}^{10}C_3 x^{18}$

${}^{10}C_4 x^{24} + {}^{10}C_5 x^{30} + {}^{10}C_6 x^{36}$

$(1-x)^{-10} = 1 + {}^{10}C_1 x + {}^{11}C_2 x^2 + \boxed{{}^{12}C_3 x^3 \; (1)}$

$+ {}^{13}C_4 x^4 + \cancel{{}^{14}C_5 x^6}$ $\; {}^{14}C_5 x^5 +$

$\boxed{{}^{15}C_6 x^6 \; (\text{III})} + {}^{16}C_7 x^7 + {}^{17}C_8 x^8 +$

$\boxed{{}^{18}C_9 x^9 \; (2)}$ $+ {}^{19}C_{10} x^{10} + {}^{20}C_{11} x^{11} + {}^{21}C_{12} x^{12}$

$+ {}^{23}C_{14} x^{15} + {}^{22}C_{13} x^{13} + {}^{22}C_{14} x^{14}$

$(x^2 + \dots x^{16})^8$

examiner wants 30 questions in which minimum each question should not be assigned less than 2 marks

8 Ques   30 marks

2 marks → 1 Question

we have $\boxed{8 \text{ Questions} \to 16 \text{ marks}}$

14 marks → 0 + 1   X

0 + 1

$(x^2 + x^3 + x^4 + \dots x^{16})^8$

$= x^{\cancel{16}} \cancel{(1 + x^2 + x^3} + x^3 \dots x^{15})$

$\cancel{x^2}\ x^3 \quad (x^2)^8 (1 + x + x^2 + x^3 + \dots x^{14})^8$

$x^{16} (1 + x + x^2 + x^3 + \dots x^{14})^8$

$= x^{16} \cancel{(1+} (1 - x^{15})^8 (1 - x)^{-8}$

$= \boxed{x^{16} (1 - x^{15})^8 (1 - x)^{-8}}$

$(1 - x^{15})^8 = 1 - \underline{8C_1 x^{15}} + 8C_2 x^{30} + 8C_3 x^{45}$

$+ 8C_4 x^{60} + 8C_5 x^{75} + 8C_6 x^{90}$

$+ 8C_7 x^{105} + 8C_8 x^{120}$

$(1 - x)^{-8} = \cancel{1 + {}^{10}C_1 x}$

$= 1 + 8C_1 x + 9C_2 x^2 + 10C_3 x^3 +$

$11C_4 x^4 + 12C_5 x^5 + 13C_6 x^6 + 14C_7 x^7 + 15C_8 x^8$

7
15
× 2
125
4
15
× 8

$\sum_{r=0}^{n} (-1)^r C(n,r)(x^m)^r$	$(1-x^{15})^8$	$r=0$	$n=8$
$\sum_{r=0}^{\infty} C(n+r-1,r)(x)^r$	$(1-x)^{-8}$	$r=14$	$n=8$

$^{15}C_9 x^9 + {}^{16}C_{10} x^{10} + {}^{17}C_{11} x^{11} +$

$^{18}C_{12} x^{12} + {}^{19}C_{13} x^{13} + {}^{20}C_{14} x^{14}$

$^{1}C_0 \quad {}^{21}C_{14}$

$\cancel{12 < A < 8} \quad 4 < A < 8$

$\cancel{4 < A} < \quad 2 < B \quad 5 < C < 2$

$(x^4 +$

1, 2, 3, 4, 5 $\qquad$ 2 $1/(3\times2)$ 12+4

50th permutation covered by fixing 1, and 2

48th $\qquad$ 4 50th

31245 $\qquad$ 31254

find 18th permutation of marks a, b, c, d in lexicographical order

$18-1 = 17 = \underline{2}\times3! + \underline{2}\times2! + \underline{1}\times1!$

Index	a b c d	
2 2 1	0 1 2	c
21	a b d	b
1	dd	d
0		

Index	order	
2 2 1	a b c d	c
21	a b d	d
1	a b	b
	a	a

18th permutation is ~~cb~~ cdba

Find 268th permutation of LISTEN in lexicographical order

$268 - 1 = 267 = 5! + 4! + 3! + 2! + 1$

$= \cancel{2 \times 5}$

$= 2 \times 5! + 1 \times 4! + 0 \times 3! + 1 \times 2! + 1 \times 1!$

21011	LISTEN	S
1011	LITEN	I
011	LTEN	L
11	LTEN	E
1	TN	N
	T	T

$\boxed{\text{SILENT}}$

6th permutation of 4321 in Lexicographical order

$6 - 1 = 5 = \cancel{2 \times 2! + 1 \times 1}! \quad 2 \times 2! + 1 \times 1!$

Sequence ~~021~~	Marks	Ans
~~4321~~ 021	4321	4
21	821	1
1	32	2
		3

4123

50th permutation of five marks 0, 1, 2, 3, 4 in reverse ~~lexi~~. lexicographical order

$50 - 1 = 49 = \underline{2} \times 4! + \underline{0} \times 3! + \underline{0} \times 2! + 1 \times 1!$

$= \underline{2001}$

~~2000~~

2001	43210	2
001	4310	4
01	310	3
1	10	0
	1	1

$\boxed{10342}$

1 2 3 4 5

$100 - 1 = 99 = 4 \times 4! \; 0 \times 3! \; 2 \times 2! \; 1 \times 1!$

4 0 2 1 1

6

$N(A_1) = 9!$ ways

$N(A_2)$ $9!$ ways

$10! - 2 \times 9! - 2 \times 9! + 2 \times 2 + 8!$

~~2~~ 3

232 850

100 th permutation of marks 1, 2, 3, 4, 5 in reverse lexicographical order

$100 - 1 = 99 = \underline{4} \times 4! + \underline{0} \times 3! + \underline{1} \times 2! + 1 \times 1! \rightarrow 4011$

4011	54321	1
011	5432	5
11	432	3
1	42	2
		4

$\boxed{42351}$

$N(a) = \frac{6300}{3} = 2100 \quad N(b) = \frac{6300}{5} = 1260$

$N(ab) = 420$

$(a'b) = 2100 + 1260 \cancel{-420}$

$(A_1) = 1$

$(A_2) = 8$

prime divisor of 70 are 2, 5 and 7

$N(a_1) = \frac{70}{2} = 35$

$N(a_2) = \frac{70}{5} = 14$

$N(a_3) = \frac{70}{7} = 10$

$a_1 = 2$
$a_2 = 5$
$a_3 = \cancel{3}7$

$N(a_1, a_2) = \cancel{7}7$ ~~N (aa₁~~

$N(a_1 a_3) = 5$

$\boxed{N(a_3 a_2) = 2}$

$N(a'b'c') = N - N(a_1) - N(a_2) - N(a_3)$
$+ N(a_1, a_2) + N(a_2, a_3) + N(a_3 a_1)$

$70 - 35 - 14 - 10 + 7 + 5 + 2$

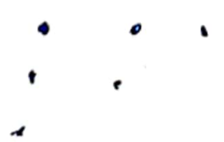
$= 25$

Ferrers Diagram

If we want to ~~take~~ a partion of a number we use this Diagram for representing partition

$\pi = \quad 6 = 4 + 2$ or

$6 = 3 + 2 + 1 \cancel{+}$



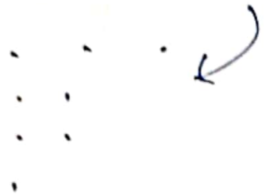
$\pi = 3$

$\pi = 2$

$\pi = 1$

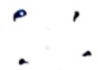
$8 = 4 + 3 + 1$

conjugate of the graph



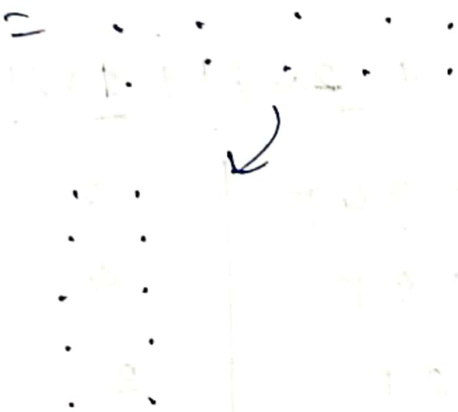
self conjugate graphs → ~~$2 = 1+1$~~ $4 = 2 + 2$

conjugate



$10 = 5 + 5$

=



27th permutation

$27 - 1 = 26 = \underline{1} \times 4! + \underline{0} \times 3! + \underline{1} \times 2! \; \underline{0} \times 1!$

Index	marks	order
1 0 1 0	1 2 3 4 5	2
0 1 0	1 3 4 5	1
1 0	3 4 5	4
0	3 5	3
	5	5

$\boxed{2\ 1\ 4\ 3\ 5}$

65th permutation

$64-1 = 63 = \underline{2} \times 4! + \underline{1} \times 3! + \underline{4} \times 2! + \underline{1} \times 1!$

Index	marks	lexicographical order
2 1 4 1	1 2 3 4 5	3
1 4 1	1 2 4 5	2
4 1	1 4 5	

$64-1 = 63 = \underline{2} \times 4! + \underline{2} \times 3! + \underline{1} \times 2! + \underline{1} \times 1!$

2 2 1 1	1 2 3 4 5	3
2 1 1	1 2 4 5	4
1 1	1 2 5	2
1	1 5	5
	1	1

$\boxed{3\ 4\ 2\ 5\ 1}$

$(p\vee q) \wedge \neg[(\neg p) \wedge (\neg q \vee \neg r)] \vee (\neg p \wedge \neg q)$
$\vee (\neg p \wedge \neg r)$

$(p\vee q) \wedge \neg[(\neg p) \wedge (\neg q \vee \neg r)]$

$(p\vee q) \wedge \neg[(\neg p) \wedge \neg(q \vee r)]$

$\cancel{\neg(p\vee q)\vee r}$

$(p\vee q) \wedge \neg((\neg p) \wedge \neg(q\vee r)] \vee (\neg p \wedge \neg q) \vee$
$\neg(p \vee r)]$

$(p\vee q) \wedge \neg((\neg p) \wedge \neg(q\vee r)] \vee (\neg(p\vee q) \vee$
$\neg(p\vee r)]$

$(p\vee q) \vee [(p\vee q) \wedge r]$ $\quad (p\vee q) \wedge (p \vee r)$

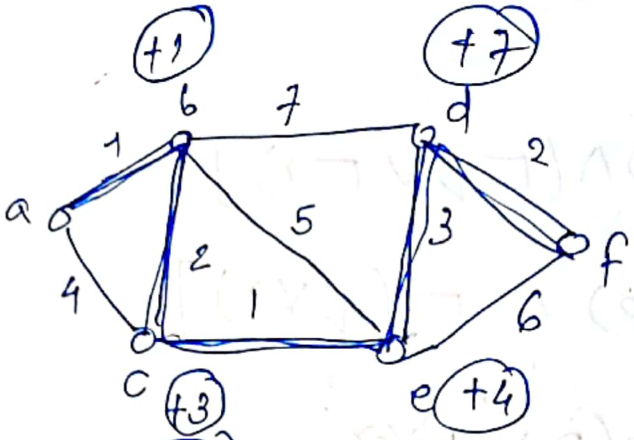
$\cancel{[p\vee q]} \vee$

$(p\vee q) \vee [(\wedge r) \wedge (p \vee r)$

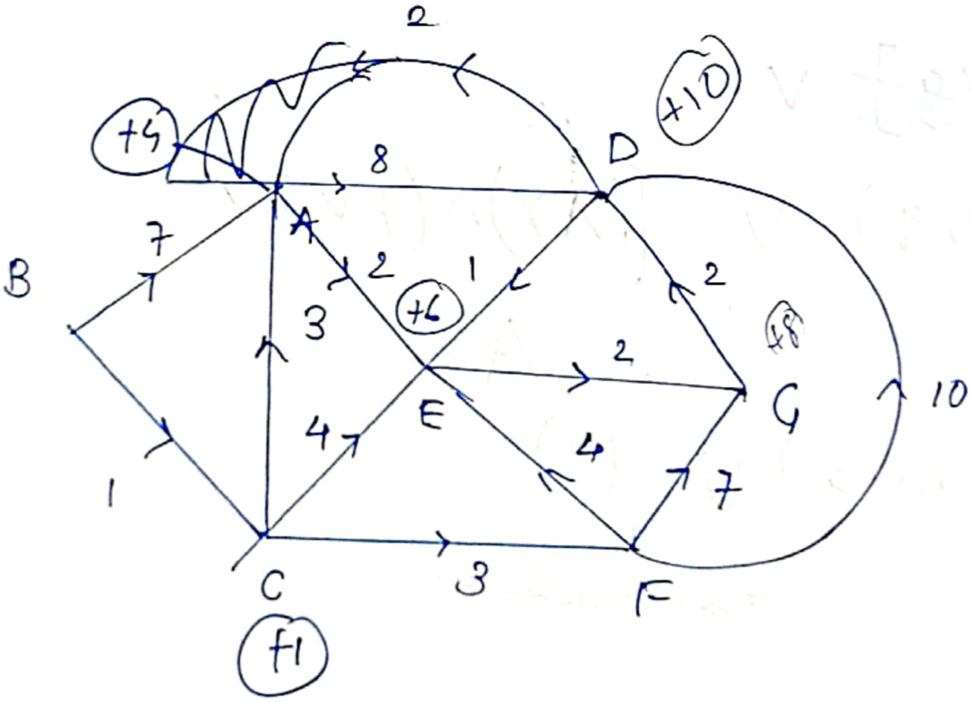
$\wedge r$

$(p\vee q) \vee (r)$

Tautology



	a	b	c	d	e	f
a	0	∞	∞	∞	∞	∞
b	0	1	4	∞	∞	∞
c	0	1	3	8	6	∞
e	0	1	3	8	4	∞
d	0	1	3	7	4	10
f	0	1	3	7	4	9



A	B	C	D	E	F	G
∞	$\boxed{0}$	∞	∞	∞	∞	∞
7	$\boxed{0}$	$\boxed{1}$	∞	∞	∞	∞
$\boxed{4}$	$\boxed{0}$	$\boxed{1}$	∞	5	$\boxed{4}$	∞
$\boxed{4}$	$\boxed{0}$	$\boxed{1}$	$\overset{12}{\infty}$	$\boxed{6}$	∞ $\boxed{4}$	∞
$\boxed{4}$	$\boxed{0}$	$\boxed{1}$	$\cancel{\infty}$ 12	$\boxed{6}$	$\cancel{\infty}$ $\boxed{4}$	$\boxed{8}$
4	$\boxed{0}$	$\boxed{1}$	$\boxed{10}$	$\boxed{6}$	$\cancel{\infty}$ $\boxed{4}$	$\boxed{8}$

$n = 5$

$2+1+1+1$

$\underbrace{2+2}+1$

$U_{10} = \{1, 3, 7, 9\}$ $\quad 9 \cdot 6 = 96 \ (10)$

·	1	3	7	9
1	1	3	7	9
3	3	9		
7				
9				

$3(1) = 3 \quad 1+1+1$

3