Engineering Economics and Financial management.

P : Present Amount

A : Year End payment

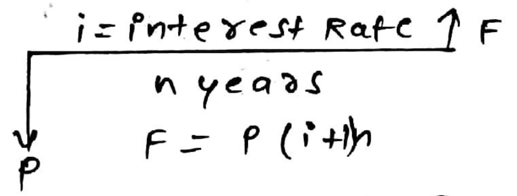
F : Future Accumulated sum / Future value

I : Interest Rate

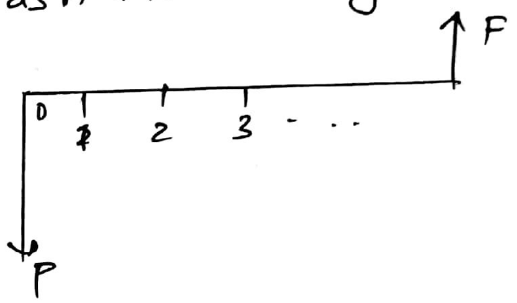
n : number of years

G : Gradient.

$\boxed{\text{1. single payment series}}$



cash flow Diagram



principal Amount : P.

for single payment series

n	Principal Amount	Interest owned princpl Amount	Total
			$P+Pi=P(i+1)$
1	P	$P \cdot i$	$P(i+1)+P(i+1)i$
2	$P(i+1)$	$P(1+i)i$	$=P(i+1)^2$
3	$P(i+1)^2$	$P(i+1)^2 i$	⋮
⋮			

$$\text{present Amount } (i+1)^{\text{no. of years}} = \text{Future Accumulated sum}$$

$$\boxed{P(i+1)^n = F}$$

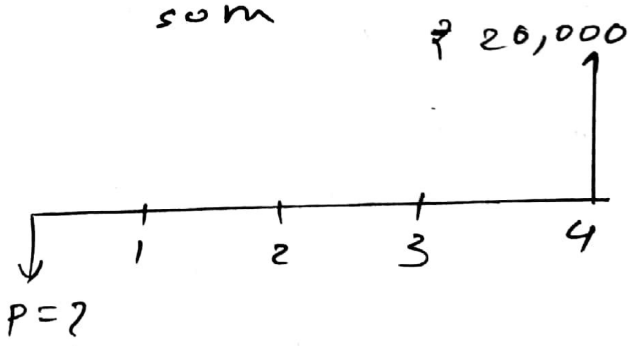
Q1) Given Data :-

$P = ?$

$i = 10\%$

Future Accumulated sum $= 20,000$

no. of years = 4 years



solution:-

present Amount $(i+1)^{\text{no. of years}}$ = Future Accumulated sum

$\therefore P(i+1)^n = F$

$\therefore P = \frac{F}{(i+1)^n}$

$\therefore P = ₹20{,}000 \times \frac{1}{(0.10+1)^4}$

$\begin{array}{r} 0.1 \\ +1 \\ \hline 1.1 \end{array}$

$\therefore P = ₹20{,}000 \times \frac{1}{(0.1+1)^4}$

$\therefore P = ₹20{,}000 \times \frac{1}{(1.1)^4}$

$\therefore P = \frac{₹20{,}000}{1.4641}$

$\therefore \boxed{P = ₹13{,}660.27}$

Alternative method to solve compound interest table

$\therefore P = F \times [P/F, i, n]$

$\therefore P = 20{,}000 [P/F, 10\%, 4]$

$\therefore P = 20{,}000 \times 0.6830$

$\therefore \boxed{P = 13660}$

Q) How much money will be accumulated in 25 years if ₹800 is deposited one year from now ₹2400 from 6th year and ₹3300 8th year.

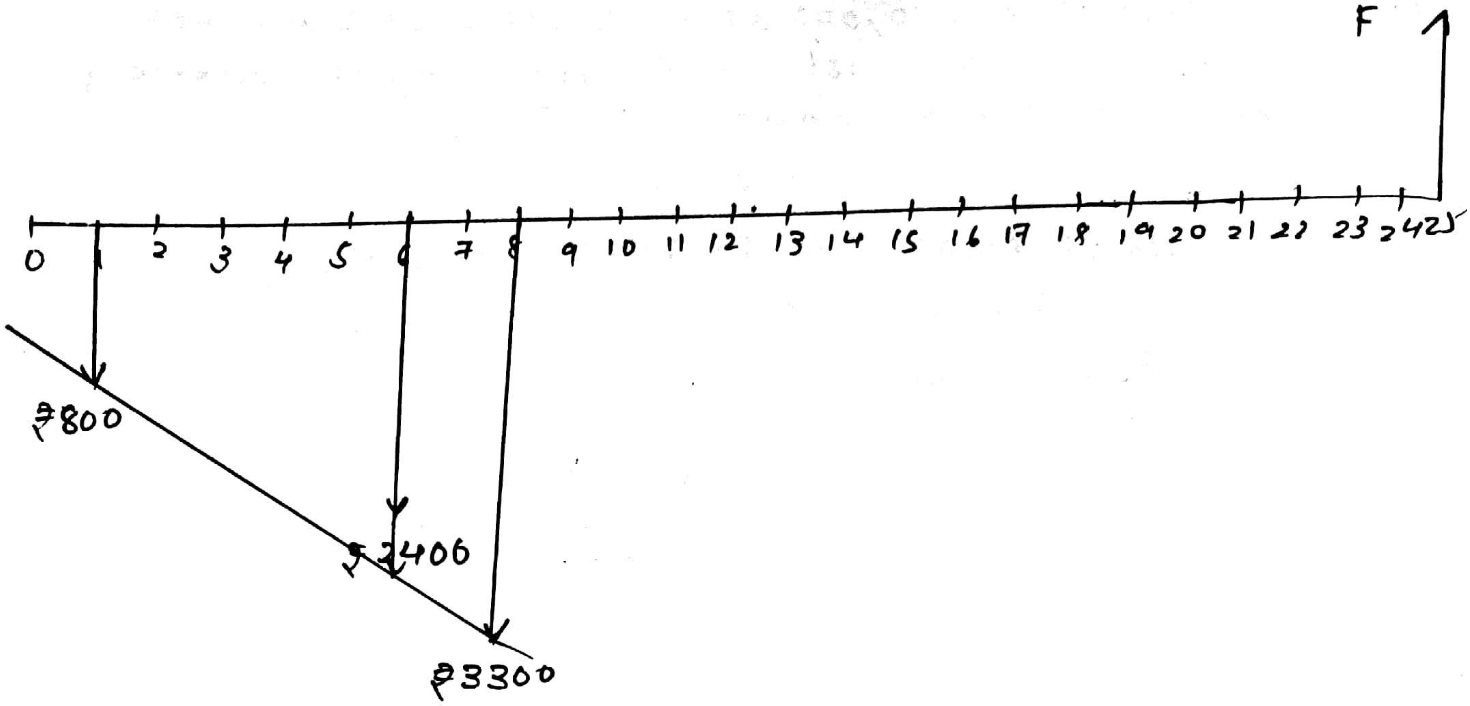
given $P_1 = 800$ at $n=1$

$P_2 = 2400$ at $n=6$

$P_3 = 3300$ at $n=8$

interest rate = 18 %

$n_{total} = 25$ years



$\therefore P_1 = F_1 \times [P/F_1, i, n]$

$\therefore P_1 = 800 \times [P/F_1, 18\%, 1]$

$\therefore P_1 = 800 \times 0.8475$

$\therefore P_1 = 678$

$P_2 = F_2 \times [P/F_2, i, n]$

$P_2 = ₹2400 \times [P/F_2, 18\%, 6]$

$P_2 = ₹2400 \times 0.3704$

$P_2 = 888.96$

$P_3 = ₹3300 \times [P/F_3, 18\%, 8]$

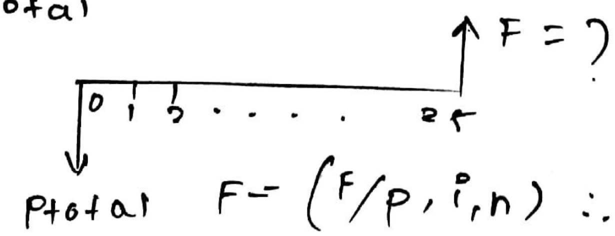
$₹3300 \times [0.266]$

$= 877.8$

$P_{total} = P_1 + P_2 + P_3$

$= 678 + 888.96 + 877.8$

$P_{total} = 2444.76$



$F = (F/P, i, n) \quad \therefore F = (F/P, 18\%, 25)$

$F = 2444.76 \times 62.669$

$\boxed{F_{total} = 153210.66}$

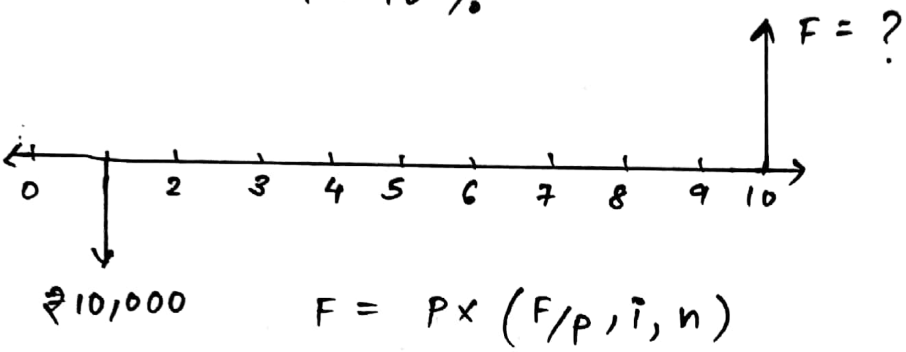
You have deposited ₹10,000 at 10% interest rate per annum on January 1st 2025. How much money will be accumulated January 1st 2035

Given :- No. of years : 2035 − 2025 = 10 years

$n = 10$ years

$P_1 = ₹10,000$

$i = 10\%$



$F = P \times (F/P, i, n)$

$F = ₹10,000 \times (F/P, 10\%, 10)$

$F = ₹10,000 \times 2.594$

$F = 25940$ RS.

Q2). Given :

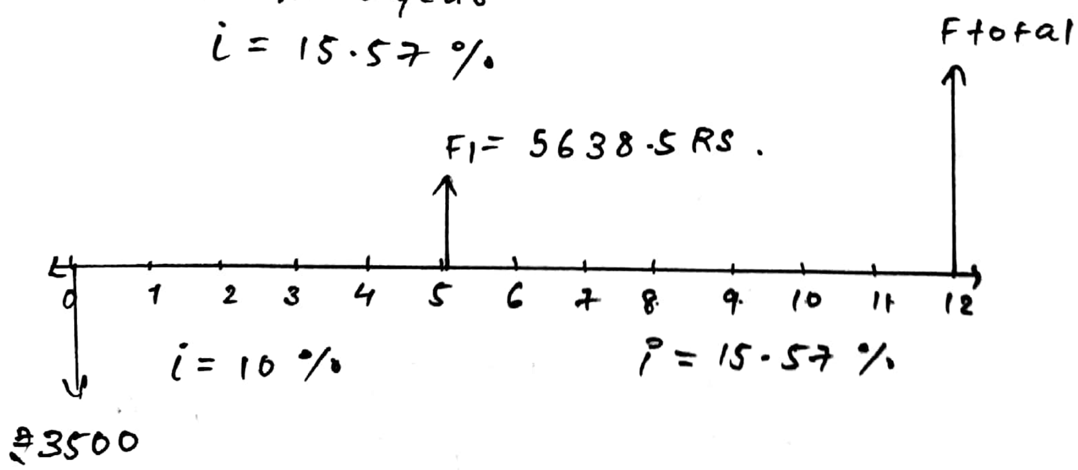
No. of years (n) = 12 years

$P = ₹3500$

$i = 10\%$ ~~for n =~~ till $n = 5$ years

from $n = 6$ year

$i = 15.57\%$



$F_1 = P \times (F/P, i, n)$

$= ₹3500 \times (F/P, 10\%, 5 \text{ years})$

$= 3500 \times (1.611)$

$F_1 = ₹5,638.5$

$F_2 = 5638.5 \times (F/P, 15.57\%, 7)$

since 15.57 not in compound interest table

$P(i+1)^n = F$

$P = 5638.5$

$i = 15.57$

$n = 7$ years

$\therefore F_{final} = (\cancel{15.57} + \cancel{1}\, 0.1557)^7 \times 5638.5$

$\therefore F_{final} = (0.1557 + 1)^7 \times 5638.5$

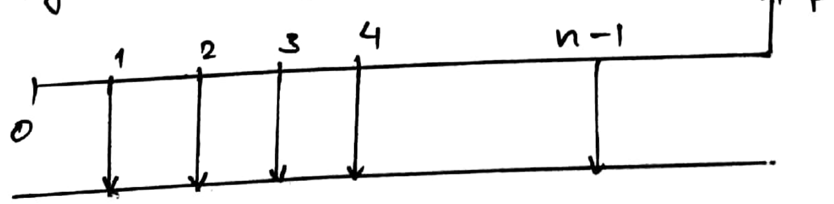
$= 2.754 \times 5638.5$

$\boxed{F_{final} = 15,528.429 \text{ RS}}$

$\therefore$ Future Accumulated sum is RS 15,528.429

Equal payment series

Equal payment compound Amount factor F=P



$F = A \times [F/A, i, n]$

A: Annual year End payment

Question on Equal Payment Series

Q1) You want to accumulate ₹5637 by making a series of 5 equal payments at interest rate of 6 %. compounded anually. what is Required Amount of each payment

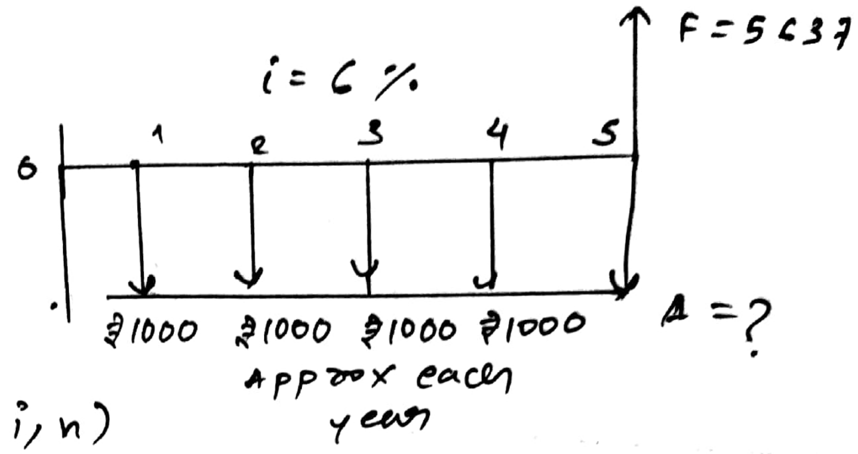
Given :-

$F = ₹5637$

$i = 6\%$

$n = 5$ years

$A = ?$



$\therefore F = A \times (F/A, i, n)$

$\therefore A = F \times (A/F, i, n)$

$\therefore A = ₹5637 \times (A/F, 6\%, 5)$

$A = 5637 \times 0.1774$

$A = 1000.0038$ RS

Q2) Determine amount P, that you should deposit into an account 2 years from now in order to withdraw ₹4000 /year For five years starting three years from now at 15 % interest rate

$P = ?$

$n = 2$

Q3)

Given :- $A = ₹\ 10,000$

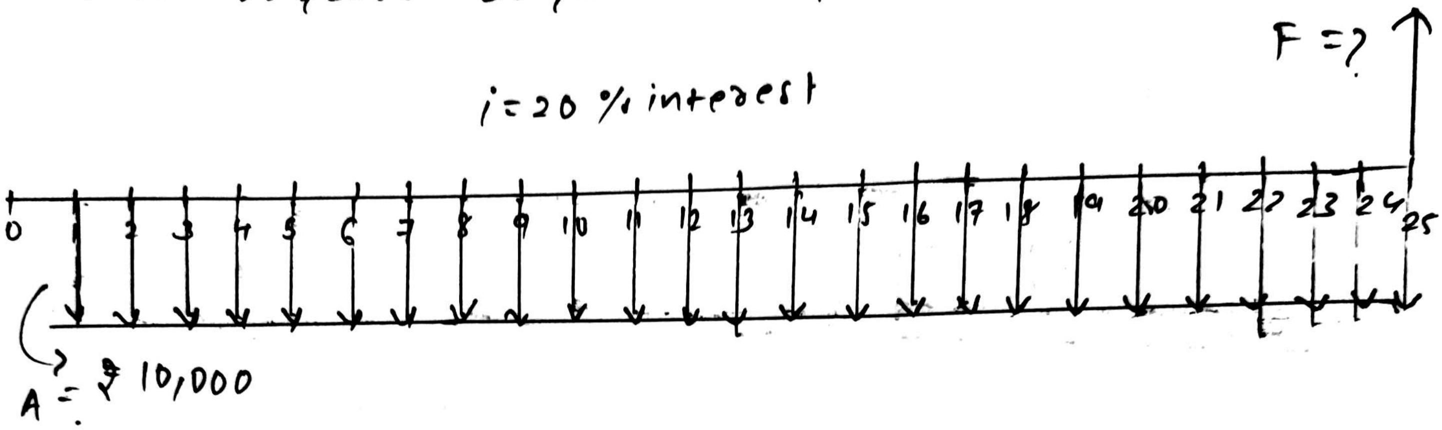
Age of person = 35 years old

$i = 20\%$

Amount after 60 years old

$\therefore n = 60 \text{ years} - 35 \text{ years} = 25 \text{ years}$

$A = ₹\ 10,000$
$n = 25$ years
$F = ?$
$P = ₹\ 10,000$



$$F = A \times (F/A, i, n)$$

$$F = ₹\ 10,000\ (F/A, 20\%, 25)$$

$$A = F \times (A/F, i, n)$$

$$₹\ 10,000 = F \times (A/F, 20\%, 25)$$

$$10,000 = F \times 0.00212$$

$$\boxed{F = 4,716981.132}$$

Nominal Interest Rate

suppose there is a savings account that pays you 6% interest compounded quaterly. if your depositing ₹500 now, then how much money will you have in your account at end of 3 years.

$P = ₹\ 500$
$i = 6\%$
$n = 3$

$F = ?$

Quaterly = 3 months

In one year 3 months × 4 = 12 months
we have 4 quaters → compounding frequency

∴ In 3 years we have $4 \times 3 = 12$ quarters

$$\frac{\text{Nominal interest Rate per annum}}{\text{Quarter}} = \frac{6\%}{4} = 1.5\%$$

$P = 500 \quad i = 1.5\% \quad n = 12$ months

$F = P \times (F/P, i, n)$

$= 500 \times [F/P, 1.5\%, 12)$

$= 500 \times 1.196$

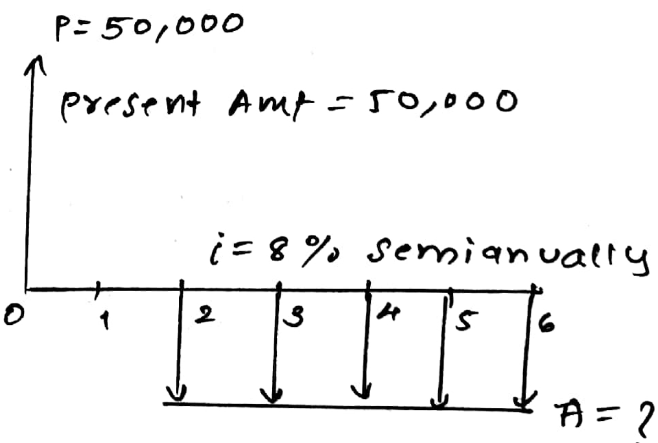
$\boxed{F = 598}$

Note:

1) The given Interest Rate (Nominal should be divided by compounding frequency which is 4 in this case

2) The time interval should be multiplied by compounding frequency

Suppose you have borrowed ₹50,000 at an interest rate of 8% per year compounded semianually and desire to repay the money with five equal end-of year payments, with the first payment made 2 years after receiving the loan amount



$$i_{effective} = \left(1 + \frac{r}{m}\right)^c - 1$$

$$i_{effective} = \left(1 + \frac{0.08}{2}\right)^c - 1$$

$c = l \times m$

$i_{effective} = (1 + 0.04)^2 - 1 \qquad = 1 \times 2$

$(1.04)^2 - 1 = 8.16\%$

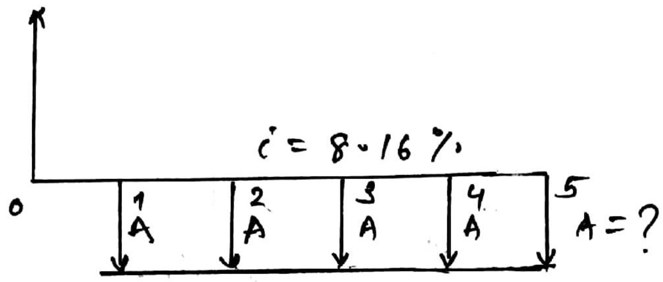
$i_{effective} = 8.16$

ii) size of annual payment

$$F = P(i+1)^n$$

$$F = 50{,}000\,(1+0.816)^1$$

$$\boxed{F = 54{,}080}$$



$$A = P_1 \times \frac{i\,(i+1)^n}{(1+i)^n - 1}$$

$$A = \frac{54{,}080 \times (1.0816)^5\,(0.0816)}{(1.0816)^5 - 1}$$

$$A = 13{,}601.85 \approx A = 13{,}602$$

Economic Analysis of Alternatives

1) Present worth Method
2) Future worth Method
3) Annual worth Method
4) Internal Rate of Return (IRR Method)

Present worth Method

The time value of the money you have now is not the same as it will be years from now and vice versa

for Example:-

~~$40~~ 1₹ : 1$ ⟶ ₹40 : $1

Need of Present worth Analysis :- Rate = 12 %

class	Machine 1.	Machine 2
Initial cost	20,000	35,000
Annual operating cost	9,000	4000
	4000	
Salvage value		7000
Life Time	5	5

which machine is preferable ?

Formulas :

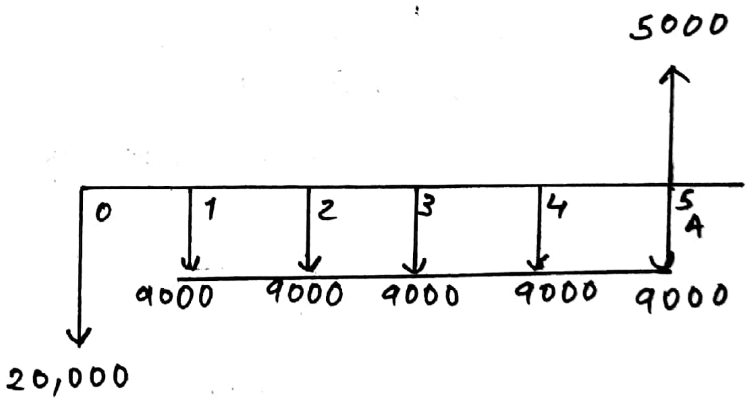
$$\text{Future value} = \text{Present value}(1+\text{rate})^{\text{Life time}}$$

$$\text{present value} = \frac{\text{Future value}}{(1+\text{rate})^{\text{life Time}}}$$

$$PV = FV / (1+r)^n$$

$$FV = PV(1+r)^n$$

class	Machine 1
Initial cost	20,000
Annual operating cost	9000
Salvage	5000
Life time	5



$$PW(\text{initial cost}) = -20,000$$

$$PW(\text{operating cost}) = -\left\{\frac{9000}{(1+0.12)^1} + 9000/(1+0.12)^2 + 9000/(1+0.12)^3 + 9000/((1+0.12)^4 + 7000/(1+0.12)^5\right\}$$

$$PW(\text{operating cost}) = -9000\left[\frac{(1+0.12)^5-1}{(1+0.12)^5\,0.12}\right] \qquad S = \left[\frac{(1+i)^m-1}{(1+i)^m i}\right]$$

$$PW(\text{operating cost}) = -9000\,[3.604]$$

$$PW(\text{operating cost}) = -32{,}442.98$$

$$PW(\text{salvage value}) = 5000/(1+0.12)^5$$

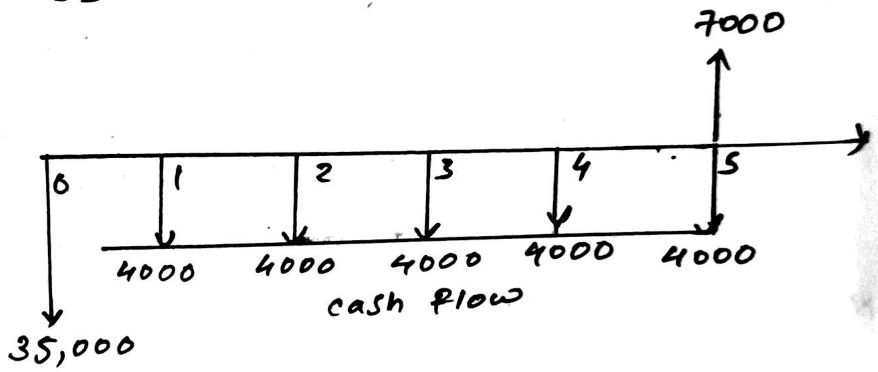
$$= +2837.13$$

Net present value = PW(initial cost) + PW(operating cost) + PW(salvage value)

$$\text{Net Present value} = -20{,}000 - 32{,}442.98 + 2837.13$$

$$= -49{,}605.85$$

class	Machine 2
Initial cost	35,000
Annual operating cost	4000
salvage	7000
Life time	5



cash flow

$$PW(\text{initial cost}) = -35{,}000$$

$$PW(\text{operating cost}) = -\{4000/(1+0.12)^1 + 4000/(1+0.12)^2 + 4000/(1+0.12)^3 + 4000/(1+0.12)^4 + 4000/(1+0.12)^5\}$$

$$-4000 \times 3.604$$

$$PW(\text{salvage value}) = +7000/(1+0.12)^5$$

$$+7000/1.7623$$

$$PW = PW_{\text{initial cost}} + PW_{\text{operating cost}} + PW_{\text{salvage value}}$$

$$PW_2 = -45{,}445$$

Machine 2 is better

$$\boxed{PW_1 = -49{,}606} \qquad \boxed{PW_2 = -45{,}445}$$

Future worth Analysis

What is Future worth Analysis?

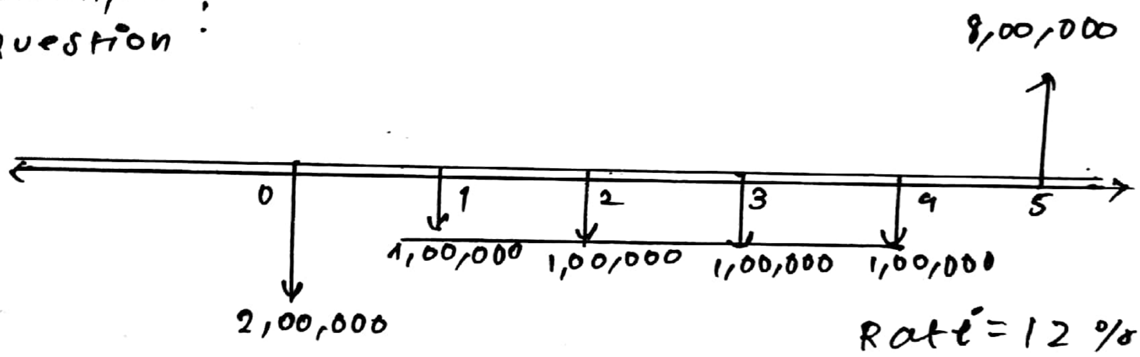
Future worth is a method of analysis in which we compute the equivalent worth at end of investment period.

How to calculate Future value of Money?

Future value = Present value $(1+r)^n$

$$\boxed{FV = PV(1+r)^n}$$

Example Question:



Rate = 12 %

Is this a good investment?

Ans: No Not a good investment.

Net Future value = FV (initial investment) + FV (annual investment)

$= -2,00,000\,(F/P, 12\%, 5)$

FV (initial investment) $= -3,52,468.33$

FV (Annual investment) $= -1,00,000\,(F/A, 12\%, 4)$

$= -\{1,00,000(1+0.12)^4 + 1,00,000(1+0.12)^3 + (1,00,000)(1+0.12)^2 + (1,00,000(1+0.12)^1\}$

$-1,00000 \left[\dfrac{(1+0.12)^4 \times 1.12}{0.12}\right]$

FV $= -1,00,000\,(5.35284731)$

$= -5,35,285.73$

Net Future value $= -3,52,468.33 - 5,35,285.73 = \boxed{-8,87,753}$

What is Annual worth Analysis?

Annual worth (AW) Analysis is defined ast the equivalent uniform annual worth of all estimated receipts (income) and disbursements (cost) during life cycle of a project.

How to calculate Annual worth

$$FV = PV(1+r)^n$$

$$PW = [A(P/A, r, n)] = [A(P/A, i, n] = A\left[\frac{(1+i)^m - 1}{(1+i)^m i}\right]$$

$$FW = [A(F/A, r, n)) = [A(P/A, i, n)] = A\left[\frac{(1+i)^m - 1}{i}\right]$$

one year ~~be~~ after Eq payment

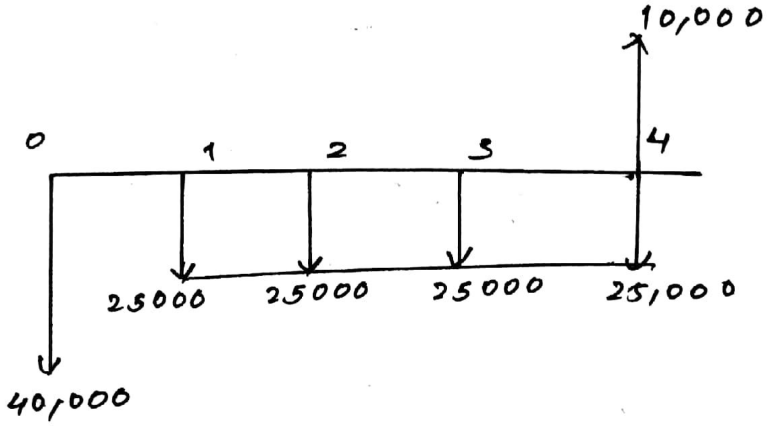
$$S = A\left[\frac{(1+i)^m - 1)}{i} \times (i+1)\right]$$

Numerical:

Q 1)

Equipment	X	Y
First cost	40,000	75,000
AOC per year	25,000	15,000
Life (in years)	4	6
Salvage value	10,000	7000
MARR 12 % per year	Select best alternative based on Annual worth	

Machine X



$AW(x) = -AW(\text{First cost}) - AW_{\text{operating cost}} + AW_{\text{salvage value}}$

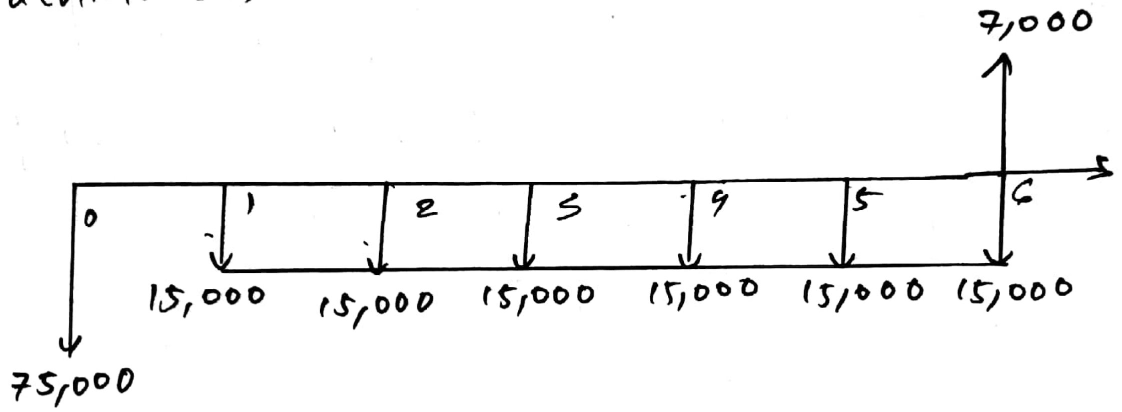
$AW(x) = -40{,}000(A/P, 12\%, 4) - 25{,}000 + 10{,}000(A/F, 12\%, 4)$

$AW(x) = -40{,}000 \times 0.3292$

$AW(x) = -13168 - 25{,}000 + 2092.34$

$\boxed{AW(x) = -36{,}077}$

Machine (Y)



$AW(Y) = -AW(\text{First cost}) - AW_{\text{operating cost}} + AW_{\text{salvage value}}$

$= -75{,}000(A/P, 12\%, 6) - 15000 + 7000(A/F, 12\%, 6)$

$-75{,}000 \times 0.2432 - 15000 + 7000 \times 0.1232$

$-18240 - 15000 + 862.4$

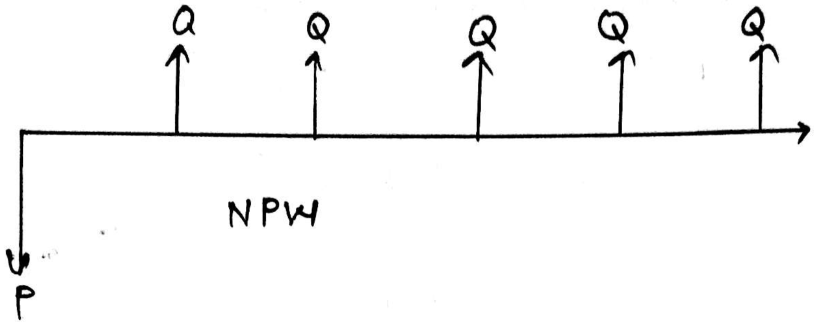
$\boxed{AW(Y) = -32{,}379}$

$AW(x) = -36{,}077$ $AW(Y) = -32{,}379$

More feasible by each year in terms of cost.

What is Rate of Return?

Rate of Return of a cash flow pattern is the interest rate at which the present worth of that cash flow pattern reduces to zero.



Net-present-worth $= -P + Q(P/A, i, n)$

$NPW = -P + Q(P/A, r, 5)$

r at which Net Present Worth $= 0$

How do we perform Rate of Return Analysis

Step 1: Calculate NPV of all cashflows in terms of r

Step 2: Do hit and trial with r to get negative NPV nearest to 0 Rate $= r_2$ and NPV is NPV_{r_2}

Do hit and trial with r to get positive NPV nearest to 0 Rate $= r_1$ and NPV is NPV_{r_1}

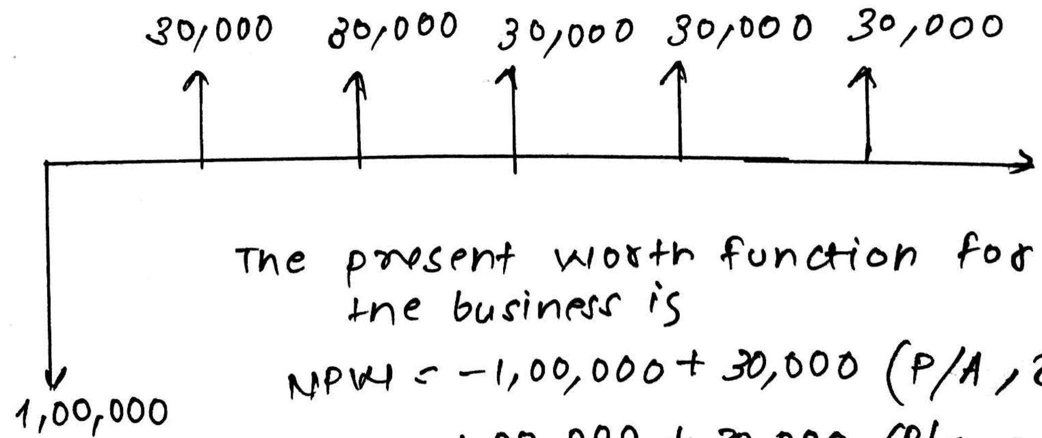
Step 3: Apply the formula to get value of k

$$\boxed{k = r_1 + \left\{ \frac{NPV_{r_1}}{(NPV_{r_1} - NPV_{r_2})} \right\} (r_2 - r_1)}$$

if $k \geq MARR$, select the project

Numerical

Q1)



The present worth function for the business is

$NPW = -1,00,000 + 30,000 \, (P/A, r, n)$

$NPW = -1,00,000 + 30,000 \, (P/A, r, 5)$

$NPW = -1,00,000 + 30,000 \left(\frac{(1+r)^5 - 1}{(1+r)^5 r} \right)$

Step 2 :

$r = 15\%$ {trial & Error}

$PW(15\%) = -1,00,000 + 30000(P/A, 15\%, 5)$

$= -1,00,000 + 30,000(3.3522)$

$PW = Rs\ 566$

$r_1 = 15\%$ $NPV_{r_1} = Rs\ 566$

$r = 16\%$

$PW(16\%) = -1,00,000 + 30,000(P/A, 16\%, 5)$

$= -1,00,000 + 30,000(3.2742)$

$= -Rs\ 1771$

$r_2 = 16\%$ $NPV_{r_2} = -1771$

$\therefore K = r_1 + \left(\frac{NPV_{r_1}}{NPV_{r_1} - NPV_{r_2}}\right)(r_2 - r_1)$

$\therefore K = 15 + \left(\frac{566}{566 - (-1771)}\right) \times (16 - 15)$

$\therefore K = 15 + \frac{566}{2337}$

$\boxed{\therefore K = 15.242\%}$

Rate of Return $K = 15.242\%$

CamScanner

Incremental IRR : Incremental Internal Rate of Return

A company must purchase a new lathe. It is considering one of three new lathes, each of which has a life of 10 years with no ~~sea~~ scrap value. Given a MARR of 15% which alternative should be chosen?

Lathe	1	2	3
First cost	\$100,000	\$150,000	\$209000
Annual saving	\$25,000	\$34,000	\$46,000

n = 10 years
MARR = 15%

Lathe 1 250

Annual saving (P/A, i^*, n) = First cost

$$25{,}000\,(P/A, i^*, 10) = 100{,}000$$

$$(P/A, i^*, 10) = \frac{100{,}000}{25{,}000}$$

$$(P/A, i^*, 10) = 4$$

Note : $(P/A, i, n) = \frac{(1+i)^n - 1}{i(1+i)^n}$

put $i = 20\%$

$$= \frac{(1+0.20)^{10} - 1}{0.20\,(1+0.20)^{10}}$$

$$\frac{6.192 - 1}{0.20 \times 6.192}$$

$$\frac{5.192}{1.2384}$$

$\boxed{NPV_{r_1} = 4.1925}$ $\boxed{r_1 = 20\%}$

put $i = 22\%$

$$P(A, i, n) = \frac{(1+0.22)^{10} - 1}{0.22\,(1+0.22)^{10}} = \frac{7.30 - 1}{0.22 \times 7.30}$$

$\boxed{NPV_{r_2} = 3.922}$ $\boxed{r_2 = 22\%}$ 6.30 / 1.606