Essentials of Management and Engineering Economics

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ENGINEERING ECONOMICS

CONTENTS

WHY ENGINEERING ECONOMICS IS NEEDED FOR AN ENGINEER?

An engineer after getting graduated in any stream of engineering must develop the technical competancies. In addition, to make him more effective in the industrial environment, he must be cost effective. To enhance cost effectiveness, he must do the best investments. Hence before any investment is done, he must evaluate and take the right decisions. This is the purpose and role of Engineering Economics. It deals with the economics of money. It helps the engineer to view all investments from economic point of view and make the right choice.

1.0 INTRODUCTION TO ENGINEERING ECONOMICS

THIS CHAPTER COVERS THE FOLLOWING TOPICS:

- INTRODUCTION TO MICRO AND MACRO ENGINEERING ECONOMICS,
- VALUE, UTILITY, CONSUMER AND PRODUCER GOODS,
- FACTORS OF PRODUCTION

1.1 Definition:

Economics can be in simple terms defined as:

"The science of useful application of wealth or natural resources"

Different economists define economics in different context as follows:

I. J. E. Cairnes – Science of wealth

"It is that body of knowledge which relates to wealth and a key position was assigned to wealth in the study of economics"

When religious sentiments and spiritual values were strong over peoples mind, this theory gave a different dimension to economics. It recognized the human role; wealth is a means and the end being human welfare.

II. Marshal – Science of material welfare

"Economics is the study of general methods by which men cooperate to meet their material needs"

It is a study of mankind in the ordinary business of life. It examines that part of individual and social action which is most closely connected with attainment and use of material requisites of wellbeing.

III. Robbins – Science of scarcity of choice

"Economics is the science which studies human behaviour as a relationship between ends and social means which have alternate uses"

Ends refer to wants. Since wants are unlimited it is termed as science of scarcity and a choice has to be made to meet the urgent wants to the less urgent ones.

IV. Keynes J. M – Modern definition

"Economics is the study of the administration of the scarce resources and the determinants of income and employment"

It studies the causes of economic fluctuations to see how economic stability is promoted.

The term "Engineering" can be defined as "That profession in which a knowledge of mathematical, and natural sciences gained by studies, experience and practice is systematically analysed and applied with judgement to develop ways to utilise economically the materials and forces of nature for the benefit of mankind."

Engineering can be termed as application of science.

1.2 Engineering Environment:

The engineering activities of analysis and design dealing with the physical environment take place to meet the needs/wants of mankind which are in the economic environment. The various phases in between the need/want recognition and satisfaction are shown in Figure 1. Engineering is concerned with the materials and forces of nature on one side and needs/wants of mankind on the other side. Since we live in an resource constrained world, engineering must be closely associated with economics. It has become absolutely essential that engineering proposals be evaluated in terms of its worth and costs before being executed. Thus "Engineering Economics" is a discipline concerned with the systematic evaluation of the costs and benefits of the proposals, technical, business projects and ventures.

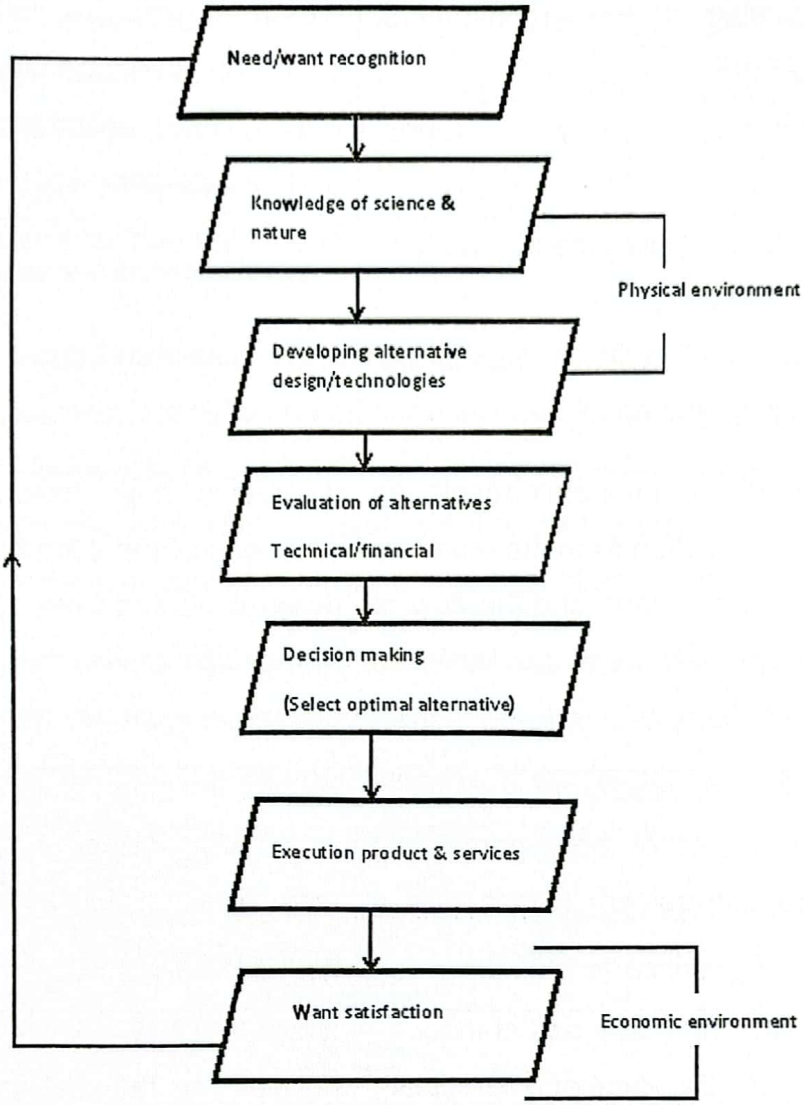


Figure 1 Physical – Engineering - Economic Environment

1.3 Nature of Economics:

In studying the nature and scope of economics it is necessary to know its subject matter i.e., micro-economics and macro-economics.

Here is a distinction between the two:

MICRO-ECONOMICS	MACRO-ECONOMICS
Price theory Resource allocation	Theory of Income & Employment
This theory studies the behaviour of individual decision making units.	It is the study of the economy of the nation as a whole. It is concerned with

• Consuming (consumers, households). • Producing (firms, farms, business). • Individual industries (cotton, textiles, steel etc.). • Resource owners (labourers, capitalists, landowners).	the aggregates of the entire economy such as national income, aggregate output, total consumption, total savings, and investments, aggregate demand and supply, general level of prices etc.
It studies the flow of economic resources or factors of production from the resource owners to business firms and the flow of goods and services from the business firms to the consumers.	It studies how these aggregates and averages of the economy as a whole are determined and what causes fluctuations in these to ensure the maximum level of income and employment in a country.
It explains the composition of such flows and how the prices of these goods and services are determined. Hence it is a Price theory (Resource allocation).	Helpful in understanding the functioning of a complicated economic system. It also gives a bird's eye view of the economic world.
It deals with the individual part of the whole economy. Ex: Price of a particular product, Income of an individual etc.	It is used in formulating useful economic policies of the nation. It is used to regulate aggregate employment, NI, and workout national wage policy etc.
It has both positive and normative approach. Positive science approach attempts to develop theories which explains only "What is" and normative science tells us "What ought to be" i.e., right/wrong of a thing.	This theory seeks to explain fluctuations in National Income(NI) output and employment. It has only positive science approach.
LIMITATIONS:	
▪ Cannot give an idea of functioning of economy as a whole.	▪ Individuals are ignored. Individual welfare is the main aim of economics, which is ignored.

▪ It assumes full employment which is a rare phenomenon at any rate in a capitalistic world. Hence unrealistic assumption.	▪ It overlooks individual differences.

1.4 Concepts of Value and Utility:

The term value has variety of meanings. In economics, "value designates the worth that a person attaches to an object or service". Hence the value of an object is not inherent in the object but is inherent in the regard that a person or people have for it. Value should not be confused with the cost or price of an object. There may not be any relation between the value and cost of an object.

Utility is the power to satisfy the human wants. The utility that an object has for an individual is determined by him. Hence the utility of an object, like its value, is not inherent in the object itself but is inherent in the regard that a person has for it. Utility and value are hence very closely related to each other.

The utility that an object has for a person is the satisfaction he derives from it. Value is an appraisal of utility in terms of a medium of exchange. Following five types of utility are exist for an object:

- Form Utility – The physical/chemical changes that make the object more valuable.
  Ex: Wood into furniture.
- Place utility – The product is made readily available/accessible to potential customers.
  Ex: Retail shops, Dealers etc.
- Time utility – The product is made available to customers when they really want it.
  Ex: Umbrellas during/just before rainy season.
- Possession utility – When a customer really buys the product/ownership title is transferred to the buyer.
- Image utility – It is the utility arising out of the emotional and psychological value that a person attaches to a product/brand (goodwill). Requires longer time to possess image utility.

1.5 Consumer and producer goods:

Two classes of goods are recognized by economists: Consumer and producer goods.

Consumer goods are products and services that directly satisfy human wants.

Ex: TV sets, Shoes, Houses, Books, etc.

Producer goods also satisfy human wants but do so indirectly as a part of the production or the construction process. Ultimate end of all engineering activity is to supply goods and services that people may consume to satisfy their desires and needs.

Ex: lathes and other machines, bull dozers, ships, rail road cars etc.

Producer goods are an intermediate step in man's effort to supply his wants. They are not desired for themselves, but because they may be instrumental in producing something that man can consume.

A person will consider two kinds of utility for consumer goods. One kind embraces the utility of goods and services that he intends to consume personally for the satisfaction he gets out of them. Thus it seems reasonable to believe that the utility a person ascribes to goods and services that are consumed directly is in large measure a result of subjective, non-logical mental processes. Here emotional appeals are more effective than factual information. Earlier automobile advertising took the form of objective information viz. design and performance but more recent practice stresses subjective aspects viz. comfort, beauty, aesthetics and prestige values.

Some kind of human wants are much more accurately predictable than others. Ex: demand for food, clothing, shelter, which are needed for bare physical existence, is much more stable and predictable than the demand for those items that satisfy human emotional needs. Subjective aspects are not much considered here instead objective factors are taken into account.

The second kind of utility that are object/service may have for a person in its utility as a means to an end. Producer goods are not consumed for the satisfaction that

can be directly derived from them but as a means of producing consumer goods. Although the utility of consumer goods is primarily determined subjectively, the utility of the producer goods as a means to an end, may be, and usually is, in large measure considered objectively.

1.6 Factors of Production:

Production is sometimes defined as a process of creation of utility or the creation of want satisfying goods and services. But creating utility without value is not important in economics. Hence creating utility is not exactly production. But creation or addition of value is production.

Following four are the major factors of production:

1. Land
2. Labour
3. Capital
4. Organization (or Enterprise).

Land – Land stands for all natural resources which yield an income or which have exchange value. It represents those natural resources which are useful and scarce, actually and potentially.

Land has certain peculiarities.

- It is nature's gift to mankind.
- It is fixed in quantity (No supply price).
- It is permanent.
- It lacks mobility (In geographical sense).
- It provides infinite variations of degrees of fertility and situation so that no two pieces of land are exactly alike.

These peculiarities have a bearing on economic rent or its value.

Labour – Following are the peculiarities of labour:

- Labour is inseperable from the labourer himself.
- Labour has to sell his labour in person.
- Labour has a very weak bargaining power.

- Changes in the price of labour react rather curiously on the supply.
- There can be no rapid adjustment of the supply of labour to demand for it, because supply cannot be increased quickly, nor can it be reduced.

These peculiarities have a intimate bearing on the determination of wages.

Following are the factors determining the efficiency of labour:

- Racial qualities – To race and hereditary characters.
- Climatic factors.
- Education and training.
- Personal qualities.
- Industrial organization and the equipment.
- Factory environment.
- Working hours.
- Fair and prompt payment of wages.
- Organization/Enterprise itself.
- Social and political factors.

Capital – Capital refers to that part of a man's wealth which is used in producing further wealth or which yields an income. The term capital refers to capital goods. Ex: Plant, Building and machinery, Tools and other accessories, Raw materials, goods in process, Fuel etc.

Land and labour considered as Primary Factors of Production.

Capital and Enterprise are the Secondary Factors of Production (Man made Factors of Production).

Distinction between Land and Capital:

	LAND	CAPITAL
1.	Not a capital item.	-
2.	Free gift of nature.	Man made.
3.	Non-perishable.	Perishable/consumed.
4.	No mobility.	Capital is mobile
5.	Land is fixed and limited.	Capital can be increased at any time.

6.	Rent on land varies.	Income from capital is uniform.

Enterprise – Enterprise/organization is supplied by the entrepreneur. The role that the entrepreneur plays consists in coordinating and consulting the other factors of production. Following functions are performed by the entrepreneur:

- Initiating a business enterprise by mobilizing and harnessing the necessary productive resources.
- Taking up the final responsibility of the business enterprise – Risk taking and uncertainty bearing.
- His role as a innovator.

No production is possible without having an appropriate organization/enterprise. Success of the enterprise depends on the entrepreneur – how far he has played his role well/functions performed.

2.0 DEMAND AND SUPPLY

THIS CHAPTER COVERS THE FOLLOWING:

- LAW OF DEMAND AND SUPPLY.
- ELASTICITY OF DEMAND AND SUPPLY.
- EQUILIBRIUM OF DEMAND AND SUPPLY.

2.1 Demand:

Demand for a commodity refers to the quantity of the commodity which and individual consumer or a household is willing to purchase per unit of time at a particular price.

Demand for a commodity implies the following:

- Desire of a consumer to buy a product.
- His willingness to buy the product.
- Sufficient purchasing power.

When we are dealing with a good demanded by an individual we call it an **Individual demand**, while if the good is demanded by the household, we call it the **Household demand**. When we consider the demand for the commodity by all the individuals and /or the households in the market taken together, we call it as **Market/Aggregate demand**.

Following are the important determinants of Demand:

General factors –

- Price of the product itself.
- Price of related goods (Substitutes and Compliments).
- Income of the consumer.
- Tastes and preferences of the consumer.

Additional factors (Related to Luxury goods and durables) –

- Consumers' expectations of future prices.
- Consumers' expectations of future income.

Additional factors (Related to market demand) –

- Population – Growth.
- Social, economic, demographic distribution of consumers

2.2 Demand Function:

A mathematical expression of the relationship between quantity demanded of the commodity and its determinants is known as demand function.

When this relationship relates to the demand by and individual consumer it is known as **Individual/household demand function** and if it relates to the market itself it is called market demand function.

Individual demand function is given as follows:

$$Q_{dx} = f(P_x, Y, P_1 \ldots\ldots P_{n-1}, T, A, E_y, E_p, u)$$

Where,

Q_{dx} = Quantity demanded of product X,

P_x = Price of product X,

Y = Level of household income,

P_1, ...P_{n-1} = Prices of all other related products in the economy,

A = Advertising,

T = Tastes and preferences,

E_y = Consumers expected future income,

E_p = Consumers expected future prices,

u = All those determinants which are not covered above.

Market demand function is given as follows:

$$Q_{dx} = f(P_x, Y, P_1 \ldots\ldots P_{n-1}, T, A, E_y, E_p, P, D, u)$$

Where,

P = Population,

D = Distribution of consumers in various categories depending on age, income, sex etc. all other remaining the same.

2.3 Law of Demand:

It states that,

"The amount demanded of a commodity and its price are inversely related, other things remaining constant".

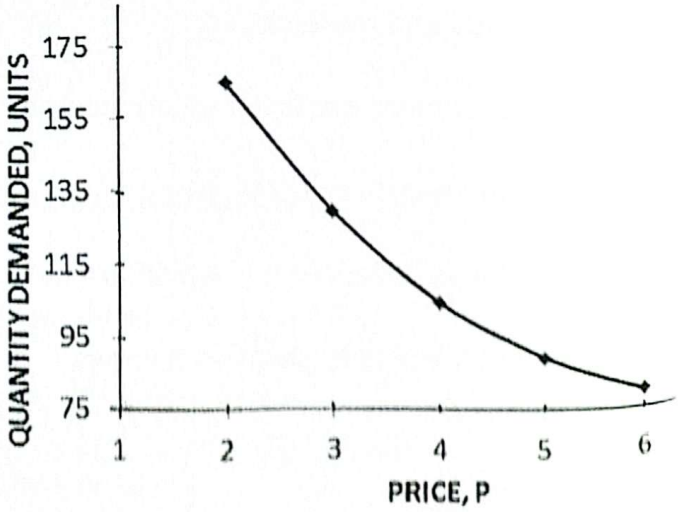
Demand schedule and demand curve –

The relationship between the price and quantity demanded can be represented in the form of a table showing quantity demanded at various prices. This is termed as demand schedule.

Price (P) Rs.	Quantity demanded (Units)
5	90
4	105
3	130
2	165

Demand Schedule of Product X

If the quantity demanded is tabulated for an individual, then it is called **Individual Demand Schedule**. If it is tabulated for the whole market, then it is termed as **Market Demand Schedule**.



Demand Curve

The same relationship can be represented in the form of a graph then it is called **Demand Curve**. If the curve is drawn for an individual, it is called **Individual Demand Curve**, and if it drawn for the market, then it is called **Market Demand Curve**.

The law of demand operates due to the underlying effects of substitution and real income changes. These are termed as follows:

- Substitution effect of a price change.
- Income effect of a price change.

When the price of the commodity falls, the consumer tends to substitute that commodity for other commodities which have not become relatively dearer (Cheaper). Similarly when the price of the commodity rises, other commodities will be used in its place, atleast to some extent. Hence fall in price of the commodity increases demand and a rise in price reduces demand. This is called **Substitution Effect**.

The fall in price of the commodity leads to and therefore is equivalent to an increase in income of the consumer, because now he has to spend less to purchase the same quantity as before. A part of the money that he has gained can be used for purchasing some more units of the commodity. Similarly when the price increases, the income of the consumer is reduced and he has to curtail his expenditure on all the commodities. This is termed as **Income Effect**.

Exceptions to Law of Demand –

- Giffen Goods – If there is an inferior good in whose case the income effect of price change is stronger than the substitution effect, the law of demand would not hold. Reduction in price of the commodity here may reduce the demand of that commodity instead of increasing the demand. Ex: Inferior goods consumed by the poor families. These goods are called the GIFFIN goods. If there is a fall in price of a giffin good, (Ex: Sweet potatoes consumed by poor people) instead of buying higher quantity of it, they may go for curtailing its consumption due to the increase in income, they may go for consuming costlier food item (like vegetables and grams).

- Commodities which are used as status symbols – Some expensive commodities like luxury cars, diamonds, ornaments, etc., which are used as status symbols to display one's wealth. The more expensive these commodities become, more will be their value and hence greater will be their demand.
- Expectations in the change in the price of the commodities – If a household expects the price of the commodity to increase, it may start purchasing greater amount of the commodity even at the presently increased price. Similarly if it expects the price to decrease, it may postpone its purchase. Hence law of demand is violated.

2.4 Supply:

Supply of a commodity refers to the various quantities of the commodity which a seller is willing and able to sell at different prices, in a given market, at a point of time, other things remaining the same.

Following are the determinants of supply which can be expressed as a supply function as follows:

$$S_x = f(P_x, P_y, P_z, \ldots\ldots P^f, O, T)$$

Where,

S_x = Amount of good supplied,

P_x = Price of good X,

P_y,P_z ,....= Prices of good y,z,……,

P^f = Prices of Factors of Production needed to produce X,

O = Objectives of the firm,

T = State of Technology used by the producer to produce X.

Law of Supply – States that,

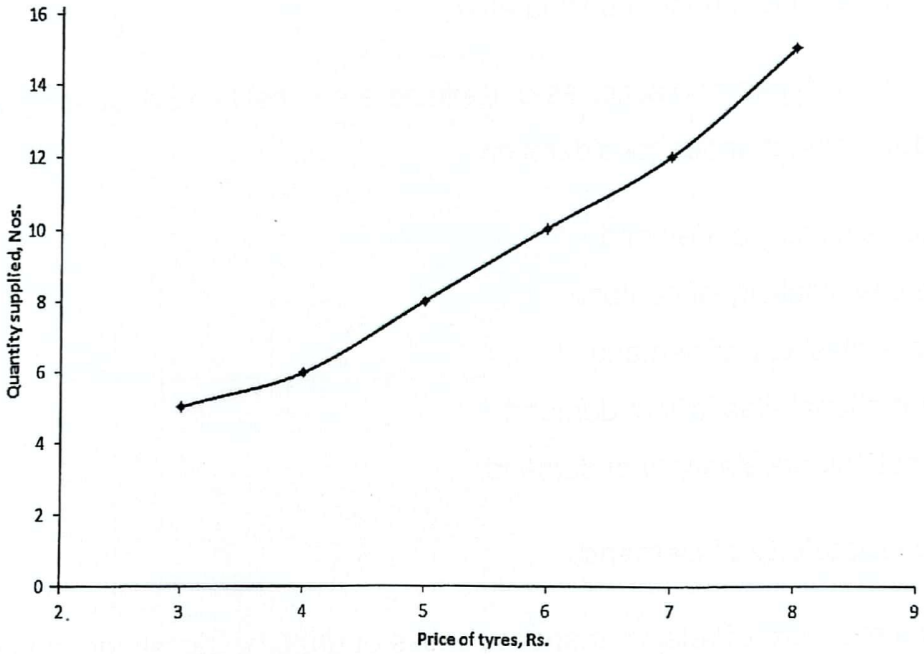
"Other things remaining constant, more of a commodity is supplied at a higher price and less of it is supplied at lower price".

A supply schedule is a tabular presentation of data on the quantity supplied and the price of the good.

Price of Tyres ('00 Rs/Tyre)	Quantity Supplied ('000 numbers)
8	15
7	12
6	10
5	8
4	6
3	5

Supply Schedule: Supply of Scooter tyres for a firm

When the supply is plotted against the price, then the plot is called the supply curve.



Supply Curve

2.5 Elasticity of Demand:

Elasticity of demand (E_d) is defined as the percentage change in quantity demanded caused by 1 percent change in demand determinant under consideration, while other determinants are held constant.

The expression is given by:

$$E_d = \frac{Percentage\ change\ in\ quantity\ demanded\ of\ good, X}{Percentage\ change\ in\ determinant, Z}$$

$$E_d = \frac{{}^{\Delta Q}/_{Q}}{{}^{\Delta Z}/_{Z}}$$

$$E_d = \frac{\Delta Q}{\Delta Z} \cdot \frac{Z}{Q}$$

Where,

Ed = Elasticity of demand,

Δ = Change,

Q = Quantity,

Z = Determinant under consideration.

Following types of elasticities of demand are considered depending upon the demand determinant under consideration:

- Price elasticity of demand
- Income elasticity of demand
- Cross elasticity of demand
- Promotional elasticity of demand
- Substitutional elasticity of demand.

2.5.1 Price Elasticity of Demand:

"The measure of relative responsiveness of quantity demanded to price along a given demand curve is known as Price elasticity of demand"

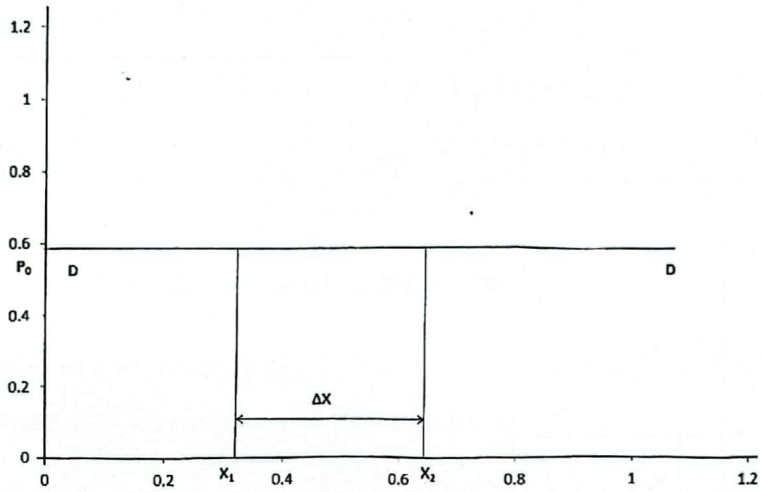
$$E_p = \frac{Proportionate\ change\ in\ quantity\ demanded\ of\ good, X}{Proportionate\ change\ in\ price\ of\ good, X}$$

$$E_p = -\frac{{}^{(Q_2 - Q_1)}/_{Q_1}}{{}^{(P_2 - P_1)}/_{P_1}}$$

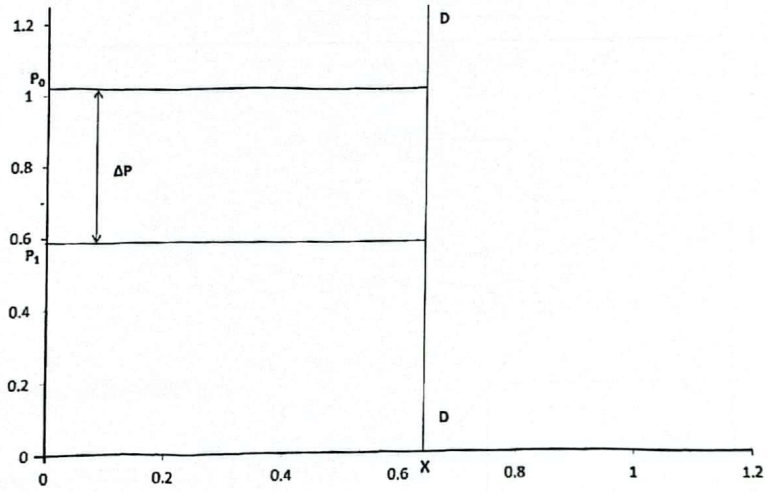
$$E_d = -\frac{\Delta Q/Q_1}{\Delta P/P_1}$$

$$E_d = -\frac{\Delta Q}{\Delta P}.\frac{P_1}{Q_1}$$

Types of Price Elasticity of Demand –



(i)



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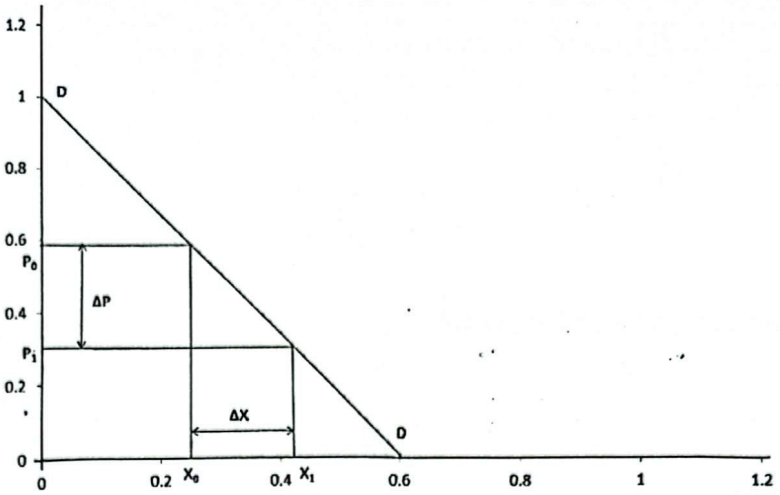

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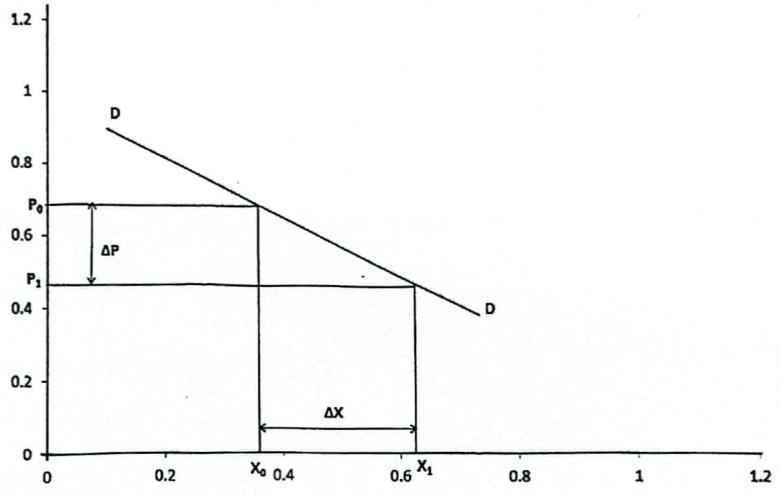

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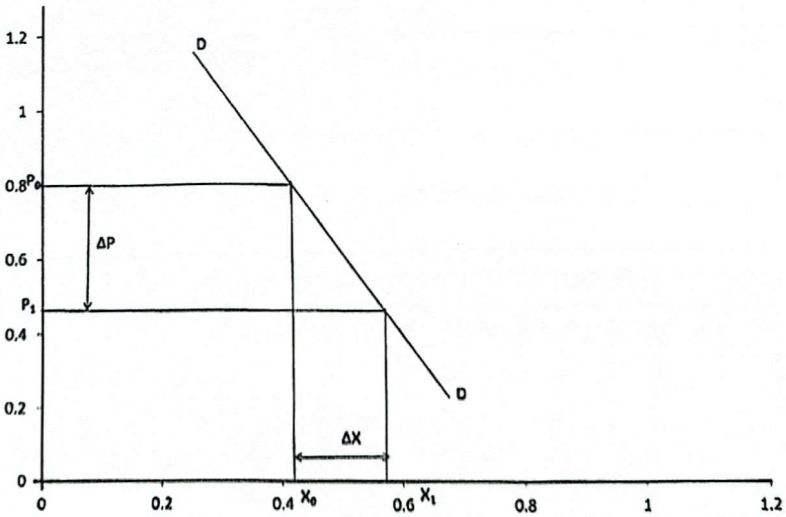


(v)

Above are shown the five types of price elasticities of demand. Their specifications are mentioned below:

Type	Numerical Expression	Description	Shape of curve
Perfectly elastic	∞	Infinite	Horizontal
Perfectly inelastic	0	Zero	Vertical
Unitary elastic	1	One	Rectangular hyperbola
Relatively elastic	>1	More than one	Flat (Approx.)
Relatively inelastic	<1	Less than one	Steep

2.5.2 Income Elasticity of Demand: Defined as:

"The ratio of percentage change in the quantity demanded of a good X, to the percentage change in the income of the consumer".

Mathematically,

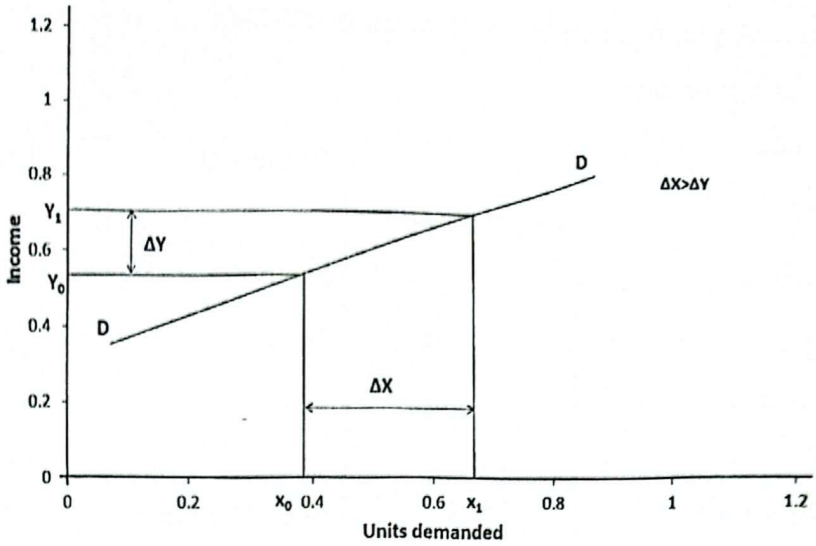
$$E_y = \frac{Percentage\ change\ in\ quantity\ demanded\ of\ good, X}{Percentage\ change\ in\ the\ income\ of\ the\ consumer}$$

$$E_y = -\frac{\Delta Q/_{Q_1}}{\Delta Y/_{Y_1}}$$

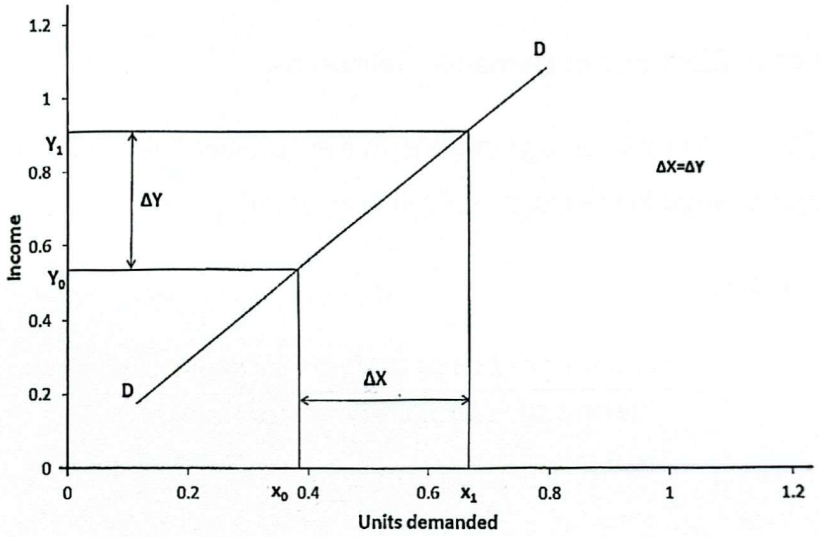
$$E_y = -\frac{\Delta Q}{\Delta Y} \cdot \frac{Y_1}{Q_1}$$

Types of income Elasticity –

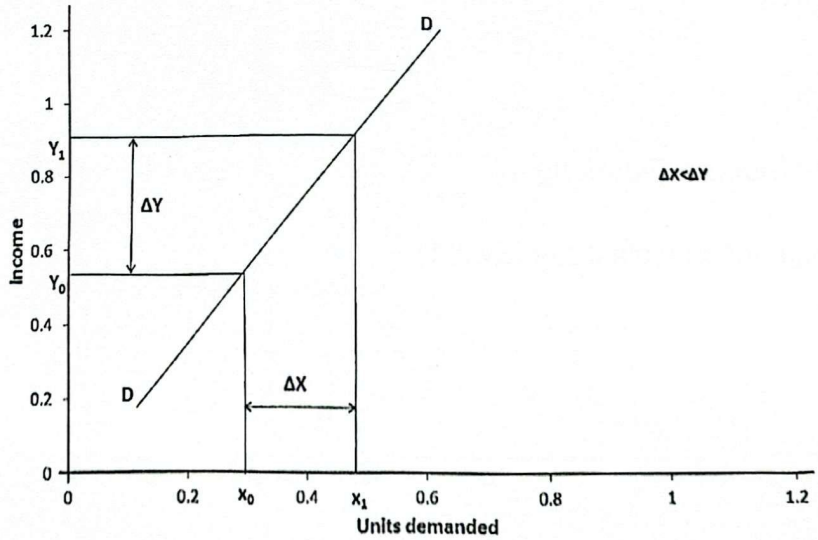
i. High income elasticity (Ey > 1)



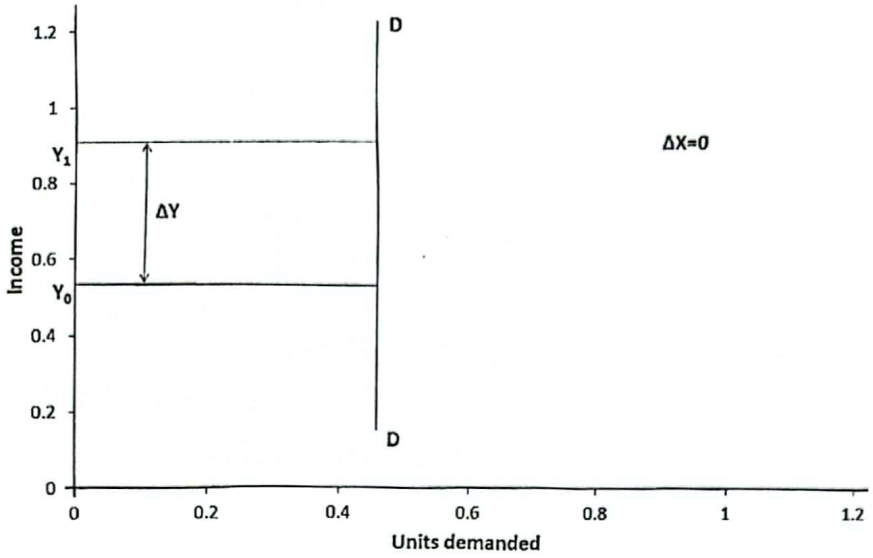
ii. Unitary income elasticity of demand ($E_y = 1$)



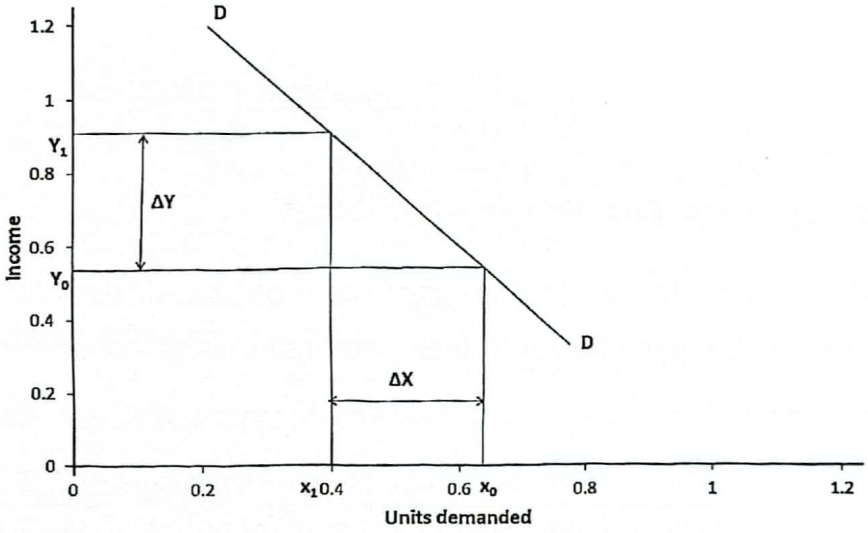
iii. Low income elasticity ($0<E_y<1$)



iv. Zero income elasticity (Ey=0)



v. Negative income elasticity (Ey<0)



Cross Elasticity of Demand –

It is defined as:

"The ratio of the percentage change in demand for one good to the percentage change in price of some other related good"

This change in demand for one good due to a change in price of some other good comes about because of the fact that the two goods may be either substitutes or compliments to each other. Once we assume that two commodities X and Y are related, the expression for cross elasticity of demand would be as follows:

$$E_{xy} = -\frac{\Delta Q_x / Q_x}{\Delta P_y / P_y}$$

$$E_{xy} = -\frac{\Delta Q_x}{\Delta P_y} \cdot \frac{P_y}{Q_x}$$

Promotional/Advertising Elasticity of Demand –

"It is a measure of the response of quantity demanded to change in expenditure on advertising and other promotion activities"

Mathematically,

$$E_A = -\frac{\Delta Q}{\Delta A} \cdot \frac{A_1}{Q_1}$$ - Point Elasticity

$$E_A = \frac{(Q_2 - Q_1)}{(Q_2 + Q_1)} \times \frac{(A_2 + A_1)}{(A_2 - A_1)}$$ - Arc elasticity

Demand Elasticity of Substitution – Defined as:

"The degree to which one good can be substituted for another as a consequence of a given change in their price ratio, if the consumer is to enjoy the same satisfaction"

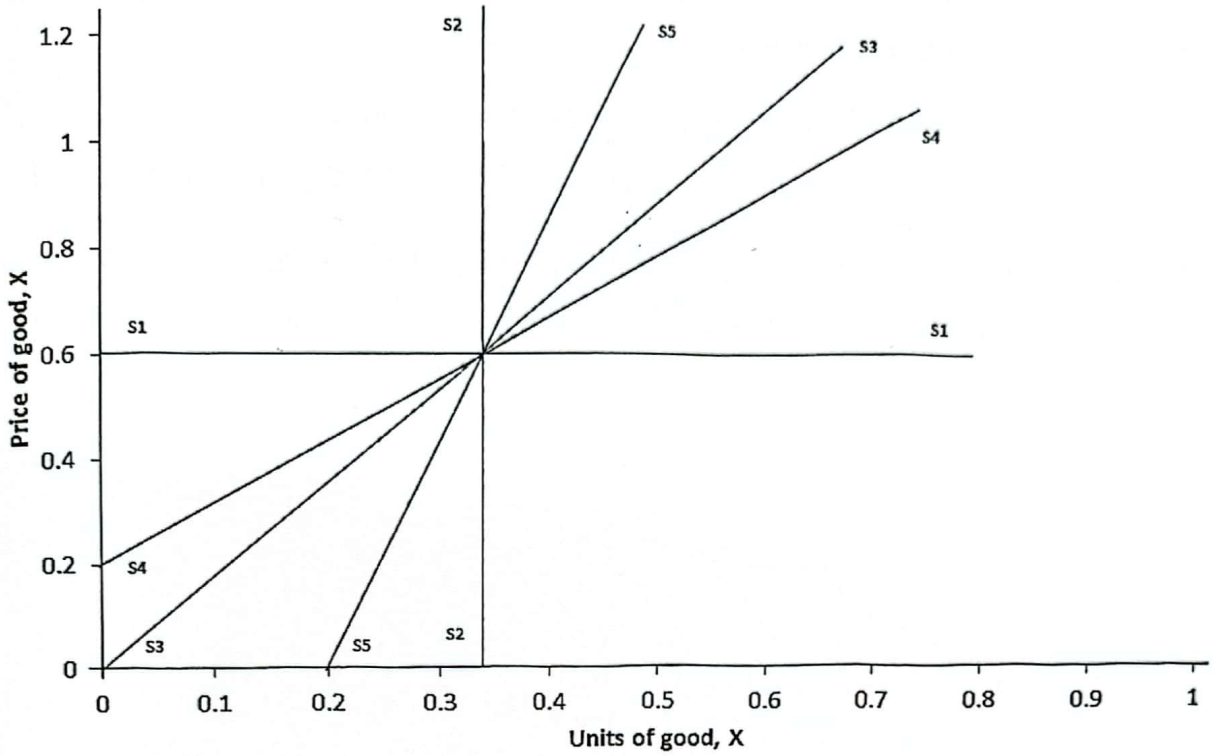
$$E_S = \frac{Proportionate\ change\ in\ quantity\ ratios\ of\ X\ \&\ Y}{Proportionate\ change\ in\ the\ price\ ratios\ of\ the\ X\ \&\ Y}$$

$$E_S = \frac{\Delta(Q_x/Q_y)}{(Q_x/Q_y)} \div \frac{\Delta(P_x/P_y)}{(P_x/P_y)}$$

2.6 Elasticity of Supply:

It is defined as the responsiveness of quantity supplied to the unit change in the price of that commodity.

Types of Elasticity of Supply –



S1-S1 – Perfectly elastic supply

S2-S2 – Perfectly Inelastic supply (Zero elastic)

S3-S3 – Unitary elastic supply

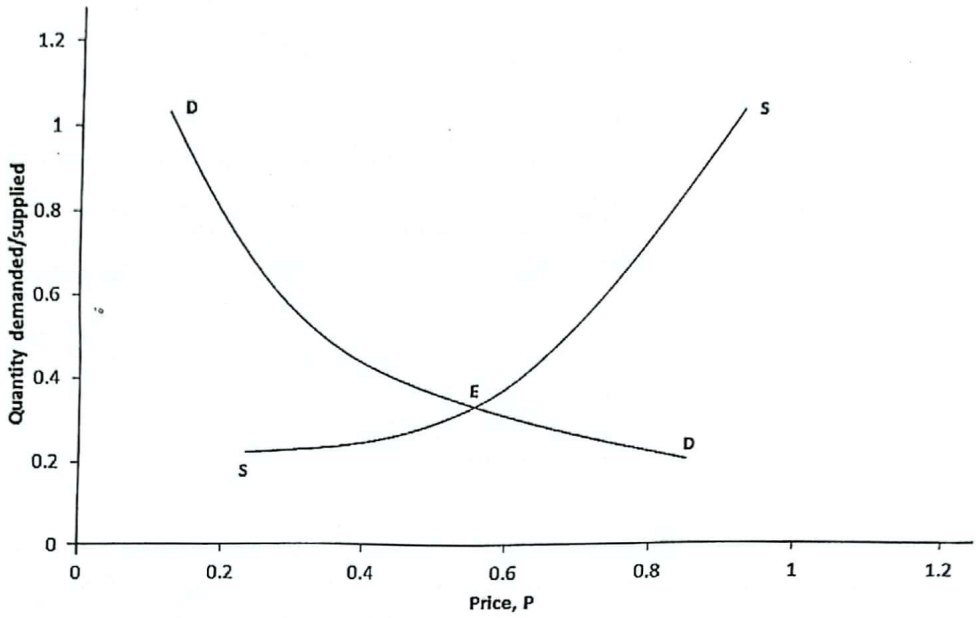
S4-S4 – Relatively elastic supply

S5-S5 – Relatively inelastic supply.

When Elasticity of supply is	It is called
Equal to infinity (∞)	Perfectly elastic supply
>1 but not infinity	Relatively elastic supply
Equal to 1	Unitary elastic supply
< one but not zero	Relatively inelastic supply
Equal to zero.	Perfectly inelastic supply.

2.7 Equilibrium of Demand and Supply:

The demand and supply of a good in the market or for an individual can be represented graphically as follows:



DD – Demand curve for the market

SS – Supply curve in the market.

Below E, demand will be higher and supply is lower. Hence more number of marketers, enter to raise it to the level of E. As a result the supply will increase and it may go beyond E, when the supply will be higher than the demand. Now, due to heavy competition, some of the small marketers will go out and hence the supply again falls down to match with the demand. At one stage, demand will be equal to the supply and this point on the curve represented by E is called the equilibrium point of demand and supply where demand is equal to the supply.

Problems:

1. If 2000 units of X are demanded at a price of Rs. 10/unit and 2500 units of it are demanded at a price of Rs. 9/unit, determine E_p.

Soln.: $$E_p = -\frac{(Q_2-Q_1)}{(P_2-P_1)} \times \frac{P_1}{Q_1}$$

$$= -\frac{(2500-2000)}{(9-10)} \times \frac{10}{2000}$$

$$= 2.5$$

E_p = 2.5, means that for 1 % decrease in price of good X, there is 2.5 % increase in quantity demanded for good X.

While computing elasticity, instead of taking Q_1 and P_1, we can take average Q and P as denominators and modify the above formula as follows and the measure of elasticity we get is better than the earlier.

$$E_p = -\frac{\Delta Q}{\Delta P} \times \frac{\left(\frac{P_1 + P_2}{2}\right)}{\left(\frac{Q_1 + Q_2}{2}\right)}$$

$$= -\frac{\Delta Q}{\Delta P} \times \frac{(P_1 + P_2)}{(Q_1 + Q_2)}$$

$$= -\frac{(2500-2000)}{(9-10)} \times \frac{(9+10)}{(2500+2000)}$$

$$= 2.11$$

2. If the consumer's demand for a commodity increases from 100 units to 200 units /week when his income rises from Rs. 2000 to Rs. 3000/-. Find his income elasticity.

Soln.:

$$E_y = \frac{(Q_2-Q_1)}{(Y_2-Y_1)} \times \frac{Y_1}{Q_1}$$

$$= \frac{(200 - 100)}{(3000 - 2000)} \times \frac{2000}{100}$$

$$= 2$$

Income elasticity of demand is always positive for all normal goods. In the case of inferior goods, the demand for the good varies inversely with income; as a result E_y will be negative.

3. The price of coffee increases from Rs. 50 to Rs. 70 per Kg and as a result demand for tea increases from 5 Kg to 10 Kg. What is the cross elasticity of demand of tea to coffee?

Soln.:

$$E_{xy} = \frac{(\Delta Q_x)}{(\Delta P_y)} \times \frac{P_y}{Q_x}$$

$$= \frac{(10-5)}{(70-50)} \times \frac{50}{5}$$

$$= 2.5$$

The sign of this measure of E_{xy} depends upon the nature of inter relationship between the two goods. If the two goods are substitutes, the value of E_{xy} will be positive. In the case of complimentary goods, the value of E_{xy} will be negative, because the change in price of one good cause opposite change in the quantity demanded of the other good.

Examples: Substitutes

- Tea and coffee
- Pencil and pen
- Coca-cola and pepsi

Compliments

- Sugar and tea
- Petrol and car

4. Firm XYZ supplied 2000 pens at a price of Rs. 8 per pen. When price increases to Rs. 10 per pen, the supply of XYZ increases to 3000 pens. Find the elasticity of supply of pens.

Soln.: Initial price (P) = Rs. 8/-

Change in price (ΔP) = 10-8 =Rs. 2

Initial quantity (Q) = 2000 pens

Change in quantity (ΔQ) = 3000-2000 = 1000 pens

Elasticity of supply, $E_{supply} = \frac{\Delta Q/Q}{\Delta P/P}$

$$= \frac{1000}{2000} \times \frac{8}{2}$$

$$= 2$$

3.0 TIME VALUE OF MONEY

THIS CHAPTER COVERS THE FOLLOWING:

- MEANING OF INTEREST.
- TIME VALUE OF MONEY.
- TYPES OF INTEREST.
- CASH FLOW DIAGRAM.
- INTEREST FORMULAE.
    - SINGLE PAYMENT COMPOUND AMOUNT INTEREST FACTOR.
    - SINGLE PAYMENT PRESENT WORTH FACTOR.
    - EQUAL PAYMENT SERIES COMPOUND AMOUNT FACTOR.
    - EQUAL PAYMENT SERIES SINKING FUND FACTOR.
    - EQUAL PAYMENT SERIES CAPITAL RECOVERY FACTOR.
    - EQUAL PAYMENT SERIES PRESENT WORTH FACTOR.

3.1 Meaning of Interest

The term interest is used to designate a rental amount charged by financial institutions for the use of money. The concept of interest can be extended to earning assets which borrow from their owners repaying through the earnings generated. This economic gain through the use of money is what gives money its time value.

Since engineering projects require the investment of money it is important that the time value of money used be properly reflected in the evaluation of these projects. An interest rate or the rate of capital growth is the rate of gain received from an investment. Usually this rate of gain is stated on a per year basis and it represents the percentage gain realized on the money committed to the undertaking.

In one aspect, interest is an amount of money received as a result of investing funds, either by loaning it or by using it in the purchase of materials, labour, or facilities. Interest received in this connection is a profit/gain. In another aspect, interest is an amount of money paid out as a result of borrowing funds. Interest paid in this connection is a cost.

Following are the major factors considered by the lender in taking a decision o interest rate:

1. What is the possibility that the borrower will not repay the loan? The answer is derived from the integrity of the borrower, his wealth, potential earnings, value

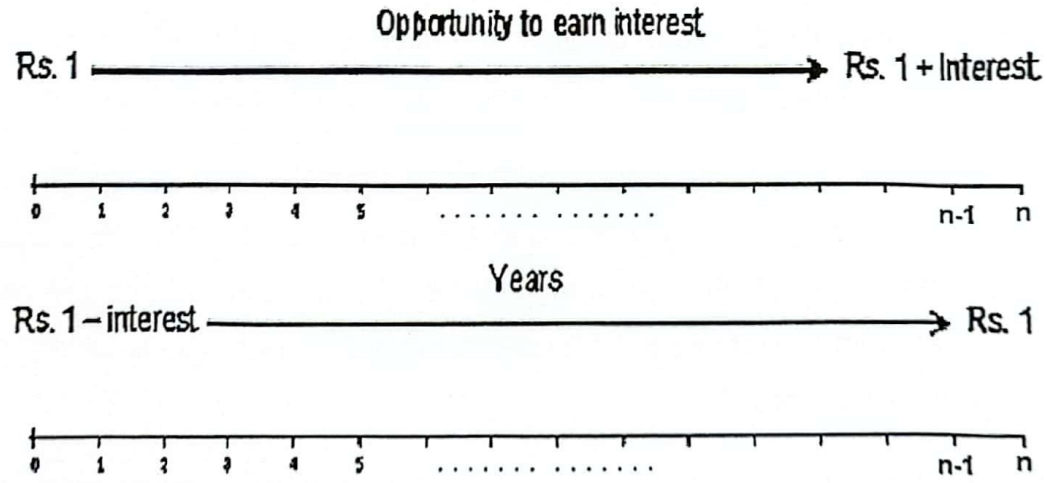
of any security etc. if the chances are 2 in 50 that the loan will not be repayed, then 4 % interest rate may be charged against risk of loss.

2. What expense will be incurred investigating the borrower, drawing up the loan agreement, transferring the funds to the borrower, and collecting the loan? If the sum of the loan is Rs. 1 lac for a period of one year, and the lender values his efforts as Rs. 2000/-, then he is justified in charging 2 % of the sum to compensate for the expenses.
3. What net amount will compensate for being deprived of electing other alternatives for disposing the money? This can be assumed as Rs. 3 per Rs. 100 invested or 3 % of the sum as adequate return.

On the above basis, the interest rate arrived at will be 4+2+3 = 9 %.

3.2 Time value of Money:

Since money can earn at a certain interest rate through its investment for a period of time, it is important to recognize that a Rupee received at some future date is not worth as a Rupee on hand at present. It is this relationship between money and time that leads to the concept of "Time value of Money". Thus the fact that the money has a time value means that equal Rupee amounts at different points in time have different value as long as interest rate that can be earned exceeds zero. This is shown schematically below:



We can argue that money also has a time value because the purchasing power of a rupee changes through time, during periods of inflation on money value are not considered here and only the opportunity to earn interest are considered.

3.2.1 Types of interest:

The rental rate (of interest) for a sum of money is usually expressed as a percentage of the sum that is to be paid for the use of the amount for a period of one year. Interest rates are quoted for periods other than one year, known as interest periods.

Simple interest –

In simple interest the interest to be paid on repayment of the loan is proportional to the length of the time the principal sum has been borrowed. The interest that may be earned may be found by using the following expression:

$$I = Pni$$

Where,

I = Simple Interest

P = Principal

n = interest period

i = interest rate.

Suppose Rs. 1000/- borrowed at a simple interest at a rate of 6% per annum for one year, then the sum (Simple interest) will be

$$I = Pni$$

$$= 1000 \times 1 \times 0.06$$

$$= \text{Rs. } 60/-$$

Compound interest –

When a loan is made for a length of time equal to several interest periods, interest is calculated at the end of each interest period. There are a number of loan repayment plans and these ranges from paying the interest when it is due to accumulating all the interest until the loan is due.

For example: The payments of 4 year loan of Rs. 1000/- at 6 % compounded interest per annum. The calculations will be as follows:

year	Amount owned, Rs	Interest to be paid, Rs.	Amount owned at the end of the year, Rs.	Amount to be paid by the borrower, Rs.
1	1000	60	1060	60
2	1000	60	1060	60
3	1000	60	1060	60
4	1000	60	1060	1060

Table 1: Compound Interest, when interest is paid annually.

Above is shown the appreciation of compound interest when interest is paid annually.

When interest is permitted to compound, the situation is as follows:

year	Amount owned, Rs	Interest to be paid, Rs.	Amount owned at the end of the year, Rs.	Amount to be paid by the borrower, Rs.
1	1000	60	1060	00
2	1000	63.60	1123.60	00
3	1000	67.42	1191.02	00
4	1000	71.46	1262.48	1262.48

Table 2: Compound interest when interest is permitted to compound.

Cash flow diagrams –

To aid in identifying and recording the economic effects of alternative investments, a graphical description of each alternative's cash transactions may be used. This pictorial description, referred to as Cash Flow Diagram (CFD), will provide us all the information necessary for analysing an investment proposal. The cash flow diagram represents any receipts received over a period of time as an upward arrow (an increase in cash) located at the period's end. The arrow's height is proportional to the magnitudes of the receipts received during the period. Similarly, the cash disbursements are represented by a downward arrow (a decrease in cash). These arrows are placed on a time scale representing the duration of the proposal.

The proposals in Table 1 & 2 can be used here to show the CFD's.

Table 1:

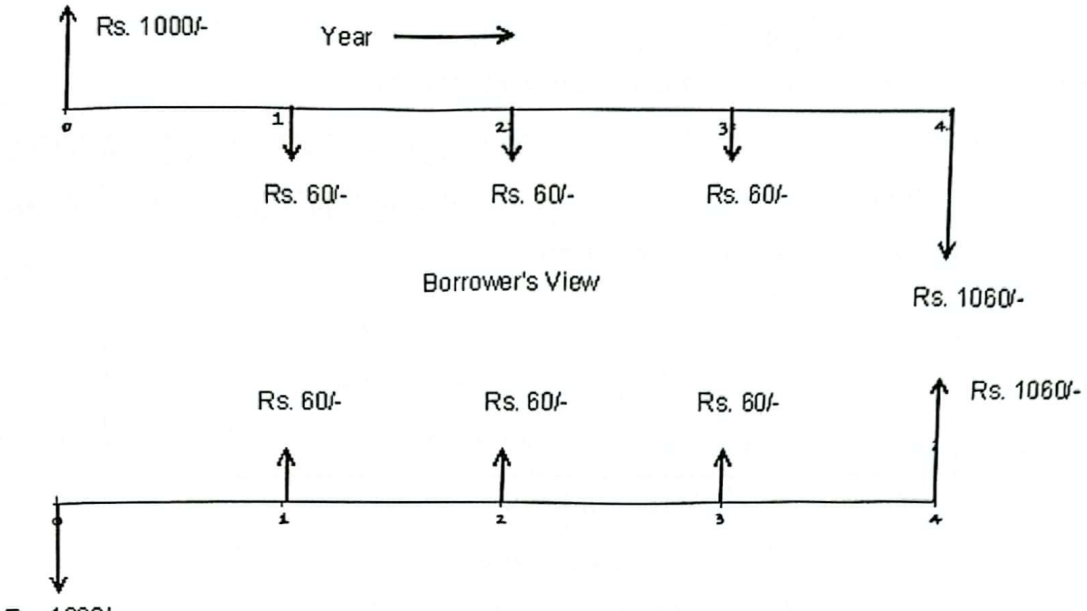
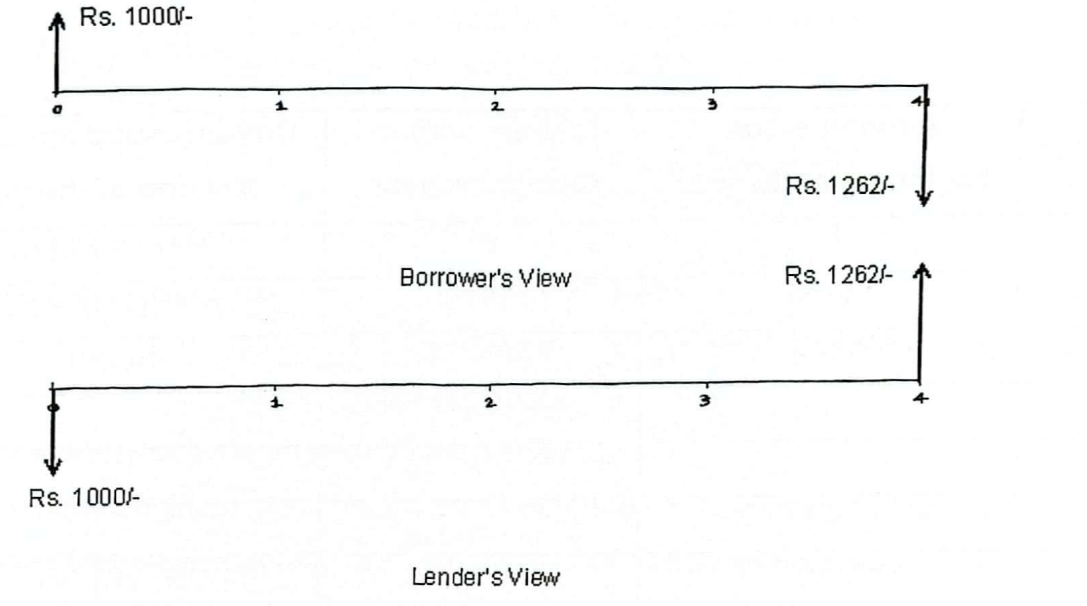


Table 2:



Cash Flow Diagrams

3.3 Interest Formulae:

The interest factors derived here apply to the common situation of annual compounding interest and annual payments. Following notations are used:

i = the annual interest rate,

n = number of annual interest periods,

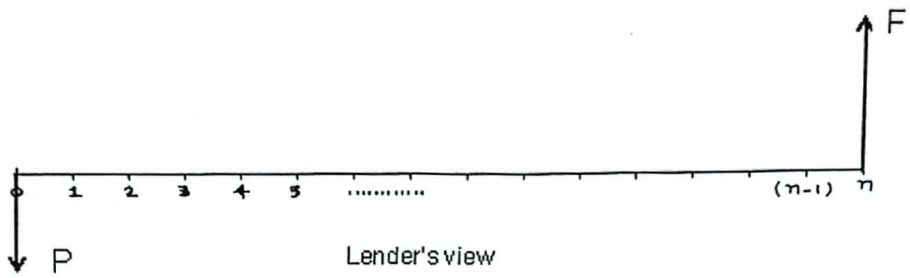
P = a present principal sum,

A = a single payment in a series of n equal payments, made at the end of each annual interest periods,

F = a future sum, n annual interest periods hence = the compound amount of present principal sum P.

Single payment compound amount interest factor –

If an amount, P is invested now with the amount earning at the rate of I per year, how much principal and interest are accumulated after n years? The cash flow diagram for this arrangement is shown below:



(Single present amount & Single future amount)

Since this investment does not provide any payments until the investment is terminated, the interest is compounded as shown below:

Year	Amount at the beginning of the year	Interest earned during the year	Compounded amount at the end of the year
1	P	Pi	P+Pi = P(1+i)
2	P(1+i)	P(1+i)i	P(1+i)+P(1+i)i = $P(1+i)^2$
3	$P(1+i)^2$	$P(1+i)^2i$	$P(1+i)^3$
4	.	.	.
.	.	.	.
.	.	.	.
n	$P(1+i)^{n-1}$	$P(1+i)^{n-1}i$	$P(1+i)^n = F$

$F = P(1+i)^n$ is known as the single payment compound amount of a present principal of P. (F/P,i,n) is called the single payment compound amount factor, which may be derived from the interest tables.

Hence, $F = P(1+i)^n$ or F = P (F/p, i, n) from tables.

Illustration: If Rs. 10000/- is invested at 6 % compounded interest in a project annually at the beginning of year 1, what will be the compounded amount of single payment at the end of year 4?

Soln.: $F = P\ (F/p, i, n)$

$= 10000 \times (1.2625)$ ----from tables

$=$ Rs. 12625/-

Single payment present worth factor –

The single payment compound amount relationship may be represented for P as follows:

$$F = P\ (1+i)^n$$

$$P = F\ [1/(1+i)^n]$$

The resulting factor, $1/(1+i)^n$ is known as the single payment present worth factor and is designated as (P/F, i, n). This factor is used to find the present worth, P of a future amount F, for the investment.

Illustration: How much must be invested now at 6 % compounded annually so that Rs. 1262/- can be received 4 years hence?

Soln.: $P = F\ [1/(1+i)^n]$

$= 1262\ [1/(1+0.06)^4]$

$=$ Rs. 1000/-

Or

$P = F\ (P/F, i, n)$

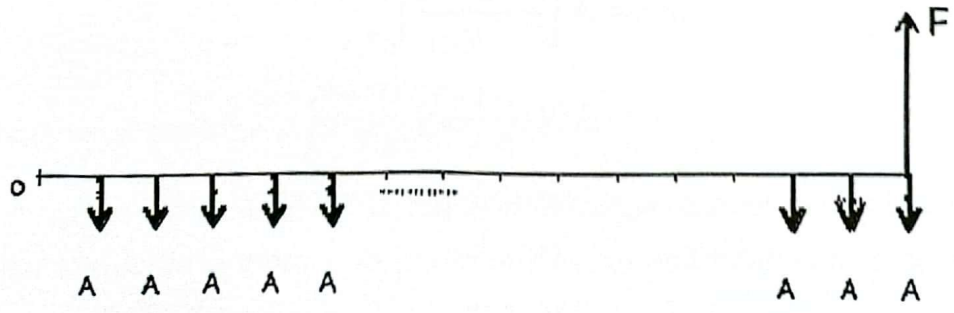
$= 1262 \times (P/F, 6, 4)$

$= 1262 \times (0.7921)$ ------ from tables.

$=$ Rs. 1000/-

Equal payment series compound amount factor –

In many engineering economy studies, it is often necessary to find the single future value that would accumulate from a series of payments occurring at the end of succeeding annual interest periods. Such a series of cash flows are presented below in the CFD:



The sum of the compound amounts of the several payments may be calculated by the use of the single payment compound amount factor.

If A is a series of n equal payments then,

$F = A(1) + A(1+i) + A(1+i)^2 + A(1+i)^3 + \ldots\ldots\ldots\ldots + A(1+i)^{n-2} + A(1+i)^{n-1}$ -------(1)

The total future amount F is equal to the sum of the individual future amounts calculated for each payment A. Multiply equation (1) by (1+i).

$F(1+i) = A(1+i) + A(1+i)^2 + A(1+i)^3 + \ldots\ldots\ldots\ldots + A(1+i)^{n-2} + A(1+i)^{n-1} + A(1+i)^n$ -----(2)

Equation (2) – (1),

$$F(1+i) - F = -A + A(1+i)^n$$

$$F(1+i-1) = A[(1+i)^n - 1]$$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

The resulting factor $\left[\frac{(1+i)^n - 1}{i}\right]$ is known as the equal payment series compound amount factor and is designated as [F/A, i, n]. This factor is used to find the compound amount, F of an equal payment series, A

Illustration: Determine the future amount of Rs. 100/- payment deposited at the end of each of the next five years and earning 6 % per annum.

Soln.:

End of year	Year end payment times compound amount factor	Compound amount at the end of 5 years.	Total compound amount Rs.
1	$100\ (1+0.06)^4$	126	126
2	$100\ (1+0.06)^3$	119	245
3	$100\ (1+0.06)^2$	113	358
4	$100\ (1+0.06)^1$	106	464
5	$100\ (1+0.06)^0$	100	564

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 100\left[\frac{(1+0.06)^5 - 1}{0.06}\right]$$

= Rs. 564/-

Or

F = A [F/A, i, n]

= 100 [5.637] ------from tables

= Rs. 564/-

Equal payment series – sinking fund factor –

The above equal payment series compound amount relationship may be expressed for A as follows:

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Or

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

The resulting factor $\left[\frac{i}{(1+i)^n-1}\right]$ is known as equal payment series sinking fund factor as is designated as [A/F, i, n]. This factor may be used to find the required year end payments, A to accumulate a future amount, F.

Illustration: It is desired to accumulate Rs. 563.70 by making a series of 5 equal annual payments at 6 % interest compounded annually. What is the required amount of each payment?

Soln.:

$$A = F\left[\frac{i}{(1+i)^n-1}\right]$$

$$= 563.70\left[\frac{0.06}{(1+0.06)^5-1}\right]$$

= Rs. 100/-

Or

A = F [A/F, i, n]

= 563.70 [A/F, 0.06, 5]

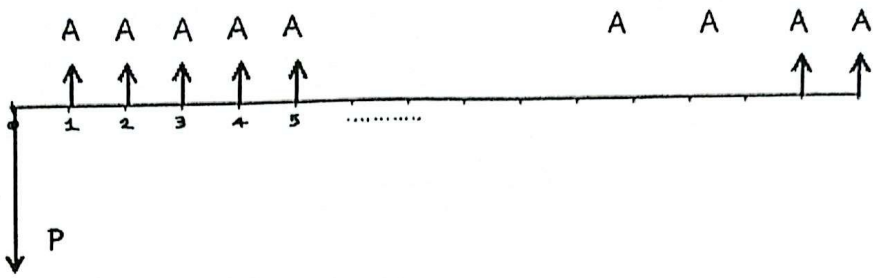
= 563.70 × [0.1774]

= Rs. 100/-

Equal payment series – capital recovery factor –

A deposit of amount P is made now at an annual interest rate of i. The depositor wishes to withdraw his principal plus earned interest in a series of equal

year end amounts over the next n years. When the last withdrawal is made, there should be no funds left in the deposit. The cash flow diagram is shown below:



It is previously determined that F is related to A by the equal payment series sinking fund factor and that F & P are linked by the single payment compound amount factor. The substitution of $F = P(1+i)^n$ in the equal payment sinking fund factor relationships results in

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= P(1+i)^n\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

The resulting factor $\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$ is called the equal payment capital recovery factor and is designated as (A/P, i, n). This factor may be used to find the year end payments, A that will be provided by the present amount, P.

Illustration: Rs. 1000/- invested now at 5 % interest compounded annually, provide for 8 equal future year end payments. Determine A.

Soln.:

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

$$= 1000\left[\frac{0.05(1+0.05)^8}{(1+0.05)^8 - 1}\right]$$

= Rs. 154.72/-

By tables,

$$A = P\ (A/P, i, n)$$

$$= 1000 \times (0.1547)$$

$$= \text{Rs. } 154.72/-$$

Equal payment series – present worth factor –

To find what single amount must be deposited now, so that equal end of year payments can be made. The equal payment series capital recovery factor may be used like this:

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

Or

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

The factor $\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$ is called the equal payment series present worth factor and designated as (P/A, i, n). This factor is used to determine the present worth, P or a series of equal annual payments.

Illustration: Determine the present worth of a series of eight equal annual payments of Rs. 154.72/- at an interest rate of 5 % compounded annually.

Soln.:

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 154.72\left[\frac{(1+0.05)^8 - 1}{i(1+0.05)^8}\right]$$

$$= \text{Rs. } 1000/-$$

Using Tables,

$$P = A\ (P/A, i, n)$$

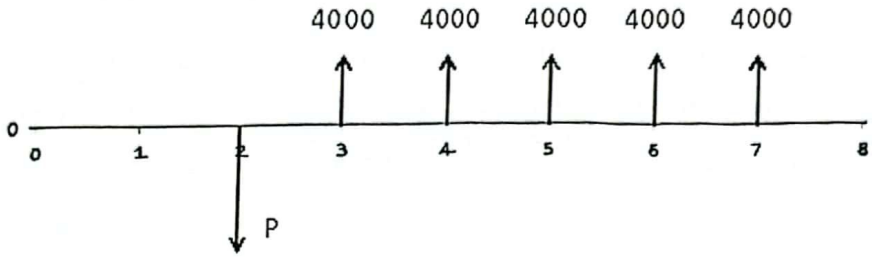
= 154.72× (6.4632)

= Rs. 1000/-

Solved Problems:

1. Determine the amount P that you should deposit into an account 2 years from now, in order to be able to withdraw Rs. 4000/- per year for 5 years starting 3 years from now, at an interest rate of 15 % per year.

   Soln.:



P = A (P/A, i, n)

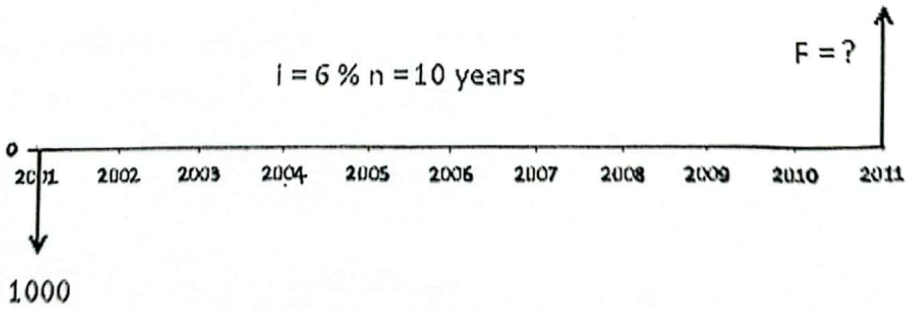
= 4000 × (P/A, 15, 5)

= 4000 × (3.3522)

= Rs. 13408.80/-

2. If Mr. X deposits Rs. 1000 in his bank account at 6 % compound interest on January 1, 2001. How much money will be accumulated on January 1, 2011?

   Soln.:



F = P (F/P, I, n)

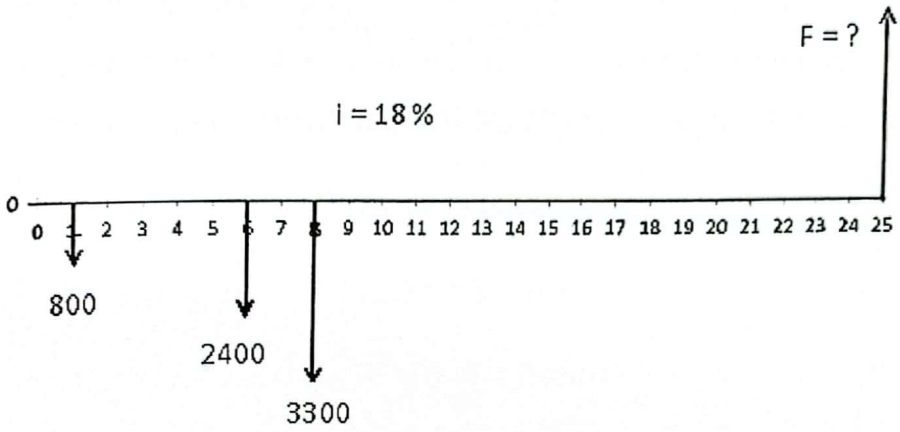
= 1000 × (F/P, 6, 10)

$= 1000 \times (1.791)$

$= \text{Rs. } 1791/-$

3. How much money will be accumulated in 25 years if Rs. 800/- is deposited one year from now, Rs. 2400/- six years from now, and Rs. 3300/- eight years from now all at an interest rate of18 % per year.

Soln.:



$F_1 = P_1 (F/P, i, n)$

$= 800 (F/P, 18, 24)$

$= 800 (53.109)$

$= \text{Rs. } 42487.20/-$

$F_2 = P_2 (F/P, i, n)$

$= 2400 (F/P, 18, 19)$

$= \text{Rs. } 55714.56/-$

$F_3 = P_3 (F/P, i, n)$

$= 3300 (F/P, 18, 17)$

$= \text{Rs. } 55018.26/-$

Therefore, $\text{Total } F = F_1+F_2+F_3$

$= 42487.20+55714.56+55018.26$

= Rs. 153220/-

Alternate method:

$$F = P(1+i)^n$$

$$F_1 = 800(1+0.18)^{24}$$

= Rs. 42487.20

$$F_2 = 2400(1+0.18)^{19}$$

= Rs. 55714.56

$$F_3 = 3300(1+0.18)^{17}$$

= Rs. 55018.26

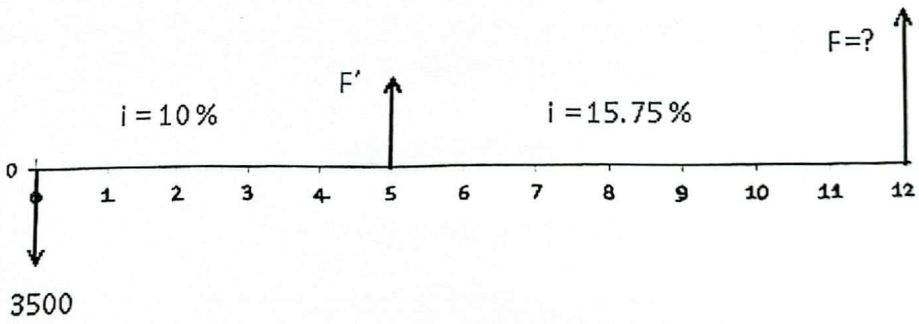
Therefore, Total $F = F_1+F_2+F_3$

= 42487.20+55714.56+55018.26

= Rs. 153220/-

4. How much money Mr. A will have in his bank account in 12 years, if he deposits Rs. 3500/- now and the interest rate is 10 % for the first 5 years and thereafter changes to 15.75 % per year.

Soln.:



$$F' = P(F/P, i, n)$$

$$= 3500(F/P, 10, 5)$$

$$= 3500(1.611)$$

= Rs. 5638.50/-

$$F = F'(F/P, i, n)$$

= 5638.50 (F/P, 15.75, 7)

= Rs. 15692/-

Or

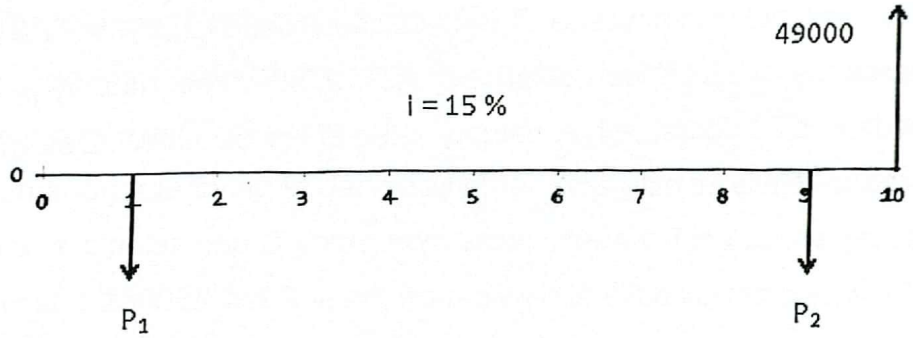
$F = P(1+i)^n$

$= 5638.50 (1+0.1575)^7$

= Rs. 15692/-

5. A company is planning to make two equal deposits such that 10 years from now the company will have $ 49000 to replace a small machine. If the first deposit is to be made 1 year from now and the second is to be made 9 years from now, how much must be deposited each time if the interest rate is 15 % per annum.

Soln.;



$F = P (F/P, i, n)$

$= P_1 (F/P, i, n) + P_2 (F/P,i,n)$

$= P_1 (F/P, 15, 9) + P_2 (F/P, 15, 1)$

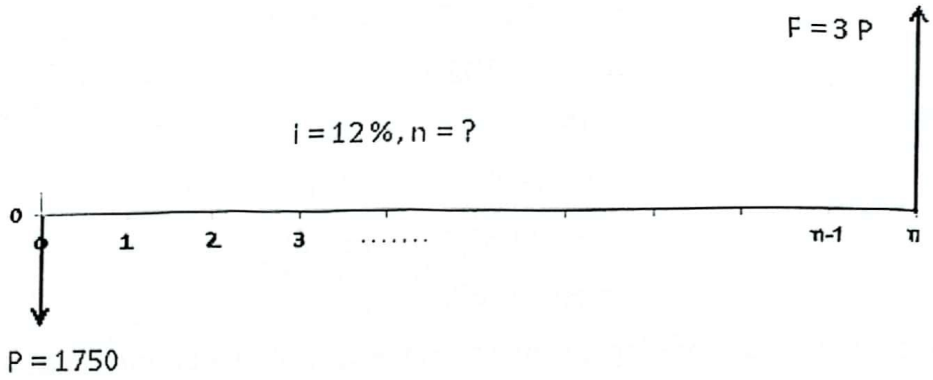
$= P (3.518) + P (1.15)$ since $P_1 = P_2 = P$

$49000 = 4.668\ P$

Therefore, $P = \$ 10497/-$

6. How many years would it take for$ 1750 to triple in value if the interest rate is 12 % per annum?

Soln.;



$$F = P\,(F/P, i, n)$$

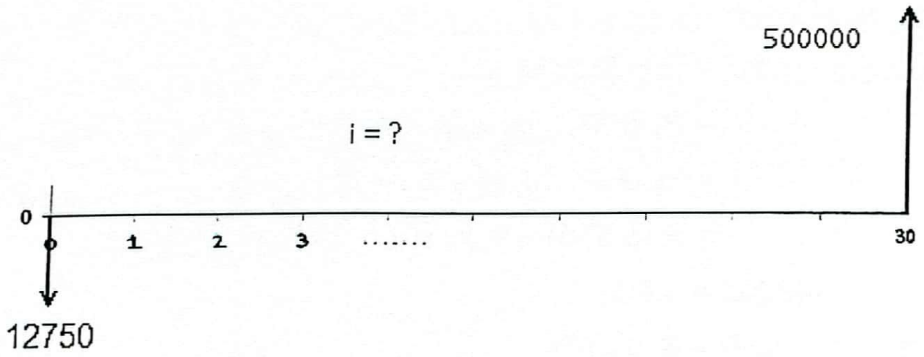
$$= P\,(1+i)^n$$

$$3 \times 1750 = 1750\,(1+ 0.12)^n$$

Therefore $n = 9.69$ years

7. IDBI came out with an issue of deep discount bonds in 2008. The bonds were offered at a deep discounted price of Rs. 12750/-. The maturity period for the bonds was 30 years, with a maturity value of Rs. 500000/-. Determine the rate of return of this investment?

Soln.;



$$F = P\,(1+i)^n$$

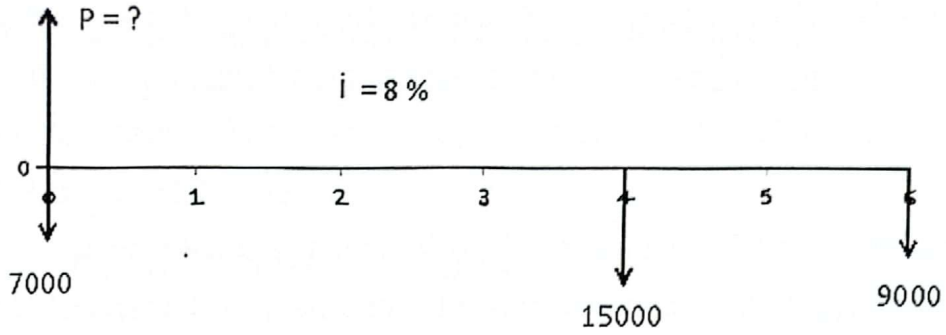
$$50000 = 12750\,(1+i)^{30}$$

$$(1+i) = (500000/12750)^{1/30}$$

Therefore, $i = 0.13$ or 13 %

8. What is the total present worth of Rs. 7000/- now, Rs. 15000/- four years from now, and Rs. 9000/- six years from now at an interest rate of 8 % per annum.

Soln.;

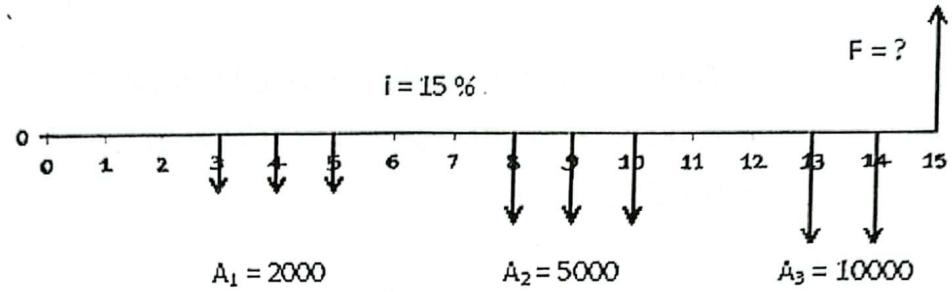


$$P = 7000 + 15000\ (P/F, 8, 4) + 9000\ (P/F, 8, 6)$$
$$= 7000 + 15000\ (0.735) + 9000\ (0.6302)$$
$$= \text{Rs. } 23696.80/-$$

Unsolved Problems

1. Mr. X plans to buy some property which his fried has generously offered to him. The payment scheme is Rs. 7000/- every other year, till year eight starting 2 years from now. What is the present worth of this generous offer if the interest rate is 17 % per year?
2. A plant manager is trying to decide whether to buy a new machine or wait and purchase a similar one 3 years from now. The machine at the present time would cost $ 25000/- but 3 years from now it is expected to cost $ 39000/-. If the interest rate the company uses is 20 % per annum, should the plant manager buy now or should he buy 3 years from now?
3. The annual deposits of Rs. 1000/- are made into a savings account for 30 years beginning one year from now. How much will be in the fund immediately after the last deposit if the fund pays an interest of 19.25 % per year.
4. If you start saving money by depositing Rs. 1000/- per year into a bank account which pays 11 % interest, how many years will it take to accumulate Rs. 10000/- if the first deposit is to be made one year from now?
5. What is the number of year end deposits that have to be made before the total value of the deposits is at least 10 times greater than the value of a single year end deposit if the interest rate is 12.5 % per year.
6. How much money Mr. A must deposit into a savings account each year starting at the end of 1985, if he wants to have $ 150000 when he retires in the year 2020? The interest rate is 16 % per year.

7. Determine the equal amounts that would have to be deposited at the end of each year for 10 years, starting one year from now, into a fund, in order to have Rs. 180000 at the end of 15th year. Assume an interest rate of 14 % per year.
8. The following CFD shows the deposits made by a person at 15 % interest rate. Calculate the compound amount accumulated at the end of 15th year.



9. A firm is available for sale for Rs. 65000/- now. The firm yields a profit of Rs. 10000/- per year for the next 10 years. If the money is worth 12 % can the firm be bought?
10. A series of 10 annual payments of Rs. 1500/- are equivalent to three equal payments at the end of the years 6, 10, and 15 at 12 % interest compounded annually. What is the amount of these three payments?
11. If Mr. X borrows Rs. 4500/- with a promise to repay it in 10 equal annual instalments staring one year from now. How much would his payments be if the interest rate is 20 % per year?
12. A person requires Rs. 20000/- at the beginning of each year from 2015 to 2019. How much should he have deposited at the end of each year from 2005 to 2010? The interest rate is 12 % per year.

4.0 ECONOMIC EVALUATION OF ALTERNATIVES

THIS CHAPTER COVERS THE FOLLOWING:

- BASES FOR COMPARISON OF ALTERNATIVES.
- NET CASH FLOWS FOR INVESTMENT ALTERNATIVES.
- PRESENT WORTH AMOUNT METHOD.
- ANNUAL EQUIVALENT COST METHOD.
- CAPITAL RECOVERY WITH RETURN.
- CAPITALIZED EQUIVALENT AMOUNT METHOD.
- FUTURE WORTH AMOUNT METHOD.

4.1 Bases for comparison of alternatives:

A basis for comparison is an index containing particular information about a series of receipts and disbursements representing an investment opportunity. The reduction of alternatives to a common base is necessary so that apparent differences become real differences, with the time value of money considered. When expressed in terms of common base, the real differences become directly comparable and may be used in decision making.

The most common basis for comparison is the present worth amount, the annual equivalent amount, the capitalized equivalent amount and the future worth amount. It is important to note that the basis for comparison represents only one element of any systematic approach used to choose between economic alternatives.

4.2 Net cash flows for investment alternatives:

An investment opportunity is viewed as the actual cash receipts and disbursements that are anticipated if the investment is undertaken. The representation of the amounts and timings of these cash receipts and disbursements is referred to as investment cash flow.

Example: An individual is planning to invest Rs. 1000/- in a SB account that pays him 5 % per annum at the end of each year. The individual anticipates that he will withdraw his initial investment at the end of 4 years. The cash flow that represents the individual's investment is shown below:

Negative sign represents cash disbursements. When an investment opportunity has both cash receipts (+ve) and cash disbursements (-ve), occurring simultaneously, a net cash flow is usually calculated for the investment opportunity.

End of year	Cash flow from individual point of view (Rs.)	Cash flow from banker's point of view (Rs.)
0	-1000	1000
1	50	-50
2	50	-50
3	50	-50
4	1050	-1050

Let F_{jt} = Net cash flow for investment proposal j at time t. If $F_{jt}<0$, then F_{jt} represents a net cash cost/disbursement. If $F_{jt}>0$, then F_{jt} represents a net cash income/receipt.

4.3 Present worth Amount:

The present worth amount is an amount at the present (t = 0) that is equivalent to an individual's cash flow for a particular interest rate, i.

$$PW(i)_j = \sum_{t=0}^{n} F_{jt}\,(1+i)^{-t} \qquad -1 < i < \infty$$

Following features of PW amount make it suitable as a basis of comparison.

- It considers the time value of money.
- It is an equivalent amount by which the equivalent receipt and equivalent payments are compared to see for +ve cash flow or –ve cash flow.
- By examining PW (i) function, considerable information useful for decision making purposes can be ascertained about the investment opportunity.

Net present worth
or
Net present value = Total present worth of cash inflows – Total present worth of cash out flows.

If NPW is +ve – Accept the proposal/project.

If NPW is –ve – Reject the proposal/project

If NPW is 0 – Situation is indifferent

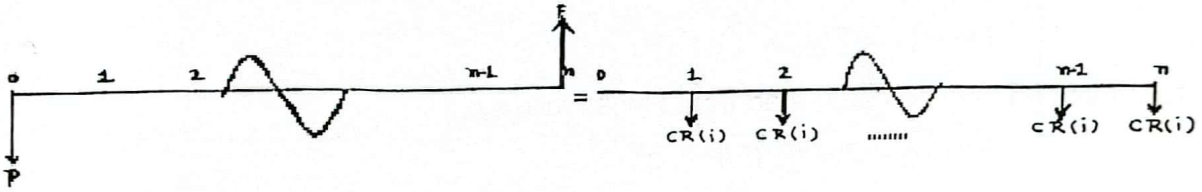
Again we have two situations occurring for decision making:

- Equal lived alternatives, having equal life of n years.
- Unequal lived alternatives, having different life for each alternative.

4.4 Annual equivalent method/Equivalent uniform annual worth or cost method (EUAW/EUAC):

While using this method for comparing alternatives having different time durations there is no need to compare them over a time period equal to the LCM of their lives. It is enough, if the EUAW is calculated for one life cycle of the alternatives, since EUAW represents equal annual worth over the life cycle of the project.

4.5 Capital recovery with return:



CR (i) = P (A/P, i, n) – F (A/F, i, n)

= P (A/P, i, n) – F [(A/P, i, n) – i] --- From Tables

= (P-F) (A/P, i, n) + Fi

4.6 Capitalized equivalent amount or Capitalized cost method:

The procedure involved in the calculation of capitalized cost is as follows:

i. Draw the CFD showing all non-recurring cash flows and at least two cycles of all recurring cash flows.
ii. Find the PW of all non-recurring cash flows using a single payment PW relationship.
iii. Find the equivalent uniform annual amount A over the life cycle for all recurring cash flows, and divide that amount by the interest rate to get the capitalized cost of recurring cash flows.

iv. Divide all the uniform cash flows occurring from year 1 to year ∞, by the interest rate to get the capitalized cost of those uniform cash flows.

v. Add the values obtained in steps ii, iii, and iv to get the total capitalized cost of the given investment.

4.7 Future worth amount method:

The future worth basis for comparison is an equivalent amount of a cash flow calculated at a future time for some interest rate. The future worth amount for proposal j at some future time for n years from the present is

$$FW(i)_j = F_{j0}\left(\frac{F}{P}, i, n\right) + F_{j1}\left(\frac{F}{P}, i, n-1\right) + \ldots\ldots + F_{j(n-1)}\left(\frac{F}{P}, i, 1\right) + F_{jn}\left(\frac{F}{P}, i, 0\right)$$

$$= \sum_{t=0}^{n} F_{jt}\left(\frac{F}{P}, i, n-t\right)$$

But since,

$(F/P, I, n\text{-}t) = (1+i)^{n-t}$

$FW(i)_j = \sum_{t=0}^{n} F_{jt}\,(1+i)^{n-t}$

But, FW can also be calculated from Pw as follows:

$FW\,(i)_j = PW\,(i)_j\,(F/P, i, n)$

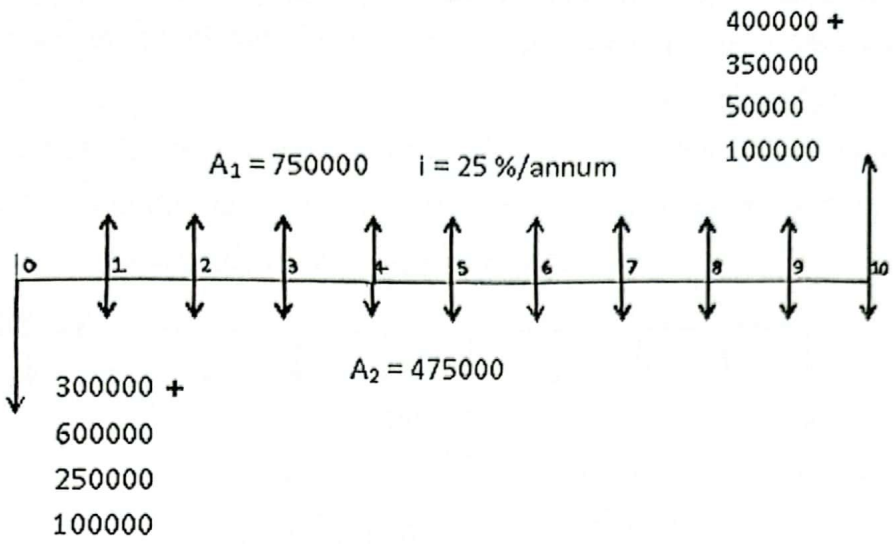
Again the following relationship is always true:

$$\frac{PW(i)_A}{PW(i)_B} = \frac{EUAC(i)_A}{EUAC(i)_B} = \frac{FW(i)_A}{FW(i)_B}$$

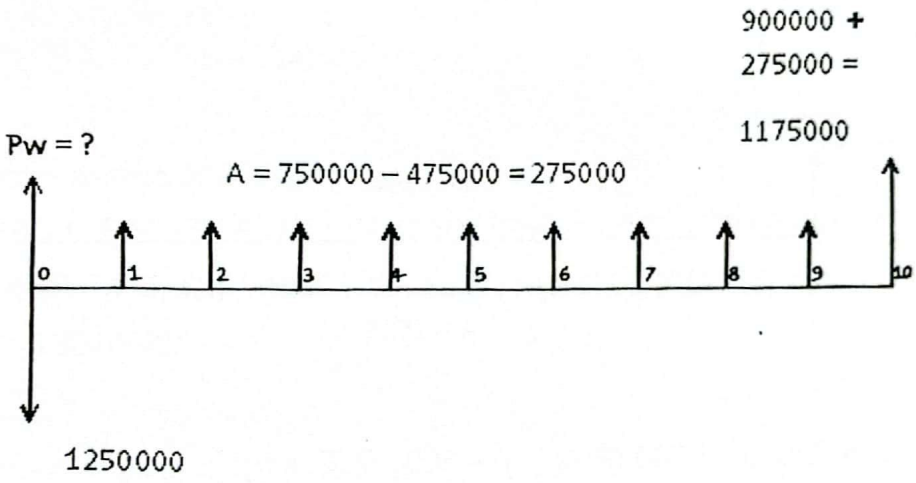
Solved Problems:

1. A company is considering a plant to manufacture a particular new product. The land costs Rs. 300000/-, the building costs Rs. 600000/-, the equipment costs Rs. 250000/-, and Rs. 100000/- working capital is required. It is expected that the product will result in sales of Rs. 750000/- per year for 10 years, at which the land can be sold for Rs. 400000/-, the building for Rs. 350000/-, the equipment for Rs. 50000/- and all of the working capital is recovered. The annual out of pocket expenses for labour, material and other items are estimated to total Rs. 475000/-. If the interest rate is 25 % per year, determine whether the company should invest in the new product line. Use present worth method.

   Soln.: CFD is:



Net CFD will be:



Pw = -1250000 + 275000 (P/A, i, n) + 1175000 (P/F, i, n)

= -1250000 + 275000 (P/A, 25, 9) + 1175000 (P/F, 25, 10)

= -1250000 + 275000 × 3.4631 + 1175000 × 0.1074

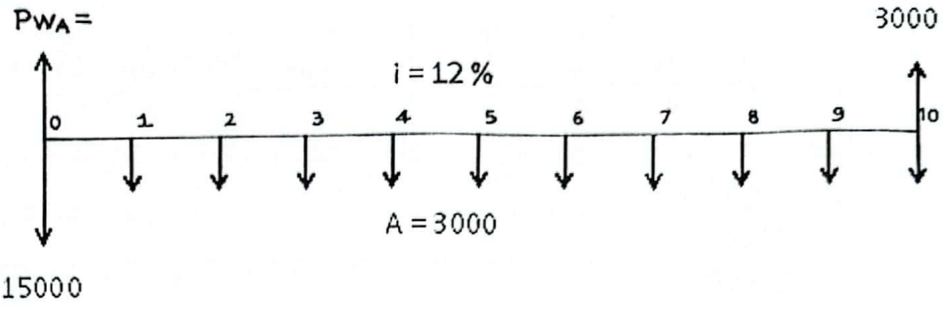
= - Rs. 171480/-

Since Pw of the proposal is negative, it is not advisable to invest in this new product line.

2. Two machines are under consideration by a metal fabricating company. Machine 'A' will have a first cost of Rs. 15000/-, annual maintenance and operation cost of Rs. 3000/- and Rs. 3000/- salvage value. Machine 'B' will have a first cost of Rs. 22000/-, annual maintenance and operation cost of Rs. 1500/- and Rs. 5000/- salvage value. If both machines are expected to last for 10 years, determine

which machine should be selected on the basis of present worth value using an interest rate of 12 % per annum.

Soln.: **Machine 'A'**

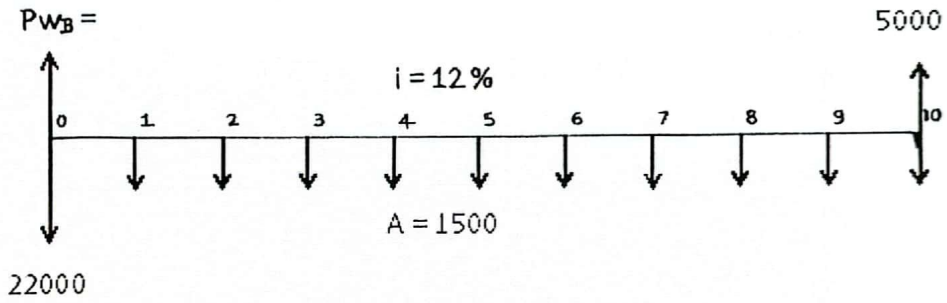


PW_A = 15000 + 3000 (P/A, i, n) – 3000 (P/F, i, n)

= 15000 + 3000 (P/A, 12, 10) – 3000 (P/F, 12, 10)

= Rs. 30984 (Cost value)

Machine 'B'



PW_B = 22000 + 1500 (P/A, i, n) – 5000 (P/F, i, n)

= 22000 + 1500 (P/A, 12, 10) – 5000 (P/F, 12, 10)

= Rs. 28865 (Cost value)

Since machine 'B' has less present worth (equivalent to present expenditure on it) **select Machine 'B'**.

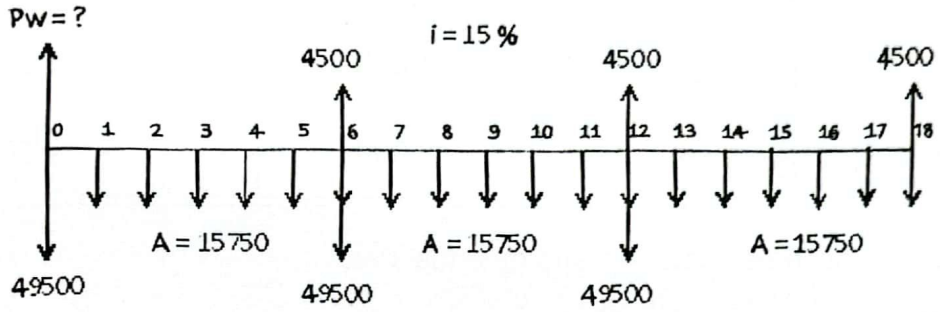
3. A plant manager is trying to decide between the machines detailed below:

	Machine 'A'	Machine 'B'
First cost	Rs. 49500/-	Rs. 63000/-
Annual operating cost	Rs. 15750/-	Rs. 13950/-
Salvage value	Rs. 4500/-	Rs. 9000
Life (years)	6	9

Determine which machine should be selected on the basis of the present worth comparison using an interest rate of 15 % per year?

Soln.: Since the two machines have different lives they must be compared over a time period equal to the LCM of their lives, which is equal to 18 years ($2 \times 3 = 6$, $3 \times 3 = 9$ LCM $= 2 \times 3 \times 3 = 18$ years).

Machine 'A'

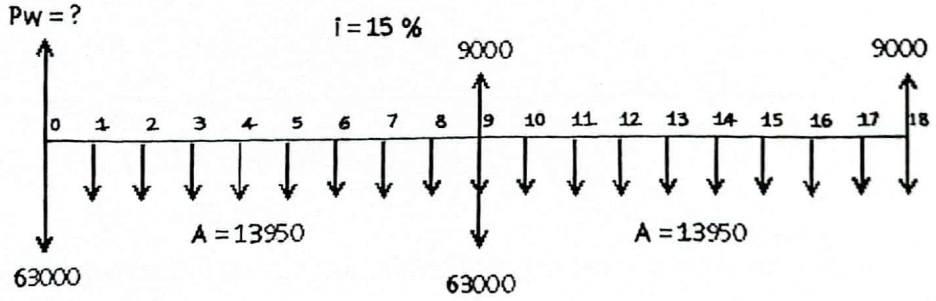


Taking costs as +ve and revenue as –ve.

Pw = 49500 + 49500 (P/F, 15, 6) + 49500 (P/F, 15, 12) – 4500 (P/F, 15, 6) – 4500 (P/F, 15, 12) – 4500 (P/F, 15, 18) + 15750 (P/A, 15, 18)

= Rs. 173516.40/-

Machine 'B'



Pw = 63000 + (63000 – 9000) (P/F, 15, 9) – 9000 (P/F, 15, 18) + 13950 (P/A, 15, 18)

= Rs. 163110.60/-

Hence **Machine 'B'** should be selected.

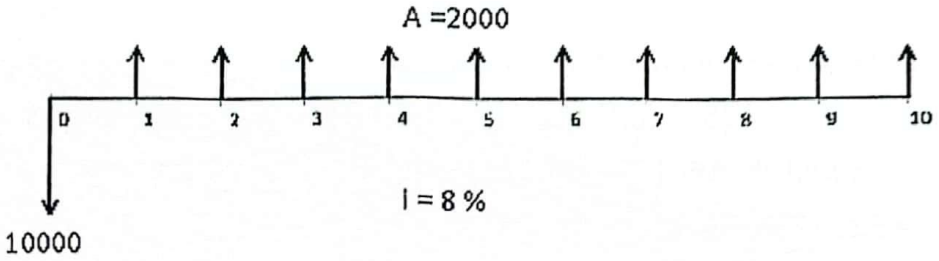
4. The following data refers to the cash flows of five investment proposals. The total money available to the company to invest is Rs. 35000/-. The company can accept any one of the proposals with the same letter or combination of proposals

with different letters subject to the budget available. Select the best alternative based on the present worth on total investment criterion. The rate of return is 8 %.

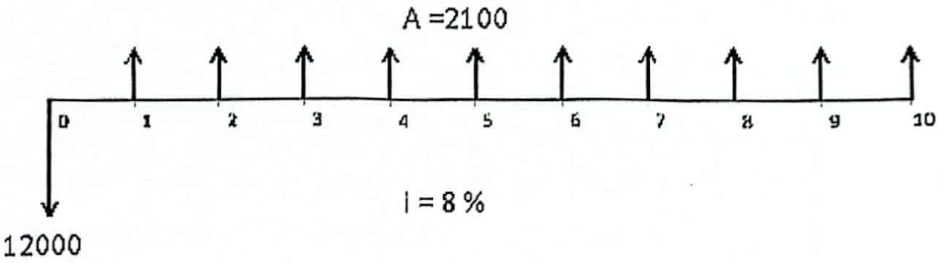
Proposal	First cost	Net income years 1-10
A_1	-10000	2000
A_2	-12000	2100
B_1	-20000	3100
B_2	-30000	5000
C_1	-35000	4500

Soln.: Assume costs –ve and revenue +ve.

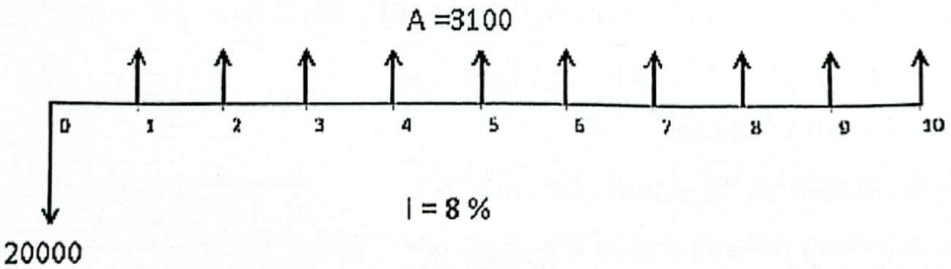
A_1:



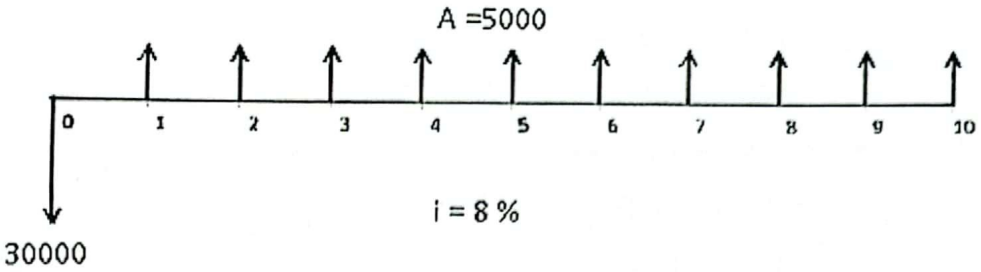
A_2:



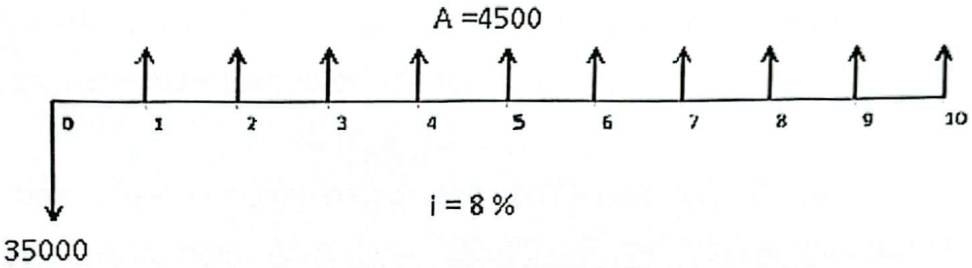
B_1:



B_2:



C_1:



$Pw(A_1) = -10000 + 2000\ (P/A, 8, 10)$

$= Rs.\ 3420/-$

$Pw(A_2) = -12000 + 2100\ (P/A, 8, 10)$

$= Rs.\ 2091/-$

$Pw(B_1) = -20000 + 3100\ (P/A, 8, 10)$

$= Rs.\ 801/-$

$Pw(B_2) = -30000 + 5000\ (P/A, 8, 10)$

$= Rs.\ 3550/-$

$Pw(C_1) = -35000 + 4500\ (P/A, 8, 10)$

$= Rs. - 480.50/-$

Proposal 'C' is rejected since the present worth is negative.

Out of the remaining proposals, the possible combinations are as follows:

$Pw\ (A_1B_1) = Pw\ (A_1) + Pw\ (B_1)$

$= 3420 + 801$

$= Rs.\ 4221/-$

$Pw\ (A_1B_2) = Pw\ (A_1) + Pw\ (B_2)$

$= 3420 + 3550$

$= Rs.\ 6970/-$

$Pw(A_2B_1) = Pw(A_2) + Pw(B_1)$

$= 2091 + 801$

$= Rs.\ 2892/-$

$Pw(A_2B_2) = Pw(A_2) + Pw(B_2)$

$= 2091 + 3550$

$= Rs.\ 5641/-$

Total investment of A_1B_1 = 10000 + 20000 = Rs. 30000/-

Total investment of A_1B_2 = 10000 + 30000 = Rs. 40000/-

Total investment of A_2B_1 = 12000 + 20000 = Rs. 32000/-

Total investment of A_2B_2 = 12000 + 30000 = Rs. 42000/-

In case of A_1B_2 and A_2B_2 combinations total investment exceeds the limit of Rs. 35000/-. Hence they cannot be considered even though the Pw is higher.

Out of the remaining two combinations, A_1B_1 and A_2B_1 where investment is less than Rs. 35000/- limit, A_1B_1 alternative gives higher Pw of Rs. 4221/-. Hence the combination A_1B_1 is to be selected.

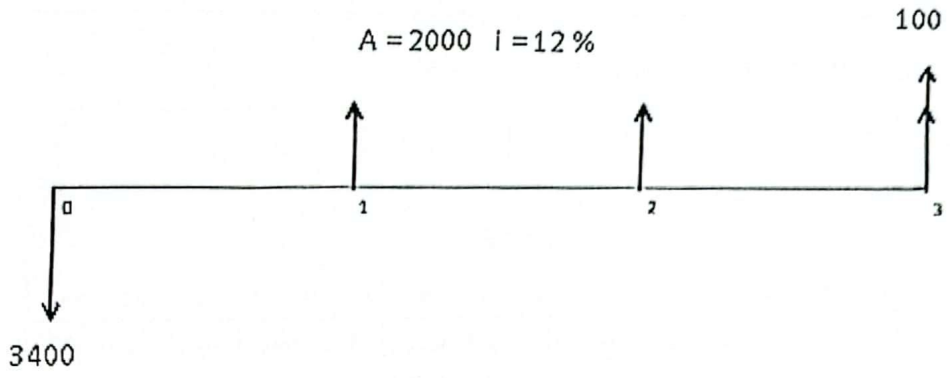
5. A firm is considering the purchase of one of the two new machines. The data on each are described below:

	Machine 'A'	**Machine 'B'**
Initial cost	3400	6500
Service life	3 years	6 years
Salvage value	100	500
Net operating cash flow after taxes	2000 / year	1800 / year

If the firm's MRR is 12 %, which machine should be selected using EUAW method.

Soln.: Assume costs –ve and revenue +ve.

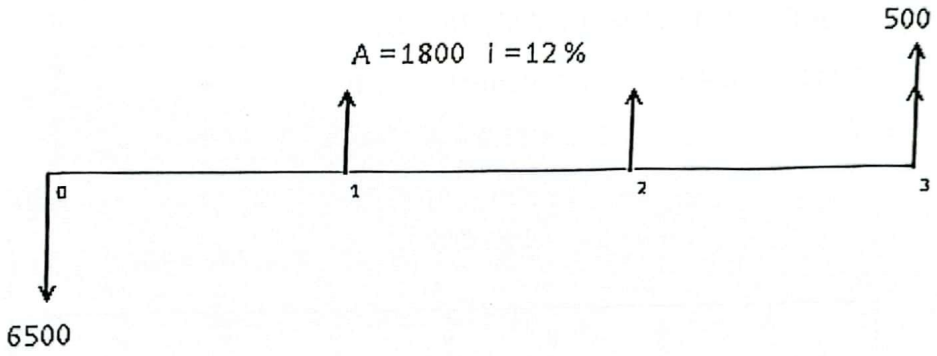
Machine 'A'



EUAW (A) = - 3400 (A/P, 12, 3) + 2000 + 100 (A/F, 12, 3)

= Rs. 640/- (Revenue value)

Machine 'B'



EUAW (B) = - 6500 (A/P, 12, 6) + 1800 + 500 (A/F, 12, 6)

= 280.60/- (Revenue value)

Here EUAW (A) > EUAW (B), **select MACHINE 'A'**.

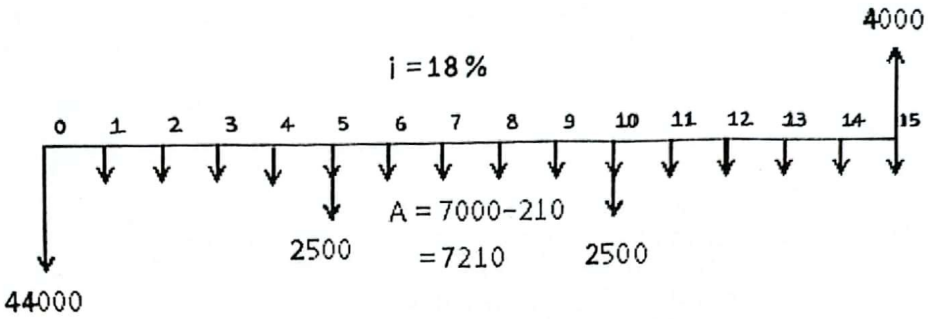
6. Compare the following machines on the basis of their equivalent annual worth, using i = 18 % per annum.

	Machine 'X'	**Machine 'Y'**
First cost	44000	23000
Annual operating cost	7000	9000
Annual repair cost	210	350
Overhaul every 2 years	-	1900
Overhaul every 5 years	2500	-
Salvage value	4000	3000

Life (years)	15	8

Soln.: Assume costs +ve and revenue –ve.

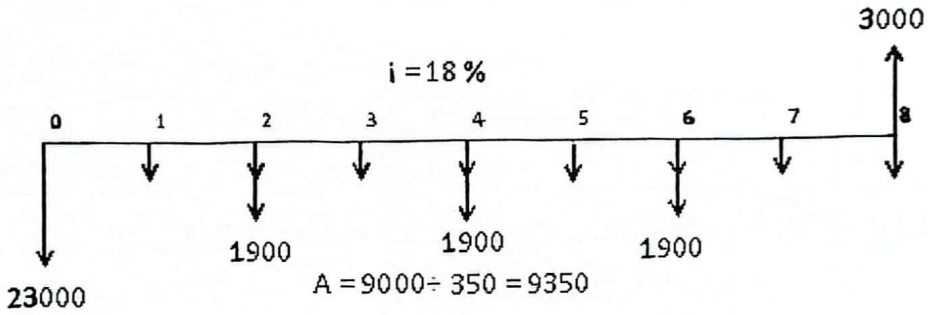
Machine 'X'



EUAC (X) = 44000 (A/P, 18, 15) + 7210 + 2500 [(P/F, 18, 5) + (P/F, 18, 10)] (A/P, 18, 15) - 4000 (A/F, 18, 15)

= Rs. 16094.40/- (Cost value)

Machine 'Y'



EUAC (Y) = 23000 (A/P, 18, 8) + 9350 + 1900 [(P/F, 18, 2) + (P/F, 18, 4) + (P/F, 18, 6)] (A/P, 18, 15) - 3000 (A/F, 18, 15)

= Rs. 15542.37/- (Cost value)

Machine Y is having lower EUAC. Hence **Select MACHINE Y.**

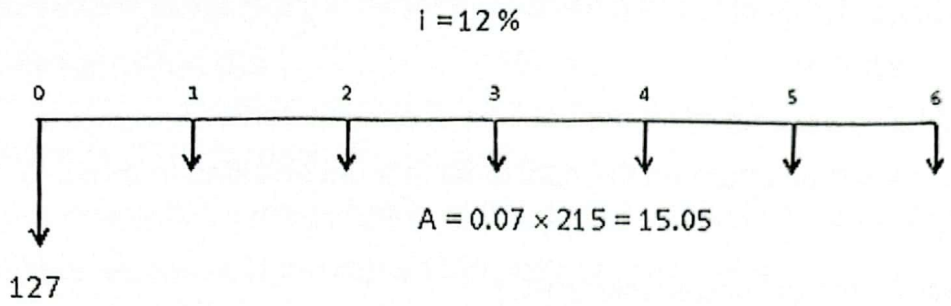
7. The heat loss through the exterior walls of a building costs $ 215/- per year. Insulation that will reduce the heat loss cost by 93 % can be installed for $ 127/- and the insulation that will reduce the heat loss by 89 % can be installed for $ 90/-. Determine which insulation is most desirable if the building is to be used for 6 years and if the interest rate is 12 %.

Soln.:

Alternative I – No insulation

EUAC (Alt I) = $ 215/- per year.

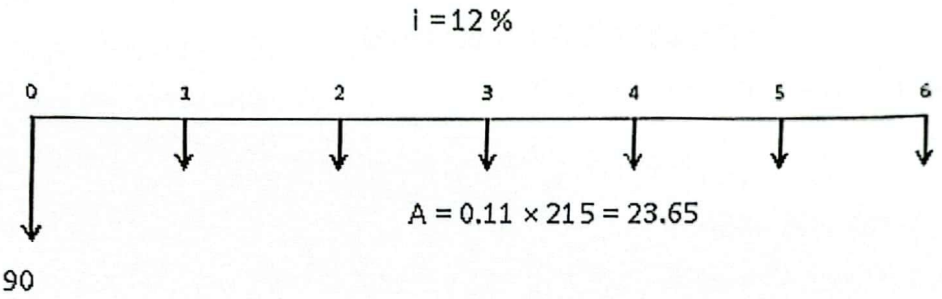
Alternative II – Insulation I



EUAC (Alt II) = 127 (A/P, 12, 6) + 15.05

= $ 45.93/- (Cost value)

Alternative III – Insulation II



EUAC (Alt III) = 90 (A/P, 12, 6) + 23.65

= $ 45.54/- (Cost value)

EUAC (Alt III) < EUAC (Alt II)

Hence select **ALTERNATIVE III**

8. A 100 HP motor required to power a large capacity blower. Two motors have been proposed with the following engineering cost data.

	Motor A	Motor B
Cost	$ 4500	$ 4000
Life	12 years	12 years
Salvage value	0	0
Efficiency ½ load	85 %	83 %
Efficiency ¾ load	92 %	89 %
Efficiency full load	89 %	88 %
Hours of use per year		
½ load	800	800
¾ load	1000	1000
Full load	600	600

Power cost per KWH is $ 0.06. Annual maintenance, taxes and insurance will amount to 1.6 % of the original cost. Interest is 10 %. Assume 1 HP = 736 W and determine

a. What is the equivalent annual cost of each motor?

b. What will be return on the additional amount invested in motor A.

Soln.:

Motor A: Annual power cost (APC)

$$APC\ (A) = \left[\frac{50}{0.85} \times \frac{736}{1000} \times 800 + \frac{75}{0.92} \times \frac{736}{1000} \times 1000 + \frac{100}{0.89} \times \frac{736}{1000} \times 600\right] \times 0.06$$

= Rs. 8655.20/-

EUAC (Motor A) = 4500 (A/P, 10, 12) + 0.016 × 4500 + APC (A)

= Rs. 9387.60/- (Cost value)

Motor B: Annual Power cost (APC)

$$APC\ (B) = \left[\frac{50}{0.83} \times \frac{736}{1000} \times 800 + \frac{75}{0.89} \times \frac{736}{1000} \times 1000 + \frac{100}{0.88} \times \frac{736}{1000} \times 600\right] \times 0.06$$

= Rs. 8860.45/-

EUAC (Motor A) = 4000 (A/P, 10, 12) + 0.016 × 4000 + APC (B)

= Rs. 9511.49/- (Cost value)

EUAC (Motor A) < EUAC (Motor B)

Hence select **MOTOR 'A'**

9. A firm requires power shovels for its open pit mining operation. The first cost is Rs. 250000/- and the salvage value Rs. 35000/- at the end of 10 years service. If the firm uses a rate of 12 % for the project evaluation how much must be earned on an equivalent annual basis so that the firm recovers its invested capital plus earns a return on the capital committed to the equivalent during its life time.

Soln.: P = Rs. 250000/-, F = Rs. 35000/-, n = 10 years, i = 12 %

Firm's earning , CR(i) = (P-F) (A/P, i, n) + Fi

= (250000 – 35000) (A/P, 12, 10) + 35000 × 0.12

= Rs. 38055/-

10. A company can invest in one of the mutually exclusive alternatives. The life of both alternatives is estimated to be 5 years with the following initial investments and salvage values.

Soln:

	A	B
Initial investment (Rs.)	10000	12000
Salvage value (Rs.)	1500	3500

a. What is CR(i) for each alternative?

b. Determine the salvage value at the end of the project life for alternative B which will result in the same CR(i) for both alternatives.

Assume i = 15 %.

Soln.:

a. **Alternative A**:

$CR(i)_A = (P-F)\,(A/P, i, n) + Fi$

$= (10000 - 1500)\,(A/P, 15, 5) + 1500 \times 0.15$

$=$ Rs. 2760.55/-

Alternative B:

$CR(i)_B = (P-F)\,(A/P, i, n) + Fi$

$= (12000 - 3500)\,(A/P, 15, 5) + 3500 \times 0.15$

$=$ Rs. 3060.55/-

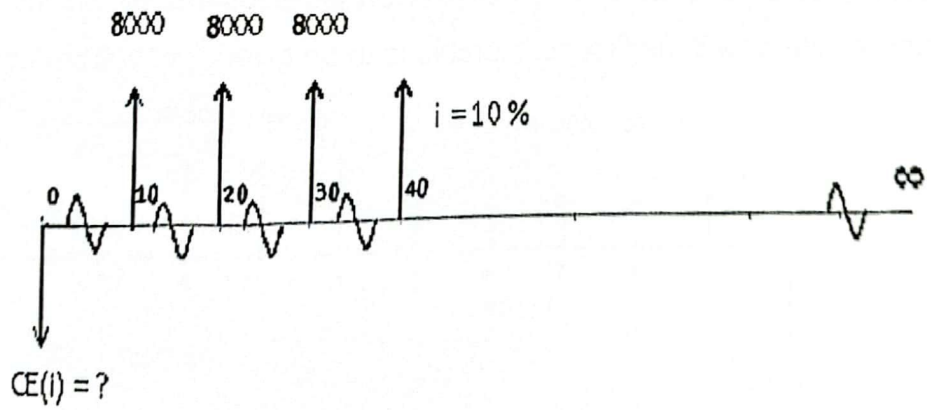
b. $CR(i)_A = CR(i)_B$

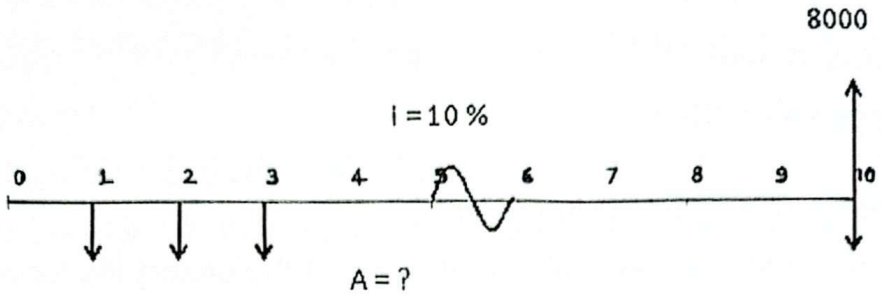
$2760.55 = (12000 - F)\,(A/P, 15, 5) + F \times 0.15$

Hence, Salvage value of B, F_B = Rs. 5222.90/-

11. Rs. 8000/- is to be withdrawn from a savings account at the end of every 10 years. Calcula capitalized equivalent amount or the single amount to be deposited now at an interest rate of 10 % per year?

Soln.:





A = 8000 (A/F, 10, 10)

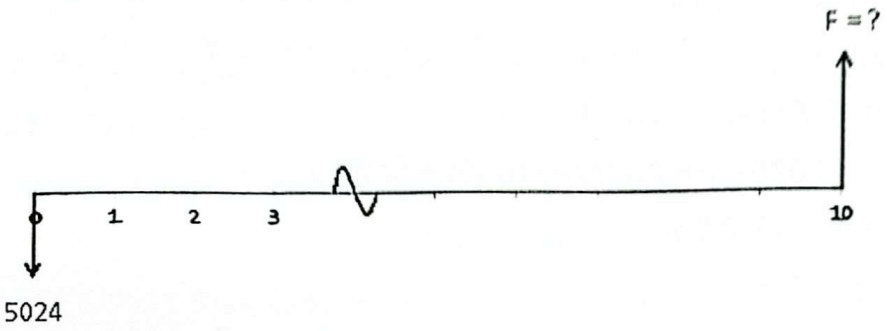
= 8000 × 0.0628

= Rs. 502.40/-

CE (i) = A/i

= 502.40/0.10

= Rs. 5024/- Capitalized equivalent amount



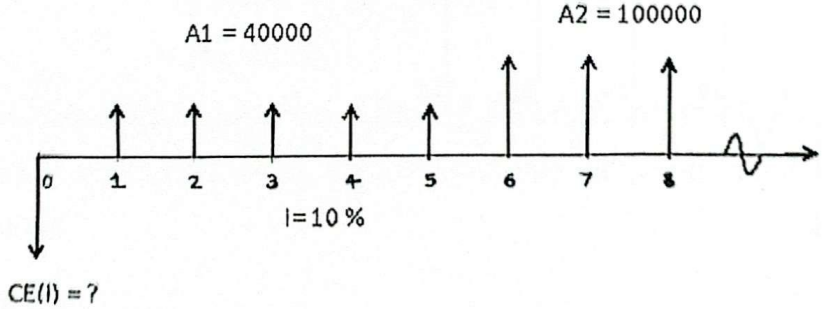
$F = P (1+i)^n$

$= 5024 (1+0.10)^{10}$

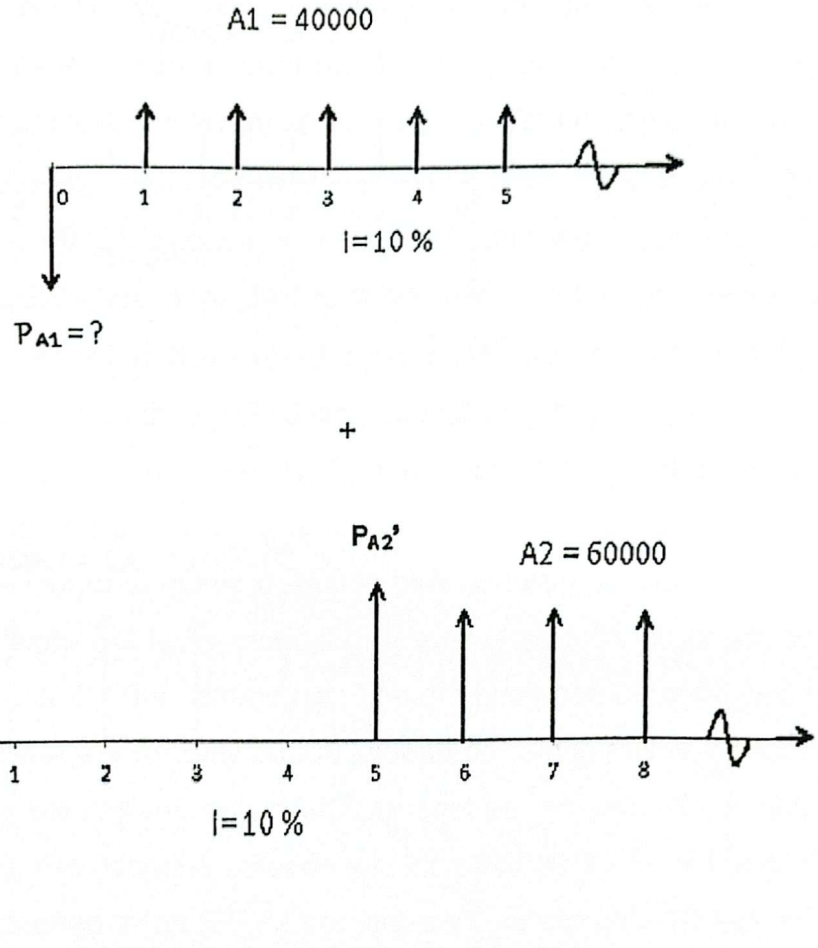
= Rs. 13030.96/-

12. A donor wishes to endow a scholarship to a certain university in the name of a certain professor. The scholarship is to provide Rs. 40000/- per year for the for the first 5 years and Rs. 100000/- per year thereafter. If the university expects to be able to earn 10 % per year on the endowment how much must the donor give now if the first scholarship is to be given 1 year from now?

Soln.:



The above CFD can be split into the following 2 CFDs and is equivalent to the sum of these CFDs..



$A_2 = 100000 - 40000 = 60000$

$CE(i) = P_{A1} + P_{A2}$

$$= \frac{A_1}{i} + P'_{A2}\left(\frac{P}{F}, 10, 5\right)$$

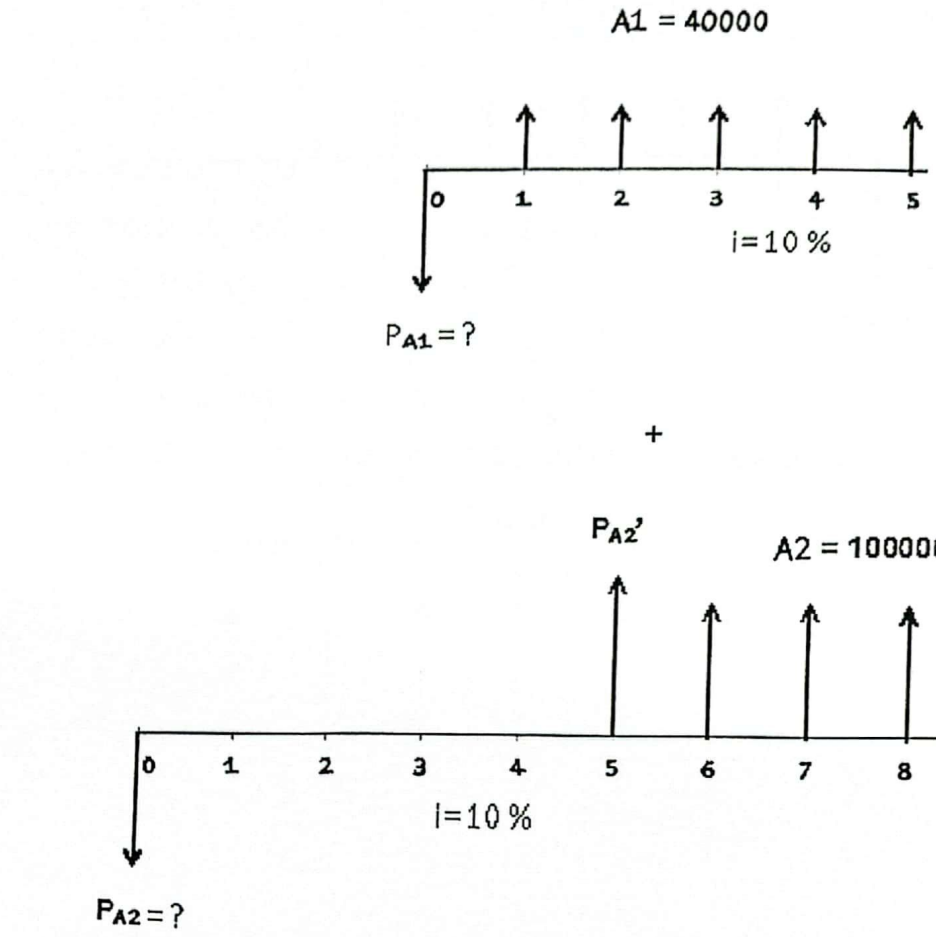
$$= \frac{A_1}{i} + \frac{A_2}{i}\left(\frac{P}{F}, 10, 5\right)$$

$$= \frac{40000}{0.1} + \frac{60000}{0.1}(0.6209)$$

= Rs. 772540/-

Alternate solution:

The above CFD can also be split as follows,



$CE(i) = P_{A1} + P_{A2}$

$$= A_1\left(\frac{P}{A}, 10, 5\right) + P'_{A2}\left(\frac{P}{F}, 10, 5\right)$$

$$= 40000 \times 3.791 + \frac{A_2}{i}\left(\frac{P}{F}, 10, 5\right)$$

$$= 151640 + \frac{100000}{0.1}(0.6209)$$

= Rs. 772540/-

Unsolved problems:

1. A couple is trying to decide between purchasing a house and renting one. They can purchase a new house with a down payment of Rs. 15000/- and a monthly payment of Rs. 750/- beginning one month from now. Taxes and insurance are expected to amount Rs. 100 per month. In addition they expect to paint the house every 4 years at a cost of Rs. 6000/-. Alternatively they can rent a house for Rs. 600/- per month payable in advance at the beginning of month with Rs. 6000/- deposit which will be returned when they vacate the house. The utilities are expected to average Rs. 135/- per month, whether they purchase or rent. If they expect to be able to sell the house in 6 years for Rs. 10000/- more than they paid down, should they buy a house or rent one if the interest rate is a nominal 12 % per year compounded monthly. Use present worth analysis.
2. An automobile owner is trying to decide between purchasing new radial tyres or having the worn out tyres recapped. Radial tyres for the car will cost $ 85 each and will last 60000/- kilometres. The old tyres can be recapped for $ 35 each but they will last for only 30000 kilometres. Since this is a second hand car it probably will register only 10000 kilometres per year. If the radial tyres are purchased, the gasoline mileage will increase by 10 %. If the cost of the gasoline is assumed to be $ 0.42 per litre and the car gets 10 kilometres per litre, what type of tyres should be purchased if the interest rate is 12 % per year. Use the present worth method and assume that the salvage value of the tyres is zero.
3. What is the capitalized cost of Rs. 75000/- now, Rs. 50000/- five years from now and a uniform annual amount of Rs. 700 per year for year 10 and every year thereafter if the interest rate is 8 % per year?
4. Calculate the capitalized cost of a project that has an initial cost of $ 150000/- and an additional investment of $ 50000/- after 10 years. The annual operating cost will be $ 5000/- for the first 4 years and $ 800/- thereafter. In addition there is expected to be a recurring major rework cost of $ 15000/- every 13 years. Assume that i = 15 % per year.
5. There are two plans A and B for a building construction. Plan A involves an initial investment of Rs. 10 lacs of which Rs. 8 lacs is to be invested on a permanent construction and Rs. 2 lacs on an investment that requires renewal

every 20 years hence. The annual upkeep cost is Rs. 10000/-. Plan B involves an initial investment of Rs. 10 lacs. A further investment of Rs. 3 lacs will be necessary 10 years hence. This construction is of permanent nature. The annual upkeep cost is Rs. 3000/- per year. Assuming 10 % interest rate which plan should be selected?

6. A city that is attempting to attract a professional football team is planning to build a new football stadium costing $ 12000000 (12 million). Annual upkeep is expected to amount to $ 25000/- per year. In addition the artificial turf will have to be replaced every 10 years at a cost of $ 150000/-. Painting every 5 years will cost $ 65000/-. If the city expects to maintain the facility indefinitely what will be its equivalent uniform annual cost? Assume i= 12 % per year.

5.0 REPLACEMENT ANALYSIS

THIS CHAPTER COVERS THE FOLLOWING:

- INTRODUCTION.
- CASH FLOW APPROACH.
- ROLE OF SUNK COST IN REPLACEMENT ANALYSIS.
- COMPARATIVE USE VALUE OF AN ASSET.
- STUDY PERIOD APPROACH.

5.1 Introduction:

Replacement analysis is concerned with the question, when is it time to replace an existing piece of equipment with a new one? The answer to this is not necessarily ``When the old one wears out.'' It is possible, after all, to keep a 1957 Chevy running up to the present day, if you're prepared to spend enough time and money on it. Conversely, it may be worth replacing an IBM XT with a Pentium well before the former breaks down. We therefore distinguish between the *physical life* of an asset and its *economic life.* The physical life will sometimes be well-defined, though in some cases, like the 1957 Chevy, we have to set an arbitrary limit on how long we're prepared to keep an obsolete asset in service. The economic life is the time after which we save money by replacing the asset. Thus, the physical life is always greater than or equal to the economic life.

The most effective way to think about the replacement interval is to consider the equivalent uniform annual cost of the asset over its life, taking various different durations for its life. The EUAC is usually made up of two factors: the initial cost of the asset, spread out over its life (the `capital recovery' annuity); and the annual cost of repairs and maintenance. The capital recovery should include a deduction for the salvage value of the asset, if any. The annual cost of repairs should, if appropriate, include a contribution representing the cost of correcting any defects in the product resulting from the use of an outmoded machine. The first factor will go down as we consider longer lifetimes, while the second will usually go up. The sum of the two will therefore (usually) have a minimum value. This minimum value is the minimum EUAC, and in most cases this will correspond to the economic life of the asset.

In many cases, we consider replacing an existing asset (the `defender') with another asset (the `challenger'). The simplest case, however, occurs when we are considering getting rid of an asset and not replacing it. Examples might include a video game machine at a corner store, or a piece of equipment that allows a company to service a particular small section of their customer base -- for example, a machine for processing Cibachrome film at a photo shop. In this simplest case, we calculate the present worth of the asset and the cash flows associated with it for a range of possible lifetimes. If any of these present worths are positive, we should retain the asset; otherwise we should get rid of it.

Following are the reasons for replacement of assets

- Physical impairment (Deterioration)
    - Continued use results in the less efficient operation, cost increases and accident occurs.
- Damage or design changes
- Inadequacy
- Technology – changes in technology
- Obsolescence
    - Obsolescence is of two types, functional and economical obsolescence.
- Financing
    - Economic opportunity changes
        - May involve tax considerations.
        - Ex. Rental (lease) of assets become more attractive than ownership.

The replacement of assets often represents economic opportunity for the firm. We compare the two alternatives:

- The asset that we own: The Defender
- The Asset that we might buy to replace it: The Challenger

Factors to consider (or ignore)

- Sunk Costs

  Sunk cost = Present book value – Present market value
- Existing Asset Value and the outsider viewpoint

- Income Tax Considerations
- Economic Life of the challenger and the defender

The Economic Life of an asset is

The period of time that minimizes the net annual cost (NAC) for the investment (when it primarily consists of costs)

or

The period of time that maximizes the net annual worth (NAW) for the investment (when it consists of costs and revenues)

The existing old asset being considered as the asset to be replaced is **DEFENDER**. The asset proposed to be the replacement is **CHALENGER**. Because of the economic characteristics of the defender and challenger, are usually so dissimilar that special attention is required when these two options are considered. One obvious feature of replacement alternatives is that the duration and magnitude of cash flows for old existing assets and new assets are quite different. New assets have higher capital costs. The reverse is true for the old existing assets due to replacement. In addition, the remaining life of an asset being considered for replacement is usually short and the future of the asset can be estimated with relative uncertainty.

5.2 Cash flow approach (Only for equal life)

The replacement problem can also be solved by cash flow approach. But this approach is applicable only when the defender and challenger have the same equal life. This approach is based on the fact that if the challenger is selected, the defender's present market value is a cash inflow to the challenger and if the defender is selected there is no actual expenditure of cash to the business organization. Hence, in this approach, the defender's first cost is taken as zero and the market value of the defender is subtracted from the challenger's first cost.

5.3 Role of including sunk cost in replacement analysis:

In spite of the fact that sunk cost cannot be recovered, charging the sunk cost of the defender to the cost of the contemplated replacement can lead to erroneous conclusions.

5.4 Comparative use value of the asset:

Use value of the old asset, is the balance of the investment remaining in it, at a point of time when it is compared with the new asset. Comparative use value of the old asset or the asset to be replaced is the value, which results in the same total equivalent annual cost as that of the challenger. The relation used for calculating the comparative use value say 'X' for the defender is given as,

$$EUAC_{defender} = EUAC_{challenger}$$

Wherein, the unknown value 'X' is calculated by substituting it as the defender's first cost.

If the comparative use value of the defender is less than its present market value then a decision to replace the defender is taken. On the other hand, if it is more than the present market value of the defender, then the defender should be retained.

5.5 Study period approach – for unequal lives:

There are two kinds of analysis here:

- Analysis of the replacement problem, by recognizing the unused value remaining in the asset at the end of the study period.
- Analysis of the replacement problem, by not recognizing the unused value remaining in the asset at the end of the study period.

Solved problems:

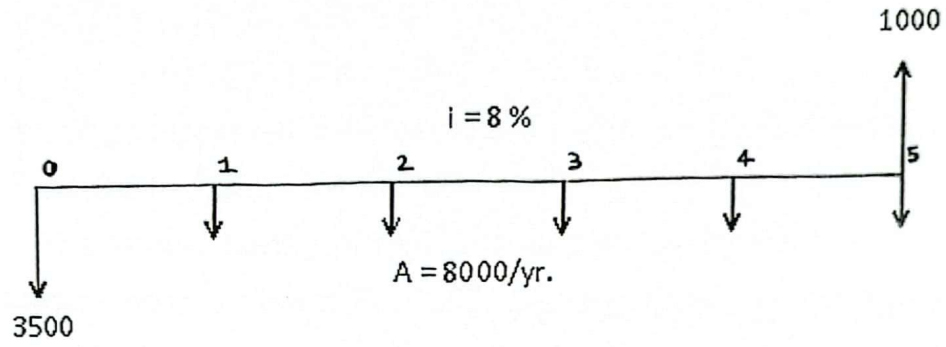
1. A company purchased machine 'X', a year ago for Rs. 8500/- with the following characteristics. Estimated life = 6 years, salvage value = Rs. 1000/-, operating expenses = Rs. 8000/- per year. At the end of the first year, a salesman offers machine 'Y' for Rs. 11500/- which has an estimated life of 5 years, a salvage value of Rs. 1500/- and an operating cost of only Rs. 5500/- per year due to improvement. The salesman offers Rs. 3500/- for machine 'X', if machine 'Y' is purchased. This appears low to the company but the best offer received elsewhere is only Rs. 3000/-. Assume an interest rate of 8 % and determine the best course of action by taking outsider's point of view.

Soln.: The outsider will consider the following data for machine 'X' and machine 'Y' (Outsider point of view)

	Defender (Machine 'X')	Challenger (Machine 'Y')
First cost (P)	3500	11500
Annual operating cost (AOC)	8000	5500
Salvage value (F)	1000	1500
Service life (n Yrs)	6-1 = 5 years	5 years

Soln.: **Machine 'X'** :

Assume cost +ve and revenue –ve.

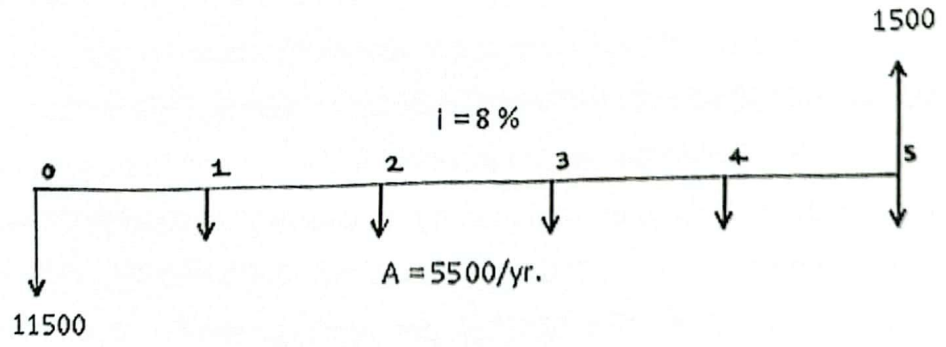


EUAC (X) = 3500 (A/P, 8, 5) + 8000 – 1000 (A/F, 8, 5)

= 3500 × 0.25046 + 8000 – 1000 × 0.17046

= Rs. 8706.15/-

Machine 'Y'



EUAC (Y) = 11500 (A/P, 8, 5) + 5500 – 1500 (A/F, 8, 5)

= 11500 × 0.25046 + 5500 – 1500 × 0.17046

= Rs. 8124.60/-

It is economical to replace machine 'X' by machine 'Y' which will result in a saving of

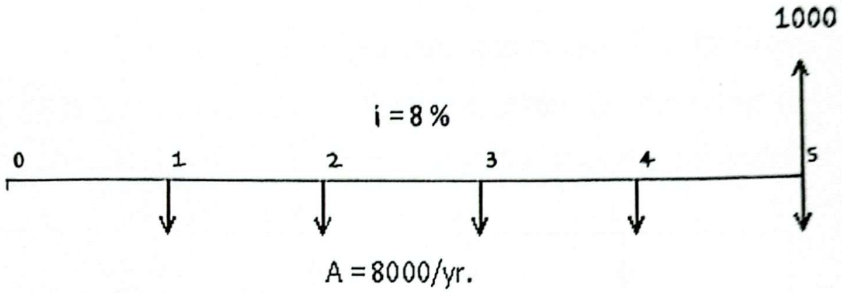
= 8706.15 – 8124.60

= Rs. 581.55/- per year by replacement.

2. Solve the same problem number 1 by Cash flow approach.

Soln: **Cash flow approach:**

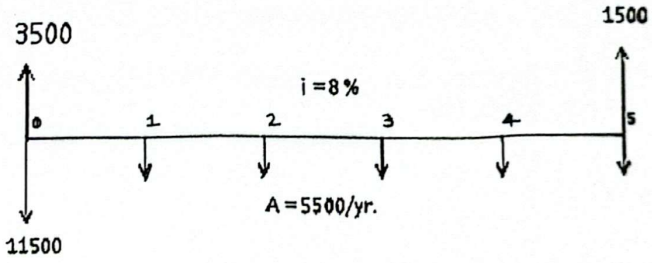
EUAC of machine 'X'



EUAC (X) = 8000 – 1000 (A/F, 8, 5)

= Rs. 7829.54/-

EUAC of machine 'Y'



EUAC (Y) = (11000-3500) (A/P, 8, 5) + 5500 – 1500 (A/F, 8, 5)

= Rs. 7247.99/-

Hence, machine 'X' is to be replaced by machine 'Y', which would result in annual savings of:

= 7829.54 – 7247.49 = Rs. 581.55/- per year.

3. In **Problem No. 1**, if the challenger has a life of 10 years, determine the best course of action.

Soln.: **Machine 'X' Defender**

P = 3500, AOC = 8000, n = 5, and F = 1000

Machine 'Y' Challenger

P = 11500, AOC = 5500, n = 10, and F = 1500

EUAC (X) = Rs. 8706.15/- Per year

EUAC (Y) = 11500 (A/P, 8,10) + 5500 – 1500 (A/F, 8, 10)

= Rs. 7110.30/-

Replacing machine 'X' by machine 'Y', saving per year is:

= 8706.15 – 7110.30 = Rs. 1595.85/- per year

Cash flow approach:

EUAC (X) = Rs. 7829.54/-

EUAC (Y) = (11500 – 3500) (A/P, 8, 10) + 5500 – 1500 (A/F, 8, 10)

= Rs. 6588.70/-

Replacing machine 'X' with machine 'Y', will result in a cost saving of:

= 7829.54 – 6588.70 = Rs. 1240.84/-

The annual saving of replacing machine 'X' by machine 'Y' considering outsider's point of view is Rs. 1595.85/- per year and by cash flow approach is Rs. 1240.84/- per year (sometimes the decision may be reverted). The error is due to the market value of machine 'X', i.e., Rs. 3500/-. It is annualized over the life of 10 years. Actually it is to be annualized over a period of 5 years only. Hence cash flow approach cannot be used when the defender and challenger have unequal lives. Only outsider's point of view is to be considered for unequal lives.

4. In Problem no. 1, if the present book value of machine 'X' is Rs. 7250/- and if the company decides to recover the sunk cost it has incurred in machine 'X' by machine 'Y', what error in equivalent annual costs will result I making the comparison of financial desirability of the two machines?

Soln.: **Machine 'X' Defender**

P = 3500, AOC = 8000, n = 5, and F = 1000

Machine 'Y' Challenger

P = 11500, AOC = 5500, n = 5, and F = 1500

EUAC using outsider's point of view:

EUAC (X) = Rs. 8706.15/-

EUAC (Y) = Rs. 8124.60/-

Hence the decision is to replace machine 'X' by machine 'Y' which will result in an annual cost saving of:

= 8706.15 – 8124.60 = Rs. 581.55/-

If **Sunk cost** is considered:

Sunk cost of machine 'X' = Present book value – Present market value

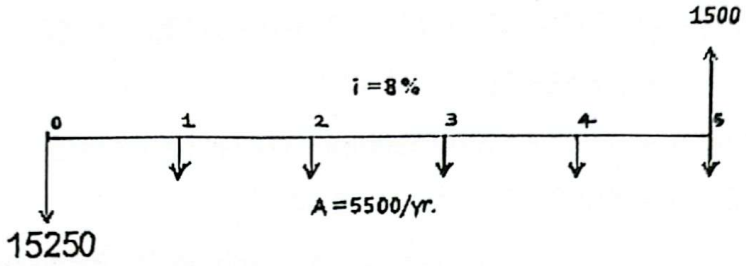
= 7250 – 3500

= Rs. 3750/-

By adding this sunk cost of machine 'X' to the first cost of machine 'Y', the challenger, following calculations of EUAC are made.

EUAC (X) = Rs. 8706.15/-

CFD of machine 'Y' will be:



EUAC (Y) = (11500 + 3750) (A/P, 8, 5) + 5500 – 1500 (A/F, 8, 5)

= Rs. 9063.83/-

When the sunk cost is considered with the first cost of the challenger (machine 'Y'), the conclusion is, it is economical to continue with the service of machine 'X', or defender. By doing so, the annual savings would be:

= 9063.83 – 8706.15 = Rs. 357.68/- per annum

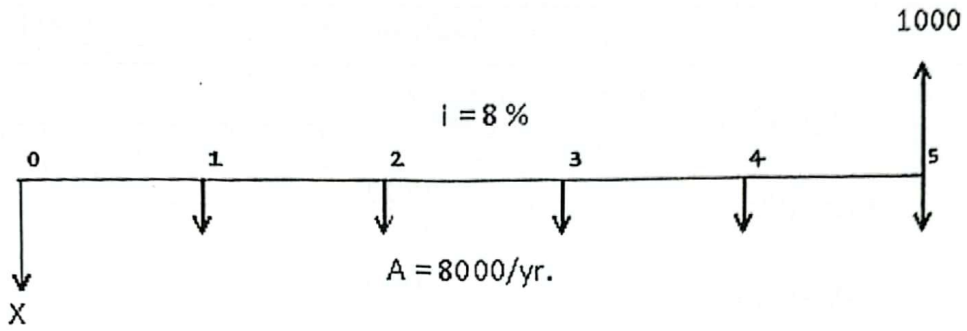
It can be seen that, the decision has been reversed when the sunk cost is added to the challenger's first cost.

By adding the sunk cost to the first cost of the challenger an attempt has been made to cover up the mistakes of past estimation, and capital to be recovered every year, would be larger due to its increased first cost, which in turn leads to wrong decision. Hence by using the outsider's point of view it is economically sound decision to replace X by Y resulting in an annual saving of Rs. 581.55/- per year.

5. A company purchased a truck three years ago for $ 12000/- with an estimated life of 8 years, salvage value of $ 1600/- and annual operating cost of $ 3000. The current book value is $ 8100. A challenger is now offered for $ 11000/-

Salvage value (F)	1000	1500
Service life (n Yrs)	6-1 = 5 years	5 years

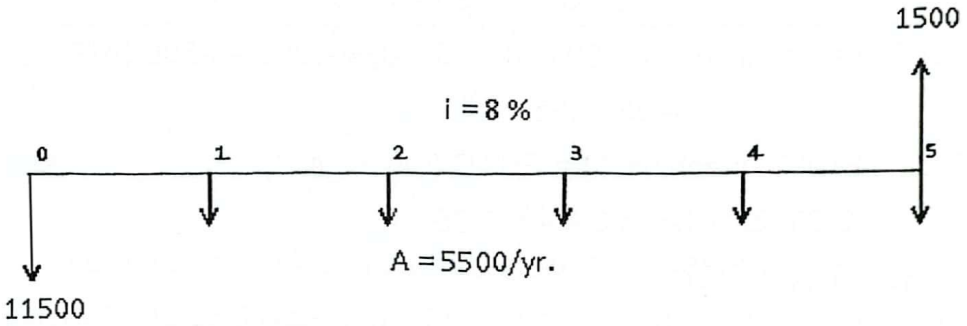
EUAC (machine X) taking P as 'X'



EUAC (X) = X (A/P, 8, 5) + 8000 – 1000 (A/F, 8, 5)

= 0.2505X + 7829.5

EUAC (machine Y)



EUAC (Y) = 115000 (A/P, 8, 5) + 5500 – 1500 (A/F, 8, 5)

= Rs. 8125/-

Setting EUAC (X) = EUAC (Y)

0.2505X + 7829.5 = 8125

Hence, X = 1179.60/-

To get same annual cost as that of the challenger, the nature of the capital investment for the defender should be Rs. 1179.60/- at now. But someone is offering Rs. 3500/- for the defender which is much higher than Rs. 1179.60/-. Hence it is economical to replace the defender.

8. Machine 'A' was purchased 4 years ago for Rs. 11000/-. It is estimated to have a life of 10 years and a salvage value of Rs. 1000/- at the end of its life. Its operative expenses had been found to be Rs. 3500/- per year. Presently a salesman is offering machine 'B' for Rs. 12000/-. Its life is estimated to be 10 years and its salvage value at the end of its life is estimated to be Rs. 1500/-,

and a trade in value of $ 7500/- for the old truck. The company estimates that challenger life is 10 years, salvage value is at $ 2000/- and annual operating cost at $ 1800/-. New estimates for the old truck are made as follows: remaining life is 3 years, salvage value $ 2000/- and same operating costs. The MARR is 10 %. Determine,

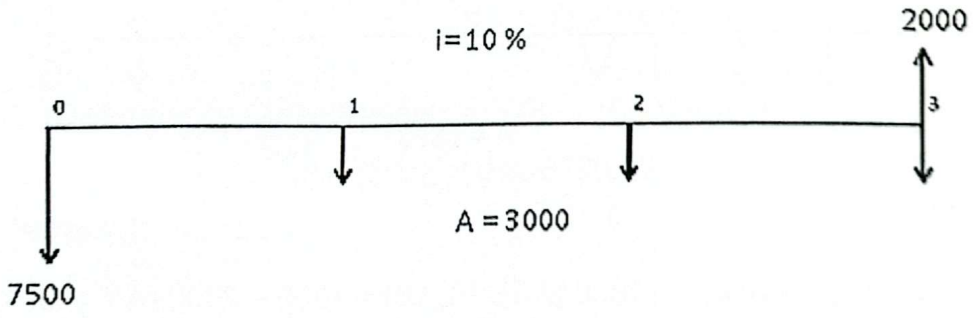
a. What values should be used for P, AOC, SV and n for conducting replacement analysis.
b. Take the consultant's point of view and determine the best course of action.
c. What error in equivalent annual costs will result if the company adds the sunk cost it has suffered to the new truck, while making the comparison of the financial desirability of the two trucks.

Soln.: the outsider will consider the following information for the two trucks:

a.

	Old truck (Defender)	New truck (Challenger)
P (Rs.)	7500	11000
SV (Rs.)	2000	2000
AOC (Rs.)	3000	1800
n (years)	3	10

b. **EUAC (Old truck):**



EUAC = 7500 (A/P, 10, 3) + 3000 – 2000 (A/F, 10, 3)

= Rs. 5411.60/-

operating expenses are Rs. 2000/- per year. The salesman offers to take the old machine for the trade in value of Rs. 3000/-. If the company's MRR is 15 %, find the comparative use value of machine 'A'.

Soln.:

	Machine 'A'	Machine 'B'
P (Rs.)	X	12000
AOC (Rs.)	3500	2000
SV (Rs.)	1000	1500
N (Years)	6	10

EUAC (machine A) = X (A/P, 15, 6) + 3500 – 1000 (A/F, 15, 6)

= 0.2642X + 3385.8

EUAC (machine B) = 12000 (A/P, 15, 10) + 2000 – 1500 (A/F, 15, 10)

= 4317.65

Setting EUAC (machine A) = EUAC (machine B)

0.2642X + 3385.8 = 4317.65

Hence, X = 3527/-

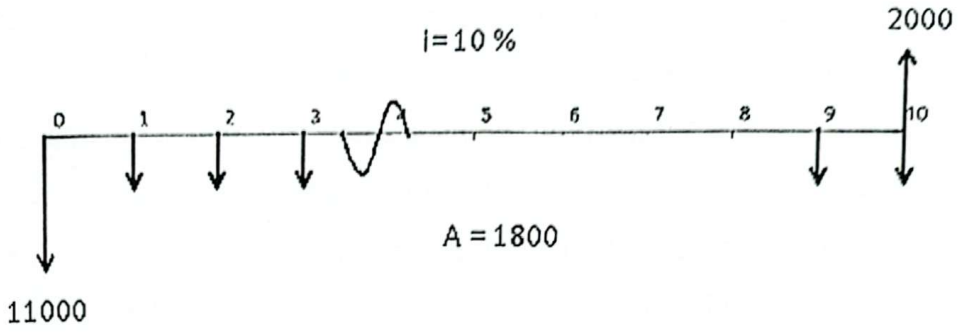
Offer price of machine 'A' is Rs. 3000/- only. Comparative use value of machine 'A' with machine 'B' is Rs. 3527/-. Hence machine 'A' may be disposed off and purchase machine 'B'

9. A certain operations now being carried on with machine 'E' whose present salvage value is $ 2000/-. The future life of machine E is estimated at 5 years, at the end of which its salvage value is estimated as Zero. Operating costs with machine E is estimated at $ 1200/- per year. it is expected that machine E will be replaced after 5 years by machine F whose initial cost, life final salvage value and annual operating costs are estimated to be $ 10000/-, 15 years, Zero and $ 600/- respectively. The desirability of replacing machine E with machine G is being considered. Machine G's initial cost, life, salvage value and annual operating costs are estimated to be $ 8000/-, 15 years, Zero and $ 900/- respectively. The interest rate is 10 % per annum. Using a study period approach of 15 years conduct the replacement analysis a) recognizing the unused value b) not recognizing the unused value.

EUAC (New truck):

EUAC = 11000 (A/P, 10, 10) + 1800 – 2000 (A/F, 10, 10)

= Rs. 3464.75/-



Hence it is economical to replace the old truck. By doing so the annual saving would be:

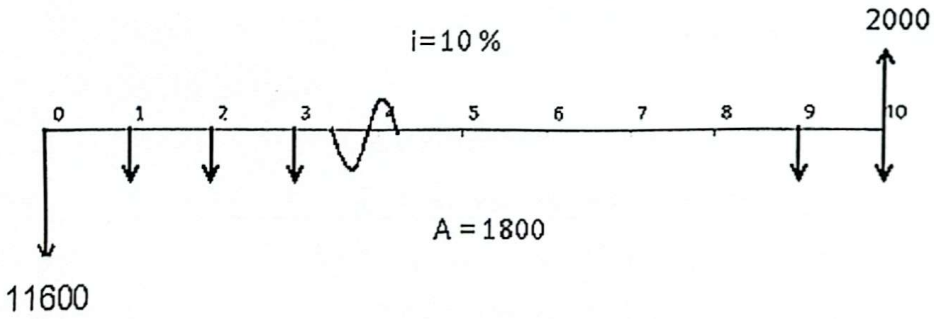
= 5411.60 – 3464.75 = Rs. 1946.85/-

c. Sunk cost = present book value – present market value

= 8100 – 7500

= Rs. 600/-

P for the new truck is = 11000 + 600 = 11600/-



EUAC (New truck) = 11600 (A/P, 10, 10) + 1800 – 2000 (A/F, 10, 10)

= Rs. 3562.40/-

The decision is till the same, but the error in EUAC is

= 3562.40 – 3464.75 = Rs. 97.65/-

The annual savings now is

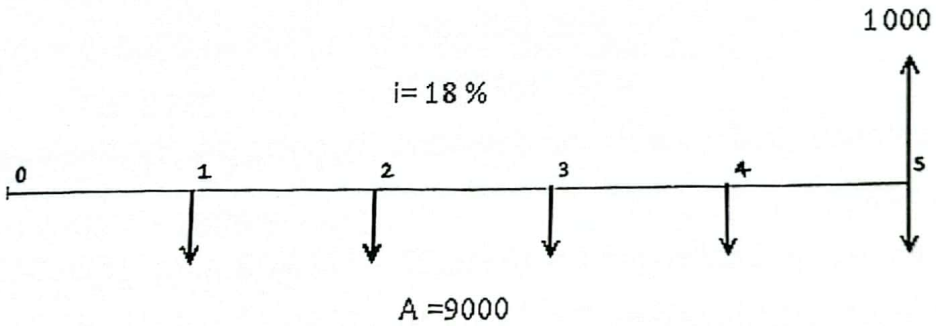
= 5411.60 – 3562.40 = Rs. 1849.20/- per annum

6. An 8 year old asset may be replaced with either of the two new assets. Current data for each alternative is given below. Using the cash flow approach and an interest rate of 18 % per year determine the best course of action.

	Current asset	Challenger 1	Challenger 2
First cost (Rs.)	-	30000	54000
Defender trade in (Rs.)	-	10500	7500
Annual cost (Rs.)	9000	4500	3600
Salvage value (Rs.)	1500	3000	1500
Life (Years)	5	5	5

Soln.: **Cash flow approach**

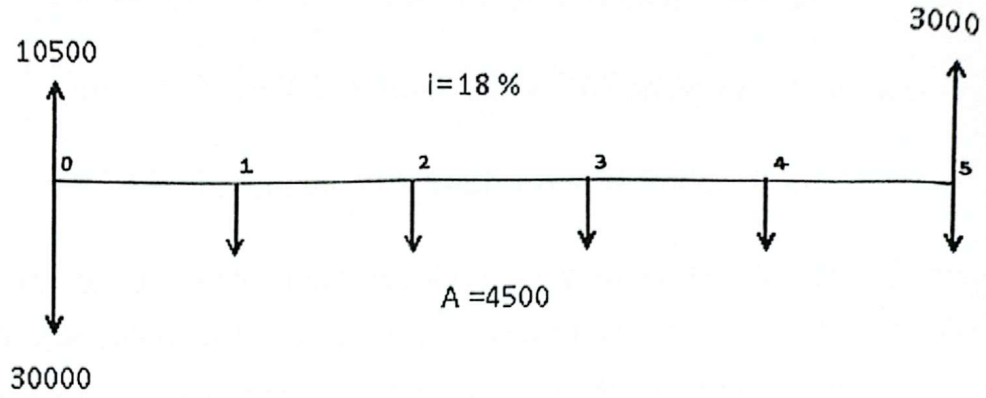
Defender:



EUAC (Defender or Current asset) = 9000 – 1500 (A/F, 18, 5)

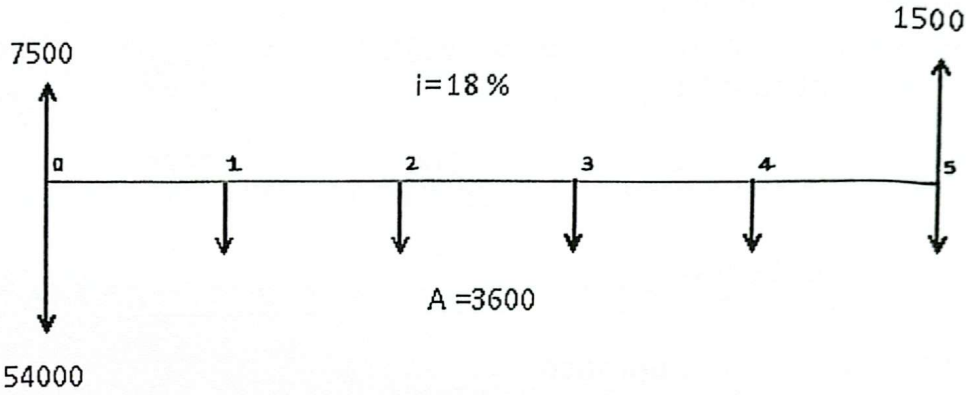
= Rs. 8790.33/-

Challenger 1:



EUAC (Challenger 1) = (30000 – 10500) (A/P, 18, 5) + 4500 –
1500 (A/F, 18, 5)
= Rs. 10316.37/-

Challenger 2:



EUAC (Challenger 2) = (54000 – 7500) (A/P, 18, 5) + 3600 –
1500 (A/F, 18, 5)
= Rs. 18260.10/-

It is economical to retain the defender. By doing so, the saving per year is as follows:

Defender v/s Challenger 1 = 10316.37 – 8790.33 = Rs. 1526.04/-

Defender v/s Challenger 2 = 18260.10 – 8790.33 = Rs. 9469.77/-

7. Calculate the comparative use value in Problem No. 1 and evaluate the assets.

Soln.:

	Defender (Machine 'X')	Challenger (Machine 'Y')
First cost (P)	3500	11500
Annual operating cost (AOC)	8000	5500

operating expenses are Rs. 2000/- per year. The salesman offers to take the old machine for the trade in value of Rs. 3000/-. If the company's MRR is 15 %, find the comparative use value of machine 'A'.

Soln.:

	Machine 'A'	Machine 'B'
P (Rs.)	X	12000
AOC (Rs.)	3500	2000
SV (Rs.)	1000	1500
N (Years)	6	10

EUAC (machine A) = X (A/P, 15, 6) + 3500 – 1000 (A/F, 15, 6)

= 0.2642X + 3385.8

EUAC (machine B) = 12000 (A/P, 15, 10) + 2000 – 1500 (A/F, 15, 10)

= 4317.65

Setting EUAC (machine A) = EUAC (machine B)

0.2642X + 3385.8 = 4317.65

Hence, X = 3527/-

Offer price of machine 'A' is Rs. 3000/- only. Comparative use value of machine 'A' with machine 'B' is Rs. 3527/-. Hence machine 'A' may be disposed off and purchase machine 'B'

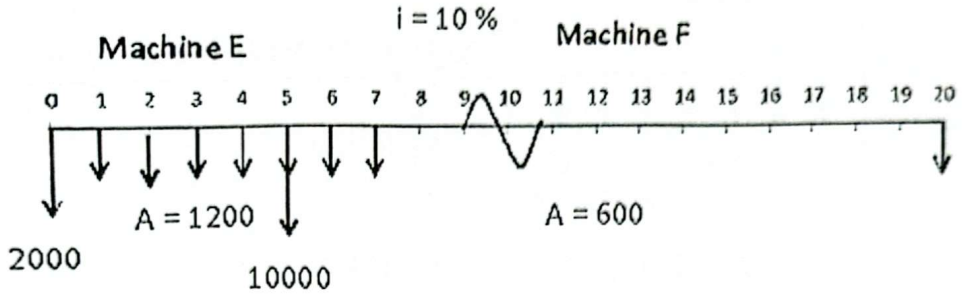
9. A certain operations now being carried on with machine 'E' whose present salvage value is $ 2000/-. The future life of machine E is estimated at 5 years, at the end of which its salvage value is estimated as Zero. Operating costs with machine E is estimated at $ 1200/- per year. it is expected that machine E will be replaced after 5 years by machine F whose initial cost, life final salvage value and annual operating costs are estimated to be $ 10000/-, 15 years, Zero and $ 600/- respectively. The desirability of replacing machine E with machine G is being considered. Machine G's initial cost, life, salvage value and annual operating costs are estimated to be $ 8000/-, 15 years, Zero and $ 900/- respectively. The interest rate is 10 % per annum. Using a study period approach of 15 years conduct the replacement analysis a) recognizing the unused value b) not recognizing the unused value.

Soln.: the problem can be solved by dividing into 2 plans, plan 1 and plan 2.

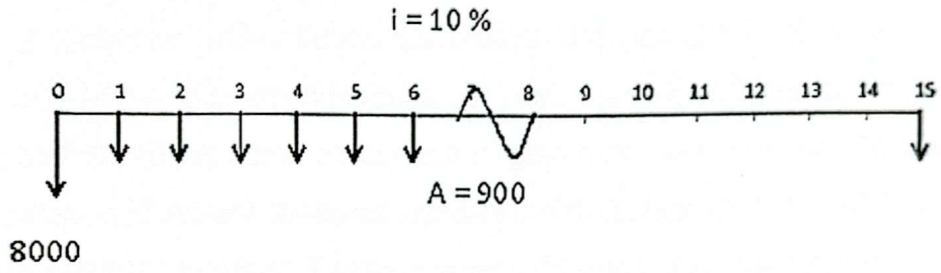
Plan 1, is retaining machine E and purchasing machine F after 5 years.

Plan 2, is purchasing machine G now.

CFD for Plan 1



CFD for Plan 2



Under Plan 1, the study period of 15 years includes 5 years of machine E and 10 years of machine F whose life extends 5 years beyond the study period. Under Plan 2, life of the machine G and the study period coincide.

a) Analysis by recognizing the unused value

By conducting the analysis by recognizing the unused value, capital recovery cost is calculated for the entire service life of the asset (machine F) even though the study period is shorter than the asset's service life (machine F only).

CR (i) = (P-F) (A/P, i, n) + Fi

= (10000-0) (A/P, 10, 15) + 0 × 0.10

= Rs. 1314.70/-.

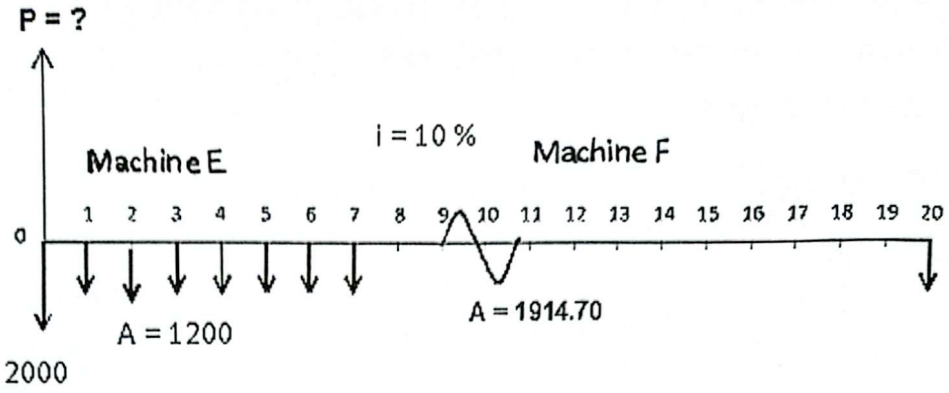
EUAC (machine F) over the useful life of 15 years

= CR (i) + AOC

= 1314.70 + 600

= Rs. 1914.70/-

For a study period of 15 years, the CFD for plan 1 is shown below:



Pw (Plan 1) = 2000 + 1200 (P/A, 10, 5) + 1914.70 (P/A, 10, 10) (P/F, 10, 5)

= Rs. 13853.42/- (Cost value)

Pw (Plan 2) = 8000 + 900 (P/A, 10, 5)

= 14845.40/- (Cost value)

On the basis of Pw, plan 1 should be chosen, i.e, machine E should be retained for another 5 years then replaced with machine F. Machine F has 5 years of unused value remaining in it since its first cost is distributed over its life of 15 years and only 10 years is considered for the analysis purposes.

The unused value remaining in machine F at the end of the study period of 15 years is:

= CR (i) (P/A, 10, 5)

= 1314.70 (3.791)

= Rs. 4984/-

b) Analysis by not recognizing the unused value

While conducting the analysis by not recognizing the unused values, the capital recovery cost is calculated assuming that the entire capital is to be recovered within the study period itself even though the service life of the asset is greater than the study period.

CR (i) for Plan 1

CR (i) = (P - F) (A/P, i, n) + Fi

= (10000 – 0) (A/P, 10, 10) + 0 × 0.10

= Rs. 1627.50/-

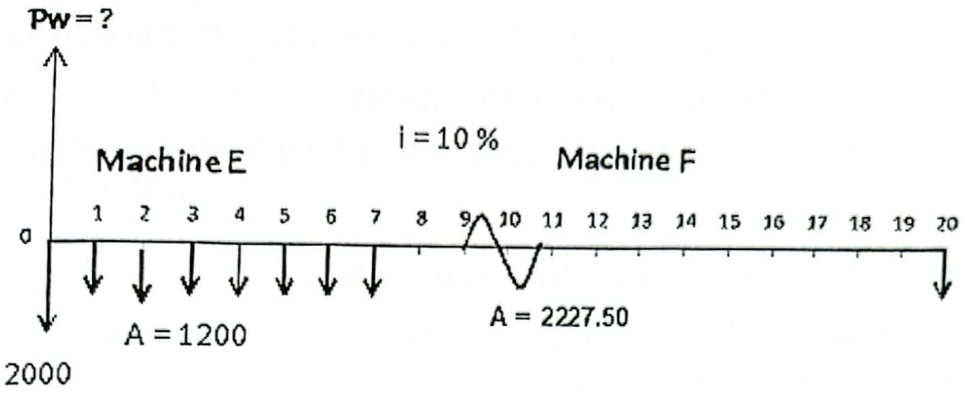
EUAc of machine F over the useful life

= CR (i) + AOC

= 1627.50 + 600

= Rs. 2227.50/-

CFD will be as follows:



Pw (Plan 1) = 2000 + 1200 (P/A, 10, 5) + 2227.50 (P/A, 10, 10) (P/F, 10, 5)

= Rs. 15047/- (Cost value)

Pw (Plan 2) = Rs. 14845.40/- (Cost value)

On the basis of Pw, here, Plan 1 is rejected and Plan 2 is selected. i.e., Machine G should be purchased.

Unsolved problems:

1. An 8 year old asset may be replaced with either of the two new assets. Current data for each alternative is given below. Using outsider's point of view and an interest rate of 18 % per year determine the best course of action.

	Current asset	Challenger 1	Challenger 2
First cost (Rs.)	-	30000	54000
Defender trade in (Rs.)	-	10500	7500
Annual cost (Rs.)	9000	4500	3600
Salvage value (Rs.)	1500	3000	1500
Life (Years)	5	5	5

2. A soft drink bottler purchased a bottling machine 2 years ago for $ 16800/-. At that time it was estimated to have a service life of 7 years with no salvage value. Annual operating cost of the machine accounted to $ 4400/-. A new bottling machine is being considered which would cost $ 20000/- but would match the output of the old machine for an annual operating cost of $ 1800/-. The new machine's service life is 5 years with no salvage value. An allowance

of $ 5000/- would be made for the old machine on the purchase of the new machine. The interest rate is 10 %.

a. Take the outsider's point of view and calculate the equipment's annual cost for each of the two alternatives.
b. What is the use value of the old machine in comparison with the new machine?
c. Should the new machine be purchased? Why?

3. A municipality 3 years ago purchased a pump for its sewage treatment plant for Rs. 18000/-. This pump had an annual operating costs of Rs. 10000/- and these are expected to continue. This pump is expected to continue to operate satisfactorily for 5 additional years at which time pump will have negligible salvage value. A new pump is available for Rs. 27000/- with an estimated life of 5 years and negligible salvage value at the end of its life and annual operating costs of Rs. 4000/-. The old pump can be sold for Rs. 2000/-. The interest rate is 10 %.

a. What error in equivalent annual costs will result if the municipality erroneously adds the sunk cost it has suffered to the cost of new pump in making a comparison of the financial desirability of the two pumps.
b. Calculate comparative use value of the old pump.

4. A firm owns 2 vans which are deteriorating faster than expected. Owning the vans or leasing on a yearly basis are the replacement options. The two vans were purchased 2 years back for Rs. 480000/- each. The company plans to keep the vans for 10 more years. Fair market value for a 2 year old van is Rs. 336000/- and for a 12 year old van is Rs. 64000/-. Annual fuel, maintenance, tax etc. costs Rs. 96000/- per year (year end payment) with annual operating charges of Rs. 112000/-. Should the company lease its vans if a 12 % per year rate of return is required?

5. Three years ago the city administration purchased a new fire truck. Due to expanded growth in a certain portion of the city, new fire fighting capacity is needed. An additional identical truck can be purchased now or a double capacity truck can replace the presently owned truck. Data for each asset is given in the table below. Compare the assets at i = 12 % per year using a) 12 year study period b) a 9 year study period which the city management believes to be more realistic due to population growth.

	Presently owned	New purchase	Double capacity
P (Rs.)	178500	203000	252000
AOC (Rs.)	5250	5250	8750
Trade in value (Rs.)	63000	-	-
SV (Rs.)	10 % of P	12 % of P	10 % of P
n (Years)	12	12	12

6. Five years ago a conveyor system was installed in a manufacturing plant at a cost of $ 27000/-. It was estimated that the system which is still in good condition would have a useful life of 20 years. Annual operating costs are $ 1350/-. The numbers of parts to be transported have doubled and will continue at the higher rate for the rest of the life of the system. An identical system can be installed for $ 22000/- or a system with a 20 year life and double the capacity can be installed for $ 31000/-. Annual operating cost is expected to be $ 2500/-. The present system can be sold for $ 6500/-. Either of the three systems will have a salvage value at retirement of 10 % of original cost. The MARR is 12 %. Compare the two alternatives of obtaining the required services on the basis of equivalent annual cost over a 15 year study period, recognizing any unused value remaining in the system at the end of that time.

6.0 DEPRECIATION

THIS CHAPTER COVERS THE FOLLOWING:

- MEANING OF DEPRECIATION.
- BOOK VALUE OF AN ASSET.
- DEPRECIATION METHODS.
  - STRAIGHT LINE METHOD.
  - DECLINING BALANCE METHOD.
  - SUM OF THE YEARS DIGITS METHOD.
  - SINKING FUND METHOD.
  - SERVICE OUTPUT METHOD.

6.1 Meaning:

Depreciation may be defined as lessening in value of a physical asset with the passage of time.

With the possible exception of land, this phenomenon is a characteristic of all physical assets.

Depreciation may be classified into the following three types:

- Physical depreciation
- Functional depreciation
- Accidents

Physical depreciation – Depreciation resulting in physical impairment of an asset is known as physical depreciation. Ex.: wearing out of particles from a bearing, corrosion of tubes of a heat exchanger etc. this type of depreciation results in the lowering of the ability of a physical asset to render its intended service. Following are the causes of physical depreciation:

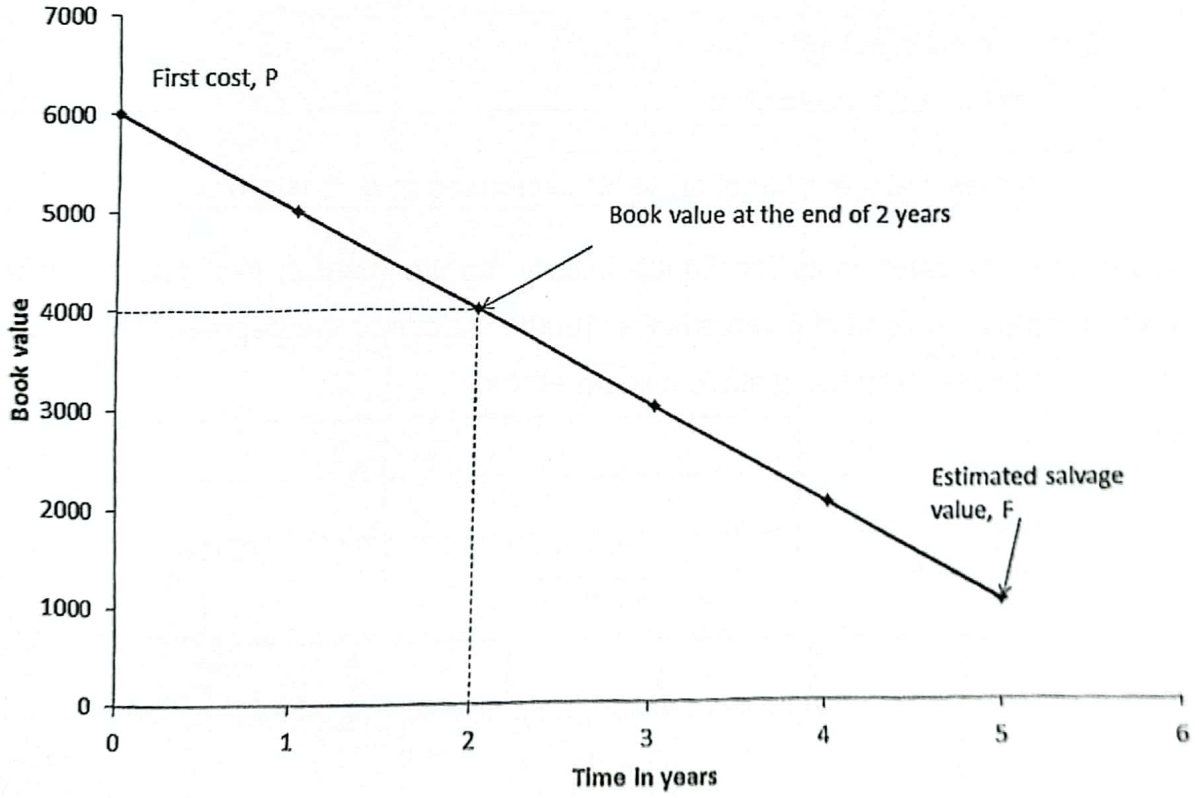
- Deterioration due to the action of elements including the corrosion of pipes, rolling of timbers, chemical decomposition, bacterial action, etc. it is independent of use.
- Wear and tear from use which subjects the asset to abrasion, shock, vibration impact etc.

Functional depreciation – It results not from deterioration in the asset's ability to serve its intended purpose, but from a change in demand for the services it can render. The demand for the services of an asset may change because it is more profitable to use a more efficient unit, there is no longer work for the asset to work, the work to be done exceeds the capacity of the asset. Depreciation resulting from these can be categorized as follows:

- Obsolescence resulting from the discovery of another asset that is sufficiently superior to make it uneconomical to continue with the existing asset. Assets also become obsolete when they are no longer needed.
- Inadequacy or the inability to meet the demand placed upon it.

6.2 Book value of an asset:

Accountants use the term book value to represent the original value of an asset less its accumulated depreciation at any point in time.



Book value at the end of the life of an asset is called the salvage value. This value may be equal to zero or any positive number, since still some value may be left in the asset even after its full life.

End of the year	Depreciation charge during the year, t	Book value at the end of year, t
0	-	5000
1	800	4200
2	800	3400
3	800	2600
4	800	1800
5	800	1000

General expressions for the calculation of depreciation and book value are as follows:

End of the year	Depreciation charge during the year, t	Book value at the end of year, t
0	-	P
1	$\left(\frac{P-F}{n}\right)$	$P-\left(\frac{P-F}{n}\right)$
2	$\left(\frac{P-F}{n}\right)$	$P-2\left(\frac{P-F}{n}\right)$
3	$\left(\frac{P-F}{Pn}\right)$	$P-3\left(\frac{P-F}{n}\right)$
.		
.		
t	$\left(\frac{P-F}{n}\right)$	$P-t\left(\frac{P-F}{n}\right)$
.		
.		
n	$\left(\frac{P-F}{n}\right)$	$P-n\left(\frac{P-F}{n}\right)$

Thus the depreciation for any year in straight line method is

$$D_t = \left(\frac{P-F}{n}\right)$$

The book value is

$$B_t = P - t\left(\frac{P-F}{n}\right)$$

and the rate of depreciation per year = $\frac{1}{n}$

6.3.2 Declining balance method (DBM):

This method assumes that an asset decreases in value at a faster rate in the early portion of the service life than in the later portion of its life. By this method a fixed percentage is multiplied times the book value of the asset at the beginning of the year to determine the depreciation charge for that year. Hence, as the book value of the asset decreases through time, so does the size of the depreciation charge. For the depreciation rate, R the general relationship between the depreciation charge for any year and its book value in the previous year is

$D_t = R.B_{t-i}$

Where,

D_t = depreciation charge for the year, t

B_{t-1} = Book value for the year (t-1)

and $B_t = B_{t-1} - D_t$ is the expression used to determine book value for the year, t

$$B_t = B_{t-1} - R.B_{t-1}$$

$$B_t = B_{t-1} (1-R)$$

Using this recursive expression it is possible to determine the general expressions for the depreciation charge and the book value for any point of time in terms of R and first cost, P.

Depreciation charge for any year, t

$$D_t = R (1-R)^{t-1}.P \quad \text{--------(1)}$$

& the book value is,

$$Bt = (1-R)^t.P \quad \text{----------(2)}$$

This is explained in the following table

End of the year	Depreciation charge during the year, t	Book value at the end of year, t
0	-	$P = B_0$
1	$R \times B_0 = RP$	$(1-R)\ B_0 = (1-R)P = B_1$
2	$R \times B_1 = R\ (1-R)\ P$	$(1-R)\ B_1 = (1-R)^2P = B_2$
3	$R \times B_2 = R\ (1-R)^2\ P$	$(1-R)\ B_2 = (1-R)^3P = B_3$
.		
.		
t	$R \times B_{t-1} = R\ (1-R)^{t-1}\ P$	$(1-R)\ B_{t-1} = (1-R)^tP = B_t$

.		
.		
n	$R \times B_{n-1} = R(1-R)^{n-1} P$	$(1-R) B_{n-1} = (1-R)^n P = B_n$

The expression for R is as follows:

$$R = 1 - \sqrt[t]{\frac{B_t}{P}} \quad \text{-------(3)}$$

6.3.3 Double declining balance method (DDB):

In this method the depreciation rate is given as the double the straight line rate that would be allowed for a particular asset being depreciated. Hence for an asset with an estimated life of n years, the depreciation rate by DDB will be $2 \times \frac{1}{n} = \frac{2}{n}$. Remaining things are the same as DBM.

6.3.4 Sum of the years digits method:

This method provides for depreciation by means of differing periodic rates which is computed as per the following formula.

If n is the estimated life of the asset, the rate is calculated for each period as a fraction in which the denominator is always the sum of the digits in L = 1, 2, 3, ..., n. For the first year it is n, 2nd year it is (n-1), 3rd year it is (n-2) and so on.

General expression is as follows:

$1 + 2 + 3 + \ldots\ldots + (n-2) + (n-1) + n = \sum_{j=1}^{n} j = \frac{n(n-1)}{n} = L$

End of the year	Depreciation charge during the year, t	Book value at the end of year, t
0	-	P
1	$(P-F) \times \frac{n}{\frac{n(n+1)}{2}}$	$P - (P-F) \times \frac{n}{\frac{n(n+1)}{2}}$
2	$(P-F) \times \frac{(n-1)}{\frac{n(n+1)}{2}}$	$P - \frac{(P-F)}{\frac{n(n-1)}{2}}\{n + (n-1)\}$
3	$(P-F) \times \frac{(n-2)}{\frac{n(n+1)}{2}}$	$P - \frac{(P-F)}{\frac{n(n-1)}{2}}\{n + (n-1) + (n-2)\}$
.		

t	$(P-F)\times\frac{(n-t+1)}{\frac{n(n+1)}{2}}$	$P-\frac{(P-F)}{\frac{n(n-1)}{2}}\{n+(n-1)+,.,.)\}$
.		
.		
n	$(P-F)\times\frac{1}{\frac{n(n+1)}{2}}$	$P-\frac{(P-F)}{\frac{n(n+1)}{2}}\times\frac{n(n+1)}{2}=F$

Here $D_t=\frac{(P-F)(n-t+1)}{\frac{n(n+1)}{2}}$

$$B_t=F+(P-F)\times\left(\frac{n-t}{n}\right)\times\left(\frac{n-t+1}{n+1}\right)$$

6.3.5 Service output method:

This method computes depreciation of an asset on the basis of the output or on the basis of its usage. When the asset is purchased, number of hours of usage or the total production units are estimated for the life time of the asset. Then from the total amount to be depreciated on hourly or unit basis depreciation rate per unit or depreciation rate per hour may be calculated and is given by the formulae:

$$Dep.\frac{rate}{unit}of\ prodn.=\frac{(P-F)}{Estimated\ life\ time\ prodn.}\ Rs./unit$$

$$Dep.\frac{rate}{hour}of\ prodn.=\frac{(P-F)}{Estimated\ life\ time\ usage}\ Rs./hour$$

The depreciation charges for any year are calculated on the basis of service rendered by the asset during that particular period. In general the formula for calculating depreciation charges for any year "t" may be written as:

$$D_t=\frac{(P-F)}{U}\times U_t$$

Where,

U = life time production

U_t = units produced during year, t

In terms of usage the depreciation charges for any year "t" is given by:

$$D_t=\frac{(P-F)}{H}\times H_t$$

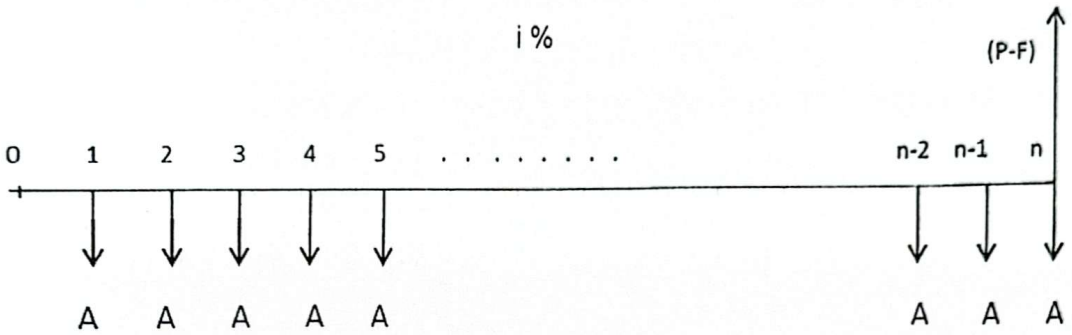
Where,

H = Total life time usage

H_t = hours utilized during year, t

6.3.6 Sinking fund method:

In this method it is assumed that when the asset is purchased sinking fund is established in which the funds are accumulated for recovering the capital invested in the asset. A series of equal amounts are assumed to be deposited into the sinking fund at the end of each year over the useful life of the asset. The sinking fund earns an interest of i % on the deposited amount and will have a balance equal to the total amount to be depreciated (P-F) at the end of the "n" years.



If the initial cost of the asset , P salvage value, F life "n" years and "i" interest rate on the sinking fund are known, then the uniform amount is to be deposited into the sinking fund at the end of each year over its useful life of the asset may be calculated as follows:

$$A = (P - F) \times \left(\frac{A}{F}, i, n\right)$$

$$A = (P - F) \times \left(\frac{i}{(1+i)^n - 1}\right)$$

The depreciation charge for any year is equal to annual deposit + the interest earned on the sinking fund balance during that particular year. The book value of the asset at the end of the year is given by:

(Purchase price – Total accumulated depreciation)

Or

(Purchase price – Sinking fund balance at the end of a particular year)

A sinking fund balance at the end of any year = sum of the depreciation charges till the end of that particular year.

General expressions for D_t and B_t for sinking fund method are given as follows:

$$D_t = (P - F)\left(\frac{A}{F}, i, n\right)\left(\frac{F}{P}, i, t-1\right)$$

$$B_t = P - (P - F)\left(\frac{A}{F}, i, n\right)\left(\frac{F}{A}, i, t\right)$$

Solved Problems:

1. Original cost of a machine is Rs. 8000/-. Its salvage value is Rs. 500/-. Life is 8 years. Evaluate its book value at the end of 5th year by using sum of the years digits method.

   Soln.:

   P = Rs. 8000/- F = Rs. 500/- n = 8 years t = 5 years.

   $$B_t = F + (P - F) \times \left(\frac{n-t}{n}\right) \times \left(\frac{n-t+1}{n+1}\right)$$

   $$= 500 + (8000 - 500) \times \left(\frac{8-5}{8}\right) \times \left(\frac{8-5+1}{8+1}\right)$$

   = Rs. 1750/-

2. A machine has a first cost of Rs. 300000/- and a salvage value of Rs. 60000/- and a life of 5 years. It is depreciated according to SLM. The management is trying to find a replacement at the end of 3 years of its useful life. What market value the management should fetch so that the capital invested in the machine is fully recovered?

   Soln.:

   P = 300000/- F = 60000/- n = 5 years t = 3 years

   $$B_t = P - t\left(\frac{P-F}{n}\right)$$

   $$= 300000 - 3\left(\frac{300000 - 60000}{5}\right)$$

   $$= Rs.\,156000/-$$

   Hence the market value to be fetched by the management should be more than Rs. 156000/-.

3. A lathe was purchased for Rs. 5 lacs. It was estimated to have a useful life of 10 years and a salvage value of Rs. 50000/-. Due to unexpected development the lathe was sold in the open market for Rs. 90000/- at the end of 8 years of the useful life. Determine how much "Sunk loss" or "Capital gain" has occurred if the asset is being depreciated according to SLM.

Soln.:

Book value at the end of 8 years,

$$B_t = P - t\left(\frac{P-F}{n}\right)$$

$$= 500000 - 8\left(\frac{500000-50000}{10}\right)$$

$$= Rs.140000/-$$

Since book value at the end of 8 years exceeds the market price (Rs. 90000/-) sunk loss has occurred. This is equal to:

Sunk Loss = 140000 – 90000

= Rs. 50000/-

4. An asset costs Rs. 5000/- and its salvage value is Rs. 1000/- estimated and an estimated service life of 5 years and a depreciation rate of 30 % per year. Determine the depreciation charges for 5 years and its book values at the end of each year.

Soln.:

$D_0 = 0$ B_0 = Rs. 5000/-

$D_1 = RP$ $B_1 = (1-R)\,P$

= 0.30×5000 = (1-0.30)×5000

= Rs. 1500/- = Rs. 3500/-

$D_2 = R(1-R)P$ $B_2 = (1-R)^2\,P$

= 0.30×(1-0.30)×5000 = $(1-0.30)^2$×5000

= Rs. 1050/- = Rs. 2450/-

$D_3 = R(1-R)^2 \times P$ $B_3 = (1-R)^3\,P$

= 0.30×$(1-0.30)^2$×5000 = $(1-0.30)^3$×5000

= Rs. 735/- = Rs. 1715/-

Similarly,

D_4 = Rs. 515/- B_4 = Rs. 1200/-

D_5 = Rs. 360/- B_5 = Rs. 840/-

5. An asset was purchased for Rs. 250000/-. It has an expected life of 10 years and a salvage value of Rs. 50000/- at the end of 10 years. What will be the un-depreciated amount of capital remaining in the asset at the end of 6^{th} year, if the asset is being depreciated according to the declining balance method (DBM)? Also calculate the depreciation charge for the 8^{th} year.

Soln.:

$$\text{Depreciation rate: } R = 1 - \sqrt[t]{\frac{B_t}{P}}$$

$$= 1 - \sqrt[10]{\frac{50000}{250000}}$$

$$= 0.1487 \; or \; 14.87\,\%$$

Un-depreciated amount of capital remaining at the end of 6th year is calculated as:

$$B_6 = (1-R)^6 \times P$$

$$= (1-0.1487)^6 \times 250000$$

$$= Rs.95156/-$$

Depreciation charge during year, 8 , D_8 is given by:

$$D_t = R(1-R)^{t-1} \times P$$

$$D_8 = 0.1487(1-0.1487)^{8-1} \times 250000$$

= Rs. 12046/-

6. An asset was purchased 10 years ago for Rs. 9600/-. It was depreciated according to SLM for an estimated life of 20 years and a salvage value of Rs. 1600/-. What is the difference in its current book value and the book value that would have been resulted if DDB method is adopted.

Soln.: By SLM:

R = 1/n = 1/20 = 0.05 or 5 %

$$B_t = P - t\left(\frac{P-F}{n}\right)$$

$$B_{10} = 9600 - 10\left(\frac{9600-1600}{20}\right)$$

= Rs. 5600/-

By DDB:

R = 2/n = 2/20 = 0.1 or 10 %

$$B_t = (1-R)^t \times P$$

$$B_{10} = (1-0.1)^{10} \times 9600$$

= Rs. 3347/-

Hence, Difference in current book value = 5600 – 3347

= Rs. 2253/-

7. An asset has a first cost of Rs. 4800/- with an estimated life of 20 years. What is the total accumulated depreciation charge during the first five years of the asset life if it is depreciated according to DDB method?

Soln.:

R = 2/n = 2/20 = 0.1 or 10 %

End of the year	Depreciation for the year, t	
0	-	
1	RP = 48000×0.1	= 4800/-
2	R(1-R)P = 0.1×0.9×48000	= 4320/-
3	$R(1-R)^2P = 0.1\times0.9^2\times48000$	= 3880/-
4	$R(1-R)^3P = 0.1\times0.9^3\times48000$	= 3499.20/-
5	$R(1-R)^4P = 0.1\times0.9^4\times48000$	= 3149.28/-
Total accumulated depreciation charge		= Rs. 19656.48/-

8. A machine has a first cost of Rs. 300000/-, a life of 5 years with no salvage value. The machine is depreciated according to the sum of the years digits method. The machine is replaced at the end of 3 years of its useful life. What market value the machine should get so that capital invested in the machine is fully recovered?

Son.:

$$B_t = F + (P - F) \times \left(\frac{n-t}{n}\right) \times \left(\frac{n-t+1}{n+1}\right)$$

$$= 0 + (300000 - 0) \times \left(\frac{5-3}{5}\right) \times \left(\frac{5-3+1}{5+1}\right)$$

$$= Rs.\,60000 \qquad - market\ value$$

9. A machine costs Rs. 220000/- and is expected to last a total of 10000 hours and then has a salvage value of Rs. 20000/-. The machine was used for 1500 hrs. and 1200 hrs. in the first and second year. If the depreciation is based on hours of usage, calculate the depreciation charge for each year and also calculate the book value of the machine at the end of first and second years.

Soln.:

$$D_t = \frac{(P-F)}{H} \times H_t$$

$$For\ the\ first\ year, D_1 = \frac{(220000-20000)}{10000} \times 1500$$

= Rs. 30000/-

B_1 = 220000-30000 = Rs. 190000/-

$$For\ the\ second\ year, D_2 = \frac{(220000-20000)}{10000} \times 1200$$

= Rs. 24000/-

B_2 = 190000-24000 = Rs. 166000/-

10. A machine is purchased at a cost of Rs. 3500000/- and an estimated life of 10 years, and a salvage value of Rs. 700000/-. Calculate the annual depreciation charges and the book values at the end of each year for the first five years of its life according to sinking fund method at the rate of 10 %.

Soln.:

$$D_t = (P-F)\left(\frac{A}{F}, i, n\right)\left(\frac{F}{P}, i, t-1\right)$$

$$D_t = (3500000-700000)\left(\frac{A}{F}, 10\ \%, 10\right)\left(\frac{F}{P}, 10\ \%, t-1\right)$$

$$= (2800000)\left(\frac{i}{(1+i)^n-1}\right)\{(1+i)^{t-1}\}$$

For t = 1,

$$D_1 = (2800000)\left(\frac{0.1}{(1+0.1)^{10}-1}\right)\{(1+0.1)^{1-1}\}$$

= Rs. 175700/-

For t = 2,

$$D_2 = (2800000)\left(\frac{0.1}{(1+0.1)^{10}-1}\right)\{(1+0.1)^{2-1}\}$$

= Rs. 193256/-

For t = 3,

$$D_3 = (2800000)\left(\frac{0.1}{(1+0.1)^{10}-1}\right)\{(1+0.1)^{3-1}\}$$

= Rs. 212597/-

For t = 4,

$$D_4 = (2800000)\left(\frac{0.1}{(1+0.1)^{10}-1}\right)\{(1+0.1)^{4-1}\}$$

= Rs. 233857/-

For t = 5,

$$D_5 = (2800000)\left(\frac{0.1}{(1+0.1)^{10}-1}\right)\{(1+0.1)^{5-1}\}$$

= Rs. 257242/-

$B_1 = P - D_1$

= 3500000 – 175700

= Rs. 3324300/-

$B_2 = B_1 - D_2$

= 3324300 – 193256

= Rs. 3131044/-

$B_3 = B_2 - D_3$

= 3131044 – 212597

= Rs. 2918447/-

$B_4 = B_3 - D_4$

= 2918447 – 233857

= Rs. 2694516/-

$B_5 = B_4 - D_5$

= 2694516 – 257242

= Rs. 2427274/-

11. An industrial plant of initial value of Rs. 4 lacs has a salvage value of Rs. 50000/- at the end of 25 years. But it is sold for Rs. 260000/- at the end of 10 years. What is the profit or the loss if sinking fund method is adopted taking an interest rate of 10 %.

Soln.: P = Rs. 400000/- n = 25 years t = 10 years

F = Rs. 50000/- i = 10 %

$$B_t = P - (P - F)\left(\frac{A}{F}, i, n\right)\left(\frac{F}{A}, i, t\right)$$

$$B_{10} = 400000 - (400000 - 50000)\left(\frac{0.1}{(1+0.1)^{25}-1}\right)\left(\frac{(1+0.1)^{10}-1}{0.1}\right)$$

= Rs. 343281.50/-

Hence, loss occurred = 343281.50 – 260000

= Rs. 83281.50/-

Unsolved Problems:

1. An asset was purchased 10 years ago for Rs. 4800/-. It was depreciated according to SLM for an estimated total life of 20 years and a salvage value of Rs. 800/-. What is the difference between its current book value and book value that would have been resulted if declining balance method is used at the rate of 10 %.
2. An asset was purchased 10 years ago for Rs. 500000/-. It is depreciated according to Double declining balance method (DDB) for an estimated life of 20 years and a salvage value of Rs. 50000/-. Calculate the current book value.
3. A machine was acquired at a cost of Rs. 35000/- has an estimated life of 5 years, a salvage value of Rs. 7000/-. Determine the annual depreciation charges and the book values at the end of each year over the useful life of the asset, if the machine is being depreciated according to the sum of the years digits method.
4. A machine is acquired at a cost of Rs. 32000/- has an estimated life of 5 years, salvage value of Rs. 3500. Determine the annual depreciation charges

during the useful life of the machine, under the sum of the years digits method.

5. A central air conditioning unit was purchased for Rs. 25000/- and it has an expected life of 10 years. The salvage value for the unit at that time is expected to be Rs. 4000/-. Compare the book value at the end of 7 years as per the SLM, DBM and sum of the years digits method.
6. A machine costs Rs. 35000/- and expected to last for a total of 17500 hours.
    a. Determine the depreciation charges if the machine is used for 1500 hours and 2200 hours in the first and second years.
    b. After 2 years if the total useful life of the machine is re-estimated to be 22500 hours, calculate the new depreciation rate for the third year and later.
7. A machine is purchased for Rs. 20000/-, life is 10 years and salvage value of Rs. 1000/-. If depreciation is charged through sinking fund method, calculate the amount in the depreciation fund at the end of 5^{th} year and 8^{th} year if the rate of interest is 5 %.

APPENDIX B

Compound Interest Tables

Values of Interest Factors When N Equals Infinity

Single Payment:
$(F/P, i, \infty) = \infty$
$(P/F, i, \infty) = 0$

Arithmetic Gradient Series:
$(A/G, i, \infty) = 1/i$
$(P/G, i, \infty) = 1/i^2$

Uniform Payment Series:
$(A/F, i, \infty) = 0$
$(A/P, i, \infty) = i$
$(F/A, i, \infty) = \infty$
$(P/A, i, \infty) = 1/i$