' BNP

Advanced Physics

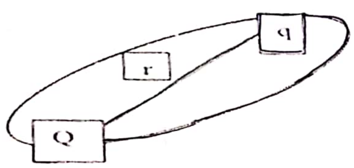
BE- Second Semester

Unit –II Electron Ballistics

Basic Definitions:

- **Electric Charge**:- It is intrinsic characteristic of fundamental particles. Any particle or object that establishes an electric field in its surrounding space is said to have a charge on it.
- **Coulomb Force**:-The force of interaction between two point charges Q_1 and Q_2 which is directly proportional to the product of two charges and inversely proportional to the square of the distance between them. It is given by $F = \dfrac{qQ}{4\pi \epsilon_0 \epsilon_r r^2}$

Where ε_0-permittivity of free space
ε_r- permittivity of the medium

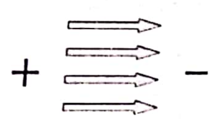


- **Electric field/ Electric field strength/ Electric field Intensity**:-
Electric field/ Electric field strength/Intensity at some point is defined as the electric force F experienced by a unit positive test charge placed at that point

$$E = \frac{F}{q}$$

E= -dV/dx (Potential gradient)

Electric field lines in Uniform electric field:-



•**Electric field lines in non-uniform electric field:-**

- **Homogeneous Electric Field /Uniform Electric Field :** An electric field having the same intensity and same direction at every point is called homogeneous electric field or uniform electric field.

- **Electric potential (V):-** Electric potential (V) at a point in space is defined as the work done by an external agent in carrying a unit positive test charge from infinity to that point against the electric force of .Thus electric potential at the point is

$$V = \frac{w}{q} = \frac{Fd}{q} = Ed \because F = qE$$

$$\therefore E = \frac{V}{d}$$

- **Lorentz Equation or Lorentz Force :** When the charged particle moves in a region where there are both electric and magnetic fields, it will experience both an electric force qE and a magnetic force q(v×B), so that the total force is the vector sum of electric force and magnetic force called Lorentz force . Thus,

$$F = F_E + F_L$$
$$= qE + q(v\times B)$$
$$= q[E+(v\times B)]$$

This expression is called as Lorentz equation and the force as Lorentz force because it was identified in this form by H. Lorentz.

Motion of Electron in Uniform Electric Field

CASE I:

Motion of (Charged particle) electron parallel to uniform electric field OR Motion of an electron in a longitudinal uniform electric field:

Consider the two plane parallel plates **A & B** of equal area separated by a distance **'d'**. If dc voltage source is connected between the plates , the plates are charged oppositely and electric field **E** is produced in the region between the plates. The electric field is directed from the positive plate A toward the negative plate B and is given by

$$E=\frac{V}{d} \text{ --------------(1)}$$

Let an electron of mass m and charge e is placed at rest in the uniform electric field and released. The electron experiences a force due to electric field given by

$$F = - eE \text{ ---------------(2)}$$

Here negative sign indicates that the force F accelerates the electron in a direction opposite to that of E.

According to second law of Newton, the acceleration is given by

$$F = ma ,$$

$$a = \frac{F}{m}$$

$$\therefore a = \frac{-eE}{m} \text{ ----------(3)}$$

...ce the equations of kinematics for uniformly accelerated motion are applicable to electron ...

$$\begin{bmatrix} v = u + at \\ S = ut + \frac{1}{2}at^2 \\ v^2 = u^2 + 2aS \end{bmatrix}$$

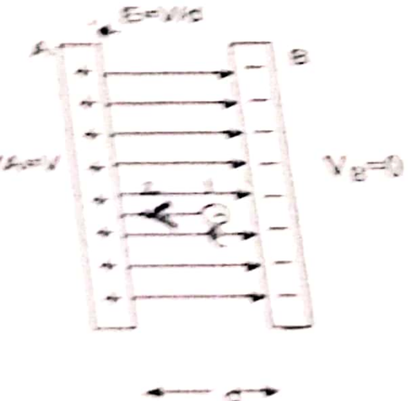


Fig 1. Electron in Uniform Electric Field

Using the initial conditions, $x_0 = 0$ and $v_0 = 0$ (electron is initially

Using 1st Kinematics equation

$$v = v_0 + at$$

$$v = 0 + at$$

$$v = at \qquad (\because\ a = \frac{eE}{m}\ )$$

$$v = \frac{eEt}{m} \text{.......... .... } (4)$$

Displacement along x direction at any time "t"

Using 2nd Kinematics equation

$$x = o + \frac{1}{2}at^2 \quad (\text{Using} \qquad s = ut + \frac{1}{2}at^2)$$

$$x = \frac{eEt^2}{2m} \text{-----------}(5)$$

Using 3rd Kinematics equation

$$v^2 = 0 + 2ax \quad (\text{using } v^2 = u^2 + 2as)$$

is called

done by
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e

e negative sign indicates that the electron is accelerated in a direction opposite to that of electric d E.

above relation the parameters e , m and E are constants and therefore electron acceleration is ıstant or uniform. The electron motion resembles that of a body freely falling in a gravitational field

ıce the equations of kinematics for uniformly accelerated motion are applicable to electron motion.

$$\begin{bmatrix} v = u + at \\ S = ut + \frac{1}{2}at^2 \\ v^2 = u^2 + 2aS \end{bmatrix}$$

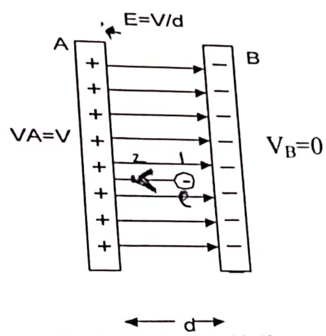


Fig 1: Electron in Uniform Electric Field

Using the initial conditions, $x_0 = 0$ and $v_0 = 0$ (electron is initially at rest)

Using 1st Kinematics equation

$$v = v_0 + at$$

$$v = 0 + at$$

$$v = at \qquad (\because a = \frac{eE}{m})$$

$$v = \frac{eEt}{m} \text{.......... ....} \quad (4)$$

Displacement along x direction at any time "t"

Using 2nd Kinematics equation

$$x = o + \frac{1}{2}at^2 \quad (\text{Using} \qquad s = ut + \frac{1}{2}at^2)$$

$$x = \frac{eEt^2}{2m} \text{----------} (5)$$

Using 3rd Kinematics equation

$$v^2 = 0 + 2ax \quad (\text{using } v^2 = u^2 + 2as)$$

$$v^2 = \frac{2Eex}{m} \text{............(6)}$$

The Kinetic Energy attained by the electron after moving a distance x is given by,

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2eEx}{m}\right) = eEx$$

$$K.E. = eEx \text{ ---------(7)}$$

So Electron follows **straight line path or Linear path** in parallel or longitudinal uniform Electric Field.

- <u>**Energy or K.E. acquired by Electron in Uniform Electric Field**</u> **(By Using Work Energy Theorem) or**
- **Show that electron gains an energy of amount eV while moving through longitudinal or parallel Uniform Electric Field or**
- **Show that the velocity acquired by the electron in an electric field is proportional to the square root of the potential difference through which it is accelerated**

The electric field E causes electron motion towards the positively charged plate A. The force on the electron is given by,

$$F = -eE = e\frac{dV}{dx} \text{ --------(1)} \quad \left(\because E = -\frac{dV}{dx} \text{ potential gradient }\right)$$

When an electron is accelerated in an electric field, the work done by the field **E** in moving the electron between two points is equal to the kinetic energy acquired by it. If **'dw'** is the work done in moving the electron through an infinitesimal distance **'dx'** it can be expressed as

$$dw = \overrightarrow{F} \cdot \overrightarrow{dx}$$

Total work done in moving the electron from position 1 to 2, is given by,

$$\therefore W_{12} = \int_1^2 \overrightarrow{F} \cdot \overrightarrow{dx} = \int_1^2 e\frac{dV}{dx}\overrightarrow{dx} = edV$$

$$\therefore W_{12} = e(V_2 - V_1) \text{ -------(2)}$$

Where **V_1** and **V_2** are the potentials at positions **1& 2** respectively. The potential energy of the electron decreases as it moves in a direction opposite to that of the electric field.
According to work energy theorem ,the work done by the field in moving the electron is equal to K.E. acquired by the electron

According to Newton's 2nd Law

$$F = ma = m\frac{dv}{dt}$$

where **'v'** is the velocity of electron along **x-** direction

$$\therefore W_{12} = \int_1^2 \overrightarrow{F} \cdot \overrightarrow{dx} = \int_1^2 m\frac{dv}{dt}\overrightarrow{dx} = \int_1^2 m\frac{dv}{dx}\frac{dx}{dt}\overrightarrow{dx} = \int_1^2 mvdv$$

$$\therefore W_{12} = \frac{1}{2}m(v_2^2 - v_1^2) \text{ -------(3)}$$

Where v_1 and v_2 are the electron velocities at positions 1 and 2 respectively.

On equating **equations (2) & (3)** we get $e(V_2 - V_1) = \frac{m}{2}(v_2^2 - v_1^2)$ ------(4)

Assuming that electron starts from rest and accelerates through a potential difference V, then $v_1 = 0$ & $v_2 = v$ and $V_1 = 0$ & $V_2 = V$ then

$$eV = \frac{1}{2}mv^2 \text{ -------}(5)$$

This equation shows that electron gains energy from the electric field.

We can write $v^2 = \frac{2eV}{m}$, $v = \sqrt{\frac{2eV}{m}}$ ----(6)

$$\therefore v \alpha \sqrt{V}$$

Thus the velocity acquired by the electron in an electric field is proportional to the square root of the potential difference through which it is accelerated.

Using numerical values of e and m of an electron in eqn (6) we get

$$\therefore v = 5.93 \times 10^5 \sqrt{V} \text{ m/s}$$

- **<u>Electron Volt:</u>**

The amount of an energy gained by an electron by accelerating in electric field is very small compared to a joule. Hence in atomic physics and Nuclear Physics, particle energies are expressed in terms of a small unit called electron – volt (eV).

Def^n: An electron volt is defined as the energy acquired by an electron when it gets accelerated through a potential difference of one volt.

1 eV = Charge on electron * 1 V

$= (1.602 \times 10^{-19} C)(1V)$

1 eV = $1.602 \times 10^{-19} J$

Units of MeV is often used in Nuclear Physics 1 MeV = 10^6 eV

<u>CASE – II:</u>

Motion of electron perpendicular (transverse) to uniform electric field :

Q. Show that an electron moving with uniform velocity follows a parabolic path in a Transverse uniform electric field.

OR

Q. Describe the motion of an electron subjected to uniform electric field acting normal to the Electron velocity.

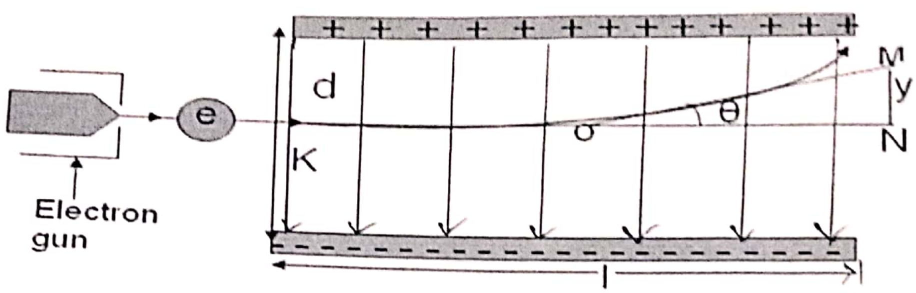
OR

Q. Show that a charged particle moving with initial uniform velocity v_o follows a parabolic Path when subjected to transverse uniform electric field.

Let A and B be the two plane parallel metal plates of length 'l' oriented horizontally , separated by a distance 'd'. A potential difference V applied between the plates produces a vertically acting uniform electric field E which is directed, say from plate A to plate B. The strength of electric field in the region between the plates is given by $E = \frac{V}{d}$ ---------(1)

D

ter its speed is



Let an electron from electron gun initially moves with constant velocity v_0 along X-direction.

$v_x = v_0$ = constant --------------(2)

The electron velocity in y direction is initially zero . At point K , the electron enters the uniform electric field between plates A and B, which is at right angles to the direction of electron. As the electric field acts in downward y direction, the electron experiences an upward Electric force and gets deflected upward in y-direction. The acceleration acquired by the electron in y direction is given by

$$a_y = \frac{F}{m} = \frac{eE}{m} ----(3)$$

The velocity attained by the electron after travelling for a time t in the electric field is (Using 1st Kinematics equation)

$$v_y = 0 + a_y t$$

$$v_y = \frac{eEt}{m} ------(4)$$

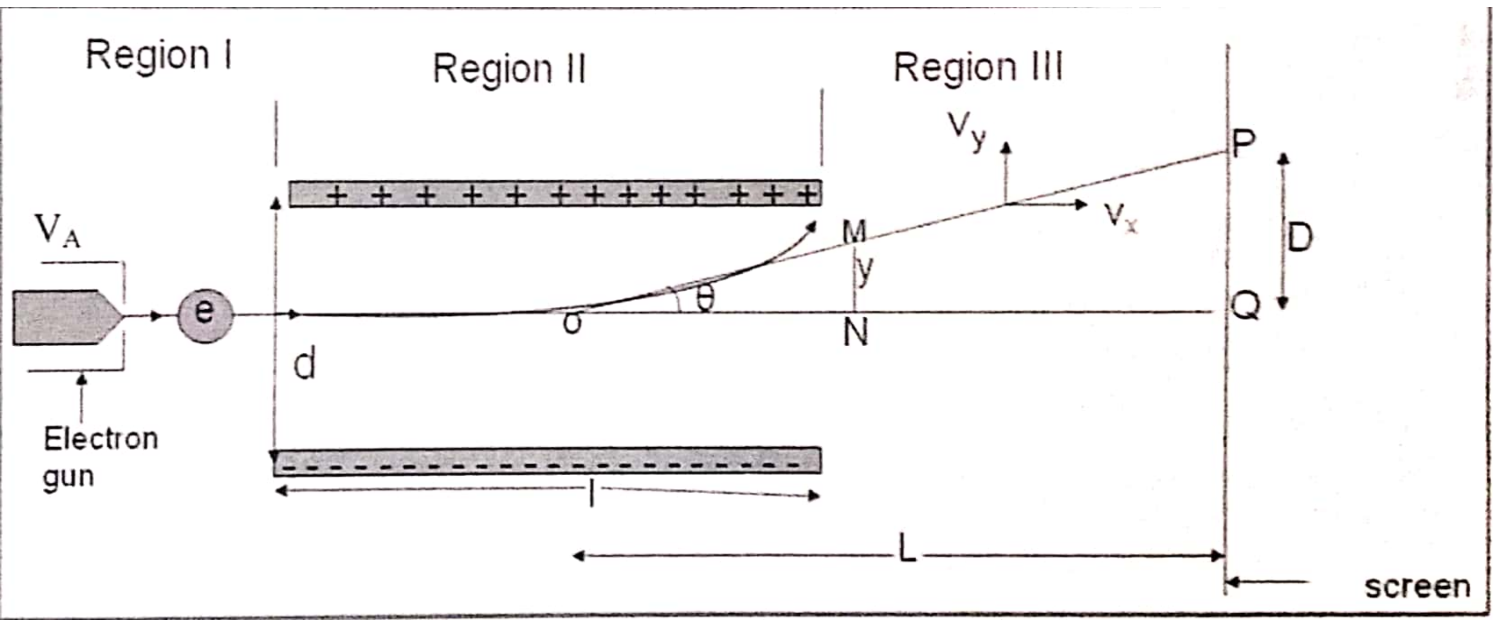
$v_y = \frac{eE}{m}t$

The displacement x of the electron in X-direction traveled by the electron in the time interval **'t'** depends on the initial velocity and is given by (Using 2nd Kinematics equation)

$$x = v_o t + \frac{1}{2}a_x t^2$$

$$x = v_o t \quad (\because a_x = 0)$$

$$t = \frac{x}{v_0} ....(5)$$



Region I
Region II
Region III
V_y
V_x
P
M
y
θ
o
N
Q
D
V_A
e
d
Electron gun
l
L
screen

As ,

$$y = \frac{eE}{2mv_0^2}x^2 \text{-----------}(2)$$

$$\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left(\frac{eEx^2}{2mv_0^2}\right) \quad \text{Therefore,} \quad \tan\theta = \frac{eEx}{mv_0^2} \text{------}(3)$$

Using $x = l$,

$$\tan\theta = \frac{eEl}{mv_0^2} \text{------}(4)$$

From eq. (1) , $ON = \frac{y}{\tan\theta} = \frac{eEl^2/2mv_0^2}{eEl/mv_0^2}$

$$ON = \frac{l}{2} \text{--------}(5)$$

This shows that the apparent source of electron striking the screen at P is at O, the centre of the electric field.

The electron travels along the axis and strikes the fluorescent screen at point Q in the absence of the electric field. It strikes the screen at point P in the presence of electric field. Therefore PQ represents the deflection of electron path occurred due to the electric field, i.e. Electrostatic Deflection D.

From ΔPOQ,

$$\tan\theta = \frac{PQ}{OQ}$$

Therefore, PQ = OQ*tanθ

D = L*tanθ

$$D = L\frac{eEl}{mv_0^2} \text{--------}(6)$$

Substituting the value of E and v_0,

$$D = Le\left(\frac{V}{d}\right)\frac{l}{m}\frac{m}{2eV_A}$$

$$D = \frac{LlV}{2dV_A} \text{--------}(7)$$

Thus electrostatic deflection D is proportional to deflecting voltage V and inversely proportional to accelerating voltage V_A.

The angular displacement of electron path in the electric field is given by

$$\tan\theta = \frac{D}{L}$$

$$\theta = \tan^{-1}\left(\frac{lV}{2dV_A}\right)$$

Transit Time: The time t spent by the electron in the electric field is known as transit time of electron and is given by

$$t = \frac{l}{v_0}$$

Deflection Sensitivity:

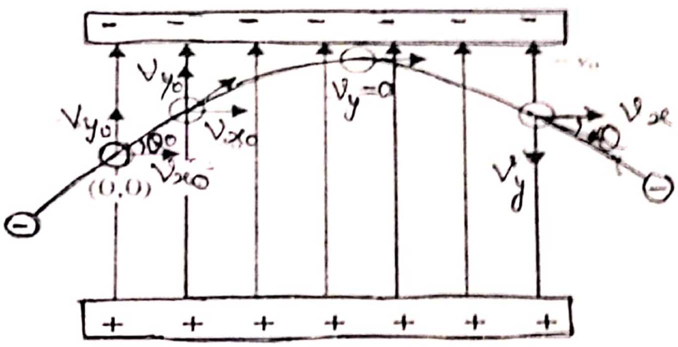
The electric deflection sensitivity S, is defined as the deflection caused by one volt of potential difference applied to deflection plates.

$$S = S_E = \frac{D}{V} = \frac{lL}{2dV_A}$$

CASE-III

- **Electron projected at an angle in uniform electric field :**
- **Discuss the motion of an electron projected into uniform electric field at an acute angle with the field direction OR**
  Describe/Discuss the motion of electron (Charged particle) when projected at an acute angle with the direction of uniform electric field and determine :
  - a. **Maximum distance reached by the electron in the direction of the field.**
  - b. **Time taken to reach maximum distance.**
  - c. **Range of charged particle.**

Suppose an electron is projected in to a uniform electric field at an acute angle with the field direction and with an initial velocity **v_0**. The electric field acts in positive **y-** direction and the electron gets accelerated in the negative y- direction. The acceleration is given by $a = \frac{F}{m} = \frac{eE}{m}$ and is constant. Thus the motion of electron will be very much similar to that of projectile in gravitational field.



Initial velocity v_0 is resolved into two components
$v_{x0} = v_0 \cos\theta_0$
$v_{y0} = v_0 \sin\theta_0$

Fig.4: Projectile Motion of an electron in uniform electric field.

The velocity component in x-direction **v_x** remains constant (So a_x =0) while **v_y decreases initially ,becomes zero at the maximum height and again increases when the electron reverses its path.**

Velocity Component along X –direction : (Using first kinematics equation)

$$v_x = v_{x0} = v_0 Cos\theta_0 + 0 = Cons\tan t \quad \Rightarrow v_x = v_{x0} + a_x . t$$

Since electron moves with constant velocity along X-direction , $a_x = 0$

$$v_x = v_0 Cos\theta_0 = Cons\tan t \quad ------(1)$$

Velocity Component along Y –direction :

$$v_y = v_{y0} + a_y t$$

$$v_y = v_0 Sin\theta_0 + a_y t \quad -------(2)$$

Using above equations we can obtain coordinates of the electron at any time **t** as

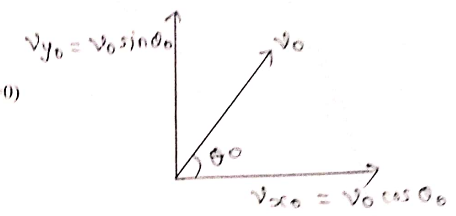
X-coordinate

$$x = v_{x0}t = v_x t + \frac{1}{2}a_x t^2$$

$$x = v_{x0}t + 0 \qquad \text{(as } a_x = 0\text{)}$$

$$x = v_{x0}t \quad ----(3)$$

$x = V_0 \cos\theta_0 \cdot t$



Y-coordinate

$$y = v_{y0}t + \frac{1}{2}a_y t^2$$

$$= (v_0 Sin\theta_0)t + \frac{1}{2}a_y t^2$$

$$= (v_0 Sin\theta_0)\left(\frac{x}{v_{x0}}\right) + \frac{1}{2}a_y\left(\frac{x}{v_{x0}}\right)^2 \quad ------(4)$$

As from eq. (3) $\quad t = \frac{x}{v_{x0}} = \frac{x}{v_0 Cos\theta_0}$

Putting this value of **t** in **Eqn. (4)** we get

$$y = (v_0 Sin\theta_0)\frac{x}{(v_0 Cos\theta_0)} + \frac{1}{2}a_y\left(\frac{x^2}{v_0^2 Cos^2\theta_0}\right)$$

$$y = (\tan\theta_0)x + \left(\frac{a_y}{2v_0^2 Cos^2\theta_0}\right)x^2 \quad ----(4)$$

where a :
b :

Eq^n (4) is of the form $y = ax + bx^2$, which represents the equation of parabola. Therefore the trajectory of an electron projected into a uniform electric field is a parabola.

The various parameter of projected charge particle in uniform electric field can be obtained as follows.

1. **Time of Ascent (t) :**The time taken by the charged particle to reach maximum height in y-direction is denoted by t given by (Using $v = u + at$)

$$v_y = v_{y0} + a_y t$$

At maximum height, $v_y = 0$

$$\therefore 0 = v_0 \sin \theta_0 + a_y t$$

$$t = \frac{v_0 Sin\theta_0}{a_y} \text{------(5)} \quad \text{(Neglecting minus sign)}$$

2. **The time of Flight:** The time taken by the charged particle to return to its original level along x-direction is called Time of flight which is given by

$$T = t + t = 2t = \frac{2v_0 Sin\theta_0}{a_y} \text{------(6)}$$

3. **Maximum Height of a projectile (Charged Particle) :** It is the maximum distance that a charged particle reaches in y-direction.

   (Using $v^2 - u^2 = 2aS$)

$$v_y^2 = v_{y0}^2 + 2a_y y$$

At maximum height, $v_y=0$

$$0 = v_{y0}^2 + 2a_y y \quad \Rightarrow \quad V_{y0}^2 = -2a_y \cdot y$$

$$y = \frac{-v_{y0}^2}{2a_y}$$

Neglecting minus sign

$$\therefore y = H_{max} = \frac{v_{y0}^2}{2a_y}$$

$$\therefore y = H_{max} = \frac{v_0^2 Sin^2\theta_0}{2a_y} \text{--------(7)}$$

4. **Range of Charged Particle :** The distance traveled along x-direction by the charged particle from the starting point to the point at which it returns to its original level along x-direction is called as range of Charged particle.

$$R = v_{xo} T = (v_0 Cos\, \theta_0)\left(\frac{2v_0 Sin\, \theta_0}{a_y}\right) = \frac{v_0^2 Sin\, 2\theta_0}{a_y} \text{------(8)}$$

Horizontal range will be maximum if $\sin 2\theta_0 = 1$ i. e. $2\theta_0 = \pi/2$

$$\theta_0 = \pi/4 = 45^\circ$$

Range will be maximum, when the particle is projected at an angle of 45° to the horizontal.

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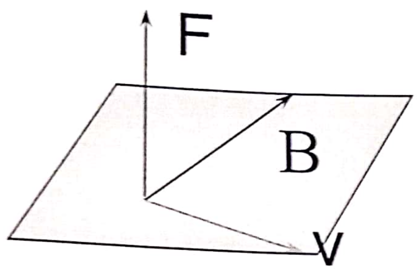
Motion of Electron in Uniform Magnetic Field

Magnetic Force:

If an electron moving with a velocity **v** enters in to the region of a magnetic field of strength **B** then it experiences a force called Magnetic Lorentz force and is given by

$$F_L = e(\vec{v} \times \vec{B}) = evBSin\theta -----(1)$$

Force vector will be at right angles to the plane containing velocity vector and field vector.



- **Work done by the magnetic force is zero:**

The static magnetic field does not act on an electron which is at rest. However when the electron moving with a velocity v enters a magnetic field, it experiences a magnetic force given by

$$F_L = e(\vec{v} \times \vec{B}) = evBSin\theta -----(1)$$

If **dx** is the infinitesimal displacement of electron during the interval of time **dt** , the work done dw on the electron by the magnetic field is given by

$$dw = \overrightarrow{F} \cdot \overrightarrow{dx} = \overrightarrow{F_L} \cdot \vec{v} \cdot dt = e(\vec{v} \times \overrightarrow{B}) \cdot \vec{v}dt = 0 \qquad (v = dx/dt)$$

which implies that no work is done by the magnetic field in moving the electron from one position to another.

Que. Show that No Energy is gained by an electron in a Magnetic Field OR
Show that Kinetic Energy in Magnetic Field remains Constant OR
Show that the magnetic field does not produce any change in either the speed or K.E.of an electron.

As the force vector is perpendicular to velocity vector, we can write

$$\vec{F_L} \cdot \vec{v} = 0$$

$$\therefore m\vec{a} \cdot \vec{v} = 0$$

$$\Rightarrow m\frac{dv}{dt}v = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{mv^2}{2}\right) = 0$$

$\frac{m \cdot 2v\,dv}{2\,dt}\, v$ $\quad \frac{m \cdot v^2}{2}$ $\quad \frac{m}{2}\,\frac{2v\,dv}{dt}$

Integrating

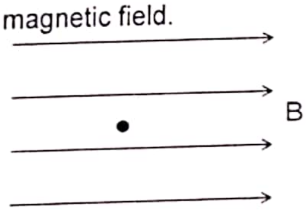
$$\therefore \frac{1}{2}mv^2 = const.$$

Or $\quad v = \text{constant}$

Kinetic Energy of charged particle in magnetic field remains constant i.e. an electron moves in the magnetic field without gaining or loosing energy. It also shows that magnetic field can not change the magnitude of speed of electron but changes the direction of motion of electron.

- **When The Charge Particle is at Rest:**

If the charged particle is at rest in the magnetic field.



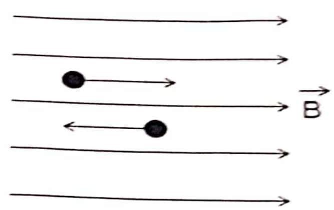
v = 0

The force on the charged particle is $F_L = e(\vec{v} \times \vec{B}) = evBSin\theta$

If **v = 0** then, $F_L = e \cdot 0 \cdot BSin\theta = 0$ indicating that magnetic force does not act on static electron or electron at rest .

i.e A stationary charge does not experience any force in magnetic field.

CASE-I:
Motion of Electron parallel to uniform Magnetic field OR Longitudinal Uniform Magnetic field :



v II B

If an electron moves parallel to the magnetic field lines, the magnetic force acting on it is zero.
If **θ = 0** then,

$$F_L = e \cdot v \cdot BSin\theta = e \cdot v \cdot B \cdot \sin 0 = 0$$

Similarly if If an electron moves anti - parallel to the magnetic field lines **θ = 180** then,

$$F_L = e \cdot v \cdot B \cdot \sin 180 = 0$$

The above condition indicates that magnetic force does not act on electron if it enters the magnetic field either parallel or anti parallel to the lines of induction and it will continue to move along the field lines with initial velocity and direction.

CASE-II:
Motion of electron perpendicular (Transverse) to uniform Magnetic field :

Q. Show that the radius of orbit of a charged particle moving at right angle to magnetic field is proportional to its momentum.

OR

Q. A charged electron of mass m moving with velocity is subjected to magnetic field Perpendicular to its direction of motion. Show that the period of its revolution is Independent of its velocity.

OR

Q. Explain why slower particles and faster particles require the same time for completing one rotation in magnetic field?

Let an electron enter with a uniform velocity **v** normally in to a magnetic field of strength **B** as shown in **Fig. 6.** The direction of B is perpendicular to the direction of motion of electron and plane of paper.
Thus the force due to magnetic field is given by

$$F_L = Bev \text{ -------(1)}$$

This force can not change the magnitude of electron velocity but deflects the electron continuously along a curvilinear path (Circular path).
Then the centripetal force required for circular motion is supplied by the magnetic force F_L ,which is given by

$$F_C = \frac{mv^2}{R} \text{ -----------(2)}$$

Where **r** is the radius of circular orbit.

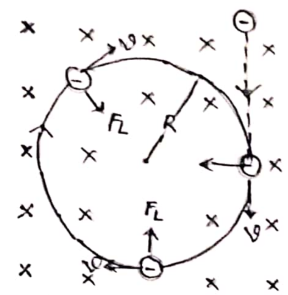


Fig 6: Motion of negatively charged particle in transverse Uniform Magnetic Field

On equating eq^{ns} (1) and (2), $F_L = F_C$ -------------(3)

$$\therefore \frac{mv^2}{R} = Bev \Rightarrow R = \frac{mv}{eB} ----(4)$$

Since all the parameters in the above relation are constant

R = constant

The locus of points located at a constant distance from a point is a circle. **The electron therefore describes a circular path in a plane perpendicular to the magnetic induction lines.**

From eqn (4),

$$R \propto mv$$

And

$$R \propto \frac{1}{B}$$

i.e. **the radius of circular path is directly proportional to the momentum (mv) of the electron and inversely proportional to the magnetic induction.**

Time period for orbital motion is

$$T = \frac{\text{Distance travelled by electron in one revolution}}{\text{Speed of Electron}}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{eB} ----(5) \quad \left(\because \text{As } R = \frac{mv}{eB}\right)$$

and frequency of revolution **f** and angular frequency **ω** of an electron are given as

$$f = \frac{1}{T} = \frac{Be}{2\pi m} \text{ and } \omega = 2\pi f = \frac{eB}{m} --- (6)$$

It is seen from **Eqns. (5)** and **(6)** that

The time period, frequency of revolution and angular frequency of electron are independent of velocity and radius of circular orbit. v and R adjust such that T and f stay constant in a fixed magnetic field B.

It implies that slower particles move in smaller circles while faster particles move in larger circles but all of them take same time for completing one revolution.

However these parameters are strongly dependent on its charge to mass ratio **(e/m)** and the strength of the magnetic field **B.**

CASE-III.

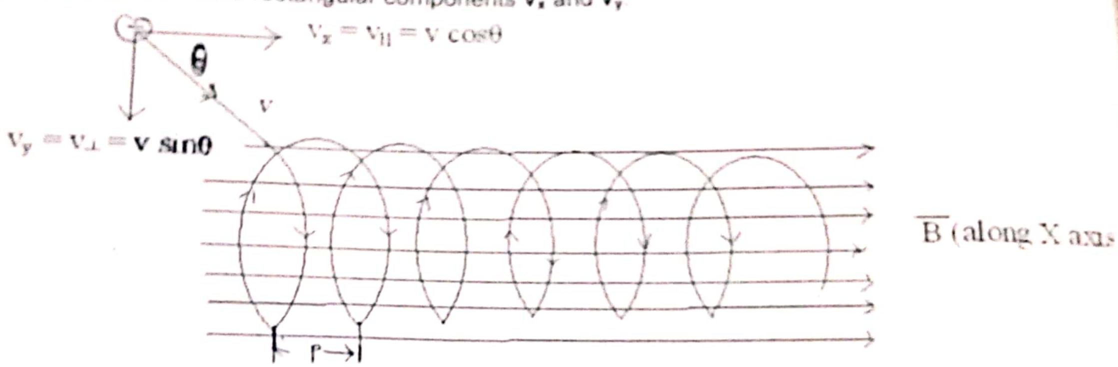
Motion of electron at an angle to uniform Magnetic field:

Q. How can a charged particles be made to travel a helical path in uniform magnetic field. Obtain an expression for pitch of the helix.

OR

Q. When and why a charged particles entering a magnetic field follows a helical path.

Let an electron moving with a uniform velocity **v** enters the magnetic field **B** at an angle **θ** with the field direction **B**. Assuming that the magnetic field B is acting in the X- direction, the electron velocity may be resolved into rectangular components v_x and v_y



1) **Parallel Component of Velocity v_x** : The component of the velocity of the electron parallel to the magnetic induction is

$v_x = v_{||} = v\cos\theta$ is not influenced by the field, as

$$F_{||} = F_{Lx} = e(v_x \times B) = 0 \quad \text{(As } \theta=0\text{)}$$

Hence electron will continue to move with a constant velocity in the X direction i.e. **Rectilinear Motion.**

2) **Perpendicular Component of Velocity v_y:**

The velocity component $v_\perp = v_y = v\sin\theta$ which give rise to a force

$$F_\perp = F_{Ly} = ev_yB = evB\sin\theta$$

Under the action of this force, the electron tends to describe a **circular path** in a plane perpendicular to magnetic field.

The radius of the circular path is given by

$$R = \frac{mv_\perp}{eB} = \frac{mv\sin\theta}{eB} ------(1)$$

Thus the time taken to complete one revolution is given by

$$T = \frac{2\pi R}{v_\perp} = \frac{2\pi R}{v\sin\theta} = \frac{2\pi mv\sin\theta}{eBv\sin\theta}$$

$$T = \frac{2\pi m}{eB} ----------(2)$$

The resultant path described by the electron is obtained by the superposition of the uniform translational motion parallel to B and the uniform circular motion in a plane normal to B. The resultant motion occurs along a helical path (spiral path); the axis of the helix being the field direction.

Pitch of the Helix:

The distance covered by the electron along the magnetic field direction in one revolution is called pitch of Helix.

Thus the pitch of the helix is given by.

$$\therefore P = v_{\parallel} \cdot T = v_x \cdot T = (v\cos\theta)T$$

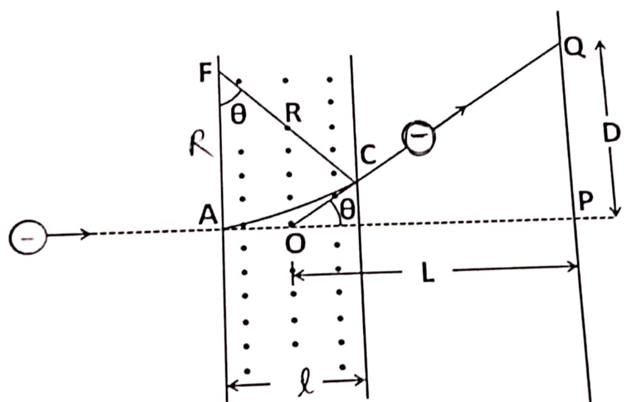
$$\therefore P = \frac{2\pi mv \cos\theta}{eB} \quad ---------- (3)$$

This relation shows that for small values of θ the pitch of the helix is independent of the angle θ. This property can be used for focusing electron beam produced by electron gun in CRT.

Magnetostatic Deflection:

Def^{n}:

The deflection produced in the path of the charged particle by a magnetic field is called magnetostatic deflection.



Suppose an electron is moving in a transverse uniform magnetic field of a limited extension in space. It is assumed that the magnetic lines are emerging out of the page which is indicated by the tips of vectors.

An electron beam travelling in the horizontal direction enters the magnetic field with its velocity perpendicular to the lines of magnetic induction. As the beam travels through the magnetic field, it bends through an arc of radius **R**. Before it completes a revolution, it emerges from the field; subsequently the beam travels along a straight line and strikes the fluorescent screen at point **Q**. In the absence of the magnetic field, the electron beam would strike the screen at point **P**. **PQ** represents the amount of deflection experienced by the electron beam.

Circular arc **AC** subtends an angle **θ** at **F**. **FA** and **FC** represents the radii of circle having its centre at **F**. **AO** and **OCQ** are tangents to the arc **AC** at points **A** and **C** respectively.

$$\therefore \angle POQ = \theta$$

BNp

$$\therefore \tan\theta = \frac{PQ}{OP}$$

$$\therefore PQ = \tan\theta \cdot OP$$

$$\therefore PQ = \tan\theta \cdot L$$

$$\therefore D = L \cdot \tan\theta$$

Where L is the distance of the screen from the centre of the magnetic field.

When θ is small, we can write,

$\therefore D = L\left(\frac{AC}{AF}\right) \Longrightarrow \therefore D \cong L\left(\frac{l}{R}\right)$

Where $l \cong AC$, is the extent of the magnetic field.

Using $R = \frac{mv}{eB}$ and $v = \sqrt{\frac{2eV_A}{m}}$

$$\therefore D = L \cdot l \cdot \frac{eB}{mv} = L \cdot l \cdot \frac{eB}{m}\sqrt{\frac{m}{2eV_A}}$$

$$\therefore D_M = D = L \cdot l \cdot B\sqrt{\frac{e}{2mV_A}}$$

$$\therefore D \propto \frac{1}{\sqrt{V_A}}$$

This shows that magnetic deflection is inversely proportional to the square root of accelerating voltage V_A.

Magnetic Deflection Sensitivity: The magnetic deflection sensitivity S, is defined as the deflection caused by one tesla of magnetic field..

$$S_M = \frac{D}{B} = L \cdot l \cdot \sqrt{\frac{e}{2mV_A}}$$

Deflection Factor: The reciprocal of magnetic deflection sensitivity is called as deflection factor.

$$G_M = \frac{1}{S_m} = \frac{1}{L \cdot l}\sqrt{\frac{2mV_A}{e}}$$

$$S_M \propto \frac{1}{\sqrt{V_A}}$$

Comparing with the electrostatic deflection, it is readily seen that for given value of accelerating voltage the amount of deflection caused by a magnetic field will be more.

Therefore larger area of fluorescent screen can be covered employing magnetic deflection than with the electrostatic deflection. In view of this magnetic deflection is employed in picture tubes of TV, Radar etc.

Electric and Magnetic Field in Crossed Configuration:

Defⁿ: When uniform electric and magnetic fields are perpendicular to each other and act over the same region they are said to be in crossed configuration.

Let two charged plane parallel plates set up a uniform electric field E in the vertical (y-direction) direction and a uniform magnetic induction B is also set up in the same region between the plates in Z-direction.

Let a stream of electrons enter the crossed field configuration with a velocity v. The electric field deflects the electrons upward where as the magnetic field deflects them downward.

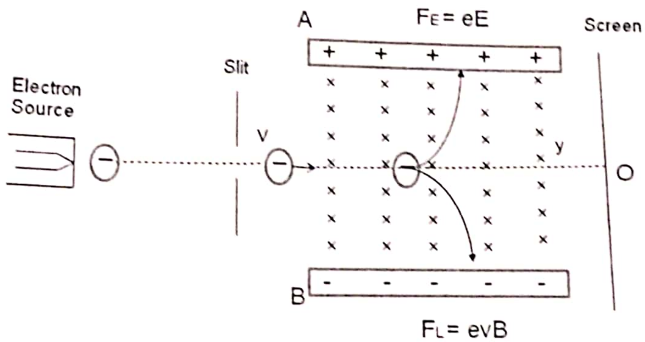


Fig 9: Crossed Field Configuration

The Force due to electric field is

$$F_E = eE ------(1)$$

And the force due to magnetic Field is

$$F_L = evB -------(2)$$

If the magnitudes of the fields E and B are adjusted such that the force they exert on electron become equal, the electron will not experience any force. Thus when

From (1) and (2), $F_E = F_L$

$$eE = evB$$

$$v = \frac{E}{B} ----(3)$$

The electrons experience a zero net force as the two forces balance each other and they will not deviate from their original straight line path and travel without change in the velocity v and strike the screen at point O.

J. J. Thomson used this method in 1897 for the determination of electron beam velocity.

Velocity Filter / Velocity Selector:

Def: **The velocity filter is an electro optic device, which uses uniform electric and magnetic field in crossed configuration for selecting a stream of charged particles of single velocity from a beam of charged particles having a wide range of velocities.**

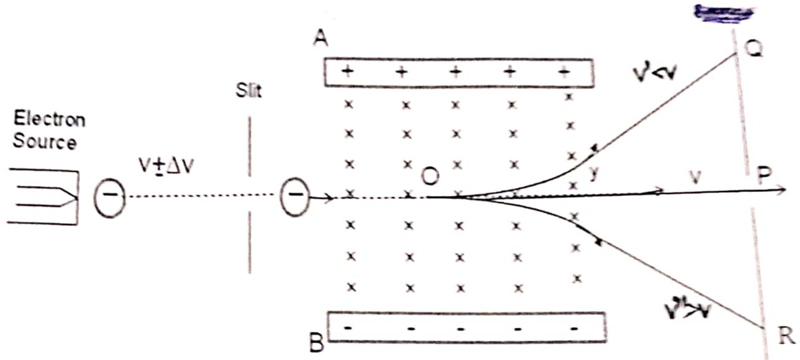


Fig 10: Velocity Filter

Suppose an Electron velocities spread around a central value v enters the crossed field configuration. The electric field E is produced in the vertical direction by a set of charged parallel plate and the uniform magnetic field B is applied perpendicular to it (acting into the page).

If the fields are adjusted such that the electric force balances the magnetic force acting on the electrons moving with velocity v, then those electrons are not deflected and continue to travel along a straight path subsequently they pass through the slit at P.

Electrons moving with a lesser velocity (v'<v) will get deflected upward along OQ due to electric force and those moving with greater velocity (v">v) will get deflected downward along OR due to magnetic force . The electrons deflected away are absorbed by the slit walls.

Thus a strictly homogeneous single velocity electrons beam travelling along OP is obtained with the help of crossed fields. This arrangement is therefore known as a velocity filter or a velocity selector. The velocity filter forms an essential component in Bainbridge Mass Spectrograph.
