$$\frac{E \; \texttt{is\_a} \; \text{string}}{[\![E]\!] = \{ \; \text{stringType()} \; \}}$$

$$\frac{E \ \mathtt{is\_a} \ \text{integer}}{[\![E]\!] = \{ \ \text{integerType()} \ \}}$$

$$\frac{E \ \texttt{is\_a} \ \text{float}}{[\![E]\!] = \{ \ \text{floatType()} \ \}}$$

$$\frac{E \equiv \text{true} \lor E \equiv \text{false}}{\llbracket E \rrbracket = \{ \text{ booleanType() } \}}$$

$$\frac{E \equiv \text{null}}{[\![E]\!] = \{\ \text{nullType()}\ \}}$$

$$\frac{E \equiv (E_1 := E_2)}{\begin{array}{c} [\![E_2]\!] <: [\![E_1]\!], \\ [\![E_1]\!] = [\![E]\!] \end{array}}$$

$$\frac{E \equiv (E_1 \ ? \ E_2 : E_3)}{[\![E]\!] = [\![E_2]\!] \vee [\![E_3]\!]} \ \text{(i)} \qquad \frac{E \equiv (E_1 \ ? : E_3)}{[\![E]\!] = [\![E_1]\!] \vee [\![E_3]\!]} \ \text{(ii)}$$

$$\frac{E_1 \ \& = E_2}{\llbracket E_1 \rrbracket = \{ \ \text{integerType() } \}} \ \text{(i)} \qquad \frac{E_1 \ | = E_2}{\llbracket E_1 \rrbracket = \{ \ \text{integerType() } \}} \ \text{(ii)} \qquad \frac{E_1 \ \hat{} = E_2}{\llbracket E_1 \rrbracket = \{ \ \text{integerType() } \}} \ \text{(iii)}$$

$$\frac{E_1 \ <<= E_2}{\llbracket E_1 \rrbracket = \{ \ \text{integerType() } \}} \ \text{(iv)} \qquad \frac{E_1 \ >>= E_2}{\llbracket E_1 \rrbracket = \{ \ \text{integerType() } \}} \ \text{(v)} \qquad \frac{E_1 \ \% = E_2}{\llbracket E_1 \rrbracket = \{ \ \text{integerType() } \}} \ \text{(vi)}$$

$$\frac{E_1 \mathrel{.=} E_2}{\llbracket E_1 \rrbracket = \{\ \text{stringType()}\ \}}\ \text{(i)} \qquad \frac{(E_1 \mathrel{.=} E_2) \qquad (\llbracket E_2 \rrbracket <: \{\ \text{objectType()}\ \})}{\llbracket E_2 \rrbracket\ \mathsf{hasMethod}\ "\_\_tostring"}\ \text{(ii)}$$

$$\frac{E_1 \ / = E_2}{\llbracket E_1 \rrbracket = \{ \text{ integerType() } \},} \ \text{(i)}$$
$$\llbracket E_2 \rrbracket \neq \{ \text{ arrayType(\_) } \}$$

$$\frac{E_1 - = E_2}{\llbracket E_1 \rrbracket = \{ \text{ integerType() } \},} \ \text{(ii)}$$
$$\llbracket E_2 \rrbracket \neq \{ \text{ arrayType(\_) } \}$$

$$\frac{E_1 \ {}^*{=}\ E_2}{[\![E_1]\!] <: \{\ \mathrm{numberType()}\ \}}\ \text{(i)} \qquad \frac{E_1\ {+}{=}\ E_2}{[\![E_1]\!] <: \{\ \mathrm{numberType()}\ \}}\ \text{(ii)}$$

$$\frac{E \equiv (+E_1)}{\llbracket E \rrbracket <: \{ \text{ numberType() } \}, \quad \llbracket E_1 \rrbracket \neq \{ \text{ arrayType(\_) } \}} \text{ (i)} \qquad \frac{E \equiv (-E_1)}{\llbracket E \rrbracket <: \{ \text{ numberType() } \}, \quad \llbracket E_1 \rrbracket \neq \{ \text{ arrayType(\_) } \}} \text{ (ii)}$$

$$\frac{E \equiv (!E_1)}{[\![E]\!] = \{\text{ booleanType() }\}} \text{ (i)}$$

$$\frac{E \equiv (\sim E_1)}{[\![E_1]\!] = \{\text{ numberType(), stringType() }\},}{[\![E]\!] = \{\text{ integerType(), stringType() }\}} \text{ (ii)}$$

$$\frac{(E \equiv (E_1 + +)) \qquad (\llbracket E_1 \rrbracket <: \{ \text{ arrayType}(\_) \})}{\llbracket E \rrbracket <: \{ \text{ arrayType}(\_) \}} \text{ (i)}$$

$$\frac{(E \equiv (E_1 + +)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ booleanType}() \})}{\llbracket E \rrbracket = \{ \text{ booleanType}() \}} \text{ (ii)}$$

$$\frac{(E \equiv (E_1 + +)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ floatType}() \})}{\llbracket E \rrbracket = \{ \text{ floatType}() \}} \text{ (iii)}$$

$$\frac{(E \equiv (E_1 + +)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ integerType}() \})}{\llbracket E \rrbracket = \{ \text{ integerType}() \}} \text{ (iv)}$$

$$\frac{(E \equiv (E_1 + +)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ nullType}() \})}{\llbracket E \rrbracket = \{ \text{ integerType}(), \text{ nullType}() \}} \text{ (v)}$$

$$\frac{(E \equiv (E_1 + +)) \qquad (\llbracket E_1 \rrbracket <: \{ \text{ objectType}() \})}{\llbracket E \rrbracket <: \{ \text{ objectType}() \}} \text{ (vi)}$$

$$\frac{(E \equiv (E_1 + +)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ resourceType}() \})}{\llbracket E \rrbracket = \{ \text{ resourceType}() \}} \text{ (vii)}$$

$$\frac{(E \equiv (E_1 + +)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ stringType}() \})}{\llbracket E \rrbracket <: \{ \text{ numberType}(), \text{ stringType}() \}} \text{ (viii)}$$

$$\frac{(E \equiv (+ + E_1)) \qquad (\llbracket E_1 \rrbracket <: \{ \text{ arrayType}(\_) \})}{\llbracket E \rrbracket <: \{ \text{ arrayType}(\_) \}} \text{ (i)}$$

$$\frac{(E \equiv (+ + E_1)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ booleanType}() \})}{\llbracket E \rrbracket = \{ \text{ booleanType}() \}} \text{ (ii)}$$

$$\frac{(E \equiv (+ + E_1)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ floatType}() \})}{\llbracket E \rrbracket = \{ \text{ floatType}() \}} \text{ (iii)}$$

$$\frac{(E \equiv (+ + E_1)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ integerType}() \})}{\llbracket E \rrbracket = \{ \text{ integerType}() \}} \text{ (iv)}$$

$$\frac{(E \equiv (+ + E_1)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ nullType}() \})}{\llbracket E \rrbracket = \{ \text{ nullType}() \}} \text{ (v)}$$

$$\frac{(E \equiv (+ + E_1)) \qquad (\llbracket E_1 \rrbracket <: \{ \text{ objectType}() \})}{\llbracket E \rrbracket <: \{ \text{ objectType}() \}} \text{ (vi)}$$

$$\frac{(E \equiv (+ + E_1)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ resourceType}() \})}{\llbracket E \rrbracket = \{ \text{ resourceType}() \}} \text{ (vii)}$$

$$\frac{(E \equiv (+ + E_1)) \qquad (\llbracket E_1 \rrbracket = \{ \text{ stringType}() \})}{\llbracket E \rrbracket <: \{ \text{ numberType}(), \text{ stringType}() \}} \text{ (viii)}$$

$$\frac{E \equiv (E_1 + E_2)}{[\![E]\!] <: \{ \text{arrayType}(\_), \text{numberType}() \}} \text{ (i)}$$

$$\frac{E \equiv (E_1 + E_2) \qquad [\![E_1]\!] <: \{ \text{arrayType}(\_) \} \wedge [\![E_2]\!] <: \{ \text{arrayType}(\_) \}}{[\![E]\!] <: \{ \text{arrayType}(\_) \}} \text{ (ii)}$$

$$\frac{E \equiv (E_1 + E_2) \qquad [\![E_1]\!] \, ! <: \{ \text{arrayType}(\_) \} \vee [\![E_2]\!] \, ! <: \{ \text{arrayType}(\_) \}}{[\![E]\!] <: \{ \text{numberType}() \}} \text{ (iii)}$$

$$\frac{E \equiv (E_1 - E_2)}{\begin{array}{l} \llbracket E \rrbracket <: \{ \text{ numberType() } \}, \\ \llbracket E_1 \rrbracket ! <: \{ \text{ arrayType(\_) } \}, \\ \llbracket E_2 \rrbracket ! <: \{ \text{ arrayType(\_) } \} \end{array}} \text{ (i)} \qquad \frac{E \equiv (E_1 * E_2)}{\begin{array}{l} \llbracket E \rrbracket <: \{ \text{ numberType() } \}, \\ \llbracket E_1 \rrbracket ! <: \{ \text{ arrayType(\_) } \}, \\ \llbracket E_2 \rrbracket ! <: \{ \text{ arrayType(\_) } \} \end{array}} \text{ (ii)}$$

$$\frac{E \equiv (E_1 / E_2)}{\begin{array}{l} \llbracket E \rrbracket <: \{ \text{ numberType() } \}, \\ \llbracket E_1 \rrbracket ! <: \{ \text{ arrayType(\_) } \}, \\ \llbracket E_2 \rrbracket ! <: \{ \text{ arrayType(\_) } \} \end{array}} \text{ (iii)}$$

$$\frac{E \equiv (E_1 \% E_2)}{[\![E]\!] = \{ \text{ integerType() } \}} \text{ (i)} \quad \frac{E \equiv (E_1 << E_2)}{[\![E]\!] = \{ \text{ integerType() } \}} \text{ (ii)} \quad \frac{E \equiv (E_1 >> E_2)}{[\![E]\!] = \{ \text{ integerType() } \}} \text{ (iii)}$$

$$\frac{E \equiv (E_1 \ \& \ E_2)}{[\![E]\!] = \{ \ \text{stringType()}, \ \text{integerType()} \ \}} \ \text{(i)}$$

$$\frac{E \equiv (E_1 \ \& \ E_2) \qquad [\![E_1]\!] = \{ \ \text{stringType()} \ \} \wedge [\![E_2]\!] = \{ \ \text{stringType()} \ \}}{[\![E]\!] = \{ \ \text{stringType()} \ \}} \ \text{(ii)}$$

$$\frac{E \equiv (E_1 \ \& \ E_2) \qquad [\![E_1]\!] \neq \{ \ \text{stringType()} \ \} \vee [\![E_2]\!] \neq \{ \ \text{stringType()} \ \}}{[\![E]\!] = \{ \ \text{integerType()} \ \}} \ \text{(iii)}$$

$$\frac{E \equiv (E_1 == E_2)}{[\![E]\!] = \{ \text{ booleanType() } \}} \text{ (i)} \qquad \frac{E \equiv (E_1 === E_2)}{[\![E]\!] = \{ \text{ booleanType() } \}} \text{ (ii)} \qquad \frac{E \equiv (E_1 \: ! = E_2)}{[\![E]\!] = \{ \text{ booleanType() } \}} \text{ (iii)}$$

$$\frac{E \equiv (E_1 <> E_2)}{[\![E]\!] = \{ \text{ booleanType() } \}} \text{ (iv)} \qquad \frac{E \equiv (E_1 \: ! == E_2)}{[\![E]\!] = \{ \text{ booleanType() } \}} \text{ (v)} \qquad \frac{E \equiv (E_1 < E_2)}{[\![E]\!] = \{ \text{ booleanType() } \}} \text{ (vi)}$$

$$\frac{E \equiv (E_1 > E_2)}{[\![E]\!] = \{ \text{ booleanType() } \}} \text{ (vii)} \qquad \frac{E \equiv (E_1 <= E_2)}{[\![E]\!] = \{ \text{ booleanType() } \}} \text{ (viii)}$$

$$\frac{E \equiv (E_1 >= E_2)}{[\![E]\!] = \{ \text{ booleanType() } \}} \text{ (ix)}$$

$$\frac{E \equiv (E_1 \ and \ E_2)}{[E] = \{ \ \text{booleanType()} \ \}} \ \text{(i)} \qquad \frac{E \equiv (E_1 \ or \ E_2)}{[E] = \{ \ \text{booleanType()} \ \}} \ \text{(ii)} \qquad \frac{E \equiv (E_1 \ xor \ E_2)}{[E] = \{ \ \text{booleanType()} \ \}} \ \text{(iii)}$$

$$\frac{E \equiv (E_1 \ \&\& \ E_2)}{[E] = \{ \ \text{booleanType()} \ \}} \ \text{(iv)} \qquad \frac{E \equiv (E_1 \ || \ E_2)}{[E] = \{ \ \text{booleanType()} \ \}} \ \text{(v)}$$

$$\frac{E \ \mathtt{is\_a} \ \mathrm{array}}{[\![E]\!] <: \{ \ \mathrm{arrayType}(\_) \ \}}$$

$$\frac{E_1[E_2] \qquad E_1 \text{ is\_a arrayAccess}}{[\![E_1]\!] \neq \{ \text{ objectType() } \}} \text{ (i)}$$

$$\frac{E \equiv (E_1[E_2]) \qquad E_1 \text{ is\_a arrayAccess} \qquad [\![E_1]\!] = \{ \text{ stringType() } \}}{[\![E]\!] = \{ \text{ stringType() } \}} \text{ (ii)}$$

$$\frac{E \equiv (E_1[E_2]) \qquad E_1 \text{ is\_a arrayAccess} \qquad [\![E_1]\!] = \{ \text{ arrayType}(E_2) \}}{[\![E]\!] = [\![E_2]\!]} \text{ (iii)}$$

$$\frac{E \equiv (E_1[E_2]) \qquad E_1 \text{ is\_a arrayAccess} \qquad [\![E_1]\!] \neq \{ \text{ stringType() } \} \qquad [\![E_1]\!] \, ! <: \{ \text{ arrayType(\_) } \}}{[\![E]\!] = \{ \text{ nullType() } \}} \text{ (iv)}$$

$$\frac{E \equiv (array)E_1}{[\![E]\!] <: \{ \text{ arrayType}(\_) \}} \text{ (i)} \qquad \frac{E \equiv (\texttt{boolean})E_1}{[\![E]\!] = \{ \text{ booleanType}() \}} \text{ (ii)} \qquad \frac{E \equiv (\texttt{bool})E_1}{[\![E]\!] = \{ \text{ booleanType}() \}} \text{ (iii)}$$

$$\frac{E \equiv (\texttt{float})E_1}{[\![E]\!] = \{ \text{ floatType}() \}} \text{ (iv)} \qquad \frac{E \equiv (\texttt{double})E_1}{[\![E]\!] = \{ \text{ floatType}() \}} \text{ (v)} \qquad \frac{E \equiv (\texttt{real})E_1}{[\![E]\!] = \{ \text{ floatType}() \}} \text{ (vi)}$$

$$\frac{E \equiv (\texttt{integer})E_1}{[\![E]\!] = \{ \text{ integerType}() \}} \text{ (vii)} \qquad \frac{E \equiv (\texttt{int})E_1}{[\![E]\!] = \{ \text{ integerType}() \}} \text{ (viii)} \qquad \frac{E \equiv (\texttt{object})E_1}{[\![E]\!] <: \{ \text{ objectType}() \}} \text{ (ix)}$$

$$\frac{E \equiv (\texttt{string})E_1}{[\![E_1]\!] = \{ \text{ stringType}() \}} \text{ (x)} \qquad \frac{E \equiv (\texttt{string})E_1 \qquad (E_1 <: \{ \text{ objectType}() \})}{[\![E_1]\!] <: \{ \texttt{hasMethod} \text{ "}\_\_tostring\text{" } \}} \text{ (xi)}$$

$$\frac{E \equiv (unset)E_1}{[\![E]\!] = \{ \text{ nullType}() \}} \text{ (xii)}$$

$$\frac{E \equiv \mathtt{clone}(E_1)}{\begin{array}{c} [\![E]\!] <: \{ \text{ objectType() } \} \\ [\![E_1]\!] <: \{ \text{ objectType() } \} \\ [\![E]\!] = [\![E_1]\!] \end{array}}$$

$$\frac{E \equiv new\ C_1()}{[\![E]\!] = \{\ \text{classType(C.decl)}\ \}}\ \text{(i)}$$

$$\frac{E \equiv new\ E_1}{[\![E]\!] <: \{\ \text{objectType()}\ \},}\ \text{(ii)}$$
$$[\![E_1]\!] <: \{\ \text{objectType()},\ \text{stringType()}\ \}$$

$$\frac{(E \equiv \texttt{self}) \in C}{\llbracket E \rrbracket <: \{ \text{objectType() } \},} \text{ (i)}$$
$$\llbracket E \rrbracket :> \{ \text{classType(C) } \}$$

$$\frac{(E \equiv \texttt{static}) \in C}{\llbracket E \rrbracket <: \{ \text{objectType() } \},} \text{ (ii)}$$
$$\llbracket E \rrbracket :> \{ \text{classType(C) } \}$$

$$\frac{(E \equiv \texttt{\$this}) \in C}{\llbracket E \rrbracket <: \{ \text{objectType() } \},} \text{ (iii)}$$
$$\llbracket E \rrbracket :> \{ \text{classType(C) } \}$$

$$\frac{(E \equiv \texttt{parent}) \in C}{\llbracket E \rrbracket <: \{ \text{objectType() } \},} \text{ (iv)}$$
$$\llbracket E \rrbracket :> \{ \text{classType(C) } \}, \llbracket E \rrbracket \mathrel{!=} \{ \text{classType(C) } \}$$

$$\frac{E_1 \to E_2 \in C \qquad E_2 \; \texttt{is\_a} \; \mathrm{expression}}{[\![E_1]\!] <: \{ \; \mathrm{objectType}() \; \}} \; (\mathrm{i})$$

$$\frac{\dfrac{E_1 \to E_2 \in C \qquad E_2 \; \texttt{is\_a} \; \mathrm{name}}{[\![E_1]\!] <: \{ \; \mathrm{objectType}() \; \}}}{[\![E_1]\!] = C.\mathrm{hasMethod}(E_2.\mathrm{name}, \mathrm{static} \notin \mathrm{Mfs})} \; (\mathrm{ii})$$

$$\frac{E \qquad E \text{ is\_a variable}}{[\![E_{definition}]\!] = [\![E_{loction}]\!]}$$

$$\frac{\text{E is\_a return} \not\subseteq f}{[\![f]\!] = nullType()} \text{ (i)} \qquad \frac{\text{is\_a return E} \subseteq \text{f} \qquad \text{E is\_a noExpr}}{[\![f]\!] = nullType()} \text{ (ii)}$$

$$\frac{(\text{return } E_1) \vee \cdots \vee (\text{return } E_k) \subseteq f \qquad E_{1\ldots k} \text{ is\_a someExpr}}{[\![f]\!] <: [\![E_1]\!] \vee \cdots \vee [\![E_k]\!]} \text{ (iii)}$$