
C335

Computer Structures

Memory Hierarchies (II)

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Adapted from Morgan Kaufmann and others

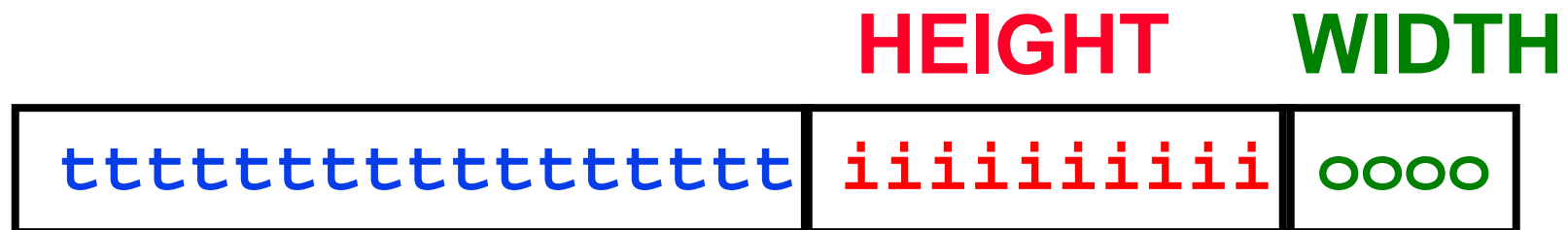
Direct-Mapped Cache Terminology

- ❑ All fields are read as unsigned integers.
- ❑ Index: specifies the cache index (which “row” of the cache we should look in)
- ❑ Offset: once we’ve found correct block, specifies which byte within the block we want -- I.e., which “column”
- ❑ Tag: the remaining bits after offset and index are determined; these are used to distinguish between all the memory addresses that map to the same location

Issues with Direct-Mapped

| Tag | Index | Offset |
|-----|-------|--------|
|-----|-------|--------|

- ❑ Since multiple memory addresses map to same cache index, how do we tell which one is in there?
- ❑ What if we have a block size > 1 byte?
- ❑ Answer: divide memory address into three fields



tag
to check
if have
correct block

index
to
select
block

byte
offset
within
block

Memory address: TIO

AREA (cache size, B)

= HEIGHT (# of blocks)

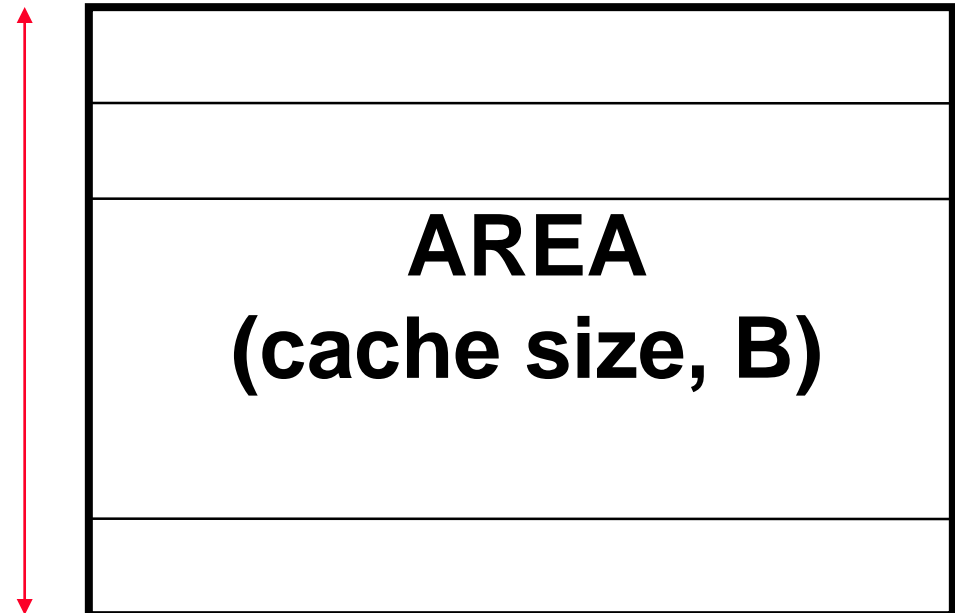
* WIDTH (size of one block, B/block)

$$2^{(I+O)} = 2^I * 2^O$$



WIDTH
(size of one block, Bytes/block, 2^O)

HEIGHT
(# of blocks, 2^I)



Caching Terminology

- ❑ When we try to read memory, 3 things can happen:
 1. cache hit:
cache block is valid and contains proper address, so read desired word
 2. cache miss:
nothing in cache in appropriate block, so fetch from memory
 3. cache miss, block replacement:
wrong data is in cache at appropriate block, so discard it and fetch desired data from memory (cache always copy)

Accessing data in a direct mapped cache

- ❑ Ex.: 16KB of data, direct-mapped, 4 word blocks
- ❑ Read 4 addresses
 1. 0x00000014
 2. 0x0000001C
 3. 0x00000034
 4. 0x00008014
- ❑ Memory values on right:

Memory

| Address (hex) | Value of Word |
|---------------|---------------|
|---------------|---------------|

| | |
|-----------------|-----|
| ... | ... |
| 00000010 | a |
| <u>00000014</u> | b |
| 00000018 | c |
| <u>0000001C</u> | d |

| | |
|-----------------|-----|
| ... | ... |
| 00000030 | e |
| <u>00000034</u> | f |
| 00000038 | g |
| 0000003C | h |

| | |
|-----------------|-----|
| ... | ... |
| 00008010 | i |
| <u>00008014</u> | j |
| 00008018 | k |
| 0000801C | l |

Accessing data in a direct mapped cache



□ 4 Addresses:

- 0x00000014, 0x0000001C,
0x00000034, 0x00008014

□ 4 Addresses divided (for convenience) into Tag, Index, Byte Offset fields

| | | |
|----------------------|------------|--------|
| 00000000000000000000 | 0000000001 | 0100 |
| 00000000000000000000 | 0000000001 | 1100 |
| 00000000000000000000 | 0000000011 | 0100 |
| 00000000000000000010 | 0000000001 | 0100 |
| Tag | Index | Offset |

16 KB Direct Mapped Cache, 16B blocks



- ❑ Valid bit: determines whether anything is stored in that row (when computer initially turned on, all entries invalid)

Valid

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| 1 | 0 | | | | |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | ... | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

1. Read 0x00000014

00000000000000000000 0000000001 0100

Tag field

Index field

Offset

Valid

| Index | Valid | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | | |
| 1 | 0 | | | | | |
| 2 | 0 | | | | | |
| 3 | 0 | | | | | |
| 4 | 0 | | | | | |
| 5 | 0 | | | | | |
| 6 | 0 | | | | | |
| 7 | 0 | | | | | |
| ... | | | ... | | | |
| 1022 | 0 | | | | | |
| 1023 | 0 | | | | | |

So we read block 1 (0000000001)

□ 000000000000000000000000 0000000001 0100

Valid **Tag field** **Index field** **Offset**

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|----------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| <u>1</u> | 0 | | | | |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | ... | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

No valid data

❑ 000000000000000000000000 0000000001 0100

Tag field

Index field

Offset

Valid

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|----------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| <u>1</u> | 0 | | | | |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

So load that data into cache, setting tag, valid

□ 00000000000000000000 0000000001 0100

Tag field

Index field

Offset

Valid

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| 1 | 1 | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | ... | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

Read from cache at offset, return word b

□ 000000000000000000000000 00000000001 0100

Valid
Tag field

Index field Offset

| Index | Tag | 0x0-3 | <u>0x4-7</u> | 0x8-b | 0xc-f |
|----------|-----|-------|--------------|-------|-------|
| 0 | 0 | | | | |
| <u>1</u> | 0 | a | <u>b</u> | c | d |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | ... | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

2. Read 0x0000001C = 0...00 0..001 1100

00000000000000000000 0000000001 1100

Tag field
Valid

Index field

Offset

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| 1 | 1 | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | ... | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

Index is Valid



000000000000000000000000 0000000001 1100

Tag field
Valid

Index field

Offset

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|----------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| <u>1</u> | 0 | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | ... | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

Index valid, Tag Matches



□ 00000000000000000000 0000000001 1100

Tag field
Valid

Index field

Offset

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| 1 | 0 | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | ... | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

Index Valid, Tag Matches, return d



□ 00000000000000000000 0000000001 1100

Tag field
Valid

Index field

Offset

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| 1 | 0 | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |

...

...

| | | | | | |
|------|---|--|--|--|--|
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

3. Read 0x00000034 = 0...00 0..011 0100

☐ 000000000000000000000000 0000000011 0100
 Valid Tag field Index field Offset

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| 1 | 1 | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | ... | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

So read block 3

000000000000000000000000 0000000011 0100

Valid Tag field Index field Offset

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| 1 | 1 | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | ... | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

No valid data

❑ 000000000000000000000000 0000000011 0100

Valid **Tag field** **Index field** **Offset**

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| 1 | 1 | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 0 | | | | |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |

...

...

| | | | | | |
|------|---|--|--|--|--|
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

Load that cache block, return word f

□ 00000000000000000000 0000000011 0100

Valid Tag field Index field Offset

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | |
| 1 | 0 | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 0 | e | f | g | h |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |

...

...

| | | | | | |
|------|---|--|--|--|--|
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

4. Read 0x00008014 = 0...10 0..001 0100

0000000000000000000010 0000000001 0100

Valid **Tag field** **Index field** **Offset**

| Index | Valid | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | | |
| 1 | 1 | 0 | a | b | c | d |
| 2 | 0 | | | | | |
| 3 | 1 | 0 | e | f | g | h |
| 4 | 0 | | | | | |
| 5 | 0 | | | | | |
| 6 | 0 | | | | | |
| 7 | 0 | | | | | |
| ... | | | ... | | | |
| 1022 | 0 | | | | | |
| 1023 | 0 | | | | | |

So read Cache Block 1, Data is Valid

□ 0000000000000000000010 0000000001 0100

Valid **Tag field** **Index field** **Offset**

| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|----------|----------|-------|-------|-------|-------|
| 0 | 0 | | | | |
| <u>1</u> | <u>0</u> | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 1 | e | f | g | h |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |

...

...

| | | | | | |
|------|---|--|--|--|--|
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

Cache Block 1 Tag does not match (0 != 2)

□ 000000000000000000010 00000000001 0100

Valid Tag field Index field Offset

Index Tag 0x0-3 0x4-7 0x8-b 0xc-f

| | | | | | |
|----------|----------|---|---|---|---|
| 0 | 0 | | | | |
| <u>1</u> | <u>0</u> | a | b | c | d |
| 2 | 0 | | | | |
| 3 | 1 | e | f | g | h |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |

...

...

| | | | | | |
|------|---|--|--|--|--|
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

Miss, so replace block 1 with new data & tag

□ 000000000000000000010 0000000001 0100

| Valid | Tag field | Index field | | | Offset |
|-------|-----------|-------------|-------|-------|--------|
| Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | | | | |
| 1 | 1 | 2 | i | j | k |
| 2 | 0 | | | | |
| 3 | 1 | 0 | e | f | g |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |

...

...

| | | | | | |
|------|---|--|--|--|--|
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

And return word j

☐ 0000000000000000000010 00000000001 0100

Valid Tag field

Index field Offset

| Index | Tag | 0x0-3 | <u>0x4-7</u> | 0x8-b | 0xc-f |
|-------|-----|-------|--------------|-------|-------|
| 0 | 0 | | | | |
| 1 | 1 | 2 | i | k | l |
| 2 | 0 | | | | |
| 3 | 1 | 0 | e | f | g |
| 4 | 0 | | | | |
| 5 | 0 | | | | |
| 6 | 0 | | | | |
| 7 | 0 | | | | |
| ... | | | ... | | |
| 1022 | 0 | | | | |
| 1023 | 0 | | | | |

Do an example yourself. What happens?

❑ Chose from: Cache Hit, Miss, Miss w. replace

Values returned: a ,b, c, d, e, ..., k, l

❑ Read address 0x00000030 ?

00000000000000000000 0000000011 0000

❑ Read address 0x0000001c ?

00000000000000000000 0000000001 1100

Cache

| Valid | Index | Tag | 0x0-3 | 0x4-7 | 0x8-b | 0xc-f |
|-------|-------|-----|-------|-------|-------|-------|
| 0 | 0 | | | | | |
| 1 | 1 | 2 | i | j | k | l |
| 0 | 2 | | | | | |
| 1 | 3 | 0 | e | f | g | h |
| 0 | 4 | | | | | |
| 0 | 5 | | | | | |
| 0 | 6 | | | | | |
| 0 | 7 | | | | | |

Answers



❑ 0x00000030 a hit

Index = 3, Tag matches, Offset = 0,
value = e

❑ 0x0000001c a miss

Index = 1, Tag mismatch, so replace
from memory,
Offset = 0xc, value = d

❑ Since reads, values
must = memory values
whether or not cached:

- 0x00000030 = e
- 0x0000001c = d

Memory

| Address | Value of Word |
|---------|---------------|
|---------|---------------|

| | |
|-----------------|-----|
| ... | ... |
| 00000010 | a |
| 00000014 | b |
| 00000018 | c |
| <u>0000001c</u> | d |

| | |
|-----------------|-----|
| ... | ... |
| <u>00000030</u> | e |
| 00000034 | f |
| 00000038 | g |
| 0000003c | h |

| | |
|----------|-----|
| ... | ... |
| 00008010 | i |
| 00008014 | j |
| 00008018 | k |
| 0000801c | l |

Another Example: (1/2)



Here is a series of address references given as **word addresses**: 2, 3, 6, 10, 7, 12, 2, 18, 11, and 3. Assuming a direct-mapped cache with 8 **two-word blocks** (total size 16 words) that is initially empty, label each reference in the list as a hit or a miss and show the final contents of the cache. (**note: the basic unit is word!**)

| Tag | Index | Data (two-word block) |
|-----|-------|-----------------------|
| | 000 | |
| | 001 | |
| | 010 | |
| | 011 | |
| | 100 | |
| | 101 | |
| | 110 | |
| | 111 | |

2 (00000010), 3(00000011), 6(00000110), 10(00001010), 7(00000111),
12(00001100), 2(00000010), 18(00010010), 11(00001011), 3(00000011)

Another Example: (2/2)



| Tag | Index | Data (two-word block) |
|-------------|-------|-----------------------|
| | 000 | |
| 0000 → 0001 | 001 | [2,3] → [18,19] |
| | 010 | |
| 0000 | 011 | [6, 7] |
| | 100 | |
| 0000 | 101 | [10, 11] |
| 0000 | 110 | [12, 13] |
| | 111 | |

2 (00000010) → miss, 3(00000011) → hit, 6(00000110) → miss,
10(00001010) → miss, 7(00000111) → hit, 12(00001100) → miss,
2(00000010) → hit, 18(00010010) → miss, 11(00001011) → hit,
3(00000011) → ?

Note: we did not present the valid bit in the table.

And in Conclusion...

❑ Direct-Mapped Cache Terminology

- Divide memory address into three field
 - Tag, Index, and Offset
- Each cache block contains Index, Tag, and Valid bit
- To check if a memory unit has been “cached” or not, using the Index field to locate a cache block (row), if Valid bit is 1, compare its Tag field with the one contained in the cache block (row).

