

1 Tidal Evolution of the Earth Moon System

For this project, python code was written to integrate the evolution of the earth and moon from when they were formed to the present day. The overall goal should show that about a billion years in the past, the two bodies collided, ie the earth moon separation goes to zero.

2 Questions

2.1 Question 1

For question one, the objective was to calculate The orbital angular momentum of the earth (L_e), The spin angular momentum of the earth (S_e) and the The orbital angular momentum of the moon (L_m).

The orbital angular momentum of the earth is given as:

$$\begin{aligned} L_e &= M_e \sqrt{(G(M_s + M_e)a_e)} \\ L_e &= 5.97 * 10^{27} \sqrt{(6.6710^{-8} - 1.988510^{33} + 5.9710^{27}) * 1.4910^{11}} \\ L_e &= 2.65395669310^{46} gcm^2 s^{-1} \end{aligned}$$

This is the angular momentum of the earth due to its revolution. It is calculated through $L = mvr$ Where m = mass of the earth v = velocity of the earth, r = the radius of the orbit The formula considers the earth as a point of mass for circular orbit.

The spin angular momentum of the earth is given as:

$$\begin{aligned} S_e &= 0.3299 * M_e R_e^2 \Omega_e \\ S_e &= 0.32995.97 * 10^{27} * 6371000^2 * 7.29 * 10^{-5} \\ S_e &= 5.829410^{36} gcm^2 s^{-1} \end{aligned}$$

This value is given by the earth's moment of inertia multiplied by the angular velocity of the earth. It is as a result of the earth spinning around its own axis which generates momentum due to its weight and velocity.

The orbital angular momentum of the moon is given as:

$$\begin{aligned} L_m &= M_m \sqrt{(G(M_e + M_m)a_m)} \\ L_m &= 7.34910^{25} \sqrt{(6.67 * 10^{-8} * (5.97 * 10^{27} + 7.34910^{25}) * 3.34 * 10^8)} \\ L_m &= 2.696548908 * 10^{40} gcm^2 s^{-1} \end{aligned}$$

This momentum comes from the moons orbiting around the earth. The Moon is a uniformly dense sphere (it isn't) of radius $1.7375 * 10^3 km$, The Moon's mass M is $7.3459 * 10^{22} kg$, The Moon's orbital period T is 27.322 days, or $2.3606 * 10^6 s$, The Moon's rotational period T is 27.322 days, or $2.3606 * 10^6 s$, and The Moon's orbit is circular (it isn't), and is 385,000 km in radius r . The moon's angular velocity (both for its orbit and its rotation, since its tidally locked in a 1:1 orbit)

2.2 Question 2

Tidal torque of the earth is the result of adding tidal force on the mass of the earth caused by tidal waves brought about by the earth-moon relation. This is enough to significantly reduce the angular velocity of the earth. It is given by:

$$T_e = \frac{3GM_e^2}{2a_e} \left(\frac{R_e}{a_e}\right)^2 \frac{K_2}{Q_e} T_e = -4.94 \times 10^{15} Nm$$

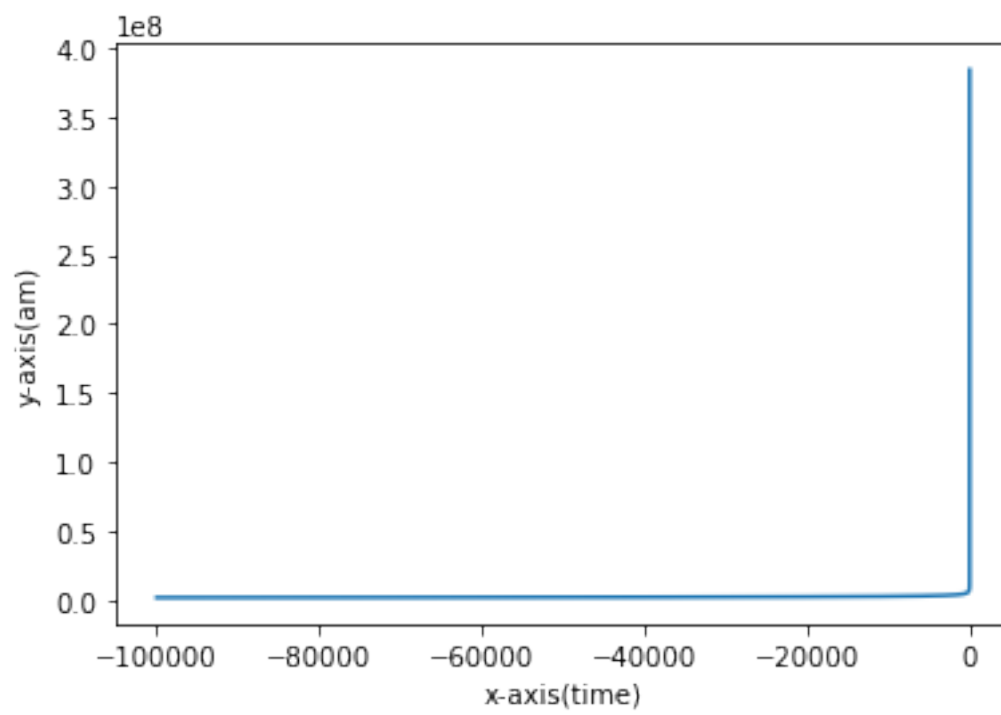
The tidal torque due to the moon is given by :

$$T_m = \frac{3GM_m^2}{2a_m} \left(\frac{R_m}{a_m}\right)^2 \frac{K_2}{Q_m} T_m = 2.569510^{31} gcm^2 s^{-2}$$

2.3 Question 3

From equation 1 time scale is $\frac{L_e}{T_s} = 4.85410^{15}s = 153.919\text{million years}$ From equation 2 time scale is $\frac{S_e}{T_s+T_m} = -187067.45s = -0.00593 \text{ years}$ From equation 3 time scale is $\frac{L_m}{T_m} = 1.04910^9s = 33\text{years}$

2.4 Q6



2.5 Question 8

Roche Radius is the distance from a celestial body, within which a secondary celestial body will begin to disintegrate as a result of the former celestial body tidal forces being in excess of the secondary's body gravitational attraction.

$$A_c = 2.44 \left(\frac{M_e}{M_m} \right)^{\frac{1}{3}} \frac{R_m^2}{R_e}$$

$$A_c = 5000 km$$

The Roche radius is given by $9492 \text{ km} / 5000 \text{ km} = 1.8984$ This means that the length of the day was about 2 hours

2.6 Question 9

The age of Earth is estimated to be 4.54 billion years with an accuracy of + or - 50 million years. The Moon is speculated to be 4.53 billion years old. The moon is considered to be a part of the earth that detached itself post collision between the earth and another celestial body. This only occurred when the earth was 50 million years old which is reflective of the earth's age in relation to the moon. Here it is noted that the two bodies are older than as expressed by the tidal wave equations. The moon's age can be used to tell how old the earth is.