**SUPPLEMENTARY INFORMATION** **for**

Atypically larger variability of resource allocation accounts for visual working memory deficits in schizophrenia

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**Supplementary Note 1: Computational models of VWM**

**Variable-precision model**. The variable-precision (VP) model has been shown as the state-of-the-art computational model of VWM. Details of the VP model have been documented in several previous studies 1,2 and the model codes are publicly available (<http://www.cns.nyu.edu/malab/resources.html>).

The VP model assumes a resource decaying function describing the decreasing trend of mean memory resource () assigned to individual items as the set size (*N*) increases 3,4:

 , (S1)

where  is the initial resources when only 1 item (*N* = 1) should be memorized and *a* is the decaying exponent. The key component of the VP model is that the memory resources  across items and trials follow a Gamma distribution with the mean and the scale parameter :

 , (S2)

Intuitively, a larger  indicates a more uneven distribution of memory resources across items or trials, with some items in some trials receiving a larger amount of resources while others receive comparative fewer. Note that a larger amount of memory resource produces a higher precision. Thus, we do not explicitly distinguish resource and precision and denote them as *J.* Defining precision as Fisher information 5, precision  can be linked to the variance of the von Mises distribution of sensory measurement:

, (S3)

where  and  are modified Bessel functions of the first kind of order 0 and 1 respectively, with the concentration parameter . Eq. S3 specifies a one-on-one mapping between precision  and variance . We can rewrite their relationship as:

 , (S4)

where  is the mapping function. The distribution of sensory measurement (*m*) given the input stimulus (*s*) can be written as:

, (S5)

We further assume that the reported color () by participants also follows a von Mises distribution:

, (S6)

where  represents the variability at the choice stage.

Given the four free parameters and stimulus color  in a trial, we can derive the probability of the observed response in a trial by marginalizing over sensory measurement  and variable precision :

  ，(S7)

Note that in Eq. S7, sensory measurement (*m*) can be analytically eliminated. Since precision is a random variable across items and trials, we sampled it 10000 times from the Gamma distribution with mean  and scale parameter . Note that van den Berg *et al*. 1 confirmed that 500 samples are enough in the model fitting. We then used all the samples to calculate response probability in each trial.

Taken together, this VP model has four free parameters: , *a*, and .

**Variable-precision-with-capacity model**. The variable-precision-with-capacity (VPcap) model inherits all parameters and the structure of the VP model above, except that an additional capacity parameter (*K*) is introduced to estimate the memory capacity of individuals. If the set size *N* is smaller than capacity *K*, the VPcap model is identical to the VP model. If the set size *N* exceeds the capacity *K*, the model assumes that the probe is stored in the VWM with the probability *K*/*N*, and out of memory with the probability 1- *K*/*N*. In the latter case, a participant randomly guesses a color. The response probability therefore can be written as:

, (S8)

where  is defined in Eq. S7. In essence, the VPcap model is a mixture model of the VP model and a random guessing process when the set size exceeds the participant’s capacity. The VPcap model has five parameters, four as the same in the VP model and the additional capacity parameter (*K*).

**Item-limit model***.* The item-limit (IL) model assumes no uncertainty in the sensory encoding stage such that the internal sensory measurement *m* is equal to the input stimulus *s*. But there exists choice variability from measurement *m* to the reported color (). Such choice variability does not vary across set size levels. The IL model also assumes a fixed capacity *K.* The response probability is:

 , (S9)

The IL model has two free parameters: choice variability , and capacity *K*.

**Mixture model***.* The mixture model (MIX) has been used in previous clinical research 6. Similar to the IL model, the MIX model only assumes the uncertainty from stimulus *s* to the reported color () and a fixed capacity *K.* The difference is that the uncertainty () reflects both sensory noise and choice variability, and thus the uncertainty is set-size dependent (each set size has one ). The response probability can be written as:

, (S10)

where and denote the uncertainty for set size 1 and 3, respectively. The MIX model has three parameters: uncertainty levels  and , and capacity *K.*

**Slots-plus-averaging model**. The slots-plus-averaging (SA) model was originally proposed in 7 and further elaborated in 1. Unlike the IL model, the SA model acknowledges the presence of noise in the sensory encoding stage. However, the memory resources are discrete chunks, and a single chunk or multiple chunks can be assigned to one item. For one item, the SA model assumes Eq. S4 still holds as the relationship between the resource assigned to that item and the width of the von Mises distribution:

 , (S11)

where *S* is the number of chunks and *Js* is the resource of one chunk. The SA model also assumes a capacity *K*.

When *N* > *K*, an item should receive either 0 or 1 chunk. Then the allocation should be similar to the IL model. the response distribution should be a mixture of a uniform and a von Mises distributions:

, (S12)

When N ≤ K, some items receive either one or more chunks. Assuming that the resource chunks should be assigned as equally as possible across items, the *S* can be calculated as:

, (S13)

where  represents the *floor* function in Matlab. The corresponding concentration parameter of von Mises distributions can be computed by Eqs. S11&13:

 (S14)

The response probability in the SA model can be written as:

, (S15)

The SA model has three free parameters: unit resource *Js*, choice variability , and capacity *K.*

**Cosine slots-plus-averaging model**. A recent paper 8 suggests that a modified version of the SA model, dubbed cosine slots-plus-average model (cosSA), outperformed the VP model to explain the delay-matching VWM behavior. To enhance the generality of our study, we also followed that work and included this model. Briefly, the cosSA model assumes that the unit memory precision is stimulus-dependent and exhibits a cosine-like periodic fluctuation:

, (S16)

where and  describe the fluctuation of unit memory precision () as a function of stimulus *s*. Note that the frequency of the cosine function was derived from the cosine-shaped bias found in our empirical data. We can convert precision  to the width of von Mises distributions according to Eq. S4. According to capacity *K*, the discrete memory resource allocation is described as Eq. S11-S14. Moreover, the cosSA model also assumes the response bias is periodic:

, (S17)

where  adjusts the magnitude of the bias. The probability of a response given the stimulus can be described as:

 , (S18)

The cosSA model has four free parameters: ,, and capacity *K*.

**Equal-precision model**. The equal-precision (EP) model is very similar to the VP model, except that an equal amount of resources is assigned to every item and in any trial. Namely, the Eq. S2 does not apply to the EP model. In the EP model, the resource assigned to one item declines as a power function (as Eq. S1). Then the resource at each set size level can be converted to the width of the von Mises distribution using (Eq. S4). The response probability is given by:

, (S19)

where *J1* is the resource when set size is 1 (initial resources). The EP model has three free parameters: initial resources , decaying exponent *a*, and choice variability .

**Supplementary Note 2: Intuitive model explanations**

Despite the mathematical details provided above, we further provide intuitive explanations for each model and highlight their differences based on cartoon illustrations in Supplementary Fig. 1. Note that all stimuli are 0 because we transformed the reported color to recall errors in each trial.

**Item-limit model.** In the IL model (Supplementary Fig. 1A), if the capacity *K* is larger than the set size N (e.g., N=2, K=3, the left panel), all items can enter working memory. The reported color follows a von Mises distribution with the mean as the color of the probed stimulus. If the capacity *K* is smaller than the set size *N* (e.g., N=2, K=3, the right panel), a probed stimulus can be stored within memory with probability *K*/*N* and out of memory with probability (1-*K*/*N*). If the probed stimulus is in memory, the same rule of von Mises distribution applies. If the probed stimulus is out of memory, a subject guesses a color (i.e., with probability 1/2π, the uniform distribution of guessing).

**Mixture model.** The mixture model (Supplementary Fig. 1B) shares all components with the IL model. The key difference is that the IL model assumes the same von Mises distribution for both set size levels (i.e., same width of the blue and the orange distributions in Supplementary Fig. 1A), while the mixture model uses two von Mises distributions with different widths for the two set size levels (i.e., different widths of the blue and the orange distributions in Supplementary Fig. 1B), to compensate the potential different level uncertainty associated with two set size levels. Thus, the mixture model has one additional free parameter than the IL model.

**Slot-plus-averaging and cosine slot-plus-averaging model.** The SA model regards memory resources as several discrete chunks (Supplementary Fig. 1C). In the example of Supplementary Fig. 1C, the subject has three (*K* = 3) chunks of resources and the blue cups stand for individual stimulus. If two stimuli are presented (i.e., two cups, set size = 2), the scenario in which the number of resource chunks is larger than the set size, two resource chunks are assigned to one cup and another chunk to the other cup. If the number of resources is smaller than the set size (e.g., four stimuli/cups), one cup will receive no resource, and the subject has to guess if this stimulus/cup is probed. The key difference between the SA model and the three models below is that the SA model assumes discrete resource chunks.

The cosSA model is a modified version of the SA model with three major changes 8. First, the unit memory precision is stimulus-dependent and follows a periodic function (see Eq. S16 and Fig. S1D). Second, it also includes a response bias that is also stimulus-dependent and periodic (see Eq. S17 above and Fig. S1D). Third, for simplicity it does not include the response variability and only includes one uncertainty (i.g., encoding precision) in the processing.

**Equal-precision, variable-precision and variable-precision-with-capacity models.** The EP, VP and VPcap models share one core assumption: memory resources are continuous, analogous to the amount of juice in a big mug (Supplementary Fig. 1E). A subject assigns the juice (i.e., resources) into different cups (i.e., stimuli). In Supplementary Fig. 1E, the orange cups stand for the mean juice amount an individual cup receives in each set size condition. We can imagine that, given the total amount of juice is fixed, the more cups (i.e., larger set size) the less juice on average each cup will receive. This is reflected by the diminishing average amount of juice as set size increases (also see Eq. S1).

Besides the core assumption of continuous resources, the three models have slightly different specifications (Supplementary Fig. 1F). In Supplementary Fig. 1F, all orange cups stand for the mean juice amount in each set size condition, and the blue cups stand for individual stimulus. The EP model assumes that in each set size condition, each cup receives an identical amount of juice (upper row in Supplementary Fig. 1F). In the VP model, however, each cup receives a variable amount of juice even though their average amount is the same as in the EP model. Using two cups as an example, the average amount of juice might be 10 ml but one cup might have 9 ml and the other one has 11 ml. Whether the amount of juice in each cup varies is the key difference between the EP and the VP models. Moreover, both EP and VP models do not constrain the total number of cups. Therefore, a cup will more or less receive a little bit juice even though there is a large number of cups (middle row). In other words, both the EP and the VP models have no concept of capacity. In contrast, the VPcap model not only inherits the assumption of variable precision and but also constraints the maximal number of cups (i.e., capacity *K*) that can receive juice. If the total number of cups (i.e., *N* stimuli) is larger than the capacity *K*, some cups will receive no juice, and the subject has to guess the color of these stimuli.

**Supplementary Note 3: Model fitting and comparisons**

**Model fitting**. The BADS optimization toolbox in MATLAB 9 was used to search the best-fit parameters that maximize the likelihood of response data in all trials. BADS has been shown to outperform other default nonlinear optimization algorithms in MATLAB, especially in the problems where gradients on loss function are not available or hard to compute 9. We fit all models separately in each participant. To avoid local minima, we repeated the optimization process with 20 different initial seeds that are equally spaced within a lower and an upper bound. Parameters bounds were set to be very broad to avoid bias. The parameters with the maximum likelihood value were used as the best-fit parameters for one subject.

**Model comparisons.** We compared the performance of all models fitted in this study. Model comparisons were performed for both groups using both Akaike information criterion (AIC) and Bayesian information criterion (BIC) 10,11 metrics (Supplementary Fig. 1). We derived the best model for each subject. Results showed that the VP model outperformed all other models over 84% of subjects in both groups under both AIC and BIC (Supplementary Fig. 2). Particularly, the VP model is the best-fitting model in 51 out of 61 (84%) HC and in 55 out of 60 SZ (92%) under the AIC. Using the BIC, the VP model is the best-fitting model in 52 out of 61 HC (85%) and 54 out of 60 (90%) SZ. These results strongly support the idea that the VP model assuming no fixed capacity better explains the VWM behavior. This result also questions the conventional theory whether capacity acts as a key determinant of limiting VWM performance in SZ.

**Supplementary Note 4: Control analyses and experiments**

**Control analyses.** One might argue that the SA, cosSA, and MIX models were worse than the VP model because AIC and BIC overly penalize the capacity parameter *K* while this parameter may not substantially improve the goodness of fitting because of low set size levels (i.e., 1/3) used here. To exclude this possibility, we further compared the SA, cos SA, and MIX models to the VP model using AIC and BIC metric but without considering the capacity *K*— that is, we kept the likelihood of the model fitting with *K* but calculated AIC and BIC without *K*. In this case, the models fully enjoyed the potential benefits endowed by *K* in model fitting but avoided overly penalizing this additional parameter. Results showed that the VP model was still the best-fitting model in the majority of subjects in both groups and under both metrics (AIC, 51 out of 61 in the HC group and 43 out of 60 in the SZ group; BIC, 52 out of 61 in the HC group and 45 out of 60 in the SZ group).

**Control experiment: color delay-estimation task.** We repeated the identical color delay-estimation task on 62 HC subjects (30 females, 19-24 years old). The stimuli and procedure were identical to the task described in the main text except that: 1) the sample array was presented for 200 ms; 2) the set sizes were 2, 4 and 6; 3) there were 100 trials in each set size.

We fitted all 7 models to each individual’s data and performed the model comparison. We found that in the total 62 HC subjects, the VP model was the best in 45 and 54 subjects using the AIC and BIC metrics, respectively.

**Control experiment: orientation delay-estimation task.**

*Subjects.* Data from twenty-six HC and nine SZ were obtained in the orientation delay-estimation task (see Supplementary Table 1 for subject information). The data from the HC subjects has been presented in ref. 12. SZ were all clinically stable inpatients who met the DSM-IV criteria for schizophrenia13. Patients having a history of any other mental or neurological disorders were excluded. All nine patients were receiving second-generation antipsychotic medication. The Positive and Negative Syndrome Scale (PANSS)14 was used to evaluate the psychotic symptoms of the group of patients. This scale includes positive symptomatology, negative symptomatology, as well as general psychopathology symptoms. HC subjects were recruited by advertisement. All HC had no current diagnosis of axis 1 or 2 disorders as well as no family history of psychosis nor substance abuse or dependence. All subjects are right-handed with normal sight.

*Stimuli and procedure.* Stimuli were presented on a 60 Hz LCD monitor through Matlab psychophysics Toolbox (Version 3). In the sample array, all items were shown on an invisible circle with 10o radius. The length and width of each item in the sample array were 3.7o and 0.6o respectively. Then orientations of the bars were randomly chosen from 1o to 180o. The procedure of this task was similar to the color delay-estimation task, except that the sample array was presented for 200 ms (Supplementary Fig. S5). In the probe array, one of the sample bars appeared as the probe, and subjects were required to adjust the orientation of the probed bar using a computer mouse.

The HC subjects completed 5 blocks with 100 trials in each block. The set size of each trial was randomly chosen from 1, 2, 3, 4, 6. The SZ patients complete the experiment in three visit sessions. In each visit, they were asked to finish 10 blocks with 40 trials in each block. The set sizes were 1, 2, 4, 6 and counter-balanced across blocks. One patient only finished 9 blocks in the second visit and another patient only finished 8 blocks in the first visit. In total, seven SZ subjects completed 300 trials for each set size, one SZ completed 290 trials and one SZ completed 280 trials.

*Results.* We fitted the models and performed the same analysis. We found that among 9 SZ subjects, the VP model was the best in 7 subjects using AIC and in all 9 subjects using BIC. Similarly, in 26 HC subjects, the VP model outperformed than other models in 21 and 24 subjects using AIC and BIC, respectively.

Most importantly, we compared the fitted parameters of the two groups. We fully replicated the results in the main experiment: resource allocation variability was statistically higher in the SZ subjects (t(33) = 5.833, p = 1.576-6, d = 2.256) and no significant group differences were detected in other parameters (initial resources, t(33) = 0.437, p = 0.665, d = 0.169; decaying exponent, t(33) = 0.145, p = 0.886, d = 0.056; choice variability, t(33) = 1.651, p = 0.108, d = 0.638).

**Supplementary Note 5: Color perception task and statistical results**

**Color perception task.** Before the main VWM task, all subjects completed a task to measure their color perception ability. The task is identical to the VWM task except for two modifications. First, only one colored object was shown in the sample array. Second, in the probe array, the colored object appeared again on the screen. A subject needed to choose its color on the color wheel while looking at it. There was 1 block with 50 trials in this task.

**Color perception results between HC and SZ.** We used the circular standard deviation (CSD) of response errors (the circular distance between the original color and chosen color in a trial) to evaluate the performance in the color task. A significant group difference was found (t(119) = -2.095, p = 0.038, d = -0.38), suggesting in general worse color perception in SZ. But this result might also be explained by potential differences in choice variability (e.g., motor control). To exclude the potential confounding of color perception, we further set CSD from the color perception as a co-variate and repeat all statistical analyses (see below).

**VWM performance.** We added the CSD in the color perception task as a co-variate to VWM performance comparison of two groups. The repeated-measure ANCOVA (see the main text for details of variables) results again showed a worse VWM performance at higher set size level (F(1,119) = 100.676, p < 0.001, partial η2 = 0.46). The group was also significant (F(1,119) = 8.902, p = 0.003, partial = 0.070), indicating that HC’s performance was better than SZ’s. The interaction between set size and group was not significant (F(1,119) = 0.324, p = 0.570, partial = 0.003). Also, the color perception ability had no influence on VWM performance (F(1,119) = 0.285, p = 0.595, partial = 0.002). These results replicated the results from the main text.

**Fitted parameters of the VP model.** Univariate general linear models were used for comparing fitted parameters between the two groups. We regressed out the factor of color perception by setting. Same as results in the main text (Fig. 5), comparable resource decay functions (Fig. 5A, initial resources, F(1,119) = 0.376, p = 0.541, partial = 0.003; decaying exponent, F(1,119) = 0.573, p = 0.451, partial = 0.005) and choice variability (Fig. 5C, F(1,119) = 1.702, p = 0.195, partial = 0.014) between SZ and HC were found in this analysis. And SZ showed larger variability in allocating resources (resource allocation variability, F(1,119) = 15.112, p < 0.001, partial = 0.114).

Supplementary Table 1. Demographics and clinical information of the subjects in the orientation delay-estimation task

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | SZ (N = 9) | |  | HC (N = 26) | |
|  | Mean | SD |  | Mean | SD |
| Age | 39.33 | 9.35 |  | 21.31 | 1.94 |
| range | 27-56 | n/a |  | 19-26 | n/a |
| Female/male | 0/9 | n/a |  | 16/26 | n/a |
| Inpatient/outpatient | 9/9 | n/a |  | n/a | n/a |
| Subjects’ education (years) | 9 | 2.12 |  | 14.12 | 1.63 |
| PANSS - positive | 11.33 | 4.33 |  | n/a | n/a |
| PANSS - negative | 22 | 8.14 |  | n/a | n/a |
| PANSS - general | 31.78 | 14.10 |  | n/a | n/a |

PANSS: ﻿The Positive and Negative Syndrome Scale14

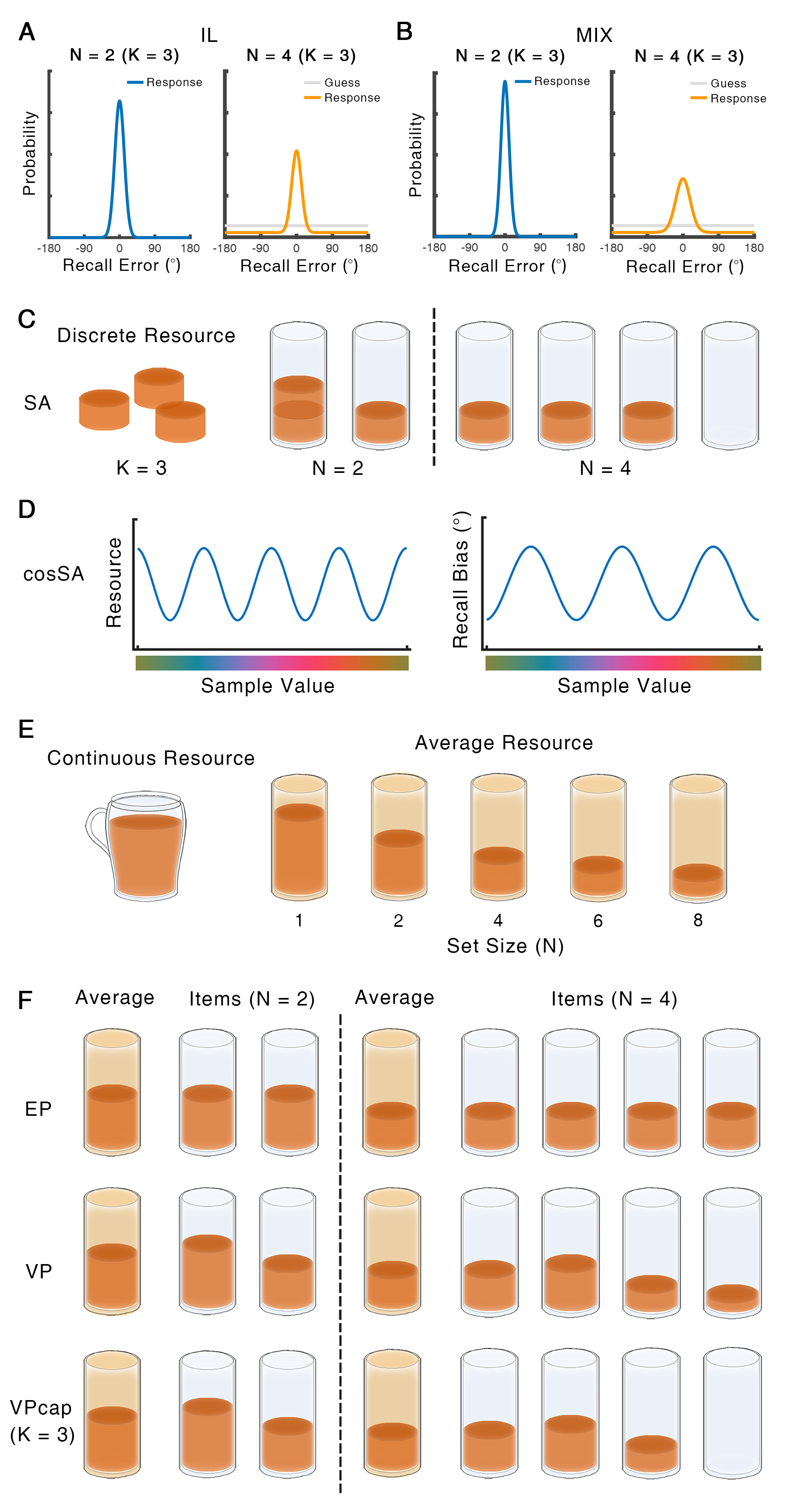


Figure S1. Cartoon illustration of all computational models considered in this study. This figure aims to aid an intuitive understanding of the models. Detailed model explanation to Supplementary Note 2. ***A***. item-limit model; ***B***. MIX model; ***C***. the principle of discrete slots and the SA model; ***D***. cosSA model; ***E***. the principle of continuous resources; ***F***, EP, VP, and VPcap models. See supplementary Note 2 for detailed explanations.



Figure S2. Negative log-likelihood (panels A, D), AIC (panels B, E) and BIC (panels C, F) values for all models. Note that here we display the negative log-likelihood values to help visually compare models since maximum negative log-likelihood values are equivalent to minimum negative log-likelihood values. As such, in all panels a lower y-axis value indicates a better model. The upper (panels A-C) and lower (panels D-F) rows depict the model comparison results for HC and SZ respectively. The best-fitting model is the VP model for both groups (also see Fig. 3 in the main text).



Figure S3. Results of model comparison in the orientation and color delay-estimation tasks. In the three independent samples of subjects (9 SZ and 26 HC in the orientation task, 62 HC in the color task), the VP model is always the best model in the majority of the subjects. Note that, in these two datasets, we measured the set size level up to 6. These results indicating that the finding of the VP model as the best model is not an artifact of low set levels used in the main experiment.

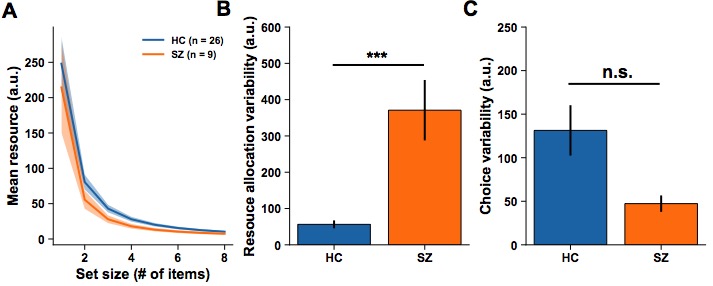


Figure S4. Comparison of fitted parameters between the two groups in the orientation task. We replicated the results in the main experiment and found a higher resource allocation variability in the SZ group. Note that we tested four set size levels (1/2/4/6) in the orientation task. These results suggest that enhanced resource allocation variability is not confined to the color task, and our findings in the main experiment are not artifacts of low set size levels.



Figure S5. Orientation delay-estimation task. This task is similar to the color delay-estimation task except that the visual stimuli are oriented bars. Subjects are required to remember their orientations and reproduce the orientation of the probed bar.



Figure S6. Simulation of the behavioral consequences of increased resource allocation variability. Based on the VP model, we simulate 4000 behavioral responses in each parameter combination. We systematically manipulate resource allocation variability and initial resource, and fix decaying exponent and choice variability to the group average of the fitted parameters of the HC group. Increased resource allocation variability leads to large response errors, indicating that low resource allocation variability is more optimal in this task.

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