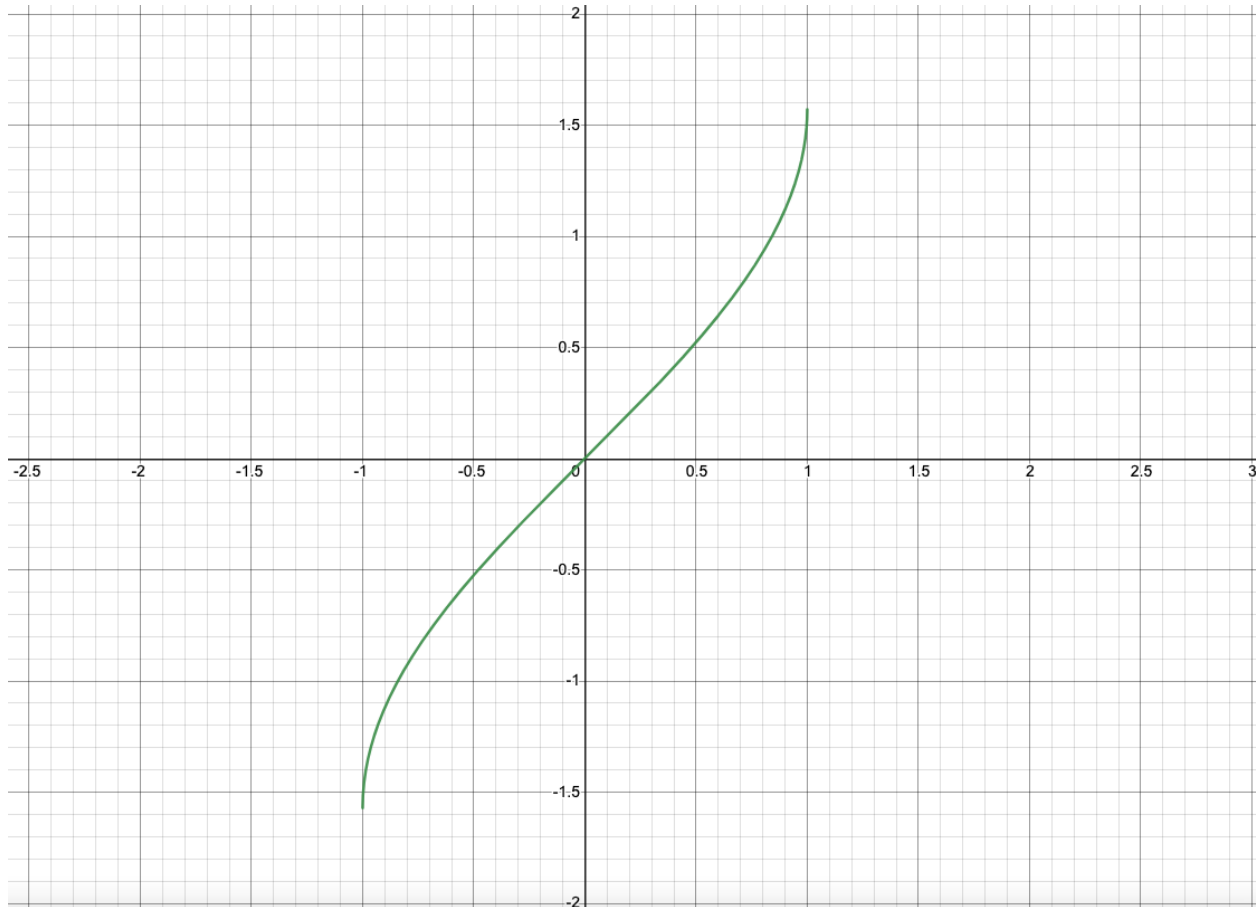


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CSE 13S Spring 2021
Assignment 2: A Small Numerical Library
Writeup Document

The program works by using Newton's method to calculate values of \arcsin , and \log . To calculate the values of \arccos and \arctan the program uses trigonometric identities to calculate in terms of \arccosine , this way only one function is being used.

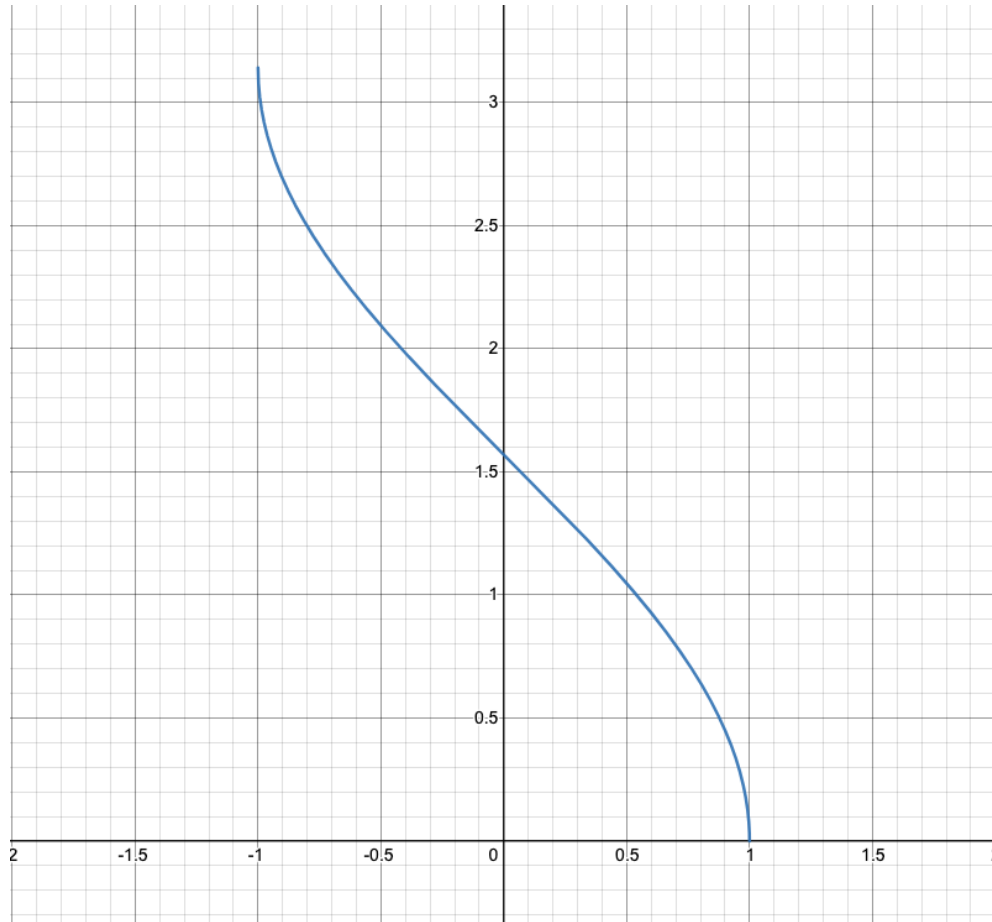
Calculating \arcsin using Newton's method uses the formula $f(x) = \sin(x) - a$ where a is a constant, the value we are trying to find \arcsin of. This method works well when the domain is from $[-0.9, 0.9]$. However, as the value reaches the extremes the method rapidly loses accuracy. This is because the vertical asymptotes of the arcsine function are at -1 and 1 so as x gets closer to the asymptotes the change in x increases drastically, in turn losing accuracy for x .



The green line is the graph of $\arcsin(x)$. As we can see as x reaches 1 and -1 the slope of x increases, and the line becomes vertical. In order to increase accuracy for arcsine we can utilize trigonometric identities. When x is close to the extremes we can use the following identity

$$\sin^{-1}(x) = \cos^{-1}(\sqrt{1-x^2}), \quad 0 \leq x \leq 1.$$

When we calculate arcsine in terms of cosine we are able to reduce the value of X. instead of having an x value that is close to the extremes we are able to reduce it by doing $\sqrt{1-x^2}$ and passing this value to the arccos function. By doing this we don't have a steep slope anymore.



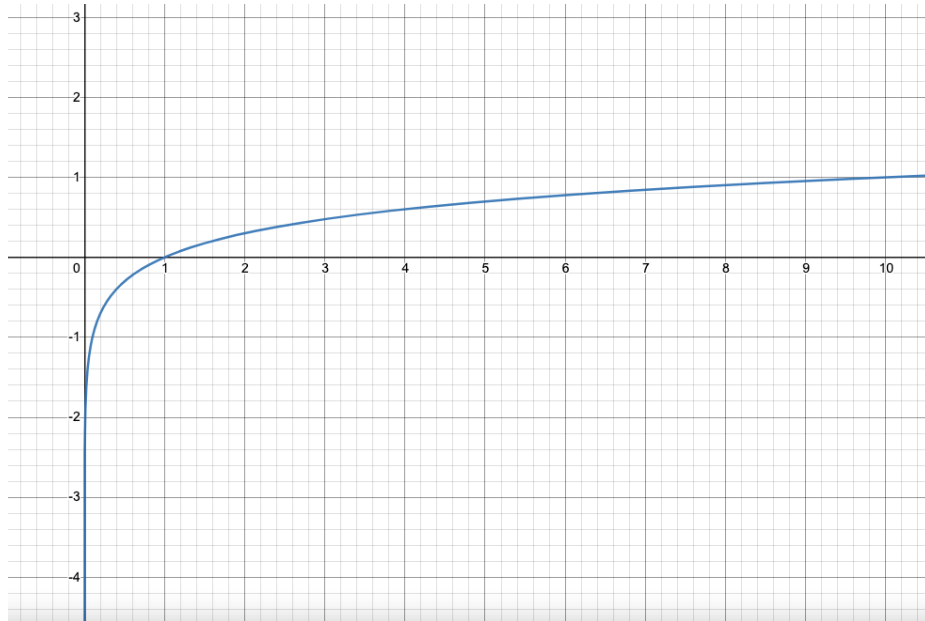
The blue line is the graph of $\arccos(x)$. As we can see the slope near the middle is much smaller than the slopes at the extremes.

Now that we have arcsin coded properly calculating the values of arccos, and arctan becomes much easier since we are using trigonometric identities.

$$\arccos(x) = \frac{\pi}{2} - \arcsin(x), \quad \arctan(x) = \arcsin\left(\frac{x}{\sqrt{x^2+1}}\right)$$

Log functions are calculated using Newton's method with the formula $f(x) = e^x - a$. Since we are only calculating the function between 1 and 10 Newton's method is accurate since the slope

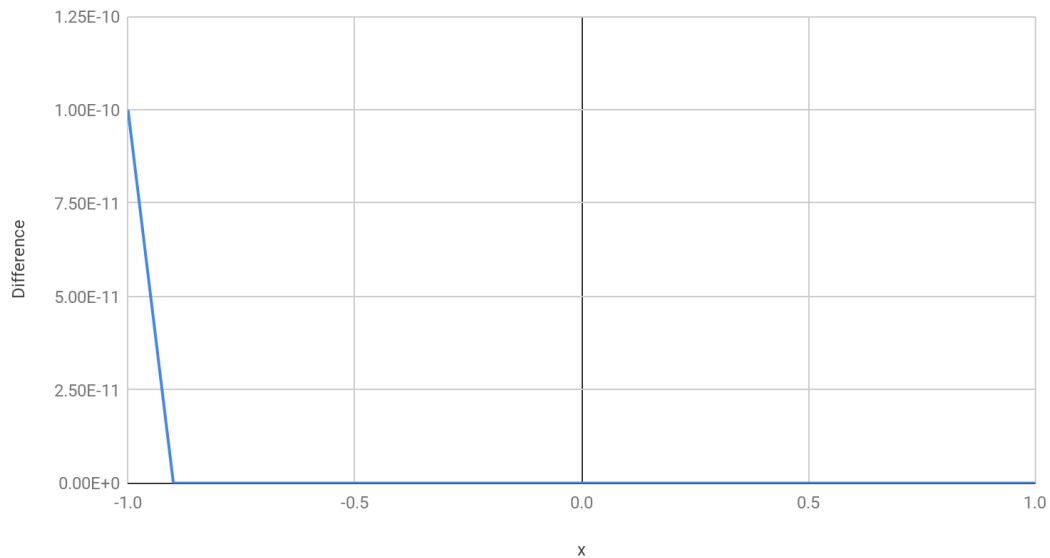
decreases as x increases.



The graph above is $\log(x)$

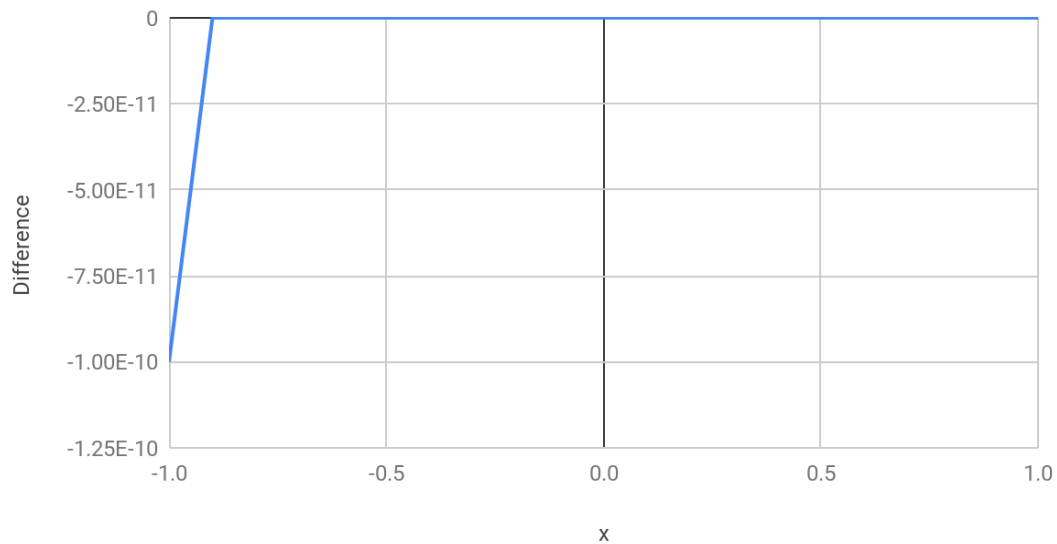
The difference between my implementation and the library implementation for the required functions are shown below. This decimal is accurate to 10 decimal places.

Difference between my implementation and library - \arcsin



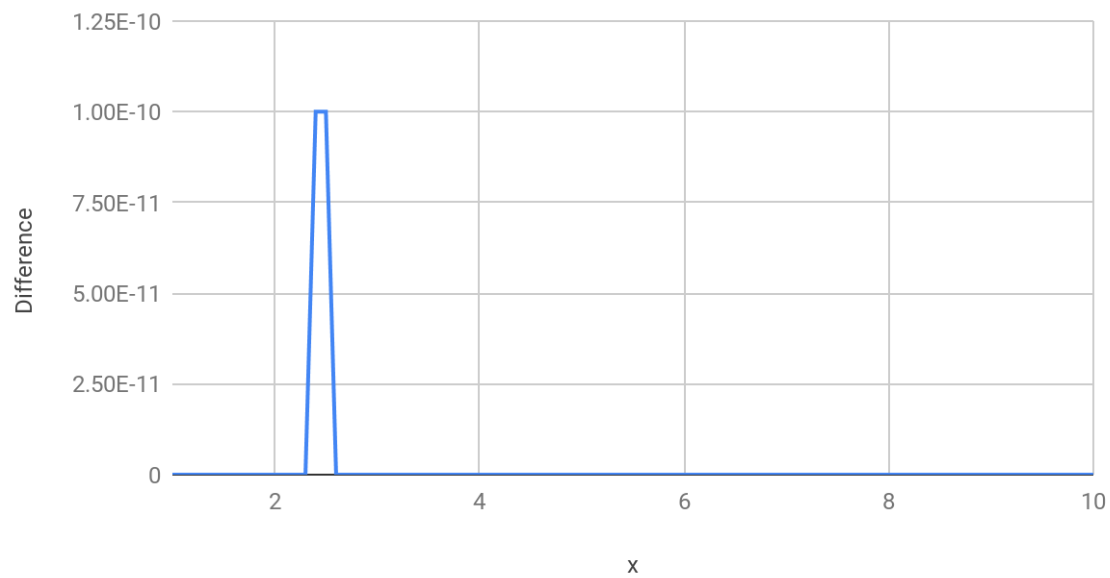
At $x = -1$ the difference is $1\text{E-}10$ because x is being rounded up when the value is smaller than epsilon.

Difference between my implementation and library - arcCos



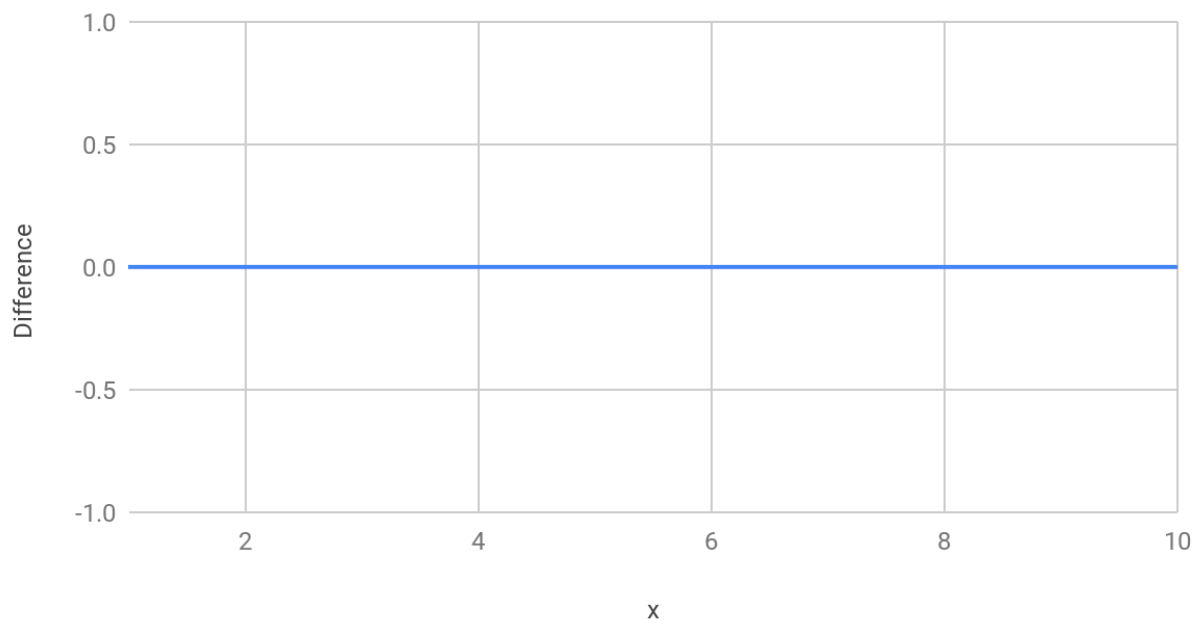
At $x=-1$ the difference is $1\text{E}-10$ because x is being rounded up when the value is smaller than epsilon.

Difference between my implementation and library - arcTan



The graph spikes at $x=2.1$ and $x=2.2$ because x is being rounded up when the value is smaller than epsilon

Difference between my implementation and library - arcTan



The difference for log is evenly at 0 since it is not being rounded up.