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Assignment 2: A Small Numerical Library
Design Document

This program utilizes Newton's method to calculate the value the inverse trig functions and log functions

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Newton's Method

Estimate roots of a function

\sin^{-1} with Newton's method

$$x = \sin^{-1}(a) \quad f(x) = \sin(x) - a$$
$$\sin(x) = a \quad x_{n+1} = x_n - \frac{\sin(x_n) - a}{\cos(x_n)}$$
$$\sin(x) - a = 0$$

\cos^{-1} ~~with~~

\cos^{-1} can be implemented by doing:

$$\frac{\pi}{2} - \arcsin(x)$$

\tan^{-1} can be implemented by doing:

$$\arcsin\left(\frac{x}{\sqrt{x^2+1}}\right)$$

\log

$$x = \ln(a) \quad f(x) = e^x - a$$
$$a = e^x \quad x_{n+1} = x_n - \frac{(e^x - a)}{e^x}$$
$$e^x - a = 0$$

For arcsin when x gets closer to the edges (1 and -1) the answer rapidly loses accuracy. To combat this when the absolute value of x greater than 0.9 then solve it using trig identities and arccos. $\arcsin(x) = \arccos(\sqrt{1-x^2})$, $0 \leq x \leq 1$. If x is negative then return the negative of the answer

I. Pseudocode

mathlib.c

```
double arcSin(double x) {
    double a;
    if x is an edge case(absolute value close to 1) {
        double a = sqrt(1-(x*x))
    } else {
        a = x;
    }
    double answer = a;
    while abs(answer) - a > epsilon {
        answer = answer - ((sin(answer) - a) / cos(answer));
    }
    if (Abs(x) > 0.9) {
        If x < 0 return -((PI / 2) - answer) else return (PI / 2) - answer
    } else {
        return answer;
    }
}

double arcCos(double x) {
    return (PI/2)-arcsin(x)
}

double arcTan(double x) {
    return arcsin(x/sqrt((x^2)+1))
}
```

```
double Log(double x) {
    double answer = a;
    while (abs(e^x - a) > epsilon) {
        answer = answer - ((e^x - a) / e^x);
    }
    return answer;
}
```

mathlib-test.c

```
void showdiff( char *name, double start, double stop, double(*i)(double) (double) (*l) double) {
```

Show the differences between the function implementation i, and the library implementation C

```
}
```

```
int main(int argc, char argv) {  
    If argc == 1 print error  
    Check if all arguments are valid. If all arguments not valid print error  
    Switch case for all arguments {  
        s: showdiff for arcsin  
        c: showdiff for arccos  
        t: showdiff for arctan  
        l: showdif for log  
        default: show difference for all functions
```