# $equation_recu$

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## 1 Introduction

$$f(t) = (y(N) - y(N-1))(t - N + 1) + y(N-1)$$
$$f(N) = ((y(N) - y(N-1))(N - N + 1) + y(N-1) = y(N)$$
$$f(N-1) = ((y(N) - y(N-1))(N - 1 - N + 1) + y(N-1) = y(N-1)$$

$$f(t-1) = (y(N-1) - y(N-2))(t - N + 2) + y(N-2)$$
$$f(t-2) = (y(N-2) - y(N-3))(t - N + 3) + y(N-3)$$

$$y(n) + b\,y(n-1) + c\,y(n-2) = 0$$
$$y(0) = \alpha \;\; y(1) = \beta$$
$$f(t) + b\,f(t-1) + c\,f(t-2) = 0$$

$$(y(N) - y(N-1))(\cancel{t-N}+1) + \cancel{y(N-1)} + b\,[(y(N-1) - y(N-2))(\cancel{t-N}+2) + \cancel{y(N-2)}] \\ + c\,[(y(N-2) - y(N-3))(\cancel{t-N}+3) + \cancel{y(N-3)}] = 0 \quad (1)$$

$$y(N) - y(N-1) + 2b(y(N-1) - y(N-2)) + 3c(y(N-2) - y(N-3)) = 0$$
$$\cancel{y(N)} - \cancel{y(N-1)} + 2b(y(N-1) - y(N-2)) + 3c(y(N-2) - y(N-3)) = 0$$
$$b(y(N-1) - y(N-2)) + 2c(y(N-2) - y(N-3)) = 0$$
$$b\,y(N-1) + (2c - b)y(N-2) - 2c\,y(N-3) = 0$$
$$y(N-1) + b\,y(N-2) + c\,y(N-3) = 0$$
$$2y(N-1) + 2b\,y(N-2) + 2c\,y(N-3) = 0$$
$$(b+2)y(N-1) + (2c+b)y(N-2) = 0$$
$$\frac{y(N-1)}{y(N-2)} = -\frac{b+2c}{b+2}$$

$$y(n) + b\,y(n-1) + c\,y(n-2) = 0$$

$$\frac{y(n)}{y(n-2)} + b\,\frac{y(n-1)}{y(n-2)} + c = 0$$

$$\left(-\frac{2c+b}{b+2}\right)^2 - b\frac{2c+b}{b+2} + c = 0$$

$$f(t) = (y(N) - y(N-1))(t - N + 1) + y(N-1)$$

$$f(t-1) = (y(N-1) - y(N-2))(t - N + 2) + y(N-2)$$

$$y(n) + b\,y(n-1) = 0$$

$$(y(N) - y(N-1))(t - N + 1) + y(N-1) + b\,(y(N-1) - y(N-2))(t - N + 2) + y(N-2) = 0$$

$$(y(N) - y(N-1))(\cancel{t - N} + 1) + \cancel{y(N-1)} + b\,(y(N-1) - y(N-2))(\cancel{t - N} + 2) + \cancel{y(N-2)} = 0$$

$$(y(N) - y(N-1)) + 2b\,(y(N-1) - y(N-2)) = 0$$

$$b\,(y(N-1) - y(N-2)) = 0$$

$$y(n) + b\,y(n-1) + c\,y(n-2) + d\,y(n-3) = 0$$

$$y(N) - y(N-1) + 2b(y(N-1) - y(N-2)) + 3c(y(N-2) - y(N-3)) + 4d(y(N-3) - y(N-4)) = 0$$

$$b(y(N-1) - y(N-2)) + 2c(y(N-2) - y(N-3)) + 3d(y(N-3) - y(N-4)) = 0$$

$$y(N-1) + b\,y(N-2) + c\,y(N-3) + d\,y(N-4) = 0$$

$$3y(N-1) + 3b\,y(N-2) + 3c\,y(N-3) + 3d\,y(N-4) = 0$$

$$b\,y(N-1) + (2c-b)y(N-2) + (3d-2c)\,y(N-3) - 3d\,y(N-4) = 0$$

$$(b+3)y(N-1) + (2c+2b)y(N-2) + (3d+c)y(N-3) = 0$$

$$y(N-1) + \frac{(2c+2b)}{(b+3)}y(N-2) + \frac{(3d+c)}{(b+3)}y(N-3) = 0$$

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$$\frac{y(N-2)}{y(N-3)} = -\frac{2\frac{(3d+c)}{(b+3)} + \frac{(2c+2b)}{(b+3)}}{\frac{(2c+2b)}{(b+3)} + 2} = -\frac{(b+2c+3d)}{(2b+c+3)}$$

Alternativa

$$f(t) = (y(N) - y(N-1))(t-N) + y(N)$$

$$f(t-1) = (y(N-1) - y(N-2))(t-N+1) + y(N-1)$$

$$f(t-2) = (y(N-2) - y(N-3))(t-N+2) + y(N-2)$$

$$\begin{aligned}(y(N) - y(N-1))(t-N) + y(N) + b\,[(y(N-1) - y(N-2))(t-N+1) + y(N-1)]\\ + c\,[(y(N-2) - y(N-3))(t-N+2) + y(N-2)] = 0 \quad (2)\end{aligned}$$

$$b\,(y(N-1) - y(N-2)) + 2c\,(y(N-2) - y(N-3)) = 0$$

$$b\,y(N-1) + (2c-b)\,y(N-2) - 2c\,y(N-3) = 0$$

$$2\,y(n-1) + 2b\,y(n-2) + 2c\,2y(n-3) = 0$$

$$(b+2)\,y(N-1) + (2c+b)\,y(N-2) = 0$$

$$\begin{aligned}(y(N) - y(N-1))(t-N) + y(N) + b\,[(y(N-1) - y(N-2))(t-N+1) + y(N-1)]\\ +c\,[(y(N-2) - y(N-3))(t-N+2) + y(N-2)] + d\,[(y(N-3) - y(N-4)(t-N+3) + y(N-3)] = 0\end{aligned} \quad (3)$$

$$b(y(N-1) - y(N-2)) + 2c(y(N-2) - y(N-3)) + 3d(y(N-3) - y(N-4)) = 0$$

$$by(N-1) + (2c-b)y(N-2) + (3d-2c)y(N-3) - 3d\,y(N-4) = 0$$

$$3y(n-1)+3b\,y(n-2)+3c\,y(n-3)+3d\,y(n-4)=0$$
$$(b+3)y(N-1)+(2c+2b)y(N-2)+(3d+c)y(N-3)=0$$
$$y(N-1)+\frac{2c+2b}{b+3}y(N-2)+\frac{3d+c}{b+3}y(N-3)=0$$
$$(-\frac{2c+2b}{b+3}+b)y(N-1)+(-\frac{2c+2b}{b+3}+b)(\frac{2c+2b}{b+3})y(N-2)+(-\frac{2c+2b}{b+3}+b)(\frac{3d+c}{b+3})y(N-3)=0$$
$$y(N)+\frac{2c+2b}{b+3}y(N-1)+\frac{3d+c}{b+3}y(N-2)=0$$
$$y(N)+by(N-1)+[(-\frac{2c+2b}{b+3}+b)(\frac{2c+2b}{b+3})+\frac{3d+c}{b+3}]y(N-2)+(-\frac{2c+2b}{b+3}+b)(\frac{3d+c}{b+3})y(N-3)=0$$
$$y(n)+b\,y(n-1)+c\,y(n-2)+d\,y(n-3)=0$$
$$(((-\frac{2c+2b}{b+3}+b)(\frac{2c+2b}{b+3})+\frac{3d+c}{b+3})-c)y(N-2)+((-\frac{2c+2b}{b+3}+b)(\frac{3d+c}{b+3})-d)y(N-3)$$

$$(b+3)y(N-1)+(2c+2b)y(N-2)+(3d+c)y(N-3)=0$$
$$(b+3)y(N)+(2c+2b)y(N-1)+(3d+c)y(N-2)=0$$
$$y(n)+b\,y(n-1)+c\,y(n-2)+d\,y(n-3)=0$$
$$(b+3)y(n)+b(b+3)\,y(n-1)+c(b+3)\,y(n-2)+d(b+3)\,y(n-3)=0$$

$$\frac{y(N-2)}{y(N-3)}=-\frac{2\frac{3d+c}{b+3}+\frac{2c+2b}{b+3}}{\frac{2c+2b}{b+3}+2}=-\frac{b+2c+3d}{2b+c+3}$$
$$\left(-\frac{b+2c+3d}{2b+c+3}\right)^2-\frac{2c+2b}{b+3}\frac{b+2c+3d}{2b+c+3}+\frac{3d+c}{b+3}$$
$$(-\frac{b+2c+3d}{2b+c+3})^3+b(-\frac{b+2c+3d}{2b+c+3})^2-c\frac{b+2c+3d}{2b+c+3}+d$$

$$f(t) = A\,t^2 + B\,t + C$$
$$f(N) = y(N) = A\,N^2 + B\,N + C$$
$$f(N-1) = y(N-1) = A\,(N-1)^2 + B\,(N-1) + C$$
$$y(N-1) = A\,(N^2 - 2\,N + 1)^2 + B\,(N-1) + C$$
$$y(N-1) = A\,N^2 - 2A\,N + A + B\,N - B + C = y(N) - 2AN + A - B$$
$$y(N) - y(N-1) = 2A(N)N - A(N) + B(N)$$

$$y(n) + b\,y(n-\alpha) + c\,y(n-\alpha) = 0$$

$$(y(N)-y(N-\alpha))(t-N)+y(N)+b\,[(y(N-\alpha)-y(N-2\alpha))(t-N+\alpha)+y(N-\alpha)] + c\,[(y(N-2\alpha) - y(N-3\alpha))(t-N+2\alpha) + y(N-2\alpha)] = 0 \quad (4)$$

$$b\alpha\,(y(N-1) - y(N-2)) + 2c\alpha\,(y(N-2) - y(N-3)) = 0$$
$$b\alpha\,y(N-1) + (2c\alpha - b\alpha)\,y(N-2) - 2c\alpha\,y(N-3) = 0$$
$$2\alpha\,y(n-1) + 2b\alpha\,y(n-2) + 2c\alpha\,2y(n-3) = 0$$
$$(b+2)\alpha\,y(N-1) + (2c+b)\alpha\,y(N-2) = 0$$

$$b\,\left(y(N-1)-y(N-2)\right)+3c\,\left(y(N-2)-y(N-3)\right)=0$$

$$b\,y(N-1)+(3c-b)\,y(N-2)-3c\,y(N-3)=0$$

$$3\,y(n-1)+3b\,y(n-2)+3c\,y(n-3)=0$$

$$(b+3)\,y(N-1)+(3c+2b)\,y(N-2)=0$$

$$\frac{y(N-1)}{y(N-2)}=-\frac{2b+3c}{b+3}$$

$$b(y(N-1)-y(N-2))+3c(y(N-2)-y(N-3))+5d(y(N-3)-y(N-4))=0$$

$$y(N-1)+b\,y(N-2)+c\,y(N-3)+d\,y(N-4)=0$$

$$5y(N-1)+5b\,y(N-2)+5c\,y(N-3)+5d\,y(N-4)=0$$

$$b\,y(N-1)+(3c-b)y(N-2)+(5d-3c)\,y(N-3)-5d\,y(N-4)=0$$

$$(b+5)\,y(N-1)+(3c+4b)y(N-2)+(5d+2c)\,y(N-3)=0$$

$$y(N-1)+\frac{(3c+4b)}{(b+5)}y(N-2)+\frac{(5d+2c)}{(b+5)}y(N-3)=0$$

$$\frac{y(N-2)}{y(N-3)}=-\frac{2\frac{(3c+4b)}{(b+5)}+3\frac{(5d+2c)}{(b+5)}}{\frac{(3c+4b)}{(b+5)}+3}=-\frac{8b+12c+15d}{15+7b+3c}$$

$$\left(-\frac{8b+12c+15d}{15+7b+3c}\right)^2-\frac{3c+4b}{b+5}\frac{8b+12c+15d}{15+7b+3c}+\frac{5d+2c}{b+5}$$

$$d=\frac{2(16b^2-4c+15bc+9c^2)}{(45(5+b))}$$

$$-\frac{8b+12c+15\frac{2(16b^2-4c+15bc+9c^2)}{(45(5+b))}}{15+7b+3c}$$

$$\left(-\frac{8b+12c+15d}{15+7b+3c}\right)^3+b\left(-\frac{8b+12c+15d}{15+7b+3c}\right)^2+c\left(-\frac{8b+12c+15d}{15+7b+3c}\right)+d$$

$$y(n) + b\, y(n-1) + c\, y(n-2) = 0$$

$$b\, (y(n-1) - y(n-3)) + 2c\, (y(n-2) - y(n-4)) = 0$$

$$b\, y(n-1) + 2c\, y(n-2) - b\, y(n-3) - 2c\, y(n-4) = 0$$

$$2y(n-2) + 2b\, y(n-3) + 2c\, y(n-4) = 0$$

$$b\, y(n-1) + (2c+2)\, y(n-2) + b\, y(n-3) = 0$$

$$y(n-1) + b\, y(n-1) + c\, y(n-3) = 0$$

$$\frac{b}{c} y(n-1) + \frac{b^2}{c}\, y(n-1) + b\, y(n-3) = 0$$

$$(b - \frac{b}{c}) y(n-1) + ((2c+2) - \frac{b^2}{c}) y(n-2) = 0$$

$$\frac{y(n-1)}{y(n-2)} = -\frac{((2c+2) - \frac{b^2}{c})}{(b - \frac{b}{c})} = \frac{b^2 - 2c - 2c^2}{b(c-1)}$$

$$(\frac{b^2 - 2c - 2c^2}{b(c-1)})^2 + b\frac{b^2 - 2c - 2c^2}{b(c-1)} + c = \frac{c(b^2 - 4c)(-1 + b - c)(1 + b + c)}{b^2(-1+c)^2}$$

$$b\ (y(n-1)-y(n-3))+2c\ (y(n-2)-y(n-4))+3d\ (y(n-3)-y(n-5))=0$$

$$b\ y(n-1)+2c\ y(n-2)+(3d-b)y(n-3)-2c\ y(n-4)-3d\ y(n-5)=0$$

$$3y(n-2)+3b\ y(n-3)+3c\ y(n-4)+3d\ y(n-5)=0$$

$$b\ y(n-1)+(2c+3)\ y(n-2)+(3d+2b)y(n-3)+c\ y(n-4)=0$$

$$\frac{bd}{c}\ y(n-1)+\frac{(2c+3)d}{c}y(n-2)+\frac{(3d+2b)d}{c}y(n-3)+d\ y(n-4)=0$$

$$y(n-1)+b\ y(n-2)+c\ y(n-3)+d\ y(n-4)=0$$

$$(\frac{bd}{c}-1)\ y(n-1)+(\frac{(2c+3)d}{c}-b)y(n-2)+(\frac{(3d+2b)d}{c}-c)y(n-3)=0$$

$$(\frac{(3d+2b)d}{c}-c)/(\frac{bd}{c}-1)=\frac{(2bd-c^2+3d^2)}{(bd-c)}$$

$$(\frac{(2c+3)d}{c}-b)/(\frac{bd}{c}-1)=\frac{(bc-2cd-3d)}{(c-bd)}$$

$$((\frac{(bc-2cd-3d)}{(c-bd)})^2-2\frac{(2bd-c^2+3d^2)}{(bd-c)}-2(\frac{(2bd-c^2+3d^2)}{(bd-c)})^2)/(\frac{(bc-2cd-3d)}{(c-bd)}(\frac{(2bd-c^2+3d^2)}{(bd-c)}-1))$$

$$b\ (y(n-1)-y(n-2))+2c\ (y(n-3)-y(n-4))=0$$

$$b\ y(n-1)-b\ y(n-2)+2c\ y(n-3)-2c\ y(n-4)=0$$

$$2y(n-2)+2b\ y(n-3)+2c\ y(n-4)=0$$

$$b\ y(n-1)+(2-b)\ y(n-2)+(2c+2b)\ y(n-3)=0$$

$$\frac{bc}{2c+2b}\ y(n-1)+\frac{(2-b)c}{2c+2b}\ y(n-2)+c\ y(n-3)=0$$

$$(\frac{bc}{2c+2b}-1)\ y(n-1)+(\frac{(2-b)c}{2c+2b}-b)\ y(n-2)=0$$

$$\frac{y(n-1)}{y(n-2)}=-\frac{(\frac{(2-b)c}{2c+2b}-b)}{(\frac{bc}{2c+2b}-1)}=\frac{(2b^2-2c+3bc)}{(b(-2+c)-2c)}$$

$$(\frac{(2b^2-2c+3bc)}{(b(-2+c)-2c)})^2+b(\frac{(2b^2-2c+3bc)}{(b(-2+c)-2c)})+c$$