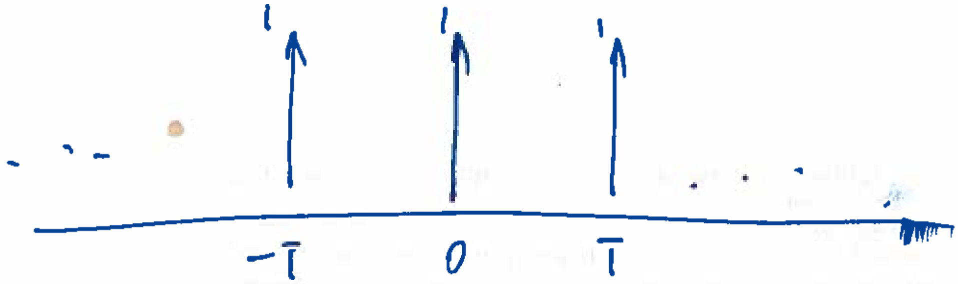
1a.



1b. $$c_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \sum_{k=-\infty}^{\infty} \delta(t-kT)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

$$c_k = \frac{1}{T}\sum_{k=-\infty}^{\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t-kT)\, e^{-j\frac{2\pi}{T}kt}\, dt$$ using picking property $\Rightarrow$

$$\int_{-\infty}^{\infty} \delta(t-t_0)\, x(t)\, dt = x(t_0)$$

$$\boxed{c_k = \frac{1}{T}}$$

1c. $$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \qquad x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{j\omega_0 k t}$$

$$X(\omega) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} c_k\, e^{jt(\omega_0 k-\omega)}\, dt \Rightarrow \sum_{k=-\infty}^{\infty} c_k \int_{-\infty}^{\infty} e^{jt(\omega_0 k-\omega)}\, dt$$

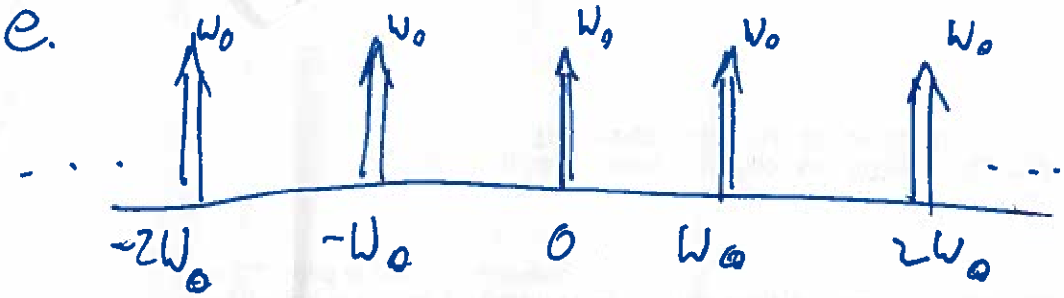
$$X(\omega) = \sum_{k=-\infty}^{\infty} c_k\, 2\pi\left(\delta(\omega-\omega_0 k)\right)$$

$$\boxed{e^{j\omega_0 t} \rightleftharpoons 2\pi\,\delta(\omega-\omega_0)}$$

Cannonical transform

1.d. $$\sum_{k=-\infty}^{\infty} \frac{1}{T}\,\delta(\omega-\omega_0 k)\, 2\pi = P(t) = \sum_{k=\infty}^{\infty} \omega_0\, \delta(\omega-\omega_0 k)$$

1.e.



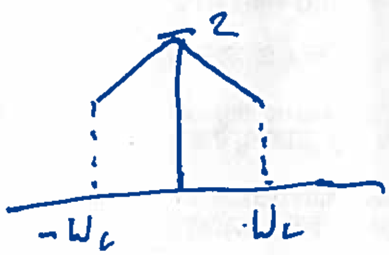
When we change T, in the time domain the impulses are further apart. In the freq domain they are scaled by $\frac{2\pi}{T}$ and closer together

$W_0 = \frac{2\pi}{T}$

2.a. $h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(w)e^{jwt}\,dw \Rightarrow \frac{1}{2\pi}\int_{-W_c}^{W_c} H(w)e^{jwt}\,dw$

$h(t) = \frac{1}{2\pi}\frac{1}{jwt}e^{jwt}\Big|_{-W_c}^{W_c} = \frac{1}{2j\pi t}\left(e^{jW_ct} - e^{-jW_ct}\right) = \boxed{\frac{1}{\pi t}\sin(W_c t)}$

2.b.

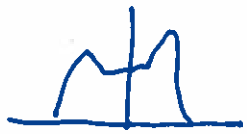


c. It is an ideal low pass filter because it removes all freq whose value are above or below W_c

d. see fig.

3.

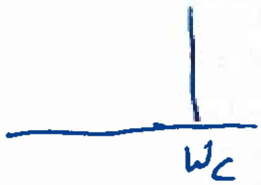
$X(W)$

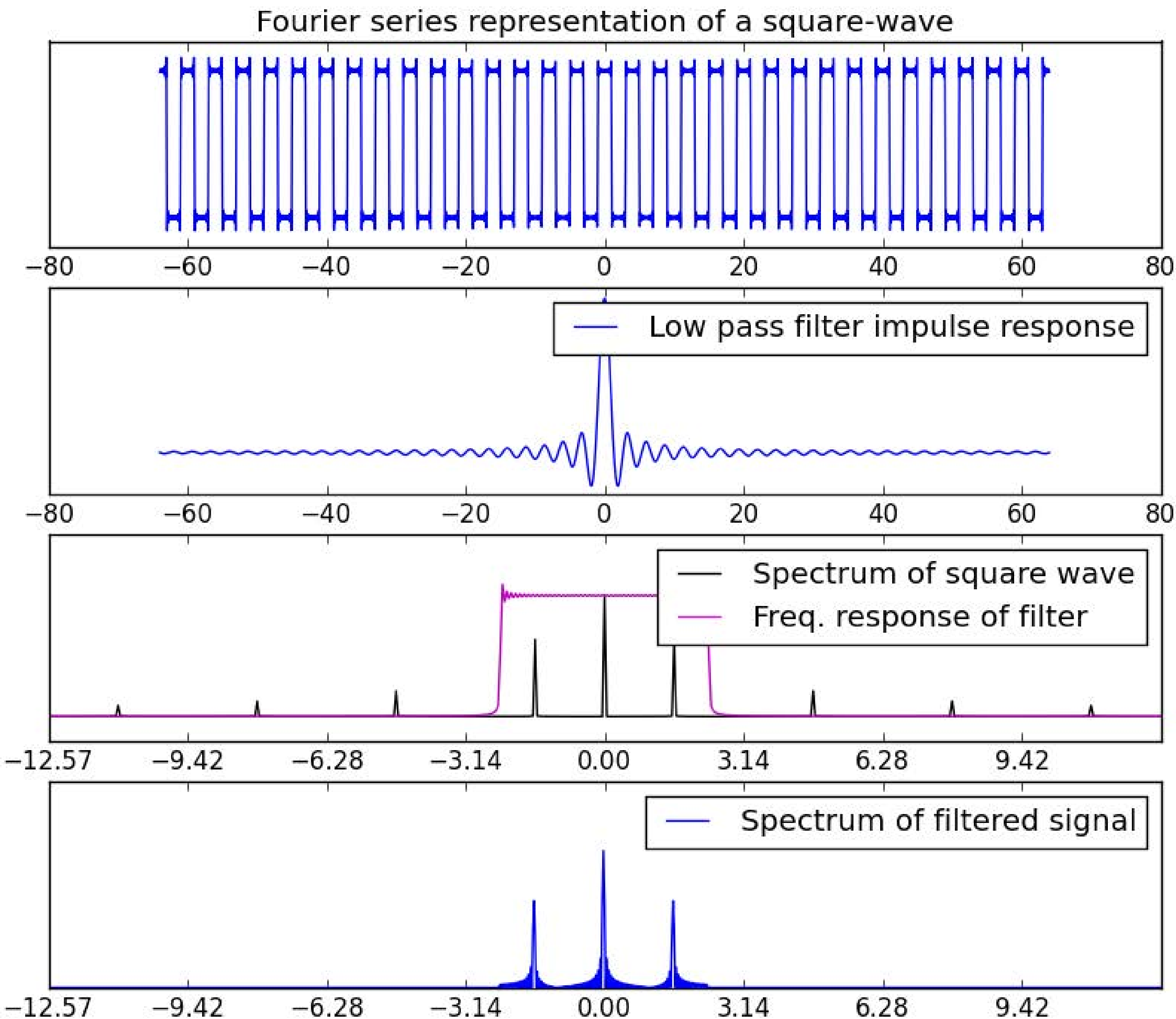


$$\boxed{x(t)\cos(W_c t) = 2\pi\,(X(W) * W_c)}$$

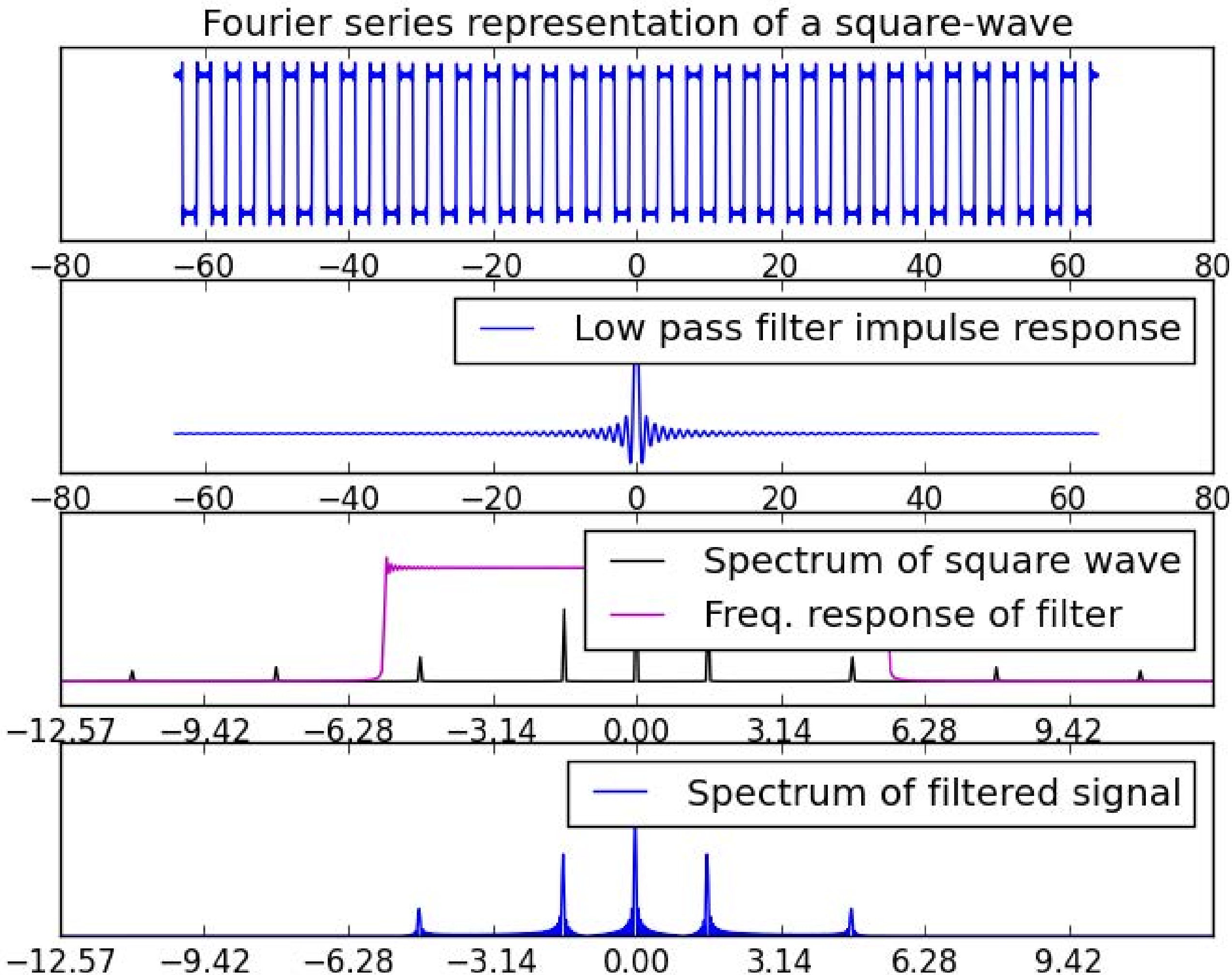
$Y(W)$

$\cos(W_c t)$





Fourier series representation of a square-wave
−80
−60
−40
−20
0
20
40
60
80
Low pass filter impulse response
−80
−60
−40
−20
0
20
40
60
80
Spectrum of square wave
Freq. response of filter
−12.57
−9.42
−6.28
−3.14
0.00
3.14
6.28
9.42
Spectrum of filtered signal
−12.57
−9.42
−6.28
−3.14
0.00
3.14
6.28
9.42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