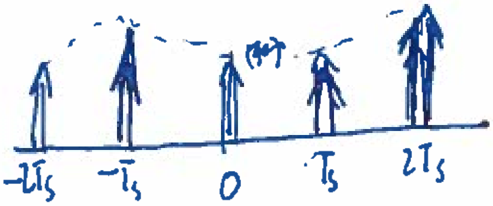
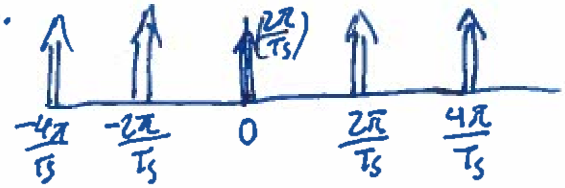
SIGSYS PS08 Radmer van der Heyde

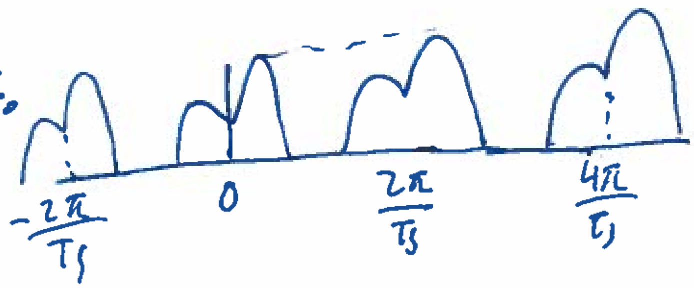
1a.



b.



c.

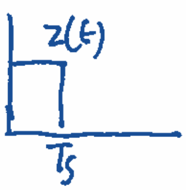


d. $-\frac{2\pi}{T_s} + W_m < -W_m$

$\frac{2\pi}{T_s} < 2W_m$ $\quad \frac{\pi}{T_s} < W_m$

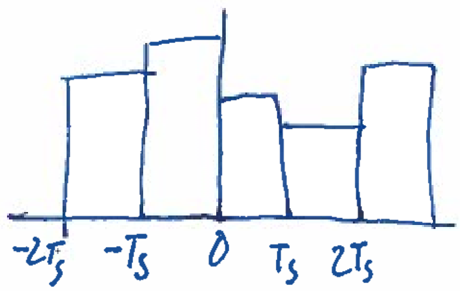
e. Use a lowpass filter with a cutoff freq at W_m

f. considered

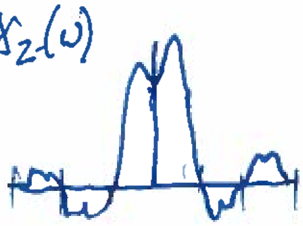


$X_Z(\omega)$

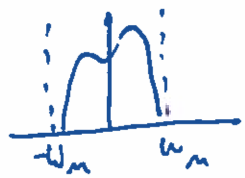
g.



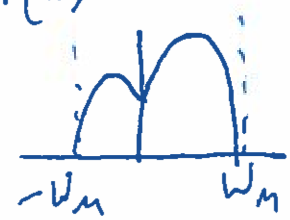
h.



i. $\bar{X}(\omega)$



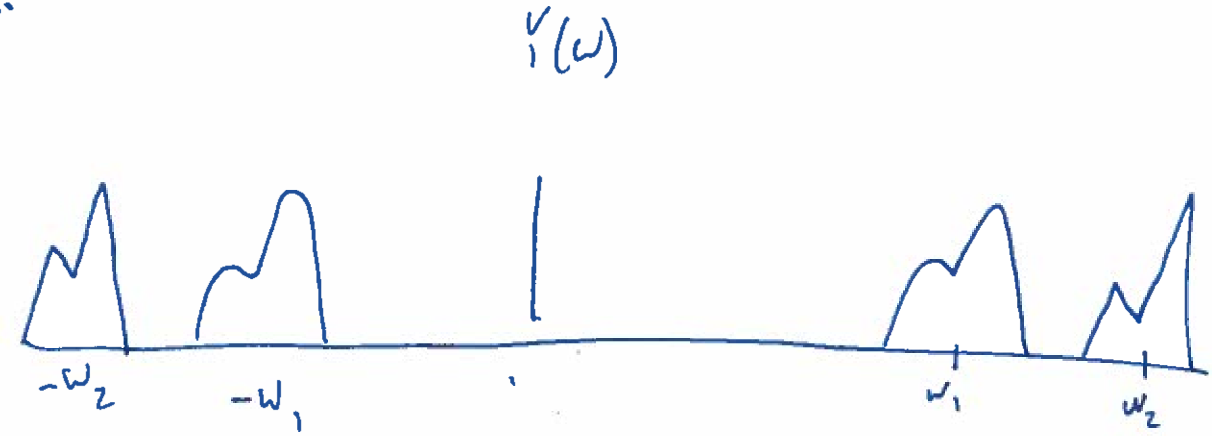
$\hat{X}(\omega)$



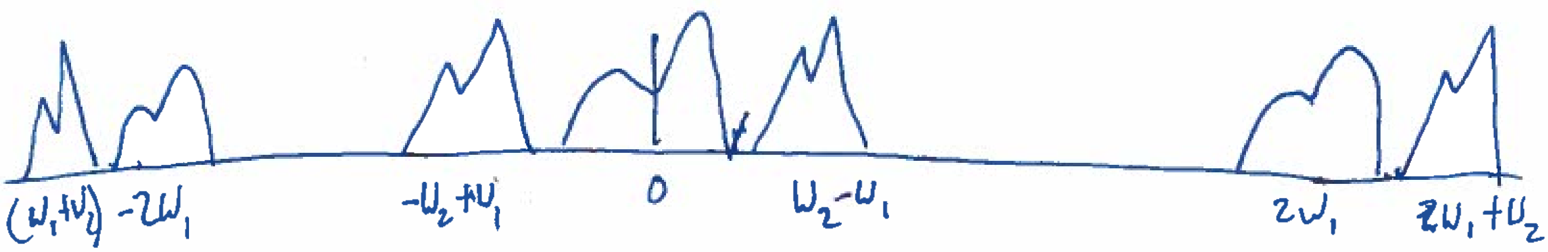
j. towards the lowpass cutoff freq. $\bar{X}(\omega)$ is scaled down.

k. at W_m $\frac{0}{0}$

2. a.

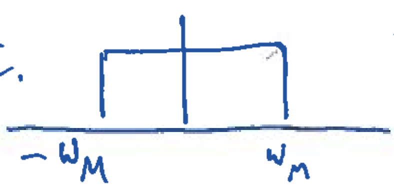


b. $y(t)\cos(W_1 t)$



$y(t)\cos(W_2 t)$

c.



Applying this lowpass filter to $y(t)\cos(\omega t)$ to recover each shifted signal with ω being some freq the channel applies to $y(t)$ before the filter.

3\. a. $V_{in}(t) = V_r(t) + V_L(t) + V_{out}(t)$

$V_r = Ri(t)$

$i(t) = C\frac{d}{dt}V_{out}(t)$

$V_L(t) = L\frac{d}{dt}i(t)$

$V_{in}(t) = RC\frac{d}{dt}V_{out}(t) + LC\frac{d^2}{dt^2}V_{out}(t) + V_{out}(t)$

b. using $V_{in}(t) = e^{j\omega t}$ and $V_{out} = H(\omega)e^{j\omega t}$

$e^{j\omega t} = RC\frac{d}{dt}H(\omega)e^{j\omega t} + LC\frac{d^2}{dt^2}H(\omega)e^{j\omega t} + \cancel{He}^{j\omega t}$

$\cancel{e^{j\omega t}} = RCj\omega H(\omega)\cancel{e^{j\omega t}} + LCj^2\omega^2 H(\omega)\cancel{e^{j\omega t}} + H(\omega)\cancel{e^{j\omega t}}$

$1 = (RCj\omega - LC\omega^2 + 1)H(\omega)$

$H(\omega) = \frac{1}{RCj\omega - LC\omega^2 + 1}$

c. $\|H(\omega)\| = \frac{1}{\sqrt{(1-LC\omega^2)^2 + (RC\omega)^2}}$

d. $0 = \frac{d}{d\omega}\left((1-LC\omega^2)^2 + (RC\omega)^2\right)^{-\frac{1}{2}} = \frac{2LC\omega(1-LC\omega^2) + 2RC(RC\omega)}{\left((1-LC\omega^2)^2 + (RC\omega)^2\right)^{3/2}}$

$0 = 2LC\omega - 2L^2C^2\omega^3 + R^2C^2\omega = (2LC + R^2C^2)\omega - 2L^2C^2\omega^3 = 0$

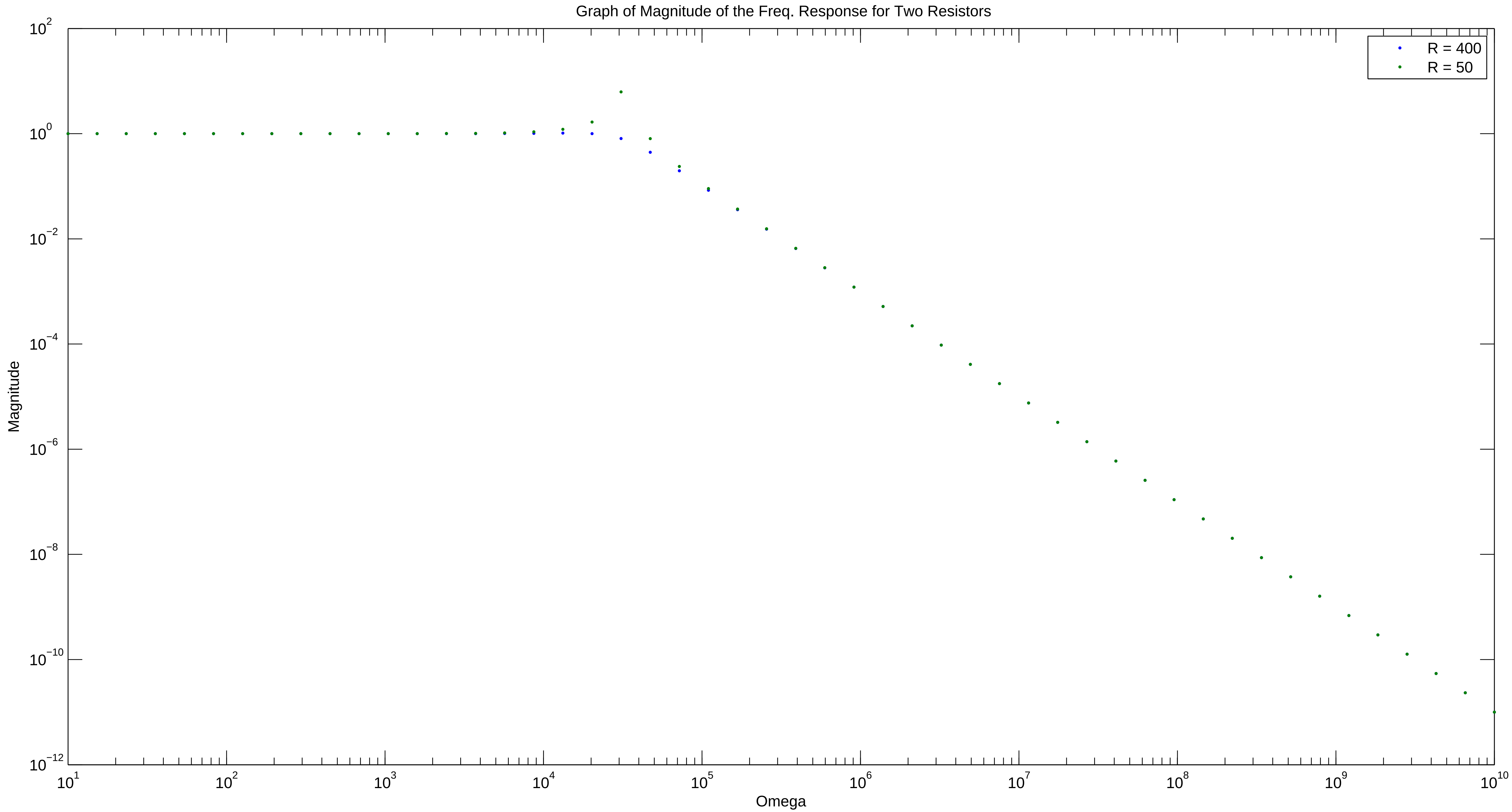
$2LC + R^2C^2 = 2L^2C^2\omega^2$

$\boxed{\sqrt{\frac{2L+R^2C}{2L^2C}} = \omega}$

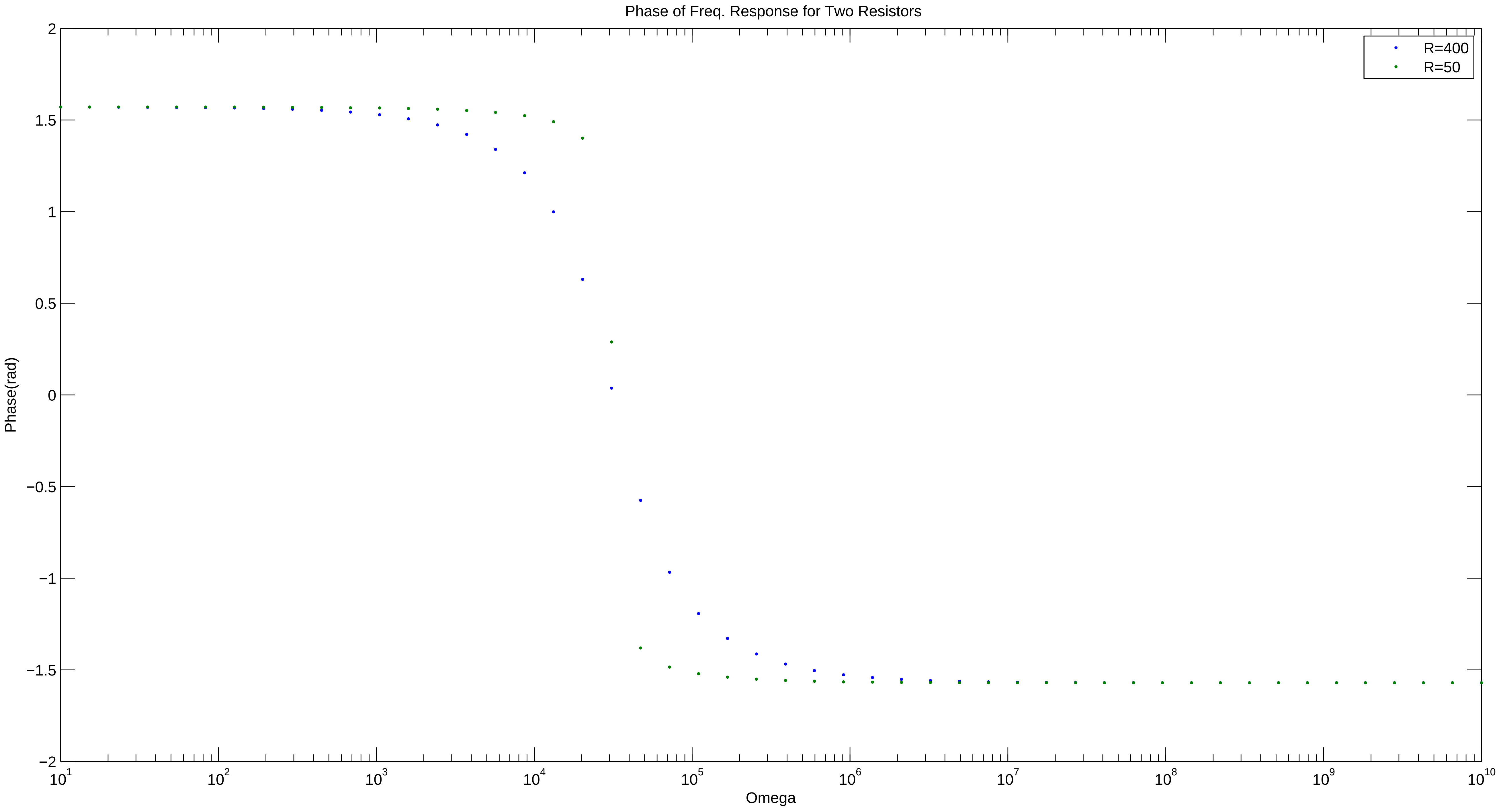
$\left(\left(1 - LC\sqrt{\frac{2L+R^2C}{2L^2C}}^{\,2}\right)^2 + \left(RC\sqrt{\frac{2L+R^2C}{2L^2C}}\right)^2\right)^{3/2} \overset{?}{\neq} 0$

$\left(\left(1 - \frac{\cancel{L}\cancel{C}(2L+R^2C)}{2L^{\cancel{2}}\cancel{C}}\right)^2 + \frac{R^2(2L+R^2C)C}{2L^2}\right)^{3/2}$

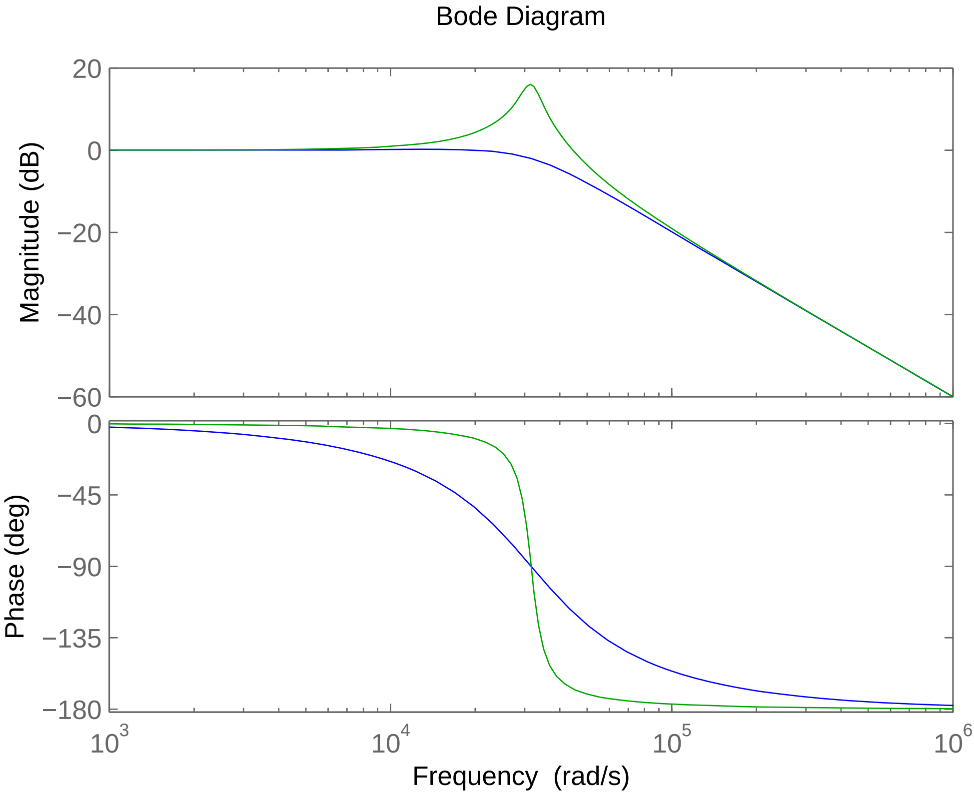
unless after a lot of algebra $L = \frac{\pm 3}{4}R^2C$ so never



Graph of Magnitude of the Freq. Response for Two Resistors
Magnitude
Omega
R = 400
R = 50



Phase of Freq. Response for Two Resistors
R=400
R=50
Phase(rad)
Omega
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}


Bode Diagram
Magnitude (dB)
20
0
−20
−40
−60
Phase (deg)
0
−45
−90
−135
−180
10^3
10^4
10^5
10^6
Frequency (rad/s)