Isotope analytical uncertainty propagation

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Here I (1) state the uncertainty propagation rules plainly, (2) apply them to our mass-balance model equation and calculate the results, and finally (3) I explore the the derivation of Genereux's (1998) isotopic tracer component uncertainty equations and how they, sadly, differ to my arithmetic propagation in section (2).

1 Uncertainty propagation rules

Our uncertainty rules are as follows. Let q be some quantity, and u_q be its uncertainty, where q is a function of variables x, y and z.

For **constants**

$$\begin{aligned} q &= Bx \\ \frac{u_q}{q} &= \frac{u_x}{x} \end{aligned} \tag{1.1}$$

For **sums**

$$\begin{aligned} q &= x \pm y \pm z \\ u_q &= \sqrt{u_x^2 + u_y^2 + u_z^2} \end{aligned} \tag{1.2}$$

And for **products/quotients**

$$\begin{aligned} q &= xyz \\ \frac{u_q}{|q|} &= \sqrt{\left(\frac{u_x}{x}\right)^2 + \left(\frac{u_y}{y}\right)^2 + \left(\frac{u_z}{z}\right)^2} \end{aligned} \tag{1.3}$$

2 Genereux's (1998) uncertainty propagation

Let p_E be the proportion of streamflow derived from rainfall according to an isotope E, where

$$p_E = \frac{E_{streamflow} - E_{baseflow}}{E_{rain} - E_{baseflow}} \tag{2.1}$$

We then average p_{δ^2H} and $p_{\delta^{18}O}$, each with their own uncertainty derived above, as follows

$$\begin{aligned} p &= \frac{p_{\delta^{18}O} + p_{\delta^2H}}{2} \\ \therefore u_p &= |p| \frac{\sqrt{u_{p_{\delta^{18}O}} + u_{p_{\delta^2H}}}}{p_{\delta^{18}O} + p_{\delta^2H}} \end{aligned} \tag{2.2}$$

For our study, we combined long term analytical precision and accuracy using Equation 1.2

$$\begin{aligned} u_{\delta^{18}O} &= \sqrt{0.0049 + 0.0169} \\ &= 0.1476482 \\ u_{\delta^2H} &= \sqrt{0.04 + 2.25} \\ &= 1.5132746 \end{aligned}$$

Knowing Equations 1.1–1.3, applying them to Equation 2.1 Genereux derived the following (see Equation 4 in Genereux 1998)

$$u_{p_E} = \sqrt{\left(u_{E_{baseflow}} \frac{E_{rain} - E_{streamflow}}{(E_{rain} - E_{baseflow})^2}\right)^2 + \left(u_{E_{rain}} \frac{E_{streamflow} - E_{baseflow}}{(E_{rain} - E_{baseflow})^2}\right)^2 + \left(u_{E_{streamflow}} \frac{-1}{E_{rain} - E_{baseflow}}\right)^2}$$

Since we know that, for our analyses, we have identical analytical uncertainty for any measurement of an isotope E, such that

$$u_{E_{streamflow}} = u_{E_{rain}} = u_{E_{baseflow}} = u_E$$

With this approach, I get $p_{\delta^{18}O} = 0.99685 \pm 0.1594934$ and $p_{\delta^2H} = 1.0259682 \pm 0.1413952$, such that following Equation 2.2 $p = 1.0114091 \pm 0.1065724$.

References

Genereux (1998) *Water Resources Research* 34(4):915–919