

With matrices $A^{(n)} = \begin{pmatrix} A_1^{(n,L)} & \dots & A_K^{(n,L)} \\ \vdots & \ddots & \vdots \\ A_1^{(n,1)} & \dots & A_K^{(n,1)} \end{pmatrix}, n = 1, 2, \dots, L$, for $n, m, l, h = 1, 2, \dots, L$, we define

$$\bar{X}^{(n,l)} = \frac{1}{K} \sum_{k=1}^K A_k^{(n,l)}$$

$$S_{(n,l)(m,h)} = \sum_{k=1}^K A_k^{(n,l)} A_k^{(m,h)} - \frac{1}{K} \left(\sum_{k=1}^K A_k^{(n,l)} \right) \left(\sum_{k=1}^K A_k^{(m,h)} \right)$$

Then, for all $n, m = 1, 2, \dots, L$, and a given l , the Pearson's coefficients are defined as

$$R_{n,m}^{(l)} = \frac{\sum_{k=1}^K (A_k^{(n,l)} - \bar{X}^{(n,l)})(A_k^{(m,l)} - \bar{X}^{(m,l)})}{\sqrt{\sum_{k=1}^K (A_k^{(n,l)} - \bar{X}^{(n,l)})^2} \cdot \sqrt{\sum_{k=1}^K (A_k^{(m,l)} - \bar{X}^{(m,l)})^2}} = \frac{S_{(n,l)(m,l)}}{\sqrt{S_{(n,l)(n,l)}} \sqrt{S_{(m,l)(m,l)}}}, l = 1, 2, \dots, L;$$

for all $l, h = 1, 2, \dots, L$, and a given n , the Pearson's coefficients are defined as

$$R_n^{(l,h)} = \frac{\sum_{k=1}^K (A_k^{(n,l)} - \bar{X}^{(n,l)})(A_k^{(n,h)} - \bar{X}^{(n,h)})}{\sqrt{\sum_{k=1}^K (A_k^{(n,l)} - \bar{X}^{(n,l)})^2} \cdot \sqrt{\sum_{k=1}^K (A_k^{(n,h)} - \bar{X}^{(n,h)})^2}} = \frac{S_{(n,l)(n,h)}}{\sqrt{S_{(n,l)(n,l)}} \sqrt{S_{(n,h)(n,h)}}}, n = 1, 2, \dots, L.$$

Thus, for given l , $R_{n,m}^{(l)}$ represent the correlation strength of the datasets between the n -th day and m -th day at level l , $l = 1, 2, \dots, L$. That is, the values of $R_{n,m}^{(l)}$ describe the dependency of the datasets between the n -th day and m -th day for those avocado samples with the shelf life of $l = 1, 2, \dots, L$.

For given n , $R_n^{(l,h)}$ represent the correlation strength of the datasets between the two shelf lives l and h on the n -th day, $n = 1, 2, \dots, L$. That is, the values of $R_n^{(l,h)}$ describe the dependency of the datasets between the two shelf lives l and h for those avocado samples on the n -th day, $n = 1, 2, \dots, L$.

We define

$$Y^{(l)} = \begin{pmatrix} R_{1,1}^{(l)} & \dots & R_{1,L}^{(l)} \\ \vdots & \ddots & \vdots \\ R_{L,1}^{(l)} & \dots & R_{L,L}^{(l)} \end{pmatrix}, \quad l = L, L-1, \dots, 2, 1, \text{ and}$$

$$Z_n = \begin{pmatrix} R_n^{(L,L)} & \dots & R_n^{(L,1)} \\ \vdots & \ddots & \vdots \\ R_n^{(1,L)} & \dots & R_n^{(1,1)} \end{pmatrix}, \quad n = 1, 2, \dots, L.$$