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SAN BERNARDINO

School of Computer Science
and Engineering

Caro-Wei Theorem

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Prerequisites

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- A graph, denoted G, is a collection of vertices, denoted V, connected by edge E.
- The independence number (also known as stability number), denoted $\alpha(G)$, is the cardinality of the largest independent vertex set.
- Independent vertex set is a set of pairwise non-adjacent vertices in a graph.

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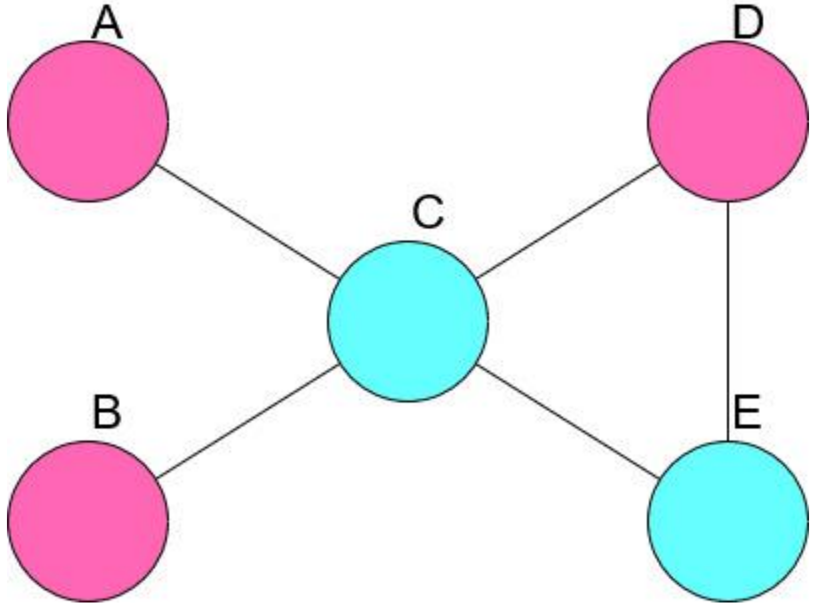
Example:

To the left is graph G, which has a vertex set,

$$V = \{A, B, C, D, E\}$$

And let S denote the independent vertex set,

$$S = \{A, B, D | S \subset V\}$$



Theorem of Caro and Wei (1981)

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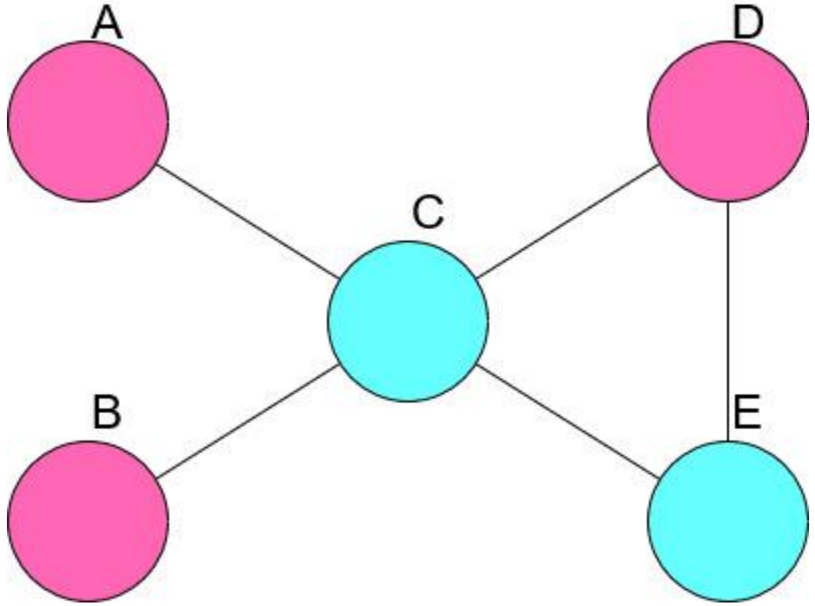
Computing a lower bound on the independence number of a graph G

$$\alpha(G) \geq w(G) \equiv \sum_{i=1}^{n} \frac{1}{d_i + 1}$$

$$3 \geq \frac{1}{5} + \frac{1}{3} + \frac{1}{3} + \frac{1}{2} + \frac{1}{2}$$

$$3 \geq 1\frac{13}{15}$$



Proof

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Erdős (1970) refined Turán's theorem (1940) which created an algorithm for producing the Caro-Wei number. This algorithm creates a collection of pairwise non-adjacent vertices $w_1, w_2, \ldots, w_s$ in graph G.

$j \leftarrow 0$

$\boldsymbol{while\ G \neq \emptyset\ do}$

  $j \leftarrow j + 1$

  $w_j \leftarrow vertex\ with\ smallest\ degree\ in\ G$

  $C_j \leftarrow \{w_j\} \cup \{v\colon\ v\ is\ adjacent\ to\ w_j\ in\ G\}$

  $G \leftarrow G - C_j$

$\boldsymbol{endwhile}$

$s \leftarrow j$

$H \leftarrow the\ graph\ in\ which\ two\ vertices\ are$
    $adjacent\ if\ and\ only\ if\ they\ belong\ to$
    $the\ same\ C_j$

Proof – Applied

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Only concerning ourselves with the while loop, we can apply $w(G)$ on line 5, for all vertices removed from G

a_1 = First iteration: $\frac{1}{2} + \frac{1}{5}$

$a_2 =$ Second iteration: $a_1 + \frac{1}{2}$

$a_3 =$ Third iteration: $a_2 + \frac{1}{3} + \frac{1}{3}$

$\therefore a3 = 1\frac{13}{15}$

$j \leftarrow 0$

$\boldsymbol{while\ G \neq \emptyset\ do}$

$\quad j \leftarrow j + 1$

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$\boldsymbol{endwhile}$

$s \leftarrow j$

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$\qquad adjacent\ if\ and\ only\ if\ they\ belong\ to$

$\qquad the\ same\ C_j$

Works Cited

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