Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	${A = \widehat{A}}$
	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right  \right\} $
5a	
6	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ L_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix}  \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & L_{10}U_{00} = \widehat{A}_{10} \\ L_{20}U_{00} = \widehat{A}_{20} \end{pmatrix} $
8	$A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$ $A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^TU_{02}) = L_{11}^{-1}(A_{12}^T - A_{10}^TA_{02})$ $A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$
7	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00} \qquad L_{00}U_{01} = \widehat{A}_{01} \qquad L_{00}U_{02} = \widehat{A}_{02} $ $ \wedge L_{10}U_{00} = \widehat{A}_{10}^T & L_{10}^TU_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} $ $ L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} $
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right] $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{pmatrix} \wedge \neg (m(A_{TL}) < n) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	{
4	
4	1
	where
2	
3	while do
2,3	
	Determine block size $b$
5a	
	where
6	
8	
7	
E la	
5b	
2	
	endwhile
2,3	
1b	(¬( ) {

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	${A = \widehat{A}}$
4	where
2	
3	while do
2,3	
5a	Determine block size $b$ where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\left\{\begin{array}{c} \wedge \\ \neg \end{array}\right\}$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

1a $\{A = \widehat{A}\}$ where  2 $\{\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{pmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{pmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix}$ Determine block size $b$ 8  8  7 $\{A_{TL} & A_{TR} & A_{TR} & A_{TR} & A_{TR} & A_{TR} & A_{TL}U_{TL} = \widehat{A}_{TL} & A_{TL}U_{TL} = \widehat{A}_{TL} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & A_{TL}U_{TL} = \widehat{A}_{TL} \\ A_{BL}U_{TL} = \widehat{A}_{BL}U_{TL} \end{pmatrix}$ 5b  2 $\{A_{TL} & A_{TR} & A_{TR} & A_{TR}U_{TL} & A_{TL}U_{TL} = \widehat{A}_{TL}U_{TL} = \widehat{A}_{TL}U_{TL}U_{TL} = \widehat{A}_{TL}U_{TL} = \widehat{A}_{TL}U_{TL}U_{TL} = \widehat{A}_{TL}U_{TL}U_{TL} = \widehat{A}_{TL}U_{TL}U_{TL} = \widehat{A}_{TL}U_{TL}U_{TL} = \widehat{A}_{TL}U_{TL}U_{TL}U_{TL} = \widehat{A}_{TL}U_{T$	Step	Algorithm: $A := LU_BLK_VAR4(A)$
where $ \begin{cases} \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{TL}} \\ \begin{pmatrix} A_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix} $ but the condition of the con	1a	$\{A = \widehat{A}\}$
where $ \begin{cases} \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{TL}} \\ \begin{pmatrix} A_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix} $ but the condition of the con		
$ \begin{bmatrix} 2 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \end{bmatrix} \\ 3 & \text{while} & \text{do} \\ 2,3 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{TL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & A_{TR} \end{pmatrix} \right. $	4	
3 while do $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		
3 while do $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{DL} & A_{DR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline I_{DL} & \widehat{A}_{DR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = A_{TL}}{I_{DL}U_{TL} = \widehat{A}_{DL}} \right\}$
$ 2.3  \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \wedge \right) $ Determine block size $b$ $ 8$ $ 8$ $ 7$ $ 5b$ $ 2  \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{TR}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  + C \cdot \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right  $ endwhile	3	
Determine block size $b$ $ \begin{array}{c} 5a \\ \hline                                   $		
$\begin{array}{c} 5a \\ \\ \hline \\ 8 \\ \hline \\ 7 \\ \hline \\ 5b \\ \hline \\ 2 \\ \hline \\ \left\{ \begin{array}{c} A_{TL}   A_{TR} \\ A_{BL}   A_{BR} \\ A_{BL}   A_{BR} \\ \end{array} \right\} = \left( \begin{array}{c} L \backslash U_{TL}   U_{TR} \\ L_{BL}   \widehat{A}_{BR} \\ \end{array} \right) \wedge \frac{L_{TL} U_{TL} = \widehat{A}_{TL}}{L_{BL} U_{TL} = \widehat{A}_{BL}} \frac{L_{TL} U_{TR} = \widehat{A}_{TR}}{L_{BL} U_{TL} = \widehat{A}_{BL}} \\ \\ \end{array}$	2,3	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) & = & \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \begin{array}{c c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right. \wedge \left. \right\}$
where $\begin{bmatrix} 6 & \\ \\ & \\ \end{bmatrix}$ $\begin{bmatrix} 7 & \\ \\ & \\ \end{bmatrix}$ $\begin{bmatrix} 5b & \\ \\ 2 & \\ \\ & \\ \\ & \\ \\ & \\ \\ & \\ \\ & \\ \\ & \\ \\ & \\ \\ \\ & \\ \\ \\ & \\ \\ \\ \\ \end{bmatrix}$ $\begin{bmatrix} L_{L}U_{TL} & U_{TR} \\ L_{BL} & L_{TL}U_{TL} & A_{TR} \\ L_{BL}U_{TL} & A_{TR} & L_{TL}U_{TR} & A_{TR} \\ L_{BL}U_{TL} & A_{BL} & L_{TL}U_{TR} & A_{TR} \\ L_{TL}U_{TR} & A_{TR} & L_{TL}U_{TR} & A_{TR} & L_{TL}U_{TR} & A_{TR} & L_{TL}U_{TR$		Determine block size b
where $\begin{bmatrix} 6 & \\ \\ & \\ \end{bmatrix}$ $\begin{bmatrix} 7 & \\ \\ & \\ \end{bmatrix}$ $\begin{bmatrix} 5b & \\ \\ 2 & \\ \\ & \\ \\ & \\ \\ & \\ \\ & \\ \\ & \\ \\ & \\ \\ & \\ \\ \\ & \\ \\ \\ & \\ \\ \\ \\ \end{bmatrix}$ $\begin{bmatrix} L_{L}U_{TL} & U_{TR} \\ L_{BL} & L_{TL}U_{TL} & A_{TR} \\ L_{BL}U_{TL} & A_{TR} & L_{TL}U_{TR} & A_{TR} \\ L_{BL}U_{TL} & A_{BL} & L_{TL}U_{TR} & A_{TR} \\ L_{TL}U_{TR} & A_{TR} & L_{TL}U_{TR} & A_{TR} & L_{TL}U_{TR} & A_{TR} & L_{TL}U_{TR$	5a.	
	- Ja	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		where
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	6	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		
$ 2  \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \text{endwhile} $	8	
$ 2  \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \text{endwhile} $		
$ 2  \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \text{endwhile} $		
$ 2  \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \text{endwhile} $		
$2  \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \text{endwhile} $	7	}
$ 2  \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \text{endwhile} $		
$ 2  \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \text{endwhile} $		
$ 2  \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \text{endwhile} $		
endwhile	5b	
endwhile		
endwhile	2	$\left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline \end{array} \right\} = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL}}{\bar{A}_{TL}} \left[ L_{TL}U_{TR} = \hat{A}_{TR} \\ \hline \end{array} \right\}$
	2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \land \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \land \right\} $
$1b  \left\{ A = L \backslash U \land LU = \widehat{A} \right\}$	1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{pmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix} \right\} $
	Determine block size $b$
5a	
	where
6	
0	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left[ \begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right] \right\} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{pmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix} \right\} $
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$A = \hat{A}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$ $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{pmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix} \right\} $
	Determine block size $b$
5a	
	where
6	
0	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left[ \begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right] \right\} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \wedge \neg (m(A_{TL}) < \right\} $
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ and $U_{TL}$ is $U$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{vmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{vmatrix} \wedge m(A_{TL}) < \right\} $
5a	Determine block size $b$ $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where $A_{11}$ is $b \times b$ , $L_{11}$ is $b \times b$ , $U_{11}$ is $b \times b$
6	
8	
7	
5b	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \dots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \dots $ $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} $
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \\ \text{endwhile} $
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{pmatrix} \wedge \neg (m(A_{TL}) < \right\} $
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right) \\ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$\begin{pmatrix} A_{BL} \mid A_{BR} \end{pmatrix} \begin{pmatrix} A_{20} \mid A_{21} \mid A_{22} \end{pmatrix} \begin{pmatrix} L_{BL} \mid L_{BR} \end{pmatrix} \begin{pmatrix} U_{BL} \mid U_{BR} \end{pmatrix}$ where $A_{11}$ is $b \times b$ , $L_{11}$ is $b \times b$ , $U_{11}$ is $b \times b$
6	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ L_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix}  \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & L_{10}U_{00} = \widehat{A}_{10} \\ L_{20}U_{00} = \widehat{A}_{20} \end{pmatrix} $
8	
7	
5b	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \cdots $ $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} $
2	
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \wedge \neg (m(A_{TL}) <  I  +  I $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$\{A = \widehat{A}\}$
	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\}  $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	
6	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ L_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} + \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ A_{10}U_{00} = \widehat{A}_{10} & L_{10}U_{00} = \widehat{A}_{10} \\ L_{20}U_{00} = \widehat{A}_{20} \end{pmatrix} $
8	
7	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00} \qquad L_{00}U_{01} = \widehat{A}_{01} \qquad L_{00}U_{02} = \widehat{A}_{02} $ $ \wedge L_{10}U_{00} = \widehat{A}_{10}^T & L_{10}^TU_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} $ $ L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} $
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right\} = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \\ \text{endwhile} $
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & \wedge \neg (m(A_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg (M_{TL}) < \sum_{TL} (M_{TL}) = \widehat{A}_{TL} & \wedge \neg ($
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$\{A = \widehat{A}\}$
	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	
6	$ \left\{ \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ L_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix}  \begin{array}{c} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & L_{10}U_{00} = \widehat{A}_{10} \\ L_{20}U_{00} = \widehat{A}_{20} \\ \end{array} \right\} $
8	$A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$ $A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^TU_{02}) = L_{11}^{-1}(A_{12}^T - A_{10}^TA_{02})$ $A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$
7	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00} \qquad L_{00}U_{01} = \widehat{A}_{01} \qquad L_{00}U_{02} = \widehat{A}_{02} $ $ \wedge L_{10}U_{00} = \widehat{A}_{10}^T & L_{10}^TU_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} $ $ L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} $
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} \left( \frac{A_{TL}}{A_{BL}} \middle  A_{TR} \right) = \left( \frac{L \backslash U_{TL}}{L_{BL}} \middle  U_{TR} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \right) \right\} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{pmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < \right\} $
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Algorithm: $A := LU_BLK_VAR4(A)$
$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
while $m(A_{TL}) < m(A)$ do
$A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$ $A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^TU_{02}) = L_{11}^{-1}(A_{12}^T - A_{10}^TA_{02})$ $A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$
$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
endwhile

## Algorithm: $A := LU_BLK_VAR4(A)$

$$A o \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \ , \ L o \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \ , \ U o \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $L_{TL}$  is  $0 \times 0$ ,  $U_{TL}$  is  $0 \times 0$ 

while  $m(A_{TL}) < m(A)$  do

Determine block size b

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots$$

where  $A_{11}$  is  $b \times b$ ,  $L_{11}$  is  $b \times b$ ,  $U_{11}$  is  $b \times b$ 

$$A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$$

$$A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^T U_{02}) = L_{11}^{-1}(A_{12}^T - A_{10}^T A_{02})$$

$$A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$$

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \cdots$$

endwhile