Step	Algorithm: $A := LU_{UNB_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \mid L_{TL} U_{TR} = \widehat{A}_{TR} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1
6	{
	update line 1
8	
	update line n
7	{ }
1	, 1
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) <) \right\} $
1b	$\left \left\{ A = L \backslash U \land LU = \widehat{A} \right\} \right $

Step	Algorithm: $A := LU_{UNB_VAR3}(A)$
1a	{
4	where
2	\text{\text{Where}}
3	while do
2,3	
5a	where
6	{
8	
7	{
5b	
2	
	endwhile
2,3	^
1b	<u>(</u> ¬()) }
	t J

Step	Algorithm: $A := LU_{UNB_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	
3	while do
2,3	
5a	where
6	{
8	
7	{
5b	
2	$\left\{ \left\{ \right. \right. \right.$
	endwhile
2,3	$\left\{ \begin{array}{c} \wedge \\ \neg \end{array} \right\}$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR3}(A)$
1a	$\left\{ A = \widehat{A} \right\}$
4	where
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \mid L_{TL} U_{TR} = \widehat{A}_{TR} \right\}$
3	while do
2,3	$\left\{ \begin{array}{c cc} \left(\begin{array}{c cc} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) & = & \left(\begin{array}{c cc} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \left L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \right\} \right\}$
5a	where
6	{
8	
7	{
5b	
2	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \land L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \land \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	
2	where $ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \left L_{TL}U_{TR} = \widehat{A}_{TR} \right \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	where
6	{
8	
7	{
5b	
2	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
	endwhile
2,3	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \left L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (\mathbf{m}(A_{TL}) < \mathbf{m}(A_{TL}) < \mathbf{m}(A_{TL}) \right \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0 $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	
	where
6	{
8	
7	{
5b	
2	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
	endwhile
2,3	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) < \right \right\} $
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR3}(A)$
1a	$\{A = \widehat{A} $
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots$ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1
6	{
8	
7	\{
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) <) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR3}(A)$
1a	$A = \widehat{A}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1
6	{
8	
7	{
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) <) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR3}(A)$
1a	$A = \widehat{A}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1
6	{
8	
7	{
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) <) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR3}(A)$
1a	$\{A = \widehat{A} $
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1
6	{
	update line 1
8	:
	update line n
7	{
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \right \right\}$
	endwhile
2,3	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left L_{TL} U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) < \right \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Algorithm: $A := LU_{UNB_VAR3}(A)$
$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
while $m(A_{TL}) < m(A)$ do
$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1
update line 1
:
update line n
$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
endwhile

Algorithm: $A := LU_{UNB_VAR3}(A)$

$$A o \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \,,\, L o \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \,,\, U o \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$$

where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|cc}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|cc}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
A_{20} & a_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|cc}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|cc}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots$$

where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1

update line 1

:

update line n

$$\left(\begin{array}{c|cccc} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|cccc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|cccc} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|cccc} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$$

endwhile