

Step	Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$
1a	$\{A = \hat{A}$
4	$A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$ <p>where <math>A_{TL}</math> is <math>0 \times 0</math></p>
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{L}_{TL} & \hat{A}_{TR} \\ \hline \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{TL} \\ \hline \hat{A}_{BL} \end{array} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{L}_{TL} & \hat{A}_{TR} \\ \hline \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{TL} \\ \hline \hat{A}_{BL} \end{array} \right) \wedge m(A_{TL}) < m(A) \right\}$
5a	<p><b>Determine block size <math>b</math></b></p> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right)$ <p>where <math>A_{11}</math> is <math>b \times b</math></p>
6	$\left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \hat{a}_{01} & \hat{A}_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \hat{a}_{12}^T \\ L_{20} & a_{21} - L_{20} l_{10} & A_{22} - L_{20} L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20} L_{00}^T \end{pmatrix} = \begin{pmatrix} \hat{A}_{00} \\ \hat{a}_{10}^T \\ \hat{A}_{20} \end{pmatrix} \right\}$
8	$\alpha_{11} := \sqrt{\alpha_{11}}$ $a_{21} := a_{21}/\alpha_{11}$ $A_{22} := A_{22} - a_{21}a_{21}^T$ update only lower triangular part
7	$\left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \hat{a}_{01} & \hat{A}_{02} \\ l_{10}^T & \lambda_{11} & \hat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T & & \\ l_{10}^T L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 & \\ L_{20} L_{00}^T & L_{20} l_{10} + \lambda_{11} l_{21} & \end{pmatrix} = \begin{pmatrix} \hat{A}_{00} \\ \hat{a}_{10}^T & \hat{\alpha}_{11} \\ \hat{A}_{20} & \hat{a}_{21} \end{pmatrix} \right\}$
5b	$\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right)$
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Step	Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$
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	where
2	{
3	while do
2,3	{ ^
5a	Determine block size $b$
	where
6	{
8	
7	{
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2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{L}_{TL} & \hat{A}_{TR} \\ \hline \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{TL} \\ \hline \hat{A}_{BL} \end{array} \right) \right\}$
3	<b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>
2,3	$\left\{ \begin{array}{l} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{L}_{TL} & \hat{A}_{TR} \\ \hline \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{TL} \\ \hline \hat{A}_{BL} \end{array} \right) \wedge m(A_{TL}) < \\ m(A) \end{array} \right\}$
5a	<p style="text-align: center;"><b>Determine block size <math>b</math></b></p> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c ccc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right)$ <p style="text-align: center; color: blue;">where <math>A_{11}</math> is <math>b \times b</math></p>
6	$\left\{ \begin{array}{c} \left( \begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L_{00} & \hat{a}_{01} & \hat{A}_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \hat{a}_{12}^T \\ L_{20} & a_{21} - L_{20}l_{10} & A_{22} - L_{20}L_{20}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{00}L_{00}^T \\ \hline l_{10}^T L_{00}^T \\ L_{20}L_{00}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{00} \\ \hline \hat{a}_{10}^T \\ \hat{A}_{20} \end{array} \right) \end{array} \right\}$
8	
7	$\left\{ \begin{array}{l} \left( \begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L_{00} & \hat{a}_{01} & \hat{A}_{02} \\ l_{10}^T & \lambda_{11} & \hat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{array} \right) \wedge \\ \left( \begin{array}{cc} L_{00}L_{00}^T & \\ l_{10}^T L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20}L_{00}^T & L_{20}l_{10} + \lambda_{11}l_{21} \end{array} \right) = \left( \begin{array}{cc} \hat{A}_{00} & \\ \hat{a}_{10}^T & \hat{\alpha}_{11} \\ \hat{A}_{20} & \hat{a}_{21} \end{array} \right) \end{array} \right\}$
5b	$\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c ccc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right)$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{L}_{TL} & \hat{A}_{TR} \\ \hline \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{TL} \\ \hline \hat{A}_{BL} \end{array} \right) \right\}$
	<b>endwhile</b>
2,3	$\left\{ \begin{array}{l} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{L}_{TL} & \hat{A}_{TR} \\ \hline \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{TL} \\ \hline \hat{A}_{BL} \end{array} \right) \wedge \neg(m(A_{TL}) < \\ m(A) \end{array} \right\}$
1b	$\{A = \text{Chol}(\hat{A})$ <span style="float: right;">}</span>

Step	<b>Algorithm:</b> $[A] := \text{CHOL\_BLK\_VAR3}(A)$
1a	$\{A = \hat{A}$ <span style="float: right;">}</span>
4	$A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$ <p style="text-align: center; color: blue;">where <math>A_{TL}</math> is <math>0 \times 0</math></p>
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{L}_{TL} & \hat{A}_{TR} \\ \hline \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{TL} \\ \hline \hat{A}_{BL} \end{array} \right) \right\}$
3	<b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>
2,3	$\left\{ \begin{array}{l} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{L}_{TL} & \hat{A}_{TR} \\ \hline \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{TL} \\ \hline \hat{A}_{BL} \end{array} \right) \wedge m(A_{TL}) < \\ m(A) \end{array} \right\}$
5a	<p style="text-align: center;"><b>Determine block size <math>b</math></b></p> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c ccc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right)$ <p style="text-align: center; color: blue;">where <math>A_{11}</math> is <math>b \times b</math></p>
6	$\left\{ \begin{array}{c} \left( \begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L_{00} & \hat{a}_{01} & \hat{A}_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \hat{a}_{12}^T \\ L_{20} & a_{21} - L_{20} l_{10} & A_{22} - L_{20} L_{20}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{00}L_{00}^T \\ \hline l_{10}^T L_{00}^T \\ L_{20}L_{00}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{00} \\ \hline \hat{a}_{10}^T \\ \hat{A}_{20} \end{array} \right) \end{array} \right\}$
8	<p style="color: red;"><math>\alpha_{11} := \sqrt{\alpha_{11}}</math></p> <p style="color: red;"><math>a_{21} := a_{21}/\alpha_{11}</math></p> <p style="color: red;"><math>A_{22} := A_{22} - a_{21}a_{21}^T</math>    <b>update only lower triangular part</b></p>
7	$\left\{ \begin{array}{l} \left( \begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L_{00} & \hat{a}_{01} & \hat{A}_{02} \\ l_{10}^T & \lambda_{11} & \hat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{array} \right) \wedge \\ \left( \begin{array}{cc} L_{00}L_{00}^T & \\ \hline l_{10}^T L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \end{array} \right) = \left( \begin{array}{cc} \hat{A}_{00} & \\ \hline \hat{a}_{10}^T & \hat{\alpha}_{11} \\ \hat{A}_{20} & \hat{a}_{21} \end{array} \right) \end{array} \right\}$
5b	$\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c ccc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right)$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{L}_{TL} & \hat{A}_{TR} \\ \hline \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{TL} \\ \hline \hat{A}_{BL} \end{array} \right) \right\}$
	<b>endwhile</b>
2,3	$\left\{ \begin{array}{l} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \hat{L}_{TL} & \hat{A}_{TR} \\ \hline \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c} \hat{A}_{TL} \\ \hline \hat{A}_{BL} \end{array} \right) \wedge \neg(m(A_{TL}) < \\ m(A) \end{array} \right\}$
1b	$\{A = \text{Chol}(\hat{A})$ <span style="float: right;">}</span>

	Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$
	$A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$ <p>where <math>A_{TL}</math> is <math>0 \times 0</math></p>
	while $m(A_{TL}) < m(A)$ do
	<p><b>Determine block size <math>b</math></b></p> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right)$ <p>where <math>A_{11}</math> is <math>b \times b</math></p>
	$\alpha_{11} := \sqrt{\alpha_{11}}$ $a_{21} := a_{21}/\alpha_{11}$ $A_{22} := A_{22} - a_{21}a_{21}^T \quad \text{update only lower triangular part}$
	$\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{cc c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right)$
	endwhile

**Algorithm:**  $[A] := \text{CHOL\_BLK\_VAR3}(A)$

$$A \rightarrow \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$$

**where**  $A_{TL}$  is  $0 \times 0$

**while**  $m(A_{TL}) < m(A)$  **do**

**Determine block size**  $b$

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right)$$

**where**  $A_{11}$  is  $b \times b$

$$\alpha_{11} := \sqrt{\alpha_{11}}$$

$$a_{21} := a_{21}/\alpha_{11}$$

$$A_{22} := A_{22} - a_{21}a_{21}^T \quad \text{update only lower triangular part}$$

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right)$$

**endwhile**