

Step	Algorithm: $A := \text{LU_UNB_VAR1}(A)$
1a	$\{A = \hat{A}$
4	$A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$ <p>where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</p>
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \hat{A}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL}}{L_{BL}U_{TL} = \hat{A}_{BL}} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \hat{A}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL}}{L_{BL}U_{TL} = \hat{A}_{BL}} \wedge m(A_{TL}) < m(A) \right\}$
5a	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$ <p>where α_{11} is 1×1, λ_{11} is 1×1, v_{11} is 1×1</p>
6	$\{$
8	update line 1 : update line n
7	$\}$
5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
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	endwhile
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \hat{A}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL}}{L_{BL}U_{TL} = \hat{A}_{BL}} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{A = L \setminus U \wedge LU = \hat{A}$
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3	while do
2,3	{ \wedge }
5a	where
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8	
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5b	
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	endwhile
2,3	{ $\wedge \neg($) }
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5a	
	where
6	$\{$
8	
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6	{
8	<p>update line 1</p> <p>:</p> <p>update line n</p>
7	}
5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \hat{A}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL}}{L_{BL}U_{TL} = \hat{A}_{BL}} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \hat{A}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL}}{L_{BL}U_{TL} = \hat{A}_{BL}} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{A = L \setminus U \wedge LU = \hat{A}$

	Algorithm: $A := \text{LU_UNB_VAR1}(A)$
	$A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$ <p>where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</p>
	while $m(A_{TL}) < m(A)$ do
	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$ <p>where α_{11} is 1×1, λ_{11} is 1×1, v_{11} is 1×1</p>
	update line 1 : update line n
	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{cc c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
	endwhile

Algorithm: $A := \text{LU_UNB_VAR1}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$$

where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c} A_{00} & a_{01} \ A_{02} \\ \hline a_{10}^T & \alpha_{11} \ a_{12}^T \\ A_{20} & a_{21} \ A_{22} \end{array} \right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$$

where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1

update line 1

:

update line n

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c} A_{00} \ a_{01} & A_{02} \\ \hline a_{10}^T \ \alpha_{11} & a_{12}^T \\ A_{20} \ a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$$

endwhile