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<tr><td>8</td><td>$a_{01} := u_{01} = L_{00}^{-1} a_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00})
$a_{10}^T := l_{10}^T = a_{10}^T U_{00}^{-1}$ (U_{00} is stored in the upper triangular part of A_{00})
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Step	Algorithm: $A := \text{LU\_UNB\_VAR1}(A)$
1a	$\{ \qquad \}$
4	where
2	$\left\{ \qquad \right\}$
3	while do
2,3	$\left\{ \qquad \wedge \qquad \right\}$
5a	where
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8	$a_{01} := u_{01} = L_{00}^{-1} a_{01}$ $a_{10}^T := l_{10}^T = a_{10}^T U_{00}^{-1}$ $\alpha_{11} := \upsilon_{11} = \alpha_{11} - l_{10}^T u_{01}$
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5b	
2	$\left\{ \qquad \right\}$
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2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{ A = L\backslash U \wedge LU = \widehat{A} \}$

Step	Algorithm: $A := \text{LU\_UNB\_VAR1}(A)$
1a	$\{ A = \widehat{A} \}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) \right\}$
5a	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1
6	$\left\{ \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ \widehat{a}_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ \widehat{A}_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{array}\right) \wedge L_{00}U_{00} = \widehat{A}_{00} \right\}$
8	$a_{01} := u_{01} = L_{00}^{-1} a_{01}$ $a_{10}^T := l_{10}^T = a_{10}^T U_{00}^{-1}$ $\alpha_{11} := \upsilon_{11} = \alpha_{11} - l_{10}^T u_{01}$
7	$\left\{ \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & u_{01} & \widehat{A}_{02} \\ l_{10}^T & \upsilon_{11} & \widehat{a}_{12}^T \\ \widehat{A}_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{array}\right) \wedge \begin{array}{cc} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} \\ l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + \upsilon_{11} = \widehat{\alpha}_{11} \end{array} \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{ A = L\backslash U \wedge LU = \widehat{A} \}$

<table>
<tr><th>Step</th><th>Algorithm: $A := \mathrm{LU\_UNB\_VAR1}(A)$</th></tr>
<tr><td>1a</td><td>$\{ A = \widehat{A} \}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$
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<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
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<tr><td>5a</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$
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	Algorithm: $A := \text{LU\_UNB\_VAR1}(A)$
	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$, $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
	while $m(A_{TL}) < m(A)$ do
	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1
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	endwhile

Algorithm: $A := \mathrm{LU\_UNB\_VAR1}(A)$

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endwhile