Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$A = \widehat{A}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\}  $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{pmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix} \wedge m(A_{TL}) < \right\} $
5a	Determine block size $b$ $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $
6	where $A_{11}$ is $b \times b$ , $L_{11}$ is $b \times b$ , $U_{11}$ is $b \times b$ $ \left\{ \qquad \qquad \right\}$
	update line 1
8	:
	update line n
7	{
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \\ \hline \right\} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}}   L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL}   L_{TL}U_{TR} = \widehat{A}_{TR} \rangle \wedge \neg (m(A_{TL}) < \right\} $
1b	$\left  \left\{ A = L \backslash U \land LU = \widehat{A} \right\} \right $

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	<b>{</b>
4	
2	where
3	while do
2,3	
5a	Determine block size $b$ where
6	<b>\</b> {
8	
7	{
5b	
2	
	endwhile
2,3	
1b	

$1a  \left\{ A = \widehat{A} \right.$	
	}
where	
2 {	
3 while do	
2,3	$\wedge$
Determine block size b  5a  where	
6 {	}
8	
7 {	}
5b	
2 {	
2 { endwhile	
	^}

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \right\}$
3	while do
2,3	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR}  \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR}  \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL}  \end{array} \right. \wedge \right\} $
5a	Determine block size $b$ where
6	{
8	
7	{
5b	
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}}  L_{TL}U_{TR} = \widehat{A}_{TR} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{pmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix} \right\} $
	Determine block size $b$
5a	
	where
6	{
8	
7	{
5b	
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left \begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array}\right\} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) < \sum_{m(A)} (m(A_{TL})) \right  \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$ $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{pmatrix} \wedge m(A_{TL}) < \right\} $
5a	
	where
6	<b>{</b>
8	
7	{
5b	
2	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  \begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right  \right\} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \wedge \neg (m(A_{TL}) < n) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{pmatrix} \wedge m(A_{TL}) < \right\} $
	Determine block size $b$
5a	$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots$ $\text{where } A_{11} \text{ is } b \times b, L_{11} \text{ is } b \times b, U_{11} \text{ is } b \times b$
6	{
8	
7	{
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} A_{20} & A_{21} & A_{22} \end{array} \right\} = \left( \begin{array}{c c} L \setminus U_{TL} & U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \left( \begin{array}{c c} L \setminus U_{TL} & U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{array} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{pmatrix} \wedge \neg (m(A_{TL}) < n) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_BLK_VAR4(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{pmatrix} \wedge m(A_{TL}) < \right\} $
	Determine block size $b$
5a	$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots$ $\text{where } A_{11} \text{ is } b \times b, L_{11} \text{ is } b \times b, U_{11} \text{ is } b \times b$
6	{
8	
7	{
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} A_{20} & A_{21} & A_{22} \end{array} \right\} = \left( \begin{array}{c c} L \setminus U_{TL} & U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \left( \begin{array}{c c} L \setminus U_{TL} & U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{array} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{pmatrix} \wedge \neg (m(A_{TL}) < n) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Algorithm: $A := LU_BLK_VAR4(A)$
$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
while $m(A_{TL}) < m(A)$ do
Determine block size $b$
$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots$ $\text{where } A_{11} \text{ is } b \times b, L_{11} \text{ is } b \times b, U_{11} \text{ is } b \times b$
update line 1
·
update line n
$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
endwhile

## Algorithm: $A := LU_BLK_VAR4(A)$

$$A o \left( egin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} 
ight) \,,\, L o \left( egin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} 
ight) \,,\, U o \left( egin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} 
ight)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $L_{TL}$  is  $0 \times 0$ ,  $U_{TL}$  is  $0 \times 0$ 

while  $m(A_{TL}) < m(A)$  do

Determine block size b

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots$$

where  $A_{11}$  is  $b \times b$ ,  $L_{11}$  is  $b \times b$ ,  $U_{11}$  is  $b \times b$ 

update line 1

:

update line n

$$\left(\begin{array}{c|cccc}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|cccc}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|cccc}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|cccc}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \cdots$$

endwhile