Q.	
Step	Algorithm: $A := LU_{UNB_VAR2}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL} U_{TL} = \widehat{A}_{TL}}{L_{BL} U_{TL} = \widehat{A}_{BL}} \right)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \\ \end{array} \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1
6	$ \begin{array}{c} \text{where } \alpha_{11} \text{ is } 1 \times 1, \lambda_{11} \text{ is } 1 \times 1, v_{11} \text{ is } 1 \times 1 \\ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ L_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} - \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} \\ L_{10}U_{00} = \widehat{a}_{10}^T \\ L_{20}U_{00} = \widehat{A}_{20} \end{pmatrix} $
8	$a_{01} := u_{01} = L_{00}^{-1} a_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00}) $\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$ $a_{21} := l_{21} = (\widehat{a}_{21} - L_{20} u_{01})/v_{11} (a_{21} - A_{20} a_{01})/\alpha_{11}$
7	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & \widehat{A}_{02} \\ l_{10}^T & v_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} & L_{00}U_{00} = \widehat{A}_{00} & \mathbb{E}_{00}u_{01} = \widehat{a}_{01} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + v_{11} = \widehat{\alpha}_{11} \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}u_{01} + l_{21}v_{11} = \widehat{a}_{21} \end{pmatrix} $
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right) $
	endwhile
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR2}(A)$
1a	{
4	where
2	
3	while do
2,3	
5a	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\left\{ \begin{array}{c} & & \\ & & \\ \end{array} \right.$
1b	{

Step	Algorithm: $A := LU_t$	$_{ m VNB_VAR2}(A)$			
1a	$\{A = \widehat{A}$				}
4	where				
2	\{\text{Whote}				igg
3	while	do			
2,3	$\bigg \bigg\{$		٨		$\bigg\}$
5a	where				
6					$\left.\begin{array}{c} \end{array}\right\}$
8					
7					$oxed{ }$
5b					
2					
	endwhile				
2,3			^ ¬()	$\left.\begin{array}{c} \\ \end{array}\right\}$
1b	$\{A = L \backslash U \land LU = \widehat{A}$				}

Step	Algorithm: $A := LU_{UNB_VAR2}(A)$
1a	$\{A = \widehat{A} $
4	where
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\}$
3	while do
2,3	$\left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \right.$
5a	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
	endwhile
2,3	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg () \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR2}(A)$	
1a	$\{A = \widehat{A}$	}
4	where	
2	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} $	igg
3	while $m(A_{TL}) < m(A)$ do	
2,3	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \right.$	$igg\}$
5a	where	
6		$\left. \begin{array}{c} \\ \end{array} \right\}$
8		
7		$\left.\begin{array}{c} \end{array}\right\}$
5b		
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. $	
	endwhile	
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A))$	igg
1b	$\left\{A = L \backslash U \land LU = \widehat{A}\right\}$	}

Step	Algorithm: $A := LU_{UNB_VAR2}(A)$
1a	$\{A = \widehat{A} $
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$ \frac{\text{where } A_{TL} \text{ is } 0 \times 0, L_{TL} \text{ is } 0 \times 0, U_{TL} \text{ is } 0 \times 0}{\left\{ \left(\frac{A_{TL}}{A_{BL}} \middle A_{BR} \right) = \left(\frac{L \setminus U_{TL}}{L_{BL}} \middle \widehat{A}_{BR} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \end{array} \right\}$
5a	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
	endwhile
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR2}(A)$	
1a	$\{A=\widehat{A}$	}
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0	
2	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} $	igg
3	while $m(A_{TL}) < m(A)$ do	
2,3	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) $	igg
5a	$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots$ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1	
6		$\left. \right\}$
8		
7		$\left. \right\}$
5b	$\langle BL BR \rangle \langle BL BR \rangle$	
2	$ \left\{ \begin{array}{c c} A_{20} & a_{21} & A_{22} \end{array} \right\} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL} & \widehat{A}_{BR}} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{BL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} $	igg
	endwhile	
2,3	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) $	igg
1b	$\{A = L \backslash U \land LU = \widehat{A}$	}

Step	Algorithm: $A := LU_{UNB_VAR2}(A)$	
1a	$A = \widehat{A}$	}
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0	
2	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} $	igg
3	while $m(A_{TL}) < m(A)$ do	
2,3	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) $	igg
5a	where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1	
6	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} & L_{00}U_{00} = \widehat{A}_{00} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T \\ L_{20}U_{00} = \widehat{A}_{20} \end{cases} $	$\left. \begin{array}{c} \\ \end{array} \right\}$
8		
7		igg
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $	
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right\} = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} $	igg
	endwhile	
2,3	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) $	$igg\}$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$	}

Step	Algorithm: $A := LU_{UNB_VAR2}(A)$
1a	$\{A = \widehat{A}\}$
4	$A ightharpoonup \left(egin{array}{c c} A_{TL} & A_{TR} \ \hline A_{BL} & A_{BR} \end{array} ight), \ L ightharpoonup \left(egin{array}{c c} L_{TL} & L_{TR} \ \hline L_{BL} & L_{BR} \end{array} ight), \ U ightharpoonup \left(egin{array}{c c} U_{TL} & U_{TR} \ \hline U_{BL} & U_{BR} \end{array} ight)$
2	where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0 $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \\ \end{array} \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & a_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1
6	$ \left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^{T} & \alpha_{11} & a_{12}^{T} \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^{T} & \widehat{\alpha}_{11} & \widehat{a}_{12}^{T} \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} \\ \wedge & l_{10}^{T}U_{00} = \widehat{a}_{10}^{T} \\ L_{20}U_{00} = \widehat{A}_{20} \end{pmatrix} $
8	
7	$ \left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & \widehat{A}_{02} \\ l_{10}^T & v_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + v_{11} = \widehat{\alpha}_{11} \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}u_{01} + l_{21}v_{11} = \widehat{a}_{21} \end{pmatrix} $
5b	$\left(\begin{array}{c c}A_{BL}&A_{BR}\end{array}\right)$ $\left(\begin{array}{c c}A_{20}&a_{21}&A_{22}\end{array}\right)$ $\left(\begin{array}{c c}L_{BL}&L_{BR}\end{array}\right)$ $\left(\begin{array}{c c}U_{BL}&U_{BR}\end{array}\right)$
2	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\}$
	endwhile
2,3	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) \right\} $
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR2}(A)$	
1a	$\{A = \widehat{A} $ }	
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0	
2	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} $	$\left. ight\}$
3	while $m(A_{TL}) < m(A)$ do	
2,3	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \\ \end{array} \right. $	$\left. ight\}$
5a	$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots$ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1	
6	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} & L_{00}U_{00} = \widehat{A}_{00} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T \\ L_{20}U_{00} = \widehat{A}_{20} \end{cases} $	$\left. \left. \right \right.$
8	$a_{01} := u_{01} = L_{00}^{-1} a_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00}) $\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$ $a_{21} := l_{21} = (\widehat{a}_{21} - L_{20} u_{01})/v_{11} (a_{21} - A_{20} a_{01})/\alpha_{11}$	
7	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & \widehat{A}_{02} \\ l_{10}^T & v_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} & L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + v_{11} = \widehat{\alpha}_{11} \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}u_{01} + l_{21}v_{11} = \widehat{a}_{21} \end{cases} $	$\left. \left. \right \right.$
5b	$\begin{pmatrix} BL & BR \end{pmatrix} \begin{pmatrix} A & A & A & A \end{pmatrix} \begin{pmatrix} BL & BR \end{pmatrix} \begin{pmatrix} BL & BR \end{pmatrix}$	
2	$ \left\{ \begin{array}{c c} A_{20} & a_{21} & A_{22} \\ \hline \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} $	$\left. \right\}$
	endwhile	
2,3	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) $	$\left. \left. \right \right.$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$	

$\begin{array}{c} \text{Algorithm: } A := \text{LU}_{\text{UNB}}\text{VAR2}(A) \end{array}$
$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
while $m(A_{TL}) < m(A)$ do
$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1
$a_{01} := u_{01} = L_{00}^{-1} a_{01} (L_{00} \text{ is stored in the strictly lower triangular part of } A_{00})$ $\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$ $a_{21} := l_{21} = (\widehat{a}_{21} - L_{20} u_{01}) / v_{11} (a_{21} - A_{20} a_{01}) / \alpha_{11}$
$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
endwhile

Algorithm: $A := LU_{UNB_VAR2}(A)$

$$A o \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \,,\, L o \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \,,\, U o \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$$

where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
A_{20} & a_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots$$

where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1

 $a_{01} := u_{01} = L_{00}^{-1} a_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00})

$$\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$$

$$a_{21} := l_{21} = (\widehat{a}_{21} - L_{20}u_{01})/v_{11} \ (a_{21} - A_{20}a_{01})/\alpha_{11}$$

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \cdots$$

endwhile