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<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$
where A_{TL} is 0×0, B_T has 0 rows, C_T has 0 rows</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + \widehat{C}_T \\ \hline A_{BL}B_T + \widehat{C}_B \end{array}\right) \right\}$</td></tr>
<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + \widehat{C}_T \\ \hline A_{BL}B_T + \widehat{C}_B \end{array}\right) \wedge m(A_{TL}) < m(A) \right\}$</td></tr>
<tr><td>5a</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ C_2 \end{array}\right)$
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<tr><td>6</td><td>$\left\{ \left(\begin{array}{c} C_0 \\ c_1^T \\ C_2 \end{array}\right) = \left(\begin{array}{c} A_{00}B_0 + \widehat{C}_0 \\ a_{10}^T B_0 + \widehat{c}_1^T \\ A_{20}B_0 + \widehat{C}_2 \end{array}\right) \right\}$</td></tr>
<tr><td>8</td><td>$C_0 := A_{00}B_0 + a_{01}b_1^T + A_{02}B_2 + C_0$
$c_1^T := a_{10}^T B_0 + \alpha_{11}b_1^T + a_{12}^T B_2 + c_1^T$
$C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$</td></tr>
<tr><td>7</td><td>$\left\{ \left(\begin{array}{c} C_0 \\ c_1^T \\ C_2 \end{array}\right) = \left(\begin{array}{c} A_{00}B_0 + (a_{10}^T)^T b_1^T + \widehat{C}_0 \\ a_{10}^T B_0 + \alpha_{11} b_1^T + \widehat{c}_1^T \\ A_{20}B_0 + a_{21} b_1^T + \widehat{C}_2 \end{array}\right) \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ c_1^T \\ \hline C_2 \end{array}\right)$</td></tr>
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<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) = \left(\begin{array}{c} A_{TL}B_T + \widehat{C}_T \\ \hline A_{BL}B_T + \widehat{C}_B \end{array}\right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$</td></tr>
<tr><td>1b</td><td>$\{C = AB + \widehat{C}\}$</td></tr>
</table>

Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), \; B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right), \; C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right)$$

where A_{TL} is 0×0, B_T has 0 rows, C_T has 0 rows

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \; \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array} \right), \; \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ C_2 \end{array} \right)$$

where α_{11} is 1×1, b_1 has 1 row, c_1 has 1 row

$$C_0 := A_{00}B_0 + a_{01}b_1^T + A_{02}B_2 + C_0$$
$$c_1^T := a_{10}^T B_0 + \alpha_{11} b_1^T + a_{12}^T B_2 + c_1^T$$
$$C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \; \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array} \right), \; \left(\begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left(\begin{array}{c} C_0 \\ c_1^T \\ \hline C_2 \end{array} \right)$$

endwhile

Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$$

where A_{TL} is 0×0, B_T has 0 rows, C_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ C_2 \end{array}\right)$$

where α_{11} is 1×1, b_1 has 1 row, c_1 has 1 row

$$C_0 := a_{01} b_1^T + C_0$$
$$c_1^T := \alpha_{11} b_1^T + c_1^T$$
$$C_2 := a_{21} b_1^T + C_2$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ c_1^T \\ \hline C_2 \end{array}\right)$$

endwhile