Step	Algorithm: $A := \mathrm{LU\_BLK\_VAR5}(A)$
1a	$\{A = \widehat{A} \}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{l} \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$
5a	**Determine block size** b $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$
6	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} - L_{10}U_{01} & \widehat{A}_{12} - L_{10}U_{02} \\ L_{20} & \widehat{A}_{21} - L_{20}U_{01} & \widehat{A}_{22} - L_{20}U_{02} \end{array}\right) \\ \wedge \begin{array}{lll} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ L_{10}U_{00} = \widehat{A}_{10} & & \\ L_{20}U_{00} = \widehat{A}_{20} & & \end{array} \end{array} \right\}$
8	$A_{11} := L\backslash U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11})$ $A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^T U_{02}) = L_{11}^{-1}A_{12}^T$ $A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1}$ $A_{22} := \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12}$
7	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & L\backslash U_{11} & U_{12} \\ L_{20} & L_{21} & \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} \end{array}\right) \\ \wedge \begin{array}{lll} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ L_{10}U_{00} = \widehat{A}_{10}^T & L_{10}^T U_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} & \end{array} \end{array} \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline \end{array} \wedge \right.$

Step	Algorithm: $A := \text{LU\_BLK\_VAR5}(A)$
1a	$\{$ $\}$
4	where
2	$\left\{ \right\}$
3	while do
2,3	$\left\{ \wedge \right\}$
5a	**Determine block size** b where
6	$\left\{ \right\}$
8	
7	$\left\{ \right\}$
5b	
2	$\left\{ \right\}$
	endwhile
2,3	$\left\{ \wedge \right.$

Step	Algorithm: $A := \text{LU\_BLK\_VAR5}(A)$
1a	$\{ A = \widehat{A} \qquad \}$
4	where
2	$\left\{ \qquad \right\}$
3	while do
2,3	$\left\{ \qquad \wedge \right\}$
5a	**Determine block size** b where
6	$\left\{ \qquad \right\}$
8	
7	$\left\{ \qquad \right\}$
5b	
2	$\left\{ \qquad \right\}$
	endwhile
2,3	$\left\{ \qquad \wedge \right.$

<table>
<tr><th>Step</th><th>Algorithm: $A := \mathrm{LU\_BLK\_VAR5}(A)$</th></tr>
<tr><td>1a</td><td>$\{A = \widehat{A}$ $\}$</td></tr>
<tr><td>4</td><td>where</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$</td></tr>
<tr><td>3</td><td>while do</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \right\}$</td></tr>
<tr><td>5a</td><td>Determine block size b
where</td></tr>
<tr><td>6</td><td>$\left\{ \right\}$</td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>7</td><td>$\left\{ \right\}$</td></tr>
<tr><td>5b</td><td></td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \end{array}\right. = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \end{array}\right. \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{array} \wedge \right.$</td></tr>
</table>

<table>
<tr><th>Step</th><th>Algorithm: $A := \text{LU\_BLK\_VAR5}(A)$</th></tr>
<tr><td>1a</td><td>$\{A = \widehat{A} \qquad \}$</td></tr>
<tr><td>4</td><td>where</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c|c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL} U_{TR} \end{array} \right) \wedge \begin{array}{c|c} L_{TL} U_{TL} = \widehat{A}_{TL} & L_{TL} U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL} U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$</td></tr>
<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \begin{array}{l} \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c|c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL} U_{TR} \end{array} \right) \wedge \begin{array}{c|c} L_{TL} U_{TL} = \widehat{A}_{TL} & L_{TL} U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL} U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$</td></tr>
<tr><td>5a</td><td>Determine block size b
where</td></tr>
<tr><td>6</td><td>$\left\{ \qquad \right\}$</td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>7</td><td>$\left\{ \qquad \right\}$</td></tr>
<tr><td>5b</td><td></td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c|c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL} U_{TR} \end{array} \right) \wedge \begin{array}{c|c} L_{TL} U_{TL} = \widehat{A}_{TL} & L_{TL} U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL} U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline \end{array} \right) = \left(\begin{array}{c|c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline \end{array} \right) \wedge \begin{array}{c|c} L_{TL} U_{TL} = \widehat{A}_{TL} & L_{TL} U_{TR} = \widehat{A}_{TR} \\ \hline \end{array} \wedge \right.$</td></tr>
</table>

<table>
<tr><th>Step</th><th>Algorithm: $A := \mathrm{LU\_BLK\_VAR5}(A)$</th></tr>
<tr><td>1a</td><td>$\{A = \widehat{A}\}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$
where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$</td></tr>
<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \begin{array}{l} \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$</td></tr>
<tr><td>5a</td><td>Determine block size b
where</td></tr>
<tr><td>6</td><td>$\left\{ \quad \right\}$</td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>7</td><td>$\left\{ \quad \right\}$</td></tr>
<tr><td>5b</td><td></td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline \end{array}\right. = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline \end{array}\right. \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline \end{array} \wedge$</td></tr>
</table>

Step	Algorithm: $A := \mathrm{LU\_BLK\_VAR5}(A)$
1a	$\{ A = \widehat{A} \}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{l} \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$
5a	**Determine block size** b $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$
6	$\left\{ \quad \right\}$
8	
7	$\left\{ \quad \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{array} \wedge \right.$

Step	Algorithm: $A := \text{LU\_BLK\_VAR5}(A)$
1a	$\{ A = \widehat{A} \quad \}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL} U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL} U_{TL} = \widehat{A}_{TL} & L_{TL} U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL} U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{l} \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL} U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL} U_{TL} = \widehat{A}_{TL} & L_{TL} U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL} U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$
5a	**Determine block size** b $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$
6	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L \backslash U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} - L_{10} U_{01} & \widehat{A}_{12} - L_{10} U_{02} \\ L_{20} & \widehat{A}_{21} - L_{20} U_{01} & \widehat{A}_{22} - L_{20} U_{02} \end{array}\right) \\ \wedge \begin{array}{lll} L_{00} U_{00} = \widehat{A}_{00} & L_{00} U_{01} = \widehat{A}_{01} & L_{00} U_{02} = \widehat{A}_{02} \\ L_{10} U_{00} = \widehat{A}_{10} & & \\ L_{20} U_{00} = \widehat{A}_{20} & & \end{array} \end{array} \right\}$
8	
7	$\left\{ \quad \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL} U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL} U_{TL} = \widehat{A}_{TL} & L_{TL} U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL} U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline \end{array}\right) = \left(\begin{array}{c\|c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline \end{array}\right) \wedge \begin{array}{c\|c} L_{TL} U_{TL} = \widehat{A}_{TL} & L_{TL} U_{TR} = \widehat{A}_{TR} \\ \hline \end{array} \wedge \right.$

<table>
<tr><th>Step</th><th>Algorithm: $A := \text{LU\_BLK\_VAR5}(A)$</th></tr>
<tr><td>1a</td><td>$\{ A = \widehat{A} \qquad \}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$
where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$</td></tr>
<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \begin{array}{l} \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$</td></tr>
<tr><td>5a</td><td>Determine block size b
$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$
where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$</td></tr>
<tr><td>6</td><td>$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} - L_{10}U_{01} & \widehat{A}_{12} - L_{10}U_{02} \\ L_{20} & \widehat{A}_{21} - L_{20}U_{01} & \widehat{A}_{22} - L_{20}U_{02} \end{array}\right) \\ \begin{array}{cccc} & L_{00}U_{00} = \widehat{A}_{00} & \text{Ł}_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & L_{10}U_{00} = \widehat{A}_{10} & & \\ & L_{20}U_{00} = \widehat{A}_{20} & & \end{array} \end{array} \right\}$</td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>7</td><td>$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & L\backslash U_{11} & U_{12} \\ L_{20} & L_{21} & \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} \end{array}\right) \\ \begin{array}{cccc} & L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & L_{10}U_{00} = \widehat{A}_{10}^{T} & L_{10}^{T}U_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} \\ & L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} & \end{array} \end{array} \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{array} \wedge \right.$</td></tr>
</table>

Step	Algorithm: $A := \text{LU\_BLK\_VAR5}(A)$
1a	$\{ A = \widehat{A} \}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{l} \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$
5a	**Determine block size** b $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$
6	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} - L_{10}U_{01} & \widehat{A}_{12} - L_{10}U_{02} \\ L_{20} & \widehat{A}_{21} - L_{20}U_{01} & \widehat{A}_{22} - L_{20}U_{02} \end{array}\right) \\ \wedge \begin{array}{lll} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ L_{10}U_{00} = \widehat{A}_{10} & & \\ L_{20}U_{00} = \widehat{A}_{20} & & \end{array} \end{array} \right\}$
8	$A_{11} := L\backslash U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11})$ $A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^T U_{02}) = L_{11}^{-1}A_{12}^T$ $A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1}$ $A_{22} := \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12}$
7	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & L\backslash U_{11} & U_{12} \\ L_{20} & L_{21} & \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} \end{array}\right) \\ \wedge \begin{array}{lll} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ L_{10}U_{00} = \widehat{A}_{10}^T & L_{10}^T U_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} & \end{array} \end{array} \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{U}_{TR} \\ \hline \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline \end{array} \wedge \right.$

Algorithm: $A := \text{LU\_BLK\_VAR5}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

Determine block size b

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$

$$A_{11} := L \backslash U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11})$$
$$A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^T U_{02}) = L_{11}^{-1} A_{12}^T$$
$$A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1}$$
$$A_{22} := \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12}$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$$

endwhile

Algorithm: $A := \mathrm{LU\_BLK\_VAR5}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

Determine block size b

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$

$$A_{11} := L\backslash U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11})$$
$$A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^{T} - L_{10}^{T}U_{02}) = L_{11}^{-1}A_{12}^{T}$$
$$A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1}$$
$$A_{22} := \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12}$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$$

endwhile