<table>
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<tr><td>8</td><td>$a_{01} := u_{01} = L_{00}^{-1} a_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00})
$\alpha_{11} := \upsilon_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$
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Step	Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$
1a	$\{$ $\}$
4	where
2	$\left\{ \right\}$
3	while do
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5a	where
6	$\left\{ \right\}$
8	
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5a	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \cdots$ where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1
6	$\left\{ \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left(\begin{array}{ccc} L\backslash U_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{array} \right) \wedge \begin{array}{c} L_{00} U_{00} = \widehat{A}_{00} \\ l_{10}^T U_{00} = \widehat{a}_{10}^T \\ L_{20} U_{00} = \widehat{A}_{20} \end{array} \right\}$
8	
7	$\left\{ \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left(\begin{array}{ccc} L\backslash U_{00} & u_{01} & \widehat{A}_{02} \\ l_{10}^T & \upsilon_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{array} \right) \wedge \begin{array}{cc} L_{00} U_{00} = \widehat{A}_{00} & \text{Ł}_{00} u_{01} = \widehat{a}_{01} \\ l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + \upsilon_{11} = \widehat{\alpha}_{11} \\ L_{20} U_{00} = \widehat{A}_{20} & L_{20} u_{01} + l_{21} \upsilon_{11} = \widehat{a}_{21} \end{array} \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \begin{array}{c} L_{TL} U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL} U_{TL} = \widehat{A}_{BL} \end{array} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \begin{array}{c} L_{TL} U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL} U_{TL} = \widehat{A}_{BL} \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{ A = L\backslash U \wedge LU = \widehat{A} \}$

<table>
<tr><th>Step</th><th>Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$</th></tr>
<tr><td>1a</td><td>$\{A = \widehat{A}\}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$
where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$</td></tr>
<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
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<tr><td>5a</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$
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<tr><td>6</td><td>$\left\{ \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{array}\right) \wedge \begin{array}{c} L_{00}U_{00} = \widehat{A}_{00} \\ l_{10}^T U_{00} = \widehat{a}_{10}^T \\ L_{20}U_{00} = \widehat{A}_{20} \end{array} \right\}$</td></tr>
<tr><td>8</td><td>$a_{01} := u_{01} = \quad L_{00}^{-1} a_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00})
$\alpha_{11} := \upsilon_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \quad \alpha_{11} - a_{10}^T a_{01}$
$a_{21} := l_{21} = (\widehat{a}_{21} - L_{20}u_{01})/\upsilon_{11} \quad (a_{21} - A_{20}a_{01})/\alpha_{11}$</td></tr>
<tr><td>7</td><td>$\left\{ \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & u_{01} & \widehat{A}_{02} \\ l_{10}^T & \upsilon_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{array}\right) \wedge \begin{array}{cc} L_{00}U_{00} = \widehat{A}_{00} & \text{Ł}_{00}u_{01} = \widehat{a}_{01} \\ l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + \upsilon_{11} = \widehat{\alpha}_{11} \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}u_{01} + l_{21}\upsilon_{11} = \widehat{a}_{21} \end{array} \right\}$</td></tr>
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<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$</td></tr>
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	Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$
	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$, $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
	while $m(A_{TL}) < m(A)$ do
	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1
	$a_{01} := u_{01} = \quad L_{00}^{-1} a_{01} \qquad$ (L_{00} is stored in the strictly lower triangular part of A_{00}) $\alpha_{11} := \upsilon_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \quad \alpha_{11} - a_{10}^T a_{01}$ $a_{21} := l_{21} = (\widehat{a}_{21} - L_{20} u_{01}) / \upsilon_{11} \quad (a_{21} - A_{20} a_{01}) / \alpha_{11}$
	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
	endwhile

Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1

$a_{01} := u_{01} = \quad L_{00}^{-1} a_{01} \qquad (L_{00}$ is stored in the strictly lower triangular part of $A_{00})$

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$a_{21} := l_{21} = (\widehat{a}_{21} - L_{20} u_{01}) / \upsilon_{11} \quad (a_{21} - A_{20} a_{01}) / \alpha_{11}$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$$

endwhile