<table>
<tr><th>Step</th><th>Algorithm: $A := \mathrm{LU\_BLK\_VAR3}(A)$</th></tr>
<tr><td>1a</td><td>$\{A = \widehat{A}\}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$
where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$</td></tr>
<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < m(A) \right\}$</td></tr>
<tr><td>5a</td><td>Determine block size b
$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$
where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$</td></tr>
<tr><td>6</td><td>$\left\{ \begin{array}{l} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & U_{01} & U_{02} \\ \widehat{A}_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \\ \wedge\ L_{00}U_{00} = \widehat{A}_{00} \quad L_{00}U_{01} = \widehat{A}_{01} \quad L_{00}U_{02} = \widehat{A}_{02} \end{array} \right\}$</td></tr>
<tr><td>8</td><td>$A_{10} := L_{10} = \widehat{A}_{10}^{T} U_{00}^{-1} = A_{10}^{T} U_{00}^{-1}$ (U_{00} is stored in the upper triangular part of A_{00})
$A_{11} := L\backslash U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(\widehat{A}_{11} - A_{10}A_{01})$
$A_{12} := U_{12}^{T} = L_{11}^{-1}(\widehat{A}_{12} - L_{10}U_{02}) = L_{11}^{-1}(A_{12} - A_{10}A_{02})$</td></tr>
<tr><td>7</td><td>$\left\{ \begin{array}{l} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & L\backslash U_{11} & U_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \\ \wedge \begin{array}{lll} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{U}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ L_{10}U_{00} = \widehat{A}_{10} & L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} \end{array} \end{array} \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg(m(A_{TL}) < m(A)) \right\}$</td></tr>
<tr><td>1b</td><td>$\{A = L\backslash U \wedge LU = \widehat{A}\}$</td></tr>
</table>

Step	Algorithm: $A := \text{LU\_BLK\_VAR3}(A)$
1a	$\{$ $\}$
4	where
2	$\left\{ \right\}$
3	while do
2,3	$\left\{ \wedge \right\}$
5a	**Determine block size** b where
6	$\left\{ \right\}$
8	
7	$\left\{ \right\}$
5b	
2	$\left\{ \right\}$
	endwhile
2,3	$\left\{ \wedge \quad \neg(\quad) \right\}$
1b	$\{$ $\}$

Step	Algorithm: $A := \text{LU\_BLK\_VAR3}(A)$
1a	$\{ A = \widehat{A} \}$
4	where
2	$\{ \quad \}$
3	while do
2,3	$\{ \quad \wedge \}$
5a	**Determine block size** b where
6	$\{ \quad \}$
8	
7	$\{ \quad \}$
5b	
2	$\{ \quad \}$
	endwhile
2,3	$\{ \quad \wedge \neg(\quad) \}$
1b	$\{ A = L \backslash U \wedge LU = \widehat{A} \}$

Step	Algorithm: $A := \text{LU\_BLK\_VAR3}(A)$
1a	$\{A = \widehat{A}$ $\}$
4	where
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
3	while do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \right\}$
5a	**Determine block size** b where
6	$\left\{ \right\}$
8	
7	$\left\{ \right\}$
5b	
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg(\quad) \right\}$
1b	$\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$

Step	Algorithm: $A := \text{LU\_BLK\_VAR3}(A)$
1a	$\{ A = \widehat{A} \}$
4	where
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \, \Big\| \, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
3	**while** $m(A_{TL}) < m(A)$ **do**
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \, \Big\| \, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < m(A) \right\}$
5a	**Determine block size b** where
6	$\{ \}$
8	
7	$\{ \}$
5b	
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \, \Big\| \, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \, \Big\| \, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{ A = L\backslash U \wedge LU = \widehat{A} \}$

Step	Algorithm: $A := \text{LU\_BLK\_VAR3}(A)$
1a	$\{A = \widehat{A}$ $\}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$, $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < m(A) \right\}$
5a	**Determine block size b** where
6	$\left\{ \right\}$
8	
7	$\left\{ \right\}$
5b	
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$

<table>
<tr><th>Step</th><th>Algorithm: $A := \mathrm{LU\_BLK\_VAR3}(A)$</th></tr>
<tr><td>1a</td><td>$\{ A = \widehat{A} \}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$
where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$</td></tr>
<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < m(A) \right\}$</td></tr>
<tr><td>5a</td><td>**Determine block size b**
$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$
where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$</td></tr>
<tr><td>6</td><td>$\left\{ \qquad \right\}$</td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>7</td><td>$\left\{ \qquad \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg(m(A_{TL}) < m(A)) \right\}$</td></tr>
<tr><td>1b</td><td>$\{ A = L\backslash U \wedge LU = \widehat{A} \}$</td></tr>
</table>

Step	Algorithm: $A := \text{LU\_BLK\_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$, $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{array} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{array} \wedge m(A_{TL}) < m(A) \right\}$
5a	**Determine block size b** $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$
6	$\left\{ \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & U_{01} & U_{02} \\ \widehat{A}_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \wedge \; L_{00}U_{00} = \widehat{A}_{00} \;\; L_{00}U_{01} = \widehat{A}_{01} \;\; L_{00}U_{02} = \widehat{A}_{02} \right\}$
8	
7	$\left\{ \quad \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{array} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{A = L\backslash U \wedge LU = \widehat{A}\}$

<table>
<tr><th>Step</th><th>Algorithm: $A := \mathrm{LU\_BLK\_VAR3}(A)$</th></tr>
<tr><td>1a</td><td>$\{A = \widehat{A}\}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$
where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\vert\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$</td></tr>
<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\vert\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < m(A) \right\}$</td></tr>
<tr><td>5a</td><td>Determine block size b
$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$
where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$</td></tr>
<tr><td>6</td><td>$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & U_{01} & U_{02} \\ \widehat{A}_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{array}\right) \\ \wedge\ L_{00}U_{00} = \widehat{A}_{00} \quad L_{00}U_{01} = \widehat{A}_{01} \quad L_{00}U_{02} = \widehat{A}_{02} \end{array} \right\}$</td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>7</td><td>$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & L\backslash U_{11} & U_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{array}\right) \\ \wedge \begin{array}{lll} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{U}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ L_{10}U_{00} = \widehat{A}_{10} & L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} \end{array} \end{array} \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\vert\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\vert\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg(m(A_{TL}) < m(A)) \right\}$</td></tr>
<tr><td>1b</td><td>$\{A = L\backslash U \wedge LU = \widehat{A}\}$</td></tr>
</table>

<table>
<tr><th>Step</th><th>Algorithm: $A := \mathrm{LU\_BLK\_VAR3}(A)$</th></tr>
<tr><td>1a</td><td>$\{ A = \widehat{A} \}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right),\ L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right),\ U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$
where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \;\Big|\; L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$</td></tr>
<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \;\Big|\; L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < m(A) \right\}$</td></tr>
<tr><td>5a</td><td>**Determine block size b**
$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right),\ \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots,\ \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$
where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$</td></tr>
<tr><td>6</td><td>$\left\{ \begin{array}{l} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & U_{01} & U_{02} \\ \widehat{A}_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \\ \quad \wedge\ L_{00}U_{00} = \widehat{A}_{00}\ \ L_{00}U_{01} = \widehat{A}_{01}\ \ L_{00}U_{02} = \widehat{A}_{02} \end{array} \right\}$</td></tr>
<tr><td>8</td><td>$A_{10} := L_{10} = \widehat{A}_{10}^T U_{00}^{-1} = A_{10}^T U_{00}^{-1}$ (U_{00} is stored in the upper triangular part of A_{00})
$A_{11} := L\backslash U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(\widehat{A}_{11} - A_{10}A_{01})$
$A_{12} := U_{12}^T = L_{11}^{-1}(\widehat{A}_{12} - L_{10}U_{02}) = L_{11}^{-1}(A_{12} - A_{10}A_{02})$</td></tr>
<tr><td>7</td><td>$\left\{ \begin{array}{l} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & L\backslash U_{11} & U_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \\ \quad \wedge \begin{array}{lll} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{U}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ L_{10}U_{00} = \widehat{A}_{10} & L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} \end{array} \end{array} \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right),\ \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots,\ \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \;\Big|\; L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \;\Big|\; L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg(m(A_{TL}) < m(A)) \right\}$</td></tr>
<tr><td>1b</td><td>$\{ A = L\backslash U \wedge LU = \widehat{A} \}$</td></tr>
</table>

Algorithm: $A := \text{LU\_BLK\_VAR3}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

Determine block size b

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$

$$A_{10} := L_{10} = \widehat{A}_{10}^T U_{00}^{-1} = A_{10}^T U_{00}^{-1} \qquad (U_{00} \text{ is stored in the upper triangular part of } A_{00})$$

$$A_{11} := L \backslash U_{11} = LU(\widehat{A}_{11} - L_{10} U_{01}) = LU(\widehat{A}_{11} - A_{10} A_{01})$$

$$A_{12} := U_{12}^T = L_{11}^{-1}(\widehat{A}_{12} - L_{10} U_{02}) = L_{11}^{-1}(A_{12} - A_{10} A_{02})$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$$

endwhile

Algorithm: $A := \mathrm{LU\_BLK\_VAR3}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), \; L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), \; U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

Determine block size b

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$

$A_{10} := L_{10} = \widehat{A}_{10}^T U_{00}^{-1} = A_{10}^T U_{00}^{-1}$ (U_{00} is stored in the upper triangular part of A_{00})

$A_{11} := L\backslash U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(\widehat{A}_{11} - A_{10}A_{01})$

$A_{12} := U_{12}^T = L_{11}^{-1}(\widehat{A}_{12} - L_{10}U_{02}) = L_{11}^{-1}(A_{12} - A_{10}A_{02})$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$$

endwhile