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<tr><td>8</td><td>$A_{01} := U_{01} = L_{00}^{-1}A_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00})
$A_{10} := L_{10} = A_{10}U_{00}^{-1}$ (U_{00} is stored in the upper triangular part of A_{00})
$A_{11} := L\backslash U_{11} = LU(A_{11} - L_{10}U_{01}$</td></tr>
<tr><td>7</td><td>$\left\{ \begin{pmatrix} A_{00} & {}_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & U_{01} & \widehat{A}_{02} \\ L_{10} & L\backslash U_{11} & \widehat{A}_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \wedge \begin{array}{cc} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} \\ L_{10}U_{00} = \widehat{A}_{10} & L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11} \end{array} \right\}$</td></tr>
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2	$\left\{ \qquad \right\}$
3	while do
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5a	**Determine block size** b where
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8	$A_{01} := U_{01} = L_{00}^{-1} A_{01}$ $A_{10} := L_{10} = A_{10} U_{00}^{-1}$ $A_{11} := L \backslash U_{11} = LU(A_{11} - L_{10} U_{01})$
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5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \cdots$
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	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \right\}$
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	Algorithm: $A := \text{LU\_BLK\_VAR1}(A)$
	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
	while $m(A_{TL}) < m(A)$ do
	Determine block size b $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$
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Algorithm: $A := \text{LU\_BLK\_VAR1}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

Determine block size b

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

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$A_{01} := U_{01} = \quad L_{00}^{-1} A_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00})

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endwhile