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| Step | Algorithm: $[\alpha] := \text{SAPDOT_UNB_VAR1}(x, y, \alpha)$ |
| 1a | $\{\alpha = \hat{\alpha}$ $\}$ |
| 4 | $x \rightarrow \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \rightarrow \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where x_T has 0 rows, y_T has 0 rows |
| 2 | $\{\alpha = x_T^T y_T + \hat{\alpha}$ $\}$ |
| 3 | while $m(x_T) < m(x)$ do |
| 2,3 | $\{ \quad \alpha = x_T^T y_T + \hat{\alpha} \wedge m(x_T) < m(x)$ $\}$ |
| 5a | $\begin{pmatrix} x_T \\ x_B \end{pmatrix} \rightarrow \begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_T \\ y_B \end{pmatrix} \rightarrow \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix}$ where χ_1 has 1 row, ψ_1 has 1 row |
| 6 | $\{ \quad \alpha = x_0^T y_0 + \hat{\alpha}$ $\}$ |
| 8 | $\alpha := \chi_1 \times \psi_1 + \hat{\alpha}$ |
| 7 | $\{ \quad \alpha = x_0^T y_0 + \chi_1 \times \psi_1 + \hat{\alpha}$ $\}$ |
| 5b | $\begin{pmatrix} x_T \\ x_B \end{pmatrix} \leftarrow \begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_T \\ y_B \end{pmatrix} \leftarrow \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix}$ |
| 2 | $\{ \quad \alpha = x_T^T y_T + \hat{\alpha}$ $\}$ |
| | endwhile |
| 2,3 | $\{\alpha = x_T^T y_T + \hat{\alpha} \wedge \neg(m(x_T) < m(x))$ $\}$ |
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| 3 | while do |
| 2,3 | { \wedge } |
| 5a | where |
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where x_T has 0 rows, y_T has 0 rows

while $m(x_T) < m(x)$ **do**

$$\begin{pmatrix} x_T \\ x_B \end{pmatrix} \rightarrow \begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_T \\ y_B \end{pmatrix} \rightarrow \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix}$$

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endwhile