Step	Algorithm: $A := LU_{UNB\_VAR3}(A)$
1a	$\left\{ A=\widehat{A}\right\}$
	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{  \frac{A_{TL}   A_{TR}}{A_{BL}   A_{BR}} \right) = \left( \frac{L \setminus U_{TL}   U_{TR}}{\widehat{A}_{BL}   \widehat{A}_{BR}} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL}   L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < 0  $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots $ where $\alpha_{11}$ is $1 \times 1$ , $\lambda_{11}$ is $1 \times 1$ , $\nu_{11}$ is $1 \times 1$
6	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ \widehat{a}_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ \widehat{A}_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} $ $ \land L_{00}U_{00} = \widehat{A}_{00}  L_{00}u_{01} = \widehat{a}_{01}  L_{00}U_{02} = \widehat{A}_{02} $
8	$a_{10}^T := l_{10}^T = \widehat{a}_{10}^T U_{00}^{-1} = a_{10}^T U_{00}^{-1} \qquad (U_{00} \text{ is stored in the upper triangular part of } A_{00})$ $\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$ $a_{12}^T := u_{12}^T = \widehat{a}_{12}^T - l_{10}^T U_{02} = a_{12}^T - a_{10}^T A_{02}$
7	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ \widehat{A}_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} \\ \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + v_{11} = \widehat{\alpha}_{11} & l_{10}^T U_{02} + u_{12}^T = \widehat{a}_{12}^T \end{pmatrix} $
5b	$ \left(\begin{array}{c cccc} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c cccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c cccc} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c cccc} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} \left( \frac{A_{TL}}{A_{BL}} \middle  A_{TR} \right) = \left( \frac{L \backslash U_{TL}}{\widehat{A}_{BL}} \middle  U_{TR} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \right. $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) < n) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR3}(A)$
1a	<b>{</b>
4	where
2	
3	while do
2,3	
5a	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\left\{egin{array}{cccccccccccccccccccccccccccccccccccc$
1b	{

Step	Algorithm: $A := LU_{UNB\_VAR3}(A)$
1a	$A = \hat{A}$
4	where
2	
3	while do
2,3	
5a	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\left\{\begin{array}{c} \\ \\ \\ \\ \end{array}\right.$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right $
3	while do
2,3	$ \left\{ \begin{array}{c c} \left( \frac{A_{TL} & A_{TR}}{A_{BL} & A_{BR}} \right) & = & \left( \frac{L \setminus U_{TL} & U_{TR}}{\widehat{A}_{BL}} & \widehat{A}_{BR} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} & \wedge \right\} $
5a	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \mid L_{TL} U_{TR} = \widehat{A}_{TR} \\ \end{array} \right\} $
	endwhile
2,3	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \wedge \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \right\} $
	endwhile
2,3	$ \left\{ \left( \frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BL} \end{vmatrix} A_{BR} \right) = \left( \frac{L \setminus U_{TL}}{\widehat{A}_{BL}} \begin{vmatrix} U_{TR} \\ \widehat{A}_{BR} \end{vmatrix} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) < A_{TL}) \right  \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	where
6	
8	
7	
5b	
2	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \right\}$
2,3	endwhile $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \backslash U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) < ) \right\} $
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots$ where $\alpha_{11}$ is $1 \times 1$ , $\lambda_{11}$ is $1 \times 1$ , $v_{11}$ is $1 \times 1$
6	
8	
7	
5b	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \cdots $
2	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \right\}$
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) < ) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where $\alpha_{11}$ is $1 \times 1$ , $\lambda_{11}$ is $1 \times 1$ , $\nu_{11}$ is $1 \times 1$
6	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ \widehat{a}_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ \widehat{A}_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} $ $ \land L_{00}U_{00} = \widehat{A}_{00}  L_{00}u_{01} = \widehat{a}_{01}  L_{00}U_{02} = \widehat{A}_{02} $
8	
7	
5b	$\left\langle \begin{array}{c c} A_{BL} & A_{BR} \end{array} \right\rangle = \left\langle \begin{array}{c c} A_{20} & a_{21} & A_{22} \end{array} \right\rangle = \left\langle \begin{array}{c c} L_{BL} & L_{BR} \end{array} \right\rangle = \left\langle \begin{array}{c c} U_{BL} & U_{BR} \end{array} \right\rangle$
2	$ \left\{ \begin{array}{c c} \left( \frac{A_{TL}}{A_{BL}} \middle  A_{TR} \right) = \left( \frac{L \setminus U_{TL}}{\widehat{A}_{BL}} \middle  \widehat{A}_{BR} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \\ \end{array} \right\} $
	endwhile
2,3	$ \left\{ \left( \frac{A_{TL}}{A_{BL}} \middle  A_{TR} \right) = \left( \frac{L \setminus U_{TL}}{\widehat{A}_{BL}} \middle  \widehat{A}_{BR} \right) \land L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \land \neg (m(A_{TL}) < \right\} \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR3}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots $ where $\alpha_{11}$ is $1 \times 1$ , $\lambda_{11}$ is $1 \times 1$ , $v_{11}$ is $1 \times 1$
6	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ \widehat{a}_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ \widehat{A}_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} $ $ \land L_{00}U_{00} = \widehat{A}_{00}  L_{00}u_{01} = \widehat{a}_{01}  L_{00}U_{02} = \widehat{A}_{02} $
8	
7	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ \widehat{A}_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} \\ \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + v_{11} = \widehat{\alpha}_{11} & l_{10}^T U_{02} + u_{12}^T = \widehat{a}_{12}^T \end{pmatrix} $
5b	$ \left(\begin{array}{c cccc} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c cccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c cccc} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c cccc} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \right\} $
	endwhile
2,3	$ \left\{ \left( \frac{A_{TL}}{A_{BL}} \middle  A_{TR} \right) = \left( \frac{L \setminus U_{TL}}{\widehat{A}_{BL}} \middle  \widehat{A}_{BR} \right) \land L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \land \neg (m(A_{TL}) < \right\} \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR3}(A)$
1a	$\{A = \widehat{A} \}$
	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \left  L_{TL} U_{TR} = \widehat{A}_{TR} \right  \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \frac{A_{TL}   A_{TR}}{A_{BL}   A_{BR}} \right) = \left( \frac{L \setminus U_{TL}   U_{TR}}{\widehat{A}_{BL}   \widehat{A}_{BR}} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL}   L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots$ where $\alpha_{11}$ is $1 \times 1$ , $\lambda_{11}$ is $1 \times 1$ , $v_{11}$ is $1 \times 1$
6	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ \widehat{a}_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ \widehat{A}_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} $ $ \land L_{00}U_{00} = \widehat{A}_{00}  L_{00}u_{01} = \widehat{a}_{01}  L_{00}U_{02} = \widehat{A}_{02} $
8	$a_{10}^T := l_{10}^T = \widehat{a}_{10}^T U_{00}^{-1} = a_{10}^T U_{00}^{-1} \qquad (U_{00} \text{ is stored in the upper triangular part of } A_{00})$ $\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$ $a_{12}^T := u_{12}^T = \widehat{a}_{12}^T - l_{10}^T U_{02} = a_{12}^T - a_{10}^T A_{02}$
7	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ \widehat{A}_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} \\ \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + v_{11} = \widehat{\alpha}_{11} & l_{10}^T U_{02} + u_{12}^T = \widehat{a}_{12}^T \end{pmatrix} $
5b	$ \left(\begin{array}{c cccc} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c cccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c cccc} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c cccc} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} \left( \frac{A_{TL}}{A_{BL}} \middle  A_{TR} \right) = \left( \frac{L \backslash U_{TL}}{\widehat{A}_{BL}} \middle  U_{TR} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \right. $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) < n) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Algorithm: $A := LU_{UNB\_VAR3}(A)$
$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
while $m(A_{TL}) < m(A)$ do
$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots $ where $\alpha_{11}$ is $1 \times 1$ , $\lambda_{11}$ is $1 \times 1$ , $v_{11}$ is $1 \times 1$
$a_{10}^T := l_{10}^T = \widehat{a}_{10}^T U_{00}^{-1} = a_{10}^T U_{00}^{-1} \qquad (U_{00} \text{ is stored in the upper triangular part of } A_{00})$ $\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$ $a_{12}^T := u_{12}^T = \widehat{a}_{12}^T - l_{10}^T U_{02} = a_{12}^T - a_{10}^T A_{02}$
$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
endwhile

## Algorithm: $A := LU_{UNB_VAR3}(A)$

$$A \to \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) , L \to \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) , U \to \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $L_{TL}$  is  $0 \times 0$ ,  $U_{TL}$  is  $0 \times 0$ 

while  $m(A_{TL}) < m(A)$  do

$$\left(\begin{array}{c|cccc}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|cccc}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
A_{20} & a_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|cccc}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|cccc}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots$$

where  $\alpha_{11}$  is  $1 \times 1$ ,  $\lambda_{11}$  is  $1 \times 1$ ,  $v_{11}$  is  $1 \times 1$ 

$$a_{10}^T := l_{10}^T = \widehat{a}_{10}^T U_{00}^{-1} = a_{10}^T U_{00}^{-1}$$
 ( $U_{00}$  is stored in the upper triangular part of  $A_{00}$ )

$$\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$$

$$a_{12}^T := u_{12}^T = \widehat{a}_{12}^T - l_{10}^T U_{02} = a_{12}^T - a_{10}^T A_{02}$$

$$\frac{a_{12}^{T} := u_{12}^{T} = \widehat{a}_{12}^{T} - l_{10}^{T} U_{02} = a_{12}^{T} - a_{10}^{T} A_{02}}{\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}} \leftarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^{T} & \alpha_{11} & a_{12}^{T} \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \cdots$$

endwhile