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<tr><td>8</td><td>$A_{01} := U_{01} = L_{00}^{-1}A_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00})
$A_{11} := L\backslash U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$
$A_{21} := L_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$
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<tr><td>7</td><td>$\left\{ \begin{array}{l} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & U_{01} & \widehat{A}_{02} \\ L_{10} & L\backslash U_{11} & \widehat{a}^T_{12} \\ L_{20} & L_{21} & \widehat{A}_{22} \end{pmatrix} \\ \quad \begin{array}{rll} & L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} \\ \wedge & L_{10}U_{00} = \widehat{A}_{10} & L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11} \\ & L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} \end{array} \end{array} \right\}$</td></tr>
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1a	$\{$ $\}$
4	where
2	$\left\{ \right\}$
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5a	**Determine block size** b where
6	$\left\{ \right\}$
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<tr><td>5a</td><td>Determine block size b
$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow$
$\cdots$
where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$</td></tr>
<tr><td>6</td><td>$\left\{ \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & \widehat{A}_{01} & \widehat{A}_{02} \\ L_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ L_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \wedge \begin{array}{c} L_{00}U_{00} = \widehat{A}_{00} \\ L_{10}U_{00} = \widehat{A}_{10} \\ L_{20}U_{00} = \widehat{A}_{20} \end{array} \right\}$</td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>7</td><td>$\left\{ \begin{array}{l} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & U_{01} & \widehat{A}_{02} \\ L_{10} & L\backslash U_{11} & \widehat{a}_{12}^T \\ L_{20} & L_{21} & \widehat{A}_{22} \end{pmatrix} \\ \wedge \begin{array}{ll} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} \\ L_{10}U_{00} = \widehat{A}_{10} & L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11} \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} \end{array} \end{array} \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow$
$\cdots$</td></tr>
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<tr><td>1b</td><td>$\{A = L\backslash U \wedge LU = \widehat{A}\}$</td></tr>
</table>

<table>
<tr><th>Step</th><th>Algorithm: $A := \mathrm{LU\_UNB\_VAR2}(A)$</th></tr>
<tr><td>1a</td><td>$\{ A = \widehat{A} \}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$
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<tr><td>5a</td><td>Determine block size b
$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow$
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<tr><td>8</td><td>$A_{01} := U_{01} = L_{00}^{-1}A_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00})
$A_{11} := L\backslash U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$
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<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow$
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Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

Determine block size b

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow$$

$\ldots$

where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$

$$A_{01} := U_{01} = L_{00}^{-1} A_{01} \quad (L_{00} \text{ is stored in the strictly lower triangular part of } A_{00})$$

$$A_{11} := L \backslash U_{11} = LU(\widehat{A}_{11} - L_{10} U_{01}) = LU(A_{11} - A_{10} A_{01})$$

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$$(U_{11} \text{ is stored in the upper triangular part of } A_{11})$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow$$

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Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

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(U_{11} is stored in the upper triangular part of A_{11})

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endwhile