

Step	Algorithm: $A := \text{LU_UNB_VAR5}(A)$
1a	$\{A = \hat{A}$
4	$A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$ <p>where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</p>
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{l} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge m(A_{TL}) < \\ m(A) \end{array} \right\}$
5a	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$ <p>where α_{11} is 1×1, λ_{11} is 1×1, v_{11} is 1×1</p>
6	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left(\begin{array}{ccc} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & \hat{\alpha}_{11} - l_{10}^T u_{01} & \hat{a}_{12}^T - l_{10}^T U_{02} \\ L_{20} & \hat{a}_{21} - L_{20} u_{01} & \hat{A}_{22} - L_{20} U_{02} \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}u_{01} = \hat{a}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge \quad l_{10}^T U_{00} = \hat{a}_{10}^T \\ L_{20}U_{00} = \hat{A}_{20} \end{array} \right\}$
8	$a_{21}^T := u_{21}^T = (\hat{a}_{21} - L_{20}u_{01})/v_{11} = a_{21}/\alpha_{11}$ $A_{22} := \hat{A}_{22} - L_{20}U_{02} - l_{21}u_{12}^T = A_{22} - a_{21}a_{12}^T$
7	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left(\begin{array}{ccc} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ L_{20} & l_{21} & \hat{A}_{22} - L_{20}U_{02} - l_{21}u_{12}^T \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}u_{01} = \hat{a}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge \quad l_{10}^T U_{00} = \hat{a}_{10}^T \quad l_{10}^T u_{01} + v_{11} = \hat{\alpha}_{11} \quad l_{10}^T U_{02} + u_{12}^T = \hat{a}_{12}^T \\ L_{20}U_{00} = \hat{A}_{20} \quad L_{20}u_{01} + v_{11}l_{21} = \hat{a}_{21} \end{array} \right\}$
5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
	endwhile
2,3	$\left\{ \begin{array}{l} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge \neg(m(A_{TL}) < \\ m(A) \end{array} \right\}$
1b	$\{A = L \setminus U \wedge LU = \hat{A}$

Step	Algorithm: $A := \text{LU_UNB_VAR5}(A)$
1a	{
4	where
2	{
3	while do
2,3	{ }
5a	where
6	{
8	
7	{
5b	
2	{
	endwhile
2,3	{ $\neg($ $)$
1b	}

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1a	{ $A = \hat{A}$ }	
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3	while	do
2,3	{	}
5a	where	
6	{	}
8		
7	{	}
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	endwhile	
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4	where
2	$\left\{ \left(\frac{A_{TL} \mid A_{TR}}{A_{BL} \mid A_{BR}} \right) = \left(\frac{L \setminus U_{TL} \mid U_{TR}}{L_{BL} \mid \hat{A}_{BR}} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \right\}$
3	while do
2,3	$\left\{ \left(\frac{A_{TL} \mid A_{TR}}{A_{BL} \mid A_{BR}} \right) = \left(\frac{L \setminus U_{TL} \mid U_{TR}}{L_{BL} \mid \hat{A}_{BR}} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge \right.$
5a	where
6	$\left\{ \right.$
8	
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5b	
2	$\left\{ \left(\frac{A_{TL} \mid A_{TR}}{A_{BL} \mid A_{BR}} \right) = \left(\frac{L \setminus U_{TL} \mid U_{TR}}{L_{BL} \mid \hat{A}_{BR}} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \right\}$
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1a	$\{A = \hat{A}$ }
4	<div style="background-color: #f0f0f0; padding: 5px;"> <p>where</p> </div>
2	$\left\{ \left(\frac{A_{TL} \mid A_{TR}}{A_{BL} \mid A_{BR}} \right) = \left(\frac{L \setminus U_{TL} \mid U_{TR}}{L_{BL} \mid \hat{A}_{BR}} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \right\}$
3	while $m(A_{TL}) < m(A)$ do
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7	$\left\{ \right\}$
5b	<div style="background-color: #f0f0f0; height: 40px;"></div>
2	$\left\{ \left(\frac{A_{TL} \mid A_{TR}}{A_{BL} \mid A_{BR}} \right) = \left(\frac{L \setminus U_{TL} \mid U_{TR}}{L_{BL} \mid \hat{A}_{BR}} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \right\}$
	endwhile
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5a	where
6	
8	
7	
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3	while $m(A_{TL}) < m(A)$ do
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6	
8	
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5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} A_{00} \ a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} \ a_{12}^T \\ \hline A_{20} \ a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
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	endwhile
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8	
7	
5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
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2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{l} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge m(A_{TL}) < \\ m(A) \end{array} \right\}$
5a	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$ <p>where α_{11} is 1×1, λ_{11} is 1×1, v_{11} is 1×1</p>
6	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left(\begin{array}{ccc} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & \hat{\alpha}_{11} - l_{10}^T u_{01} & \hat{a}_{12}^T - l_{10}^T U_{02} \\ L_{20} & \hat{a}_{21} - L_{20} u_{01} & \hat{A}_{22} - L_{20} U_{02} \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}u_{01} = \hat{a}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge \quad l_{10}^T U_{00} = \hat{a}_{10}^T \\ L_{20}U_{00} = \hat{A}_{20} \end{array} \right\}$
8	
7	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left(\begin{array}{ccc} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ L_{20} & l_{21} & \hat{A}_{22} - L_{20}U_{02} - l_{21}u_{12}^T \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}u_{01} = \hat{a}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge \quad l_{10}^T U_{00} = \hat{a}_{10}^T \quad l_{10}^T u_{01} + v_{11} = \hat{\alpha}_{11} \quad l_{10}^T U_{02} + u_{12}^T = \hat{a}_{12}^T \\ L_{20}U_{00} = \hat{A}_{20} \quad L_{20}u_{01} + v_{11}l_{21} = \hat{a}_{21} \end{array} \right\}$
5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
	endwhile
2,3	$\left\{ \begin{array}{l} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge \neg(m(A_{TL}) < \\ m(A) \end{array} \right\}$
1b	$\{A = L \setminus U \wedge LU = \hat{A}$

Step	Algorithm: $A := \text{LU_UNB_VAR5}(A)$
1a	$\{A = \hat{A}$
4	$A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$ <p>where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</p>
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{l} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge m(A_{TL}) < \\ m(A) \end{array} \right\}$
5a	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$ <p>where α_{11} is 1×1, λ_{11} is 1×1, v_{11} is 1×1</p>
6	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left(\begin{array}{ccc} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & \hat{\alpha}_{11} - l_{10}^T u_{01} & \hat{a}_{12}^T - l_{10}^T U_{02} \\ L_{20} & \hat{a}_{21} - L_{20} u_{01} & \hat{A}_{22} - L_{20} U_{02} \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}u_{01} = \hat{a}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge \quad l_{10}^T U_{00} = \hat{a}_{10}^T \\ L_{20}U_{00} = \hat{A}_{20} \end{array} \right\}$
8	$a_{21}^T := u_{21}^T = (\hat{a}_{21} - L_{20}u_{01})/v_{11} = a_{21}/\alpha_{11}$ $A_{22} := \hat{A}_{22} - L_{20}U_{02} - l_{21}u_{12}^T = A_{22} - a_{21}a_{12}^T$
7	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right) = \left(\begin{array}{ccc} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ L_{20} & l_{21} & \hat{A}_{22} - L_{20}U_{02} - l_{21}u_{12}^T \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}u_{01} = \hat{a}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge \quad l_{10}^T U_{00} = \hat{a}_{10}^T \quad l_{10}^T u_{01} + v_{11} = \hat{\alpha}_{11} \quad l_{10}^T U_{02} + u_{12}^T = \hat{a}_{12}^T \\ L_{20}U_{00} = \hat{A}_{20} \quad L_{20}u_{01} + v_{11}l_{21} = \hat{a}_{21} \end{array} \right\}$
5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
	endwhile
2,3	$\left\{ \begin{array}{l} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \hat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge \neg(m(A_{TL}) < \\ m(A) \end{array} \right\}$
1b	$\{A = L \setminus U \wedge LU = \hat{A}$

Algorithm: $A := \text{LU_UNB_VAR5}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$$

where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$$

where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1

$$a_{21}^T := u_{21}^T = (\hat{a}_{21} - L_{20}u_{01})/v_{11} = a_{21}/\alpha_{11}$$

$$A_{22} := \hat{A}_{22} - L_{20}U_{02} - l_{21}u_{12}^T = A_{22} - a_{21}a_{12}^T$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$$

endwhile

Algorithm: $A := \text{LU_UNB_VAR5}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$$

where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c} A_{00} & a_{01} \ A_{02} \\ \hline a_{10}^T & \alpha_{11} \ a_{12}^T \\ \hline A_{20} & a_{21} \ A_{22} \end{array} \right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$$

where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1

$$a_{21}^T := u_{21}^T = (\hat{a}_{21} - L_{20}u_{01})/v_{11} = a_{21}/\alpha_{11}$$

$$A_{22} := \hat{A}_{22} - L_{20}U_{02} - l_{21}u_{12}^T = A_{22} - a_{21}a_{12}^T$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c} A_{00} \ a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} \ a_{12}^T \\ \hline A_{20} \ a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$$

endwhile