Step	Algorithm: $A := LU_{UNB_VAR2}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \end{array} \right\}$
5a	Determine block size $b$ $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots $
	where $A_{11}$ is $b \times b$ , $L_{11}$ is $b \times b$ , $U_{11}$ is $b \times b$
6	$ \left( \begin{array}{c} A_{00} & A_{01} & A_{02} \\ A & A & A \end{array} \right) = \left( \begin{array}{c} L \setminus U_{00} & A_{01} & A_{02} \\ I & \widehat{A} & \widehat{A} \end{array} \right) \left( \begin{array}{c} L_{00} U_{00} = A_{00} \\ A & I & II \end{array} \right) $
	$ \left\{ \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & \widehat{A}_{01} & \widehat{A}_{02} \\ L_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ L_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} - \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} \\ L_{10}U_{00} = \widehat{A}_{10} \\ L_{20}U_{00} = \widehat{A}_{20} \end{pmatrix} $
	$A_{01} := U_{01} = L_{00}^{-1} A_{01}$ ( $L_{00}$ is stored in the strictly lower triangular part of $A_{00}$ )
8	$A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$
	$A_{21} := L_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$
7	$ \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & \widehat{A}_{02} \\ L_{10} & L \setminus U_{11} & \widehat{a}_{12}^T \\ L_{20} & L_{21} & \widehat{A}_{22} \end{pmatrix} $ $ \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} \\ A_{10}U_{00} = \widehat{A}_{10} & L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11} \end{pmatrix} $
	$L_{20}U_{00} = \widehat{A}_{20} \ L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21}$
5b	$(A_{00} A_{01} A_{02}) (A_{02} A_{01} A_{02})$
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
	endwhile
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & A_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = A_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR2}(A)$
1a	{
4	where
2	
3	while do
2,3	
	Determine block size b
5a	
	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\left\{ \qquad \qquad \land \neg ( \qquad \qquad ) \right\}$
1b	}

Step	Algorithm: $A := LU_{UNB\_VAR2}(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	
3	while do
2,3	
	Determine block size $b$
5a	
	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\left\{ \qquad \qquad \land \neg ( \qquad ) \qquad \right\}$
1b	$\{A = L \setminus U \land LU = \widehat{A}\}$

Step	Algorithm: $A := LU_{UNB\_VAR2}(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\}$
3	while do
2,3	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \right. $
	Determine block size $b$
5a	
Ja	
	where
6	}
8	
7	$  \; \langle \;   \;   \;   \;   \;   \;   \;   \;   \; $
5b	
2	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\}$
	endwhile
0.0	$\int \left( A_{TL} \middle  A_{TR} \right)  \left( L \backslash U_{TL} \middle  \widehat{A}_{TR} \right)  L_{TL} U_{TL} = \widehat{A}_{TL}  . $
2,3	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg ( )  $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR2}(A)$	
1a	$\{A = \widehat{A}\}$	
4	where	
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $	$\left.  ight\}$
3	while $m(A_{TL}) < m(A)$ do	
2,3	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \\ \end{array} \right\} $	$\left.  ight\}$
	Determine block size $b$	
5a		
Ja		
	where	
6		}
		J
8		
7		}
		,
5b		
2	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right.$	}
	endwhile	1
0.0		
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) \right\}$	$\int$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$	

Step	Algorithm: $A := LU_{UNB\_VAR2}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$ $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \end{array} \right\}$
	Determine block size $b$
F -	
5a	
	where
6	}
8	
7	}
5b	
	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \end{array} \right\} = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{} $
2	$ \left( \begin{array}{c c} A_{BL} & A_{BR} \end{array} \right)  \left( \begin{array}{c c} L_{BL} & \widehat{A}_{BR} \end{array} \right)  L_{BL} U_{TL} = \widehat{A}_{BL} $
	endwhile
2,3	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB\_VAR2}(A)$
1a	$A = \widehat{A}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \end{array} \right\}$
5a	Determine block size $b$ $ \left(\begin{array}{c c}A_{TL} & A_{TR}\\\hline A_{BL} & A_{BR}\end{array}\right) \rightarrow \left(\begin{array}{c c}A_{00} & A_{01} & A_{02}\\\hline A_{10} & A_{11} & A_{12}\\\hline A_{20} & A_{21} & A_{22}\end{array}\right), \left(\begin{array}{c c}L_{TL} & L_{TR}\\\hline L_{BL} & L_{BR}\end{array}\right) \rightarrow \cdots, \left(\begin{array}{c c}U_{TL} & U_{TR}\\\hline U_{BL} & U_{BR}\end{array}\right) \rightarrow \cdots $
	where $A_{11}$ is $b \times b$ , $L_{11}$ is $b \times b$ , $U_{11}$ is $b \times b$
6	
8	
7	
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow $
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $ and while
2,3	endwhile $ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) \right\} $
1b	$ \begin{cases} A = L \setminus U \land LU = \widehat{A} \end{cases} $

Step	Algorithm: $A := LU_{UNB\_VAR2}(A)$	
1a	${A = \hat{A}}$	
4	where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$	
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $	
3	while $m(A_{TL}) < m(A)$ do	
2,3	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \end{array} \right\}$	
5a	Determine block size $b$ $ \left(\begin{array}{c c}A_{TL} & A_{TR}\\\hline A_{BL} & A_{BR}\end{array}\right) \rightarrow \left(\begin{array}{c c}A_{00} & A_{01} & A_{02}\\\hline A_{10} & A_{11} & A_{12}\\\hline A_{20} & A_{21} & A_{22}\end{array}\right), \left(\begin{array}{c c}L_{TL} & L_{TR}\\\hline L_{BL} & L_{BR}\end{array}\right) \rightarrow \cdots, \left(\begin{array}{c c}U_{TL} & U_{TR}\\\hline U_{BL} & U_{BR}\end{array}\right) \rightarrow \cdots $ $ \cdots $	
6	$ \begin{cases}                                    $	<u> </u>
8		
7		<b>}</b>
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right) , \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots , \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow $	
2	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right) $	}
	endwhile	
2,3	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) \right\} $	}
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$	

Step	Algorithm: $A := LU_{UNB_VAR2}(A)$	
1a	$A = \hat{A}$	}
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$	
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} $	
3	while $m(A_{TL}) < m(A)$ do	
2,3	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \right.$	
	Determine block size $b$ $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) - \cdots $	$\rightarrow$
	where $A_{11}$ is $b \times b$ , $L_{11}$ is $b \times b$ , $U_{11}$ is $b \times b$	
6	$ \begin{cases}                                    $	
8		
7	$ \begin{cases} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{cases} = \begin{pmatrix} L \setminus U_{00} & U_{01} & \widehat{A}_{02} \\ L_{10} & L \setminus U_{11} & \widehat{a}_{12}^T \\ L_{20} & L_{21} & \widehat{A}_{22} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00} \qquad L_{00}U_{01} = \widehat{A}_{01} $ $ \wedge L_{10}U_{00} = \widehat{A}_{10} & L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11} $ $ L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} $	
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right) , \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots , \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $	_
2	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \\ \text{endwhile} $	
2,3	endwhile $ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) $	
1b	$\{A = L \backslash U \land LU = \widehat{A}$	}

Step	Algorithm: $A := LU_{UNB\_VAR2}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL} U_{TL} = \widehat{A}_{TL}}{L_{BL} U_{TL} = \widehat{A}_{BL}} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge m(A_{TL}) < m(A) \end{array} \right\}$
5a	Determine block size $b$ $ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots $
6	$ \begin{cases}                                    $
8	$A_{01} := U_{01} = L_{00}^{-1} A_{01}  (L_{00} \text{ is stored in the strictly lower triangular part of } A_{00})$ $A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$ $A_{21} := L_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$ $(U_{11} \text{ is stored in the upper triangular part of } A_{11})$
7	$ \begin{cases} A_{00} A_{01} A_{02} \\ A_{10} A_{11} A_{12} \\ A_{20} A_{21} A_{22} \end{cases} = \begin{pmatrix} L \setminus U_{00} U_{01} & \widehat{A}_{02} \\ L_{10} L \setminus U_{11} & \widehat{a}_{12}^{T} \\ L_{20} L_{21} & \widehat{A}_{22} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00} \qquad L_{00}U_{01} = \widehat{A}_{01} $ $ \wedge L_{10}U_{00} = \widehat{A}_{10} L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11} $ $ L_{20}U_{00} = \widehat{A}_{20} L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} $
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
	endwhile
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Algorithm: $A := LU_{UNB_VAR2}(A)$
$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
while $m(A_{TL}) < m(A)$ do
Determine block size $b$ $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where $A$ is $b \times b$ $L$ is $b \times b$ $U$ is $b \times b$
where $A_{11}$ is $b \times b$ , $L_{11}$ is $b \times b$ , $U_{11}$ is $b \times b$
$A_{01} := U_{01} = L_{00}^{-1} A_{01}  (L_{00} \text{ is stored in the strictly lower triangular part of } A_{00})$ $A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$ $A_{21} := L_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$ $(U_{11} \text{ is stored in the upper triangular part of } A_{11})$
$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
endwhile

Algorithm:  $A := LU_{UNB_VAR2}(A)$ 

$$A o \left( egin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} 
ight) \,,\, L o \left( egin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} 
ight) \,,\, U o \left( egin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} 
ight)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $L_{TL}$  is  $0 \times 0$ ,  $U_{TL}$  is  $0 \times 0$ 

while  $m(A_{TL}) < m(A)$  do

Determine block size b

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots$$

where  $A_{11}$  is  $b \times b$ ,  $L_{11}$  is  $b \times b$ ,  $U_{11}$  is  $b \times b$ 

 $A_{01} := U_{01} = L_{00}^{-1} A_{01}$  ( $L_{00}$  is stored in the strictly lower triangular part of  $A_{00}$ )

$$A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$$

$$A_{21} := L_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$$

 $(U_{11} \text{ is stored in the upper triangular part of } A_{11})$ 

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \cdots$$

endwhile