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| 8 | $A_{00}B_0 +$ $A_{20}^T B_2 +$<br>$A_{21}^T B_1 +$<br>$C_2 := A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2$ |
| 7 | $\left\{ \left( \begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array} \right) = \left( \begin{array}{r} A_{10}^T B_1 + A_{20}^T B_2 + \widehat{C}_0 \\ A_{10}B_0 + A_{11}B_1 + A_{21}^T B_2 + \widehat{C}_1 \\ A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + \widehat{C}_2 \end{array} \right) \right\}$ |
| 5b | $\left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline B_1 \\ B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline C_1 \\ C_2 \end{array} \right)$ |
| 2 | $\left\{ \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{r} A_{BL}^T B_B + \widehat{C}_T \\ \hline A_{BL}B_T + A_{BR}B_B + \widehat{C}_B \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{r} A_{BL}^T B_B + \widehat{C}_T \\ \hline A_{BL}B_T + A_{BR}B_B + \widehat{C}_B \end{array} \right) \wedge \neg (m(A_{BR}) < m(A)) \right\}$ |
| 1b | $\{ C = AB + \widehat{C} \}$ |

| Step | Algorithm: $C := AB + C$ where $A$ is symmetric and stored in the lower triangular part |
|---|---|
| 1a | $\{ C = \widehat{C} \}$ |
| 4 | $A \rightarrow \left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$ <br> where $A_{BR}$ is $0 \times 0$, $B_B$ has 0 rows, $C_B$ has 0 rows |
| 2 | $\left\{ \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} A_{BL}^T B_B + \widehat{C}_T \\ \hline A_{BL} B_T + A_{BR} B_B + \widehat{C}_B \end{array} \right) \right\}$ |
| 3 | while $m(A_{BR}) < m(A)$ do |
| 2,3 | $\left\{ \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} A_{BL}^T B_B + \widehat{C}_T \\ \hline A_{BL} B_T + A_{BR} B_B + \widehat{C}_B \end{array} \right) \wedge m(A_{BR}) < m(A) \right\}$ |
| 5a | **Determine block size** $b$ <br> $\left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ B_1 \\ \hline B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ C_1 \\ \hline C_2 \end{array} \right)$ <br> where $A_{11}$ is $b \times b$, $B_1$ has $b$ rows, $C_1$ has $b$ rows |
| 6 | $\left\{ \left( \begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array} \right) = \left( \begin{array}{r} A_{20}^T B_2 + \widehat{C}_0 \\ A_{21}^T B_2 + \widehat{C}_1 \\ A_{20} B_0 + A_{21} B_1 + A_{22} B_2 + \widehat{C}_2 \end{array} \right) \right\}$ |
| 8 | $C_0 := A_{00} B_0 + A_{10}^T B_1 + A_{20}^T B_2 + C_0$ <br> $C_1 := A_{10} B_0 + A_{11} B_1 + A_{21}^T B_1 + C_1$ <br> $C_2 := A_{20} B_0 + A_{21} B_1 + A_{22} B_2 + C_2$ |
| 7 | $\left\{ \left( \begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array} \right) = \left( \begin{array}{r} A_{10}^T B_1 + A_{20}^T B_2 + \widehat{C}_0 \\ A_{10} B_0 + A_{11} B_1 + A_{21}^T B_2 + \widehat{C}_1 \\ A_{20} B_0 + A_{21} B_1 + A_{22} B_2 + \widehat{C}_2 \end{array} \right) \right\}$ |
| 5b | $\left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline B_1 \\ B_2 \end{array} \right), \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline C_1 \\ C_2 \end{array} \right)$ |
| 2 | $\left\{ \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} A_{BL}^T B_B + \widehat{C}_T \\ \hline A_{BL} B_T + A_{BR} B_B + \widehat{C}_B \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) = \left( \begin{array}{c} A_{BL}^T B_B + \widehat{C}_T \\ \hline A_{BL} B_T + A_{BR} B_B + \widehat{C}_B \end{array} \right) \wedge \neg (m(A_{BR}) < m(A)) \right\}$ |
| 1b | $\{ C = AB + \widehat{C} \}$ |

Algorithm: $C := AB + C$ where $A$ is symmetric and stored in the lower triangular part

$$A \rightarrow \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), \; B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), \; C \rightarrow \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right)$$

where $A_{BR}$ is $0 \times 0$, $B_B$ has 0 rows, $C_B$ has 0 rows

while $m(A_{BR}) < m(A)$ do

**Determine block size** $b$

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right), \; \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ B_1 \\ \hline B_2 \end{array} \right), \; \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \rightarrow \left( \begin{array}{c} C_0 \\ C_1 \\ \hline C_2 \end{array} \right)$$

where $A_{11}$ is $b \times b$, $B_1$ has $b$ rows, $C_1$ has $b$ rows

$$C_0 := A_{00}B_0 + A_{10}^T B_1 + A_{20}^T B_2 + C_0$$

$$C_1 := A_{10}B_0 + A_{11}B_1 + A_{21}^T B_1 + C_1$$

$$C_2 := A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2$$

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \; \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline B_1 \\ B_2 \end{array} \right), \; \left( \begin{array}{c} C_T \\ \hline C_B \end{array} \right) \leftarrow \left( \begin{array}{c} C_0 \\ \hline C_1 \\ C_2 \end{array} \right)$$

endwhile

Algorithm: $C := AB + C$ where $A$ is symmetric and stored in the lower triangular part

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$$

where $A_{BR}$ is $0 \times 0$, $B_B$ has $0$ rows, $C_B$ has $0$ rows

while $m(A_{BR}) < m(A)$ do

**Determine block size** $b$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ C_1 \\ \hline C_2 \end{array}\right)$$

where $A_{11}$ is $b \times b$, $B_1$ has $b$ rows, $C_1$ has $b$ rows

$$C_0 := A_{10}^T B_1 + C_0$$

$$C_1 := A_{10} B_0 + A_{11} B_1 + C_1$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ C_2 \end{array}\right)$$

endwhile