$ \begin{array}{c} \text{Step} \text{Algorithm: } A := \text{LU-unr.var1}(A) \\ 1a \left\{A = \widehat{A} \right. \\ 4 A \rightarrow \left(\begin{array}{c} A_{TL} \\ A_{BL} \\ A_{BR} \\ A_{BL} \\ A_{BR} \\ A_{BR} \\ A_{BL} \\ A_{BR} \\ A_{BR} \\ A_{BL} \\ A_{BR} \\ A_{BL} \\ A_{BR} \\ A_{BL} \\ A_{BR} \\ A_{BR} \\ A_{BL} \\ A_{BR} \\ A_{BL} \\ A_{BR} \\ A_{BL} \\ A_{BR} \\ A_{BR} \\ A_{BL} $		
$ \begin{array}{c} 4 & A \rightarrow \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \rightarrow \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \rightarrow \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \\ & \text{where } A_{TL} \text{ is } 0 \times 0, L_{TL} \text{ is } 0 \times 0, U_{TL} \text{ is } 0 \times 0 \\ \hline 2 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline 3 & \text{while } m(A_{TL}) < m(A) \text{ do} \\ \hline 2,3 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) \\ \hline 5a & \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots \\ \hline 6 & \left\{ \\ & \text{update line 1} \\ \hline 5b & \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \cdots \\ \hline 2 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \cdots \\ \hline 2 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \\ \hline 2 & \text{endwhile} \\ \hline 2 & 3 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 2 & 0 & 0 $	Step	Algorithm: $A := LU_{UNB_VAR1}(A)$
$ \begin{array}{c} \text{where } A_{TL} \text{ is } 0 \times 0, \ L_{TL} \text{ is } 0 \times 0, \ U_{TL} \text{ is } 0 \times 0 \\ \hline 2 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline 3 & \text{while } m(A_{TL}) < m(A) \text{ do} \\ \hline 2,3 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) \\ \hline 5a & \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \\ A_{20} & A_{21} & A_{$	1a	$\{A = \widehat{A} \}$
$ \begin{array}{c} 3 \text{while } m(A_{TL}) < m(A) \text{ do} \\ \\ 2,3 \left\{ \begin{array}{c} \left(\frac{A_{TL}}{A_{BL}} \middle A_{TR} \right) = \left(\frac{L \setminus U_{TL}}{\widehat{A}_{BL}} \middle \widehat{A}_{TR} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) \end{array} \right. \\ \\ \begin{array}{c} \text{Determine block size } b \\ \left(\frac{A_{TL}}{A_{BL}} \middle A_{AR} \right) \rightarrow \left(\frac{A_{00}}{A_{10}} \middle A_{01} \middle A_{11} \middle A_{12} \right) \\ \left(\frac{A_{20}}{A_{21}} \middle A_{21} \middle A_{22} \right) \\ \text{where } A_{11} \text{ is } b \times b, L_{11} \text{ is } b \times b, U_{11} \text{ is } b \times b \end{array} \right) \rightarrow \cdots, \left(\frac{U_{TL}}{U_{BL}} \middle U_{BR} \right) \rightarrow \cdots \\ \\ \begin{array}{c} \text{b} \\ \text{oupdate line 1} \end{array} \\ 8 : \\ \text{update line n} \end{array} $ $ \begin{array}{c} \text{b} \\ \text{b} \\ \left(\frac{A_{TL}}{A_{BL}} \middle A_{RR} \right) \leftarrow \left(\frac{A_{00}}{A_{01}} \middle A_{02} \\ A_{10} & A_{11} \middle A_{12} \\ A_{20} & A_{21} \middle A_{22} \right), \left(\frac{L_{TL}}{L_{TR}} \middle L_{TR} \\ L_{BL} \middle L_{BR} \right) \leftarrow \cdots, \left(\frac{U_{TL}}{U_{TR}} \middle U_{TR} \\ U_{BL} \middle U_{BR} \right) \leftarrow \cdots \\ \\ \text{condense of the end while} \\ \\ 2,3 \left\{ \left(\frac{A_{TL}}{A_{RL}} \middle A_{TR} \\ A_{BL} \middle A_{BR} \right) = \left(\frac{L \setminus U_{TL}}{\widehat{A}_{RR}} \middle \widehat{A}_{RR} \\ \widehat{A}_{BL} \middle \widehat{A}_{BR} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \\ \end{array} \right. \\ \end{array} \right\} $	4	
$ 2.3 \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right\} = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) $ $ \begin{array}{c c} \textbf{Determine block size } b \\ \hline 5a & \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \cdots $ $ \begin{array}{c c} \textbf{where } A_{11} \text{ is } b \times b, L_{11} \text{ is } b \times b, L_{11} \text{ is } b \times b, L_{11} \text{ is } b \times b \end{array} \right. $ $ \begin{array}{c c} \textbf{1} & \textbf{2} & \textbf{3} \\ \hline \textbf{2} & \textbf{3} \\ \hline \textbf{3} & \textbf{4} \\ \hline \textbf{3} & \textbf{4} \\ \hline \textbf{4} & $	2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	3	while $m(A_{TL}) < m(A)$ do
	2,3	$ \left\{ \begin{array}{c c} \left(\frac{A_{TL} & A_{TR}}{A_{BL} & A_{BR}} \right) = \left(\frac{L \setminus U_{TL} & \widehat{A}_{TR}}{\widehat{A}_{BL} & \widehat{A}_{BR}} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) \\ \end{array} \right. $
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	5a	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \to \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \to \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \to \cdots $
8 : update line n 7 { $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \cdots$ 2 $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} $ endwhile 2,3 $ \begin{cases} \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) $	6	
update line n 7 { $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \cdots$ 2 $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} $ endwhile 2,3 $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{A}_{TR} \\ \widehat{A}_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) $		update line 1
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	8	
5b $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \cdots $ $ 2 \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} $ $ \text{endwhile} $ $ 2,3 \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \right\} $		update line n
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	7	{
endwhile $2,3 \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \right\}$	5b	$\langle A_{20} A_{21} A_{22} \rangle$
endwhile $2,3 \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \right\}$	2	$ \left\{ \begin{array}{c c} \left(\frac{A_{TL}}{A_{BL}} \middle A_{TR} \right) = \left(\frac{L \backslash U_{TL}}{\widehat{A}_{BL}} \middle \widehat{A}_{BR} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \\ \end{array} \right\} $
$\left(\begin{array}{c c} A_{BL} & A_{BR} \end{array}\right) \left(\begin{array}{c c} \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \stackrel{\text{TE-TE}}{=} \stackrel{\text{TE-TE-TE}}{=} \stackrel{\text{TE-TE-TE-TE}}{=} \stackrel{\text{TE-TE-TE}}{=} \stackrel{\text{TE-TE-TE-TE}}{=} \stackrel{\text{TE-TE-TE-TE-TE}}{=} \text{TE-TE-TE-TE-TE-TE-TE-TE-TE-TE-TE-TE-TE-T$		endwhile
$1b \left\{ A = L \backslash \overline{U \wedge LU = \widehat{A}} \right\}$	2,3	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) $
	1b	$\left\{ A = L \backslash U \wedge L \overline{U} = \widehat{A} \right\}$

Step	Algorithm: $A := LU$	_UNB_VAR $1(A)$				
1a	{					}
4	where					
2						
3	while	do				
2,3				٨		
	Determine blo	ck size b				
5a						
	where					
6	{					}
8						
7	{					}
5b						
2	$\left\{ \right.$					$\bigg\}$
	endwhile					
2,3			^	\ ¬()	igg
1b	{					}

Step	Algorithm: $A := LU_{UNB_VAR1}(A)$		
1a	$A = \widehat{A}$		}
4	where		
2			
3	while do		,
2,3		٨	
	Determine block size b		
5a			
	where		
6	{		}
8			
7	{		}
5b			
2			}
	endwhile		,
2,3		^¬(
1b	$\{A = L \backslash U \land LU = \widehat{A}$		}

Step	Algorithm: $A := LU_{UNB_VAR1}(A)$
1a	$\{A = \widehat{A} $
4	where
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL}$
3	while do
2,3	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \right.$
	Determine block size b
5a	
	where
6	{
8	
7	{
5b	
2	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \end{array} \right\}$
	endwhile
2,3	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg () $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR1}(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) \end{array} \right\}$
5a	
	where
6	{
8	
7	{
5b	
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \\ \end{array} \right\} $
	endwhile
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR1}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) \end{array} \right\}$
	Determine block size b
5a	
	where
6	{
8	
7	{
5b	
2	$ \left\{ \begin{array}{c c} \left(\frac{A_{TL}}{A_{BL}} \middle A_{TR} \right) = \left(\frac{L \backslash U_{TL}}{\widehat{A}_{BL}} \middle \widehat{A}_{BR} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \\ \end{array} \right\} $
	endwhile
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR1}(A)$]
1a	$\{A = \widehat{A}\}$	
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0	
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL}$	$\left. \right\}$
3	while $m(A_{TL}) < m(A)$ do	
2,3	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) $	
5a		
6	{	
8		
7	{	
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $	
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} $	
	endwhile	
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) $	$\left. \right\}$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$	

Step	Algorithm: $A := LU_{UNB_VAR1}(A)$	
1a	$A = \hat{A}$	
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0	
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL}$	
3	while $m(A_{TL}) < m(A)$ do	
2,3	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) \end{array} \right\}$	
5a		
6	{	
8		
7	{	
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $	
2	$ \left\{ \begin{array}{c c} \left(\frac{A_{TL} & A_{TR}}{A_{BL} & A_{BR}} \right) = \left(\frac{L \setminus U_{TL} & \widehat{A}_{TR}}{\widehat{A}_{BL} & \widehat{A}_{BR}} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \\ \end{array} \right\} $	
	endwhile	
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \right\}$	
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$	

Step	Algorithm: $A := LU_{UNB_VAR1}(A)$	
1a	$A = \hat{A}$	
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0	
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL}$	
3	while $m(A_{TL}) < m(A)$ do	
2,3	$\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) \end{array} \right\}$	
5a		
6	{	
8		
7	{	
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $	
2	$ \left\{ \begin{array}{c c} \left(\frac{A_{TL} & A_{TR}}{A_{BL} & A_{BR}} \right) = \left(\frac{L \setminus U_{TL} & \widehat{A}_{TR}}{\widehat{A}_{BL} & \widehat{A}_{BR}} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \\ \end{array} \right\} $	
	endwhile	
2,3	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) \right\}$	
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$	

Algorithm: $A := LU_{UNB_VAR1}(A)$
$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
while $m(A_{TL}) < m(A)$ do
Determine block size b
$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$
update line 1
:
update line n
$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
endwhile

Algorithm: $A := LU_{UNB_VAR1}(A)$

$$A o \left(egin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}
ight) \, , \, L o \left(egin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}
ight) \, , \, U o \left(egin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}
ight)$$

where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

Determine block size b

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots$$

where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$

update line 1

:

update line n

$$\left(\begin{array}{c|cccc}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|cccc}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|cccc}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|cccc}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \cdots$$

endwhile