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$A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^T U_{02}) = L_{11}^{-1}(A_{12}^T - A_{10}^T A_{02})$
$A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$</td></tr>
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Step	Algorithm: $A := \text{LU\_BLK\_VAR4}(A)$
1a	$\{$ $\}$
4	where
2	$\left\{ \right\}$
3	while do
2,3	$\left\{ \wedge \right\}$
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6	$\left\{ \right\}$
8	
7	$\left\{ \right\}$
5b	
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2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{ A = L\backslash U \wedge LU = \widehat{A} \}$

Step	Algorithm: $A := \text{LU\_BLK\_VAR4}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge m(A_{TL}) < m(A) \right\}$
5a	**Determine block size** b $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$
6	$\left\{ \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ L_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{array}\right) \wedge \begin{array}{ccc} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ L_{10}U_{00} = \widehat{A}_{10} & & \\ L_{20}U_{00} = \widehat{A}_{20} & & \end{array} \right\}$
8	
7	$\left\{ \left(\begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & u_{01} & U_{02} \\ l_{10}^T & \upsilon_{11} & u_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{array}\right) \begin{array}{cccc} & L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & L_{10}U_{00} = \widehat{A}_{10}^T & L_{10}^TU_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} \\ & L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} & \end{array} \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{A = L\backslash U \wedge LU = \widehat{A}\}$

<table>
<tr><th>Step</th><th>Algorithm: $A := \mathrm{LU\_BLK\_VAR4}(A)$</th></tr>
<tr><td>1a</td><td>$\{A = \widehat{A}\}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$, $U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$
where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0</td></tr>
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<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge m(A_{TL}) < m(A) \right\}$</td></tr>
<tr><td>5a</td><td>Determine block size b
$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$, $\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$
where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$</td></tr>
<tr><td>6</td><td>$\left\{ \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ L_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \wedge \begin{array}{lll} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ L_{10}U_{00} = \widehat{A}_{10} & & \\ L_{20}U_{00} = \widehat{A}_{20} & & \end{array} \right\}$</td></tr>
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$A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^TU_{02}) = L_{11}^{-1}(A_{12}^T - A_{10}^TA_{02})$
$A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = (A_{21} - A_{20}A_{01})U_{11}^{-1}$</td></tr>
<tr><td>7</td><td>$\left\{ \begin{array}{l} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & u_{01} & U_{02} \\ l_{10}^T & \upsilon_{11} & u_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} \\ \begin{array}{llll} & L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & L_{10}U_{00} = \widehat{A}_{10}^T & L_{10}^TU_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} \\ & L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} & \end{array} \end{array} \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$, $\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$</td></tr>
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Algorithm: $A := \text{LU\_BLK\_VAR4}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

Determine block size b

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$

$$A_{11} := L\backslash U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11} - A_{10}A_{01})$$
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$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$$

endwhile

Algorithm: $A := \text{LU\_BLK\_VAR4}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), \; L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), \; U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Determine block size b

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

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endwhile