Step	Algorithm: $A := LU_{UNB_VAR4}(A)$
1a	$\{A = \widehat{A} $ }
	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1
6	$ \left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} \begin{array}{c} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T \\ L_{20}U_{00} = \widehat{A}_{20} \end{pmatrix} \right\} $
8	$\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$ $a_{12}^T := u_{12}^T = \widehat{a}_{12}^T - l_{10}^T U_{02} = a_{12}^T - a_{10}^T A_{02}$ $a_{21}^T := u_{21}^T = (\widehat{a}_{21} - L_{20} u_{01}) / v_{11} = (a_{21} - A_{20} a_{01}) / \alpha_{11}$
7	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} \\ L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + v_{11} = \widehat{\alpha}_{11} & l_{10}^T U_{02} + u_{12}^T = \widehat{a}_{12}^T \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}u_{01} + v_{11}l_{21} = \widehat{a}_{21} \end{cases} $
5b	$\begin{pmatrix} BE & BR \end{pmatrix} \begin{pmatrix} A_{20} & a_{21} & A_{22} \end{pmatrix} \begin{pmatrix} BE & BR \end{pmatrix}$
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array} \right\} = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left[\begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right] $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{pmatrix} \wedge \neg (m(A_{TL}) < \right\} $
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR4}(A)$
1a	{
4	where
2	
3	while do
2,3	
5a	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\left\{\begin{array}{c} \wedge \\ \neg (\end{array}\right.$
1b	{

Step	Algorithm: $A := LU_{UNB_VAR4}(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	
3	while do
2,3	
5a	where
6	
8	
7	
5b	
2	
	endwhile
2,3	
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR4}(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left(\begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right) \right\}$
3	while do
2,3	$ \left\{ \begin{array}{c c} \left(\frac{A_{TL}}{A_{BL}} \middle A_{TR} \right) & = & \left(\frac{L \setminus U_{TL}}{L_{BL}} \middle U_{TR} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \middle L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \right\} $
5a	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left L_{TL}U_{TR} = \widehat{A}_{TR} \right \\ \hline Conduction \qquad \qquad$
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \land \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \land \right\} $
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR4}(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left[\begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right] \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{pmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix} \right\} $
5a	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\} $
	endwhile
2,3	$\left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{pmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < \right\}$
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR4}(A)$
1a	$\{A = \widehat{A}$
4	$A \rightarrow \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \rightarrow \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \rightarrow \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0 $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) < \right\} \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

1a $\{A = \widehat{A}\}$ $ A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} $ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0 $ 2 \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $	}
where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0	
$ 2 \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. $	
3 while $m(A_{TL}) < m(A)$ do	
$ \begin{array}{c c} 2,3 & \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right\} = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left L_{TL}U_{TR} = \widehat{A}_{TR} \wedge n \right \\ + \left(\begin{array}{c c} C & C & C & C \\ \hline L_{BL} & A_{BR} & C & C \\ \hline L_{BL} & A_{BR} & C & C \\ \hline L_{BL} & C & C $	$n(A_{TL}) < $
5a $\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots$ where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1	,
6 {	
8	
7	
5b $ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \cdots $ $ 2 \left\{ \begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right. $	
endwhile	
	$i(A_{TL}) < $
$1b \{A = L \setminus U \land LU = \widehat{A}$	}

Step	Algorithm: $A := LU_{UNB_VAR4}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , ν_{11} is 1×1
6	$ \left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} \begin{array}{c} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T \\ L_{20}U_{00} = \widehat{A}_{20} \end{pmatrix} \right\} $
8	
7	
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right\} = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left[\begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right] $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left L_{TL}U_{TR} = \widehat{A}_{TR} \right \wedge \neg (m(A_{TL}) < \right\} $
1b	$\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR4}(A)$
1a	$\{A = \widehat{A} $
	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right. \\ \left. \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{TL}} \right. \\ \left. \left(\begin{array}{c c} L \backslash U_{TL} & U_{TL} \\ \hline L_{TL} & U_{TL} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL}} \right. $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{vmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{vmatrix} \wedge m(A_{TL}) < \right\} $
5a	where α_{11} is 1×1 λ_{11} is 1×1 ν_{11} is 1×1
6	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} - \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \lambda & l_{10}^T U_{00} = \widehat{a}_{10}^T \\ L_{20}U_{00} = \widehat{A}_{20} \end{pmatrix} $
8	
7	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00} \qquad L_{00}u_{01} = \widehat{a}_{01} \qquad L_{00}U_{02} = \widehat{A}_{02} $ $ \wedge l_{10}^T U_{00} = \widehat{a}_{10}^T \qquad l_{10}^T u_{01} + v_{11} = \widehat{\alpha}_{11} \qquad l_{10}^T U_{02} + u_{12}^T = \widehat{a}_{12}^T $ $ L_{20}U_{00} = \widehat{A}_{20} L_{20}u_{01} + v_{11}l_{21} = \widehat{a}_{21} $
5b	$\begin{pmatrix} A_{BL} & A_{BR} \end{pmatrix} \begin{pmatrix} A_{20} & a_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{BL} & B_{BR} \end{pmatrix} \begin{pmatrix} B_{BL} & B_{BR} \end{pmatrix}$
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left L_{TL}U_{TR} = \widehat{A}_{TR} \right \\ \hline \right\} $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} & L_{TL}U_{TR} = \widehat{A}_{TR} \end{pmatrix} \wedge \neg (m(A_{TL}) < \left\{ \begin{array}{c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array} \right\} = \left(\begin{array}{c} L \setminus U_{TL} & U_{TR} \\ A_{BR} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{BL}} \right\} \wedge \neg (m(A_{TL}) < \left\{ \begin{array}{c} A_{TL} & A_{TR} \\ A_{BL} & \widehat{A}_{BR} \end{array} \right\} = \left(\begin{array}{c} L \setminus U_{TL} & U_{TR} \\ A_{BR} & \widehat{A}_{BR} \end{array} \right) \wedge \neg (m(A_{TL}) < \left\{ \begin{array}{c} A_{TL} & A_{TR} \\ A_{BL} & \widehat{A}_{BR} \end{array} \right\} = \left(\begin{array}{c} L \setminus U_{TL} & A_{TR} \\ A_{BR} & \widehat{A}_{BR} \end{array} \right) \wedge \neg (m(A_{TL}) < \left\{ \begin{array}{c} A_{TL} & A_{TR} \\ A_{TL} & A_{TR} \end{array} \right\} = \left(\begin{array}{c} L \setminus U_{TL} & A_{TR} \\ A_{BR} & \widehat{A}_{BR} \end{array} \right) \wedge \neg (m(A_{TL}) < \left\{ \begin{array}{c} A_{TL} & A_{TR} \\ A_{TL} & A_{TR} \end{array} \right\} = \left(\begin{array}{c} L \setminus U_{TL} & A_{TR} \\ A_{TL} & \widehat{A}_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) < \left\{ \begin{array}{c} A_{TL} & A_{TR} \\ A_{TL} & A_{TR} \end{array} \right\} = \left(\begin{array}{c} L \setminus U_{TL} & A_{TR} \\ A_{TR} & \widehat{A}_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) < \left\{ \begin{array}{c} A_{TL} & A_{TR} \\ A_{TR} & \widehat{A}_{TR} \end{array} \right\} = \left(\begin{array}{c} L \setminus U_{TL} & A_{TR} \\ A_{TR} & \widehat{A}_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) < \left\{ \begin{array}{c} A_{TL} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL}) > \left\{ \begin{array}{c} A_{TR} & A_{TR} \\ A_{TR} & A_{TR} \end{array} \right) \wedge \neg (m(A_{TL$
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Step	Algorithm: $A := LU_{UNB_VAR4}(A)$
1a	$\{A = \widehat{A} $ }
	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0
2	$ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left L_{TL}U_{TR} = \widehat{A}_{TR} \right \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \begin{vmatrix} L_{TL}U_{TR} = \widehat{A}_{TR} \\ L_{BL}U_{TL} = \widehat{A}_{BL} \end{vmatrix} \wedge m(A_{TL}) < \right\} $
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1
6	$ \left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} \begin{array}{c} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T \\ L_{20}U_{00} = \widehat{A}_{20} \end{pmatrix} \right\} $
8	$\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$ $a_{12}^T := u_{12}^T = \widehat{a}_{12}^T - l_{10}^T U_{02} = a_{12}^T - a_{10}^T A_{02}$ $a_{21}^T := u_{21}^T = (\widehat{a}_{21} - L_{20} u_{01}) / v_{11} = (a_{21} - A_{20} a_{01}) / \alpha_{11}$
7	$ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & u_{01} & U_{02} \\ l_{10}^T & v_{11} & u_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} \\ L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \wedge & l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + v_{11} = \widehat{\alpha}_{11} & l_{10}^T U_{02} + u_{12}^T = \widehat{a}_{12}^T \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}u_{01} + v_{11}l_{21} = \widehat{a}_{21} \end{cases} $
5b	$\begin{pmatrix} BE & BR \end{pmatrix} \begin{pmatrix} A_{20} & A_{22} \end{pmatrix} \begin{pmatrix} BE & BR \end{pmatrix} \begin{pmatrix} BE & BR \end{pmatrix}$
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array} \right\} = \left(\begin{array}{c c} L \setminus U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left[\begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right] $
	endwhile
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & U_{TR} \\ L_{BL} & \widehat{A}_{BR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left L_{TL}U_{TR} = \widehat{A}_{TR} \right \wedge \neg (m(A_{TL}) < \right\} $
1b	$\left\{ A = L \backslash U \land LU = \widehat{A} \right\}$

Algorithm: $A := LU_{UNB_VAR4}(A)$	
$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0	
while $m(A_{TL}) < m(A)$ do	
$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \to \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \to \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \to \cdots $ where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1	
$\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$ $a_{12}^T := u_{12}^T = \widehat{a}_{12}^T - l_{10}^T U_{02} = a_{12}^T - a_{10}^T A_{02}$ $a_{21}^T := u_{21}^T = (\widehat{a}_{21} - L_{20} u_{01}) / v_{11} = (a_{21} - A_{20} a_{01}) / \alpha_{11}$	
$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $	
endwhile	

Algorithm: $A := LU_{UNB_VAR4}(A)$

$$A o \left(egin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}
ight) \,,\, L o \left(egin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}
ight) \,,\, U o \left(egin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}
ight)$$

where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|cccc}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|cccc}
A_{00} & a_{01} & A_{02} \\
\hline
a_{10}^T & \alpha_{11} & a_{12}^T \\
A_{20} & a_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|cccc}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|cccc}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots$$

where α_{11} is 1×1 , λ_{11} is 1×1 , v_{11} is 1×1

$$\alpha_{11} := v_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$$

$$a_{12}^T := u_{12}^T = \widehat{a}_{12}^T - l_{10}^T U_{02} = a_{12}^T - a_{10}^T A_{02}$$

$$a_{21}^T := u_{21}^T = (\widehat{a}_{21} - L_{20}u_{01})/v_{11} = (a_{21} - A_{20}a_{01})/\alpha_{11}$$

$$\frac{a_{21}^{T} := u_{21}^{T} = (\widehat{a}_{21} - L_{20}u_{01})/v_{11} = (a_{21} - A_{20}a_{01})/\alpha_{11}}{\left(\frac{A_{TL} \mid A_{TR}}{A_{BL} \mid A_{BR}}\right) \leftarrow \left(\frac{A_{00} \mid a_{01} \mid A_{02}}{a_{10}^{T} \mid \alpha_{11} \mid a_{12}^{T}}\right), \left(\frac{L_{TL} \mid L_{TR}}{L_{BL} \mid L_{BR}}\right) \leftarrow \cdots, \left(\frac{U_{TL} \mid U_{TR}}{U_{BL} \mid U_{BR}}\right) \leftarrow \cdots$$

endwhile