Step	Algorithm: $A := \text{LU\_UNB\_VAR3}(A)$
1a	$\{A = \widehat{A}$ $\}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$, $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < m(A) \right\}$
5a	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1
6	$\{$ $\}$
8	update line 1 : update line n
7	$\{$ $\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$

Step	Algorithm: $A := \text{LU\_UNB\_VAR3}(A)$
1a	$\{$ $\}$
4	where
2	$\left\{ \right\}$
3	while do
2,3	$\left\{ \wedge \right\}$
5a	where
6	$\{$ $\}$
8	
7	$\{$ $\}$
5b	
2	$\left\{ \right\}$
	endwhile
2,3	$\left\{ \wedge \; \neg(\quad) \right\}$
1b	$\{$ $\}$

Step	Algorithm: $A := \text{LU\_UNB\_VAR3}(A)$
1a	$\{A = \widehat{A}$ $\}$
4	where
2	$\{$ $\}$
3	while do
2,3	$\{$ $\wedge$ $\}$
5a	where
6	$\{$ $\}$
8	
7	$\{$ $\}$
5b	
2	$\{$ $\}$
	endwhile
2,3	$\{$ $\wedge$ $\neg($ $)$ $\}$
1b	$\{A = L \backslash U \wedge LU = \widehat{A}$ $\}$

Step	Algorithm: $A := \text{LU\_UNB\_VAR3}(A)$
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4	where
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge\ L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\vert\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
3	while do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge\ L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\vert\, L_{TL}U_{TR} = \widehat{A}_{TR} \ \wedge \right\}$
5a	where
6	$\{$ $\}$
8	
7	$\{$ $\}$
5b	
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge\ L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\vert\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge\ L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\vert\, L_{TL}U_{TR} = \widehat{A}_{TR} \ \wedge \ \neg(\quad) \right\}$
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3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \; L_{TL} U_{TL} = \widehat{A}_{TL} \, \big\| \, L_{TL} U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < m(A) \right\}$
5a	where
6	$\{ \}$
8	
7	$\{ \}$
5b	
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \; L_{TL} U_{TL} = \widehat{A}_{TL} \, \big\| \, L_{TL} U_{TR} = \widehat{A}_{TR} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \; L_{TL} U_{TL} = \widehat{A}_{TL} \, \big\| \, L_{TL} U_{TR} = \widehat{A}_{TR} \wedge \neg (m(A_{TL}) < m(A)) \right\}$
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1a	$\{ A = \widehat{A} \}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \, \Big\| \, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
3	while $m(A_{TL}) < m(A)$ do
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5a	where
6	$\{ \}$
8	
7	$\{ \}$
5b	
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \, \Big\| \, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$
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3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < m(A) \right\}$
5a	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1
6	$\{ \}$
8	
7	$\{ \}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
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6	$\{ \}$
8	
7	$\{ \}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \cdots$
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2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c\|c} L \backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \; L_{TL} U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL} U_{TR} = \widehat{A}_{TR} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
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8	
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	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \; L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big\|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
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<table>
<tr><td>Step</td><td>Algorithm: $A := \text{LU\_UNB\_VAR3}(A)$</td></tr>
<tr><td>1a</td><td>$\{ A = \widehat{A} \}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$
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<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$</td></tr>
<tr><td>3</td><td>while $m(A_{TL}) < m(A)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge m(A_{TL}) < m(A) \right\}$</td></tr>
<tr><td>5a</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$
where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1</td></tr>
<tr><td>6</td><td>$\{ \quad \}$</td></tr>
<tr><td>8</td><td>update line 1
:
update line n</td></tr>
<tr><td>7</td><td>$\{ \quad \}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c|c} L\backslash U_{TL} & U_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \,\Big|\, L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \neg(m(A_{TL}) < m(A)) \right\}$</td></tr>
<tr><td>1b</td><td>$\{ A = L\backslash U \wedge LU = \widehat{A} \}$</td></tr>
</table>

Algorithm: $A := \text{LU\_UNB\_VAR3}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1

update line 1

:

update line n

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$$

endwhile

Algorithm: $A := \text{LU\_UNB\_VAR3}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1

update line 1

:

update line n

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$$

endwhile