Step	Algorithm: $A := LU_BLK_VAR5(A)$
1a	$A = \widehat{A}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left[ L_{TL}U_{TR} = \widehat{A}_{TR} \right] \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array} \right\} = \left( \begin{array}{c c} L \setminus U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \left\{ \begin{array}{c c} L_{TL}U_{TL} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\} $
5a	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
6	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} - L_{10}U_{01} & \widehat{A}_{12} - L_{10}U_{02} \\ L_{20} & \widehat{A}_{21} - L_{20}U_{01} & \widehat{A}_{22} - L_{20}U_{02} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00}  \mathbb{E}_{00}U_{01} = \widehat{A}_{01}  L_{00}U_{02} = \widehat{A}_{02} $ $ \wedge L_{10}U_{00} = \widehat{A}_{10} $ $ L_{20}U_{00} = \widehat{A}_{20} $
8	$A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11})$ $A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^TU_{02}) = L_{11}^{-1}A_{12}^T$ $A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1}$ $A_{22} := \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12}$
7	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & L \setminus U_{11} & U_{12} \\ L_{20} & L_{21} & \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00}  L_{00}U_{01} = \widehat{A}_{01}  L_{00}U_{02} = \widehat{A}_{02} $ $ \wedge L_{10}U_{00} = \widehat{A}_{10}^{T}  L_{10}^{T}U_{01} + L_{11}U_{11} = \widehat{A}_{11}  L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} $ $ L_{20}U_{00} = \widehat{A}_{20}  L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} $
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left[ \begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right] \right\} $
	endwhile
9.9	$\left  \begin{array}{c c} A_{TL} & A_{TR} \end{array} \right  = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{\widehat{A}_{TL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR}}{\widehat{A}_{TL}} \right  \wedge \frac{1}{2} \left  \frac{1}{2$

Step	Algorithm: $A := LU_BLK_VAR5(A)$
1a	{
4	
	where
2	}
3	while do
0.0	
2,3	^}
	Determine block size $b$
5a	
	1
	where
6	}
8	
0	
7	
·	
F1	
5b	
2	<b>\</b>
	endwhile

Step	Algorithm: $A := LU_BLK_VAR5(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	
3	while do
2,3	
	Determine block size $b$
5a	
	where
6	
8	
7	
5b	
2	
	endwhile
0.0	

Step	Algorithm: $A := LU_BLK_VAR5(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left( \begin{array}{c c} L_{TL}U_{TR} & \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} & \widehat{A}_{BL} \end{array} \right) \right\} $
3	while do
2,3	$ \left\{ \begin{array}{c c} \left( \frac{A_{TL} \mid A_{TR}}{A_{BL} \mid A_{BR}} \right) = \left( \frac{L \setminus U_{TL} \mid \widehat{U}_{TR}}{L_{BL} \mid \widehat{A}_{BR} - L_{BL}U_{TR}} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \right. $
	Determine block size $b$
5a	
	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  \right\} $
	endwhile
0.0	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \end{array}\right) = \left(\begin{array}{c c} L \setminus U_{TL} \end{array}\right) \widehat{U}_{TR} $ $ \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} L_{TL}U_{TR} = \widehat{A}_{TR} $

Step	Algorithm: $A := LU_BLK_VAR5(A)$
1a	$\{A = \widehat{A}\}$
4	where
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left( \begin{array}{c c} L_{TL}U_{TR} & \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} & \widehat{A}_{BL} \end{array} \right) \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right\} = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \left\{ \begin{array}{c c} C & C & C & C \\ \hline C & C & C & C \\ \hline C & C & C & C \\ \hline C & C & C & C \\ \hline C & C & C & C \\ \hline C & C & C & C \\ \hline C & C & C & C \\ \hline C & C & C & C \\ \hline C & C & C & C \\ \hline C & C & C & C \\ \hline C & C & C & C \\ \hline C & C & C $
	Determine block size b
5a	
6	where
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} \left( \frac{A_{TL}}{A_{BL}} \middle  A_{TR} \right) = \left( \frac{L \setminus U_{TL}}{L_{BL}} \middle  \widehat{U}_{TR} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \middle  L_{TL}U_{TR} = \widehat{A}_{TR} \right) \\ \text{endwhile} $
0.0	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \end{array}\right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \end{array}\right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TR} = \widehat{A}_{TR}} \wedge $

Step	Algorithm: $A := LU_BLK_VAR5(A)$
1a	$A = \widehat{A}$
4	$A \rightarrow \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \rightarrow \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \rightarrow \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$ $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{U}_{TR} \\ L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{array}{c c} \left( \frac{A_{TL} \mid A_{TR}}{A_{BL} \mid A_{BR}} \right) = \left( \frac{L \setminus U_{TL} \mid \widehat{U}_{TR}}{L_{BL} \mid \widehat{A}_{BR} - L_{BL}U_{TR}} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL} \mid L_{TL}U_{TR} = \widehat{A}_{TR}}{L_{BL}U_{TL} = \widehat{A}_{BL} \mid} \wedge \right\} $
5a	
Ja	
	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left \begin{array}{c c} L_{TL}U_{TR} = \widehat{A}_{TR} \end{array}\right\} $
	endwhile
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \end{array}\right) = \left(\begin{array}{c c} L \setminus U_{TL} \end{array}\right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{} L_{TL}U_{TR} = \widehat{A}_{TR}} \wedge $

Step	Algorithm: $A := LU_BLK_VAR5(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$ $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{U}_{TR} \\ L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{L}_{TL}U_{TR} \\ \hline L_{BL}U_{TL} & \widehat{A}_{BL} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TL} = \widehat{A}_{BL}} \wedge \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} & \widehat{A}_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{TL}U_{TR}} \wedge \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{TL}U_{TL} & \widehat{A}_{TR} \\ \hline L_{TL}U_{TR} & \widehat{A}_{TR} \\ \hline L_{TL}U_{$
	Determine block size b
5a	$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots$ $\text{where } A_{11} \text{ is } b \times b, L_{11} \text{ is } b \times b, U_{11} \text{ is } b \times b$
6	
8	
7	
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} A_{20} & A_{21} & A_{22} \end{array} \right) \left( \begin{array}{c c} BL & BL \end{array} \right) \left( \begin{array}{c c} BL & BL \end{array} \right) \\ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left( \begin{array}{c c} L \setminus U_{TR} & \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} & \widehat{A}_{BL} \end{array} \right) \right\} $
	endwhile
2 2	$ \left  \int \left( \begin{array}{c c} A_{TL} & A_{TR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \end{array} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} L_{TL}U_{TR} = \widehat{A}_{TR} \wedge \right) $

Step	Algorithm: $A := LU_BLK_VAR5(A)$
1a	$\{A = \widehat{A}\}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left( \begin{array}{c c} L_{TL}U_{TR} & \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} & \widehat{A}_{BL} \end{array} \right) \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \begin{cases} \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{U}_{TR} \\ L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \\ M(A_{TL}) < M(A) \end{cases} $
	Determine block size b
5a	$ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where $A_{11}$ is $b \times b$ , $L_{11}$ is $b \times b$ , $U_{11}$ is $b \times b$
6	$\begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} - L_{10}U_{01} & \widehat{A}_{12} - L_{10}U_{02} \\ L_{20} & \widehat{A}_{21} - L_{20}U_{01} & \widehat{A}_{22} - L_{20}U_{02} \end{pmatrix} \\ L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ & \wedge L_{10}U_{00} = \widehat{A}_{10} \\ L_{20}U_{00} = \widehat{A}_{20} \end{cases}$
8	
7	
5b	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} A_{20} & A_{21} & A_{22} \\ \hline \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array}\right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
	endwhile
9.9	$ \left  \int \left( \begin{array}{c c} A_{TL} & A_{TR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \end{array} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TL} = \widehat{A}_{TL}} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TL}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TL}} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \widehat{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \left( L_{TL}U_{TR} = \widehat{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \left( L_{TL}U_{TR} = \underbrace{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \left( L_{TL}U_{TR} = \underbrace{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \left( L_{TL}U_{TR} = \underbrace{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \left( L_{TL}U_{TR} = \underbrace{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \left( L_{TL}U_{TR} = \underbrace{A}_{TR} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = \underbrace{A}_{TR}} \right) \wedge \underbrace{L_{TL}U_{TR} = $

Step	Algorithm: $A := LU_BLK_VAR5(A)$
1a	$A = \widehat{A}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left( \begin{array}{c c} L_{TL}U_{TR} & \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} & \widehat{A}_{BL} \end{array} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \setminus U_{TL} & \widehat{U}_{TR} \\ L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{pmatrix} \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \right\} $
5a	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
6	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} - L_{10}U_{01} & \widehat{A}_{12} - L_{10}U_{02} \\ L_{20} & \widehat{A}_{21} - L_{20}U_{01} & \widehat{A}_{22} - L_{20}U_{02} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00}  \mathbb{E}_{00}U_{01} = \widehat{A}_{01}  L_{00}U_{02} = \widehat{A}_{02} $ $ \wedge L_{10}U_{00} = \widehat{A}_{10} $ $ L_{20}U_{00} = \widehat{A}_{20} $
8	
7	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & L \setminus U_{11} & U_{12} \\ L_{20} & L_{21} & \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \land L_{10}U_{00} = \widehat{A}_{10}^{T} & L_{10}^{T}U_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} \end{cases} $
5b	A20 A21 A22 /
2	$ \left\{ \begin{array}{c c} \left( \frac{A_{TL} & A_{TR}}{A_{BL} & A_{BR}} \right) = \left( \frac{L \setminus U_{TL}}{L_{BL}} & \widehat{U}_{TR} \\ \overline{A}_{BR} - L_{BL}U_{TR} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \overline{A}_{BL} & \overline{A}_{BR} - \overline{A}_{BL}U_{TR} & \overline{A}_{BL}U_{TL} = \widehat{A}_{BL} & \overline{A}_{BL}U_{TL} = \widehat{A}_{BL} & \overline{A}_{BL}U_{TL} = \widehat{A}_{BL} & \overline{A}_{BL}U_{TL} = \widehat{A}_{BL} & \overline{A}_{BL}U_{TL} = \widehat{A}_{BL}U_{TL} & \overline{A}_{BL}U_{TL} = \widehat{A}_{BL}U_{TL} & \overline{A}_{BL}U_{TL} = \widehat{A}_{BL}U_{TL} & \overline{A}_{BL}U_{TL} = \widehat{A}_{BL}U_{TL} & \overline{A}_{BL}U_{TL} & \overline$
	endwhile
9.9	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \end{array}\right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{U}_{TR} \end{array}\right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{\widehat{A}_{TL}} L_{TL}U_{TR} = \widehat{A}_{TR}}{\widehat{A}_{TR}} \wedge \right) $

Step	Algorithm: $A := LU_BLK_VAR5(A)$
1a	$A = \hat{A}$
4	$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
2	$ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} L \backslash U_{TL} & \widehat{U}_{TR} \\ L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{pmatrix} \land \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \right\} $
3	while $m(A_{TL}) < m(A)$ do
2,3	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right\} = \left( \begin{array}{c c} L \backslash U_{TL} & \widehat{U}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \wedge \left\{ \begin{array}{c c} L \backslash U_{TR} & \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\} $
	Determine block size b
5a	$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots$ $\text{where } A_{11} \text{ is } b \times b, L_{11} \text{ is } b \times b, U_{11} \text{ is } b \times b$
6	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \widehat{A}_{11} - L_{10}U_{01} & \widehat{A}_{12} - L_{10}U_{02} \\ L_{20} & \widehat{A}_{21} - L_{20}U_{01} & \widehat{A}_{22} - L_{20}U_{02} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00}  L_{00}U_{01} = \widehat{A}_{01}  L_{00}U_{02} = \widehat{A}_{02} $ $ \wedge L_{10}U_{00} = \widehat{A}_{10} $ $ L_{20}U_{00} = \widehat{A}_{20} $
8	$A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11})$ $A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^TU_{02}) = L_{11}^{-1}A_{12}^T$ $A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1}$ $A_{22} := \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12}$
7	$ \begin{cases} \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & L \setminus U_{11} & U_{12} \\ L_{20} & L_{21} & \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} \end{pmatrix} $ $ L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ \land L_{10}U_{00} = \widehat{A}_{10}^T & L_{10}^TU_{01} + L_{11}U_{11} = \widehat{A}_{11} & L_{10}U_{02} + L_{11}U_{12} = \widehat{A}_{12} \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}U_{01} + L_{21}U_{11} = \widehat{A}_{21} \end{cases} $
5b	$ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $
2	$ \left\{ \begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array} \right\} = \left( \begin{array}{c c} L \setminus U_{TL} & \widehat{U}_{TR} \\ L_{BL} & \widehat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{L_{BL}U_{TL} = \widehat{A}_{BL}} \left  L_{TL}U_{TR} = \widehat{A}_{TR} \right  $
	endwhile
	$ \left(\begin{array}{c c} A_{TL} & A_{TR} \end{array}\right) = \left(\begin{array}{c c} L \setminus U_{TL} & \widehat{U}_{TR} \end{array}\right) \wedge \frac{L_{TL}U_{TL} = \widehat{A}_{TL}}{\widehat{A}_{TL}} L_{TL}U_{TR} = \widehat{A}_{TR}} \wedge \left(\begin{array}{c c} C & C & C & C & C & C & C & C & C & C &$

Algorithm: $A := LU_BLK_VAR5(A)$
$A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$ , $L_{TL}$ is $0 \times 0$ , $U_{TL}$ is $0 \times 0$
while $m(A_{TL}) < m(A)$ do
$A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11})$ $A_{12} := U_{12} = L_{11}^{-1}(\widehat{A}_{12}^T - L_{10}^T U_{02}) = L_{11}^{-1} A_{12}^T$ $A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1}$
$A_{22} := \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12}$
$\begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{pmatrix} \leftarrow \cdots$
endwhile

## Algorithm: $A := LU_BLK_VAR5(A)$

$$A o \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \ , \ L o \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \ , \ U o \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $L_{TL}$  is  $0 \times 0$ ,  $U_{TL}$  is  $0 \times 0$ 

while  $m(A_{TL}) < m(A)$  do

## Determine block size b

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots$$

where  $A_{11}$  is  $b \times b$ ,  $L_{11}$  is  $b \times b$ ,  $U_{11}$  is  $b \times b$ 

$$A_{11} := L \setminus U_{11} = LU(\widehat{A}_{11} - L_{10}U_{01}) = LU(A_{11})$$

$$A_{12} := U_{12} = L_{11}^{-1} (\widehat{A}_{12}^T - L_{10}^T U_{02}) = L_{11}^{-1} A_{12}^T$$

$$A_{21} := U_{21} = (\widehat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1}$$

$$A_{22} := \widehat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12}$$

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \cdots$$

endwhile