| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part   |
|------|---|
|      | $\{C = \widehat{C}$   |
|      | $A \to \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), B \to \left(\begin{array}{c c} B_T \\ \hline B_B \end{array}\right), C \to \left(\begin{array}{c c} C_T \\ \hline C_B \end{array}\right)$ where $A_{BR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows   |
| 2    | $ \left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \right\} $  |
| 3    | while $m(A_{BR}) < m(A)$ do   |
| 2,3  | $ \left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \land m(A_{BR}) < m(A) \right\} $   |
|      | Determine block size b  |
| 5a   | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ C_1 \\ \hline C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows |
|      | $ \begin{pmatrix} C_0 \\ C_0 \end{pmatrix} \begin{pmatrix} A_{20}^T B_2 + \hat{C}_0 \end{pmatrix} $   |
| 6    | $ \left\{ \begin{array}{c} C_1 \\ C_2 \end{array} \right\} = \left( \begin{array}{c} A_{21}^T B_2 + \widehat{C}_1 \\ A_{22} B_2 + \widehat{C}_2 \end{array} \right) $   |
|      | $C_0 := A_{00}B_0 + A_{10}^TB_1 + A_{20}^TB_2 + C_0$  |
| 8    | $C_1 := A_{10}B_0 + A_{11}B_1 + A_{21}^TB_1 + C_1$  |
|      | $C_2 := A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2$  |
| 7    | $ \begin{cases} \begin{pmatrix} C_0 \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} A_{10}^T B_1 + A_{20}^T B_2 + \widehat{C}_0 \\ A_{11} B_1 + A_{21}^T B_2 + \widehat{C}_1 \\ A_{21} B_1 + A_{22} B_2 + \widehat{C}_2 \end{pmatrix} $   |
| 5b   | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ B_1 \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array}\right) $  |
| 2    | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B}\right) \end{array} \right\}$   |
|      | endwhile  |
| 2,3  | $ \left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \land \neg (m(A_{BR}) < m(A)) \right\} $  |
| 1b   | $\{C = AB + \hat{C} $   |

| Step | Algorithm: $C := AB + C$              | where $A$ is s | symmetric and st | tored in the lower tria | angular part |
|------|---------------------------------------|----------------|------------------|-------------------------|--------------|
| 1a   | {                                     |                |                  |                         | }            |
| 4    | where                                 |                |                  |                         |              |
| 2    |                                       |                |                  |                         | $\bigg\}$    |
| 3    | while d                               | 0              |                  |                         |              |
| 2,3  |                                       |                | ٨                |                         | }            |
| 5a   | Determine block s where               | ize b          |                  |                         |              |
| 6    |                                       |                |                  |                         | $\bigg\}$    |
| 8    | $A_{00}B_0 + A_{10}B_0 + A_{20}B_0 +$ |                |                  |                         |              |
| 7    |                                       |                |                  |                         |              |
| 5b   |                                       |                |                  |                         |              |
| 2    |                                       |                |                  |                         |              |
|      | endwhile                              |                |                  |                         |              |
| 2,3  | $\bigg   \bigg\{$                     |                | ^ ¬(             | )                       | }            |
| 1b   | {                                     |                |                  |                         | }            |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |                                 |
|------|---|---------------------------------|
| 1a   | $\{C = \widehat{C}\}$   |                                 |
| 4    | where   |                                 |
| 2    |   | $\left. ight\}$                 |
| 3    | while do  |                                 |
| 2,3  |   | }                               |
| 5a   | Determine block size $b$ where  |                                 |
| 6    |   | $\left. \left. \right  \right.$ |
| 8    | $A_{00}B_0 + A_{20}^T B_2 +$ $A_{10}B_0 + A_{21}^T B_1 +$ $A_{20}B_0 + A_{22}B_2 +$   |                                 |
| 7    |   | $\left. \right\}$               |
| 5b   |   |                                 |
| 2    |   | $\left. ight\}$                 |
|      | endwhile  |                                 |
| 2,3  | $\bigg   \bigg\{ \hspace{1cm} \wedge \neg ( \hspace{1cm} )$                           |                                 |
| 1b   | $\{C = AB + \widehat{C}\}$  | _                               |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part   |                                 |
|------|---|---------------------------------|
| 1a   | ${C = \widehat{C}}$   |                                 |
| 4    | where   |                                 |
| 2    | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \right.$   | $\left.  ight\}$                |
| 3    | while do  |                                 |
| 2,3  | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B}\right) \wedge \end{array} \right.$               | $\left.  ight\}$                |
| 5a   |   |                                 |
|      | where   |                                 |
| 6    |   | $\left. \left. \right  \right.$ |
| 8    | $A_{00}B_0 + A_{20}^T B_2 +$ $A_{10}B_0 + A_{21}^T B_1 +$ $A_{20}B_0 + A_{22}B_2 +$   |                                 |
| 7    |   | $\left. \overline{ ight\}}$     |
| 5b   |   |                                 |
| 2    | $ \left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B}\right) \\ \text{endwhile} \end{array} \right. $ | $\left.  ight\}$                |
| 2,3  | $ \left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B} \right) \land \neg ( ) \right\} $                                     | $\left.  ight\}$                |
| 1b   | $\{C = AB + \widehat{C}\}$  |                                 |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part   |
|------|---|
| 1a   | ${C = \widehat{C}}$   |
| 4    | where   |
| 2    | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \right\}$  |
| 3    | while $m(A_{BR}) < m(A)$ do   |
| 2,3  | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B}\right) \wedge m(A_{BR}) < m(A) \end{array} \right\}$ |
|      | Determine block size $b$  |
| 5a   | where   |
|      | where   |
| 6    |   |
|      | $A_{00}B_0 + A_{20}^T B_2 +$  |
| 8    | $A_{10}B_0 + A_{21}^T B_1 +$  |
|      | $A_{20}B_0+ A_{22}B_2+$   |
| 7    |   |
| 5b   |   |
|      | $ \left\{ \begin{array}{c} \left( \frac{C_T}{C_T} \right) = \left( \frac{A_{BL}^T B_B + \hat{C}_T}{C_T} \right) \end{array} \right\} $  |
| 2    | $ \left( \begin{array}{c} C_B \end{array} \right)  \left( \begin{array}{c} A_{BR}B_B + \widehat{C}_B \end{array} \right) $  |
|      | endwhile $AT R + \widehat{C}$   |
| 2,3  | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \land \neg (m(A_{BR}) < m(A)) \right\}$                    |
| 1b   | $\{C = AB + \widehat{C} $   |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part  |
|------|--|
| 1a   | $\{C = \widehat{C}$  |
| 4    | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where $A_{BR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows |
| 2    | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \right\}$   |
| 3    | while $m(A_{BR}) < m(A)$ do  |
| 2,3  | $ \left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B} \right) \land m(A_{BR}) < m(A) \right\} $  |
| 5a   |  |
|      | where  |
| 6    |  |
| 8    | $A_{00}B_0 + A_{20}^T B_2 +$ $A_{10}B_0 + A_{21}^T B_1 +$ $A_{20}B_0 + A_{22}B_2 +$  |
| 7    |  |
| 5b   |  |
| 2    | $ \left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B}\right) \\ A_{BR} B_B + \widehat{C}_B \end{array} \right\} $  |
| 2,3  | endwhile $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B} \right) \land \neg (m(A_{BR}) < m(A)) \right\}$  |
| 1b   | $\{C = AB + \widehat{C} $ }  |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part  |
|------|--|
| 1a   | $\{C = \widehat{C}$  |
| 4    | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where $A_{BR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows   |
| 2    | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \right\}$   |
| 3    | while $m(A_{BR}) < m(A)$ do  |
| 2,3  | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B} \right) \land m(A_{BR}) < m(A) \right\}$  |
| 5a   | Determine block size $b$ $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} B_T \\ B_B \end{pmatrix} \rightarrow \begin{pmatrix} B_0 \\ B_1 \\ B_2 \end{pmatrix}, \begin{pmatrix} C_T \\ C_B \end{pmatrix} \rightarrow \begin{pmatrix} C_0 \\ C_1 \\ C_2 \end{pmatrix} $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows |
| 6    |  |
| 8    | $A_{00}B_0 + A_{20}^T B_2 + A_{10}B_0 + A_{21}^T B_1 + A_{20}B_0 + A_{22}B_2 +$  |
| 7    |  |
| 5b   | $ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ B_1 \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array}\right) $         |
| 2    | $ \left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B}\right) \\ \end{array} \right\} $   |
|      | endwhile   |
| 2,3  | $ \left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B} \right) \land \neg (m(A_{BR}) < m(A)) \right\} $   |
| 1b   | $\{C = AB + \widehat{C} $  |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part   |
|------|---|
| 1a   | $\{C = \widehat{C}\}$   |
| 4    | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where $A_{BR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows  |
| 2    | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B} \right) \right\}$  |
| 3    | while $m(A_{BR}) < m(A)$ do   |
| 2,3  | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \land m(A_{BR}) < m(A) \right\}$   |
| 5a   | Determine block size $b$ $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows |
| 6    | $ \left\{  \begin{pmatrix} C_0 \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} A_{20}^T B_2 + \widehat{C}_0 \\ A_{21}^T B_2 + \widehat{C}_1 \\ A_{22} B_2 + \widehat{C}_2 \end{pmatrix} $   |
| 8    | $A_{00}B_0 + A_{20}^T B_2 + A_{10}B_0 + A_{21}^T B_1 + A_{20}B_0 + A_{22}B_2 +$   |
| 7    |   |
| 5b   | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ B_1 \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array}\right) $  |
| 2    | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B}\right) \end{array} \right\}$   |
|      | endwhile  |
| 2,3  | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \land \neg (m(A_{BR}) < m(A)) \right\}$  |
| 1b   | $\{C = AB + \widehat{C} $   |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part  |
|------|--|
| 1a   | $\{C = \widehat{C}\}$  |
| 4    | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where $A_{BR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows   |
| 2    | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B} \right) \right\}$   |
| 3    | while $m(A_{BR}) < m(A)$ do  |
| 2,3  | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \land m(A_{BR}) < m(A) \right\}$  |
| 5a   | Determine block size $b$ $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows  |
| 6    | $ \begin{cases} \begin{pmatrix} C_0 \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} A_{20}^T B_2 + \widehat{C}_0 \\ A_{21}^T B_2 + \widehat{C}_1 \\ A_{22} B_2 + \widehat{C}_2 \end{pmatrix} $   |
| 8    | $A_{00}B_0 + A_{20}^T B_2 +$ $A_{10}B_0 + A_{21}^T B_1 +$ $A_{20}B_0 + A_{22}B_2 +$  |
| 7    | $ \left\{  \begin{pmatrix} C_0 \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} A_{10}^T B_1 + A_{20}^T B_2 + \widehat{C}_0 \\ A_{11} B_1 + A_{21}^T B_2 + \widehat{C}_1 \\ A_{21} B_1 + A_{22} B_2 + \widehat{C}_2 \end{pmatrix} \right\} $  |
| 5b   | $A_{00} A_{01} A_{02} A_{02} A_{01} A_{02} A_{02} A_{02} A_{01} A_{02} $ |
| 2    | $ \left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B}\right) \\ \end{array} \right\} $   |
|      | endwhile   |
| 2,3  | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \land \neg (m(A_{BR}) < m(A)) \right\}$   |
| 1b   | $\{C = AB + \widehat{C} $  |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part   |
|------|---|
| 1a   | ${C = \widehat{C}}$   |
| 4    | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where $A_{BR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows  |
| 2    | $\left\{ \begin{pmatrix} C_T \\ C_B \end{pmatrix} = \begin{pmatrix} A_{BL}^T B_B + \hat{C}_T \\ A_{BR} B_B + \hat{C}_B \end{pmatrix} \right\}$  |
| 3    | while $m(A_{BR}) < m(A)$ do   |
| 2,3  | $\left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \land m(A_{BR}) < m(A) \right\}$   |
| 5a   | Determine block size $b$ $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ B_1 \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ C_1 \\ \hline C_2 \end{array}\right) $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows |
| 6    | $ \begin{cases} \begin{pmatrix} C_0 \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} A_{20}^T B_2 + \hat{C}_0 \\ A_{21}^T B_2 + \hat{C}_1 \\ A_{22} B_2 + \hat{C}_2 \end{pmatrix} $  |
| 8    | $C_0 := A_{00}B_0 + A_{10}^T B_1 + A_{20}^T B_2 + C_0$ $C_1 := A_{10}B_0 + A_{11}B_1 + A_{21}^T B_1 + C_1$ $C_2 := A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2$   |
| 7    | $ \begin{cases} \begin{pmatrix} C_0 \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} A_{10}^T B_1 + A_{20}^T B_2 + \widehat{C}_0 \\ A_{11} B_1 + A_{21}^T B_2 + \widehat{C}_1 \\ A_{21} B_1 + A_{22} B_2 + \widehat{C}_2 \end{pmatrix} $   |
| 5b   | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ B_1 \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array}\right) $  |
| 2    | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{A_{BL}^T B_B + \hat{C}_T}{A_{BR} B_B + \hat{C}_B}\right) \end{array} \right\}$   |
|      | endwhile  |
| 2,3  | $ \left\{ \left( \frac{C_T}{C_B} \right) = \left( \frac{A_{BL}^T B_B + \widehat{C}_T}{A_{BR} B_B + \widehat{C}_B} \right) \land \neg (m(A_{BR}) < m(A)) \right\} $  |
| 1b   | $\{C = AB + \widehat{C} $   |

| Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part  |
|--|
|  |
| $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where $A_{BR}$ is $0 \times 0$ , $B_B$ has 0 rows, $C_B$ has 0 rows   |
|  |
| while $m(A_{BR}) < m(A)$ do  |
|  |
| Determine block size $b$ $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} B_T \\ B_B \end{pmatrix} \rightarrow \begin{pmatrix} B_0 \\ B_1 \\ B_2 \end{pmatrix}, \begin{pmatrix} C_T \\ C_B \end{pmatrix} \rightarrow \begin{pmatrix} C_0 \\ C_1 \\ C_2 \end{pmatrix} $ where $A_{11}$ is $b \times b$ , $B_1$ has $b$ rows, $C_1$ has $b$ rows |
|  |
| $C_0 := A_{00}B_0 + A_{10}^T B_1 + A_{20}^T B_2 + C_0$ $C_1 := A_{10}B_0 + A_{11}B_1 + A_{21}^T B_1 + C_1$ $C_2 := A_{20}B_0 + A_{21}B_1 + A_{22}B_2 + C_2$  |
|  |
| $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ B_1 \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ C_1 \\ C_2 \end{array}\right) $           |
|  |
| endwhile   |
|  |
|  |
|  |

Algorithm: C := AB + C where A is symmetric and stored in the lower triangular part

$$A o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) , B o \left( \begin{array}{c|c} B_T \\ \hline B_B \end{array} \right) , C o \left( \begin{array}{c|c} C_T \\ \hline C_B \end{array} \right)$$

where  $A_{BR}$  is  $0 \times 0$ ,  $B_B$  has 0 rows,  $C_B$  has 0 rows

while  $m(A_{BR}) < m(A)$  do

Determine block size b

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
B_B
\end{array}\right) \to \left(\begin{array}{c}
B_0 \\
B_1 \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
C_B
\end{array}\right) \to \left(\begin{array}{c}
C_0 \\
C_1 \\
C_2
\end{array}\right)$$

where  $A_{11}$  is  $b \times b$ ,  $B_1$  has b rows,  $C_1$  has b rows

$$C_0 := A_{10}^T B_1 + C_0$$

$$C_1 := A_{11}B_1 + C$$

$$C_2 := A_{21}B_1 + C_2$$

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
B_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
B_0 \\
B_1 \\
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
C_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
C_0 \\
C_1 \\
C_2
\end{array}\right)$$

endwhile