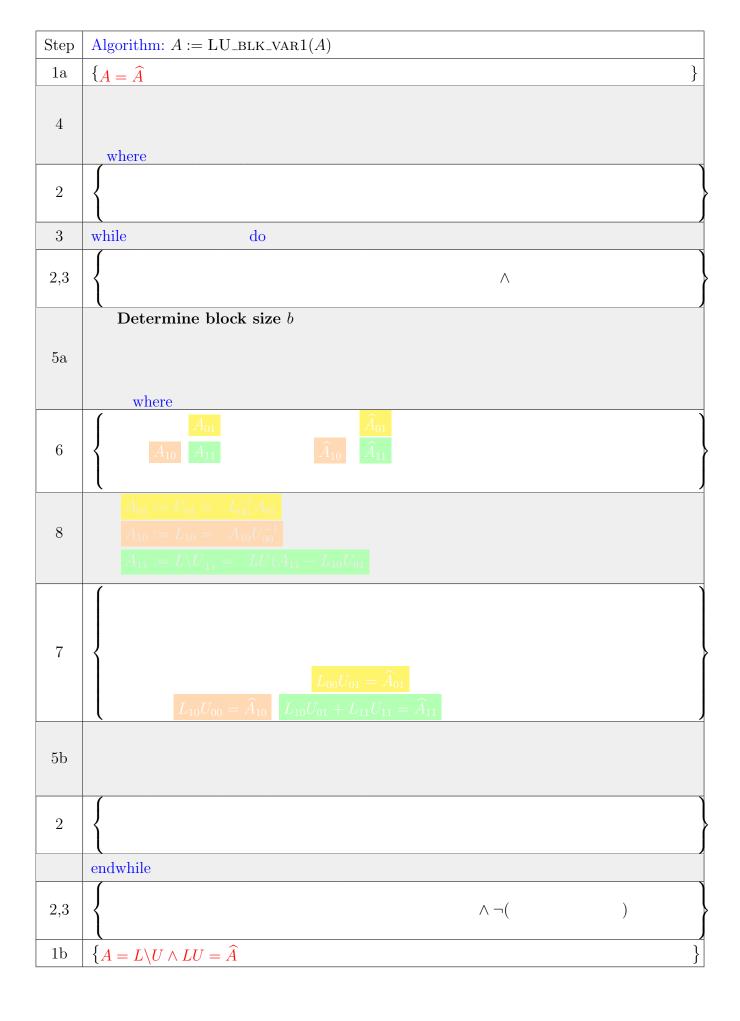
| Step | Algorithm: $A := LU_BLK_VAR1(A)$ |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1a | $\{A = \widehat{A}\}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0 |
| 2 | $ \left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} $ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A) \end{array} \right\}$ |
| 5a | Determine block size b $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \rightarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \cdots $ where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$ |
| 6 | $ \left\{ \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & a_{12}^{T} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & \widehat{A}_{01} & \widehat{A}_{02} \\ \widehat{A}_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \wedge L_{00}U_{00} = \widehat{A}_{00} $ |
| 8 | $A_{01} := U_{01} = L_{00}^{-1} A_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00}) $A_{10} := L_{10} = A_{10} U_{00}^{-1}$ (U_{00} is stored in the upper triangular part of A_{00}) $A_{11} := L \setminus U_{11} = LU(A_{11} - L_{10}U_{01})$ |
| 7 | $ \begin{cases} \begin{pmatrix} A_{00} &_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & U_{01} & \widehat{A}_{02} \\ L_{10} & L \setminus U_{11} & \widehat{A}_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \\ \begin{pmatrix} L_{00}U_{00} = \widehat{A}_{00} & L_{00}U_{01} = \widehat{A}_{01} \\ L_{10}U_{00} = \widehat{A}_{10} & L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11} \end{pmatrix} $ |
| 5b | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots $ |
| 2 | $ \left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \\ \end{array} \right\} $ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) $ |
| 1b | $\left\{ A = L \backslash U \wedge LU = \widehat{A} \right\}$ |

| Step | Algorithm: $A := LU_BLK_VAR1(A)$ |
|------|-----------------------------------------------------------------------------------|
| 1a | { |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | |
| | Determine block size b |
| 5a | |
| | |
| | where |
| 6 | $egin{array}{c ccccccccccccccccccccccccccccccccccc$ |
| | $A_{01} := U_{01} = L_{00}^{-1} A_{01}$ |
| 8 | $A_{10} := L_{10} = A_{10}U_{00}^{-1}$ |
| | $A_{11} := L \setminus U_{11} = LU(A_{11} - L_{10}U_{01})$ |
| | |
| | |
| 7 | |
| | $L_{00}U_{01} = \widehat{A}_{01}$ |
| | $L_{10}U_{00} = \widehat{A}_{10} L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11}$ |
| 5b | |
| 9.0 | |
| | |
| 2 | |
| | endwhile |
| 2,3 | |
| ۷,3 | |
| 1b | { |



```
Algorithm: A := LU_BLK_VAR1(A)
Step
           \{A=\widehat{A}
 1a
  4
               where
                                                 L \backslash U_{TL} \mid \widehat{A}_{TR} \mid
                            A_{TR}
                                                                           \wedge L_{TL}U_{TL} = \widehat{A}_{TL}
  2
           while
                                                 do
  3
                                                                   \widehat{A}_{TR}
                                                      L \backslash U_{TL}
                                 A_{TR}
                                                                               \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge
2,3
                 Determine block size b
 5a
                     where
  6
  8
  7
 5b
                                                                                \wedge L_{TL}U_{TL} = \widehat{A}_{TL}
  2
           endwhile
                            A_{TR}
                                                                          \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (
 2,3
                                                   \widehat{A}_{BL}
           \{A = L \backslash U \land LU = \widehat{A}\}
 1b
```

| Step | Algorithm: $A := LU_BLK_VAR1(A)$ | |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|
| 1a | $\{A = \widehat{A}\}$ | } |
| 4 | where | |
| 2 | $\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL}$ | |
| 3 | while $m(A_{TL}) < m(A)$ do | |
| 2,3 | $\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A)$ | |
| | Determine block size b | |
| 5a | | |
| | wh one | |
| | where \widehat{A}_{01} \widehat{A}_{01} | |
| 6 | $egin{array}{ c c c c c c c c c c c c c c c c c c c$ | |
| | $A_{01} := U_{01} = L_{00}^{-1} A_{01}$ | |
| 8 | | |
| | | |
| 7 | $L_{10}U_{01} = \hat{A}_{01}$ $L_{10}U_{00} = \hat{A}_{10} L_{10}U_{01} + L_{11}U_{11} = \hat{A}_{11}$ | } |
| 5b | | |
| 2 | $\left\{ \begin{array}{c c} \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL}$ | |
| | endwhile | |
| 2,3 | $\left\{ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left(\begin{array}{c c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A))$ | } |
| 1b | $\{A = L \setminus U \land LU = \widehat{A}$ | 1 |

```
Algorithm: A := LU_BLK_VAR1(A)
Step
              {A = \widehat{A}}
  1a
                            egin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \\ \hline \end{array} , L 
ightarrow \left( egin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \\ \hline \end{array} 
ight) , U 
ightarrow \left( egin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \\ \hline \end{array} 
ight)
   4
                  where A_{TL} is 0 \times 0, L_{TL} is 0 \times 0, U_{TL} is 0 \times 0
                                                         \left\langle \begin{array}{c|c} L \setminus U_{TL} & \widehat{A}_{TR} \\ \hline \end{array} \right\rangle \wedge L_{TL} U_{TL} = \widehat{A}_{TL}
                      A_{TL} \mid A_{TR}
   2
             while m(A_{TL}) < m(A) do
   3
                                                                 L \backslash U_{TL} \mid \widehat{A}_{TR} \mid
                            A_{TL} \mid A_{TR}
                                                                                              2,3
                     Determine block size b
  5a
                         where
   6
   8
   7
  5b
                                                                   \frac{L \backslash U_{TL} \mid \widehat{A}_{TR}}{\widehat{}} \wedge L_{TL} U_{TL} = \widehat{A}_{TL}
   2
             endwhile
                                                           L\backslash U_{TL} \mid \widehat{A}_{TR} \mid
                                  A_{TR}
                                                                                        2,3
                                                              \widehat{A}_{BL}
  1b
              \{A = L \backslash U \land LU = \widehat{A}\}
```

```
Algorithm: A := LU_BLK_VAR1(A)
Step
                     \{A = \widehat{A}\}
   1a

\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right), L \to \left(\begin{array}{c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right), U \to \left(\begin{array}{c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right)

    4
                                                                                        \left(\begin{array}{c|c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL}
    2
                     while m(A_{TL}) < m(A) do
    3
                                                                                                    L \setminus U_{TL} \mid \widehat{A}_{TR} \rangle \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A)
                                             A_{TL} \mid A_{TR}
  2,3
                                Determine block size b
                                                                                        \left( egin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline \end{array} \right) \, , \, \left( egin{array}{c|c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) 
ightarrow \cdots \, , \, \left( egin{array}{c|c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) 
ightarrow \cdots \, .
   5a
                                      where A_{11} is b \times b, L_{11} is b \times b, U_{11} is b \times b
    6
    8
    7
                                                                                        \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \cdots, \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \cdots
   5b
                                                                                                        \begin{array}{c|c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \end{array} \Big) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} 
    2
                     endwhile
                                                                                           \frac{L \setminus U_{TL} \mid \widehat{A}_{TR}}{\widehat{A}_{TR}} \setminus \Lambda L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A))
  2,3
                     {A = L \setminus U \land LU = \widehat{A}}
   1b
```

```
Step
                         Algorithm: A := LU_BLK_VAR1(A)
                         \{A = \widehat{A}\}
    1a
                       A 	o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \,,\, L 	o \left( \begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \,,\, U 	o \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)
     4
                                                                                                  \left(\begin{array}{c|c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL}
     2
                        while m(A_{TL}) < m(A) do
     3

\frac{L \setminus U_{TL} \mid \widehat{A}_{TR}}{\widehat{A}_{BL} \mid \widehat{A}_{BR}} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A)

                                                   A_{TL} \mid A_{TR}
  2,3
                                      Determine block size b

\begin{pmatrix}
A_{TL} & A_{TR} \\
A_{BL} & A_{BR}
\end{pmatrix} \to \begin{pmatrix}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{pmatrix}, \begin{pmatrix}
L_{TL} & L_{TR} \\
L_{BL} & L_{BR}
\end{pmatrix} \to \cdots, \begin{pmatrix}
U_{TL} & U_{TR} \\
U_{BL} & U_{BR}
\end{pmatrix} \to \cdots

    5a
                                             where A_{11} is b \times b, L_{11} is b \times b, U_{11} is b \times b
                                                         \begin{array}{c|cccc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & a_{12}^T \\ A_{20} & A_{21} & A_{22} \\ \end{array} \right) = \left( \begin{array}{c|cccc} L \backslash U_{00} & \widehat{A}_{01} & \widehat{A}_{02} \\ \hline \widehat{A}_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ \hline \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \\ \end{array} \right) \wedge L_{00}U_{00} = \widehat{A}_{00} 
     6
     8
     7

\left(\begin{array}{c|cccc}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|cccc}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|cccc}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|cccc}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \cdots

   5b
                                                                                                                       \frac{L \setminus U_{TL} \mid \widehat{A}_{TR}}{\widehat{A}_{BL} \mid \widehat{A}_{BR}} \wedge L_{TL}U_{TL} = \widehat{A}_{TL}
     2
                        endwhile
                                                                                                         \frac{L \setminus U_{TL} \mid \widehat{A}_{TR}}{\widehat{\Box} \mid \widehat{\Box}} \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A))
  2,3
                         {A = L \setminus U \land LU = \widehat{A}}
   1b
```

```
Step
                        Algorithm: A := LU_BLK_VAR1(A)
                        {A = \widehat{A}}
    1a
                      A 	o \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \,,\, L 	o \left( \begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \,,\, U 	o \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)
     4
                                       \frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}} = \left( \frac{L \setminus U_{TL} | \widehat{A}_{TR}}{\widehat{A}_{BL} | \widehat{A}_{BR}} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} 
     2
                       while m(A_{TL}) < m(A) do
     3
                                                                                               = \left( \begin{array}{c|c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \overline{m(A_{TL})} < m(A)
                                                  A_{TL} \mid A_{TR}
  2,3
                                     Determine block size b

\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots

    5a
                                           where A_{11} is b \times b, L_{11} is b \times b, U_{11} is b \times b
                                                       \begin{array}{c|cccc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & a_{12}^T \\ A_{20} & A_{21} & A_{22} \end{array} \right) = \left( \begin{array}{c|cccc} L \backslash U_{00} & \widehat{A}_{01} & \widehat{A}_{02} \\ \hline \widehat{A}_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ \hline \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{array} \right) \wedge L_{00}U_{00} = \widehat{A}_{00} 
     6
     8
                                              7

\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \cdots

   5b
     2
                       endwhile
                                                                                = \left( \begin{array}{c|c} L \backslash U_{TL} & \widehat{A}_{TR} \\ \hline \widehat{A}_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge L_{TL} U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A)) 
  2,3
   1b
                        \{A = L \setminus U \wedge LU = \widehat{A}\}
```

```
Algorithm: A := LU_BLK_VAR1(A)
Step
                      {A = \widehat{A}}
   1a
                    A 	o \left( egin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} 
ight) \,,\, L 	o \left( egin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} 
ight) \,,\, U 	o \left( egin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} 
ight)
                              where A_{TL} is 0 \times 0, L_{TL} is 0 \times 0, U_{TL} is 0 \times 0
                                     \frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}} = \left( \frac{L \setminus U_{TL} | \widehat{A}_{TR}}{\widehat{A}_{BL} | \widehat{A}_{BR}} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL}
     2
                      while m(A_{TL}) < m(A) do
     3
                                              \frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}} = \left( \frac{L \setminus U_{TL} | \widehat{A}_{TR}}{\widehat{A}_{BL} | \widehat{A}_{BR}} \right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge m(A_{TL}) < m(A)
  2,3
                                   Determine block size b

\begin{pmatrix}
A_{TL} & A_{TR} \\
A_{BL} & A_{BR}
\end{pmatrix} \to \begin{pmatrix}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{pmatrix}, \begin{pmatrix}
L_{TL} & L_{TR} \\
L_{BL} & L_{BR}
\end{pmatrix} \to \cdots, \begin{pmatrix}
U_{TL} & U_{TR} \\
U_{BL} & U_{BR}
\end{pmatrix} \to \cdots

   5a
                                         \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & a_{12}^T \\ A_{20} & A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \setminus U_{00} & \widehat{A}_{01} & \widehat{A}_{02} \\ \widehat{A}_{10} & \widehat{A}_{11} & \widehat{A}_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix} \wedge L_{00}U_{00} = \widehat{A}_{00}
     6
                                      A_{01} := U_{01} = L_{00}^{-1} A_{01} ( L_{00} is stored in the strictly lower triangular part of A_{00})
     8
                                      A_{10} := L_{10} = A_{10}U_{00}^{-1} (U_{00} is stored in the upper triangular part of A_{00})
                                      A_{11} := L \setminus U_{11} = LU(A_{11} - L_{10}U_{01})
                                          \begin{pmatrix} A_{00} & {}_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L \backslash U_{00} & U_{01} & \widehat{A}_{02} \\ L_{10} & L \backslash U_{11} & \widehat{A}_{12} \\ \widehat{A}_{20} & \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix}
     7

  \begin{array}{c|c}
    & L_{00}U_{00} = \widehat{A}_{00} \\
    & L_{10}U_{01} = \widehat{A}_{01}
  \end{array}

  \begin{array}{c|c}
    & L_{10}U_{01} = \widehat{A}_{01} \\
    & L_{10}U_{01} + L_{11}U_{11} = \widehat{A}_{11}
  \end{array}

\left(\begin{array}{c|c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots

   5b
                                                  \frac{A_{TL} | A_{TR}}{A_{BL} | A_{BR}} = \begin{pmatrix} \frac{L \setminus U_{TL} | \widehat{A}_{TR}}{\widehat{A}_{BL} | \widehat{A}_{BR}} \end{pmatrix} \wedge L_{TL}U_{TL} = \widehat{A}_{TL}
     2
                      endwhile
                                                                                               \frac{\left(\begin{array}{c|c}L\backslash U_{TL} & \widehat{A}_{TR}\\ \widehat{A}_{BL} & \widehat{A}_{BR}\end{array}\right) \wedge L_{TL}U_{TL} = \widehat{A}_{TL} \wedge \neg (m(A_{TL}) < m(A))
  2,3
   1b
                       \{A = L \setminus U \land LU = \widehat{A}\}
```

| Algorithm: $A := LU_BLK_VAR1(A)$ |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| |
| $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, L \to \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix}, U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}$ where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0 |
| |
| while $m(A_{TL}) < m(A)$ do |
| |
| |
| |
| $A_{01} := U_{01} = L_{00}^{-1} A_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00}) $A_{10} := L_{10} = A_{10} U_{00}^{-1}$ (U_{00} is stored in the upper triangular part of A_{00}) $A_{11} := L \setminus U_{11} = LU(A_{11} - L_{10}U_{01})$ |
| |
| $\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right), \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$ |
| |
| endwhile |
| |
| |

Algorithm: $A := LU_BLK_VAR1(A)$

$$A o \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \,, \, L o \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \,, \, U o \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$$

where A_{TL} is 0×0 , L_{TL} is 0×0 , U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

Determine block size b

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \to \cdots, \left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \to \cdots$$

where A_{11} is $b \times b$, L_{11} is $b \times b$, U_{11} is $b \times b$

 $A_{01} := U_{01} = L_{00}^{-1} A_{01}$ (L_{00} is stored in the strictly lower triangular part of A_{00})

 $A_{10} := L_{10} = A_{10}U_{00}^{-1}$ (U_{00} is stored in the upper triangular part of A_{00})

 $A_{11} := L \setminus U_{11} = LU(A_{11} - L_{10}U_{01})$

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c|c|c}
L_{TL} & L_{TR} \\
\hline
L_{BL} & L_{BR}
\end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c|c}
U_{TL} & U_{TR} \\
\hline
U_{BL} & U_{BR}
\end{array}\right) \leftarrow \cdots$$

endwhile