| Step | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ , $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$ , $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ <br> where $A_{TL}$ is $0 \times 0$, $L_{TL}$ is $0 \times 0$, $U_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ <br> where $\alpha_{11}$ is $1 \times 1$, $\lambda_{11}$ is $1 \times 1$, $\upsilon_{11}$ is $1 \times 1$ |
| 6 | $\{$ $\}$ |
| 8 | update line 1 <br> : <br> update line n |
| 7 | $\{$ $\}$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$ |

| Step | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
|---|---|
| 1a | { } |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | $\wedge$ |
| 5a | where |
| 6 | { } |
| 8 | |
| 7 | { } |
| 5b | |
| 2 | |
| | endwhile |
| 2,3 | $\wedge \neg($ ) |
| 1b | { } |

| Step | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | where |
| 2 | $\left\{ \right.$ $\left. \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \right.$ $\wedge$ $\left. \right\}$ |
| 5a | where |
| 6 | { } |
| 8 | |
| 7 | { } |
| 5b | |
| 2 | $\left\{ \right.$ $\left. \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \right.$ $\wedge \neg($ $)$ $\left. \right\}$ |
| 1b | $\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$ |

| Step | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | where |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge \right\}$ |
| 5a | where |
| 6 | $\{$ $\}$ |
| 8 | |
| 7 | $\{$ $\}$ |
| 5b | |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge \neg( \quad ) \right\}$ |
| 1b | $\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$ |

| Step | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | where |
| 2 | $\left\{ \left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | where |
| 6 | $\{$ $\}$ |
| 8 | |
| 7 | $\{$ $\}$ |
| 5b | |
| 2 | $\left\{ \left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array} \right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$ |

| Step | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ , $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$ , $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ <br> where $A_{TL}$ is $0 \times 0$, $L_{TL}$ is $0 \times 0$, $U_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | where |
| 6 | $\{$ $\}$ |
| 8 | |
| 7 | $\{$ $\}$ |
| 5b | |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$ |

| Step | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ , $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$ , $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ <br> where $A_{TL}$ is $0 \times 0$, $L_{TL}$ is $0 \times 0$, $U_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ <br> where $\alpha_{11}$ is $1 \times 1$, $\lambda_{11}$ is $1 \times 1$, $\upsilon_{11}$ is $1 \times 1$ |
| 6 | $\{$ $\}$ |
| 8 | |
| 7 | $\{$ $\}$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$ |

| Step | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ , $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$ , $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ <br> where $A_{TL}$ is $0 \times 0$, $L_{TL}$ is $0 \times 0$, $U_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ <br> where $\alpha_{11}$ is $1 \times 1$, $\lambda_{11}$ is $1 \times 1$, $\upsilon_{11}$ is $1 \times 1$ |
| 6 | $\{$ $\}$ |
| 8 | |
| 7 | $\{$ $\}$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$ |

| Step | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ , $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$ , $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ <br> where $A_{TL}$ is $0 \times 0$, $L_{TL}$ is $0 \times 0$, $U_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ <br> where $\alpha_{11}$ is $1 \times 1$, $\lambda_{11}$ is $1 \times 1$, $\upsilon_{11}$ is $1 \times 1$ |
| 6 | $\{$ $\}$ |
| 8 | |
| 7 | $\{$ $\}$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = L\backslash U \wedge LU = \widehat{A}$ $\}$ |

| Step | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
|---|---|
| 1a | $\{ A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ , $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$ , $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ <br> where $A_{TL}$ is $0 \times 0$, $L_{TL}$ is $0 \times 0$, $U_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ <br> where $\alpha_{11}$ is $1 \times 1$, $\lambda_{11}$ is $1 \times 1$, $\upsilon_{11}$ is $1 \times 1$ |
| 6 | $\{$ $\}$ |
| 8 | update line 1 <br> : <br> update line n |
| 7 | $\{$ $\}$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & \widehat{A}_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{l} L_{TL}U_{TL} = \widehat{A}_{TL} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{ A = L\backslash U \wedge LU = \widehat{A}$ $\}$ |

| | |
|---|---|
| | Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$ |
| | |
| | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ , $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$ , $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$<br>where $A_{TL}$ is $0 \times 0$, $L_{TL}$ is $0 \times 0$, $U_{TL}$ is $0 \times 0$ |
| | |
| | while $m(A_{TL}) < m(A)$ do |
| | |
| | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$<br>where $\alpha_{11}$ is $1 \times 1$, $\lambda_{11}$ is $1 \times 1$, $\upsilon_{11}$ is $1 \times 1$ |
| | |
| | update line 1<br>:<br>update line n |
| | |
| | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$ , $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$ , $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$ |
| | |
| | endwhile |
| | |
| | |

Algorithm: $A := \text{LU\_UNB\_VAR2}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where $A_{TL}$ is $0 \times 0$, $L_{TL}$ is $0 \times 0$, $U_{TL}$ is $0 \times 0$

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

where $\alpha_{11}$ is $1 \times 1$, $\lambda_{11}$ is $1 \times 1$, $\upsilon_{11}$ is $1 \times 1$

update line 1

:

update line n

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$$

endwhile