



Step	<b>Algorithm:</b> $A := \text{LU\_BLK\_VAR5}(A)$
1a	$\{A = \hat{A}$
4	$A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$ <p>where <math>A_{TL}</math> is <math>0 \times 0</math>, <math>L_{TL}</math> is <math>0 \times 0</math>, <math>U_{TL}</math> is <math>0 \times 0</math></p>
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
3	<b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>
2,3	$\left\{ \begin{array}{l} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$
5a	<p><b>Determine block size</b> <math>b</math></p> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c} A_{00} & A_{01} \ A_{02} \\ \hline A_{10} & A_{11} \ A_{12} \\ A_{20} & A_{21} \ A_{22} \end{array} \right), \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$ <p>where <math>A_{11}</math> is <math>b \times b</math>, <math>L_{11}</math> is <math>b \times b</math>, <math>U_{11}</math> is <math>b \times b</math></p>
6	$\left\{ \begin{array}{l} \left( \begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \hat{A}_{11} - L_{10}U_{01} & \hat{A}_{12} - L_{10}U_{02} \\ L_{20} & \hat{A}_{21} - L_{20}U_{01} & \hat{A}_{22} - L_{20}U_{02} \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \ L_{00}U_{01} = \hat{A}_{01} \ L_{00}U_{02} = \hat{A}_{02} \\ \wedge \ L_{10}U_{00} = \hat{A}_{10} \\ L_{20}U_{00} = \hat{A}_{20} \end{array} \right\}$
8	$\begin{aligned} A_{11} &:= L \setminus U_{11} = LU(\hat{A}_{11} - L_{10}U_{01}) = LU(A_{11}) \\ A_{12} &:= U_{12} = L_{11}^{-1}(\hat{A}_{12} - L_{10}U_{02}) = L_{11}^{-1}A_{12} \\ A_{21} &:= U_{21} = (\hat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1} \\ A_{22} &:= \hat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12} \end{aligned}$
7	$\left\{ \begin{array}{l} \left( \begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & L \setminus U_{11} & U_{12} \\ L_{20} & L_{21} & \hat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}U_{01} = \hat{A}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge \ L_{10}U_{00} = \hat{A}_{10} \ L_{10}^T U_{01} + L_{11}U_{11} = \hat{A}_{11} \ L_{10}U_{02} + L_{11}U_{12} = \hat{A}_{12} \\ L_{20}U_{00} = \hat{A}_{20} \ L_{20}U_{01} + L_{21}U_{11} = \hat{A}_{21} \end{array} \right\}$
5b	$\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c} A_{00} \ A_{01} \ A_{02} \\ \hline A_{10} \ A_{11} \ A_{12} \\ A_{20} \ A_{21} \ A_{22} \end{array} \right), \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
	<b>endwhile</b>
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge$

Step	Algorithm: $A := \text{LU\_BLK\_VAR5}(A)$
1a	{
4	where
2	{
3	while do
2,3	{ ^ }
5a	Determine block size $b$  where
6	{
8	
7	{
5b	
2	{
	endwhile
2,3	{ ^ }

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1a	{ $A = \hat{A}$ }
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3	while do
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL}} \wedge \right\}$
5a	Determine block size $b$
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2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL}} \right\}$
	endwhile
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL}} \wedge \right\}$

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3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge \right.$ $\left. m(A_{TL}) < m(A) \right\}$
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5a	<p><b>Determine block size <math>b</math></b></p> <p>where</p>
6	
8	
7	
5b	
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \right\}$
	endwhile
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge \right.$

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3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{l} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$
5a	<p><b>Determine block size <math>b</math></b></p> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$ <p>where <math>A_{11}</math> is <math>b \times b</math>, <math>L_{11}</math> is <math>b \times b</math>, <math>U_{11}</math> is <math>b \times b</math></p>
6	
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2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
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2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \backslash U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge$



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3	<b>while</b> $m(A_{TL}) < m(A)$ <b>do</b>
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5a	<p><b>Determine block size <math>b</math></b></p> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$ <p>where <math>A_{11}</math> is <math>b \times b</math>, <math>L_{11}</math> is <math>b \times b</math>, <math>U_{11}</math> is <math>b \times b</math></p>
6	$\left\{ \begin{array}{l} \left( \begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \hat{A}_{11} - L_{10}U_{01} & \hat{A}_{12} - L_{10}U_{02} \\ L_{20} & \hat{A}_{21} - L_{20}U_{01} & \hat{A}_{22} - L_{20}U_{02} \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}U_{01} = \hat{A}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge L_{10}U_{00} = \hat{A}_{10} \\ L_{20}U_{00} = \hat{A}_{20} \end{array} \right\}$
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5b	$\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
	<b>endwhile</b>
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge$

Step	Algorithm: $A := \text{LU\_BLK\_VAR5}(A)$
1a	$\{A = \hat{A}$
4	$A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$ <p>where <math>A_{TL}</math> is <math>0 \times 0</math>, <math>L_{TL}</math> is <math>0 \times 0</math>, <math>U_{TL}</math> is <math>0 \times 0</math></p>
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{l} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$
5a	<p><b>Determine block size <math>b</math></b></p> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$ <p>where <math>A_{11}</math> is <math>b \times b</math>, <math>L_{11}</math> is <math>b \times b</math>, <math>U_{11}</math> is <math>b \times b</math></p>
6	$\left\{ \begin{array}{l} \left( \begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \hat{A}_{11} - L_{10}U_{01} & \hat{A}_{12} - L_{10}U_{02} \\ L_{20} & \hat{A}_{21} - L_{20}U_{01} & \hat{A}_{22} - L_{20}U_{02} \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}U_{01} = \hat{A}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge L_{10}U_{00} = \hat{A}_{10} \\ L_{20}U_{00} = \hat{A}_{20} \end{array} \right\}$
8	
7	$\left\{ \begin{array}{l} \left( \begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & L \setminus U_{11} & U_{12} \\ L_{20} & L_{21} & \hat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}U_{01} = \hat{A}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge L_{10}U_{00} = \hat{A}_{10} \quad L_{10}^T U_{01} + L_{11}U_{11} = \hat{A}_{11} \quad L_{10}U_{02} + L_{11}U_{12} = \hat{A}_{12} \\ L_{20}U_{00} = \hat{A}_{20} \quad L_{20}U_{01} + L_{21}U_{11} = \hat{A}_{21} \end{array} \right\}$
5b	$\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
	endwhile
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge$

Step	Algorithm: $A := \text{LU\_BLK\_VAR5}(A)$
1a	$\{A = \hat{A}$
4	$A \rightarrow \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$ <p>where <math>A_{TL}</math> is <math>0 \times 0</math>, <math>L_{TL}</math> is <math>0 \times 0</math>, <math>U_{TL}</math> is <math>0 \times 0</math></p>
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \begin{array}{l} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge \\ m(A_{TL}) < m(A) \end{array} \right\}$
5a	<p><b>Determine block size <math>b</math></b></p> $\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c c} A_{00} & A_{01} \ A_{02} \\ \hline A_{10} & A_{11} \ A_{12} \\ A_{20} & A_{21} \ A_{22} \end{array} \right), \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$ <p>where <math>A_{11}</math> is <math>b \times b</math>, <math>L_{11}</math> is <math>b \times b</math>, <math>U_{11}</math> is <math>b \times b</math></p>
6	$\left\{ \begin{array}{l} \left( \begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & \hat{A}_{11} - L_{10}U_{01} & \hat{A}_{12} - L_{10}U_{02} \\ L_{20} & \hat{A}_{21} - L_{20}U_{01} & \hat{A}_{22} - L_{20}U_{02} \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \ L_{00}U_{01} = \hat{A}_{01} \ L_{00}U_{02} = \hat{A}_{02} \\ \wedge \ L_{10}U_{00} = \hat{A}_{10} \\ L_{20}U_{00} = \hat{A}_{20} \end{array} \right\}$
8	$\begin{aligned} A_{11} &:= L \setminus U_{11} = LU(\hat{A}_{11} - L_{10}U_{01}) = LU(A_{11}) \\ A_{12} &:= U_{12} = L_{11}^{-1}(\hat{A}_{12} - L_{10}U_{02}) = L_{11}^{-1}A_{12} \\ A_{21} &:= U_{21} = (\hat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1} \\ A_{22} &:= \hat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12} \end{aligned}$
7	$\left\{ \begin{array}{l} \left( \begin{array}{ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right) = \left( \begin{array}{ccc} L \setminus U_{00} & U_{01} & U_{02} \\ L_{10} & L \setminus U_{11} & U_{12} \\ L_{20} & L_{21} & \hat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} \end{array} \right) \\ L_{00}U_{00} = \hat{A}_{00} \quad L_{00}U_{01} = \hat{A}_{01} \quad L_{00}U_{02} = \hat{A}_{02} \\ \wedge \ L_{10}U_{00} = \hat{A}_{10} \ L_{10}^T U_{01} + L_{11}U_{11} = \hat{A}_{11} \ L_{10}U_{02} + L_{11}U_{12} = \hat{A}_{12} \\ L_{20}U_{00} = \hat{A}_{20} \ L_{20}U_{01} + L_{21}U_{11} = \hat{A}_{21} \end{array} \right\}$
5b	$\left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c c} A_{00} \ A_{01} \ A_{02} \\ \hline A_{10} \ A_{11} \ A_{12} \\ A_{20} \ A_{21} \ A_{22} \end{array} \right), \left( \begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left( \begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$
2	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid}$
	endwhile
2,3	$\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} L \setminus U_{TL} & \hat{U}_{TR} \\ \hline L_{BL} & \hat{A}_{BR} - L_{BL}U_{TR} \end{array} \right) \wedge \frac{L_{TL}U_{TL} = \hat{A}_{TL} \mid L_{TL}U_{TR} = \hat{A}_{TR}}{L_{BL}U_{TL} = \hat{A}_{BL} \mid} \wedge$

Algorithm:  $A := \text{LU\_BLK\_VAR5}(A)$

$$A \rightarrow \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left( \begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$$

where  $A_{TL}$  is  $0 \times 0$ ,  $L_{TL}$  is  $0 \times 0$ ,  $U_{TL}$  is  $0 \times 0$

while  $m(A_{TL}) < m(A)$  do

**Determine block size  $b$**

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$$

where  $A_{11}$  is  $b \times b$ ,  $L_{11}$  is  $b \times b$ ,  $U_{11}$  is  $b \times b$

$$A_{11} := L \backslash U_{11} = LU(\hat{A}_{11} - L_{10}U_{01}) = LU(A_{11})$$

$$A_{12} := U_{12} = L_{11}^{-1}(\hat{A}_{12} - L_{10}U_{02}) = L_{11}^{-1}A_{12}$$

$$A_{21} := U_{21} = (\hat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1}$$

$$A_{22} := \hat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12}$$

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$$

endwhile

**Algorithm:**  $A := \text{LU\_BLK\_VAR5}(A)$

$$A \rightarrow \left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), L \rightarrow \left( \begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right), U \rightarrow \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right)$$

**where**  $A_{TL}$  is  $0 \times 0$ ,  $L_{TL}$  is  $0 \times 0$ ,  $U_{TL}$  is  $0 \times 0$

**while**  $m(A_{TL}) < m(A)$  **do**

**Determine block size**  $b$

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \rightarrow \dots, \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \rightarrow \dots$$

**where**  $A_{11}$  is  $b \times b$ ,  $L_{11}$  is  $b \times b$ ,  $U_{11}$  is  $b \times b$

$$A_{11} := L \setminus U_{11} = LU(\hat{A}_{11} - L_{10}U_{01}) = LU(A_{11})$$

$$A_{12} := U_{12} = L_{11}^{-1}(\hat{A}_{12} - L_{10}U_{02}) = L_{11}^{-1}A_{12}$$

$$A_{21} := U_{21} = (\hat{A}_{21} - L_{20}U_{01})U_{11}^{-1} = A_{21}U_{11}^{-1}$$

$$A_{22} := \hat{A}_{22} - L_{20}U_{02} - L_{21}U_{12} = A_{22} - A_{21}A_{12}$$

$$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right), \left( \begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array} \right) \leftarrow \dots, \left( \begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array} \right) \leftarrow \dots$$

**endwhile**