Step	Algorithm: Solve $Ux = y$ overwriting y with x. U is upper triangular.
1a	$\{ y = \widehat{y}$ $\}$
4	$U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$, $x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right)$, $y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where U_{BR} is 0×0, x_B has 0 rows, y_B has 0 rows
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \right\}$
3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	$\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{cc\|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$ where υ_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row
6	$\left\{ \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) = \left(\begin{array}{c} \widehat{y}_0 - U_{02}x_2 \\ \widehat{\psi}_1 - u_{12}^T x_2 \\ x_2 \end{array}\right) \wedge U_{22}x_2 = \widehat{y}_2 \right\}$
8	$y_0 := \widehat{y}_0 - \chi_1 u_{01} - U_{02}x_2 = y_0 - \chi_1 u_{01}$ $\psi_1 := \chi_1 = (\widehat{\psi}_1 - u_{12}^T x_2)/\upsilon_{11} = \psi_1/\upsilon_{11}$
7	$\left\{ \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) = \left(\begin{array}{c} \widehat{y}_0 - \chi_1 u_{01} - U_{02}x_2 \\ \chi_1 \\ x_2 \end{array}\right) \wedge \begin{array}{r} \upsilon_{11}\chi_1 + u_{12}^T x_2 = \widehat{\psi}_1 \\ U_{22}x_2 = \widehat{y}_2 \end{array} \right\}$
5b	$\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg(m(U_{BR}) < m(U)) \right\}$
1b	$\{ y = x \wedge Ux = \widehat{y}$ $\}$

Step	Algorithm: Solve $Ux = y$ overwriting y with x. U is upper triangular.
1a	{ }
4	where
2	
3	while do
2,3	$\wedge$
5a	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\wedge\ \neg($ $)$
1b	{ }

Step	Algorithm: Solve $Ux = y$ overwriting y with x. U is upper triangular.
1a	$\{y = \widehat{y}$ $\}$
4	where
2	$\left\{ \right.$ $\left. \right\}$
3	while do
2,3	$\left\{ \right.$ $\wedge$ $\left. \right\}$
5a	where
6	$\left\{ \right.$ $\left. \right\}$
8	
7	$\left\{ \right.$ $\left. \right\}$
5b	
2	$\left\{ \right.$ $\left. \right\}$
	endwhile
2,3	$\left\{ \right.$ $\wedge \neg($ $)$ $\left. \right\}$
1b	$\{y = x \wedge Ux = \widehat{y}$ $\}$

Step	Algorithm: Solve $Ux = y$ overwriting y with x. U is upper triangular.
1a	$\{y = \widehat{y}$ $\}$
4	where
2	$\left\{ \begin{pmatrix} y_T \\ \hline y_B \end{pmatrix} = \begin{pmatrix} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{pmatrix} \wedge U_{BR}x_B = \widehat{y}_B \right\}$
3	while do
2,3	$\left\{ \begin{pmatrix} y_T \\ \hline y_B \end{pmatrix} = \begin{pmatrix} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{pmatrix} \wedge U_{BR}x_B = \widehat{y}_B \wedge \right.$
5a	where
6	$\left\{ \right.$
8	
7	$\left. \right\}$
5b	
2	$\left\{ \begin{pmatrix} y_T \\ \hline y_B \end{pmatrix} = \begin{pmatrix} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{pmatrix} \wedge U_{BR}x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \begin{pmatrix} y_T \\ \hline y_B \end{pmatrix} = \begin{pmatrix} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{pmatrix} \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg(\right.$)
1b	$\{y = x \wedge Ux = \widehat{y}$ $\}$

Step	Algorithm: Solve $Ux = y$ overwriting y with x. U is upper triangular.
1a	$\{y = \widehat{y}$ $\}$
4	where
2	$\left\{ \left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T - U_{TR}x_B}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B \right\}$
3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T - U_{TR}x_B}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	where
6	$\left\{ \right.$ $\left. \right\}$
8	
7	$\left\{ \right.$ $\left. \right\}$
5b	
2	$\left\{ \left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T - U_{TR}x_B}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T - U_{TR}x_B}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg(m(U_{BR}) < m(U)) \right\}$
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Step	Algorithm: Solve $Ux = y$ overwriting y with x. U is upper triangular.
1a	$\{y = \widehat{y}$ $\}$
4	$U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$, $x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right)$, $y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where U_{BR} is 0×0, x_B has 0 rows, y_B has 0 rows
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \right\}$
3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	where
6	$\left\{ \right\}$
8	
7	$\left\{ \right\}$
5b	
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg(m(U_{BR}) < m(U)) \right\}$
1b	$\{y = x \wedge Ux = \widehat{y}$ $\}$

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3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR} x_B \\ \hline x_B \end{array}\right) \wedge U_{BR} x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	$\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{cc\|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$ where υ_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row
6	$\left\{ \right.$ $\left. \right\}$
8	
7	$\left\{ \right.$ $\left. \right\}$
5b	$\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR} x_B \\ \hline x_B \end{array}\right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR} x_B \\ \hline x_B \end{array}\right) \wedge U_{BR} x_B = \widehat{y}_B \wedge \neg(m(U_{BR}) < m(U)) \right\}$
1b	$\{ y = x \wedge Ux = \widehat{y}$ $\}$

Step	Algorithm: Solve $Ux = y$ overwriting y with x. U is upper triangular.
1a	$\{ y = \widehat{y}$ $\}$
4	$U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$, $x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right)$, $y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where U_{BR} is 0×0, x_B has 0 rows, y_B has 0 rows
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \right\}$
3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	$\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{cc\|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$ where υ_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row
6	$\left\{ \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) = \left(\begin{array}{c} \widehat{y}_0 - U_{02}x_2 \\ \hline \widehat{\psi}_1 - u_{12}^T x_2 \\ \hline x_2 \end{array}\right) \wedge U_{22}x_2 = \widehat{y}_2 \right\}$
8	
7	$\left\{ \right\}$
5b	$\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|cc} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg(m(U_{BR}) < m(U)) \right\}$
1b	$\{ y = x \wedge Ux = \widehat{y}$ $\}$

Step	Algorithm: Solve $Ux = y$ overwriting y with x. U is upper triangular.
1a	$\{ y = \widehat{y}$ $\}$
4	$U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$, $x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right)$, $y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where U_{BR} is 0×0, x_B has 0 rows, y_B has 0 rows
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR} x_B \\ \hline x_B \end{array}\right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR} x_B \\ \hline x_B \end{array}\right) \wedge U_{BR} x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	$\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{cc\|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$ where υ_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row
6	$\left\{ \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 - U_{02} x_2 \\ \widehat{\psi}_1 - u_{12}^T x_2 \\ x_2 \end{pmatrix} \wedge U_{22} x_2 = \widehat{y}_2 \right\}$
8	
7	$\left\{ \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 - \chi_1 u_{01} - U_{02} x_2 \\ \chi_1 \\ x_2 \end{pmatrix} \wedge \begin{array}{r} \upsilon_{11}\chi_1 + u_{12}^T x_2 = \widehat{\psi}_1 \\ U_{22} x_2 = \widehat{y}_2 \end{array} \right\}$
5b	$\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|cc} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & \upsilon_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right)$
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR} x_B \\ \hline x_B \end{array}\right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR} x_B \\ \hline x_B \end{array}\right) \wedge U_{BR} x_B = \widehat{y}_B \wedge \neg(m(U_{BR}) < m(U)) \right\}$
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1a	$\{ y = \widehat{y}$ $\}$
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2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \right\}$
3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	$\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{cc\|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$ where υ_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row
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8	$y_0 := \widehat{y}_0 - \chi_1 u_{01} - U_{02}x_2 = y_0 - \chi_1 u_{01}$ $\psi_1 := \chi_1 = (\widehat{\psi}_1 - u_{12}^T x_2)/\upsilon_{11} = \psi_1/\upsilon_{11}$
7	$\left\{ \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) = \left(\begin{array}{c} \widehat{y}_0 - \chi_1 u_{01} - U_{02}x_2 \\ \chi_1 \\ x_2 \end{array}\right) \wedge \begin{array}{r} \upsilon_{11}\chi_1 + u_{12}^T x_2 = \widehat{\psi}_1 \\ U_{22}x_2 = \widehat{y}_2 \end{array} \right\}$
5b	$\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|cc} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & \upsilon_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{array}\right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right)$
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} \widehat{y}_T - U_{TR}x_B \\ \hline x_B \end{array}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg(m(U_{BR}) < m(U)) \right\}$
1b	$\{ y = x \wedge Ux = \widehat{y}$ $\}$

Algorithm: Solve $Ux = y$ overwriting y with x. U is upper triangular.

$$U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right), \; x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), \; y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$$

where U_{BR} is 0×0, x_B has 0 rows, y_B has 0 rows

while $m(U_{BR}) < m(U)$ do

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{cc|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \; \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \; \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$$

where υ_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row

$$y_0 := \widehat{y}_0 - \chi_1 u_{01} - U_{02} x_2 = y_0 - \chi_1 u_{01}$$
$$\psi_1 := \chi_1 = (\widehat{\psi}_1 - u_{12}^T x_2)/\upsilon_{11} = \psi_1/\upsilon_{11}$$

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|cc} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & \upsilon_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{array}\right), \; \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \; \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$$

endwhile

Algorithm: Solve $Ux = y$ overwriting y with x. U is upper triangular.

$$U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right), x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$$

where U_{BR} is 0×0, x_B has 0 rows, y_B has 0 rows

while $m(U_{BR}) < m(U)$ do

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{cc|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$$

where υ_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row

$$y_0 := \widehat{y}_0 - \chi_1 u_{01} - U_{02} x_2 = y_0 - \chi_1 u_{01}$$
$$\psi_1 := \chi_1 = (\widehat{\psi}_1 - u_{12}^T x_2)/\upsilon_{11} = \psi_1/\upsilon_{11}$$

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & \upsilon_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$$

endwhile