| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ where $A_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | **Determine block size** $b$<br>$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right)$<br>where $A_{11}$ is $b \times b$ |
| 6 | $\left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} - L_{20}l_{10} & A_{22} - L_{20}L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20}L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} \right\}$ |
| 8 | $\alpha_{11} := \sqrt{\alpha_{11}}$<br>$a_{21} := a_{21}/\alpha_{11}$<br>$A_{22} := A_{22} - a_{21}a_{21}^T$ update only lower triangular part |
| 7 | $\left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T & \\ l_{10}^T L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20}L_{00}^T & L_{20}l_{10} + \lambda_{11}l_{21} \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} & \\ \widehat{a}_{10}^T & \widehat{\alpha}_{11} \\ \widehat{A}_{20} & \widehat{a}_{21} \end{pmatrix} \right\}$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = \text{Chol}(\widehat{A})$ $\}$ |

| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$ |
|---|---|
| 1a | { } |
| 4 | where |
| 2 | |
| 3 | while do |
| 2,3 | $\wedge$ |
| 5a | **Determine block size** $b$<br>where |
| 6 | |
| 8 | |
| 7 | |
| 5b | |
| 2 | |
| | endwhile |
| 2,3 | $\wedge$<br>$\neg($ ) |
| 1b | { } |

| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | where |
| 2 | $\left\{ \right.$ $\left. \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \right.$ $\wedge$ $\left. \right\}$ |
| 5a | **Determine block size** $b$ where |
| 6 | $\left\{ \right.$ $\left. \right\}$ |
| 8 | |
| 7 | $\left\{ \right.$ $\left. \right\}$ |
| 5b | |
| 2 | $\left\{ \right.$ $\left. \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \right.$ $\wedge$ $\neg($ $)$ $\left. \right\}$ |
| 1b | $\{A = \text{Chol}(\widehat{A})$ $\}$ |

| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | where |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \right.$ |
| 5a | **Determine block size** $b$ where |
| 6 | $\left\{ \right.$ |
| 8 | |
| 7 | $\left\{ \right.$ |
| 5b | |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \right.$ $\neg($ $)$ |
| 1b | $\{A = \text{Chol}(\widehat{A})$ $\}$ |

| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | where |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | **Determine block size** $b$ where |
| 6 | $\left\{ \right\}$ |
| 8 | |
| 7 | $\left\{ \right\}$ |
| 5b | |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = \text{Chol}(\widehat{A})$ $\}$ |

| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ where $A_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | **Determine block size** $b$<br>where |
| 6 | $\left\{ \right\}$ |
| 8 | |
| 7 | $\left\{ \right\}$ |
| 5b | |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = \text{Chol}(\widehat{A})$ $\}$ |

| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ where $A_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | **Determine block size** $b$ <br> $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$ <br> where $A_{11}$ is $b \times b$ |
| 6 | $\left\{ \right.$ |
| 8 | |
| 7 | $\left\{ \right.$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = \text{Chol}(\widehat{A})$ $\}$ |

| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ where $A_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | **Determine block size** $b$ <br> $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right)$ <br> where $A_{11}$ is $b \times b$ |
| 6 | $\left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} - L_{20}l_{10} & A_{22} - L_{20}L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20}L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} \right\}$ |
| 8 | |
| 7 | |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = \text{Chol}(\widehat{A})$ $\}$ |

| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ where $A_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | **Determine block size** $b$ <br> $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right)$ <br> where $A_{11}$ is $b \times b$ |
| 6 | $\left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} - L_{20}l_{10} & A_{22} - L_{20}L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20}L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} \right\}$ |
| 8 | |
| 7 | $\left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T & \\ l_{10}^T L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20}L_{00}^T & L_{20}l_{10} + \lambda_{11}l_{21} \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} & \\ \widehat{a}_{10}^T & \widehat{\alpha}_{11} \\ \widehat{A}_{20} & \widehat{a}_{21} \end{pmatrix} \right\}$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = \text{Chol}(\widehat{A})$ $\}$ |

| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$ |
|---|---|
| 1a | $\{A = \widehat{A}$ $\}$ |
| 4 | $A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$ where $A_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | **Determine block size** $b$<br>$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right)$<br>where $A_{11}$ is $b \times b$ |
| 6 | $\left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} - L_{20}l_{10} & A_{22} - L_{20}L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20}L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} \right\}$ |
| 8 | $\alpha_{11} := \sqrt{\alpha_{11}}$<br>$a_{21} := a_{21}/\alpha_{11}$<br>$A_{22} := A_{22} - a_{21}a_{21}^T$ update only lower triangular part |
| 7 | $\left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T & \\ l_{10}^T L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20}L_{00}^T & L_{20}l_{10} + \lambda_{11}l_{21} \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} & \\ \widehat{a}_{10}^T & \widehat{\alpha}_{11} \\ \widehat{A}_{20} & \widehat{a}_{21} \end{pmatrix} \right\}$ |
| 5b | $\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \neg(m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{A = \text{Chol}(\widehat{A})$ $\}$ |

Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$$

where $A_{TL}$ is $0 \times 0$

while $m(A_{TL}) < m(A)$ do

**Determine block size** $b$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$$

where $A_{11}$ is $b \times b$

$\alpha_{11} := \sqrt{\alpha_{11}}$

$a_{21} := a_{21}/\alpha_{11}$

$A_{22} := A_{22} - a_{21}a_{21}^T$ update only lower triangular part

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$$

endwhile

Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$$

where $A_{TL}$ is $0 \times 0$

while $m(A_{TL}) < m(A)$ do

**Determine block size** $b$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right)$$

where $A_{11}$ is $b \times b$

$\alpha_{11} := \sqrt{\alpha_{11}}$

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$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$$

endwhile