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| 4 | where |
| 2 | $\left\{ \quad \right\}$ |
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| 8 | |
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<tr><td>2,3</td><td>$\left\{ \left( \begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left( \begin{array}{r} A_{BL}^T x_B + \widehat{y}_T \\ \hline A_{BL} x_T + A_{BR} x_B + \widehat{y}_B \end{array} \right) \wedge m(A_{BR}) < m(A) \right\}$</td></tr>
<tr><td>5a</td><td>$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left( \begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array} \right), \left( \begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left( \begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array} \right)$<br>where $\alpha_{11}$ is $1 \times 1$, $\chi_1$ has 1 row, $\psi_1$ has 1 row</td></tr>
<tr><td>6</td><td>$\left\{ \left( \begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array} \right) = \left( \begin{array}{r} A_{20}^T x_2 + \widehat{y}_0 \\ a_{21}^T x_2 + \widehat{\psi}_1 \\ A_{20} x_0 + \chi_1 a_{21} + A_{22} x_2 + \widehat{y}_2 \end{array} \right) \right\}$</td></tr>
<tr><td>8</td><td>$y_0 := \chi_1 (a_{10}^T)^T + y_0$<br>$\psi_1 := a_{10}^T x_0 + \alpha_{11} \chi_1 + \psi_1$</td></tr>
<tr><td>7</td><td>$\left\{ \left( \begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array} \right) = \left( \begin{array}{r} \chi_1 (a_{10}^T)^T + A_{20}^T x_2 + \widehat{y}_0 \\ a_{10}^T x_0 + \alpha_{11} \chi_1 + a_{21}^T x_2 + \widehat{\psi}_1 \\ A_{20} x_0 + \chi_1 a_{21} + A_{22} x_2 + \widehat{y}_2 \end{array} \right) \right\}$</td></tr>
<tr><td>5b</td><td>$\left( \begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \left( \begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left( \begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array} \right), \left( \begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left( \begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array} \right)$</td></tr>
<tr><td>2</td><td>$\left\{ \left( \begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left( \begin{array}{r} A_{BL}^T x_B + \widehat{y}_T \\ \hline A_{BL} x_T + A_{BR} x_B + \widehat{y}_B \end{array} \right) \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left( \begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left( \begin{array}{r} A_{BL}^T x_B + \widehat{y}_T \\ \hline A_{BL} x_T + A_{BR} x_B + \widehat{y}_B \end{array} \right) \wedge \neg (m(A_{BR}) < m(A)) \right\}$</td></tr>
<tr><td>1b</td><td>$\{ y = Ax + \widehat{y} \}$</td></tr>
</table>

| | |
|---|---|
| | Algorithm: $y := Ax + y$    ($A$ symmetric stored in lower triangular part) |
| | |
| | $A \rightarrow \left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$ , $x \rightarrow \left( \begin{array}{c} x_T \\ \hline x_B \end{array} \right)$ , $y \rightarrow \left( \begin{array}{c} y_T \\ \hline y_B \end{array} \right)$ <br> where $A_{BR}$ is $0 \times 0$, $x_B$ has 0 rows, $y_B$ has 0 rows |
| | |
| | while $m(A_{BR}) < m(A)$ do |
| | |
| | $\left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$ , $\left( \begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left( \begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array} \right)$ , $\left( \begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left( \begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array} \right)$ <br> where $\alpha_{11}$ is $1 \times 1$, $\chi_1$ has 1 row, $\psi_1$ has 1 row |
| | |
| | $y_0 := \chi_1 (a_{10}^T)^T + y_0$ <br> $\psi_1 := a_{10}^T x_0 + \alpha_{11} \chi_1 + \psi_1$ |
| | |
| | $\left( \begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right)$ , $\left( \begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left( \begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array} \right)$ , $\left( \begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left( \begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array} \right)$ |
| | |
| | endwhile |
| | |
| | |

**Algorithm:** $y := Ax + y$ ($A$ symmetric stored in lower triangular part)

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$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), \; x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), \; y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$$

where $A_{BR}$ is $0 \times 0$, $x_B$ has 0 rows, $y_B$ has 0 rows

**while** $m(A_{BR}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \; \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array}\right), \; \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array}\right)$$

where $\alpha_{11}$ is $1 \times 1$, $\chi_1$ has 1 row, $\psi_1$ has 1 row

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$y_0 := \chi_1 (a_{10}^T)^T + y_0$

$\psi_1 := a_{10}^T x_0 + \alpha_{11}\chi_1 + \psi_1$

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$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \; \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \; \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right)$$

**endwhile**