| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Q.   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Step | Algorithm: $[A] := CHOL_BLK_VAR3(A)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| $ \begin{array}{c} \text{where } A_{TL} \text{ is } 0 \times 0 \\ 2 \\ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \\ \widehat{L}_{BL} & \widehat{L}_{TL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \\ 3 \\ \text{while } m(A_{TL}) < m(A) \text{ do} \\ 2,3 \\ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{TL}^T \end{pmatrix} - \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge m(A_{TL}) < m(A) \\ \end{array} \right. \\ 5a \\ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} \\ \text{where } A_{11} \text{ is } b \times b \\ \\ 6 \\ \left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ L_{10} & \alpha_{21} - L_{20}L_{10} & A_{22} - L_{20}L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ L_{0L}L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} \\ \hat{A}_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ L_{20} & a_{21} - L_{20}L_{20} & A_{22} - L_{20}L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ L_{20}L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} \\ \hat{A}_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ L_{10} & A_{11} & \widehat{a}_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ L_{10} & A_{11} & \widehat{a}_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ L_{20} & L_{20}L_{20}^T - L_{20}L_{20}^T - L_{20}L_{20}^T - L_{20}L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ L_{10}L_{00}^T & L_{10}^T L_{10} + \lambda_{11}^2 \\ L_{20}L_{10}^T & A_{11} & \widehat{a}_{12}^T \\ A_{20} & \widehat{a}_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{01} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} L_{00}L_{10}^T & L_{10}L_{21}^T \\ L_{20}L_{20}^T & L_{20}L_{20}^T - L_{20}L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{11}L_{11}^T \\ L_{20}L_{10}^T & L_{10}L_{11}^T \end{pmatrix} + \begin{pmatrix} A_{11}L_{11} & A_{12} \\ L_{20}L_{10}^T & A_{11}L_{21}^T \end{pmatrix} \end{pmatrix} = \begin{pmatrix} A_{11}L_{11} & A_{12} \\ L_{20}L_{11}^T & A_{11}^T & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} L_{11}L_{1$ | 1a   | $A = \widehat{A}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| $ \begin{cases} \begin{cases} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{cases} = \begin{pmatrix} \hat{L}_{TL} & \hat{A}_{TR} \\ \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \\ \hat{L}_{BL} & \hat{L}_{TL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \hat{A}_{TL} \\ \hat{A}_{BL} \end{pmatrix} \\ \end{cases} $ $ \text{while } m(A_{TL}) < m(A) \text{ do} $ $ 2.3 & \begin{cases} \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \hat{L}_{TL} & \hat{A}_{TR} \\ \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \hat{A}_{TL} \\ \hat{A}_{BL} \end{pmatrix} \wedge m(A_{TL}) < m(A) \end{cases} $ $ \text{Determine block size } b \\ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} \\ \text{where } A_{11} \text{ is } b \times b \end{cases} $ $ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \hat{a}_{01} & \hat{A}_{02} \\ L_{20} & a_{21} - L_{20}l_{10} & A_{22} - L_{20}L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}I_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20}I_{20}^T \end{pmatrix} = \begin{pmatrix} \hat{A}_{00} \\ \hat{a}_{10}^T \\ \hat{A}_{20} \end{pmatrix} $ $ \begin{cases} \alpha_{11} := \sqrt{\alpha_{11}} \\ \alpha_{21} := a_{21}/\alpha_{11} \\ A_{22} := A_{22} - a_{21}a_{21}^T \end{aligned} \text{ update only lower triangular part} $ $ \begin{cases} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ a_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \hat{a}_{01} & \hat{A}_{02} \\ L_{20} & l_{21}^T & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^T L_{00}^T & l_{10}^T + \lambda_{11}^2 \\ L_{20}L_{20}^T & A_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^T L_{00}^T & l_{10}^T + \lambda_{11}^2 \\ L_{20}L_{10}^T & A_{11} & A_{12} \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & a_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} A_{00} & A_{01} $                                                                                                                                                            | 4    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 2    | $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \right\} $                                                                                                                                                                                                                                                   |
| $ \begin{array}{c} \textbf{Determine block size } b \\ & \begin{array}{c} a \\ & \begin{array}{c} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \\ \end{array} \end{array} \right) \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \\ \end{array} \\ & \begin{array}{c} \textbf{where } A_{11} \text{ is } b \times b \\ & \begin{array}{c} a \\ A_{00} & a_{01} & A_{02} \\ A_{20} & a_{21} & A_{22} \\ \end{array} \right) \\ & \begin{array}{c} \textbf{where } A_{11} \text{ is } b \times b \\ \end{array} \\ & \begin{array}{c} a \\ a_{10} & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \\ \end{array} \right) = \begin{pmatrix} L_{00} & \hat{a}_{01} & \hat{A}_{02} \\ l_{10}^T & \alpha_{11} & l_{10}^T \\ l_{20} & a_{21} - L_{20}l_{10} & A_{22} - L_{20}L_{20}^T \\ \end{array} \right) \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^TL_{00}^T \\ l_{10}^TL_{00}^T \\ L_{20}L_{20}^T \\ \end{array} \right) \\ & \begin{array}{c} \alpha_{11} := \sqrt{\alpha_{11}} \\ \end{array} \\ & \begin{array}{c} a_{21} := a_{21}/\alpha_{11} \\ A_{22} := A_{22} - a_{21}a_{21}^T \\ \end{array} \right) \text{ update only lower triangular part} \\ & \begin{array}{c} A_{00} & a_{01} & A_{02} \\ l_{10}^T & \alpha_{11} & \hat{a}_{12}^T \\ A_{20} & a_{21} & A_{22} \\ \end{array} \right) = \begin{pmatrix} L_{00} & \hat{a}_{01} & \hat{A}_{02} \\ l_{10}^T & \lambda_{11} & \hat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \\ \end{array} \right) \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^Tl_{10} + \lambda_{11}^2 \\ L_{20}l_{10} + \lambda_{11}^2 \\ L_{20}l_{21}^T & A_{22} - L_{20}l_{20}^T - l_{21}l_{21}^T \\ \end{array} \right) \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^Tl_{10} + \lambda_{11}^2 \\ L_{20}l_{10} + \lambda_{11}^2 \\ L_{20}l_{21}^T & A_{22} - L_{20}l_{20}^T - l_{21}l_{21}^T \\ \end{array} \right) \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^Tl_{10} + \lambda_{11}^2 \\ L_{20}l_{21} + \lambda_{11}l_{21} \\ L_{20}l_{21}^T & A_{22} - L_{20}l_{20}^T - l_{21}l_{21}^T \\ \end{array} \right) \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^Tl_{10} + \lambda_{11}^2 \\ L_{20}l_{21}^T & A_{22} - L_{20}l_{20}^T - l_{21}l_{21}^T \\ \end{array} \right) \\ = \begin{pmatrix} A_{11} & A_{12} \\ A_{20} & a_{21} \\ A_{21} & A_{12} \\ A_{20} & A_{21} \\ A_{22} & A_{22} \\ \end{array} \right) \\ = \begin{pmatrix} A_{11} & A_{12} \\ A_{20} & A_{21} \\ A_{22} & A_{22} \\ \end{array} \right) \\ = \begin{pmatrix} A_{11} & A_{12} \\ A_{20} & A_{21} \\ A_{22} & A_{22} \\ \end{array} \right) \\ = \begin{pmatrix} A_{11} & A_{12} \\ A_{20} & A_{21} \\ A_{20$                                                                                                                                                                                                                                                                                                                                                                         | 3    | while $m(A_{TL}) < m(A)$ do                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 2,3  | $\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \wedge m(A_{TL}) < m(A) \end{array} \right\}$                                                                                                  |
| $ \begin{array}{c} \text{where } A_{11} \text{ is } b \times b \\ \hline \\ 6 \end{array} \left\{ \begin{array}{c} A_{00}  a_{01}  A_{02} \\ a_{10}^T  \alpha_{11}  a_{12}^T \\ A_{20}  a_{21}  A_{22} \end{array} \right\} = \begin{pmatrix} L_{00}  \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T  \alpha_{11} - l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20}  a_{21} - L_{20} l_{10}  A_{22} - L_{20} L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20} L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{a}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} $ $ \begin{array}{c} \alpha_{11} := \sqrt{\alpha_{11}} \\ 8 \end{array}  \begin{array}{c} \alpha_{11} := \sqrt{\alpha_{11}} \\ a_{21} := a_{21}/\alpha_{11} \\ A_{22} := A_{22} - a_{21} a_{21}^T  \text{update only lower triangular part} \end{array} $ $ \begin{array}{c} \begin{pmatrix} A_{00}  a_{01}  A_{02} \\ a_{10}^T  \alpha_{11}  a_{12}^T \\ A_{20}  a_{21}  A_{22} \end{pmatrix} = \begin{pmatrix} L_{00}  \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T  \lambda_{11} & \widehat{a}_{12}^T \\ L_{20}  l_{21}  A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}  l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20} L_{10}  \lambda_{11} & \widehat{a}_{12}^T \\ L_{20}  l_{21}  A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}  l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20} L_{10}  k_{11} l_{21} \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00}  A_{01} \\ \widehat{A}_{02} \\ A_{21}  A_{21}  A_{22} \end{pmatrix} $ $ \begin{array}{c} 5b  \begin{pmatrix} A_{TL}  A_{TR} \\ A_{BL}  A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00}  A_{01}  A_{02} \\ A_{10}  A_{11}  A_{12} \\ A_{20}  A_{21}  A_{22} \end{pmatrix} \\ 2  \begin{pmatrix} A_{TL}  A_{TR} \\ A_{BL}  A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL}  \widehat{A}_{TR} \\ \widehat{L}_{BL}  \widehat{A}_{BR} - L_{BL} L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL} L_{TL}^T \\ L_{BL} L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < m(A)) \\ \end{array} $ $ \begin{array}{c} end \\ end \\ end \\ \end{array} $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |      | Determine block size $b$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| $ \begin{cases} \begin{cases} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{cases} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} - L_{20} l_{10} & A_{22} - L_{20} L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{10}^T \\ l_{10}^T L_{00}^T \\ L_{20} L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} $ $ \begin{cases} \alpha_{11} := \sqrt{\alpha_{11}} \\ \alpha_{21} := a_{21}/\alpha_{11} \\ A_{22} := A_{22} - a_{21} a_{21}^T \end{aligned}  \text{update only lower triangular part} $ $ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20} L_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20} L_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20} L_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{10} + \lambda_{11} l_{21} \\ L_{20} L_{10}^T & \lambda_{11} & \lambda_{12} \\ L_{20} L_{20}^T & \lambda_{20} l_{20} + \lambda_{11} l_{21} \end{pmatrix} = \begin{pmatrix} A_{00} L_{00} & A_{01} & A_{02} \\ A_{10} A_{11} & A_{12} \\ A_{20} A_{21} & A_{22} \end{pmatrix} $ $2 \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL} L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL} L_{TL}^T \\ L_{BL} L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < m(A)) $ endwhile $2.3 \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{L}_{BL} & \widehat{L}_{BL} \end{pmatrix} \wedge \begin{pmatrix} L_{TL} L_{TL}^T \\ \widehat{L}_{BL} & \widehat{A}_{BL} & \widehat{L}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < m(A))$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 5a   | (20  21  22 /                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| $ \begin{array}{ll} \alpha_{11} := \sqrt{\alpha_{11}} \\ a_{21} := a_{21}/\alpha_{11} \\ A_{22} := A_{22} - a_{21}a_{21}^T  \text{update only lower triangular part} \\ \\ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \\ \\ 5b & \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} \\ \\ 2 & \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \\ \\ endwhile \\ \\ 2,3 & \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < m(A)) \\ \\ \end{pmatrix} $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |      | where $A_{11}$ is $\theta \times \theta$ $ \left( \begin{array}{ccc} A_{00} & a_{01} & A_{00} \\ A_{00} & a_{01} & A_{00} \end{array} \right) \left( \begin{array}{ccc} A_{00} & \widehat{a}_{01} \\ A_{00} & \widehat{a}_{01} \end{array} \right) \left( \begin{array}{ccc} \widehat{A}_{00} \\ \widehat{A}_{00} \end{array} \right) $                                                                                                                                                                                                                                                                        |
| $ \begin{array}{ll} \alpha_{11} := \sqrt{\alpha_{11}} \\ a_{21} := a_{21}/\alpha_{11} \\ A_{22} := A_{22} - a_{21}a_{21}^T  \text{update only lower triangular part} \\ \\ \begin{cases} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \\ \\ 5b & \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} \\ \\ 2 & \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \\ \\ endwhile \\ \\ 2,3 & \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < m(A)) \\ \\ \end{pmatrix} $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 6    | $ \left\{ \begin{array}{cccc} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} B_{00} & a_{01} & A_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} - L_{20} l_{10} & A_{22} - L_{20} L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} B_{00} B_{00} \\ l_{10}^T L_{00}^T \\ L_{20} L_{00}^T \end{pmatrix} = \begin{pmatrix} A_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} \right\} $                                                                           |
| $ 8 \qquad a_{21} := a_{21}/\alpha_{11} \\ A_{22} := A_{22} - a_{21}a_{21}^T  \text{update only lower triangular part}                                    $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 8    | ·                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |      | $A_{22} := A_{22} - a_{21}a_{21}^T$ update only lower triangular part                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| $ 2 \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \\ \text{endwhile} \\ 2,3 \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \wedge \neg (m(A_{TL}) < m(A)) \right\} $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 7    | $ \left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} & l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^TL_{00}^T & l_{10}^Tl_{10} + \lambda_{11}^2 \\ L_{20}L_{01}^T & L_{20}L_{10} + \lambda_{11}l_{21} \end{pmatrix} = \left\{ \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T & \widehat{\alpha}_{11} \end{pmatrix} \right\} $ |
| $ 2 \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \\ \text{endwhile} \\ 2,3 \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \wedge \neg (m(A_{TL}) < m(A)) \right\} $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 5b   | $\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$                                                                                                                                                                                                                                                                                                                                                              |
| $2,3  \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \wedge \neg (m(A_{TL}) < m(A))$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 2    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| $\left(\begin{array}{c c} A_{BL} & A_{BR} \end{array}\right)  \left(\begin{array}{c c} \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right)  \left(\begin{array}{c c} \widehat{L}_{BL}L_{TL}^T \end{array}\right)  \left(\begin{array}{c c} \widehat{A}_{BL} \end{array}\right)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 2,3  | $\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \wedge \neg (m(A_{TL}) < m(A))  \right\}$                                                                                                                         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 1b   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |

| Step | Algorithm: $[A] := CHOL_BLK_VAR3(A)$                       |
|------|------------------------------------------------------------|
| 1a   | {                                                          |
| 4    | where                                                      |
|      | where                                                      |
| 2    |                                                            |
| 3    | while do                                                   |
| 2,3  |                                                            |
|      | Determine block size $b$                                   |
| 5a   |                                                            |
|      | where                                                      |
| 6    |                                                            |
| 8    |                                                            |
| 7    |                                                            |
| 5b   |                                                            |
| 2    |                                                            |
|      | endwhile                                                   |
| 2,3  | $\left\{ egin{array}{cccccccccccccccccccccccccccccccccccc$ |
| 1b   | {                                                          |

| Step | Algorithm: $[A] := CHOL_BLK_VAR3(A)$                    |
|------|---------------------------------------------------------|
| 1a   | $\{A = \widehat{A}\}$                                   |
| 4    | where                                                   |
| 2    | $\left\{ \begin{array}{c} \end{array} \right\}$         |
| 3    | while do                                                |
| 2,3  |                                                         |
|      | Determine block size $b$                                |
| 5a   |                                                         |
|      | where                                                   |
| 6    |                                                         |
| 8    |                                                         |
| 7    |                                                         |
| 5b   |                                                         |
| 2    |                                                         |
|      | endwhile                                                |
| 2,3  |                                                         |
| 1b   | $\left\{ A = \operatorname{Chol}(\widehat{A}) \right\}$ |

| Step | Algorithm: $[A] := CHOL_BLK_VAR3(A)$                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1a   | $\{A = \widehat{A}\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| 4    | where                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| 2    | $\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right)$                                            |
| 3    | while do                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 2,3  | $\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) & = & \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) & = & \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \wedge \right\}$ |
|      | Determine block size $b$                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 5a   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|      | where                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| 6    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| 8    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| 7    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| 5b   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| 2    | $ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) $                       |
|      | endwhile ( ) ( ) ( )                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| 2,3  | $ \left\{ \left( \frac{A_{TL}}{A_{BL}} \begin{vmatrix} A_{TR} \\ A_{BL} \end{vmatrix} A_{BR} \right) = \left( \frac{\widehat{L}_{TL}}{\widehat{L}_{BL}} \begin{vmatrix} \widehat{A}_{RR} \\ \widehat{L}_{BL} \end{vmatrix} \widehat{A}_{BR} - L_{BL}L_{BL}^T \right) \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \left( \frac{\widehat{A}_{TL}}{\widehat{A}_{BL}} \right) \wedge \right\} $                                                                            |
| 1b   | $\left\{ A = \operatorname{Chol}(\widehat{A}) \right\}$                                                                                                                                                                                                                                                                                                                                                                                                                                  |

| Step | Algorithm: $[A] := CHOL_BLK_VAR3(A)$                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1a   | $\{A = \widehat{A}\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 4    | where                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 2    | $\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \right\}$                       |
| 3    | while $m(A_{TL}) < m(A)$ do                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 2,3  | $\left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge m(A_{TL}) < \right\}$                                                                                                |
|      | Determine block size b                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| 5a   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|      | where                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 6    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 8    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 7    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 5b   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 2    | $\left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL} L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL} L_{TL}^T \\ \hline L_{BL} L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \right\}$ |
|      | endwhile                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| 2,3  | $\left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < \right\}$                                                                                          |
| 1b   | $\left\{ A = \operatorname{Chol}(\widehat{A}) \right\}$                                                                                                                                                                                                                                                                                                                                                                                                                      |

| Step | Algorithm: $[A] := CHOL_BLK_VAR3(A)$                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1a   | $\{A = \widehat{A}\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| 4    | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$                                                                                                                                                                                                                                                                                                                                                                     |
| 2    | $ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \right\} $                    |
| 3    | while $m(A_{TL}) < m(A)$ do                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 2,3  | $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge m(A_{TL}) < \right\} $                                                                                             |
|      | Determine block size b                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 5a   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|      | where                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| 6    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| 8    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| 7    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| 5b   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| 2    | $ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \right\} $ |
|      | endwhile                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| 2,3  | $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < \right\} $                                                                                                 |
| 1b   | $\left\{ A = \operatorname{Chol}(\widehat{A}) \right\}$                                                                                                                                                                                                                                                                                                                                                                                                                     |

| Step | Algorithm: $[A] := \text{CHOL\_BLK\_VAR3}(A)$                                                                                                                                                                                                                                                                                                                                                                                                   |
|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1a   | $A = \widehat{A}$                                                                                                                                                                                                                                                                                                                                                                                                                               |
| 4    | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$                                                                                                                                                                                                                                                                                                                                         |
| 2    | $ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) $ |
| 3    | while $m(A_{TL}) < m(A)$ do                                                                                                                                                                                                                                                                                                                                                                                                                     |
| 2,3  | $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \left( \frac{\widehat{A}_{TL}}{\widehat{A}_{BL}} \right) \wedge m(A_{TL}) < \right\} $                                                                                     |
| 5a   | $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} $ where $A_{11}$ is $b \times b$                                                                                                                                                                                                                  |
| 6    |                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 8    |                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 7    |                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 5b   | $ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right) $                                                                                                                                                                                                    |
| 2    | $ \left\{ \begin{array}{c c} \left( \frac{A_{TL}}{A_{BL}} \middle  A_{TR} \right) = \left( \frac{\widehat{L}_{TL}}{\widehat{L}_{BL}} \middle  \widehat{A}_{RR} \right) \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \left( \frac{\widehat{A}_{TL}}{\widehat{A}_{BL}} \right) \\ \end{array} \right\} $                                                                                                                         |
|      | endwhile                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 2,3  | $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \left( \frac{\widehat{A}_{TL}}{\widehat{A}_{BL}} \right) \wedge \neg (m(A_{TL}) < \right\} $                                                                               |
| 1b   | $\left\{ A = \operatorname{Chol}(\widehat{A}) \right\}$                                                                                                                                                                                                                                                                                                                                                                                         |

| Step | Algorithm: $[A] := CHOL_BLK_VAR3(A)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1a   | $A = \widehat{A}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| 4    | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| 2    | $ \left\{ \left( \begin{array}{c c} A_{TL} & \text{Is } 0 \times 0 \\ \hline A_{BL} & A_{RR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \\ \hline \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \\ \hline \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \\ \hline \end{array} \right) $                           |
| 3    | while $m(A_{TL}) < m(A)$ do                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 2,3  | $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge m(A_{TL}) < \right\} $                                                                                                                                                                  |
|      | Determine block size $b$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| 5a   | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \to \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) $                                                                                                                                                                                                                                                                                                                |
|      | where $A_{11}$ is $b \times b$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| 6    | $ \left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} - L_{20} l_{10} & A_{22} - L_{20} L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20} L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} \right\} $ |
| 8    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 7    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 5b   | $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$                                                                                                                                                                                                                                                                                                    |
| 2    | $ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) \right\} $                                                                      |
|      | endwhile                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| 2,3  | $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \left( \frac{\widehat{A}_{TL}}{\widehat{A}_{BL}} \right) \wedge \neg (m(A_{TL}) < \right\} $                                                                                                                                                                                |
| 1b   | $\left\{ A = \operatorname{Chol}(\widehat{A}) \right\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |

| $ \begin{array}{cccccccccccccccccccccccccccccccccccc$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Step | Algorithm: $[A] := CHOL_BLK_VAR3(A)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| $ \begin{array}{c} \text{where } A_{TL} \text{ is } 0 \times 0 \\ 2 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{RR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \\ \widehat{L}_{BL} & \widehat{L}_{BL} & A_{BL} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \\ \widehat{L}_{BL} & \widehat{L}_{BL} & A_{BL} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{CO} & A_{O1} & A_{O2} \\ A_{O0} & A_{O1} & A_{O2} \\ A_{O2} & A_{O2} & A_{O2} & A_{O2} \\ A_{O2} & A_{O2} & A_{O2} & A_{O2} \\ A_{O$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 1a   | $A = \widehat{A}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |  |
| $ \begin{array}{c} 2  \left\{ \left( \frac{A_{TL}}{A_{BL}} \right  \frac{A_{TR}}{A_{BR}} \right) = \left( \frac{\widehat{L}_{TL}}{\widehat{L}_{BL}} \right  \frac{\widehat{A}_{TR}}{\widehat{L}_{BL}} - L_{BL}L_{BL}^T \right) \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \left( \frac{\widehat{A}_{TL}}{\widehat{A}_{BL}} \right) \\ 3  \text{while } m(A_{TL}) < m(A) \text{ do} \\ \\ 2,3  \left\{ \begin{array}{c} A_{TL} \mid A_{TR} \\ A_{BL} \mid A_{BR} \end{array} \right) = \left( \frac{\widehat{L}_{TL}}{\widehat{L}_{BL}} \right  \frac{\widehat{A}_{TR}}{\widehat{L}_{BL}} - L_{BL}L_{RL}^T \right) \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \left( \frac{\widehat{A}_{TL}}{\widehat{A}_{BL}} \right) \wedge m(A_{TL}) < \\ 8  \left( \frac{A_{TL}}{A_{BL}} \right  \frac{A_{TR}}{A_{BR}} \right) \rightarrow \left( \frac{A_{O0}}{A_{O1}} \right  \frac{A_{O2}}{A_{O1}} \right) \\ A_{O0}  A_{O1} \mid A_{O2} \\ A_{O2} \mid A_{O1} \mid A_{O2} \right) \\ A_{O1}  A_{O2} \mid A_{O2} \mid A_{O2} \right) \\ A_{O2}  A_{O2} \mid A_{O2} \mid A_{O2} \right) \rightarrow \left( \frac{L_{O0} \mid \widehat{A}_{O2}}{L_{O0} \mid A_{O2}} \right) \wedge \left( \frac{L_{O0}L_{O0}^T}{L_{O0}^T} \right) = \left( \frac{\widehat{A}_{O0}}{\widehat{A}_{O1}} \right) \\ A_{O2}  A_{O1} \mid A_{O2} \mid A_{O2} \right) \\ A_{O3}  A_{O1} \mid A_{O2} \mid A_{O2} \mid A_{O2} \right) \rightarrow \left( \frac{L_{O0} \mid \widehat{A}_{O2}}{L_{O0}^T} \right) \wedge \left( \frac{L_{O0}L_{O0}^T}{L_{O0}^T} \right) = \left( \frac{\widehat{A}_{O0}}{\widehat{A}_{O1}} \right) \\ A_{O2}  A_{O1}  A_{O2} \mid A_{O$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 4    | where $A_{TL}$ is $0 \times 0$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 2    | $\left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right)$                                                                                                                                                                                                                                                                                                                                                                               |  |
| $ \begin{array}{c} \text{Determine block size } b \\ & \left( \begin{array}{c} A_{TL} \mid A_{TR} \\ A_{BL} \mid A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c} A_{00} \mid A_{01} \mid A_{02} \\ A_{10} \mid A_{11} \mid A_{12} \\ A_{20} \mid A_{21} \mid A_{22} \end{array} \right) \\ & \text{where } A_{11} \text{ is } b \times b \\ & \left\{ \begin{array}{c} \left( \begin{array}{c} A_{00} \mid a_{01} \mid A_{02} \\ A_{20} \mid a_{21} \mid A_{22} \end{array} \right) = \left( \begin{array}{c} L_{00} \mid \widehat{a}_{01} \mid \widehat{A}_{02} \\ l_{10}^T \mid \alpha_{11} \mid a_{12}^T \\ A_{20} \mid a_{21} \mid A_{22} \end{array} \right) + \left( \begin{array}{c} L_{00} \mid \widehat{a}_{01} \mid \widehat{A}_{02} \\ l_{10}^T \mid \alpha_{11} \mid a_{12}^T \\ A_{20} \mid a_{21} \mid A_{22} \end{array} \right) + \left( \begin{array}{c} L_{00} \mid \widehat{a}_{01} \mid \widehat{A}_{02} \\ l_{10}^T \mid \widehat{A}_{11} \mid \widehat{A}_{12}^T \\ A_{20} \mid a_{21} \mid A_{22} \end{array} \right) + \left( \begin{array}{c} A_{00} \mid \widehat{a}_{01} \mid \widehat{A}_{02} \\ a_{10}^T \mid \widehat{A}_{11} \mid \widehat{A}_{12}^T \\ A_{20} \mid \widehat{a}_{21} \mid A_{22} \end{array} \right) + \left( \begin{array}{c} A_{00} \mid \widehat{a}_{01} \mid \widehat{A}_{02} \\ l_{10}^T \mid \widehat{A}_{11} \mid \widehat{A}_{12}^T \\ A_{20} \mid \widehat{a}_{21} \mid A_{22} \end{array} \right) + \left( \begin{array}{c} A_{00} \mid \widehat{a}_{01} \mid \widehat{A}_{02} \\ A_{10} \mid \widehat{A}_{11} \mid \widehat{A}_{12} \\ A_{20} \mid \widehat{a}_{21} \end{array} \right) + \left( \begin{array}{c} A_{00} \mid \widehat{a}_{01} \mid \widehat{A}_{02} \\ \widehat{a}_{10}^T \mid \widehat{A}_{11} \mid \widehat{A}_{12} \\ \widehat{A}_{20} \mid \widehat{a}_{21} \end{array} \right) + \left( \begin{array}{c} A_{00} \mid \widehat{A}_{01} \mid \widehat{A}_{02} \\ \widehat{a}_{10}^T \mid \widehat{A}_{11} \mid \widehat{A}_{12} \\ \widehat{A}_{20} \mid \widehat{a}_{21} \end{array} \right) + \left( \begin{array}{c} A_{00} \mid \widehat{A}_{01} \mid \widehat{A}_{02} \\ A_{10} \mid \widehat{A}_{11} \mid \widehat{A}_{12} \\ \widehat{A}_{20} \mid \widehat{A}_{21} \mid \widehat{A}_{22} \end{array} \right) + \left( \begin{array}{c} A_{11} \mid \widehat{A}_{12} \\ \widehat{A}_{20} \mid \widehat{A}_{21} \mid \widehat{A}_{22} \\ \widehat{A}_{20} \mid \widehat{A}_{21} \mid \widehat{A}_{22} \end{array} \right) + \left( \begin{array}{c} A_{11} \mid \widehat{A}_{12} \\ \widehat{A}_{20} \mid \widehat{A}_{21} \mid \widehat{A}_{22} \\ \widehat{A}_{20} \mid \widehat{A}_{21} \mid \widehat{A}_{$ | 3    | while $m(A_{TL}) < m(A)$ do                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |  |
| $ \begin{array}{c} 5a & \left( \begin{array}{c ccc} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right) \\ & \text{where } A_{11} \text{ is } b \times b \\ \hline \\ 6 & \left\{ \begin{array}{c ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right\} = \left( \begin{array}{c ccc} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \alpha_{11} & l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} & L_{20} l_{10} & A_{22} - L_{20} L_{20}^T \right) \wedge \left( \begin{array}{c} L_{00} L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20} L_{20}^T \end{array} \right) = \left( \begin{array}{c ccc} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ A_{20} & \alpha_{21} & A_{22} \end{array} \right) \\ \hline \\ 8 & \\ \hline \\ 7 & \left\{ \begin{array}{c ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right\} = \left( \begin{array}{c ccc} L_{00} \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ A_{20} & l_{10}^T l_{10} + \lambda_{11} \\ L_{20} L_{21}^T & A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{array} \right) \wedge \left( \begin{array}{c ccc} A_{00} & A_{01} & A_{02} \\ l_{10}^T & \lambda_{11} & \widehat{a}_{12}^T \\ A_{20} & A_{21} & A_{22} \end{array} \right) \\ \hline \\ 5b & \left( \begin{array}{c ccc} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c ccc} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array} \right) \\ \hline \\ endwhile & \\ \hline \\ 2,3 & \left\{ \left( \begin{array}{c ccc} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c ccc} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL} L_{BL}^T \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL} L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c ccc} L_{TL} L_{TL}^T \\ L_{BL} L_{TL}^T \end{array} \right) = \left( \begin{array}{c ccc} \widehat{A}_{TL} \\ \widehat{A}_{BL} \\ \widehat{A}_{BL} \end{array} \right) \wedge \gamma(m(A_{TL}) < \left\{ \begin{array}{c ccc} A_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL} L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c ccc} L_{TL} L_{TL}^T \\ L_{BL} L_{TL}^T \end{array} \right) = \left( \begin{array}{c ccc} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{array} \right) \wedge \gamma(m(A_{TL}) < \left\{ \begin{array}{c ccc} A_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL} L_{BL}^T \end{array} \right\} \wedge \left( \begin{array}{c ccc} L_{TL} L_{TL}^T \\ L_{BL} L_{TL}^T \end{array} \right) = \left( \begin{array}{c ccc} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{array} \right) \wedge \gamma(m(A_{TL}) < \left\{ \begin{array}{c ccc} A_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} \end{array} \right\} \right\} + \left( \begin{array}{c ccc} A_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{TL} & \widehat{A}_{TL} & \widehat{A}_{TL} \end{array} \right) + \left( \begin{array}{c ccc} A_{TL} & \widehat{A}_{T$                                                                                                                                                                                                                                                                                                                                                                                              | 2,3  | $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge m(A_{TL}) < \right\} $                                                                                                                                                                                                                                                                                                                                                                                                                                             |  |
| $ \begin{array}{c} \text{where } A_{11} \text{ is } b \times b \\ \begin{cases} A_{00} \ a_{01} \ A_{02} \\ a_{10}^T \ \alpha_{11} \ a_{12}^T \\ A_{20} \ a_{21} \ A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} \ \widehat{a}_{01} \ \widehat{A}_{01} \\ l_{10}^T \ \alpha_{11} - l_{10}^T l_{10} \\ L_{20} \ a_{21} - L_{20} l_{10} \ A_{22} - L_{20} L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20} L_{20}^T L_{20} \end{pmatrix} \\ \\ 8 \end{cases} $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |      | Determine block size b                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |
| $ \begin{cases} \begin{cases} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{cases} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \alpha_{11} & l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} - L_{20}l_{10} & A_{22} - L_{20}L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20}L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} $ $ \begin{cases} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} l_{21} & A_{22} - L_{20}L_{20}^T - l_{21}l_{21}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00}L_{00}^T \\ L_{00}L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20}L_{10}^T & L_{20}l_{10} + \lambda_{11}l_{21} \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T & \widehat{a}_{11} \\ \widehat{A}_{20} & \widehat{a}_{21} \end{pmatrix} $ $ \begin{cases} A_{TL} \begin{vmatrix} A_{TR} \\ A_{BL} \end{vmatrix} A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} A_{01} & A_{02} \\ A_{10} A_{11} & A_{12} \\ A_{20} A_{21} & A_{22} \end{pmatrix} $ $ \begin{cases} A_{TL} \begin{vmatrix} A_{TR} \\ A_{BL} \end{vmatrix} A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} \begin{vmatrix} \widehat{A}_{TR} \\ \widehat{L}_{BL} \end{vmatrix} \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge -(m(A_{TL}) < m(A_{TL}) <$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 5a   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |  |
| $ \begin{cases}           \begin{pmatrix}             A_{00} & a_{01} & A_{02} \\             a_{10}^{T} & \alpha_{11} & a_{12}^{T} \\             A_{20} & a_{21} & A_{22}       \end{pmatrix} = \begin{pmatrix}             L_{00} \hat{a}_{01} & \hat{A}_{02} \\             l_{10}^{T} & \lambda_{11} & \hat{a}_{12}^{T} \\             L_{20} & l_{21} & A_{22} - L_{20}L_{20}^{T} - l_{21}l_{21}^{T}       \end{pmatrix} \wedge \\           \begin{pmatrix}             L_{00}L_{00}^{T} & l_{10}^{T}l_{10} + \lambda_{11}^{2} \\             L_{20}l_{10}^{T} & L_{20}l_{10} + \lambda_{11}l_{21}       \end{pmatrix} = \begin{pmatrix}             \hat{a}_{10}^{T} & \hat{a}_{11} \\             \hat{A}_{20} & \hat{a}_{21}       \end{pmatrix} $ $           \begin{pmatrix}             A_{TL} & A_{TR} \\             A_{BL} & A_{BR}       \end{pmatrix} \leftarrow \begin{pmatrix}             A_{00} & A_{01} & A_{02} \\             A_{10} & A_{11} & A_{12} \\             A_{20} & A_{21} & A_{22}       \end{pmatrix} $ $           \begin{pmatrix}             A_{TL} & A_{TR} \\             A_{BL} & A_{BR}       \end{pmatrix} = \begin{pmatrix}             \hat{L}_{TL} & \hat{A}_{TR} \\             \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^{T}       \end{pmatrix} \wedge \begin{pmatrix}             L_{TL}L_{TL}^{T} \\             L_{BL}L_{TL}^{T}       \end{pmatrix} = \begin{pmatrix}             \hat{A}_{TL} & \hat{A}_{TL} \\             \hat{A}_{BL} & \hat{A}_{BR}       \end{pmatrix} $ $           = \text{endwhile} $ $           \begin{pmatrix}             A_{TL} & A_{TR} \\             A_{BL} & A_{BR}       \end{pmatrix} = \begin{pmatrix}             \hat{L}_{TL} & \hat{A}_{TR} \\             \hat{L}_{BL} & \hat{A}_{BR} - L_{BL}L_{BL}^{T}       \end{pmatrix} \wedge \begin{pmatrix}             L_{TL}L_{TL}^{T} \\             L_{BL}L_{TL}^{T}       \end{pmatrix} = \begin{pmatrix}             \hat{A}_{TL} \\             \hat{A}_{BL}       \end{pmatrix} \wedge \neg (m(A_{TL}) < \frac{1}{A_{BL}} \\             M(A)) $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |      | where $A_{11}$ is $b \times b$ $ \begin{pmatrix} A_{12} & A_{22} & A_{23} \\ A_{23} & A_{23} \end{pmatrix} \begin{pmatrix} I_{23} & \widehat{A}_{23} \\ \widehat{A}_{23} & \widehat{A}_{23} \end{pmatrix} \begin{pmatrix} I_{23} I^T \\ \widehat{A}_{23} \end{pmatrix} $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |  |
| $ \begin{cases} A_{00} \ a_{01} \ A_{02} \\ a_{10}^{T} \ \alpha_{11} \ a_{12}^{T} \\ A_{20} \ a_{21} \ A_{22} \end{cases} = \begin{pmatrix} L_{00} \ \hat{a}_{01} & \hat{A}_{02} \\ l_{10}^{T} \ \lambda_{11} & \hat{a}_{12}^{T} \\ L_{20} \ l_{21} \ A_{22} - L_{20} L_{20}^{T} - l_{21} l_{21}^{T} \end{pmatrix} \land \begin{pmatrix} L_{00} L_{00}^{T} \\ l_{10}^{T} L_{00}^{T} & l_{10}^{T} l_{10} + \lambda_{11}^{2} \\ L_{20} l_{21} & A_{22} - L_{20} L_{20}^{T} - l_{21} l_{21}^{T} \end{pmatrix} \land \begin{pmatrix} L_{00} L_{00}^{T} & l_{10}^{T} l_{10} + \lambda_{11}^{2} \\ L_{20} l_{21} & A_{22} - L_{20} L_{20}^{T} - l_{21} l_{21}^{T} \end{pmatrix} $ $ 5b \qquad \begin{pmatrix} A_{TL} A_{TR} \\ A_{BL} A_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} A_{00} A_{01} & A_{02} \\ A_{10} A_{11} & A_{12} \\ A_{20} A_{21} & A_{22} \end{pmatrix} $ $ 2 \qquad \begin{pmatrix} A_{TL} A_{TR} \\ A_{BL} A_{BR} \end{pmatrix} = \begin{pmatrix} \hat{L}_{TL} & \hat{A}_{TR} \\ \hat{L}_{BL} & \hat{A}_{BR} - L_{BL} L_{BL}^{T} \end{pmatrix} \land \begin{pmatrix} L_{TL} L_{TL}^{T} \\ L_{BL} L_{TL}^{T} \end{pmatrix} = \begin{pmatrix} \hat{A}_{TL} \\ \hat{A}_{BL} \end{pmatrix} $ $ endwhile $ $ 2,3 \qquad \begin{cases} \begin{pmatrix} A_{TL} A_{TR} \\ A_{BL} A_{BR} \end{pmatrix} = \begin{pmatrix} \hat{L}_{TL} & \hat{A}_{TR} \\ \hat{L}_{BL} & \hat{A}_{BR} - L_{BL} L_{BL}^{T} \end{pmatrix} \land \begin{pmatrix} L_{TL} L_{TL}^{T} \\ L_{BL} L_{TL}^{T} \end{pmatrix} = \begin{pmatrix} \hat{A}_{TL} \\ \hat{A}_{BL} \end{pmatrix} \land \neg (m(A_{TL}) < \\ M(A)) $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 6    | $ \left\{ \begin{array}{c} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} B_{00} & a_{01} & A_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} - L_{20} l_{10} & A_{22} - L_{20} L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} B_{00} L_{00} \\ l_{10}^T L_{00}^T \\ L_{20} L_{00}^T \end{pmatrix} = \begin{pmatrix} A_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} \right\} $                                                                                                                                                                                                                                                                                           |  |
| 5b $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$ $2 \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right)$ endwhile $2,3 \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \neg (m(A_{TL}) < \left(\begin{array}{c c} A_{TL} & \widehat{A}_{TR} \\ \hline A_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |  |
| 5b $\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$ $2 \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right)$ endwhile $2,3 \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right) \wedge \left(\begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array}\right) = \left(\begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array}\right) \wedge \neg (m(A_{TL}) < \left(\begin{array}{c c} A_{TL} & \widehat{A}_{TR} \\ \hline A_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array}\right)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 7    | $ \begin{cases} A_{00} \ a_{01} \ A_{02} \\ a_{10}^T \ \alpha_{11} \ a_{12}^T \\ A_{20} \ a_{21} \ A_{22} \end{cases} =  \begin{pmatrix} L_{00} \ \hat{a}_{01} & \hat{A}_{02} \\ l_{10}^T \ \lambda_{11} & \hat{a}_{12}^T \\ L_{20} \ l_{21} \ A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \land $ $ \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T \lambda_{11} & \hat{a}_{12}^T \\ L_{20} \ l_{21} \ A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \Rightarrow $ $ \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T \lambda_{11} & \hat{a}_{12}^T \\ L_{20} \ l_{21} \ A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \Rightarrow $ $ \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T \lambda_{11} & \hat{a}_{12}^T \\ L_{20} \ l_{21} \ A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} $ |  |
| endwhile $2,3  \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < ) \right\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 5b   | $\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |  |
| $ \begin{array}{c c} 2,3 & \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \left( \frac{\widehat{A}_{TL}}{\widehat{A}_{BL}} \right) \wedge \neg (m(A_{TL}) < \right\} \end{array} $                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 2    | $ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) $                                                                                                                                                                                                                                                                                                                                                          |  |
| (m(A))                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |      | endwhile                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |  |
| $1b  \left\{ A = \text{Chol}(\widehat{A}) \right\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 2,3  | $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge \neg (m(A_{TL}) < \right\} $                                                                                                                                                                                                                                                                                                                                                                                                                                       |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 1b   | $A = \operatorname{Chol}(\widehat{A})$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |  |

| Step | Algorithm: $[A] := \text{Chol_blk_var}(A)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1a   | $\{A = \widehat{A}\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|      | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| 2    | $ \left\{ \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) $                                                                                                                                                                                    |
| 3    | while $m(A_{TL}) < m(A)$ do                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 2,3  | $ \left\{ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} = \begin{pmatrix} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{pmatrix} \wedge \begin{pmatrix} L_{TL}L_{TL}^T \\ L_{BL}L_{TL}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{TL} \\ \widehat{A}_{BL} \end{pmatrix} \wedge m(A_{TL}) < \right\} $                                                                                                                                                                                                                                                    |
| 5a   | Determine block size $b$ $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix} $ where $A_{11}$ is $b \times b$                                                                                                                                                                                                                                                                                                                                                                            |
| 6    | $ \left\{  \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T & \alpha_{11} - l_{10}^T l_{10} & \widehat{a}_{12}^T \\ L_{20} & a_{21} - L_{20} l_{10} & A_{22} - L_{20} L_{20}^T \end{pmatrix} \wedge \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}^T \\ L_{20} L_{00}^T \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \\ \widehat{A}_{20} \end{pmatrix} \right\} $                                                                                  |
| 8    | $lpha_{11}:=\sqrt{lpha_{11}}$ $a_{21}:=a_{21}/lpha_{11}$ $A_{22}:=A_{22}-a_{21}a_{21}^T  \text{update only lower triangular part}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 7    | $\begin{cases} A_{00} \ a_{01} \ A_{02} \\ a_{10}^T \ \alpha_{11} \ a_{12}^T \\ A_{20} \ a_{21} \ A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} \ \widehat{a}_{01} & \widehat{A}_{02} \\ l_{10}^T \ \lambda_{11} & \widehat{a}_{12}^T \\ L_{20} \ l_{21} \ A_{22} - L_{20} L_{20}^T - l_{21} l_{21}^T \end{pmatrix} \land \\ \begin{pmatrix} L_{00} L_{00}^T \\ l_{10}^T L_{00}^T & l_{10}^T l_{10} + \lambda_{11}^2 \\ L_{20} L_{20}^T \ L_{20} l_{10} + \lambda_{11} l_{21} \end{pmatrix} = \begin{pmatrix} \widehat{A}_{00} \\ \widehat{a}_{10}^T \ \widehat{\alpha}_{11} \\ \widehat{A}_{20} \ \widehat{a}_{21} \end{pmatrix}$ |
| 5b   | $\left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right)$                                                                                                                                                                                                                                                                                                                                                                                    |
| 2    | $ \left\{ \begin{array}{c c} \left( \begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) = \left( \begin{array}{c c} \widehat{L}_{TL} & \widehat{A}_{TR} \\ \hline \widehat{L}_{BL} & \widehat{A}_{BR} - L_{BL}L_{BL}^T \end{array} \right) \wedge \left( \begin{array}{c c} L_{TL}L_{TL}^T \\ \hline L_{BL}L_{TL}^T \end{array} \right) = \left( \begin{array}{c c} \widehat{A}_{TL} \\ \hline \widehat{A}_{BL} \end{array} \right) $                                                                                                                                                                 |
|      | endwhile                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| 2,3  | $ \left\{ \left( \frac{A_{TL}   A_{TR}}{A_{BL}   A_{BR}} \right) = \left( \frac{\widehat{L}_{TL}   \widehat{A}_{TR}}{\widehat{L}_{BL}   \widehat{A}_{BR} - L_{BL}L_{BL}^T} \right) \wedge \left( \frac{L_{TL}L_{TL}^T}{L_{BL}L_{TL}^T} \right) = \left( \frac{\widehat{A}_{TL}}{\widehat{A}_{BL}} \right) \wedge \neg (m(A_{TL}) < \right\} $                                                                                                                                                                                                                                                                                      |
| 1b   | $\left\{ A = \operatorname{Chol}(\widehat{A}) \right\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |

| Algorithm: $[A] := CHOL_BLK_VAR3(A)$                                                                                                                                                                                                                                                              |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                                                                                                                                                                                                                                                                   |
| $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}$ where $A_{TL}$ is $0 \times 0$                                                                                                                                                                                           |
| while $m(A_{TL}) < m(A)$ do                                                                                                                                                                                                                                                                       |
|                                                                                                                                                                                                                                                                                                   |
| Determine block size $b$ $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{array}\right) $ where $A_{11}$ is $b \times b$ |
|                                                                                                                                                                                                                                                                                                   |
| $lpha_{11}:=\sqrt{lpha_{11}}$ $a_{21}:=a_{21}/lpha_{11}$ $A_{22}:=A_{22}-a_{21}a_{21}^T  \text{update only lower triangular part}$                                                                                                                                                                |
|                                                                                                                                                                                                                                                                                                   |
| $ \left(\begin{array}{c c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array}\right) $                                                      |
|                                                                                                                                                                                                                                                                                                   |
| endwhile                                                                                                                                                                                                                                                                                          |
|                                                                                                                                                                                                                                                                                                   |
|                                                                                                                                                                                                                                                                                                   |

## Algorithm: $[A] := CHOL_BLK_VAR3(A)$

$$A \to \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$$

where  $A_{TL}$  is  $0 \times 0$ 

while  $m(A_{TL}) < m(A)$  do

Determine block size b

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \to \left(\begin{array}{c|c}
A_{00} & A_{01} & A_{02} \\
\hline
A_{10} & A_{11} & A_{12} \\
A_{20} & A_{21} & A_{22}
\end{array}\right)$$

where  $A_{11}$  is  $b \times b$ 

$$\alpha_{11} := \sqrt{\alpha_{11}}$$

$$a_{21} := a_{21}/\alpha_{11}$$

 $A_{22} := A_{22} - a_{21}a_{21}^T$  update only lower triangular part

$$\left(\begin{array}{c|c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c|c}
A_{00} & A_{01} & A_{02} \\
A_{10} & A_{11} & A_{12} \\
\hline
A_{20} & A_{21} & A_{22}
\end{array}\right)$$

endwhile