Step Algorithm: Solve $Ux = y$ overwriting y with x . U is upper triangular. 1a $\{y = \widehat{y}\}$ 4 $U \rightarrow \left(\frac{U_{TL}}{U_{BL}} \frac{U_{TR}}{U_{BR}}\right)$, $x \rightarrow \left(\frac{x_T}{x_B}\right)$, $y \rightarrow \left(\frac{y_T}{y_B}\right)$ where U_{BR} is 0×0 , x_B has 0 rows, y_B has 0 rows 2 $\left\{\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y_T}}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B$ 3 while $m(U_{BR}) < m(U)$ do 2,3 $\left\{\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y_T}}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U)$ 5a $\left(\frac{U_{TL}}{U_{BL}} U_{TR}\right) \rightarrow \left(\frac{U_{00}}{u_{10}} u_{12}\right) + \left(\frac{x_T}{u_{12}}\right) + \left(\frac{x_T}{x_B}\right) \rightarrow \left(\frac{x_1}{x_2}\right) + \left(\frac{y_T}{y_B}\right) \rightarrow \left(\frac{y_0}{y_1}\right) + \left(\frac{y_0}{y_2}\right)$ where v_{11} is 1×1 , x_1 has 1 row, ψ_1 has 1 row 6 $\left\{\left(\frac{y_0}{\psi_1}\right) = \left(\widehat{\psi_0}\right) \wedge U_{22}x_2 = \widehat{y}_2$ 8 $\psi_1 := \chi_1 = (\widehat{\psi_1} - u_{12}^Tx_2)/v_{11} = (\psi_1 - u_{12}^Ty_2)/v_{11}$ 7 $\left\{\left(\frac{y_0}{\psi_1}\right) = \left(\widehat{\psi_0}\right) \wedge V_{11}\chi_1 + u_{12}^Tx_2 = \widehat{\psi_1} + U_{22}x_2 = \widehat{y}_2$ 5b $\left(\frac{U_{TL}}{U_{BL}} U_{TR}\right) \leftarrow \left(\frac{U_{00}}{u_{10}} u_{01} U_{02} + U_{02} u_{21} U_{22}\right) + \left(\frac{x_T}{x_B}\right) \leftarrow \left(\frac{x_0}{\chi_1}\right) + \left(\frac{y_T}{y_B}\right) \leftarrow \left(\frac{y_0}{\psi_1}\right)$ 2 $\left\{\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y_T}}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B$ endwhile 2,3 $\left\{\left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y_T}}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg (m(U_{BR}) < m(U))$ 1b $\{y = x \wedge Ux = \widehat{y}\}$	~	
$ \begin{array}{ll} 4 & U \rightarrow \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}, x \rightarrow \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \rightarrow \begin{pmatrix} y_T \\ y_B \end{pmatrix} \\ & \text{where } U_{BR} \text{ is } 0 \times 0, x_B \text{ has } 0 \text{ rows}, y_B \text{ has } 0 \text{ rows} \\ 2 & \left\{ \begin{pmatrix} \frac{y_T}{y_B} \end{pmatrix} = \begin{pmatrix} \hat{y}_T \\ x_B \end{pmatrix} \wedge U_{BR} x_B = \hat{y}_B \\ 3 & \text{while } m(U_{BR}) < m(U) \text{ do} \\ 2.3 & \left\{ \begin{pmatrix} \frac{y_T}{y_B} \end{pmatrix} = \begin{pmatrix} \hat{y}_T \\ x_B \end{pmatrix} \wedge U_{BR} x_B = \hat{y}_B \wedge m(U_{BR}) < m(U) \\ \frac{U_{TL}}{U_{BL}} & U_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{pmatrix}, \begin{pmatrix} x_T \\ x_B \end{pmatrix} \rightarrow \begin{pmatrix} x_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}, \begin{pmatrix} y_T \\ y_B \end{pmatrix} \rightarrow \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} \\ & \text{where } v_{11} \text{ is } 1 \times 1, \chi_1 \text{ has } 1 \text{ row}, \psi_1 \text{ has } 1 \text{ row} \\ & \left\{ \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \hat{y}_0 \\ \hat{\psi}_1 \\ \chi_2 \end{pmatrix} \wedge U_{22} x_2 = \hat{y}_2 \\ & \\ 8 & \psi_1 := \chi_1 = (\hat{\psi}_1 - u_{12}^T x_2)/v_{11} = (\psi_1 - u_{12}^T y_2)/v_{11} \\ & V_{22} x_2 = \hat{y}_2 \\ & \\ 5b & \begin{pmatrix} U_{TL} & U_{TR} \\ V_{BL} & U_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} U_{00} & u_{01} & U_{02} \\ u_{10} & v_{11} & v_{12}^T \\ v_{20} & u_{21} & U_{22} \end{pmatrix}, \begin{pmatrix} x_T \\ x_T \end{pmatrix} \leftarrow \begin{pmatrix} x_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}, \begin{pmatrix} y_T \\ y_B \end{pmatrix} \leftarrow \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} \\ & \\ 2 & \begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} \hat{y}_T \\ x_B \end{pmatrix} \wedge U_{BR} x_B = \hat{y}_B \\ & \text{endwhile} \\ & \\ 2.3 & \begin{cases} \left(\frac{y_T}{y_B} \right) = \begin{pmatrix} \hat{y}_T \\ x_B \end{pmatrix} \wedge U_{BR} x_B = \hat{y}_B \wedge \neg (m(U_{BR}) < m(U)) \\ & \\ \end{array} \right. \end{array}$	Step	Algorithm: Solve $Ux = y$ overwriting y with x . U is upper triangular.
$ \begin{cases} \left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B \\ 3 & \text{while } m(U_{BR}) < m(U) \text{ do} \end{cases} $ $ 2.3 \left\{ \left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right. $ $ 5a \left(\frac{U_{TL}}{U_{BL}} \frac{U_{TR}}{U_{BR}}\right) \rightarrow \left(\frac{U_{00}}{u_{01}} \frac{U_{02}}{u_{12}}\right) \wedge \left(\frac{x_T}{x_B}\right) \rightarrow \left(\frac{x_0}{x_1}\right) \wedge \left(\frac{y_T}{y_B}\right) \rightarrow \left(\frac{y_0}{y_1}\right) \\ \text{where } v_{11} \text{ is } 1 \times 1, \chi_1 \text{ has } 1 \text{ row}, \ \psi_1 \text{ has } 1 \text{ row} \end{cases} $ $ 6 \left\{ \left(\frac{y_0}{y_1}\right) = \left(\frac{\widehat{y}_0}{\widehat{y}_1}\right) \wedge U_{22}x_2 = \widehat{y}_2 \\ 8 \psi_1 := \chi_1 = (\widehat{\psi}_1 - u_{12}^Tx_2)/v_{11} = (\psi_1 - u_{12}^Ty_2)/v_{11} \\ 7 \left\{ \left(\frac{y_0}{y_1}\right) = \left(\frac{\widehat{y}_0}{y_1}\right) \wedge v_{11}\chi_1 + u_{12}^Tx_2 = \widehat{\psi}_1 \\ U_{22}x_2 = \widehat{y}_2 \\ \end{cases} \right\} $ $ 5b \left(\frac{U_{TL}}{U_{BL}} \frac{U_{TR}}{U_{BR}}\right) \leftarrow \left(\frac{U_{00}}{u_{10}} \frac{u_{10}}{v_{11}} \frac{U_{02}}{u_{12}^T} \\ U_{20} \frac{v_{21}}{v_{21}} \frac{v_{22}}{v_{22}} \right), \left(\frac{x_T}{x_B}\right) \leftarrow \left(\frac{x_0}{\chi_1}\right), \left(\frac{y_T}{y_B}\right) \leftarrow \left(\frac{y_0}{\psi_1}\right) \\ 2 \left\{ \left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg(m(U_{BR}) < m(U)) \\ \end{cases} $ $ 2.3 \left\{ \left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{x_B}\right) \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg(m(U_{BR}) < m(U)) \right\} $	1a	$\{y = \widehat{y} \}$
3 while $m(U_{BR}) < m(U)$ do 2,3 $\left\{\begin{array}{c} \left(\frac{y_T}{y_B}\right) = \left(\frac{\widehat{y}_T}{x_B}\right) \land U_{BR}x_B = \widehat{y}_B \land m(U_{BR}) < m(U) \\ \end{array}\right.$ 5a $\left(\begin{array}{c} \left(\frac{U_{TL}}{U_{BR}}\right) U_{TR} \\ U_{BL} \mid U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c} \left(\frac{U_{00}}{u_{10}} u_{11} \mid u_{12}^T \\ u_{20} \mid u_{21} \mid U_{22} \end{array}\right), \left(\frac{x_T}{x_B}\right) \rightarrow \left(\frac{x_0}{x_1}\right), \left(\frac{y_T}{y_B}\right) \rightarrow \left(\frac{y_0}{y_1}\right) \\ \frac{v_1}{y_2} \mid v_1 \mid v_1 \mid v_1 \mid v_2 \mid v_2 \mid v_3 \mid v_4 \mid $	4	
	2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
$5a \qquad \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{pmatrix}, \begin{pmatrix} x_T \\ x_B \end{pmatrix} \rightarrow \begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_T \\ y_B \end{pmatrix} \rightarrow \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix}$ $where v_{11} \text{ is } 1 \times 1, \chi_1 \text{ has } 1 \text{ row}, \psi_1 \text{ has } 1 \text{ row}$ $6 \qquad \begin{cases} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 \\ \widehat{\psi}_1 \\ x_2 \end{pmatrix} \wedge U_{22}x_2 = \widehat{y}_2 \\ \chi_2 \end{pmatrix} \wedge U_{21}x_1 = (\widehat{\psi}_1 - u_{12}^T y_2)/v_{11}$ $7 \qquad \begin{cases} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} \wedge v_{11}\chi_1 + u_{12}^T x_2 = \widehat{\psi}_1 \\ U_{22}x_2 = \widehat{y}_2 \end{pmatrix}$ $5b \qquad \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{pmatrix}, \begin{pmatrix} x_T \\ x_B \end{pmatrix} \leftarrow \begin{pmatrix} x_0 \\ \chi_1 \\ \chi_2 \end{pmatrix}, \begin{pmatrix} y_T \\ y_B \end{pmatrix} \leftarrow \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix}$ $2 \qquad \begin{cases} \begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} \widehat{y}_T \\ x_B \end{pmatrix} \wedge U_{BR}x_B = \widehat{y}_B \\ \text{endwhile} \end{cases}$ $2,3 \qquad \begin{cases} \begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} \widehat{y}_T \\ x_B \end{pmatrix} \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg (m(U_{BR}) < m(U)) \end{cases}$	3	while $m(U_{BR}) < m(U)$ do
where v_{11} is 1×1 , χ_{1} has 1 row, ψ_{1} has 1 row $\begin{cases} y_{0} \\ \psi_{1} \\ y_{2} \end{cases} = \begin{pmatrix} \widehat{y}_{0} \\ \widehat{\psi}_{1} \\ x_{2} \end{pmatrix} \wedge U_{22}x_{2} = \widehat{y}_{2} \\ 8 \psi_{1} := \chi_{1} = (\widehat{\psi}_{1} - u_{12}^{T}x_{2})/v_{11} = (\psi_{1} - u_{12}^{T}y_{2})/v_{11} \\ 7 \begin{cases} y_{0} \\ \psi_{1} \\ y_{2} \end{cases} = \begin{pmatrix} \widehat{y}_{0} \\ \chi_{1} \\ \chi_{2} \end{cases} \wedge v_{11}\chi_{1} + u_{12}^{T}x_{2} = \widehat{\psi}_{1} \\ U_{22}x_{2} = \widehat{y}_{2} \end{cases}$ $5b \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} U_{00} & u_{01} & U_{02} \\ u_{10}^{T} & v_{11} & u_{12}^{T} \\ U_{20} & u_{21} & U_{22} \end{pmatrix}, \begin{pmatrix} x_{T} \\ x_{B} \end{pmatrix} \leftarrow \begin{pmatrix} x_{0} \\ \chi_{1} \\ \chi_{2} \end{pmatrix}, \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} \leftarrow \begin{pmatrix} y_{0} \\ \psi_{1} \\ y_{2} \end{pmatrix}$ $2 \begin{cases} (y_{T} \\ y_{B}) = (\widehat{y}_{T} \\ x_{B}) \wedge U_{BR}x_{B} = \widehat{y}_{B} \end{cases}$ endwhile $2.3 \begin{cases} (y_{T} \\ y_{B}) = (\widehat{y}_{T} \\ x_{B}) \wedge U_{BR}x_{B} = \widehat{y}_{B} \wedge \neg (m(U_{BR}) < m(U))$	2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
$ \begin{cases} \begin{cases} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 \\ \widehat{\psi}_1 \\ x_2 \end{pmatrix} \wedge U_{22}x_2 = \widehat{y}_2 \\ \end{cases} \\ 8 \qquad \psi_1 := \chi_1 = (\widehat{\psi}_1 - u_{12}^T x_2)/v_{11} = (\psi_1 - u_{12}^T y_2)/v_{11} \\ 7 \qquad \begin{cases} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 \\ \chi_1 \\ x_2 \end{pmatrix} \wedge \begin{pmatrix} v_{11}\chi_1 + u_{12}^T x_2 = \widehat{\psi}_1 \\ U_{22}x_2 = \widehat{y}_2 \end{cases} \end{cases} \\ 5b \qquad \begin{pmatrix} \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} \begin{pmatrix} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{pmatrix}, \begin{pmatrix} \frac{x_T}{x_B} \end{pmatrix} \leftarrow \begin{pmatrix} \frac{x_0}{\chi_1} \\ \chi_2 \end{pmatrix}, \begin{pmatrix} \frac{y_T}{y_B} \end{pmatrix} \leftarrow \begin{pmatrix} \frac{y_0}{\psi_1} \\ y_2 \end{pmatrix} \end{cases} \\ 2 \qquad \begin{cases} \begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} \hat{y}_T \\ x_B \end{pmatrix} \wedge U_{BR}x_B = \widehat{y}_B \end{cases} \\ \text{endwhile} \\ 2,3 \qquad \begin{cases} \begin{pmatrix} \frac{y_T}{y_B} \end{pmatrix} = \begin{pmatrix} \hat{y}_T \\ x_B \end{pmatrix} \wedge U_{BR}x_B = \widehat{y}_B \wedge \neg (m(U_{BR}) < m(U)) \end{cases} \end{cases} $	5a	
$ \begin{cases} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 \\ \chi_1 \\ x_2 \end{pmatrix} \land v_{11}\chi_1 + u_{12}^T x_2 = \widehat{\psi}_1 \\ U_{22}x_2 = \widehat{y}_2 \end{cases} $ $ \begin{cases} \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{pmatrix}, \begin{pmatrix} x_T \\ x_B \end{pmatrix} \leftarrow \begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_T \\ y_B \end{pmatrix} \leftarrow \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} $ $ \begin{cases} \begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} \widehat{y}_T \\ x_B \end{pmatrix} \land U_{BR}x_B = \widehat{y}_B \end{cases} $ endwhile $ 2,3 \begin{cases} \begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} \widehat{y}_T \\ x_B \end{pmatrix} \land U_{BR}x_B = \widehat{y}_B \land \neg (m(U_{BR}) < m(U)) \end{cases} $	6	$(y_0 \setminus \widehat{y_0})$
$ \begin{cases} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 \\ \chi_1 \\ x_2 \end{pmatrix} \land v_{11}\chi_1 + u_{12}^T x_2 = \widehat{\psi}_1 \\ U_{22}x_2 = \widehat{y}_2 \end{cases} $ $ \begin{cases} \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{pmatrix}, \begin{pmatrix} x_T \\ x_B \end{pmatrix} \leftarrow \begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_T \\ y_B \end{pmatrix} \leftarrow \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} \end{cases} $ $ \begin{cases} \begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} \widehat{y}_T \\ x_B \end{pmatrix} \land U_{BR}x_B = \widehat{y}_B \land \neg(m(U_{BR}) < m(U)) \end{cases} $ $ \begin{cases} \begin{pmatrix} y_T \\ y_B \end{pmatrix} = \begin{pmatrix} \widehat{y}_T \\ x_B \end{pmatrix} \land U_{BR}x_B = \widehat{y}_B \land \neg(m(U_{BR}) < m(U)) \end{cases} $	8	$\psi_1 := \chi_1 = (\widehat{\psi}_1 - u_{12}^T x_2) / v_{11} = (\psi_1 - u_{12}^T y_2) / v_{11}$
5b $ \left(\frac{U_{TL}}{U_{BL}} \middle U_{TR} \middle U_{TR} \middle U_{DR} \middle V_{11} \middle U_{12} \middle U_{12} \middle U_{12} \middle U_{22} \middle V_{13} \middle U_{22} \middle V_{22} \middle V_{23} \middle V_{24} \middle U_{24} \middle V_{25} \middle V_{25}$	7	
endwhile $2,3 \left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \land U_{BR} x_B = \widehat{y}_B \land \neg (m(U_{BR}) < m(U)) \right\}$	5b	$ \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $
2,3 $\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \land U_{BR} x_B = \widehat{y}_B \land \neg (m(U_{BR}) < m(U)) \right\}$	2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
$(\setminus y_B) \setminus x_B)$		endwhile
$1b \{y = x \land Ux = \widehat{y} $	2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \land U_{BR} x_B = \widehat{y}_B \land \neg (m(U_{BR}) < m(U)) \right\}$
	1b	$\{y = x \land Ux = \widehat{y} \}$

Step	Algorithm: Solve $Ux = y$ overwriting y with x . U is upper triangular.
1a	{
4	where
2	
3	while do
2,3	
5a	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\left\{ \begin{array}{c} \wedge \neg (\end{array} \right.)$
1b	{

Step	Algorithm: Solve $Ux = y$ over	erwriting y with x . U is upper triangu	lar.
1a	$\{y=\widehat{y}$		}
4	where		
2			
3	while do		
2,3		\wedge	
5a	where		
6			
8			
7			
5b			
2			
	endwhile		
2,3		$\wedge \neg (\hspace{1cm})$	
1b	$\{y = x \land Ux = \widehat{y}$		}

Step	Algorithm: Solve $Ux = y$ overwriting y with x . U is upper triangular.
1a	$\{y = \widehat{y} $
4	where
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
3	while do
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \wedge \right\}$
5a	where
6	
8	
7	
5b	
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \land U_{BR} x_B = \widehat{y}_B \land \neg () \right\}$
1b	$\{y = x \land Ux = \widehat{y} $ }

Step	Algorithm: Solve $Ux = y$ overwriting y with x . U is upper triangular.
1a	$ \{y = \widehat{y} \} $
4	where
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	where
6	
8	
7	
5b	
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \land U_{BR} x_B = \widehat{y}_B \land \neg (m(U_{BR}) < m(U)) \right\}$
1b	$\{y = x \land Ux = \widehat{y} \}$

	live $Ux = y$ overwriting y with x . U is upper triangular.
$1a \{y = \widehat{y} \}$	}
$ \begin{array}{c c} 4 & U \to \left(\frac{U_{TL}}{U_{BL}}\right) \\ & \text{where } U_{BR} \end{array} $	$\left(\frac{U_{TR}}{U_{BR}}\right), x \to \left(\frac{x_T}{x_B}\right), y \to \left(\frac{y_T}{y_B}\right)$ is $0 \times 0, x_B$ has 0 rows, y_B has 0 rows
$2 \qquad \left\{ \left(\frac{y_T}{y_B} \right) = 1 \right\}$	$\left(\frac{\widehat{y}_T}{x_B}\right) \wedge U_{BR} x_B = \widehat{y}_B$
3 while $m(U_{BR})$	< m(U) do
$ 2,3 \left\{ \left(\frac{y_T}{y_B}\right) \right. $	$= \left(\frac{\widehat{y}_T}{x_B}\right) \wedge U_{BR} x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U)$
5a	
6 {	
8	
7 {	
5b	
$2 \qquad \left\{ \qquad \left(\frac{y_T}{y_B} \right) \right.$	$= \left(\frac{\widehat{y}_T}{x_B}\right) \wedge U_{BR} x_B = \widehat{y}_B$
endwhile	
$2,3 \left\{ \left(\frac{y_T}{y_B} \right) = 1 \right\}$	$\left(\frac{\widehat{y}_T}{x_B}\right) \wedge U_{BR} x_B = \widehat{y}_B \wedge \neg (m(U_{BR}) < m(U))$
$1b \{y = x \land Ux = 0\}$	$=\widehat{y}$

Step	Algorithm: Solve $Ux = y$ overwriting y with x . U is upper triangular.
1a	$\{y=\widehat{y}$
4	$U \to \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right), x \to \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where U_{BR} is 0×0 , x_B has 0 rows, y_B has 0 rows
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	$ \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{pmatrix}, \begin{pmatrix} x_T \\ x_B \end{pmatrix} \rightarrow \begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_T \\ y_B \end{pmatrix} \rightarrow \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} $ where v_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
6	
8	
7	
5b	$ \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right) $
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \land U_{BR} x_B = \widehat{y}_B \land \neg (m(U_{BR}) < m(U)) \right\}$
1b	$\{y = x \land Ux = \widehat{y} $ }

Step	Algorithm: Solve $Ux = y$ overwriting y with x . U is upper triangular.	
1a	$\{y = \widehat{y}\}$	}
4	$U \to \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right), x \to \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where U_{BR} is 0×0 , x_B has 0 rows, y_B has 0 rows	
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \land U_{BR} x_B = \widehat{y}_B \right.$	
3	while $m(U_{BR}) < m(U)$ do	
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$	
5a	$ \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \to \left(\begin{array}{c c c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right) , \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \to \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array}\right) , \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \to \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array}\right) $	
6	where v_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row $\begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 \\ \widehat{\psi}_1 \\ x_2 \end{pmatrix} \wedge U_{22}x_2 = \widehat{y}_2$	
8		
7		$\left. \begin{array}{c} \\ \end{array} \right\}$
5b	$ \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c c c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right) $	
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right.$	
	endwhile	
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \land U_{BR} x_B = \widehat{y}_B \land \neg (m(U_{BR}) < m(U)) \right\}$	
1b	$\{y = x \land Ux = \widehat{y}$	}

Step	Algorithm: Solve $Ux = y$ overwriting y with x . U is upper triangular.
1a	$\{y = \widehat{y} $
4	$U \to \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right), x \to \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where U_{BR} is 0×0 , x_B has 0 rows, y_B has 0 rows
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	$ \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $ where v_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
6	$ \left\{ \begin{array}{c} \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array} \right) = \left(\begin{array}{c} \widehat{y}_0 \\ \widehat{\psi}_1 \\ x_2 \end{array} \right) \wedge U_{22} x_2 = \widehat{y}_2 \\ \end{array} \right\} $
8	
7	$ \left\{ \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 \\ \chi_1 \\ x_2 \end{pmatrix} \land \begin{array}{l} \upsilon_{11}\chi_1 + u_{12}^T x_2 = \widehat{\psi}_1 \\ U_{22}x_2 = \widehat{y}_2 \end{array} \right\} $
5b	$ \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c c c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \land U_{BR} x_B = \widehat{y}_B \land \neg (m(U_{BR}) < m(U)) \right\}$
1b	$\{y = x \land Ux = \widehat{y} $ }

Step	Algorithm: Solve $Ux = y$ overwriting y with x . U is upper triangular.
1a	$\{y = \widehat{y} $
4	$U \to \begin{pmatrix} U_{TL} & U_{TR} \\ U_{BL} & U_{BR} \end{pmatrix}, x \to \begin{pmatrix} x_T \\ x_B \end{pmatrix}, y \to \begin{pmatrix} y_T \\ y_B \end{pmatrix}$ where U_{BR} is 0×0 , x_B has 0 rows, y_B has 0 rows
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
3	while $m(U_{BR}) < m(U)$ do
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \wedge m(U_{BR}) < m(U) \right\}$
5a	$ \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array}\right) $ where v_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
6	$ \left\{ \begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array} \right) = \begin{pmatrix} \widehat{y}_0 \\ \widehat{\psi}_1 \\ x_2 \end{pmatrix} \land U_{22}x_2 = \widehat{y}_2 \\ $
8	$\psi_1 := \chi_1 = (\widehat{\psi}_1 - u_{12}^T x_2) / v_{11} = (\psi_1 - u_{12}^T y_2) / v_{11}$
7	$ \left\{ \begin{array}{c} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{y}_0 \\ \chi_1 \\ x_2 \end{pmatrix} \wedge \begin{array}{c} \upsilon_{11}\chi_1 + u_{12}^T x_2 = \widehat{\psi}_1 \\ U_{22}x_2 = \widehat{y}_2 \end{array} \right. $
5b	$ \left(\begin{array}{c c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c c c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right) $
2	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \wedge U_{BR} x_B = \widehat{y}_B \right\}$
	endwhile
2,3	$\left\{ \left(\frac{y_T}{y_B} \right) = \left(\frac{\widehat{y}_T}{x_B} \right) \land U_{BR} x_B = \widehat{y}_B \land \neg (m(U_{BR}) < m(U)) \right\}$
1b	$\{y = x \land Ux = \widehat{y} $ }

Algorithm: Solve $Ux = y$ overwriting y with x . U is upper triangular.
$U \to \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right), x \to \left(\begin{array}{c c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c c} y_T \\ \hline y_B \end{array}\right)$ where U_{BR} is 0×0 , x_B has 0 rows, y_B has 0 rows
while $m(U_{BR}) < m(U)$ do
$ \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \to \left(\begin{array}{c c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \to \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \to \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $ where v_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
$\psi_1 := \chi_1 = (\widehat{\psi}_1 - u_{12}^T x_2) / v_{11} = (\psi_1 - u_{12}^T y_2) / v_{11}$
$ \left(\begin{array}{c c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} U_{00} & u_{01} & U_{02} \\ \hline u_{10}^T & v_{11} & u_{12}^T \\ U_{20} & u_{21} & U_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array}\right) $
endwhile

Algorithm: Solve Ux = y overwriting y with x. U is upper triangular.

$$U \to \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right), x \to \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$$

where U_{BR} is 0×0 , x_B has 0 rows, y_B has 0 rows

while $m(U_{BR}) < m(U)$ do

$$\left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \to \left(\begin{array}{c|c} U_{00} & u_{01} & U_{02} \\ u_{10}^T & v_{11} & u_{12}^T \\ \hline U_{20} & u_{21} & U_{22} \end{array}\right) , \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \to \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array}\right) , \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \to \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array}\right)$$

where v_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row

$$\psi_1 := \chi_1 = (\widehat{\psi}_1 - u_{12}^T x_2) / v_{11} = (\psi_1 - u_{12}^T y_2) / v_{11}$$

$$\psi_{1} := \chi_{1} = (\widehat{\psi}_{1} - u_{12}^{T} x_{2}) / v_{11} = (\psi_{1} - u_{12}^{T} y_{2}) / v_{11}
\left(\frac{U_{TL} | U_{TR}}{U_{BL} | U_{BR}}\right) \leftarrow \begin{pmatrix} U_{00} | u_{01} | U_{02} \\ u_{10}^{T} | v_{11} | u_{12}^{T} \\ U_{20} | u_{21} | U_{22} \end{pmatrix}, \begin{pmatrix} x_{T} \\ x_{B} \end{pmatrix} \leftarrow \begin{pmatrix} x_{0} \\ \chi_{1} \\ x_{2} \end{pmatrix}, \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} \leftarrow \begin{pmatrix} y_{0} \\ \psi_{1} \\ y_{2} \end{pmatrix}$$

endwhile