Step Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.  1a $\{y = \hat{y}\}$ $L \to \left(\frac{L_{TL}}{L_{BR}}\right), x \to \left(\frac{x_T}{x_B}\right), y \to \left(\frac{y_T}{y_B}\right)$ where $L_{TL}$ is $0 \times 0$ , $x_T$ has $0$ rows, $y_T$ has $0$ rows  2 $\left\{\left(\frac{y_T}{y_B}\right) = \left(\frac{x_T}{\hat{y}_B - L_{BL}x_T}\right) \land L_{TL}x_T = y_T$ 3 while $m(L_{TL}) < m(L)$ do  2.3 $\left\{\left(\frac{y_T}{y_B}\right) = \left(\frac{x_T}{\hat{y}_B - L_{BL}x_T}\right) \land L_{TL}x_T = y_T \land m(L_{TL}) < m(L)\right\}$ $\frac{L_{TL}}{L_{BL}} \frac{L_{TR}}{L_{BR}} \to \left(\frac{L_{00}}{\hat{y}_B - L_{BL}x_T}\right) \land L_{TL}x_T = y_T \land m(L_{TL}) < m(L)$ 5a $\left(\frac{L_{TL}}{L_{BL}} \frac{L_{TR}}{L_{BR}}\right) \to \left(\frac{L_{00}}{\hat{y}_B} \frac{l_{01}}{l_{01}} \frac{L_{02}}{L_{22}}\right), \left(\frac{x_T}{x_B}\right) \to \left(\frac{x_0}{x_1}\right), \left(\frac{y_T}{y_B}\right) \to \left(\frac{y_0}{\psi_1}\right)$ where $\lambda_{11}$ is $1 \times 1$ , $\lambda_1$ has $1$ row, $\psi_1$ has $1$ row  6 $\left\{\begin{pmatrix}y_0\\\psi_1\\y_2\end{pmatrix} = \begin{pmatrix}x_0\\\hat{\psi}_1 - l_{10}^Tx_0\\\hat{y}_2 - L_{20}^Tx_0\end{pmatrix} \land L_{00}x_0 = \hat{y}_0$ 8 $y_2 := \hat{y}_2 - L_{00}x_0 - \chi_1 l_{21} = y_2 - \psi_1 l_{21}$ 7 $\left\{\begin{pmatrix}y_0\\\psi_1\\y_2\end{pmatrix} = \begin{pmatrix}x_0\\\chi_1\\\hat{y}_2 - L_{00}x_0 - \chi_1 l_{21}\end{pmatrix} \land \begin{pmatrix}L_{00}x_0 - \hat{y}_0\\I_{10}^Tx_0 + \chi_1 = \hat{\psi}_1\\I_{20}x_0 + \chi_1 = \hat{\psi}_1\end{pmatrix}\right\}$ 5b $\left(\frac{L_{TL}}{L_{TR}} \frac{L_{TR}}{L_{BR}}\right) \leftarrow \left(\frac{L_{00}}{l_1} \frac{l_{10}}{L_{22}}\right), \left(\frac{x_T}{x_B}\right) \leftarrow \left(\frac{x_0}{x_1}\right), \left(\frac{y_T}{y_B}\right) \leftarrow \left(\frac{y_0}{\psi_1}\right)$ 2 $\left\{\begin{pmatrix}y_T\\y_B\end{pmatrix} = \left(\frac{x_T}{\hat{y}_B - L_{BL}x_T}\right) \land L_{TL}x_T = y_T$ endwhile  2.3 $\left\{\begin{pmatrix}y_T\\y_B\end{pmatrix} = \left(\frac{x_T}{\hat{y}_B - L_{BL}x_T}\right) \land L_{TL}x_T = y_T \land \neg(m(L_{TL}) < m(L))$ 1b $\{y = x \land Lx = \hat{y}\}$		
$ \begin{array}{ll} 4 & L \rightarrow \left( \begin{array}{c} L_{TL} \mid L_{TR} \\ L_{BL} \mid L_{BR} \end{array} \right), x \rightarrow \left( \begin{array}{c} x_T \\ x_B \end{array} \right), y \rightarrow \left( \begin{array}{c} y_T \\ y_B \end{array} \right) \\ & \text{where } L_{TL} \text{ is } 0 \times 0, x_T \text{ has } 0 \text{ rows, } y_T \text{ has } 0 \text{ rows} \end{array} $ $ 2 & \left\{ \left( \begin{array}{c} \frac{y_T}{y_B} \right) = \left( \begin{array}{c} x_T \\ \widehat{y_B} - L_{BL}x_T \end{array} \right) \wedge L_{TL}x_T = y_T \end{array} \right. \\ 3 & \text{while } m(L_{TL}) < m(L) \text{ do} \\ 2,3 & \left\{ \left( \begin{array}{c} \frac{y_T}{y_B} \right) = \left( \begin{array}{c} x_T \\ \widehat{y_B} - L_{BL}x_T \end{array} \right) \wedge L_{TL}x_T = y_T \wedge m(L_{TL}) < m(L) \end{array} \right. \\ 5a & \left( \begin{array}{c} L_{TL} \mid L_{TR} \\ L_{BL} \mid L_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c} L_{00} \mid l_{01} \mid L_{02} \\ l_{10} \mid l_{11} \mid l_{12} \\ L_{20} \mid l_{21} \mid L_{22} \end{array} \right), \left( \begin{array}{c} x_T \\ x_B \end{array} \right) \rightarrow \left( \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} \right), \left( \begin{array}{c} y_T \\ y_B \end{array} \right) \rightarrow \left( \begin{array}{c} y_0 \\ \psi_1 \\ \psi_2 \end{array} \right) \\ & \text{where } \lambda_{11} \text{ is } 1 \times 1, \chi_1 \text{ has } 1 \text{ row, } \psi_1 \text{ has } 1 \text{ row} \end{array} \right. \\ 6 & \left\{ \begin{array}{c} \left( \begin{array}{c} y_0 \\ y_1 \\ y_2 \end{array} \right) = \left( \begin{array}{c} x_0 \\ \widehat{\psi}_1 - l_{10}^T x_0 \\ \widehat{y}_2 - L_{20}^T x_0 \end{array} \right) \wedge L_{00} x_0 = \widehat{y}_0 \\ & \left( \begin{array}{c} x_0 \\ y_1 \\ y_2 \end{array} \right) = \left( \begin{array}{c} x_0 \\ \chi_1 \\ \widehat{y}_2 - L_{00} x_0 - \chi_1 l_{21} \end{array} \right) \wedge L_{10} x_0 + \chi_1 = \widehat{\psi}_1 \end{array} \right. \\ 7 & \left\{ \begin{array}{c} \left( \begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array} \right) = \left( \begin{array}{c} x_0 \\ \chi_1 \\ \widehat{y}_2 - L_{00} x_0 - \chi_1 l_{21} \end{array} \right) - \left( \begin{array}{c} L_{00} x_0 \\ l_{10}^T x_0 + \chi_1 = \widehat{\psi}_1 \end{array} \right) + \left( \begin{array}{c} x_0 \\ \chi_1 \\ \chi_2 \end{array} \right), \left( \begin{array}{c} y_T \\ y_B \end{array} \right) \leftarrow \left( \begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array} \right) \\ 2 & \left\{ \begin{array}{c} \left( \begin{array}{c} x_T \\ y_B \end{array} \right) = \left( \begin{array}{c} x_T \\ \widehat{y_B} - L_{BL} x_T \end{array} \right) \wedge L_{TL} x_T = y_T \\ \text{endwhile} \end{array} \right. \\ 2 , 3 & \left\{ \left( \begin{array}{c} \frac{y_T}{y_B} \right) = \left( \begin{array}{c} x_T \\ \widehat{y_B} - L_{BL} x_T \end{array} \right) \wedge L_{TL} x_T = y_T \wedge \neg (m(L_{TL}) < m(L)) \right. \end{array} \right.$	Step	Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.
$ \begin{array}{c} \text{where } L_{TL} \text{ is } 0 \times 0, x_{T} \text{ has } 0 \text{ rows}, y_{T} \text{ has } 0 \text{ rows} \\ 2 & \left\{ \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} = \begin{pmatrix} x_{T} \\ \widehat{y}_{B} - L_{BL}x_{T} \end{pmatrix} \wedge L_{TL}x_{T} = y_{T} \\ 3 & \text{while } m(L_{TL}) < m(L) \text{ do} \\ 2,3 & \left\{ \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} = \begin{pmatrix} x_{T} \\ \widehat{y}_{B} - L_{BL}x_{T} \end{pmatrix} \wedge L_{TL}x_{T} = y_{T} \wedge m(L_{TL}) < m(L) \\ 5a & \left( \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \right) \rightarrow \begin{pmatrix} L_{00} & l_{01} & L_{02} \\ l_{10}^{T} & \lambda_{11} & l_{12}^{T} \\ L_{20} & l_{21} & L_{22} \end{pmatrix}, \begin{pmatrix} x_{T} \\ x_{B} \end{pmatrix} \rightarrow \begin{pmatrix} x_{0} \\ \chi_{1} \\ \chi_{2} \end{pmatrix}, \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} \rightarrow \begin{pmatrix} y_{0} \\ \psi_{1} \\ y_{2} \end{pmatrix} \\ \text{where } \lambda_{11} \text{ is } 1 \times 1, \chi_{1} \text{ has } 1 \text{ row}, \psi_{1} \text{ has } 1 \text{ row} \\ 6 & \left\{ \begin{pmatrix} y_{0} \\ \psi_{1} \\ y_{2} \end{pmatrix} = \begin{pmatrix} x_{0} \\ \chi_{1} \\ \chi_{2} \end{pmatrix} \wedge L_{00}x_{0} = \widehat{y}_{0} \\ \widehat{y}_{2} - L_{20}^{T}x_{0} \end{pmatrix} \wedge L_{00}x_{0} = \widehat{y}_{0} \\ 8 & y_{2} := \widehat{y}_{2} - L_{00}x_{0} - \chi_{1}l_{21} = y_{2} - \psi_{1}l_{21} \\ 7 & \left\{ \begin{pmatrix} y_{0} \\ \psi_{1} \\ y_{2} \end{pmatrix} = \begin{pmatrix} x_{0} \\ \chi_{1} \\ \widehat{y}_{2} - L_{00}x_{0} - \chi_{1}l_{21} \end{pmatrix} \wedge L_{00}x_{0} = \widehat{y}_{0} \\ l_{10}^{T}x_{0} + \chi_{1} = \widehat{\psi}_{1} \\ k_{DL} & L_{DR} \end{pmatrix} \leftarrow \begin{pmatrix} l_{00} & l_{01} & l_{02} \\ l_{10}^{T} & \lambda_{11} & l_{12}^{T} \\ l_{20} & l_{21} & l_{22} \end{pmatrix}, \begin{pmatrix} x_{T} \\ x_{B} \end{pmatrix} \leftarrow \begin{pmatrix} x_{0} \\ \chi_{1} \\ \chi_{2} \end{pmatrix}, \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} \leftarrow \begin{pmatrix} y_{0} \\ \psi_{1} \\ y_{2} \end{pmatrix} \\ 2 & \left\{ \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} = \begin{pmatrix} x_{T} \\ \widehat{y}_{B} - L_{BL}x_{T} \end{pmatrix} \wedge L_{TL}x_{T} = y_{T} \wedge \neg(m(L_{TL}) < m(L)) \\ \end{pmatrix} \right. \\ \\ 2,3 & \left\{ \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} = \begin{pmatrix} x_{T} \\ \widehat{y}_{B} - L_{BL}x_{T} \end{pmatrix} \wedge L_{TL}x_{T} = y_{T} \wedge \neg(m(L_{TL}) < m(L)) \\ \end{pmatrix} \right. \\ \end{array}$	1a	$\{y = \widehat{y} \}$
3 while $m(L_{TL}) < m(L)$ do  2,3 $ \begin{cases} \left(\frac{y_T}{y_B}\right) = \left(\frac{x_T}{\hat{y}_B - L_{BL}x_T}\right) \land L_{TL}x_T = y_T \land m(L_{TL}) < m(L) \end{cases} $ 5a $ \left(\frac{L_{TL}}{L_{BL}} \begin{vmatrix} L_{TR} \\ L_{BL} \end{vmatrix} \rightarrow \left(\frac{L_{00}}{l_{10}} \begin{vmatrix} l_{01} & L_{02} \\ L_{20} & l_{21} & L_{22} \end{pmatrix}, \left(\frac{x_T}{x_B}\right) \rightarrow \left(\frac{x_0}{\chi_1}\right), \left(\frac{y_T}{y_B}\right) \rightarrow \left(\frac{y_0}{\psi_1}\right) \\                                    $	4	
	2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \right\}$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	3	while $m(L_{TL}) < m(L)$ do
$\begin{array}{lll} & \text{where } \lambda_{11} \text{ is } 1 \times 1, \chi_{1} \text{ has } 1 \text{ row, } \psi_{1} \text{ has } 1 \text{ row} \\ & \begin{cases} y_{0} \\ \psi_{1} \\ y_{2} \end{cases} = \begin{pmatrix} x_{0} \\ \widehat{\psi}_{1} - l_{10}^{T} x_{0} \\ \widehat{y}_{2} - L_{20}^{T} x_{0} \end{pmatrix} \wedge L_{00} x_{0} = \widehat{y}_{0} \\ & \end{cases} \\ & 8 \qquad y_{2} := \widehat{y}_{2} - L_{00} x_{0} - \chi_{1} l_{21} = y_{2} - \psi_{1} l_{21} \\ & 7 \qquad \begin{cases} \begin{pmatrix} y_{0} \\ \psi_{1} \\ y_{2} \end{pmatrix} = \begin{pmatrix} x_{0} \\ \chi_{1} \\ \widehat{y}_{2} - L_{00} x_{0} - \chi_{1} l_{21} \end{pmatrix} \wedge \begin{pmatrix} L_{00} x_{0} = \widehat{y}_{0} \\ l_{10}^{T} x_{0} + \chi_{1} = \widehat{\psi}_{1} \end{pmatrix} \\ & 5 \text{b} \qquad \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} L_{00} & l_{01} & L_{02} \\ l_{10}^{T} & \lambda_{11} & l_{12}^{T} \\ L_{20} & l_{21} & L_{22} \end{pmatrix}, \begin{pmatrix} x_{T} \\ x_{B} \end{pmatrix} \leftarrow \begin{pmatrix} x_{0} \\ \chi_{1} \\ x_{2} \end{pmatrix}, \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} \leftarrow \begin{pmatrix} y_{0} \\ \psi_{1} \\ y_{2} \end{pmatrix} \\ & 2 \qquad \begin{cases} \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} = \begin{pmatrix} x_{T} \\ \widehat{y}_{B} - L_{BL} x_{T} \end{pmatrix} \wedge L_{TL} x_{T} = y_{T} \\ & \text{endwhile} \end{cases} \\ & 2,3 \qquad \begin{cases} \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} = \begin{pmatrix} x_{T} \\ \widehat{y}_{B} - L_{BL} x_{T} \end{pmatrix} \wedge L_{TL} x_{T} = y_{T} \wedge \neg (m(L_{TL}) < m(L)) \end{cases} \end{aligned}$	2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL}x_T} \right) \land L_{TL}x_T = y_T \land m(L_{TL}) < m(L) \right\}$
$ \begin{array}{ll} 8 & y_{2} := \widehat{y}_{2} - L_{00}x_{0} - \chi_{1}l_{21} = y_{2} - \psi_{1}l_{21} \\ 7 & \begin{cases} \begin{pmatrix} y_{0} \\ \psi_{1} \\ y_{2} \end{pmatrix} = \begin{pmatrix} x_{0} \\ \chi_{1} \\ \widehat{y}_{2} - L_{00}x_{0} - \chi_{1}l_{21} \end{pmatrix} \wedge \begin{pmatrix} L_{00}x_{0} &= \widehat{y}_{0} \\ l_{10}^{T}x_{0} + \chi_{1} &= \widehat{\psi}_{1} \end{pmatrix} \\ 5b & \begin{pmatrix} L_{TL} & L_{TR} \\ L_{BL} & L_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} L_{00} & l_{01} & L_{02} \\ l_{10}^{T} & \lambda_{11} & l_{12}^{T} \\ L_{20} & l_{21} & L_{22} \end{pmatrix}, \begin{pmatrix} x_{T} \\ x_{B} \end{pmatrix} \leftarrow \begin{pmatrix} x_{0} \\ \chi_{1} \\ x_{2} \end{pmatrix}, \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} \leftarrow \begin{pmatrix} y_{0} \\ \psi_{1} \\ y_{2} \end{pmatrix} \\ 2 & \begin{cases} \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} = \begin{pmatrix} x_{T} \\ \widehat{y}_{B} - L_{BL}x_{T} \end{pmatrix} \wedge L_{TL}x_{T} = y_{T} \\ & \text{endwhile} \\ 2,3 & \begin{cases} \begin{pmatrix} y_{T} \\ y_{B} \end{pmatrix} = \begin{pmatrix} x_{T} \\ \widehat{y}_{B} - L_{BL}x_{T} \end{pmatrix} \wedge L_{TL}x_{T} = y_{T} \wedge \neg (m(L_{TL}) < m(L)) \\ \end{cases} $	5a	$( \begin{array}{c c} D_{20} & v_{21} & D_{22} \end{array}) $
$ \begin{cases} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_0 \\ \chi_1 \\ \widehat{y}_2 - L_{00}x_0 - \chi_1 l_{21} \end{pmatrix} \land \begin{pmatrix} L_{00}x_0 = \widehat{y}_0 \\ l_{10}^T x_0 + \chi_1 = \widehat{\psi}_1 \end{pmatrix} \\ 5b \begin{pmatrix} \left( \frac{L_{TL}}{L_{TR}} \right) L_{TR} \\ L_{BL} \mid L_{BR} \end{pmatrix} \leftarrow \begin{pmatrix} \left( \frac{L_{00}}{l_{10}} \right) l_{12} \\ l_{10}^T \lambda_{11} \mid l_{12}^T \\ L_{20} \mid l_{21} \mid L_{22} \end{pmatrix}, \begin{pmatrix} \left( \frac{x_T}{x_B} \right) \leftarrow \begin{pmatrix} \left( \frac{y_T}{y_B} \right) + \left( \frac{y_0}{y_B} \right) \end{pmatrix} \\ 2 \begin{pmatrix} \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL}x_T} \right) \land L_{TL}x_T = y_T \\ \end{cases} $ endwhile $ 2,3 \begin{pmatrix} \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL}x_T} \right) \land L_{TL}x_T = y_T \land \neg (m(L_{TL}) < m(L)) $	6	$ \left\{ \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_0 \\ \widehat{\psi}_1 - l_{10}^T x_0 \\ \widehat{y}_2 - L_{20}^T x_0 \end{pmatrix} \land L_{00} x_0 = \widehat{y}_0 \right\} $
5b $\left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} L_{00} & t_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$ $2  \left\{\begin{array}{c c} \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} x_T \\ \hline \widehat{y}_B - L_{BL}x_T \end{array}\right) \wedge L_{TL}x_T = y_T \\ \text{endwhile} \\ 2,3  \left\{\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} x_T \\ \hline \widehat{y}_B - L_{BL}x_T \end{array}\right) \wedge L_{TL}x_T = y_T \wedge \neg (m(L_{TL}) < m(L)) \right\}$	8	$y_2 := \widehat{y}_2 - L_{00}x_0 - \chi_1 l_{21} = y_2 - \psi_1 l_{21}$
5b $\left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} L_{00} & t_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$ $2  \left\{\begin{array}{c c} \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} x_T \\ \hline \widehat{y}_B - L_{BL}x_T \end{array}\right) \wedge L_{TL}x_T = y_T \\ \text{endwhile} \\ 2,3  \left\{\left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) = \left(\begin{array}{c} x_T \\ \hline \widehat{y}_B - L_{BL}x_T \end{array}\right) \wedge L_{TL}x_T = y_T \wedge \neg (m(L_{TL}) < m(L)) \right\}$	7	$ \left\{ \begin{array}{c} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_0 \\ \chi_1 \\ \widehat{y}_2 - L_{00}x_0 - \chi_1 l_{21} \end{pmatrix} \land \begin{array}{c} L_{00}x_0 = \widehat{y}_0 \\ l_{10}^T x_0 + \chi_1 = \widehat{\psi}_1 \end{array} \right\} $
$ \begin{cases} \left(\frac{y_T}{y_B}\right) = \left(\frac{x_T}{\widehat{y}_B - L_{BL}x_T}\right) \land L_{TL}x_T = y_T \end{cases} $ endwhile $ 2,3  \left\{\left(\frac{y_T}{y_B}\right) = \left(\frac{x_T}{\widehat{y}_B - L_{BL}x_T}\right) \land L_{TL}x_T = y_T \land \neg(m(L_{TL}) < m(L)) \end{cases} $	5b	$ \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} L_{00} & l_{01} & L_{02} \\ l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array}\right) $
2,3 $\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \land \neg (m(L_{TL}) < m(L)) \right\}$	2	
$ \frac{2,3}{y_B} = \frac{1}{\widehat{y}_B - L_{BL}x_T} \wedge L_{TL}x_T = y_T \wedge \neg (m(L_{TL}) < m(L)) $		endwhile
1b $\{y = x \land Lx = \widehat{y}\}$	2,3	1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 -
	1b	$\{y = x \land Lx = \widehat{y} $

Step	Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.
1a	<b>\</b> {
4	where
2	
3	while do
2,3	
5a	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\left\{ \begin{array}{c} & & \\ & & \\ \end{array} \right.$
1b	{

Step	Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.	
1a	$\{y=\widehat{y}$	}
4	where	
2		
3	while do	
2,3		
5a	where	
6		
8		
7		
5b		
2		
	endwhile	
2,3	$\left\{ \begin{array}{c} \\ \\ \\ \end{array} \right. $	
1b	$\{y = x \land Lx = \widehat{y}$	}

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Step	Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.
1a	$ \{y=\widehat{y} $
4	where
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \wedge L_{TL} x_T = y_T \right\}$
3	while do
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \wedge L_{TL} x_T = y_T \wedge \right\}$
5a	where
6	
8	
7	
5b	
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \wedge L_{TL} x_T = y_T \right\}$
	endwhile
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \land \neg ( ) \right\}$
1b	$\{y = x \land Lx = \widehat{y} $

Step	Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.
1a	$\{y = \widehat{y} $
4	where
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \wedge L_{TL} x_T = y_T \right\}$
3	while $m(L_{TL}) < m(L)$ do
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL}x_T} \right) \wedge L_{TL}x_T = y_T \wedge m(L_{TL}) < m(L) \right\}$
5a	where
6	
8	
7	
5b	
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \wedge L_{TL} x_T = y_T \right\}$
	endwhile
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \land \neg (m(L_{TL}) < m(L)) \right\}$
1b	$\{y = x \land Lx = \widehat{y} \}$

Step	Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.
1a	
4	$L \to \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), x \to \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where $L_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \wedge L_{TL} x_T = y_T \right\}$
3	while $m(L_{TL}) < m(L)$ do
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL}x_T} \right) \wedge L_{TL}x_T = y_T \wedge m(L_{TL}) < m(L) \right\}$
5a	where
	where
6	
8	
7	
5b	
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \wedge L_{TL} x_T = y_T \right\}$
	endwhile
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \land \neg (m(L_{TL}) < m(L)) \right\}$
1b	$\{y = x \land Lx = \widehat{y} \}$

Step	Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.	
1a	$\{y=\widehat{y}$	}
4	$L \to \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), x \to \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where $L_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows	
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \right.$	
3	while $m(L_{TL}) < m(L)$ do	
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \wedge L_{TL} x_T = y_T \wedge m(L_{TL}) < m(L) \right\}$	
5a	$ \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \to \left(\begin{array}{c c} L_{00} & l_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \to \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \to \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $ where $\lambda$ is 1 × 1, $\lambda$ has 1 row $\lambda$ has 1 row.	
6	where $\lambda_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row $\left\{\begin{array}{cccccccccccccccccccccccccccccccccccc$	
8		
7		
5b	$ \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} L_{00} & l_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $	
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \right.$	
	endwhile	
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \land \neg (m(L_{TL}) < m(L)) \right\}$	$igg\}$
1b	$\{y = x \land Lx = \widehat{y}$	}

Step	Algorithm: Solve $Lx = y$ overwriting y with x. L is unit lower triangular.	
1a	$\{y=\widehat{y}$	}
4	$L \to \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), x \to \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where $L_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows	
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \right\}$	
3	while $m(L_{TL}) < m(L)$ do	
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL}x_T} \right) \wedge L_{TL}x_T = y_T \wedge m(L_{TL}) < m(L) \right\}$	
5a	$egin{pmatrix} L_{BL} & L_{BR} \end{pmatrix} & igg( L_{20} & l_{21} & L_{22} \end{pmatrix} & igg( x_B \end{pmatrix} & igg( x_2 \end{pmatrix} & igg( y_B \end{pmatrix} & igg( y_2 \end{pmatrix}$	
6	where $\lambda_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row $\begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_0 \\ \widehat{\psi}_1 - l_{10}^T x_0 \\ \widehat{y}_2 - L_{20}^T x_0 \end{pmatrix} \wedge L_{00} x_0 = \widehat{y}_0$	
8		
7		$\left. \begin{array}{c} \\ \end{array} \right\}$
5b	$ \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} L_{00} & l_{01} & L_{02} \\ l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array}\right) $	
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \right.$	
	endwhile	
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \land \neg (m(L_{TL}) < m(L)) \right\}$	
1b	$\{y = x \land Lx = \widehat{y}$	}

Step	Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.
1a	$\{y = \widehat{y} $
4	$L \to \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), x \to \left(\begin{array}{c c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c c} y_T \\ \hline y_B \end{array}\right)$ where $L_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \right\}$
3	while $m(L_{TL}) < m(L)$ do
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL}x_T} \right) \land L_{TL}x_T = y_T \land m(L_{TL}) < m(L) \right\}$
5a	$ \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} L_{00} & l_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $ where $\lambda_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row
6	$ \left\{ \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_0 \\ \widehat{\psi}_1 - l_{10}^T x_0 \\ \widehat{y}_2 - L_{20}^T x_0 \end{pmatrix} \wedge L_{00} x_0 = \widehat{y}_0 \right\} $
8	
7	$ \left\{ \begin{array}{c} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_0 \\ \chi_1 \\ \widehat{y}_2 - L_{00}x_0 - \chi_1 l_{21} \end{pmatrix} \wedge \begin{array}{c} L_{00}x_0 = \widehat{y}_0 \\ l_{10}^T x_0 + \chi_1 = \widehat{\psi}_1 \end{array} \right\} $
5b	$ \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} L_{00} & l_{01} & L_{02} \\ l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array}\right) $
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \wedge L_{TL} x_T = y_T \right\}$
	endwhile
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \land \neg (m(L_{TL}) < m(L)) \right\}$
1b	$\{y = x \land Lx = \widehat{y} \}$

Step	Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.
1a	$\{y = \widehat{y} $
4	$L \to \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), x \to \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where $L_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \right\}$
3	while $m(L_{TL}) < m(L)$ do
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL}x_T} \right) \wedge L_{TL}x_T = y_T \wedge m(L_{TL}) < m(L) \right\}$
5a	$ \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} L_{00} & l_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right) $ where $\lambda_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row
6	$ \left\{ \begin{array}{c} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_0 \\ \widehat{\psi}_1 - l_{10}^T x_0 \\ \widehat{y}_2 - L_{20}^T x_0 \end{pmatrix} \wedge L_{00} x_0 = \widehat{y}_0 \\ \end{array} \right\} $
8	$y_2 := \widehat{y}_2 - L_{00}x_0 - \chi_1 l_{21} = y_2 - \psi_1 l_{21}$
7	$ \left\{ \begin{array}{c} \begin{pmatrix} y_0 \\ \psi_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_0 \\ \chi_1 \\ \widehat{y}_2 - L_{00}x_0 - \chi_1 l_{21} \end{pmatrix} \wedge \begin{array}{c} L_{00}x_0 = \widehat{y}_0 \\ l_{10}^T x_0 + \chi_1 = \widehat{\psi}_1 \end{array} \right\} $
5b	$ \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} L_{00} & l_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right) $
2	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \right\}$
	endwhile
2,3	$\left\{ \left( \frac{y_T}{y_B} \right) = \left( \frac{x_T}{\widehat{y}_B - L_{BL} x_T} \right) \land L_{TL} x_T = y_T \land \neg (m(L_{TL}) < m(L)) \right\}$
1b	$\{y = x \land Lx = \widehat{y} \}$

Algorithm: Solve $Lx = y$ overwriting $y$ with $x$ . $L$ is unit lower triangular.
$L \to \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), x \to \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$ where $L_{TL}$ is $0 \times 0$ , $x_T$ has 0 rows, $y_T$ has 0 rows
while $m(L_{TL}) < m(L)$ do
$ \left(\begin{array}{c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} L_{00} & l_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right) $ where $\lambda_{11}$ is $1 \times 1$ , $\chi_1$ has 1 row, $\psi_1$ has 1 row
$y_2 := \widehat{y}_2 - L_{00}x_0 - \chi_1 l_{21} = y_2 - \psi_1 l_{21}$
$ \left(\begin{array}{c c c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} L_{00} & l_{01} & L_{02} \\ l_{10}^T & \lambda_{11} & l_{12}^T \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array}\right) $
endwhile

Algorithm: Solve Lx = y overwriting y with x. L is unit lower triangular.

$$L \to \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), x \to \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \to \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$$

where  $L_{TL}$  is  $0 \times 0$ ,  $x_T$  has 0 rows,  $y_T$  has 0 rows

while  $m(L_{TL}) < m(L)$  do

$$\left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \to \left(\begin{array}{c|c} L_{00} & l_{01} & L_{02} \\ \hline l_{10}^T & \lambda_{11} & l_{12}^T \\ L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \to \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \to \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array}\right)$$

where  $\lambda_{11}$  is  $1 \times 1$ ,  $\chi_1$  has 1 row,  $\psi_1$  has 1 row

$$y_2 := \widehat{y}_2 - L_{00}x_0 - \chi_1 l_{21} = y_2 - \psi_1 l_{21}$$

$$y_{2} := \widehat{y}_{2} - L_{00}x_{0} - \chi_{1}l_{21} = y_{2} - \psi_{1}l_{21}$$

$$\left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c} L_{00} & l_{01} & L_{02} \\ \hline l_{10}^{T} & \lambda_{11} & l_{12}^{T} \\ \hline L_{20} & l_{21} & L_{22} \end{array}\right), \left(\begin{array}{c} x_{T} \\ \hline x_{B} \end{array}\right) \leftarrow \left(\begin{array}{c} x_{0} \\ \chi_{1} \\ \hline x_{2} \end{array}\right), \left(\begin{array}{c} y_{T} \\ \hline y_{B} \end{array}\right) \leftarrow \left(\begin{array}{c} y_{0} \\ \psi_{1} \\ \hline y_{2} \end{array}\right)$$

endwhile