Step	Algorithm: $A := \mathrm{LU\_UNB\_VAR4}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$, $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge m(A_{TL}) < m(A) \right\}$
5a	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1
6	$\left\{ \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & u_{01} & U_{02} \\ l_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{pmatrix} \wedge \begin{array}{ccc} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ l_{10}^T U_{00} = \widehat{a}_{10}^T & & \\ L_{20}U_{00} = \widehat{A}_{20} & & \end{array} \right\}$
8	$\alpha_{11} := \upsilon_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$ $a_{12}^T := u_{12}^T = \widehat{a}_{12}^T - l_{10}^T U_{02} = a_{12}^T - a_{10}^T A_{02}$ $a_{21}^T := u_{21}^T = (\widehat{a}_{21} - L_{20}u_{01})/\upsilon_{11} = (a_{21} - A_{20}a_{01})/\alpha_{11}$
7	$\left\{ \begin{array}{l} \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L\backslash U_{00} & u_{01} & U_{02} \\ l_{10}^T & \upsilon_{11} & u_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{pmatrix} \\ \wedge \begin{array}{ccc} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + \upsilon_{11} = \widehat{\alpha}_{11} & l_{10}^T U_{02} + u_{12}^T = \widehat{a}_{12}^T \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}u_{01} + \upsilon_{11} l_{21} = \widehat{a}_{21} & \end{array} \end{array} \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
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2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{A = L\backslash U \wedge LU = \widehat{A}\}$

Step	Algorithm: $A := \text{LU\_UNB\_VAR4}(A)$
1a	$\{ \quad \}$
4	where
2	$\left\{ \quad \right\}$
3	while do
2,3	$\left\{ \quad \wedge \right\}$
5a	where
6	$\left\{ \quad \right\}$
8	
7	$\left\{ \quad \right\}$
5b	
2	$\left\{ \quad \right\}$
	endwhile
2,3	$\left\{ \quad \wedge \quad \neg(\quad) \right\}$
1b	$\{ \quad \}$

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1a	$\{A = \widehat{A}$ $\}$
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Step	Algorithm: $A := \text{LU\_UNB\_VAR4}(A)$
1a	$\{A = \widehat{A}\}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $L \rightarrow \left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$, $U \rightarrow \left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$ where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge m(A_{TL}) < m(A) \right\}$
5a	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$ where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1
6	$\left\{ \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & u_{01} & U_{02} \\ l_{10}^T & \widehat{\alpha}_{11} & \widehat{a}_{12}^T \\ L_{20} & \widehat{a}_{21} & \widehat{A}_{22} \end{array}\right) \wedge \begin{array}{lll} L_{00}U_{00} = \widehat{A}_{00} & \text{Ł}_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ l_{10}^T U_{00} = \widehat{a}_{10}^T & & \\ L_{20}U_{00} = \widehat{A}_{20} & & \end{array} \right\}$
8	
7	$\left\{ \begin{array}{l} \left(\begin{array}{ccc} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right) = \left(\begin{array}{ccc} L\backslash U_{00} & u_{01} & U_{02} \\ l_{10}^T & \upsilon_{11} & u_{12}^T \\ L_{20} & l_{21} & \widehat{A}_{22} \end{array}\right) \\ \wedge \begin{array}{lll} L_{00}U_{00} = \widehat{A}_{00} & L_{00}u_{01} = \widehat{a}_{01} & L_{00}U_{02} = \widehat{A}_{02} \\ l_{10}^T U_{00} = \widehat{a}_{10}^T & l_{10}^T u_{01} + \upsilon_{11} = \widehat{\alpha}_{11} & l_{10}^T U_{02} + u_{12}^T = \widehat{a}_{12}^T \\ L_{20}U_{00} = \widehat{A}_{20} & L_{20}u_{01} + \upsilon_{11} l_{21} = \widehat{a}_{21} & \end{array} \end{array} \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right)$, $\left(\begin{array}{c\|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \cdots$, $\left(\begin{array}{c\|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \leftarrow \cdots$
2	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \right\}$
	endwhile
2,3	$\left\{ \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c\|c} L\backslash U_{TL} & U_{TR} \\ \hline L_{BL} & \widehat{A}_{BR} \end{array}\right) \wedge \begin{array}{c\|c} L_{TL}U_{TL} = \widehat{A}_{TL} & L_{TL}U_{TR} = \widehat{A}_{TR} \\ \hline L_{BL}U_{TL} = \widehat{A}_{BL} & \end{array} \wedge \neg(m(A_{TL}) < m(A)) \right\}$
1b	$\{A = L\backslash U \wedge LU = \widehat{A}\}$

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Algorithm: $A := \mathrm{LU\_UNB\_VAR4}(A)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right), U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

where α_{11} is 1×1, λ_{11} is 1×1, υ_{11} is 1×1

$$\alpha_{11} := \upsilon_{11} = \widehat{\alpha}_{11} - l_{10}^T u_{01} = \alpha_{11} - a_{10}^T a_{01}$$
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endwhile

Algorithm: $A := \text{LU\_UNB\_VAR4}(A)$

$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right)$, $L \rightarrow \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right)$, $U \rightarrow \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right)$

where A_{TL} is 0×0, L_{TL} is 0×0, U_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \cdots, \left(\begin{array}{c|c} U_{TL} & U_{TR} \\ \hline U_{BL} & U_{BR} \end{array}\right) \rightarrow \cdots$$

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endwhile