| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
|------|---|
| 1a | ${C = \widehat{C}}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land m(A_{TL}) < m(A) \right\}$ |
| 5a | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ C_2 \end{array}\right) $ where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row |
| 6 | $ \begin{cases} \begin{pmatrix} C_0 \\ c_1^T \\ C_2 \end{pmatrix} = \begin{pmatrix} \hat{C}_1 \\ A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + \hat{C}_2 \end{pmatrix} $ |
| 8 | $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ $c_1^T := a_{10}^T B_0 + \alpha_{11}b_1^T + a_{21}^T B_2 + c_1^T$ $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| 7 | $ \begin{cases} \begin{pmatrix} C_0 \\ c_1^T \\ C_2 \end{pmatrix} = \begin{pmatrix} \hat{C}_0 \\ a_{10}^T B_0 + \alpha_{11} b_1^T + a_{21}^T B_2 + \hat{c}_1^T \\ A_{20} B_0 + a_{21} b_1^T + A_{22} B_2 + \hat{C}_2 \end{pmatrix} $ |
| 5b | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ c_1^T \\ C_2 \end{array}\right) $ |
| 2 | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B}\right) \end{array} \right\}$ |
| | endwhile |
| 2,3 | $ \left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land \neg (m(A_{TL}) < m(A)) \right\} $ |
| 1b | $\{C = AB + \widehat{C} $ |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
|------|---|
| 1a | { |
| | |
| 4 | |
| | where |
| 2 | |
| 3 | while do |
| 2,3 | $\left\{ \begin{array}{c} \wedge \end{array} \right.$ |
| | |
| 5a | |
| | where |
| | |
| 6 | |
| | |
| 8 | $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ |
| | $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| | |
| 7 | } |
| | |
| 5b | |
| อม | |
| | |
| 2 | |
| | endwhile |
| 0.0 | |
| 2,3 | |
| 1b | { |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
|------|---|
| 1a | $\{C = \widehat{C}\}$ |
| | |
| 4 | |
| | where |
| 2 | |
| 3 | while do |
| 2,3 | ^ |
| | |
| | |
| 5a | |
| | 1 |
| | where |
| 6 | |
| | |
| | $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ |
| 8 | |
| | $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| _ | |
| 7 | |
| |) |
| 5b | |
| 3.5 | |
| | |
| 2 | |
| | endwhile |
| | |
| 2,3 | |
| 1b | $\{C = AB + \widehat{C} $ |
| | • |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
|------|--|
| 1a | ${C = \widehat{C}}$ |
| 4 | where |
| 2 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B}\right) \wedge \end{array} \right\}$ |
| 5a | where |
| 6 | |
| 8 | $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| 7 | |
| 5b | |
| 2 | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B}\right) \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land \neg () \right\}$ |
| 1b | $\{C = AB + \widehat{C} $ } |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
|------|--|
| 1a | $\{C = \widehat{C}\}$ |
| 4 | where |
| 2 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \wedge m(A_{TL}) < m(A) \right\}$ |
| 5a | where |
| 6 | |
| 8 | $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| 7 | |
| 5b | |
| 2 | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B}\right) \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C} $ |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
|------|--|
| 1a | $\{C = \widehat{C}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $ \left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \right\} $ |
| 3 | $ \mathbf{w}_{h} \mathbf{w}_{h$ |
| 2,3 | $ \left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land m(A_{TL}) < m(A) \right\} $ |
| 5a | where |
| | where) |
| 6 | |
| 8 | $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| 7 | |
| 5b | |
| 2 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C} $ |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
|------|---|
| 1a | $\{C = \widehat{C}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land m(A_{TL}) < m(A) \right\}$ |
| 5a | $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} B_T \\ B_B \end{pmatrix} \rightarrow \begin{pmatrix} B_0 \\ b_1^T \\ B_2 \end{pmatrix}, \begin{pmatrix} C_T \\ C_B \end{pmatrix} \rightarrow \begin{pmatrix} C_0 \\ C_1^T \\ C_2 \end{pmatrix} $ where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row |
| 6 | |
| 8 | $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| 7 | |
| 5b | $\left(\begin{array}{c c} A_{BL} & A_{BR} \end{array}\right) = \left(\begin{array}{c c} A_{20} & a_{21} & A_{22} \end{array}\right) = \left(\begin{array}{c c} B_B \end{array}\right) = \left(\begin{array}{c c} C_B \end{array}\right) = \left(\begin{array}{c c} C_B \end{array}\right)$ |
| 2 | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B}\right) \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C} $ |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
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| 1a | $\{C = \widehat{C}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land m(A_{TL}) < m(A) \right\}$ |
| 5a | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ C_1^T \\ C_2 \end{array}\right) $ where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row |
| 6 | $ \begin{cases} \begin{pmatrix} C_0 \\ c_1^T \\ C_2 \end{pmatrix} = \begin{pmatrix} & \hat{C}_0 \\ & \hat{c}_1^T \\ & A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + \hat{C}_2 \end{pmatrix} $ |
| 8 | $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| 7 | |
| 5b | $\begin{pmatrix} A_{20} & a_{21} & A_{22} \end{pmatrix}$ $\begin{pmatrix} B_{2} & B_{2} \end{pmatrix}$ $\begin{pmatrix} B_{2} & C_{2} & C_{2} \end{pmatrix}$ |
| 2 | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B}\right) \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C} $ |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
|------|--|
| 1a | $\{C = \widehat{C}$ |
| 4 | $A \to \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), B \to \left(\begin{array}{c c} B_T \\ \hline B_B \end{array}\right), C \to \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land m(A_{TL}) < m(A) \right\}$ |
| 5a | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ C_1^T \\ C_2 \end{array}\right) $ where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row |
| 6 | $ \begin{cases} \begin{pmatrix} C_0 \\ c_1^T \\ C_2 \end{pmatrix} = \begin{pmatrix} & \hat{C}_0 \\ & \hat{c}_1^T \\ & A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + \hat{C}_2 \end{pmatrix} $ |
| 8 | $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| 7 | $ \begin{cases} \begin{pmatrix} C_0 \\ c_1^T \\ C_2 \end{pmatrix} = \begin{pmatrix} \hat{C}_0 \\ a_{10}^T B_0 + \alpha_{11} b_1^T + a_{21}^T B_2 + \hat{c}_1^T \\ A_{20} B_0 + a_{21} b_1^T + A_{22} B_2 + \hat{C}_2 \end{pmatrix} $ |
| 5b | $A_{BL} A_{BR} $ $A_{BR} $ $A_{20} A_{21} A_{22}$ $A_{22} $ $A_{22} $ $A_{23} A_{22}$ $A_{22} $ |
| 2 | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B}\right) \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C} $ |

| Step | Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
|------|---|
| 1a | $\{C = \widehat{C}$ |
| 4 | $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| 2 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \right\}$ |
| 3 | while $m(A_{TL}) < m(A)$ do |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land m(A_{TL}) < m(A) \right\}$ |
| 5a | $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ C_2 \end{array}\right) $ where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row |
| 6 | $ \left\{ \begin{array}{c} \begin{pmatrix} C_0 \\ c_1^T \\ C_2 \end{pmatrix} = \begin{pmatrix} & \widehat{C}_0 \\ & \widehat{c}_1^T \\ & A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + \widehat{C}_2 \end{pmatrix} \right\} $ |
| 8 | $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ $c_1^T := a_{10}^T B_0 + \alpha_{11}b_1^T + a_{21}^T B_2 + c_1^T$ $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| 7 | $ \begin{cases} \begin{pmatrix} C_0 \\ c_1^T \\ C_2 \end{pmatrix} = \begin{pmatrix} \hat{C}_0 \\ a_{10}^T B_0 + \alpha_{11} b_1^T + a_{21}^T B_2 + \hat{c}_1^T \\ A_{20} B_0 + a_{21} b_1^T + A_{22} B_2 + \hat{C}_2 \end{pmatrix} $ |
| 5b | $A_{BL} A_{BR} $ $A_{BR} $ $A_{20} A_{21} A_{22}$ $A_{22} $ $A_{22} $ $A_{23} A_{22}$ $A_{22} $ |
| 2 | $\left\{ \begin{array}{c} \left(\frac{C_T}{C_B}\right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B}\right) \end{array} \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\frac{C_T}{C_B} \right) = \left(\frac{\widehat{C}_T}{A_{BL}B_T + A_{BR}B_B + \widehat{C}_B} \right) \land \neg (m(A_{TL}) < m(A)) \right\}$ |
| 1b | $\{C = AB + \widehat{C} $ |

| Algorithm: $C := AB + C$ where A is symmetric and stored in the lower triangular part |
|--|
| |
| $A \to \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_T \\ C_B \end{pmatrix}$ where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows |
| |
| while $m(A_{TL}) < m(A)$ do |
| |
| $ \begin{pmatrix} A_{TL} & A_{TR} \\ A_{BL} & A_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{pmatrix}, \begin{pmatrix} B_T \\ B_B \end{pmatrix} \rightarrow \begin{pmatrix} B_0 \\ b_1^T \\ B_2 \end{pmatrix}, \begin{pmatrix} C_T \\ C_B \end{pmatrix} \rightarrow \begin{pmatrix} C_0 \\ C_1^T \\ C_2 \end{pmatrix} $ where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row |
| |
| $C_0 := A_{00}B_0 + (a_{10}^T)^T b_1^T + A_{20}^T B_2 + C_0$ $c_1^T := a_{10}^T B_0 + \alpha_{11}b_1^T + a_{21}^T B_2 + c_1^T$ $C_2 := A_{20}B_0 + a_{21}b_1^T + A_{22}B_2 + C_2$ |
| |
| $ \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ c_1^T \\ C_2 \end{array}\right) $ |
| |
| endwhile |
| |
| |

Algorithm: C := AB + C where A is symmetric and stored in the lower triangular part

$$A o \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) , B o \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) , C o \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right)$$

where A_{TL} is 0×0 , B_T has 0 rows, C_T has 0 rows

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \to \left(\begin{array}{c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \to \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \to \left(\begin{array}{c} C_0 \\ \hline c_1^T \\ \hline C_2 \end{array}\right)$$

where α_{11} is 1×1 , b_1 has 1 row, c_1 has 1 row

$$c_1^T := a_{10}^T B_0 + \alpha_{11} b_1^T + a_{21}^T B_2 + c_1^T$$

$$\left(\begin{array}{c|c}
A_{TL} & A_{TR} \\
\hline
A_{BL} & A_{BR}
\end{array}\right) \leftarrow \left(\begin{array}{c|c}
A_{00} & a_{01} & A_{02} \\
a_{10}^T & \alpha_{11} & a_{12}^T \\
\hline
A_{20} & a_{21} & A_{22}
\end{array}\right), \left(\begin{array}{c}
B_T \\
B_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
B_0 \\
b_1^T \\
\hline
B_2
\end{array}\right), \left(\begin{array}{c}
C_T \\
C_B
\end{array}\right) \leftarrow \left(\begin{array}{c}
C_0 \\
c_1^T \\
\hline
C_2
\end{array}\right)$$

endwhile