Numerical study of nonlinear full wave acoustic propagation

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Westervelt equation

M. F. Hamilton and C. L. Morfey. *Model equations*, Chap. 3, in M. F. Hamilton and D. T. Blackstock, *Nonlinear acoustics*, 1998.

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\delta}{c_0^4}\frac{\partial^3 p}{\partial t^3} - \frac{\beta}{\rho_0 c_0^4}\frac{\partial^2 p^2}{\partial t^2}$$

- Compressible fluid
- Nonlinearity, β
- Thermoviscous dissipation, δ
- Not restricted in direction of propagation

A finite volume method cannot be applied directly

Reformulation as conservation laws

(or balance laws)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot \left(\rho_0 \vec{u} \otimes \vec{u} + c_0^2 \mathbf{I} \left(\rho + \frac{1}{\rho_0} (\beta - 1)(\rho')^2 \right) \right) = \rho_0 \delta \nabla^2 \vec{u}$$

Obtained with the same hypotheses used to obtain Westervelt equation, except for one: we didn't drop the Lagragian density term.

This equations are in the appropriate form to apply a finite volume method.

Numeric method

R. LeVeque, *Finite Volume Methods for Hyperbolic Problems*, 2002, Cambridge University Press.

We rewrite our system of equations as:

$$q_t + f'(q)q_x + g'(q)q_y = \mathcal{B}$$

Then we split it to use a fractional step method:

$$q_t = -f'(q)q_x$$
$$q_t = -g'(q)q_y$$
$$q_t = \mathcal{B}$$

Finite volume method used for the first and second equations, with:

- high resolution
- Roe linearization

Validation against analytic solution

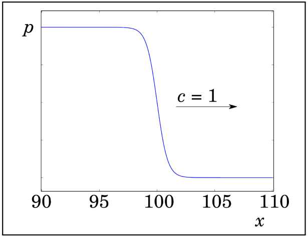
Taylor shock

The Taylor shock is just an hyperbolic tangent solution

$$p = \frac{-\delta}{\beta} \tanh(x - t)$$

We implemented it, in a 2D Cartesian mesh, for different:

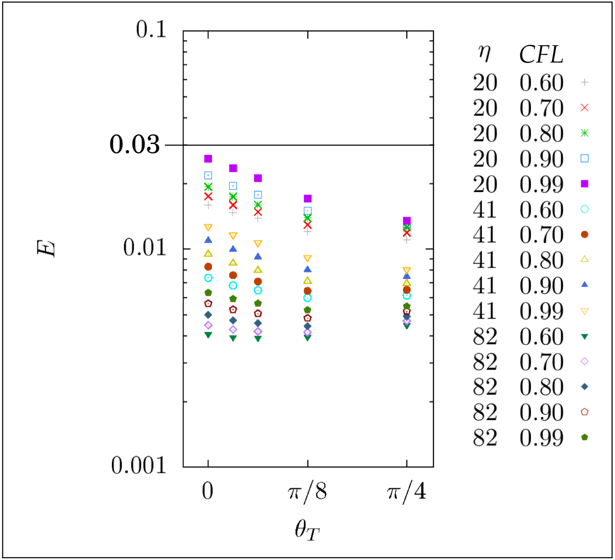
- mesh refinements,
- propagation angles θ_T.



Then the error was evaluated, at $t = 100$, as

$$E = \frac{\|\text{numeric solution} - \text{reference}\|}{\|\text{reference}\|}$$

Taylor shock, error analysis



η is the number of cells across the shock

Validation against another numeric result

Full wave diffraction

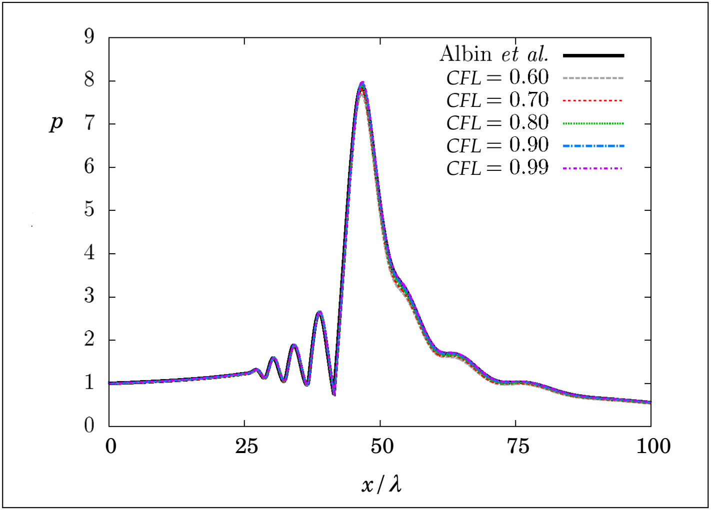
N. Albin et. al, *Fourier continuation methods for high-fidelity simulation of nonlinear acoustic beams*, J. Acoust. Soc. Am., 132 (2012): 2371.

This kind of simulation corresponds to a High Intensity Focused Ultrasound (HIFU) system.

see video: hifu.avi

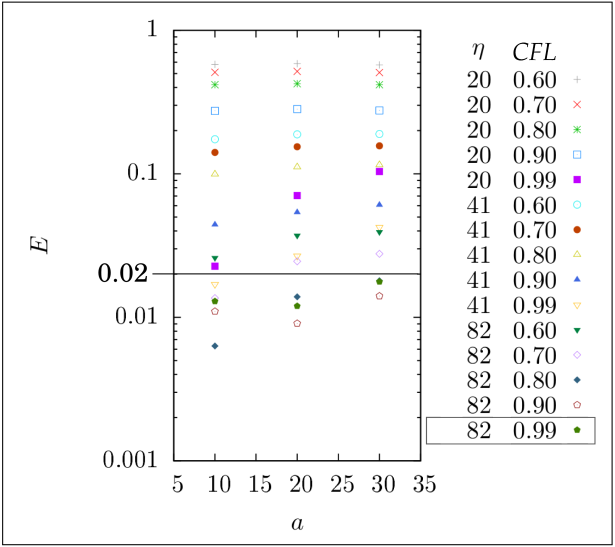
Against another numeric result

Comparison: maximum pressure over the propagation axis



$\eta = 82$ cells per wavelength

HIFU, error analysis



Performance

Our code is written C++/CUDA, it is inspired on CLAWPACK code, but it does not incorporate or depend on it.

	Device	Cores	Exec. time
Our code	GPU C2075	448	10s
CLAWPACK	CPU i3-550	1	10min

	Nodes	Cores	Δx	Exec. t.
Our code	1	448	$\lambda/82$ time dom.	31 min
Albin et al.	16?	128	$\lambda/21$ freq. dom.	14 min

Conclusions

- A set of conservation laws is presented, it is model for nonlinear acoustic propagation at least as general as Westervelt equation.
- A very good agreement is observed between our numeric results and the references.
- Limitations of the model are observed: when amplitudes are too large solutions become complex.
- Limitations of GPU are found: memory. Then hybrid schemes should be considered: clusters of GPUs.
- Details can be found in `http://arxiv.org/abs/1311.3004`.
- Code will be published soon as FOSS. `http://github.com/rvelseg/FiVoNAGI`.

Potential application: parametric acoustic array

work in progress

In collaboration with:

- Ricardo R. Bulloza
- Pablo L. Rendón
- Antonio Peréz-López
- Ricardo Dorantes
- Israel González

see video: paa.avi

Boundary condition at the window:

$$g(t) = A\sin(2\pi t)\sin\left(\frac{2\pi}{16}t\right)\exp\left(-(t/72)^{10}\right)$$

$$q^1 = g(t) + 1$$

$$q^2 = q_1 g(t)$$

$$q^3 = 0$$

We expect to have experimental comparison

Thank you