

kreher-stinson

**Algorithms from the book implemented
in GAP**

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Chapter 1

Generating Combinatorial Objects

1.1 Subsets

1.1.1 KSSubsetLexRank

▷ `KSSubsetLexRank(number, subset)` (function)

Returns the rank of *subset* as a subset of the set of numbers from 1 to *number* (Algorithm 2.1).

1.1.2 KSSubsetLexUnrank

▷ `KSSubsetLexUnrank(number, rank)` (function)

Returns the subset of $\{1..number\}$ whose rank is *rank*. (Algorithm 2.2).

1.1.3 KSkSubsetLexRank

▷ `KSkSubsetLexRank(T, k, n)` (function)

Finds the rank of *T*, among all *k*-subsets of an *n*-set.

1.1.4 KSkSubsetLexUnrank

▷ `KSkSubsetLexUnrank(r, k, n)` (function)

Given an integer *r* between 0 and $\binom{n}{k} - 1$, returns the *k*-subset of an *n*-set with rank *r*.

Chapter 2

Bactracking

2.1 Knapsack

2.1.1 KSCheckKnapsackInput

▷ `KSCheckKnapsackInput(profits, weights, capacity)` (function)

Checks for valid input data for the Knapsack problems (Problems 1.1-1.4).

2.1.2 KSKnapsack1

▷ `KSKnapsack1(profits, weights, capacity)` (function)

Implementation of Algorithm 4.1.

2.1.3 KSKnapsack2

▷ `KSKnapsack2(profits, weights, capacity)` (function)

Implementation of Algorithm 4.3.

2.2 Generating all cliques

2.2.1 KSAllCliques

▷ `KSAllCliques(graph)` (function)

Implementation of Algorithm 4.4. A graph G is defined by the list *graph*, which must be a list of subsets of $\{1, \dots, n\}$, for some integer n . The neighbors of vertex i are the elements of *graph*[i].

2.3 Exact cover

2.3.1 KSExactCover

▷ KSExactCover(*number*, *cover*) (function)

Finds an subcollection of *cover* (which is a set of subsets of $\{1, \dots, \textit{number}\}$) that is an exact cover of $\{1, \dots, \textit{number}\}$, if it exists.

2.4 Bounding functions

2.4.1 KSSortForRationalKnapsack

▷ KSSortForRationalKnapsack(*profits*, *weights*) (function)

Given two vectors *profits*, *weights* of the same length, this function returns a vector of the two vectors, sorted in non-decreasing order of values of $\textit{profits}[i] / \textit{weights}[i]$.

2.4.2 KSRationalKnapsackSorted

▷ KSRationalKnapsackSorted(*profits*, *weights*, *capacity*) (function)

Solves the rational Knapsack problem with parameters given. The vectors *profits*, *weights* must already be sorted.

2.4.3 KSKnapsack3

▷ KSKnapsack3(*profits*, *weights*, *capacity*) (function)

Solves the Knapsack problem with parameters given, using the function KSRationalKnapsackSorted as bounding function.

2.4.4 KSRandomKnapsackInstance

▷ KSRandomKnapsackInstance(*size*, *maximum_weight*) (function)

Returns a random instance of a Knapsack problem, for *size* objects. The maximum weight is *maximum_weight*. For each *i*, the profit $P[i]$ is $2 * W[i] * \epsilon$, where ϵ is a random number between 0.9 and 1.1.

2.4.5 KSRandomTSPInstance

▷ KSRandomTSPInstance(*n*, *wmax*) (function)

Returns a random instance of the TSP problem, which is a symmetric *n* by *n* matrix, such that its *ij* entry is the cost to travel from city *i* to city *j*. The entries in the diagonal are made equal to ∞ . Each cost is a random integer between 1 and *wmax*.

2.4.6 KSTSP1

▷ `KSTSP1(G)` (function)

Solves the TSP problem, for the instance G , traversing the whole tree space.

2.4.7 KSMinCostBound

▷ `KSMinCostBound(V , G)` (function)

A bounding function for the TSP problem.

2.4.8 KSReduce

▷ `KSReduce(M)` (function)

Reduce function for matrices, which will be useful to implement a second bounding function for the TSP problem.

2.4.9 KSReduceBound

▷ `KSReduceBound(V , M)` (function)

A second bounding function for the TSP problem. V is a partial solution, and M is the problem instance. This implements Algorithm 4.12.

2.4.10 KSTSP2

▷ `KSTSP2(G , F)` (function)

Solves the TSP problem for instance G , using the bounding function F .

2.4.11 KSMaxClique1

▷ `KSMaxClique1(G)` (function)

Adapts the function that lists the complete subgraphs of G , to find the size of the largest clique of G . This implements Algorithm 4.14.

2.4.12 KSMaxClique2

▷ `KSMaxClique2(G , F)` (function)

Finds the size of the maximum clique in the graph G , using the bounding function F . This implements Algorithm 4.19.

2.4.13 KSSizeBound

▷ `KSSizeBound(XX, G, Cl)` (function)

A bounding function for the MaxClique problem. *XX* is a complete subgraph of *G*, and *Cl* is the set of candidates to extend *XX*.

2.4.14 KSGenerateRandomGraph

▷ `KSGenerateRandomGraph(n)` (function)

Returns a list of edges of a random graph on *n* vertices. This implements Algorithm 4.20.

2.4.15 KSEdgeListToAdjacencyList

▷ `KSEdgeListToAdjacencyList(Ged, n)` (function)

Given the list of edges *Ged* of a graph with *n* vertices, returns the adjacency list of such graph.

2.4.16 KSGreedyColor

▷ `KSGreedyColor(G)` (function)

Colors the vertices of a graph *G* using a greedy strategy. This implements Algorithm 4.16.

2.4.17 KSSamplingBound

▷ `KSSamplingBound(XX, G, Cl)` (function)

A bounding function for the MaxClique problem. *XX* is a complete subgraph of *G*, and *Cl* is the set of candidates to extend *XX*. This function uses a fixed greedy coloring of the graph *G*. Implements Algorithm 4.17.

2.4.18 KSInducedSubgraph

▷ `KSInducedSubgraph(G, L)` (function)

Returns the adjacency list of the subgraph of *G* induced by the vertices in *L*.

2.4.19 KSGreedyBound

▷ `KSGreedyBound(XX, G, Cl)` (function)

A bounding function for the MaxClique problem. *XX* is a complete subgraph of *G*, and *Cl* is the set of candidates to extend *XX*. This uses a greedy coloring of the subgraph of *G* induced by *L*.

2.4.20 KSGenerateRandomGraph2

▷ `KSGenerateRandomGraph2(n, delta)` (function)

Returns the list of edges of a random graph on n vertices with edge density *delta*.

2.4.21 KSTSP3

▷ `KSTSP3(G, F)` (function)

Solves the TSP problem for instance G , using bounding function F , applying the branch and bound technique.

2.5 Exercises

2.5.1 KSQueens

▷ `KSQueens(size)` (function)

Solves the n queens problem for a $size \times size$ board.

Example

```
gap> KSQueens(4);  
[ 2, 4, 1, 3 ]  
[ 3, 1, 4, 2 ]
```

Chapter 3

Heuristic Search

3.1 Uniform graph partition

3.1.1 KSRandomkSubset

▷ `KSRandomkSubset(k , n)` (function)

Returns a randomly chosen k -subset of the set of integers from 1 to n .

3.1.2 KSelectPartition

▷ `KSelectPartition(n)` (function)

Returns a random partition of the set $\{1, 2, \dots, 2n\}$ into two subsets of size n each. (Algorithm 5.7)

3.1.3 KSCost

▷ `KSCost(G , P)` (function)

Returns the cost of the partition P of the vertices of the weighted graph G .

3.1.4 KSGain

▷ `KSGain(G , P , u , v)` (function)

P is a partition in equal parts of the vertices of G . This function calculates the change in the value of the cost function when interchanging the vertex u from the first set in the partition P with the vertex v which is in the second set of the partition.

3.1.5 KSRandomCostMatrix

▷ `KSRandomCostMatrix(n , w_{max})` (function)

Returns a symmetric n by n matrix, such that its entries are random integers from 0 to w_{max} , and with zeros in the main diagonal.

3.1.6 KSAscend

▷ `KSAscend(G , P)` (function)

Given a partition P of the vertices of the weighted graph G , it returns a partition Q with less cost than P , by exchanging one vertex of the partition, if such partition exists. Otherwise, returns the same partition P .

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