kreher-stinson

Algorithms from the book implemented in GAP

Version 0.1

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Bertín Hernández-Trejo
Rafael Villarroel-Flores
Citlalli Zamora-Mejía

Bertín Hernández-Trejo Email: bertin13@gmail.com

Rafael Villarroel-Flores Email: rvf0068@gmail.com
Homepage: http://rvf0068.github.io

Citlalli Zamora-Mejía Email: cizame@gmail.com

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Contents

Chapter 1

Generating Combinatorial Objects

1.1 Subsets

1.1.1 KSSubsetLexRank

▷ `KSSubsetLexRank(number, subset)` (function)

Returns the rank of *subset* as a subset of the set of numbers from 1 to *number* (Algorithm 2.1).

1.1.2 KSSubsetLexUnrank

▷ `KSSubsetLexUnrank(number, rank)` (function)

Returns the subset of $\{1..number\}$ whose rank is *rank*. (Algorithm 2.2).

1.1.3 KSkSubsetLexRank

▷ `KSkSubsetLexRank(T, k, n)` (function)

Finds the rank of T, among all k-subsets of an n-set.

1.1.4 KSkSubsetLexUnrank

▷ `KSkSubsetLexUnrank(r, k, n)` (function)

Given an integer r between 0 and $\binom{n}{k}-1$, returns the k-subset of an n-set with rank r.

Chapter 2

Bactracking

2.1 Knapsack

2.1.1 KSCheckKnapsackInput

▷ `KSCheckKnapsackInput(`*`profits, weights, capacity`*`)` (function)

Checks for valid input data for the Knapsack problems (Problems 1.1-1.4).

2.1.2 KSKnapsack1

▷ `KSKnapsack1(`*`profits, weights, capacity`*`)` (function)

Implementation of Algorithm 4.1.

2.1.3 KSKnapsack2

▷ `KSKnapsack2(`*`profits, weights, capacity`*`)` (function)

Implementation of Algorithm 4.3.

2.2 Generating all cliques

2.2.1 KSAllCliques

▷ `KSAllCliques(`*`graph`*`)` (function)

Implementation of Algorithm 4.4. A graph G is defined by the list *`graph`*, which must be a list of subsets of $\{1,...,n\}$, for some integer n. The neighbors of vertex i are the elements of *`graph[i]`*.

2.3 Exact cover

2.3.1 KSExactCover

▷ `KSExactCover(number, cover)` (function)

Finds an subcollection of *cover* (which is a set of subsets of $\{1,..,number\}$) that is an exact cover of $\{1,..,number\}$, if it exists.

2.3.2 KSRandomSubsetOfSubsets

▷ `KSRandomSubsetOfSubsets(n, delta)` (function)

Generates a random subset of the set of all subsets of $\{1..n\}$, with density *delta*. This can be used as an instance of the ExactCover problem.

2.4 Bounding functions

2.4.1 KSSortForRationalKnapsack

▷ `KSSortForRationalKnapsack(profits, weights)` (function)

Given two vectors *profits*, *weights* of the same length, this function returns a vector of the two vectors, sorted in non-decreasing order of values of *profits[i]* / *weights[i]*.

2.4.2 KSRationalKnapsackSorted

▷ `KSRationalKnapsackSorted(profits, weights, capacity)` (function)

Solves the rational Knapsack problem with parameters given. The vectors *profits*, *weights* must already be sorted.

2.4.3 KSKnapsack3

▷ `KSKnapsack3(profits, weights, capacity)` (function)

Solves the Knapsack problem with parameters given, using the function KSRationalKnapsack-Sorted as bounding function.

2.4.4 KSRandomKnapsackInstance

▷ `KSRandomKnapsackInstance(size, maximum_weight)` (function)

Returns a random instance of a Knapsack problem, for *size* objects. The maximum weight is *maximum_weight*. For each i, the profit $P[i]$ is $2 * W[i] * \varepsilon$, where ε is a random number between 0.9 and 1.1.

2.4.5 KSRandomTSPInstance

▷ KSRandomTSPInstance(*n*, *Wmax*) (function)

Returns a random instance of the TSP problem, which is a symmetric n by n matrix, such that its ij entry is the cost to travel from city i to city j. The entries in the diagonal are made equal to ∞. Each cost is a random integer between 1 and *Wmax*.

2.4.6 KSTSP1

▷ KSTSP1(*G*) (function)

Solves the TSP problem, for the instance *G*, traversing the whole tree space.

2.4.7 KSMinCostBound

▷ KSMinCostBound(*V*, *G*) (function)

A bounding function for the TSP problem.

2.4.8 KSReduce

▷ KSReduce(*M*) (function)

Reduce function for matrices, which will be useful to implement a secound bounding function for the TSP problem.

2.4.9 KSReduceBound

▷ KSReduceBound(*V*, *M*) (function)

A second bounding function for the TSP problem. *V* is a partial solution, and *M* is the problem instance. This implements Algorithm 4.12.

2.4.10 KSTSP2

▷ KSTSP2(*G*, *F*) (function)

Solves the TSP problem for instance *G*, using the bounding function *F*.

2.4.11 KSMaxClique1

▷ KSMaxClique1(*G*) (function)

Adapts the function that lists the complete subgraphs of *G*, to find the size of the largest clique of *G*. This implements Algorithm 4.14.

2.4.12 KSMaxClique2

▷ `KSMaxClique2(G, F)` (function)

Finds the size of the maximum clique in the graph *G*, using the bounding function *F*. This implements Algorithm 4.19.

2.4.13 KSSizeBound

▷ `KSSizeBound(XX, G, Cl)` (function)

A bounding function for the MaxClique problem. *XX* is a complete subgraph of *G*, and *Cl* is the set of candidates to extend *XX*.

2.4.14 KSGenerateRandomGraph

▷ `KSGenerateRandomGraph(n)` (function)

Returns a list of edges of a random graph on *n* vertices. This implements Algorithm 4.20.

2.4.15 KSEdgeListToAdjacencyList

▷ `KSEdgeListToAdjacencyList(Ged, n)` (function)

Given the list of edges *Ged* of a graph with *n* vertices, returns the adjacency list of such graph.

2.4.16 KSGreedyColor

▷ `KSGreedyColor(G)` (function)

Colors the vertices of a graph *G* using a greedy strategy. This implements Algorithm 4.16.

2.4.17 KSSamplingBound

▷ `KSSamplingBound(XX, G, Cl)` (function)

A bounding function for the MaxClique problem. *XX* is a complete subgraph of *G*, and *Cl* is the set of candidates to extend *XX*. This function uses a fixed greedy coloring of the graph *G*. Implements Algorithm 4.17.

2.4.18 KSInducedSubgraph

▷ `KSInducedSubgraph(G, L)` (function)

Returns the adjacency list of the subgraph of *G* induced by the vertices in *L*.

2.4.19 KSGreedyBound

▷ `KSGreedyBound(XX, G, Cl)` (function)

A bounding function for the MaxClique problem. *XX* is a complete subgraph of *G*, and *Cl* is the set of candidates to extend *XX*. This uses a greedy coloring of the subgraph of *G* induced by *L*.

2.4.20 KSGenerateRandomGraph2

▷ `KSGenerateRandomGraph2(n, delta)` (function)

Returns the list of edges of a random graph on *n* vertices with edge density *delta*.

2.4.21 KSTSP3

▷ `KSTSP3(G, F)` (function)

Solves the TSP problem for instance *G*, using bounding function *F*, applying the branch and bound technique.

2.5 Exercises

2.5.1 KSQueens

▷ `KSQueens(size)` (function)

Solves the n queens problem for a *size* $\times$ *size* board. (Exercise 4.1.(a))

Example
```
gap> KSQueens(4);
[ 2, 4, 1, 3 ]
[ 3, 1, 4, 2 ]
```

2.5.2 KSWalks

▷ `KSWalks(number)` (function)

Finds all the walks in the plane of lenght *number*. (Exercise 4.1.(b))

Chapter 3

Heuristic Search

3.1 Uniform graph partition

3.1.1 KSRandomkSubset

▷ `KSRandomkSubset(k, n)` (function)

Returns a randomly chosen k-subset of the set of integers from 1 to n.

3.1.2 KSSelectPartition

▷ `KSSelectPartition(n)` (function)

Returns a random partition of the set $\{1,2,\ldots,2n\}$ into two subsets of size n each. (Algorithm 5.7)

3.1.3 KSCost

▷ `KSCost(G, P)` (function)

Returns the cost of the partition P of the vertices of the weighted graph G.

3.1.4 KSGain

▷ `KSGain(G, P, u, v)` (function)

P is a partition in equal parts of the vertices of G. This function calculates the change in the value of the cost function when interchanging the vertex u from the first set in the partition P with the vertex v which is in the second set of the partition.

3.1.5 KSRandomCostMatrix

▷ `KSRandomCostMatrix(n, Wmax)` (function)

Returns a symmetric n by n matrix, such that its entries are random integers from 0 to $Wmax$, and with zeros in the main diagonal.

3.1.6 KSAscend

▷ `KSAscend(G, P)` (function)

Given a partition *P* of the vertices of the weighted graph *G*, it returns a partition *Q* with less cost than *P*, by exchanging one vertex of the partition, if such partition exists. Otherwise, returns the same partition *P*.

3.2 Steiner systems

3.2.1 KSConstructBlocks

▷ `KSConstructBlocks(v, other)` (function)

Constructs a list of blocks of length *v* from the list of lists *other*. (Algorithm 5.12)

3.2.2 KSRevisedStinsonAlgorithm

▷ `KSRevisedStinsonAlgorithm(v)` (function)

Constructs a Steiner triple system with *v* points, using a hill-climbing algorithm. Implements Algorithm 5.19.

Index