Hausdorff spaces

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- Any totally ordered set with the order topology is Hausdorff.
- The product of two Hausdorff spaces is Hausdorff.
- Any subspace of a Hausdorff space is Hausdorff.
- The Sierpiński space is not Hausdorff.
- Is the cofinite topology on $\mathbb{N}$ Hausdorff?

Properties of Hausdorff spaces

Theorem (Points in Hausdorff)

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For $x \in X$, let $y \notin \{x\}$. If U, V are disjoint neigborhoods of x, y respectively, then $V \subseteq X - \{x\}$. Hence $X - \{x\}$ is open.

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Theorem (Limit points in Hausdorff)

Let X be a Hausdorff space and $A \subseteq X$. Then $x \in X$ is a limit point of A if and only if every neigborhood of x contains infinitely many points of A.

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Exercises

- We say that a space is a T_1 space if for any two different points x, y, each has a neighborhood that does not contain the other point. Show that all singletons are closed in X if and only if X is T_1.

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- We say that a space is a T_1 space if for any two different points x, y, each has a neighborhood that does not contain the other point. Show that all singletons are closed in X if and only if X is T_1.
- Show that If X is Hausdorff, then X is T_1. Give an example of a T_1 space that is not Hausdorff.
- We say that a space is a T_0 space if for any two different points x, y, one of them has a neighborhood that does not contain the other point. Show that a T_1 space is T_0. Give an example of a T_0 space that is not T_1. Give an example of a space that is not T_0.

Links

- Hausdorff space - Wikipedia, the free encyclopedia