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Regular dodecahedron

A **regular dodecahedron** or **pentagonal dodecahedron** is a dodecahedron that is regular, which is composed of 12 regular pentagonal faces, three meeting at each vertex. It is one of the five Platonic solids. It has 12 faces, 20 vertices, 30 edges, and 160 diagonals (60 face diagonals, 100 space diagonals).[2] It is represented by the Schläfli symbol {5,3}.

Dimensions

If the edge length of a regular dodecahedron is a, the radius of a circumscribed sphere (one that touches the regular dodecahedron at all vertices) is

$$r_u = a\frac{\sqrt{3}}{4}\left(1+\sqrt{5}\right) \approx 1.401\,258\,538 \cdot a$$

(sequence A179296 in the OEIS)

and the radius of an inscribed sphere (tangent to each of the regular dodecahedron's faces) is

$$r_i = a\frac{1}{2}\sqrt{\frac{5}{2}+\frac{11}{10}\sqrt{5}} \approx 1.113\,516\,364 \cdot a$$

while the midradius, which touches the middle of each edge, is

$$r_m = a\frac{1}{4}\left(3+\sqrt{5}\right) \approx 1.309\,016\,994 \cdot a$$

These quantities may also be expressed as

$$r_u = a\,\frac{\sqrt{3}}{2}\phi$$
$$r_i = a\,\frac{\phi^2}{2\sqrt{3-\phi}}$$

$$r_m = a\,\frac{\phi^2}{2}$$

where ϕ is the golden ratio.

Note that, given a regular dodecahedron of edge length one, r_u is the radius of a circumscribing sphere about a cube of edge length ϕ, and r_i is the apothem of a regular pentagon of edge length ϕ.

Surface area and volume

The surface area A and the volume V of a regular dodecahedron of edge length a are:

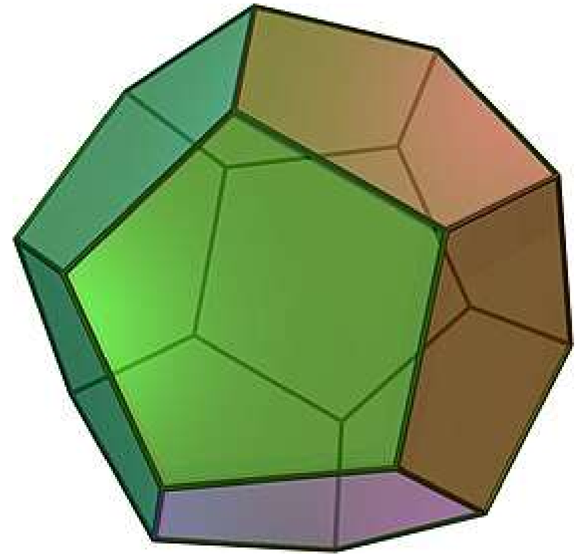
$$A = 3\sqrt{25+10\sqrt{5}}a^2 \approx 20.645\,728\,807a^2$$
$$V = \frac{1}{4}(15+7\sqrt{5})a^3 \approx 7.663\,118\,9606a^3$$

Additionally, the surface area and volume of a regular dodecahedron are related to the golden ratio. A dodecahedron with an edge length of one unit has the properties:[3]

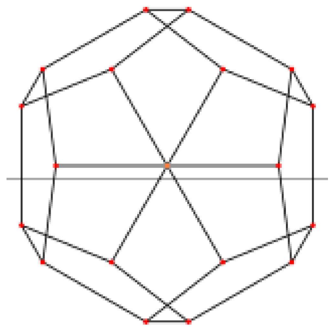
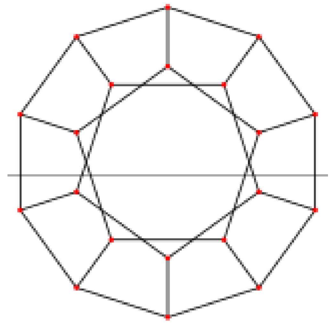
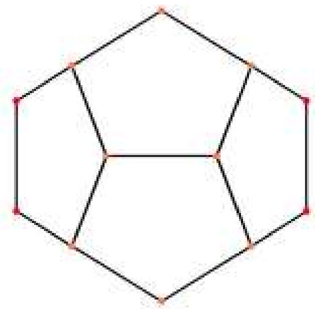
$$A = \frac{15\phi}{\sqrt{3-\phi}}$$
$$V = \frac{5\phi^3}{6-2\phi}$$

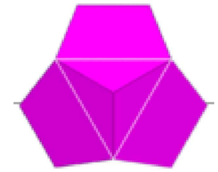
Two-dimensional symmetry projections

The *regular dodecahedron* has two high orthogonal projections, centered, on vertices and pentagonal faces, correspond to the A_2 and H_2 Coxeter planes. The edge-center projection has two orthogonal lines of reflection.

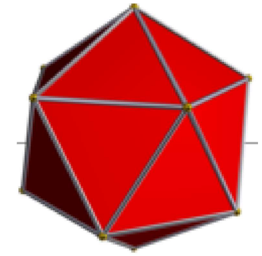
Regular dodecahedron	
 (Click here for rotating model)	
Type	Platonic solid
Elements	F = 12, E = 30 V = 20 (χ = 2)
Faces by sides	12{5}
Conway notation	D
Schläfli symbols	{5,3}
Face configuration	V3.3.3.3.3
Wythoff symbol	3 | 2 5
Coxeter diagram	
Symmetry	I_h, H_3, [5,3], (*532)
Rotation group	I, $[5,3]^+$, (532)
References	U_{23}, C_{26}, W_5
Properties	regular, convex
Dihedral angle	116.56505° = arccos($-1/\sqrt{5}$)

Orthogonal projections

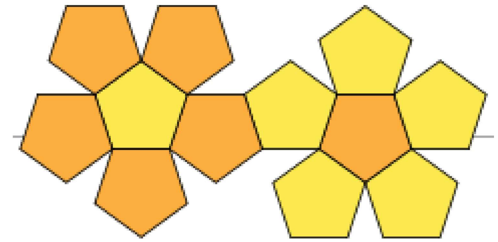
Centered by	Vertex	Face	Edge
Image			
Projective symmetry	[[3]] = [6]	[[5]] = [10]	[2]



5.5.5
(Vertex figure)

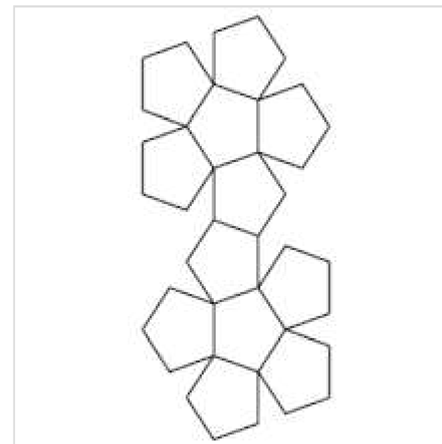


Regular icosahedron
(dual polyhedron)

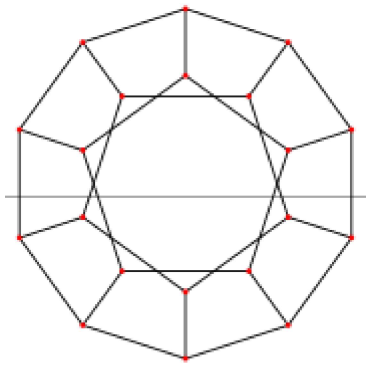
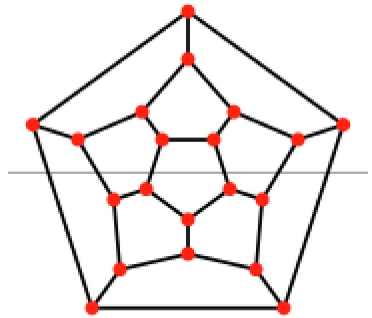
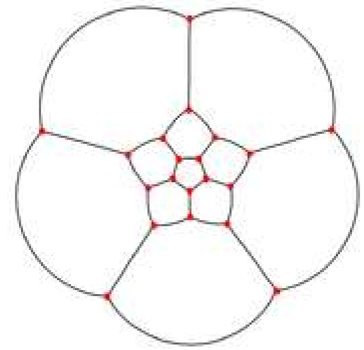
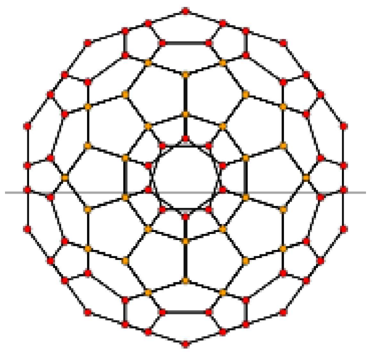
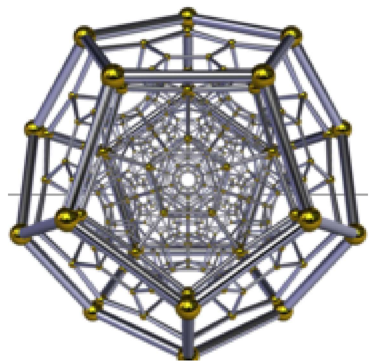
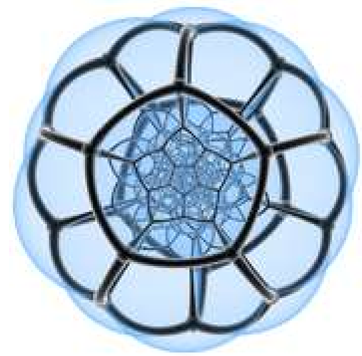


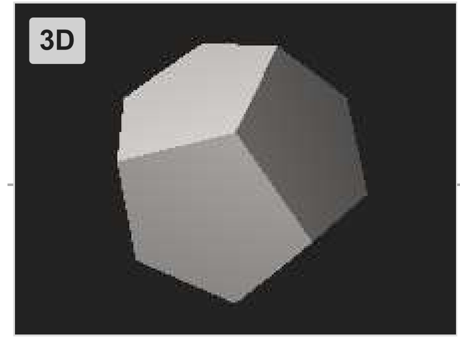
Net

In perspective projection, viewed on top of a pentagonal face, the regular dodecahedron can be seen as a linear-edged Schlegel diagram, or stereographic projection as a spherical polyhedron. These projections are also used in showing the four-dimensional 120-cell, a regular 4-dimensional polytope, constructed from 120 dodecahedra, projecting it down to 3-dimensions.

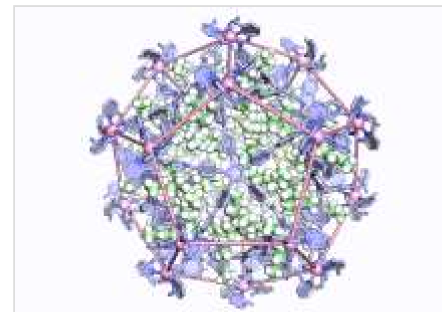


Animation of a net of a regular (pentagonal) dodecahedron being folded

Projection	Orthogonal projection	Perspective projection	
		Schlegel diagram	Stereographic projection
Regular dodecahedron			
Dodecaplex (120-cell)			



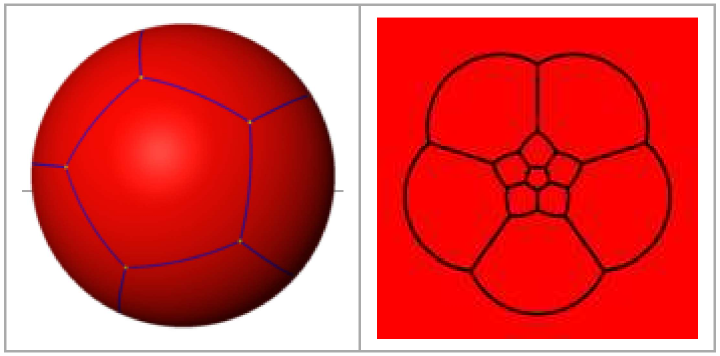
3D model of a regular dodecahedron



Crystal structure of **Co20L12** dodecahedron reported by Kai Wu, Jonathan Nitschke and co-workers at University of Cambridge in *Nat. Synth.* **2023**, DOI:10.1038/s44160-023-00276-9 [1]

Spherical tiling

The regular dodecahedron can also be represented as a spherical tiling.



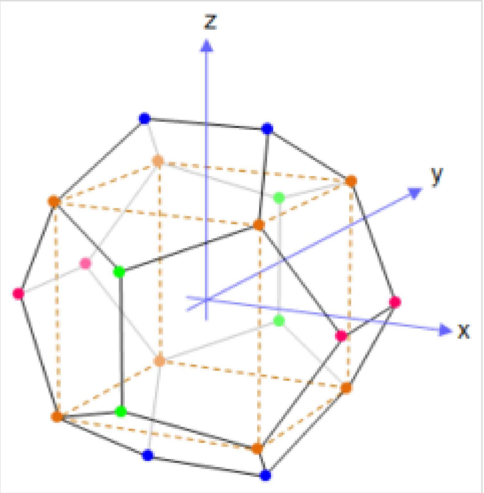
Orthographic projection	Stereographic projection

Cartesian coordinates

The following Cartesian coordinates define the 20 vertices of a regular dodecahedron centered at the origin and suitably scaled and oriented:[4]

$(\pm 1, \pm 1, \pm 1)$
$(0, \pm\phi, \pm\frac{1}{\phi})$
$(\pm\frac{1}{\phi}, 0, \pm\phi)$
$(\pm\phi, \pm\frac{1}{\phi}, 0)$

where $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$ is the golden ratio. The edge length is $\frac{2}{\phi} = \sqrt{5} - 1$. The circumradius is $\sqrt{3}$.



Vertex coordinates:

- The orange vertices lie at $(\pm 1, \pm 1, \pm 1)$ and form a cube (dotted lines).
- The green vertices lie at $(0, \pm\phi, \pm\frac{1}{\phi})$ and form a rectangle on the *yz*-plane.
- The blue vertices lie at $(\pm\frac{1}{\phi}, 0, \pm\phi)$ and form a rectangle on the *xz*-plane.
- The pink vertices lie at $(\pm\phi, \pm\frac{1}{\phi}, 0)$ and form a rectangle on the *xy*-plane.

The distance between adjacent vertices is $\frac{2}{\phi}$, and the distance from the origin to any vertex is $\sqrt{3}$.
$\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.

Facet-defining equations

Similar to the symmetry of the vertex coordinates, the equations of the twelve facets of the regular dodecahedron also display symmetry in their coefficients:

$x \pm \phi y = \pm\phi^2$
$y \pm \phi z = \pm\phi^2$
$z \pm \phi x = \pm\phi^2$

Properties

- The dihedral angle of a regular dodecahedron is 2 arctan(ϕ) or approximately 116.565° (where again $\phi = \frac{1+\sqrt{5}}{2}$, the golden ratio). OEIS: A137218 Note that the tangent of the dihedral angle is exactly −2.
- If the original regular dodecahedron has edge length 1, its dual icosahedron has edge length ϕ.
- If the five Platonic solids are built with same volume, the regular dodecahedron has the shortest edges. It is the *roundest* of the five Platonic solids, enclosing the most volume within the same radius.

- It has 43,380 nets.
- The map-coloring number of a regular dodecahedron's faces is 4.
- The distance between the vertices on the same face not connected by an edge is ϕ times the edge length, because the diagonal of a pentagon is ϕ times its edge length.
- If two edges share a common vertex, then the midpoints of those edges form a 36-72-72 golden triangle with the body center.

As a configuration

This configuration matrix represents the dodecahedron. The rows and columns correspond to vertices, edges, and faces. The diagonal numbers say how many of each element occur in the whole dodecahedron. The nondiagonal numbers say how many of the column's element occur in or at the row's element.[5][6]

$$\begin{bmatrix} 20 & 3 & 3 \\ 2 & 30 & 2 \\ 5 & 5 & 12 \end{bmatrix}$$

Here is the configuration expanded with k-face elements and k-figures. The diagonal element counts are the ratio of the full Coxeter group H_3, order 120, divided by the order of the subgroup with mirror removal.

H_3		*k*-face	f_k	f_0	f_1	f_2	*k*-fig	Notes
A_2	×	( )	f_0	20	3	3	{3}	$H_3/A_2 = 120/6 = 20$
A_1A_1	×	{ }	f_1	2	30	2	{ }	$H_3/A_1A_1 = 120/4 = 30$
H_2	×	{5}	f_2	5	5	12	( )	$H_3/H_2 = 120/10 = 12$

Geometric relations

The *regular dodecahedron* is the third in an infinite set of truncated trapezohedra which can be constructed by truncating the two axial vertices of a pentagonal trapezohedron.

The stellations of the regular dodecahedron make up three of the four Kepler–Poinsot polyhedra.

A rectified regular dodecahedron forms an icosidodecahedron.

The regular dodecahedron has icosahedral symmetry I_h, Coxeter group [5,3], order 120, with an abstract group structure of $A_5 \times Z_2$.

Relation to the regular icosahedron

The dodecahedron and icosahedron are dual polyhedra. A regular dodecahedron has 12 faces and 20 vertices, whereas a regular icosahedron has 20 faces and 12 vertices. Both have 30 edges.

When a regular dodecahedron is inscribed in a sphere, it occupies more of the sphere's volume (66.49%) than an icosahedron inscribed in the same sphere (60.55%).

A regular dodecahedron with edge length 1 has more than three and a half times the volume of an icosahedron with the same length edges (7.663... compared with 2.181...), which ratio is approximately 3.512 461 179 75, or in exact terms: $\frac{3}{5}(3\phi + 1)$ or $(1.8\phi + 0.6)$.

Relation to the nested cube

A cube can be embedded within a regular dodecahedron, affixed to eight of its equidistant vertices, in five different positions.[7] In fact, five cubes may overlap and interlock inside the regular dodecahedron to result in the compound of five cubes.

The ratio of the edge of a regular dodecahedron to the edge of a cube embedded inside such a regular dodecahedron is $1 : \phi$, or $(\phi - 1) : 1$.

The ratio of a regular dodecahedron's volume to the volume of a cube embedded inside such a regular dodecahedron is $1 : \frac{2}{2 + \phi}$, or $\frac{1 + \phi}{2} : 1$, or $(5 + \sqrt{5}) : 4$.

For example, an embedded cube with a volume of 64 (and edge length of 4), will nest within a regular dodecahedron of volume $64 + 32\phi$ (and edge length of $4\phi - 4$).

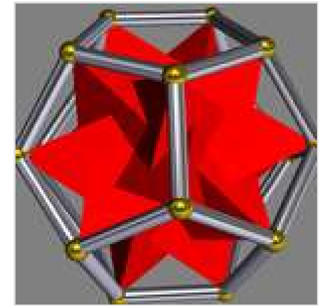
Thus, the difference in volume between the encompassing regular dodecahedron and the enclosed cube is always one half the volume of the cube times ϕ.

From these ratios are derived simple formulas for the volume of a regular dodecahedron with edge length a in terms of the golden mean:

$$V = (a\phi)^3 \cdot \tfrac{1}{4}(5 + \sqrt{5})$$

$V = \frac{1}{4}(14\phi + 8)a^3$

Relation to the regular tetrahedron



Five tetrahedra inscribed in a dodecahedron. Five opposing tetrahedra (not shown) can also be inscribed.

As two opposing tetrahedra can be inscribed in a cube, and five cubes can be inscribed in a dodecahedron, ten tetrahedra in five cubes can be inscribed in a dodecahedron: two opposing sets of five, with each set covering all 20 vertices and each vertex in two tetrahedra (one from each set, but not the opposing pair).

> Just as a tetrahedron can be inscribed in a cube, so a cube can be inscribed in a dodecahedron. By reciprocation, this leads to an octahedron circumscribed about an icosahedron. In fact, each of the twelve vertices of the icosahedron divides an edge of the octahedron according to the "golden section". Given the icosahedron, the circumscribed octahedron can be chosen in five ways, giving a compound of five octahedra, which comes under our definition of stellated icosahedron. (The reciprocal compound, of five cubes whose vertices belong to a dodecahedron, is a stellated triacontahedron.) Another stellated icosahedron can at once be deduced, by stellating each octahedron into a stella octangula, thus forming a compound of ten tetrahedra. Further, we can choose one tetrahedron from each stella octangula, so as to derive a compound of five tetrahedra, which still has all the rotation symmetry of the icosahedron (i.e. the icosahedral group), although it has lost the reflections. By reflecting this figure in any plane of symmetry of the icosahedron, we obtain the complementary set of five tetrahedra. These two sets of five tetrahedra are enantiomorphous, i.e. not directly congruent, but related like a pair of shoes. [Such] a figure which possesses no plane of symmetry (so that it is enantiomorphous to its mirror-image) is said to be *chiral*.[8]

Relation to the golden rectangle

Golden rectangles of ratio (ϕ + 1) : 1 and ϕ : 1 also fit perfectly within a regular dodecahedron.[9] In proportion to this golden rectangle, an enclosed cube's edge is ϕ, when the long length of the rectangle is $\phi + 1$ (or ϕ^2) and the short length is 1 (the edge shared with the regular dodecahedron).

In addition, the center of each face of the regular dodecahedron form three intersecting golden rectangles.[10]

Relation to the 6-cube and rhombic triacontahedron

It can be projected to 3D from the 6-dimensional 6-demicube using the same basis vectors that form the hull of the rhombic triacontahedron from the 6-cube. Shown here including the inner 12 vertices, which are not connected by the outer hull edges of 6D norm length $\sqrt{2}$, form a regular icosahedron.

Projection of 6-demicube into regular dodecahedral envelope

The 3D projection basis vectors [u,v,w] used are:

$$u = (1, \phi, 0, -1, \phi, 0)$$
$$v = (\phi, 0, 1, \phi, 0, -1)$$
$$w = (0, 1, \phi, 0, -1, \phi)$$

History and uses

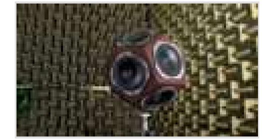
Roman dodecahedron

Regular dodecahedral objects have found some practical applications, and have also played a role in the visual arts and in philosophy.

Iamblichus states that Hippasus, a Pythagorean, perished in the sea, because he boasted that he first divulged "the sphere with the twelve pentagons".[11] In *Theaetetus*, a dialogue of Plato, Plato hypothesized that the classical elements were made of the five uniform regular solids; these later became known as the platonic solids. Of the fifth Platonic solid, the dodecahedron, Plato obscurely remarked, "...the god used [it] for arranging the constellations on the whole heaven". Timaeus (c. 360 BC), as a personage of Plato's dialogue, associates the other four platonic solids with the four classical elements, adding that there is a fifth solid pattern which, though commonly associated with the regular dodecahedron, is never directly mentioned as such; "this God used in the delineation of the universe."[12] Aristotle also postulated that the heavens were made of a fifth element, which he called aithêr (*aether* in Latin, *ether* in American English).

Theaetetus gave a mathematical description of all five and may have been responsible for the first known proof that no other convex regular polyhedra exist. Euclid completely mathematically described the Platonic solids in the *Elements*, the last book (Book XIII) of which is devoted to their properties. Propositions 13–17 in Book XIII describe the construction of the tetrahedron, octahedron, cube, icosahedron, and dodecahedron in that order. For each solid Euclid finds the ratio of the diameter of the circumscribed sphere to the edge length. In Proposition 18 he argues that there are no further convex regular polyhedra.

Regular dodecahedra have been used as dice and probably also as divinatory devices. During the Hellenistic era, small, hollow bronze Roman dodecahedra were made and have been found in various Roman ruins in Europe. Their purpose is not certain.



Omnidirectional sound source

In 20th-century art, dodecahedra appear in the work of M. C. Escher, such as his lithographs *Reptiles* (1943) and *Gravitation* (1952). In Salvador Dalí's painting *The Sacrament of the Last Supper* (1955), the room is a hollow regular dodecahedron. Gerard Caris based his entire artistic oeuvre on the regular dodecahedron and the pentagon, which is presented as a new art movement coined as Pentagonism.

A climbing wall consisting of three dodecahedral pieces

In modern role-playing games, the regular dodecahedron is often used as a twelve-sided die, one of the more common polyhedral dice.

Immersive Media Company, a former Canadian digital imaging company, made the Dodeca 2360 camera, the world's first 360° full-motion camera which captures high-resolution video from every direction simultaneously at more than 100 million pixels per second or 30 frames per second. It is based on regular dodecahedron.

The Megaminx twisty puzzle, alongside its larger and smaller order analogues, is in the shape of a regular dodecahedron.

In the children's novel *The Phantom Tollbooth*, the regular dodecahedron appears as a character in the land of Mathematics. Each of his faces wears a different expression – *e.g.* happy, angry, sad – which he swivels to the front as required to match his mood.

In nature and supramolecules

The fossil coccolithophore *Braarudosphaera bigelowii* (see figure), a unicellular coastal phytoplanktonic alga, has a calcium carbonate shell with a regular dodecahedral structure about 10 micrometers across.[13]

Some quasicrystals and cages have dodecahedral shape (see figure). Some regular crystals such as garnet and diamond are also said to exhibit "dodecahedral" habit, but this statement actually refers to the rhombic dodecahedron shape.[14] [1]

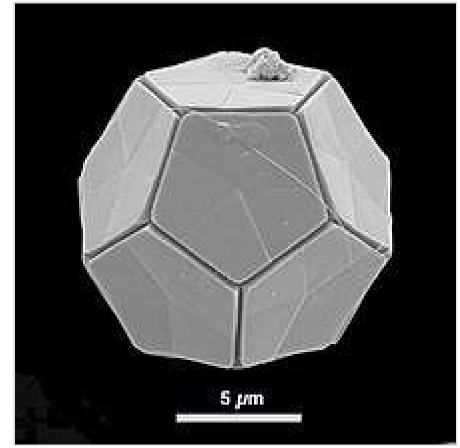
Shape of the universe

Various models have been proposed for the global geometry of the universe. In addition to the primitive geometries, these proposals include the Poincaré dodecahedral space, a positively curved space consisting of a regular dodecahedron whose opposite faces correspond (with a small twist). This was proposed by Jean-Pierre Luminet and colleagues in 2003,[15][16] and an optimal orientation on the sky for the model was estimated in 2008.[17]

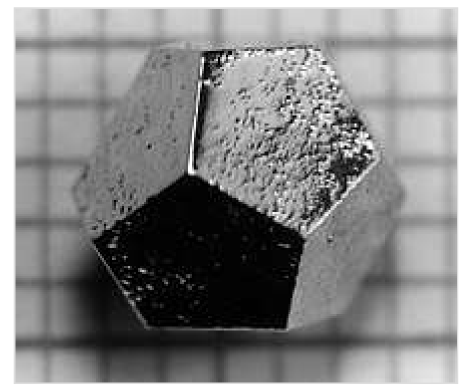
In Bertrand Russell's 1954 short story "The Mathematician's Nightmare: The Vision of Professor Squarepunt", the number 5 said: "I am the number of fingers on a hand. I make pentagons and pentagrams. And but for me dodecahedra could not exist; and, as everyone knows, the universe is a dodecahedron. So, but for me, there could be no universe."

Space filling with cube and bilunabirotunda

Regular dodecahedra fill space with cubes and bilunabirotundas (Johnson solid 91), in the ratio of 1 to 1 to 3.[18][19] The dodecahedra alone make a lattice of edge-to-edge pyritohedra. The bilunabirotundas fill the rhombic gaps. Each cube meets six bilunabirotundas in three orientations.

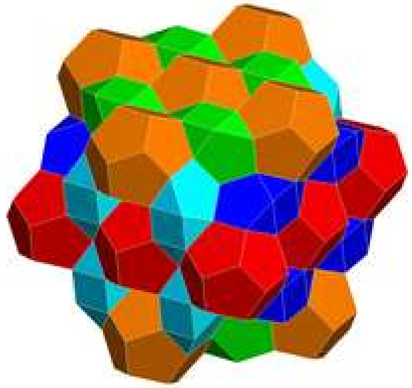


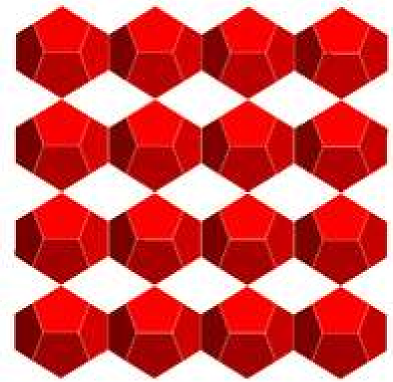
The fossil record of the coccolithophore *Braarudosphaera bigelowii* goes back 100 million years

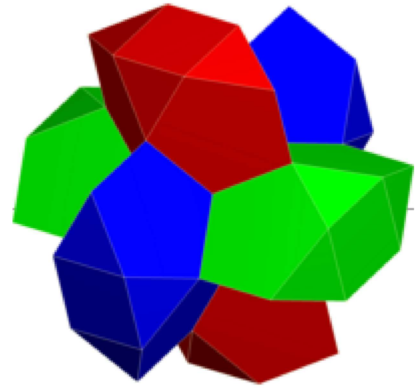


The faces of a Holmium–magnesium–zinc (Ho-Mg-Zn) quasicrystal are true regular pentagons


Block model

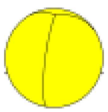
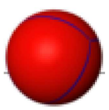
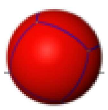
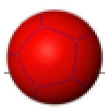
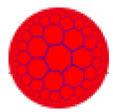
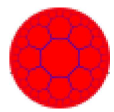
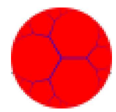
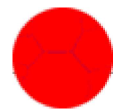
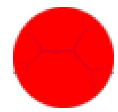
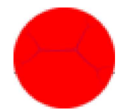
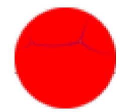



Lattice of dodecahedra

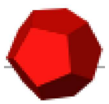
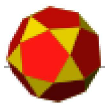
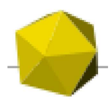
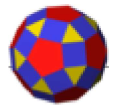
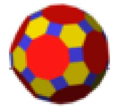
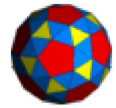
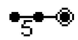
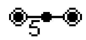
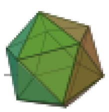
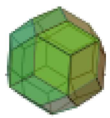
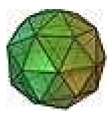
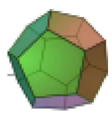
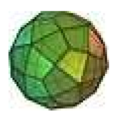

6 bilunabirotundas around a cube

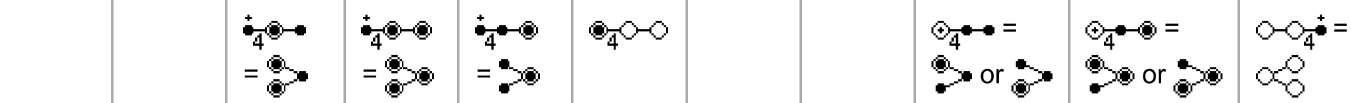
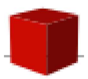
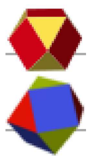
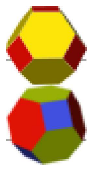
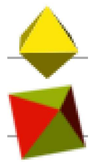
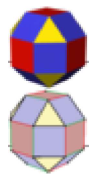
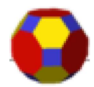
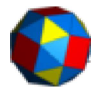
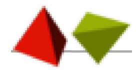
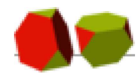
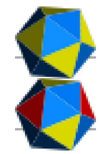
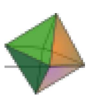
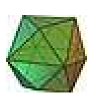
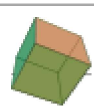
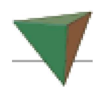
Related polyhedra and tilings

The regular dodecahedron is topologically related to a series of tilings by vertex figure n^3.

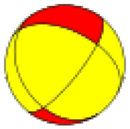
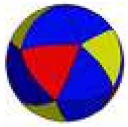
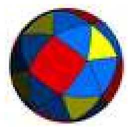
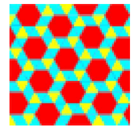
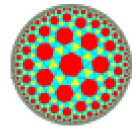
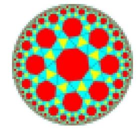
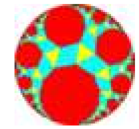
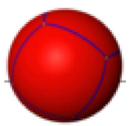
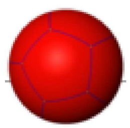
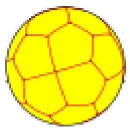
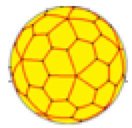
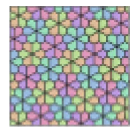
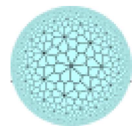
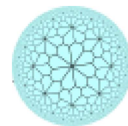
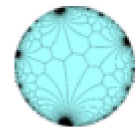
***n*32 symmetry mutation of regular tilings: {*n*,3}** [show]											
Spherical				**Euclidean**	**Compact hyperb.**		**Paraco.**	**Noncompact hyperbolic**			
											
{2,3}	{3,3}	{4,3}	{5,3}	{6,3}	{7,3}	{8,3}	{∞,3}	{12i,3}	{9i,3}	{6i,3}	{3i,3}

The regular dodecahedron can be transformed by a truncation sequence into its dual, the icosahedron:

Family of uniform icosahedral polyhedra							[show]
Symmetry: [5,3], (*532)							$[5,3]^+$, (532)
							
							
{5,3}	t{5,3}	r{5,3}	t{3,5}	{3,5}	rr{5,3}	tr{5,3}	sr{5,3}
Duals to uniform polyhedra							
							
V5.5.5	V3.10.10	V3.5.3.5	V5.6.6	V3.3.3.3.3	V3.4.5.4	V4.6.10	V3.3.3.3.5

Uniform octahedral polyhedra [show]										
Symmetry: [4,3], (*432)							$[4,3]^+$ (432)	$[1^+,4,3]$ = [3,3] (*332)		$[3^+,4]$ (3*2)
{4,3}	t{4,3}	r{4,3} $r\{3^{1,1}\}$	t{3,4} $t\{3^{1,1}\}$	{3,4} $\{3^{1,1}\}$	rr{4,3} $s_2\{3,4\}$	tr{4,3}	sr{4,3}	h{4,3} {3,3}	$h_2\{4,3\}$ t{3,3}	s{3,4} $s\{3^{1,1}\}$
										
										
										
Duals to uniform polyhedra										
$V4^3$	$V3.8^2$	$V(3.4)^2$	$V4.6^2$	$V3^4$	$V3.4^3$	V4.6.8	$V3^4.4$	$V3^3$	$V3.6^2$	$V3^5$
										
										
										

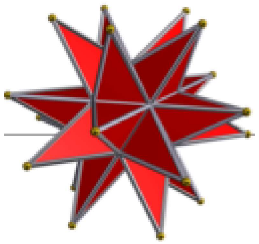
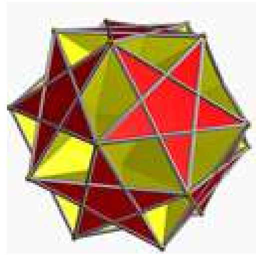
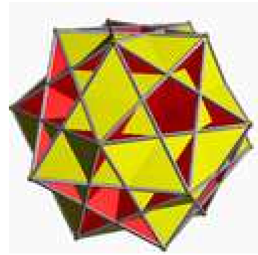
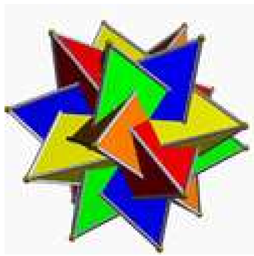
The regular dodecahedron is a member of a sequence of otherwise non-uniform polyhedra and tilings, composed of pentagons with face configurations (V3.3.3.3.*n*). (For $n > 6$, the sequence consists of tilings of the hyperbolic plane.) These face-transitive figures have (*n*32) rotational symmetry.

*n*32 symmetry mutations of snub tilings: *3.3.3.3.n* [show]								
Symmetry *n*32	**Spherical**				**Euclidean**	**Compact hyperbolic**		**Paracomp.**
	232	**332**	**432**	**532**	**632**	**732**	**832**	**∞32**
Snub figures								
Config.	**3.3.3.3.2**	**3.3.3.3.3**	**3.3.3.3.4**	**3.3.3.3.5**	**3.3.3.3.6**	**3.3.3.3.7**	**3.3.3.3.8**	**3.3.3.3.∞**
Gyro figures								
Config.	V3.3.3.3.2	V3.3.3.3.3	V3.3.3.3.4	V3.3.3.3.5	V3.3.3.3.6	V3.3.3.3.7	V3.3.3.3.8	V3.3.3.3.∞

Vertex arrangement

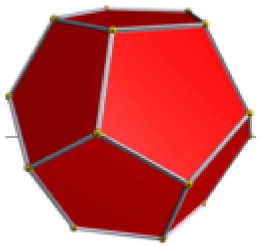
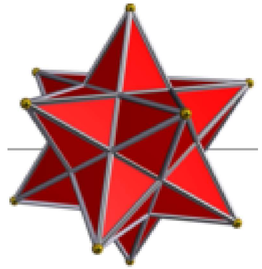
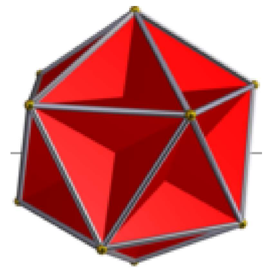
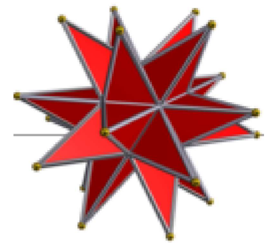
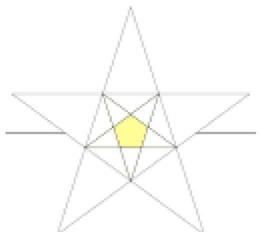
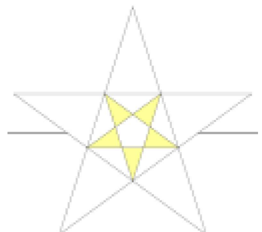
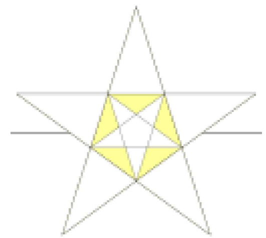
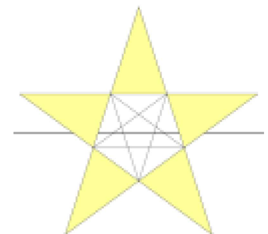
The regular dodecahedron shares its vertex arrangement with four nonconvex uniform polyhedra and three uniform polyhedron compounds.

Five cubes fit within, with their edges as diagonals of the regular dodecahedron's faces, and together these make up the regular polyhedral compound of five cubes. Since two tetrahedra can fit on alternate cube vertices, five and ten tetrahedra can also fit in a regular dodecahedron.

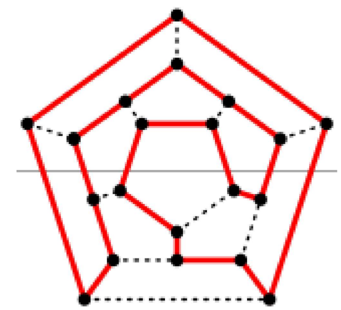
 Great stellated dodecahedron	 Small ditrigonal icosidodecahedron	 Ditrigonal dodecadodecahedron	 Great ditrigonal icosidodecahedron
 Compound of five cubes	 Compound of five tetrahedra	 Compound of ten tetrahedra	

Stellations

The 3 stellations of the regular dodecahedron are all regular (nonconvex) polyhedra: (Kepler–Poinsot polyhedra)

	0	1	2	3
Stellation	 *Regular dodecahedron*	 Small stellated dodecahedron	 Great dodecahedron	 Great stellated dodecahedron
Facet diagram				

Dodecahedral graph

Regular dodecahedron graph	
 A Hamiltonian cycle in a dodecahedron.	
Vertices	20
Edges	30
Radius	5

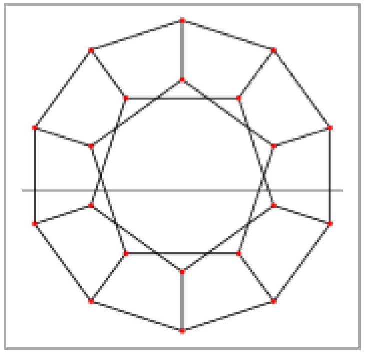
The skeleton of the dodecahedron (the vertices and edges) form a graph. It is one of 5 Platonic graphs, each a skeleton of its Platonic solid.

This graph can also be constructed as the generalized Petersen graph $G(10,2)$ where the vertices of a decagon are connected to those of two pentagons, one pentagon connected to odd vertices of the decagon and the other pentagon connected to the even vertices. Geometrically, this can be visualized as the 10-vertex equatorial belt of the dodecahedron connected to the two 5-vertex polar regions, one on each side.

The high degree of symmetry of the polygon is replicated in the properties of this graph, which is distance-transitive, distance-regular, and symmetric. The automorphism group has order 120. The vertices can be colored with 3 colors, as can the edges, and the diameter is 5.[21]

The dodecahedral graph is Hamiltonian – there is a cycle containing all the vertices. Indeed, this name derives from a mathematical game invented in 1857 by William Rowan Hamilton, the icosian game. The game's object was to find a Hamiltonian cycle along the edges of a dodecahedron.

Orthogonal projection



Diameter	5
Girth	5
Automorphisms	120 ($A_5 \times Z_2$)[20]
Chromatic number	3
Properties	Hamiltonian, regular, symmetric, distance-regular, distance-transitive, 3-vertex-connected, planar graph

Table of graphs and parameters

See also

- 120-cell, a regular polychoron (4D polytope whose surface consists of 120 dodecahedral cells)
- *Braarudosphaera bigelowii* − A dodecahedron shaped coccolithophore (a unicellular phytoplankton algae).
- Dodecahedrane (molecule)
- Pentakis dodecahedron
- Snub dodecahedron
- Truncated dodecahedron

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- The Uniform Polyhedra (http://www.mathconsult.ch/showroom/unipoly/)
- Origami Polyhedra (https://www.flickr.com/photos/pascalin/sets/72157594234292561/) – Models made with Modular Origami
- Dodecahedron (http://polyhedra.org/poly/show/3/dodecahedron) – 3-d model that works in your browser
- Virtual Reality Polyhedra (http://www.georgehart.com/virtual-polyhedra/vp.html) The Encyclopedia of Polyhedra
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- Stella: Polyhedron Navigator (http://www.software3d.com/Stella.php): Software used to create some of the images on this page.
- How to make a dodecahedron from a Styrofoam cube (http://video.fc2.com/content/20141015mMG9QR5R)
- The Greek, Indian, and Chinese Elements – Seven Element Theory (http://www.friesian.com/elements.htm)

Fundamental convex regular and uniform polytopes in dimensions 2–10					[hide]
Family	A_n	B_n	$I_2(p)$ / D_n	E_6 / E_7 / E_8 / F_4 / G_2	H_n
Regular polygon	Triangle	Square	p-gon	Hexagon	Pentagon
Uniform polyhedron	Tetrahedron	Octahedron • Cube	Demicube		Dodecahedron • Icosahedron
Uniform polychoron	Pentachoron	16-cell • Tesseract	Demitesseract	24-cell	120-cell • 600-cell
Uniform 5-polytope	5-simplex	5-orthoplex • 5-cube	5-demicube		
Uniform 6-polytope	6-simplex	6-orthoplex • 6-cube	6-demicube	1_{22} • 2_{21}	
Uniform 7-polytope	7-simplex	7-orthoplex • 7-cube	7-demicube	1_{32} • 2_{31} • 3_{21}	
Uniform 8-polytope	8-simplex	8-orthoplex • 8-cube	8-demicube	1_{42} • 2_{41} • 4_{21}	
Uniform 9-polytope	9-simplex	9-orthoplex • 9-cube	9-demicube		
Uniform 10-polytope	10-simplex	10-orthoplex • 10-cube	10-demicube		
Uniform *n*-polytope	*n*-simplex	*n*-orthoplex • *n*-cube	*n*-demicube	1_{k2} • 2_{k1} • k_{21}	*n*-pentagonal polytope
Topics: Polytope families • Regular polytope • List of regular polytopes and compounds					

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