MANE 6760 (FEM for Fluid Dyn.) Fall 2022: Final Project

Nov 25, 2022

Due: 11pm on Fri/Dec 9, 2022

Weight is 25% of the total grade points
In each problem state all the assumptions/choices and show the necessary steps
Submissions must be made on Gradescope
Refer to the following link for necessary input files and updates:
`https://www.scorec.rpi.edu/~sahni/MANE6760/F22/Projects/Final/question/`

1. (10 points) Consider the unsteady, 1D, linear, scalar AD equation. Use the SUPG formulation for linear finite elements and backward Euler time-integration scheme. Set problem parameters as: $a_x = 1.0, \kappa = 2.5e-2$ and $s = 0$. Consider a domain length of $L = 1$, i.e., $x \in [0, L = 1]$, and time duration of $T = L^2/\kappa$, i.e., $t \in [0, T = L^2/\kappa = 40.0]$. Set BCs as: $\phi(x = 0, t) = \phi_0(t) = 0$ and $\phi(x = L, t) = \phi_L(t) = g(t)$, where $g(t) = \min(1.25t/T, 1)$, and IC as: $\phi_{IC}(x, t = 0) = 0$. Use a uniform mesh with $N_e = 10$ elements and a uniform time-step size with $N_t = 50$. Provide the following:

   (a) Provide the plot of the FE solution at $n = 0$ (IC), 10, 20, 30, 40 and 50

   (b) Provide the Python code

2. (40 points) Consider the steady, 1D, non-linear, scalar ADR equation:

$$\mathcal{L}(\phi) = a_x\phi_{,x} - \kappa\phi_{,xx} + c(\phi)\phi = s$$

   Use the VMS formulation for linear finite elements such that:

$$\hat{\mathcal{L}}(\cdot) = \mathcal{L}^*(\cdot) = -a_x(\cdot)_{,x} + c(\phi)(\cdot)$$

   Set problem parameters as: $a_x = 1.0, \kappa = 1.0e-1$, $c = c_0(1.0 + 0.01\phi)$, $c_0 = 1.0e+2$ and $s = 10.0$. Consider a domain length of $L = 1$, i.e., $x \in [0, L = 1]$. Set BCs as: $\phi(x = 0) = \phi_0 = 0$ and $\phi(x = L) = \phi_L = 1.0$. Use a uniform mesh with $N_e = 8$ elements. Set non-linear tolerances to be $1.0e-6$ and max non-linear iterations to be 100, and take initial guess for the FE solution to be one at all *interior* mesh nodes (and make sure to satisfy BCs). Make use of an appropriate quadrature rule. Derive and provide the following:

   (a) Derive the expression for G_a^e

   (b) Derive the expression for $\frac{\partial G_a^e}{\partial \hat{\phi}_b^e}$

   (c) Provide the non-linear convergence history as well as the plot of the FE solution

   (d) Provide the Python code