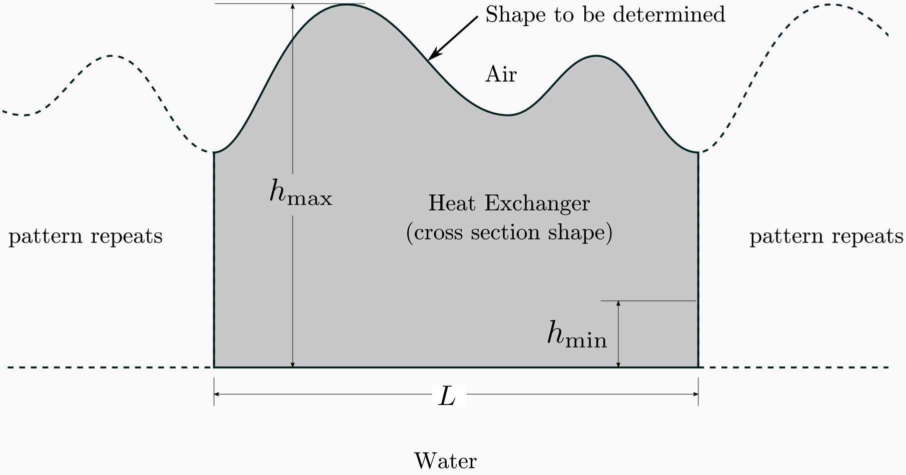
Project #1 Model

Reminder

Objective: heat-flux per unit length



The Heat Equation

We will model the flow of energy from the water to the air using the 2-dimensional steady heat equation

Fourier's Law

$$\begin{aligned} \nabla \cdot (k \nabla T) &= 0, \qquad \forall x \in \Omega \\ T(x, y = 0) &= T_{\text{water}}, \\ T(x, y(x)) &= T_{\text{air}}, \\ T(x, y) &= T(x + L, y) \end{aligned}$$

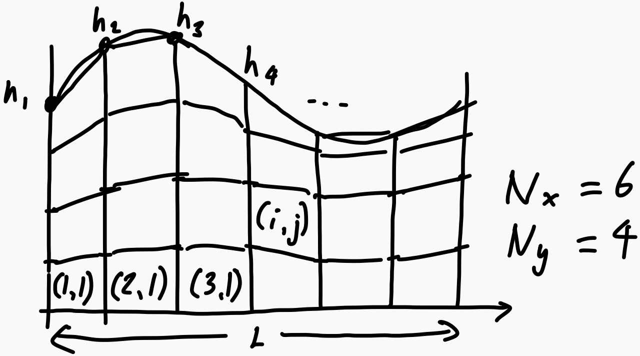
where T is the temperature in Kelvin and k is the thermal conductivity in $\mathrm{W/(mK)}$. Ω denotes the region of the heat exchanger that we are modeling.

Finite-Volume Discretization

The heat equation is approximated using a finite-volume discretization:

First, we subdivide the domain of interest into a finite number of quadrilaterals.

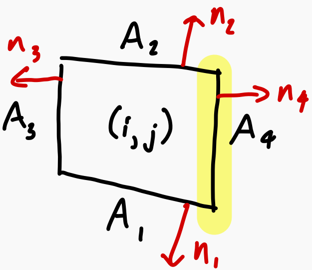
- Each quadrilateral is a finite "volume" over which the heat equation must be satisfied.



Finite-Volume Discretization (cont.)

Next, we integrate the heat equation over on of these quadrilaterals. For example, consider the volume located at the spatial indices (i, j):

$$\iint_{V_{i,j}} \nabla \cdot (k \nabla T) \; dV = \int_{A_{i,j}} k(\nabla T) \cdot \vec{n} dA$$
$$= \sum_{f=1}^{4} \int_{A_f} k(\nabla T) \cdot \vec{n} dA.$$



Finite-Volume Discretization (cont.)

The "surface" integrals are approximated by replacing the normal derivatives $(\nabla T) \cdot \vec{n}$ with a finite-difference approximation.

For example, for a vertical side on the right of the quadrilateral,

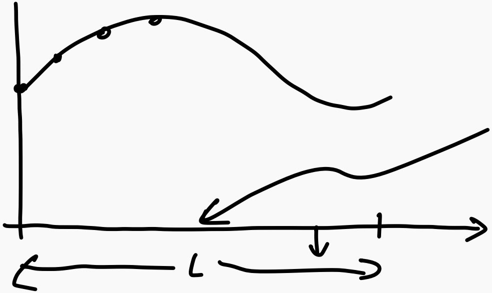
$$\int_{A_f} k(\nabla T) \cdot \vec{n} dA \approx k \frac{T_{i+1,j} - T_{i,j}}{\Delta x} \Delta y$$

At the boundaries, we account for the boundary temperatures in an appropriate way.

Finite-Volume Discretization (cont.)

After all the finite-volume equations are determined, we end up with a large linear system of equations, which we can solve to find the $T_{i,j}$, the temperatures at the center of each volume.

Once we have the temperatures, we can compute the heat flux, which is the integral $\int k(\nabla T) \cdot \vec{n} dA$ over the top or bottom of the domain.



$$\int k(\nabla T) \cdot \hat{n}\, dA$$
$$= \int k \frac{\partial T}{\partial y} dx = \text{Flux}$$

Matlab Implementation

This finite-volume model is implemented by the (top-level) Matlab function `CalcFlux`

```matlab
function [flux,T,dTdx,xy] = CalcFlux(L, h, Nx, Ny, kappa, Ttop, Tbot)
% Solves for the temperature in a simple domain, and returns the heat flux
% from the water to the air
% Inputs:
%   L - length of domain in x direction
%   h - height as a function of x; note that size(h,1) must be Nx+1
%   ...
```

I recommend the following parameterization of the height h (you are welcome to use others):

$$h(x) = a_1 + \sum_{k=2}^{n} a_k \sin\left(\frac{2\pi(k-1)x}{L}\right)$$