W.D. Henshaw **Math 6800: Problem Set 1** **Due:** Thurs. Sept. 15, 2022 (two weeks).

NLA = the text-book *Numerical Linear Algebra*, by Trefethen and Bau

1. NLA exercise 1.1 *Let B be a 4×4 matrix ...*

2. NLA exercise 2.2 *The Pythagorean theorem asserts that for a set...*

3. NLA exercise 2.3 Let $A \in \mathbb{C}^{m \times m}$ be hermitian. An eigenvector ...

4. NLA exercise 2.5 Let $S \in \mathbb{C}^{m \times m}$ be skew-hermitian...

5. NLA exercise 2.6 If u and v are $m - vectors$, the matrix $I + uv^*$ is known...

6. NLA exercise 3.1 Prove that if W is an arbitrary nonsingular matrix, ...

7. NLA exercise 3.3 *Vector and matrix p-norms are related by various inequalities, ...*

8. Let $A \in \mathbb{C}^{m \times n}$ with columns a_i, and $B \in \mathbb{C}^{p \times n}$ with columns b_i

$$A = \left[\begin{array}{c|c|c|c} & & & \\ a_1 & a_2 & \cdots & a_n \\ & & & \end{array}\right], \qquad B = \left[\begin{array}{c|c|c|c} & & & \\ b_1 & b_2 & \cdots & b_n \\ & & & \end{array}\right],$$

Show that

$$AB^* = a_1 b_1^* + a_2 b_2^* + \ldots + a_n b_n^*,$$

in <u>two</u> ways : (a) first using the component-wise definition for the elements of the product of two matrices (i.e. if $D = AC$ then $d_{ij} = \sum_k a_{ik} c_{kj}$) and (b) secondly using block-matrix multiplication (hint: choose the blocks to be the columns of A and the rows of B^*).