MANE 6760 (FEM for Fluid Dyn.) Fall 2022: Midterm Project

Oct 13, 2022

Due: 11pm on Thr/Oct 27, 2022

Weight is 15% of the total grade points
In each problem state all the assumptions/choices and show the necessary steps
Submissions must be made on Gradescope
Refer to the following link for necessary input files and updates:
`https://www.scorec.rpi.edu/~sahni/MANE6760/F22/Projects/Midterm/question/`

1. (20 points) Consider the Python code provided in the course for the stabilized finite element (FE) method for steady, 1D, linear, scalar AD equation. Use the following stabilization parameter: $\tau_{exact1} = \frac{h}{2|a_x|}(\frac{1+e^{-2.0Pe^e}}{1-e^{-2.0Pe^e}} - \frac{1}{Pe^e})$. Consider the following meshes:

   (a) M0 (uniform): $Ne = 8$ with node locations of: $\{0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1\}$

   (b) M1 (non-uniform): $Ne = 4$ with node locations of: $\{0, 0.5, 0.75, 0.875, 1.0\}$

   (c) M2 (non-uniform): $Ne = 2$ with node locations of: $\{0, 0.875, 1.0\}$

   Keep all the other settings the same (i.e., $a_x = 1.0$, $\kappa = 1.0e-4$, $\phi(x=0) = \phi_0 = 0$, $\phi(x=L) = \phi_L = 1.0$ and $s = 0$). Provide the plot of the FE solutions (one plot for all settings, or a separate plot for each of the settings, with clear labels). Also, provide the updated Python code.
   (Additional suggested exercise (*not required*): increase and/or decrease diffusivity by a factor of 10).

2. (20 points) Consider the Python code provided in the course for the stabilized finite element (FE) method for steady, 1D, linear, scalar AD equation. Use the following stabilization parameter: $\tau_{exact1} = \frac{h}{2|a_x|}(\frac{1+e^{-2.0Pe^e}}{1-e^{-2.0Pe^e}} - \frac{1}{Pe^e})$. Set $\phi(x=L) = \phi_L = 0$ and $s = 1.0$. Consider the following meshes:

   (a) M0 (uniform): $Ne = 8$ with node locations of: $\{0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1\}$

   (b) M1: $Ne = 4$ with node locations of: $\{0, 0.5, 0.75, 0.875, 1.0\}$

   (c) M2: $Ne = 2$ with node locations of: $\{0, 0.875, 1.0\}$

   Keep all the other settings the same (i.e., $a_x = 1.0$, $\kappa = 1.0e-4$ and $\phi(x=0) = \phi_0 = 0$). Provide the plot of the FE solutions (one plot for all settings, or a separate plot for each of the settings, with clear labels). Also, provide the updated Python code.

3. (20 points) Consider the Python code provided in the course for the stabilized finite element (FE) method for steady, 1D, linear, scalar ADR equation. Use the VMS formulation (i.e., $\hat{\mathcal{L}}(\cdot) = \mathcal{L}^*(\cdot)$). Set $\kappa = 1.0e-1$, $c = 1.0e+2$ and $s = 10.0$. Consider the following meshes:

   (a) M0 (uniform): $Ne = 8$ with node locations of: $\{0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1\}$

   (b) M3 (non-uniform): $Ne = 6$ with node locations of: $\{0, 0.125, 0.25, 0.5, 0.75, 0.875, 1\}$

   Keep all the other settings the same (i.e., $a_x = 1.0$, $\phi(x=0) = \phi_0 = 0$ and $\phi(x=L) = \phi_L = 1.0$). Provide the plot of the FE solutions (one plot for all settings, or a separate plot for each of the settings, with clear labels). Also, provide the updated Python code.