Project #3 Model

Reminder

Objective: Your objective is to maximize the standard deviation of the angular velocity of each car on the platform, $f = \sigma\left(\frac{d\phi}{dt}\right)$

Tilt-A-Whirl as Dynamical System

If we treat each car as a point mass, we can model the car's motion as a dynamical system.

For the equation of motion, refer to *"Chaos at the amusement park: Dynamics of the Tilt-A-Whirl"*, R. L. Kautz and B. M. Huggard, Am. J. Phys. 62 (1), January 1994.

- The equation we will use is (27) on page 63.

I will post this paper on LMS

Tilt-A-Whirl as Dynamical System (cont.)

Equation (27) is a second-order in time ordinary differential equation (ODE), of the form

$$\frac{d^2\phi}{d\tau^2} = F\left[\phi, \frac{d\phi}{d\tau}\right],$$

where

- ϕ is the car's angle with respect to the beam, and;
- $\tau = 3\omega t$ is a nondimensional time.

Tilt-A-Whirl as Dynamical System (cont.)

To solve this numerically, rewrite it as a first-order system.

- Define the state vector

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{d\phi}{d\tau} \\ \phi \end{bmatrix}$$

- Add a new ODE to the system:

$$\frac{d\phi}{d\tau} = \frac{dy_2}{d\tau} = y_1$$

Tilt-A-Whirl as Dynamical System (cont.)

Then, the first-order system of ODEs to be solved is

$$\frac{dy}{d\tau} = \frac{d}{d\tau}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} F\left(y_2, y_1\right) \\ y_1 \end{bmatrix}$$

- You can use Matlab's `ODE45` or other appropriate ODE solver to solve this system.

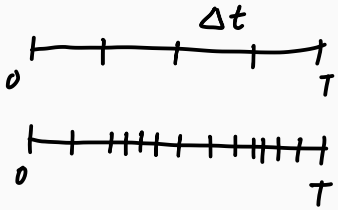
Calculating the Objective

The objective is the standard deviation of $\frac{d\phi}{dt}$ over time:

$$f(x) = 3\omega \sqrt{\frac{1}{T} \int_0^T (y_1 - \bar{y}_1)^2 \, d\tau}$$

where

- $\bar{y}_1$ is the mean angular velocity, $\frac{1}{T}\int_0^T y_1 \, d\tau$, and;
- T is the total (non-dimensional) period of simulation.



The Matlab function `trapz` will come in handy for these calculations.

trapz(tau, y1)

output from ode45

Verify, Verify, Verify!

$$\frac{d\phi}{dt} = \frac{d\phi}{d\tau}\frac{d\tau}{dt}$$

You must provide evidence that your analysis is working.

- Do ϕ and $d\phi/dt$ behave as expected for $\omega << 1$?
- Do ϕ and $d\phi/dt$ behave as expected for $\omega >> 1$?
- Does your analysis agree with the results in the paper?

$\hookrightarrow$ Does your analysis agree with data posted on Piazza?

Some other Pointers

- Check that your integration period T is sufficiently long that the statistics are converged.
- To reduce the influence of the initial conditions, you should run your analysis for a short "spin-up" period, T_{spin}, the final solution of which becomes the initial condition for the actual statistics-gathering run.

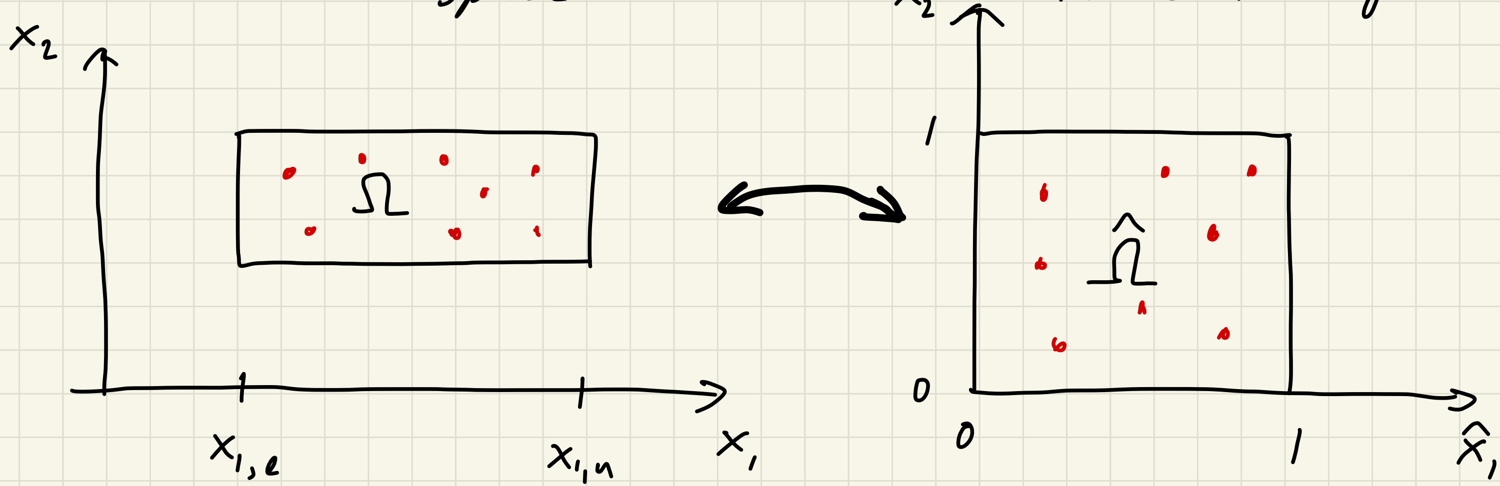
beware of numerator & denominators

Eq. (26) $\quad \varepsilon = r_1/9\, r_2 \qquad \cancel{(r_1/9) * r_2}$

$$\frac{r_1}{9 r_2}$$

Method 1: samples are scaled to lie within the true (physical) feasible space.

Method 2: samples are in uniform $[0, 1]^3$ space, and they are transformed inside the objective.



Example of how to transform between the two feasible spaces $\longrightarrow$

$$\hat{x}_1 = \frac{(x_1 - x_{1,\ell})}{(x_{1,u} - x_{1,\ell})}, \quad x_1 = \hat{x}_1 (x_{1,u} - x_{1,\ell}) + x_{1,\ell}$$