Interpolation With RBFs

Interpolation With RBFs Is Similar To Any Other GLM

If we choose the basis centers to be the training points (so $s = p$), then we get a system of equations of the form

$$\hat{f}(x^{(j)}, \alpha) = \sum_{k=1}^{p} \alpha_k \phi(\|x^{(j)} - x^{(k)}\|) = f(x^{(j)}), \qquad \forall j = 1, 2, \ldots, s$$

or, in matrix form, $K\alpha = y$ where α and y are defined as before and

$$K = \underbrace{\left.\begin{bmatrix} \phi(0) & \phi(\|x^{(1)} - x^{(2)}\|) & \cdots & \phi(\|x^{(1)} - x^{(p)}\|) \\ \phi(\|x^{(2)} - x^{(1)}\|) & \phi(0) & \cdots & \phi(\|x^{(2)} - x^{(p)}\|) \\ \vdots & \vdots & \ddots & \vdots \\ \phi(\|x^{(s)} - x^{(1)}\|) & \phi(\|x^{(s)} - x^{(2)}\|) & \cdots & \phi(0) \end{bmatrix}\right\}\text{each sample gets a row}}_{\text{each basis gets a column}}$$

The Matrix K Is Called the Gram Matrix In The Context of RBFs

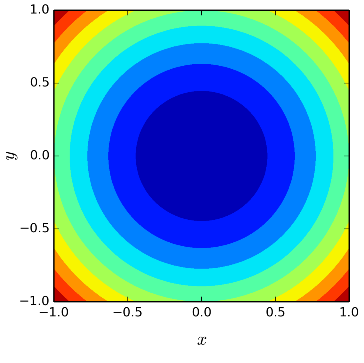
If K is invertible, then the parameters are given by

$$\alpha = K^{-1}y$$

- Unfortunately, K is only guaranteed to be invertible if Gaussian RBF are chosen; even then, K can be highly ill-conditioned
- If we wish to use ϕ that are not Gaussian, a polynomial term must be included in the interpolation; see [KN05, pg. 229] for further details.

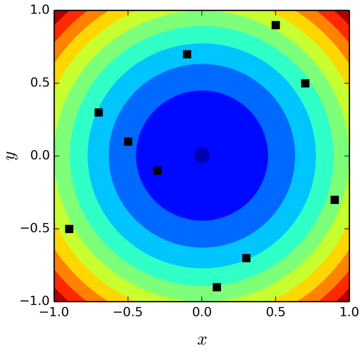
Let's Consider An RBF Surrogate Using Gaussian Basis Functions

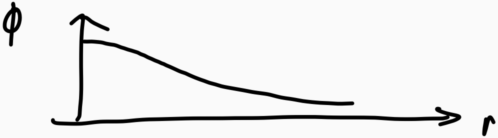
Exact Function: $f(x) = x^2 + y^2$



Let's Consider An RBF Surrogate Using Gaussian Basis Functions (cont.)

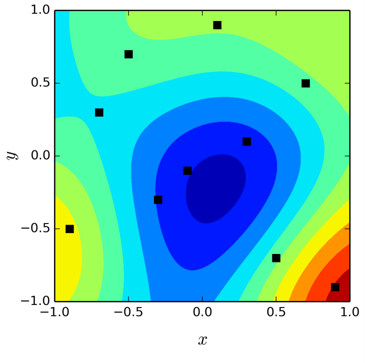
Gaussian RBF: $s = 10$ and $\theta = 10$

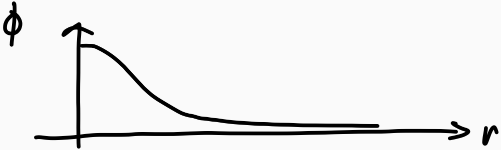




Let’s Consider An RBF Surrogate Using Gaussian Basis Functions (cont.)

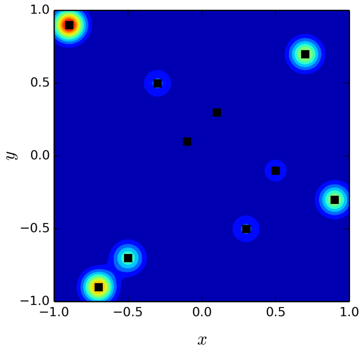
Gaussian RBF: $s = 10$ and $\theta = 1$





Let’s Consider An RBF Surrogate Using Gaussian Basis Functions (cont.)

Gaussian RBF: $s = 10$ and $\theta = 0.1$



References

Andy J. Keane and Prasanth B. Nair, *Computational Approaches for Aerospace Design: The Pursuit of Excellence*, John Wiley & Sons, Ltd, Chichester, UK, June 2005.