MANE 6760 - FEM for Fluid Dyn. - Lecture 03

Prof. Onkar Sahni, RPI

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FE Form: AD equation

For brevity we set FE form as: find $\bar{\phi} \in \bar{\mathcal{S}} \subset \mathcal{S}$ such that

$$\int_\Omega \bar{w} \frac{\partial \bar{\phi}}{\partial t} dV + \int_\Omega \nabla \bar{w} \cdot (\boldsymbol{a} \bar{\phi} - \kappa \nabla \bar{\phi}) dV - \int_{\Gamma_h} \bar{w} d_h = (\bar{w}, s)$$

for all $\bar{w} \in \bar{\mathcal{W}} \subset \mathcal{W}$

A number of simplifications:

- Steady
- 1D domain: $x \in [0, L]$
- No source term: s=0
- Only Dirichlet/essential boundary conditions and no Neumann/flux boundary condition:
  $\Gamma_h = \emptyset$, i.e., $\Gamma_g = \partial\Omega = \{x = 0, x = L\}$

After simplifications, find $\bar{\phi} \in \bar{\mathcal{S}} \subset \mathcal{S}$ such that

$$\boxed{\int_0^L \bar{w}_{,x} (a_x \bar{\phi} - \kappa \bar{\phi}_{,x}) dx = 0}$$

for all $w^h = \bar{w} \in \mathcal{W}^h = \bar{\mathcal{W}} \subset \mathcal{W}$

FE Setup and Procedure

Consider a mesh with N_n nodes/vertices and N_e elements:

Global and local/element views of the mesh (with n_n^e nodes/vertices for any element):

FE Setup and Procedure

Solution/trial or weight/test function representation (with N_s and n_s^e basis/shape functions):

Real and element/parent coordinates (i.e., $\boldsymbol{x}$ and $\boldsymbol{\xi}$):

Global and local/element views of linear basis/shape functions (where $N_s = N_n$ and $n_s^e = n_n^e$):

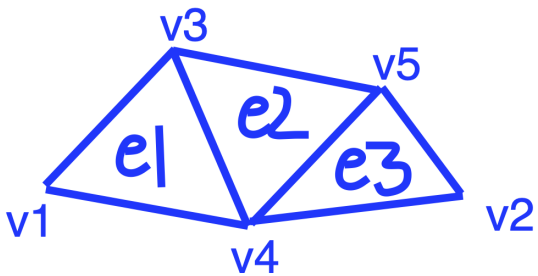
FE Setup and Procedure

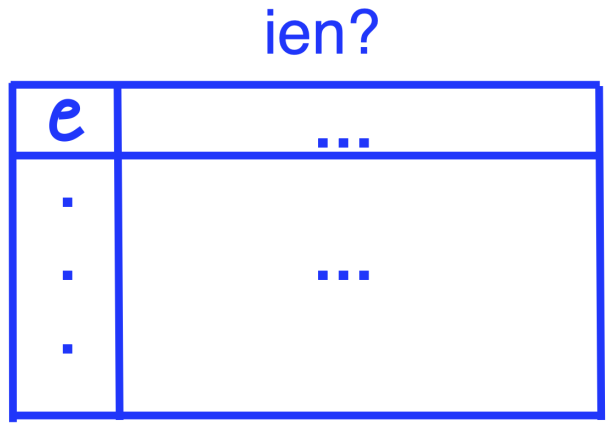
Numerical integration (with n_l^e integration points):

Element-node connectivity (*ien* array/table):

FE Setup and Procedure

Exercise on element-node connectivity:





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