MANE 6760 (FEM for Fluid Dyn.) Fall 2022: HW4

Nov 15, 2022

Due: 11pm on Tue/Nov 22, 2022

Weight is 15% of the total grade points
In each problem state all the assumptions/choices and show the necessary steps
Submissions must be made on Gradescope
Refer to the following link for necessary input files and updates:
`https://www.scorec.rpi.edu/~sahni/MANE6760/F22/HWs/HW4/question/`

1. (20 points) Consider the formulation provided in the course for the stabilized/SUPG finite element (FE) method for the transient, 1D, scalar AD equation. Use the template code provided to implement the backward Euler scheme. Try three different values of $N_t = 10$, 50 and 250. Keep all the other settings the same (e.g., a_x, κ, N_e, etc.). Provide the updated Python code and the three solution plots (one for each value of N_t).

2. (10 points) For the transient, 1D, non-linear Burgers equation: $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \nu\frac{\partial^2 u}{\partial x^2} = s$, consider the stabilized/SUPG finite element (FE) method leading to the following semi-discrete non-linear weak residual:

$$G_A = \int_0^L (\ldots + N_{A,x}\bar{u}\tau\bar{u}_{,t} + \ldots + \ldots)\,dx$$

   Find the contribution of the (only) term shown above to the tangent/LHS matrix $\frac{\partial \mathcal{G}_A}{\partial \hat{\phi}_B^{(n+1)}}$ for the fully discrete form based on the following two time integration schemes:

   (a) Forward Euler

   (b) Backward Euler