GLM With A Quadratic Basis

GLMs with Quadratic Basis

GLMs with linear basis are (perhaps) suitable for approximating the constraints

$$\hat{c}(x, \alpha) \approx c(x).$$

However, a linear surrogate may be less appropriate for the objective

A Quadratic Basis Is Better Suited For Modeling the Objective

Definition: GLM with a quadratic basis

A generalized linear model with a quadratic basis takes the form

$$\hat{f}(x, \alpha) = \alpha_0 + \sum_{k=1}^{n} \alpha_k x_k + \sum_{m=1}^{n} \sum_{k=m}^{n} \alpha_{n-1+m+k} x_m x_k$$

The Vandermonde Matrix Needs To Reflect The Quadratic Basis

Suppose we are interested in a least-squares fit using the quadratic GLM. Then, the residual vector is given by

$$R(\alpha) = V\alpha - y \neq 0.$$

where y is defined as before (for linear regression) but

$$V = \begin{bmatrix} 1 & x_1^{(1)} & x_2^{(1)} & \cdots & x_n^{(1)} & (x_1^{(1)})^2 & \cdots & (x_n^{(1)})^2 \\ 1 & x_1^{(2)} & x_2^{(2)} & \cdots & x_n^{(2)} & (x_1^{(2)})^2 & \cdots & (x_n^{(2)})^2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & & \\ 1 & x_1^{(s)} & x_2^{(s)} & \cdots & x_n^{(s)} & (x_1^{(s)})^2 & \cdots & (x_n^{(s)})^2 \end{bmatrix}$$

Least-Squares Parameter Estimation for Polynomial GLMs

Definition: Least-squares α in polynomial GLM

The parameters for a GLM using a polynomial basis can be determined by solving the overdetermined system

$$V\alpha = y$$

where V is the Vandermonde matrix corresponding to the basis functions evaluated at the sample points.

Example

For the following data points, find the GLM with a quadratic basis using least-squares regression.

$$\begin{bmatrix} x^{(1)} & x^{(2)} & x^{(3)} & x^{(4)} \\ f^{(1)} & f^{(2)} & f^{(3)} & f^{(4)} \end{bmatrix} = \begin{bmatrix} 0.55 & 0.76 & 0.48 & 0.51 \\ 0.45 & 0.99 & 0.99 & 0.64 \end{bmatrix}$$