MANE 6760 - FEM for Fluid Dyn. - Lecture 18

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Simplified: 1D Unsteady/Transient AD (TAD) Eqn

A number of simplifications:

- 1D (spatial) domain: $x \in [0, L]$
- Only Dirichlet/essential boundary conditions

Strong form:

$$
\begin{aligned}
R(\phi) = \mathcal{L}(\phi) - s = \frac{\partial \phi}{\partial t} + a_x \frac{\partial \phi}{\partial x} - \kappa \frac{\partial^2 \phi}{\partial x^2} - s = 0, \qquad & \phi \in \mathcal{S}_{strong} \\
x \in [0, L] & \\
t \in [t_0, t_1] & \\
\phi(x, t = t_0) = \phi_{IC}(x) \, \forall x & \\
\phi(x = 0, t) = \phi_0(t) \quad \text{on} \quad x = 0 \, \forall t & \\
\phi(x = L, t) = \phi_L(t) \quad \text{on} \quad x = L \, \forall t &
\end{aligned}
$$

FE Setup and Procedure

Stabilization Parameter: TAD equation

τ approximation in 1D: algebraic version by Shakib *et al.* (1991):

$$\tau_{alg,skb} = \tau_{alg1} : (\tau_{alg,skb})^{-2} = \left(\frac{\Delta t}{2}\right)^{-2} + \left(\frac{(h/2)}{|a_x|}\right)^{-2} + 9\left(\frac{(h/2)^2}{\kappa}\right)^{-2}$$

$$= \left(\frac{\Delta t}{2}\right)^{-2} + \left(\frac{2|a_x|}{h}\right)^{2} + 9\left(\frac{4\kappa}{h^2}\right)^{2}$$

$$\tau_{alg,skb} = \tau_{alg1} = \frac{1}{\sqrt{(\frac{2}{\Delta t})^2 + \left(\frac{2|a_x|}{h}\right)^2 + 9\left(\frac{4\kappa}{h^2}\right)^2}}$$

τ approximation in multiple dimensions:

$$(\tau_{alg,skb})^{-2} = (\tau_{alg1})^{-2} = \left(\frac{\Delta t}{2}\right)^{-2} + a_i g_{ij} a_j + c_{diff}^2 g_{ij} g_{ij} \kappa^2$$

$$\tau_{alg,skb} = \tau_{alg1} = \sqrt{\frac{1}{(\frac{2}{\Delta t})^2 + a_i g_{ij} a_j + c_{diff}^2 g_{ij} g_{ij} \kappa^2}}$$

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