MANE 6760 (FEM for Fluid Dyn.) Fall 2022: HW3

Nov 8, 2022

Due: 11pm on Tue/Nov 15, 2022

Weight is 12% of the total grade points
In each problem state all the assumptions/choices and show the necessary steps
Submissions must be made on Gradescope
Refer to the following link for necessary input files and updates:
`https://www.scorec.rpi.edu/~sahni/MANE6760/F22/HWs/HW3/question/`

Consider the formulation and Python code provided in the course for the stabilized finite element (FE) method for steady, 1D, non-linear, scalar AD equation.

1. (10 points) Consider $\kappa = \kappa_0(1 + \frac{1}{1+\phi_{,x}^2})$ (i.e., instead of $\kappa = \kappa_0\phi$ or $\kappa = \kappa_0\phi^2$). The non-linear weak residual is given as:

$$G_A = \int_0^L \left(\ldots + \ldots + N_{A,x}\kappa\bar{\phi}_{,x} + \ldots + \ldots\right) dx$$

   Find the contribution (only) of the term shown above to the tangent/LHS matrix $\frac{\partial G_A}{\partial \bar{\phi}_B}$. Hint: note that $\kappa = \kappa(\phi_{,x})$ (instead of $\kappa = \kappa(\phi)$) and thus, consider $\frac{\partial \kappa}{\partial \phi_{,x}}$.

2. (20 points) Update the code for the above equation (i.e., $\kappa = \kappa_0(1 + \frac{1}{1+\phi_{,x}^2})$). Keep all the other settings the same (e.g., a_x, κ_0, s, N_e, etc.). Provide the updated solution plot and the updated Python code.