MANE 6760 (FEM for Fluid Dyn.) Fall 2022: HW5

Nov 29, 2022

Due: 11pm on Tue/Dec 6, 2022

Weight is 5% of the total grade points
In each problem state all the assumptions/choices and show the necessary steps
Submissions must be made on Gradescope
Refer to the following link for necessary input files and updates:
`https://www.scorec.rpi.edu/~sahni/MANE6760/F22/HWs/HW5/question/`

1. (5 points) For the compressible Navier-Stokes equations in pressure-primitive variables and with Nobel-Able equation of state: $\rho = \rho(p,T) = \frac{p}{RT+bp}$, where R and b are some constants. Determine: $(A_0)_{l=1,m=1}$ in terms of p and T (recall that $\boldsymbol{\mathcal{A}}_0 = \boldsymbol{\mathcal{U}}_{,\boldsymbol{Y}}$).

2. (10 points) For 1D, steady compressible Navier-Stokes equations with no source terms, consider the stabilized FE form for linear finite elements to be:

$$B_{stab}(\bar{\boldsymbol{W}},\bar{\boldsymbol{Y}}) = \sum_e \int_{\Omega_e} \boldsymbol{\mathcal{A}}_1^T \bar{\boldsymbol{W}}_{,1} \cdot \boldsymbol{\tau} \boldsymbol{\mathcal{A}}_1 \bar{\boldsymbol{Y}}_{,1} d\Omega_e = \sum_e \int_{\Omega_e} \bar{\boldsymbol{W}}_{,1} \cdot \boldsymbol{\mathcal{K}}_{num} \bar{\boldsymbol{Y}}_{,1} d\Omega_e$$

   where $\boldsymbol{\mathcal{A}}_1$, $\boldsymbol{\tau}$ and $\boldsymbol{\mathcal{K}}_{num}$ are $(n_{sd}+2)\times(n_{sd}+2) = 3\times 3$ matrices. Expand out $(\mathcal{K}_{num})_{lm}$ in terms of entries of the $\boldsymbol{\mathcal{A}}_1$ and $\boldsymbol{\tau}$ matrices, i.e., in terms of $(\mathcal{A}_1)_{11}$, $(\mathcal{A}_1)_{12}$, ..., τ_{11}, τ_{12}, ..., leading to a form such as: $(\mathcal{K}_{num})_{lm} = (\mathcal{A}_1)_{??}\tau_{??}(\mathcal{A}_1)_{??} + (\mathcal{A}_1)_{??}\tau_{??}(\mathcal{A}_1)_{??} + \ldots + (\mathcal{A}_1)_{??}\tau_{??}(\mathcal{A}_1)_{??}$. Specifically, expand out the following

   (a) $(\mathcal{K}_{num})_{l=2,m=3}$