W.D. Henshaw **Math 6800: Problem Set 7** **Due:** Thurs. Nov. 17, 2022

NLA = the text-book *Numerical Linear Algebra*, by Trefethen and Bau

1. A matrix D is block tridiagonal if it is of the form

$$D = \begin{bmatrix} B_1 & C_1 & & & & \\ A_2 & B_2 & C_2 & & & \\ & A_3 & B_3 & C_3 & & \\ & & A_4 & B_4 & C_4 & \\ & & & \ddots & \ddots & \ddots \\ & & & & A_n & B_n \end{bmatrix}$$

where each A_i, B_i and C_i is a small matrix of size $p \times p$ (p is the *block size*). Derive a block LU decomposition (i.e. an LU decomposition that uses operations involving $p \times p$ matrices instead of scalars), assuming no pivoting. What are the conditions you need for this LU decomposition to exist?

2. NLA 20.3 *Suppose an $m \times m$ matrix A ...*

3. The matrix $A \in \mathbb{C}^{m\times m}$ is *diagonally dominant* if

$$|a_{ii}| > \sum_{j=1,j\neq i}^{m} |a_{ij}|, \quad i = 1, 2, \ldots, m.$$

(a) Prove that if A is diagonally dominant, then any principal submatrix of A is diagonally dominant.
(b) Prove that if A is diagonally dominant, then A is nonsingular.
(c) Prove that if A is diagonally dominant then it will have an LU decomposition (you may use the result of NLA 20.1).

4. Write a Matlab code `[L,U,P]=lufactor(A)` that takes an $m \times m$ matrix A and computes the LU factorization, $PA = LU$, using partial pivoting. Write a second Matlab code `x = lusolve( b, L,U,P)` that solves the system $Ax = b$, for x, given b, using the output from `lufactor`. For this exercise you should only use elementary arithmetic, vector and matrix operations (e.g. no backslash operators to solve the triangular systems).
(a) Test your `lufactor` routine using the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{bmatrix}$$

Output A, L, U and P. Check that $PA = LU$.

(b) Test your function `lusolve` by solving $Ax = b$ where A is from part (a) and $b = [7, 23, 69, 79]^T$. Output x and check that $Ax = b$.