MANE 6760 - FEM for Fluid Dyn. - Lecture 14

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Non-linear (Steady) Equations: Non-linear FE

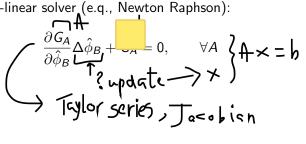
Derive a finite-element based non-linear weak residual:

$$\hat{w}_A G_A = 0$$
, where $G_A = G_A^{galk}$ or $G_A = G_A^{stab}$

Derive a non-linear system of (algebraic) equations:

$$G_A = 0, \forall A$$

Setup a non-linear solver (e.q., Newton Raphson):



Non-linear (Steady) Equations: AD/ADR Examples

Transport of heat with $\phi = T$ (temperature):

$$\nabla \cdot (\boldsymbol{a}T - \kappa(T)\nabla T) = s$$

Transport of momentum with $\phi = \boldsymbol{u}$ (velocity vector):

$$\mathbf{u} \cdot \nabla \mathbf{u} - \nu \nabla^2 \mathbf{u} = \mathbf{s}$$

Transport for turbulence modeling with $\phi = \nu_T$ (eddy viscosity):

$$\boldsymbol{u} \cdot \nabla \nu_T - k_1 \nabla \cdot ((\nu + \nu_T) \nabla \nu_T) - k_2 \nabla \nu_T \cdot \nabla \nu_T + c(\nu_T) \nu_T = s$$

FE Setup and Procedure

Element level operations (including assembly):

$$A^{e} = \frac{\partial G_{c}^{e}}{\partial \mathcal{S}_{b}^{e}} ; b^{e} = -G_{c}^{e}$$

$$G_{c}^{e}(\alpha) ff = \int_{\Lambda_{c}^{e}} N_{c} \times K \overline{\phi} J(N_{c} \times \hat{\phi}_{c}) dx$$

$$\overline{\phi}_{,x}$$

think of Transport of heat equation from previous slide for \phi

$$\psi_{\mathbf{x}} = \int_{--\frac{9\sqrt{3}}{4}} \left(\right) + --\kappa \frac{9\sqrt{3}}{4} \right)$$

FE Setup and Procedure

Global level operations (including BCs):

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