MANE 6760 - FEM for Fluid Dyn. - Lecture 14

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Non-linear (Steady) Equations: Non-linear FE

Derive a finite-element based non-linear weak residual:

$$\hat{w}_A G_A = 0, \qquad \text{where } G_A = G_A^{galk} \text{ or } G_A = G_A^{stab}$$

Derive a non-linear system of (algebraic) equations:

$$G_A = 0, \qquad \forall A$$

Setup a non-linear solver (e.q., Newton Raphson):

$$\left.\overbrace{\frac{\partial G_A}{\partial \hat{\phi}_B}}^{A} \underbrace{\Delta \hat{\phi}_B}_{} + G_A = 0, \qquad \forall A \right\} Ax = b$$

? update $\rightarrow x$

Taylor series, Jacobian

Non-linear (Steady) Equations: AD/ADR Examples

Transport of heat with $\phi = T$ (temperature):

$$\nabla \cdot (\boldsymbol{a} T - \kappa(T) \nabla T) = s$$

Transport of momentum with $\boldsymbol{\phi} = \boldsymbol{u}$ (velocity vector):

$$\boldsymbol{u} \cdot \nabla \boldsymbol{u} - \nu \nabla^2 \boldsymbol{u} = s$$

Transport for turbulence modeling with $\phi = \nu_T$ (eddy viscosity):

$$\boldsymbol{u} \cdot \nabla \nu_T - k_1 \nabla \cdot ((\nu + \nu_T) \nabla \nu_T) - k_2 \nabla \nu_T \cdot \nabla \nu_T + c(\nu_T) \nu_T = s$$

FE Setup and Procedure

Element level operations (including assembly):

$$A^e = \frac{\partial G_a^e}{\partial \hat{\phi}_b^e} \quad ; \quad b^e = -G_a^e$$

$$G_a^e(\mathrm{diff})\Big|_{\bar{\phi}} = \int_{\Omega^e} N^e_{a,jx}\, \kappa(\bar{\phi}) \underbrace{\left(N_{c,jx}\, \hat{\phi}_c\right)}_{\bar{\phi}_{,jx}} d\Omega$$

think of Transport of heat equation from previous slide for \phi

$$A^e = \int -- \frac{\partial \kappa}{\partial \hat{\phi}_c} \left(\quad \right) + --\kappa \frac{\partial \bar{\phi}_{,jx}}{\partial \hat{\phi}_c}$$

FE Setup and Procedure

Global level operations (including BCs):

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