Radial Basis Functions

A Popular GLM For Surrogate Modeling Uses Radial Basis Functions

Definition: Radial Basis Function (RBF)

A radial basis function is a function $\phi : \mathbb{R}^{\geq 0} \to \mathbb{R}$, i.e. it maps non-negative real numbers to the real numbers.

A Popular GLM For Surrogate Modeling Uses Radial Basis Functions (cont.)

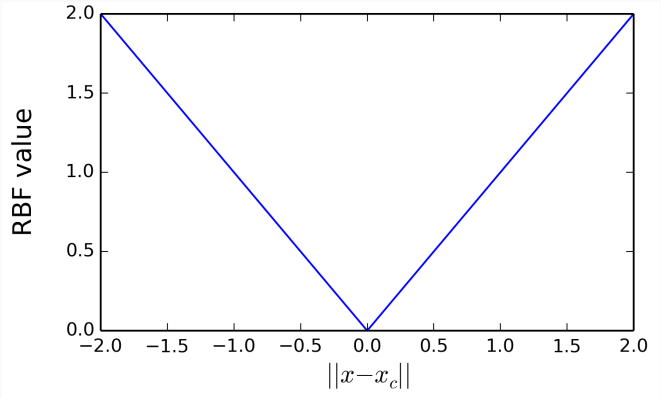
A (GLM) surrogate model using radial basis functions takes the form

$$\hat{f}(x, \alpha) = \sum_{k=1}^{p} \alpha_k \phi(\|x - x^{(k)}\|)$$

where $\phi : \mathbb{R}^{\geq 0} \to \mathbb{R}$ is an RBF.

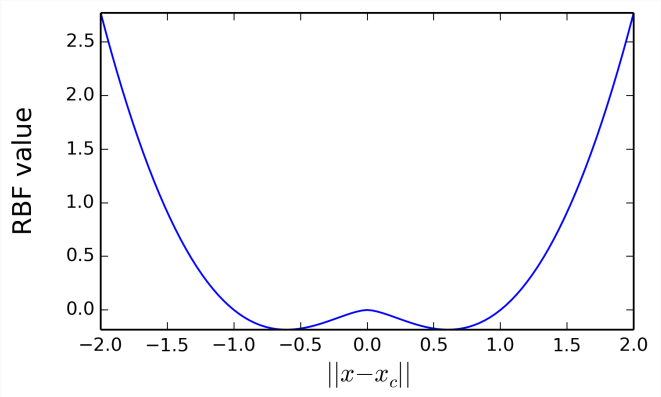
Common Basis Functions Used For RBF Surrogates

Linear splines: $\phi(\|x - x_{(k)}\|) = \|x - x^{(k)}\|$



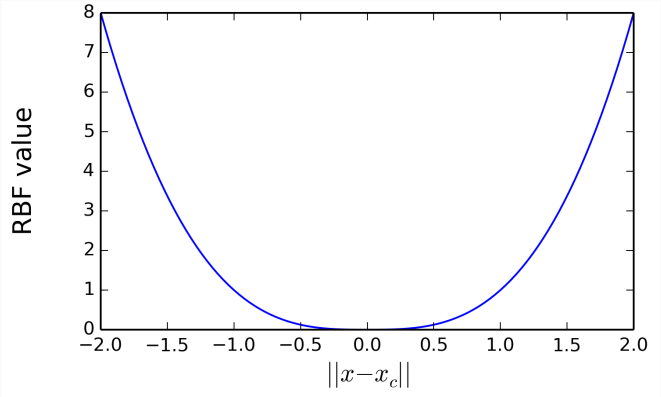
Common Basis Functions Used For RBF Surrogates (cont.)

Thin-plate splines: $\phi(\|x - x_{(k)}\|) = \|x - x^{(k)}\|^p \ln \|x - x^{(k)}\|, \qquad p \in [2, 4, 6, \ldots$



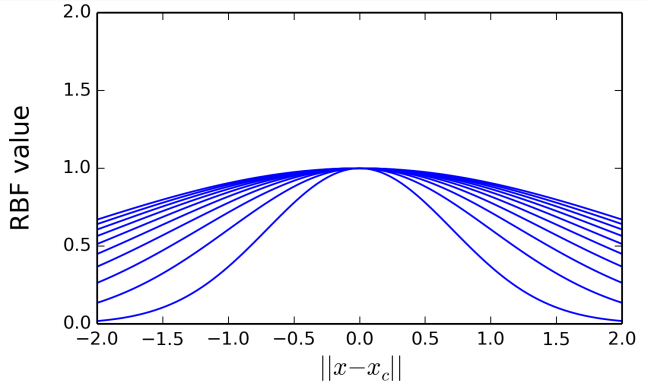
Common Basis Functions Used For RBF Surrogates (cont.)

Cubic splines: $\phi(\|x - x_{(k)}\|) = \|x - x^{(k)}\|^3$



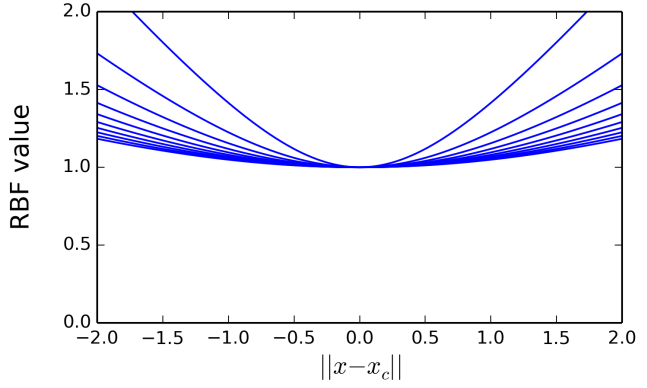
Common Basis Functions Used For RBF Surrogates (cont.)

Gaussian: $\phi(\|x - x_{(k)}\|) = \exp\left(-\frac{\|x-x^{(k)}\|^2}{\theta}\right)$



Common Basis Functions Used For RBF Surrogates (cont.)

Multiquadrics: $\phi(\|x - x_{(k)}\|) = \sqrt{1 + \frac{\|x - x^{(k)}\|^2}{\theta}}$



Common Basis Functions Used For RBF Surrogates (cont.)

Inverse Multiquadrics: $\phi(\|x - x_{(k)}\|) = 1/\sqrt{1 + \frac{\|x-x^{(k)}\|^2}{\theta}}$