MANE 6760 (FEM for Fluid Dyn.) Fall 2022: HW2

Oct 4, 2022

Due: 11pm on Tue/Oct 11, 2022

Weight is 10% of the total grade points
In each problem state all the assumptions/choices and show the necessary steps
Submissions must be made on Gradescope
Refer to the following link for necessary input files and updates:
`https://www.scorec.rpi.edu/~sahni/MANE6760/F22/HWs/HW2/question/`

Consider the Python code provided in the course for the stabilized finite element (FE) method for steady, 1D, linear, scalar AD equation.

1. (5 points) Implement and use the following two stabilization parameters: $\tau_{alg,skb} = \tau_{alg1} = \frac{1}{\sqrt{\left(\frac{2|a_x|}{h}\right)^2 + 9\left(\frac{4\kappa}{h^2}\right)^2}}$ and $\tau_{alg,cod} = \tau_{alg2} = \frac{1}{\frac{2|a_x|}{h} + \frac{4\kappa}{h^2}}$. Set $a_x = 1.0$ and $\kappa = 1.0e-4$. Keep all the other settings the same. Provide the plot of the FE solutions (one plot for both settings, or a separate plot for each of the two settings, with clear labels). Also, provide the updated Python code.

2. (15 points) Implement and use the following four stabilization parameters:

   (a) $\tau_{exact1} = \frac{h}{2|a_x|}(\coth(Pe^e) - \frac{1}{Pe^e})$ with $\coth(Pe^e) = \frac{1+e^{-2.0Pe^e}}{1-e^{-2.0Pe^e}}$

   (b) $\tau_{exact2} = \frac{h}{2|a_x|}(\coth(Pe^e) - \frac{1}{Pe^e})$ with $\coth(Pe^e) = \frac{e^{Pe^e}+e^{-Pe^e}}{e^{Pe^e}-e^{-Pe^e}}$

   (c) $\tau_{alg,skb} = \tau_{alg1} = \frac{1}{\sqrt{\left(\frac{2|a_x|}{h}\right)^2 + 9\left(\frac{4\kappa}{h^2}\right)^2}}$

   (d) $\tau_{alg,cod} = \tau_{alg2} = \frac{1}{\frac{2|a_x|}{h} + \frac{4\kappa}{h^2}}$

   Consider the following values of a_x and κ:

   (a) Case A: $a_x = 1.0e-8$ and $\kappa = 1.0$ (note: $Pe^G = 1.0e-8$)

   (b) Case B: $a_x = 1.0$ and $\kappa = 1.0e+8$ (note: $Pe^G = 1.0e-8$)

   (c) Case C: $a_x = 1.0e-4$ and $\kappa = 1.0$ (note: $Pe^G = 1e-4$)

   (d) Case D: $a_x = 1.0e-4$ and $\kappa = 1.0e-8$ (note: $Pe^G = 1e+4$)

   (e) Case E: $a_x = 1.0$ and $\kappa = 1.0e-8$ (note: $Pe^G = 1e+8$)

   For 20 combinations of 4 stabilization parameters and 5 cases, determine whether: i) the stabilization parameter is computed properly, and ii) the stabilized FE calculation yields useful a result. For example, no overflow or significant precision/rounding issues. **Specifically check for both, the stabilization parameter and the stabilized FE solution**.

Provide the updated Python code and summarize the result for each combination in the following tabular form. Each cell will contain one of the four possibilities: (yes-τ, yes-$\bar{\phi}$), (yes-τ, no-$\bar{\phi}$), (no-τ, yes-$\bar{\phi}$) and (no-τ, no-$\bar{\phi}$). yes-τ implies the stabilization parameter is computed properly while no-τ implies otherwise. Similarly, yes-$\bar{\phi}$) implies a useful stabilized FE solution is obtained and vice-versa.

	τ_{exact1}	τ_{exact2}	τ_{alg1}	τ_{alg2}
Case A	?	?	?	?
Case B	?	?	?	?
Case C	?	?	?	?
Case D	?	?	?	?
Case E	?	?	?	?

3. (10 points) Implement and use the following three stabilization parameters:
$\tau_{exact1} = \frac{h}{2|a_x|}(\coth(Pe^e) - \frac{1}{Pe^e})$ with $\coth(Pe^e) = \frac{1+e^{-2.0Pe^e}}{1-e^{-2.0Pe^e}}$,
$\tau_{alg,skb} = \tau_{alg1} = \frac{1}{\sqrt{\left(\frac{2|a_x|}{h}\right)^2 + 9\left(\frac{4\kappa}{h^2}\right)^2}}$
and $\tau_{alg,cod} = \tau_{alg2} = \frac{1}{\frac{2|a_x|}{h} + \frac{4\kappa}{h^2}}$
Set $a_x = 1.0$ and $\kappa = 2.0e-3$. Keep all the other settings the same.

Note that exact solution for this case is given as: $\phi^{exact} = \frac{e^{-Pe^G(1-\frac{x}{L})} - e^{-Pe^G}}{1-e^{-Pe^G}}$.

For each of the three stabilization parameters, provide a semi-log plot (log scale only in the vertical axis) for quantity: $|\phi^{exact} - \bar{\phi}|$, at mesh vertices/nodes (i.e., absolute value of the nodal error, or difference between the exact solution and FE solution evaluated at mesh nodes, in log scale). Three plots in total. Also, provide the updated Python code.