MANE 6760 - FEM for Fluid Dyn. - Lecture 22

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Transient Equations: Fully discrete form

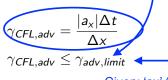
CFL numbers and conditions/restrictions on time step size (due to stability; note that a condition/restriction on time step size due to accuracy is taken into account separately)

Depends on the scheme (time integration scheme)

Advective regime

$$\Delta t \leq \left(- - - \right)$$

Diffusive regime



number)
Given: 'ax' (or 'kappa') and 'dx',
then how large can one go for

$$\gamma_{CFL,diff} = \frac{\kappa \Delta t}{\Delta^2 x}$$
 $\gamma_{CFL,diff} \leq \gamma_{diff,limit}$

Depends on the scheme (time integration scheme)

Upper limit/bound (max allowable value for CFL number)

Upper limit/bound (max allowable value for CFL

Transient Non-Linear System of Equations: AD (Conservative Variables)

Consider the vector of conserved solution variables (e.g., species energy): $\psi = [\psi_1, \psi_2, \dots, \psi_M]^T$

Physicists
$$\frac{\partial \psi}{\partial t} + \underbrace{\nabla \cdot \mathbf{F}}_{\nabla \cdot (\mathbf{F}^{adv} + \mathbf{F}^{diff})} = \mathbf{s} \qquad \mathcal{L}(\circ) = \mathbf{S}$$

$$\frac{\partial \psi}{\partial t} + \nabla \cdot \left(\tilde{\mathcal{A}}\psi - \tilde{\mathcal{K}}\nabla\psi\right) = \mathbf{s} \qquad \text{quasi-linear}$$

$$\psi_{l,t} + \left((\tilde{\mathcal{A}}_{lm})_{i}\psi_{m} - (\tilde{\mathcal{K}}_{lm})_{ij}\phi_{m,j}\right)_{,i} = \mathbf{s}_{l}$$

$$\mathcal{L}(\circ) = \underbrace{\mathcal{L}(\circ)}_{\mathbf{L}(\bullet)} + \underbrace{\mathcal{L}(\circ)}_{\mathbf{L}(\bullet)} = \mathbf{S}$$

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Transient Non-Linear System of Equations: AD (Primitive Variables)

Consider the vector of primitive solution variables (e.g., species temperature): $\phi = [\phi_1, \phi_2, \dots, \phi_M]^T$ such that $\mathcal{A}_0 = \frac{\partial \psi}{\partial \phi}$

Engineer

$$\frac{\partial \phi}{\partial t} = \frac{\partial \mathbf{V}}{\nabla \cdot (\mathbf{F}^{adv} + \mathbf{F}^{diff})} = \mathbf{s}$$

$$\mathcal{A}_{0} \frac{\partial \phi}{\partial t} + \nabla \cdot (\mathcal{A}\phi - \mathcal{K}\nabla\phi) = \mathbf{s}$$

$$(\mathcal{A}_{0})_{lm}\phi_{m,t} + ((\mathcal{A}_{lm})_{i}\phi_{m} - (\mathcal{K}_{lm})_{ij}\phi_{m,j})_{,i} = \mathbf{s}_{l}$$

Transient Non-Linear System of Equations: AD (Entropy Variables)

Consider the vector of entropy solution variables (e.g., species "entropy"): $\boldsymbol{\vartheta} = [\vartheta_1, \vartheta_2, \dots, \vartheta_M]^T$ such that $\hat{\boldsymbol{\mathcal{A}}}_0 = \frac{\partial \psi}{\partial \boldsymbol{\vartheta}}$

Mathematician

$$egin{aligned} \hat{oldsymbol{\mathcal{A}}}_0 rac{\partial oldsymbol{artheta}}{\partial t} + \sum_{
abla \cdot (oldsymbol{F}^{adv} + oldsymbol{F}^{diff})} = oldsymbol{s} \ \hat{oldsymbol{\mathcal{A}}}_0 rac{\partial oldsymbol{artheta}}{\partial t} +
abla \cdot \left(\hat{oldsymbol{\mathcal{A}}} \partial_{lm} \partial_{m} + \left((\hat{oldsymbol{\mathcal{A}}}_{lm})_{ij} \partial_{m} - (\hat{oldsymbol{\mathcal{K}}}_{lm})_{ij} \partial_{m,j}
ight)_{ij} = oldsymbol{s}_{l} \end{aligned}$$

Transient Non-Linear System of Equations: Navier-Stokes

Conservative variables:

$$\mathcal{U} = [\rho, \rho u_1, \dots, \rho e_{tot}]^T$$

$$rac{\partial \mathcal{U}}{\partial t} +
abla \cdot \left(ilde{\mathcal{A}} \mathcal{U} - ilde{\mathcal{K}}
abla \mathcal{U}
ight) = \mathbf{S}$$

Primitive variables:

$$\mathbf{Y} = [p, u_1, \dots, T]$$
 (pressure) or $[\rho, u_1, \dots, T]^T$ (density)

$$\mathcal{A}_0 \frac{\partial \mathbf{Y}}{\partial t} + \nabla \cdot (\mathcal{A}\mathbf{Y} - \mathcal{K}\nabla \mathbf{Y}) = \mathbf{S}$$

Entropy variables:

$$\mathbf{V} = [\vartheta_1, \vartheta_2, \dots, \vartheta_{n_{sd}+2}]^T$$

$$\hat{\mathcal{A}}_0 \frac{\partial \mathbf{V}}{\partial t} + \nabla \cdot (\hat{\mathcal{A}} \mathbf{V} - \hat{\mathcal{K}} \nabla \mathbf{V}) = \mathbf{S}$$

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