MANE 6760 - FEM for Fluid Dyn. - Lecture 13

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FE Setup and Procedure: Multiple Dimensions

Assembly of element-level (FE) matrices and vectors:

Global level operations (including BCs):

Non-linear (Steady) Equations

Consider a non-linear, steady, scalar equation:

$$R(\phi) = \mathcal{L}(\phi) - s = 0, \qquad \phi \in \mathcal{S}_{strong}$$

Derive (using integration by parts) a non-linear/semi-linear weak form:

$$a(w, \phi) = (w, s), \qquad \phi \in \mathcal{S}, \forall w \in \mathcal{W}$$

Derive a finite-element non-linear/semi-linear weak form:

$$a(\bar{w}, \bar{\phi}) = (\bar{w}, s), \qquad \bar{\phi} \in \bar{\mathcal{S}}, \forall \bar{w} \in \bar{\mathcal{W}}$$

Non-linear (Steady) Equations

Derive a finite-element based non-linear weak residual:

$$\hat{w}_A G_A = 0$$

Derive a non-linear system of (algebraic) equations:

$$G_A = 0, \qquad \forall A$$

Setup a non-linear solver (e.q., Newton Raphson):

$$\frac{\partial G_A}{\partial \hat{\phi}_B} \Delta \hat{\phi}_B + G_A = 0, \qquad \forall A$$

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