

Project -1 Analysis Write up

a) Estimation of Heat-Flux per unit length

The heat exchanger here describes a 2-D heat transfer problem. This can be estimated using finite difference methods after discretizing the 2-D surface area of the heat exchanger into nodal points where mass is clumped. Heat transfer is defined as heat that is transferred between one nodal point to another through the rod that is connecting the nodes.

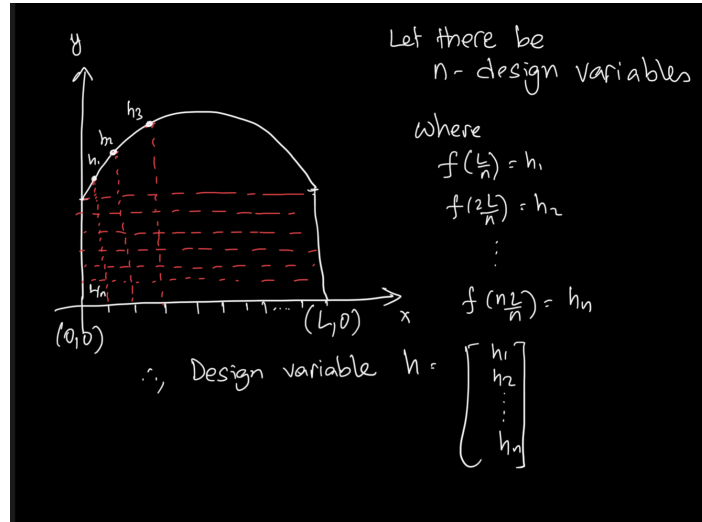
The catch here is that the top profile of the heat exchanger can be optimized to get maximum heat flux.

Heat flux is given by : $q = -k\nabla T$ where T is the temperature profile $T(x, y)$ of the heat exchanger across the nodal points.

$$\Rightarrow q = -k\left(\frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j}\right)$$

From conservation of energy, the heat that transfers from the water on to the heat exchanger is equal to the heat transferred to air from the exchanger.

b) Geometry definition of the problem



Boundary conditions are :

$$T(\frac{L}{n}, 0) = T(\frac{2L}{n}, 0) = \dots = T(L, 0) = T_{water} = 90^\circ C$$

$$T(\frac{L}{n}, h_1) = T(\frac{2L}{n}, h_2) = \dots = T(L, h_n) = T_{air} = 20^\circ C$$

The top profile is a function of the 1 - D design variable $h = [h_1 h_2 \dots h_n]^T$

Optimization problem is :

$$\max_h q(h) = \min_h -q(h) = \min_h |k\nabla T(h)|$$

with constraints

$$c(h) = h_{min} \leq h \leq h_{max}$$

Once the Temperature profile is found using a Finite Difference method based solver, the Heat flux out of the Heat exchanger can be computed easily. As the top profile changes, the heat flux out of the Heat exchanger also changes and that is exactly why the objective function is a function of how the geometry is defined.