NLA = the text-book *Numerical Linear Algebra*, by Trefethen and Bau

1. NLA exercise 6.5 *Let $P \in \mathbb{C}^{m\times m}$ be a nonzero projector...*
Hint: To show that $\|P\|_2 = 1$ implies that P is an orthogonal projector show that for any oblique projector there will be some vector v such that $\|Pv\|_2 > \|v\|_2$ (drawing a picture in $\mathbb{R}^2$ may help).

2. NLA exercise 7.1 *Consider again...*

3. NLA exercise 7.5 *Let A be an $m \times n$ matrix ...*

4. Write matlab functions `[Qc, Rc]=clgs(A)` and `[Qm, Rm]=mgs(A)` that implement the reduced QR factorization using the classical Gram-Schmidt and modified Gram-Schmidt algorithms, respectively. Test the implementations (in parts (a) and (b) below) by computing the QR factorization for the $m \times m$ Vandermonde matrix

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & \ldots & x_1^{m-1} \\ 1 & x_2 & x_2^2 & \ldots & x_2^{m-1} \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 1 & x_m & x_m^2 & \ldots & x_m^{m-1} \end{bmatrix},$$

for points $x_i \equiv (i-1)/(m-1)$, and compare to the results from the built-in Matlab function `[Q,R]=qr(A)`.

WARNING: The matlab `qr` algorithm may return R with negative diagonal entries, r_{ii}. If $r_{ii} < 0$, you can change the sign of column i of Q and row i of R to obtain a QR factorization with r_{ii} positive.

(a) For $m = 5$, compute $\|A - QR\|_2$ for each of the three approximations. Also compute the 2-norm differences between the classical and modified results compared to the Matlab results, $\|Qc - Q\|_2$, $\|Rc - R\|_2$, $\|Qm - Q\|_2$, $\|Rm - R\|_2$, and also compute the error $\|Q^*Q - I\|_2$ for each of the three approximations to Q. (Hint: for $m = 5$ the errors should all be small).

(b) Repeat (a) but with $m = 100$.