### MANE 6760 - FEM for Fluid Dyn. - Lecture 21

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### Simplified: 1D Non-linear (NL) TAD Eqn

A number of simplifications:

- ▶ 1D (spatial) domain:  $x \in [0, L]$
- Only Dirichlet/essential boundary conditions

 $\phi(x = 0, t) = \phi_0(t)$  on  $x = 0 \forall t$  $\phi(x = L, t) = \phi_L(t)$  on  $x = L \forall t$ 

#### Strong form:

$$\begin{split} R(\phi) &= \mathcal{L}(\phi) - s = \frac{\partial \phi}{\partial t} + a_{x} \frac{\partial \phi}{\partial x} - \frac{\partial}{\partial x} \left( \kappa(\phi) \frac{\partial \phi}{\partial x} \right) - s = 0, \quad \phi \in \mathcal{S}_{\textit{strong}} \\ &\quad x \in [0, L] \\ &\quad t \in [t_{\textit{min}}, t_{\textit{max}}] \\ &\quad \phi(x, t = t_{\textit{min}}) = \phi_{\textit{IC}}(x) \, \forall x \end{split}$$

### Non-linear Transient Equations: (Semi-discrete) FE for NL TAD

Derive a finite-element based non-linear weak residual:

$$\hat{w}_A G_A = 0$$
, where  $G_A = G_A^{galk}$  or  $G_A = G_A^{stab} = \sum_{k=1}^{N} N_k y_k$ 

Derive a non-linear system of (ordinary differential) equations:

$$G_A(\dot{\hat{\Phi}},\hat{\Phi})=G_A^L(\dot{\hat{\Phi}},\hat{\Phi})+G_A^{NL}(\dot{\hat{\Phi}},\hat{\Phi})=0, \hspace{0.5cm} orall A$$

 $G_A(\hat{\hat{\Phi}}, \hat{\Phi}) = G_A^T(\hat{\hat{\Phi}}, \hat{\Phi}) + G_A^S(\hat{\Phi}) = M_{AB}\hat{\phi}_B + G_A^S(\hat{\Phi}) = 0,$ 

fully-discrete  $\mathcal{G}_{A}(\hat{\mathbf{\Phi}}^{(n+1)},\hat{\mathbf{\Phi}}^{(n+1)},\hat{\mathbf{\Phi}}^{(n)}) = M_{AB}\frac{\hat{\phi}_{B}^{(n+1)} - \hat{\phi}_{B}^{(n)}}{t_{n+1} - t_{n}} + G_{A}^{S}(\hat{\mathbf{\Phi}}^{(0)}) = 0,$ 

Use a non-linear solver (e.g., Newton Raphson) for each time step:
$$\frac{\partial \mathcal{G}_A}{\partial x} = \frac{\partial \mathcal{G}_A}{\partial x} = \frac$$

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## Stabilized FE Form: (Simplified) 1D NL TAD Eqn

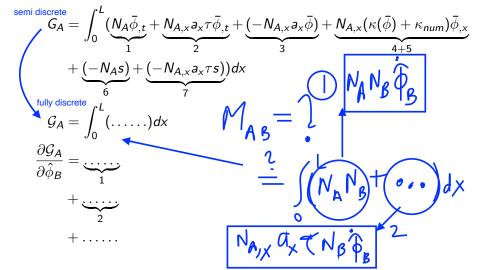
Stabilized/SUPG FE semi-discrete form (with  $\hat{\mathcal{L}}(\cdot) = -\mathcal{L}^{adv}(\cdot)$ ): find  $\bar{\phi} \in \bar{\mathcal{S}} \subset \mathcal{S}$  such that

$$\int_{0}^{L} \underbrace{\left(\overline{w}\overline{\phi}_{,t} + \underline{\overline{w}_{,x}}a_{x}\tau\overline{\phi}_{,t}}_{1} + \underbrace{\left(-\overline{w}_{,x}a_{x}\overline{\phi}\right)}_{3} + \underbrace{\overline{w}_{,x}\kappa(\overline{\phi})\overline{\phi}_{,x}}_{4} + \underbrace{\overline{w}_{,x}}_{knum} \underbrace{\kappa_{num} = a_{x}\tau a_{x}}_{5} \\ + \underbrace{\left(-\overline{w}s\right)}_{6} + \underbrace{\left(-\overline{w}_{,x}a_{x}\tau s\right)}_{6}\right)dx = 0$$

for all  $\bar{w} \in \bar{\mathcal{W}} \subset \mathcal{W}$ 

### Non-linear Iterations: (Simplified) 1D NL TAD Eqn

Non-linear iterations: require *fully discrete* non-linear weak residual and tangent matrix at every iteration within each time step



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