MFE 405 Computational Methods in Finance Project 2

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1 Lecture 4: Binomial-tree, Trinomial-tree models

1.1 Problem 1

1.1.1 (a)

The value of the option seems to converge to **$10.69**

1.1.2 (b)

The value of the option seems to converge to **$10.68**

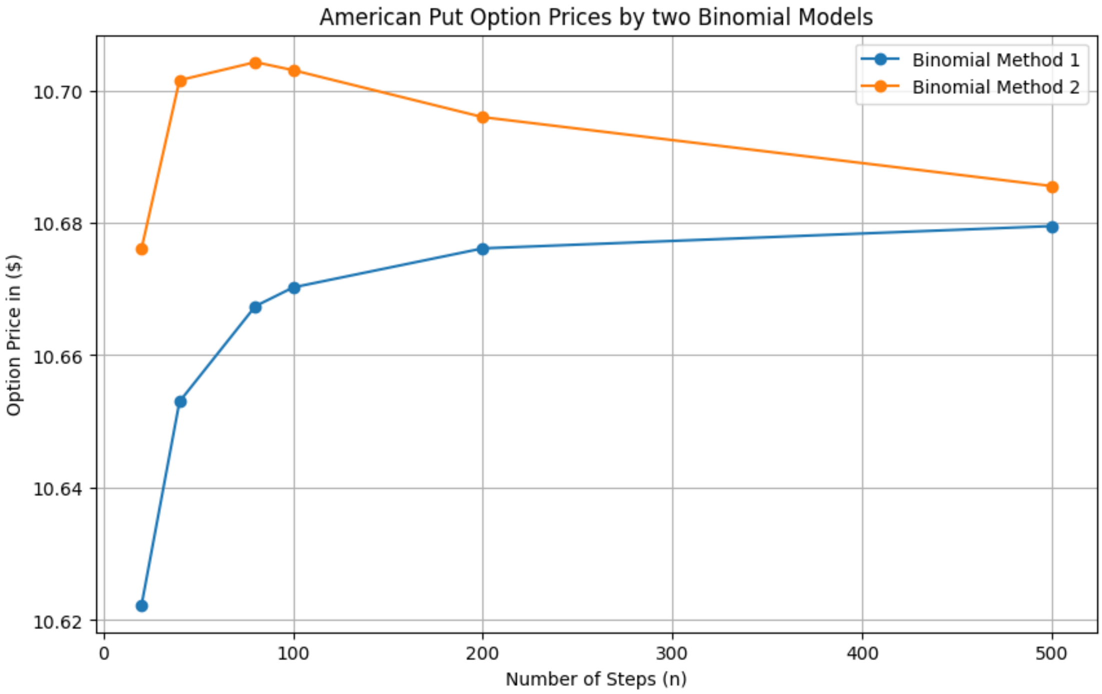
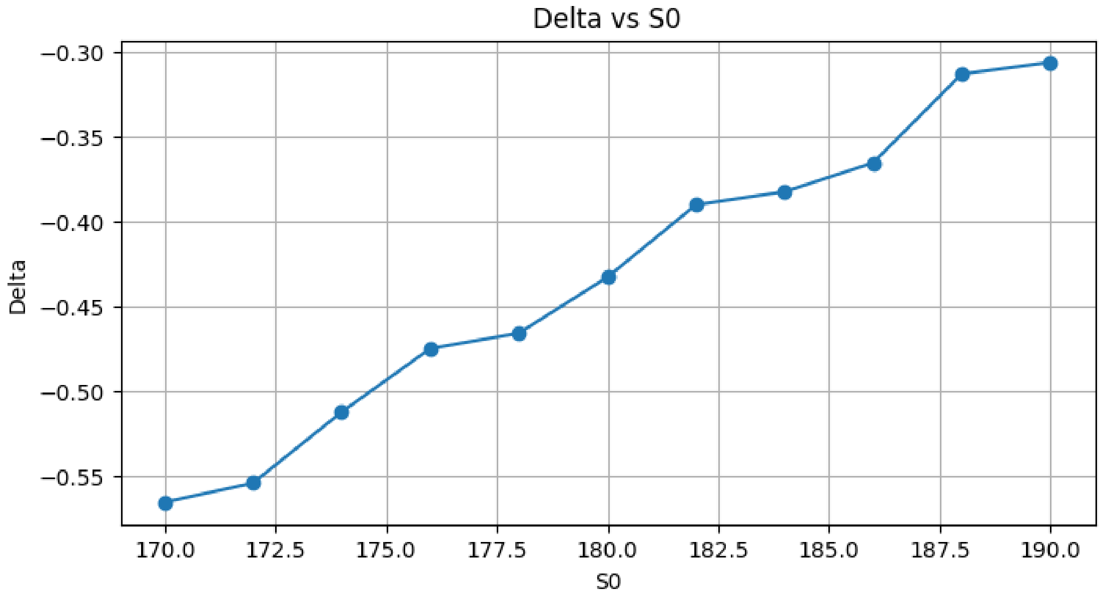


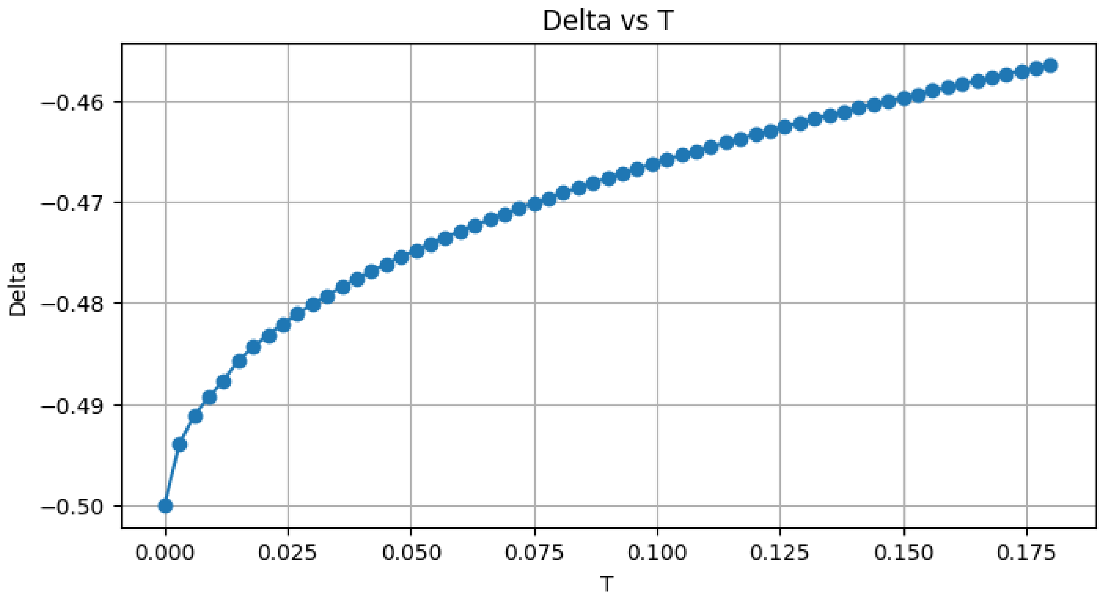
Figure 1: Option Prices using the two methods

1.2 Problem 2

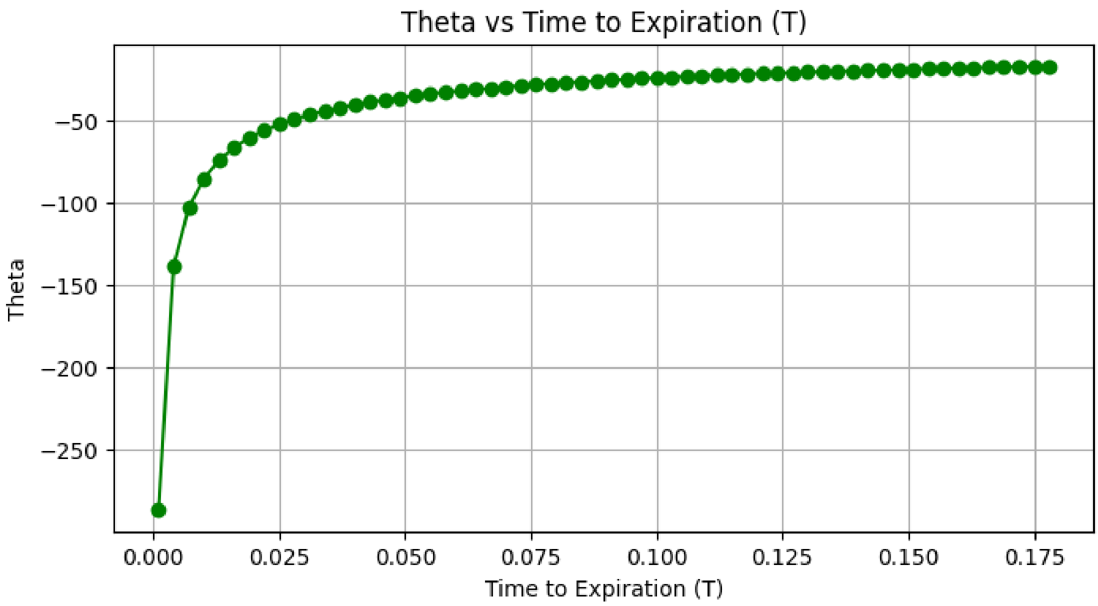
1.2.1 (a)



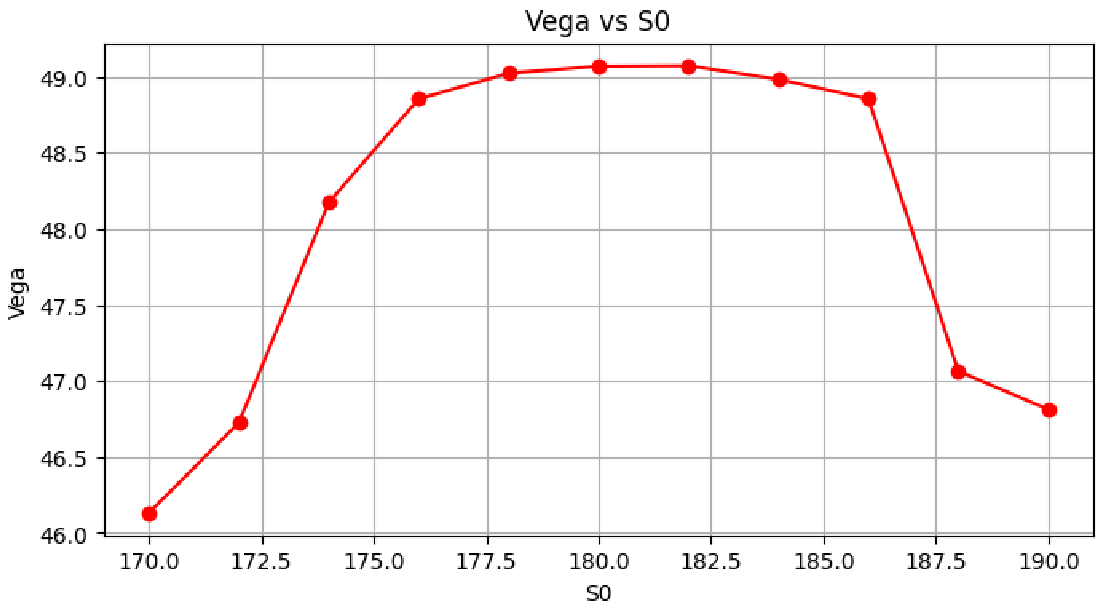
1.2.2 (b)



1.2.3 (c)



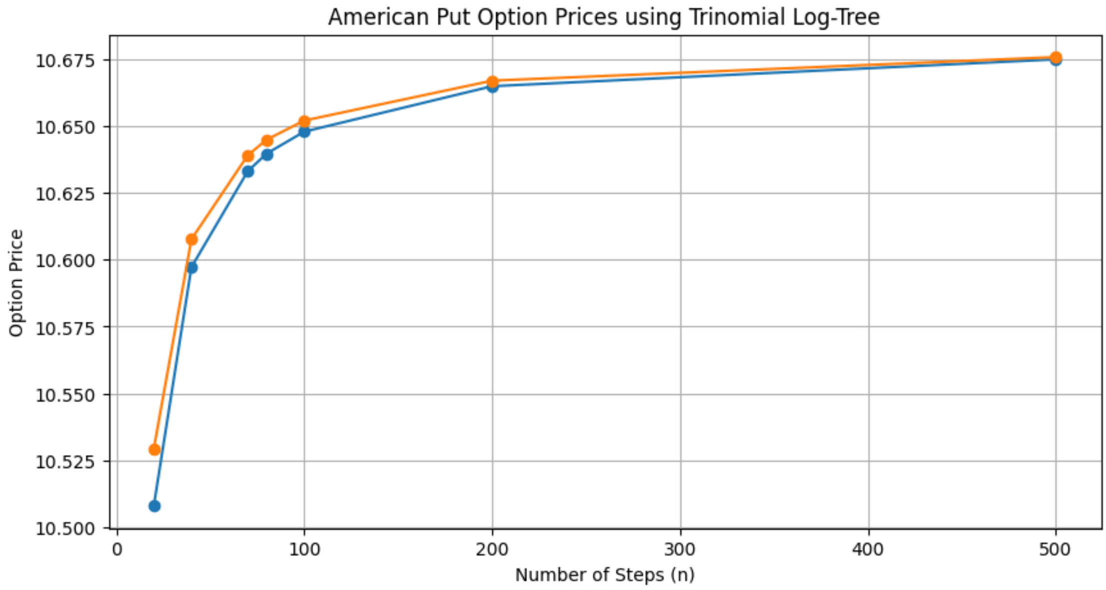
1.2.4 (d)



1.3 Problem 3

The values seem to merge at **$10.675**

1.3.1 (a) and (b)



2 Lecture 5: Least Square Monte Carlo Method

2.1 Problem 4

2.1.1 (a) and (b) and (c)

Table 1: Option Prices for Laguerre, Hermite, and Monomial Polynomials

2*k	Laguerre Polynomials		Hermite Polynomials		Simple Monomials	
	0.5 years	1.5 years	0.5 years	1.5 years	0.5 years	1.5 years
2	10.665795	16.372413	10.579403	16.162716	10.607738	16.115300
3	10.717950	16.350273	10.685534	16.502300	10.684104	16.492821
4	10.717825	16.527741	10.705109	16.504105	10.715626	16.502325
5	10.754864	16.560802	10.728074	16.502705	10.722171	16.540917

The study compared the convergence, computational efficiency, and accuracy of three different polynomial bases - Laguerre, Hermite, and Simple Monomials - in pricing an American Put option using the Least Squares Monte Carlo (LSMC) method. The option had a strike price of $180 and maturities of 0.5 and 1.5 years.

The Simple Monomials demonstrated the most effective convergence among the tested bases. Their straightforward nature and alignment with linear and non-complex patterns commonly found in financial modeling contributed to their robust performance in achieving stable pricing efficiently.

In contrast, the Laguerre and Hermite polynomials exhibited slower convergence compared to Simple Monomials. Despite the suitability of Laguerre polynomials for models with exponential decay and Hermite polynomials for Gaussian distributions, they did not match the simplicity and directness offered by the monomial basis in this application.

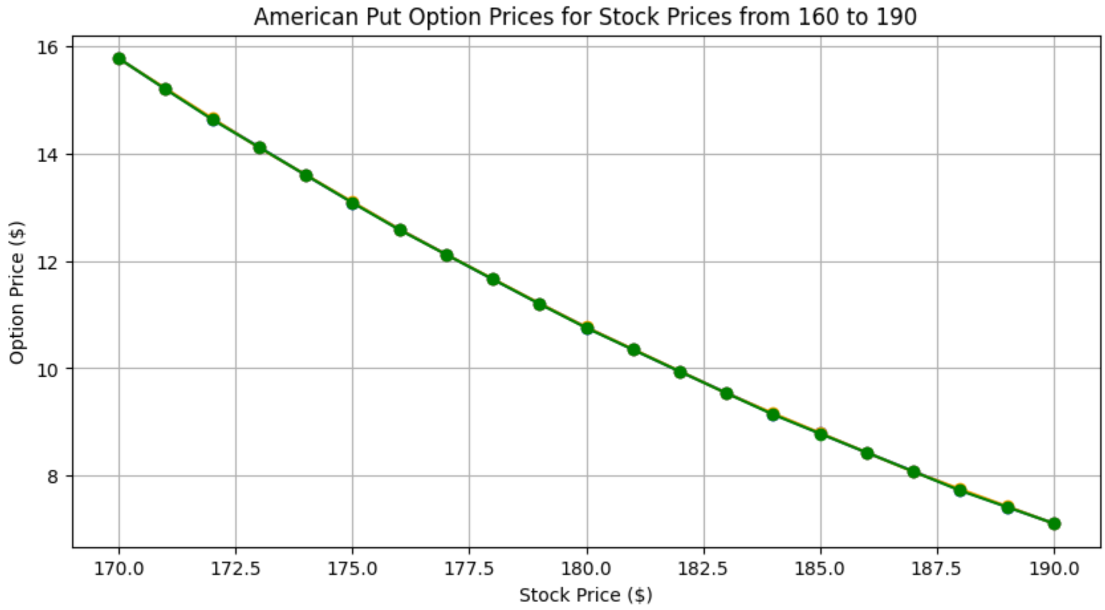
The study also found that as k increases, the convergence improves, comparing the Binomial and Trinomial models. Furthermore, the computation time required for convergence indicated that Simple Monomials not only converged faster but also utilized fewer computational resources due to their simplicity and straightforward calculations.

Based on the comparative analysis, the study recommends using Simple Monomials in LSMC methods for pricing American Put options. Their superior convergence properties and computational efficiency make them an ideal choice for practitioners seeking a reliable and straightforward modeling approach.

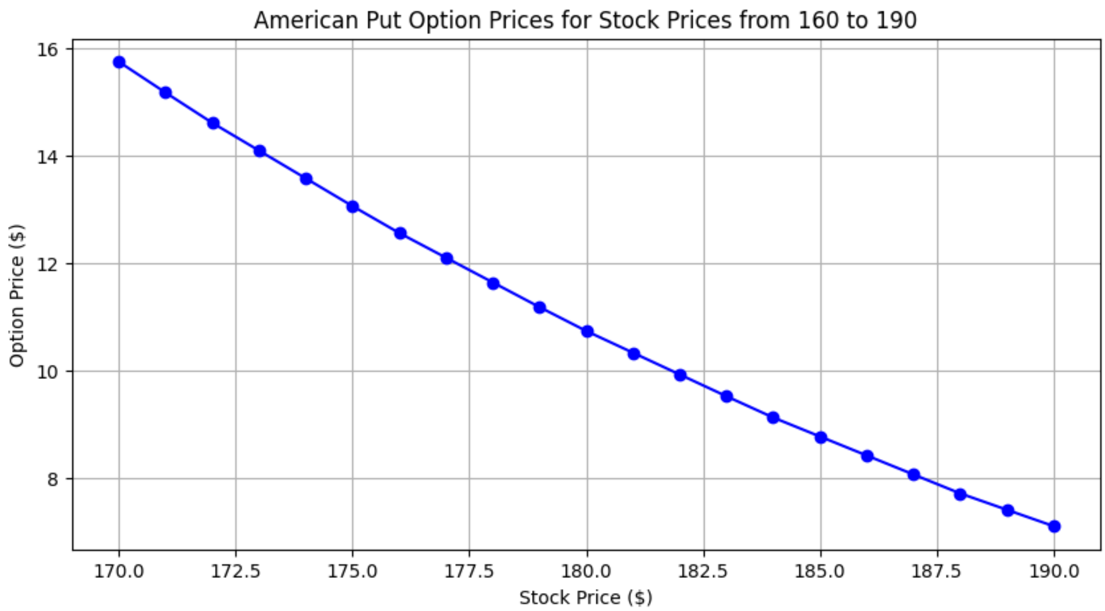
3 Lecture 6: Numerical PDE Method

3.1 Problem 5

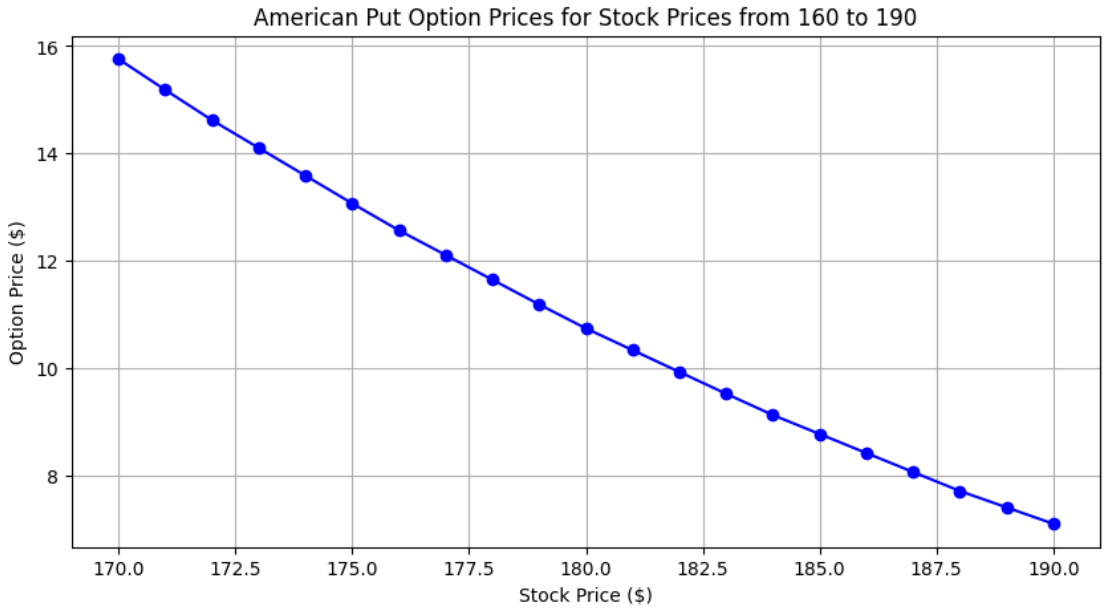
3.1.1 (a)



3.1.2 (b)



3.1.3 (c)



Recommendation and Error Analysis

Based on the analysis, the **Crank-Nicolson method** is recommended for pricing American Put options in environments where accuracy and stability are paramount. It provides a robust pricing mechanism suitable for financial markets that require precise and reliable option valuation.

For Errors:

The Trinomial method is giving the optional value of approximately 10.65 at a stock price of 180. Comparing it to our values in Question 5 below:

For the stock price at 180, using the grid transformation $\Delta X = \sigma\sqrt{\Delta t}$, the calculated option values from each finite-difference method are as follows:

- Explicit Finite-Difference Method: \$10.67
- Implicit Finite-Difference Method: \$10.66
- Crank-Nicolson Method: \$10.67

The error for the Crank Nicolson Method is 0.02.

For the stock price at 180, using the grid transformation $\Delta X = \sigma\sqrt{3\Delta t}$, the calculated option values from each finite-difference method are as follows:

- Explicit Finite-Difference Method: \$10.67
- Implicit Finite-Difference Method: \$10.65
- Crank-Nicolson Method: \$10.66

The error for the Crank Nicolson Method is 0.01.

For the stock price at 180, using the grid transformation $\Delta X = \sigma\sqrt{4\Delta t}$, the calculated option values from each finite-difference method are as follows:

- Explicit Finite-Difference Method: \$10.66

- Implicit Finite-Difference Method: \$10.64
- Crank-Nicolson Method: \$10.65

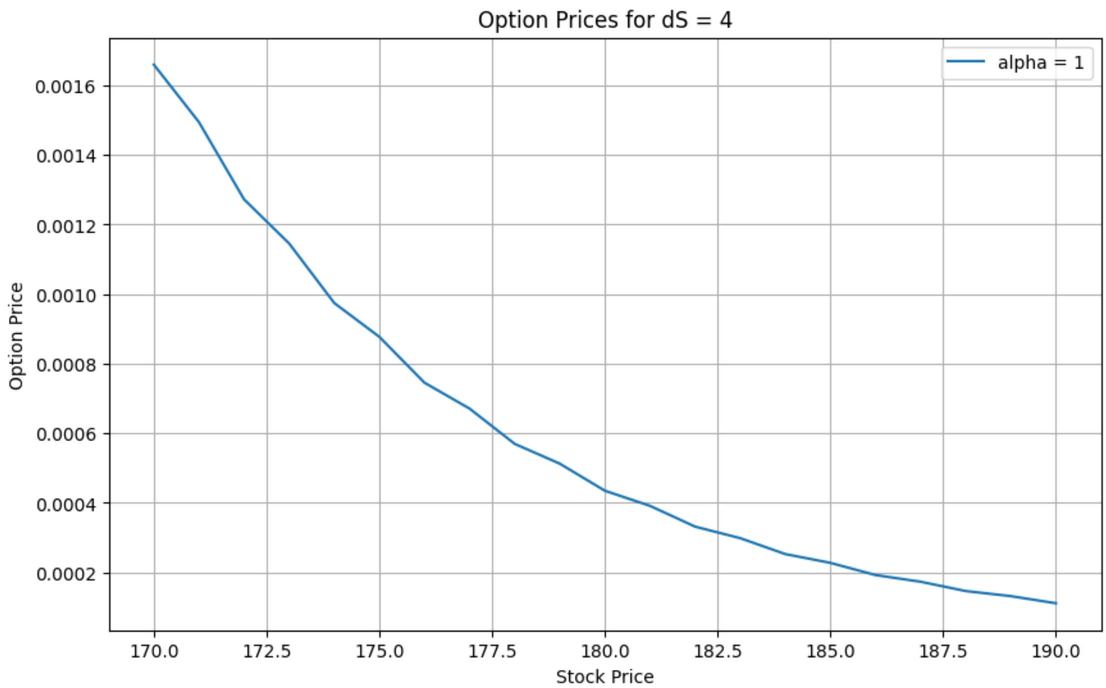
The error for the Crank Nicolson Method is 0.

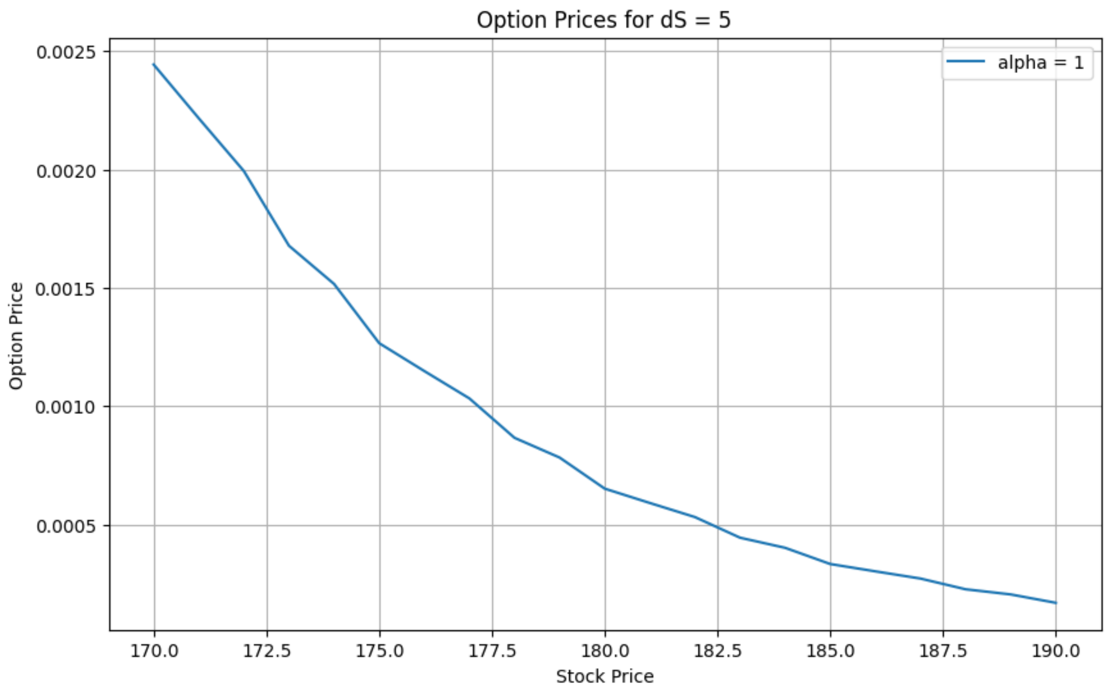
If we look at the error it is the least for the Crank-Nicolson Method for $\Delta X = \sigma\sqrt{4\Delta t}$, the error is the least as expected, for $\Delta X > \sigma\sqrt{3\Delta t}$, the stability is better and the convergence is the best for the Crank-Nicolson Method.

3.2 Problem 6

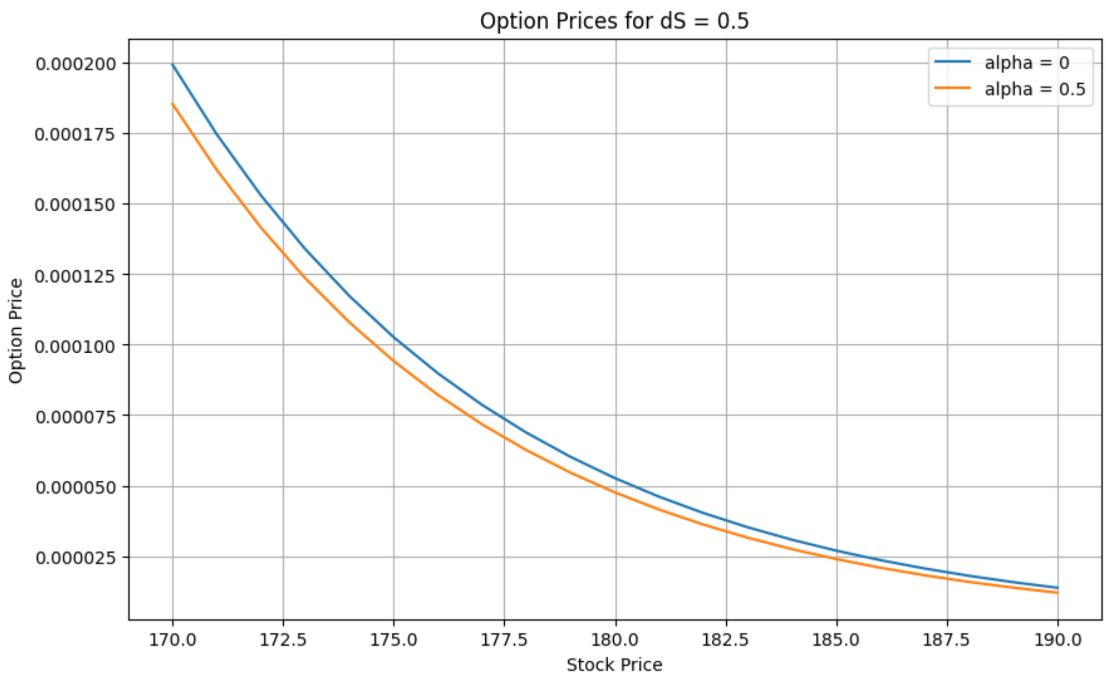
3.2.1 (a) and (b) and (c)

For alpha = 1, I could not get convergence for dS = 0.5 or 1.0. So I tried for dS = 4 and 5 to get some convergence.





Option Prices for dS = 5
alpha = 1
Option Price
Stock Price
0.0025
0.0020
0.0015
0.0010
0.0005
170.0
172.5
175.0
177.5
180.0
182.5
185.0
187.5
190.0



Option Prices for dS = 0.5
alpha = 0
alpha = 0.5
Option Price
Stock Price
0.000200
0.000175
0.000150
0.000125
0.000100
0.000075
0.000050
0.000025
170.0
172.5
175.0
177.5
180.0
182.5
185.0
187.5
190.0

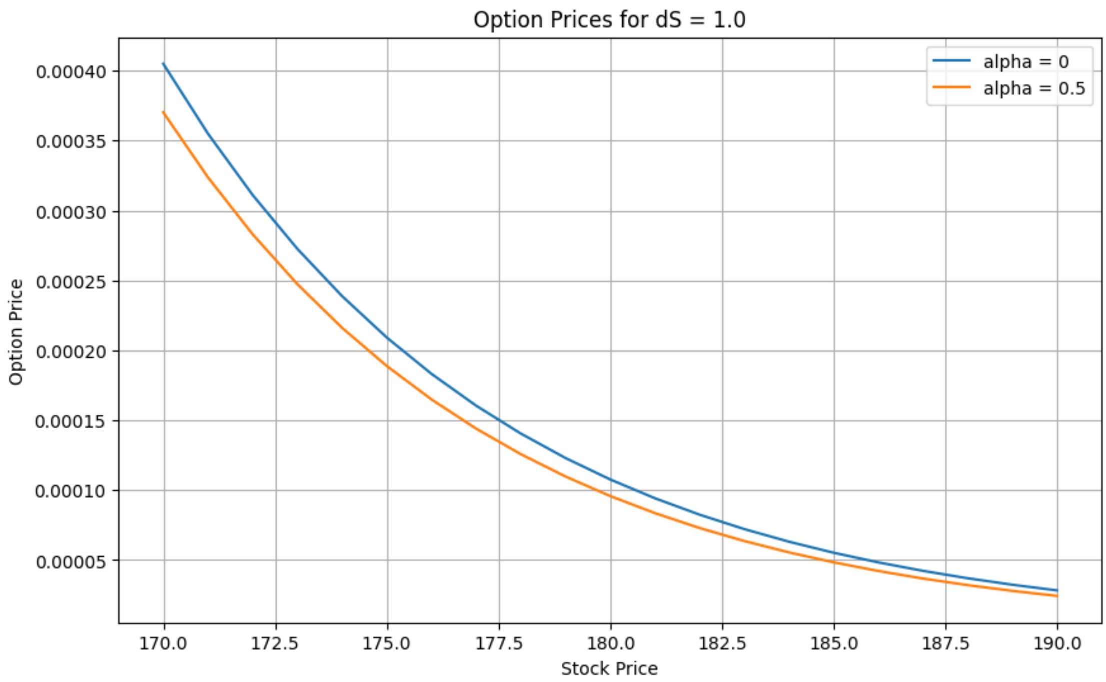


Table 2: Option Prices for Different Alpha Values

Stock Price	Alpha = 0	Alpha = 0.5
170	16.254369	16.266147
171	15.639052	15.650857
172	15.036799	15.048612
173	14.447692	14.459497
174	13.871810	13.883594
175	13.309221	13.320961
176	12.759986	12.771662
177	12.224151	12.235741
178	11.701753	11.713239
179	11.192823	11.204188
180	10.697377	10.708602
181	10.215421	10.226493
182	9.746954	9.757854
183	9.291958	9.302664
184	8.850406	8.860904
185	8.422261	8.432525
186	8.007463	8.017479
187	7.605948	7.615702
188	7.217643	7.227116
189	6.842451	6.851635
190	6.480278	6.489158