PROJECT 2

MGMTMFE 405
SPRING 2024

You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them and interpreting the results. Code quality, speed, and accuracy will determine the grades.

LECTURE-4 [Binomial-tree, Trinomial-tree models]

1. Compare the convergence rates of the four methods below by doing the following:
Use the Binomial Method to price a 6-month American Put option with the following information: the risk-free interest rate is 5.5% per annum; the volatility is 25% per annum; the current stock price is \$180; and the strike price is \$180. Divide the time interval into n parts to estimate the price of this option. Use $n = 20, 40, 80, 100, 200, 500$, to estimate the price and draw all resulting prices in one graph, where the horizontal axis measures n, and the vertical one the price of the option.

(a) Use the binomial method in which

$$u = \frac{1}{d}, \qquad d = c - \sqrt{c^2 - 1}, \qquad c = \frac{1}{2}\left(e^{-r\Delta} + e^{(r+\sigma^2)\Delta}\right), \qquad p = \frac{e^{r\Delta} - d}{u - d}$$

(b) Use the binomial method in which

$$u = e^{\left(r-\frac{\sigma^2}{2}\right)\Delta + \sigma\sqrt{\Delta}}, \qquad d = e^{\left(r-\frac{\sigma^2}{2}\right)\Delta - \sigma\sqrt{\Delta}}, \qquad p = 1/2$$

Outputs: Graphs: Two plots in one graph.

2. Consider the following information on the stock of a company and American put options on it:
$S_0 = \$180,\ X = \$180,\ r = 0.055,\ \sigma = 0.25,\ T = 6 \text{ months},\ \mu = 0.15.$
Using the CRR Binomial tree method, estimate the following and draw their graphs:

(i) **Delta** of the put option as a function of S_0, for S_0 ranging from \$170 to \$190, in increments of \$2.

(ii) **Delta** of the put option, as a function of T (time to expiration), T ranging from 0 to 0.18 in increments of 0.003.

(iii) **Theta** of the put option, as a function of T (time to expiration), T ranging from 0 to 0.18 in increments of 0.003.

(iv) **Vega** of the put option, as a function of S_0, for S_0 ranging from \$170 to \$190, in increments of \$2.

Note: In the CRR model, $u = e^{\sigma\sqrt{\Delta}},\ d = e^{-\sigma\sqrt{\Delta}},\ ,\ p = \frac{e^{r\Delta} - d}{u - d}$.

Outputs: Graphs: 4 separate graphs.

3. Compare the convergence rates of the two methods, (a) and (b), described below, by doing the following:

Use the Trinomial-tree method to price a 6-month American put option with the following information: the risk-free interest rate is 5.5% per annum, the volatility is 25% per annum, the current stock price is \$180, and the strike price is \$180.

Divide the time interval into n equal parts to estimate the option price. Use n = 20, 40, 70, 80, 100, 200, 500; to estimate option prices and draw them all in one graph, where the horizontal axis measures n, and the vertical one measures option price.

The two methods are in (a) and (b) below:

(a) Use the Trinomial-tree method applied to the stock price-process (S_t) in which

$$u = \frac{1}{d}, \; d = e^{-\sigma\sqrt{3\Delta}}, \;\; p_d = \frac{r\Delta(1-u)+(r\Delta)^2+\sigma^2\Delta}{(u-d)(1-d)}, \;\; p_u = \frac{r\Delta(1-d)+(r\Delta)^2+\sigma^2\Delta}{(u-d)(u-1)}, \;\; p_m = 1 - p_u - p_d$$

(b) Use the Trinomial-tree method applied to the Log-stock price-process (X_t) in which

$$\Delta X_u = \sigma\sqrt{3\Delta}, \quad \Delta X_d = -\sigma\sqrt{3\Delta},$$

$$p_d = \frac{1}{2}\left(\frac{\sigma^2\Delta+\left(r-\frac{\sigma^2}{2}\right)^2\Delta^2}{{\Delta X_u}^2} - \frac{\left(r-\frac{\sigma^2}{2}\right)\Delta}{\Delta X_u}\right), \quad p_u = \frac{1}{2}\left(\frac{\sigma^2\Delta+\left(r-\frac{\sigma^2}{2}\right)^2\Delta^2}{{\Delta X_u}^2} + \frac{\left(r-\frac{\sigma^2}{2}\right)\Delta}{\Delta X_u}\right), \; p_m = 1 - p_u - p_d$$

Outputs: Graphs: plot in a graph.

LECTURE - 5 [Least Square Monte Carlo method]

4. Consider the following information on the stock of company XYZ: The current stock price is $180, and the volatility of the stock price is $\sigma = 25\%$ per annum. Assume the prevailing risk-free rate is $r = 5.5\%$ per annum. Use the following method to price the specified option:

(a) Use the **LSMC** method with N=100,000 paths simulations (50,000 plus 50,000 antithetic variates) and a time step of $\Delta = \frac{1}{\sqrt{N}}$ to price an American Put option with strike price of $K = \$180$ and maturity of 0.5-years and 1.5-years. Use the first k of the **Laguerre Polynomials** for $k = 2, 3, 4, 5$. (That is, you will compute 8 prices here). Compare the prices for the 4 cases, $k = 2, 3, 4, 5$ and comment on the choice of k.

(b) Use the **LSMC** method with N=100,000 paths simulations (50,000 plus 50,000 antithetic variates) and a time step of $\Delta = \frac{1}{\sqrt{N}}$ to price an American Put option with strike price of $K = \$180$ and maturity of 0.5-years and 1.5-years. Use the first k of the **Hermite Polynomials** for $k = 2, 3, 4, 5$. (That is, you will compute 8 prices here). Compare the prices for the 4 cases, $k = 2, 3, 4, 5$ and comment on the choice of k.

(c) Use the **LSMC** method with N=100,000 paths simulations (50,000 plus 50,000 antithetic variates) and a time step of $\Delta = \frac{1}{\sqrt{N}}$ to price an American Put option with strike price of $K = \$180$ and maturity of 0.5-years and 1.5-years. Use the first k of the **Simple Monomials** for $k = 2, 3, 4, 5$. (That is, you will compute 8 prices here). Compare the prices for the 4 cases, $k = 2, 3, 4, 5$ and comment on the choice of k.

(d) Compare all your findings above and comment.

Note: You will need to use the weighted-polynomials as it was done by the authors of the method.

Inputs: $S_0, K, T, r, \sigma, N, k$
Outputs: Values of Option Prices; writeup: comments.

LECTURE-6 [Numerical PDE method]

5. Consider the following information on the stock of company XYZ: The volatility of the stock price is $\sigma = 25\%$ per annum. Assume the prevailing risk-free rate is $r = 5.5\%$ per annum. Use the $X = ln(S)$ transformation of the Black-Scholes PDE, and $\Delta t = 0.002$, with $\Delta X = \sigma\sqrt{\Delta t}$; then with $\Delta X = \sigma\sqrt{3\Delta t}$; then with $\Delta X = \sigma\sqrt{4\Delta t}$, and a *uniform* grid (on X) to price an American Put option with strike price of $K = \$180$, expiration of 6 months and current stock prices ranging from \$170 to \$190; using the specified methods below:

(a) *Explicit Finite-Difference method,*
(b) *Implicit Finite-Difference method,*
(c) *Crank-Nicolson Finite-Difference method.*

Inputs: K, σ, T, Δt
Outputs:

i. Values: Pa, Pb and Pc for the Put option using each of the methods (a), (b) and (c).
ii. Writeup: compare the three methods from (a), (b) and (c) and comment. To compare, calculate the relative error with respect to the prices estimated in earlier problems of this set. Do this for current stock prices of \$170 to \$190 in \$1 increments and put them in a table. Put the table and your comments in a .pdf file.

6. Consider the following information on the stock of company XYZ: The volatility of the stock price is $\sigma = 25\%$ per annum. Assume the prevailing risk-free rate is $r = 5.5\%$ per annum.
Use the Black-Scholes PDE (for S) to price American Put options with strike prices of $K = \$180$, expiration of 6 months and current stock prices for a range from \$170 to \$190; using the specified methods below:

(a) *Explicit Finite-Difference method,*
(b) *Implicit Finite-Difference method,*
(c) *Crank-Nicolson Finite-Difference method.*

Choose $\Delta t = 0.002$, with $\Delta S = 0.5$, or with $\Delta S = 1$.

Inputs: K, σ, T, Δt
Outputs:

i. Values: Ca, Cb, Cc, Pa, Pb and Pc and for the American put options using each of the methods (a), (b) and (c).
ii. Graphs: Plot the American Put option price as a function of the current stock price from \$170 to \$190 in \$1 increments for methods (a), (b) and (c) on the same graph. Use a color legend or linestyles to differentiate the plots. Place the two graphs in a .pdf file.