MFE 405 Computational Methods in Finance Project 3

Vinamra Rai

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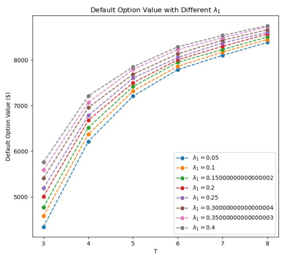
1 Lecture 7: Exotic Options, Variance Swaps, Jump-Diffusions

1.1 Problem 1

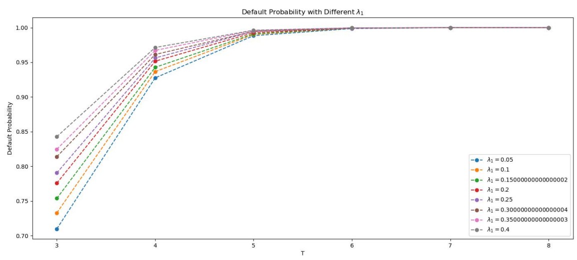
1.1.1 (a)

Price of the default option is **$7498.8009**



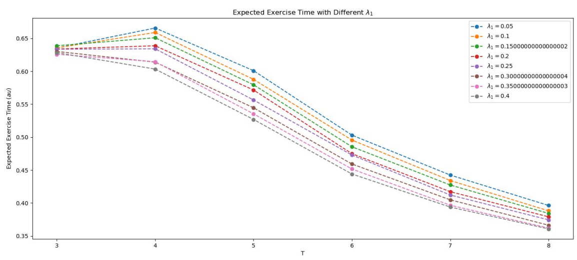
1.1.2 (b)

Default Intensity is **0.9926**



1.1.3 (c)

Expected Exercise Time is **0.5716 years**



1.2 Problem 2

1.2.1 (a)

Price of the do put option with a fixed barrier is **$0.009513**

1.2.2 (b)

Price of the do put option with a time-dependent moving barrier is **$0.002135**

1.2.3 (c)

The price of the down-and-out put option with a time-dependent barrier is **$0.013583**

Comparison of Different Do Put Option Prices

The prices P_1, P_2, and P_3 correspond to down-and-out put options with different barrier types:

- P_1 ($0.009513): Fixed barrier at $94
- P_2 ($0.002135): Increasing time-dependent barrier, starting at $91 and ending at $97
- P_3 ($0.013583): Decreasing time-dependent barrier, starting at $97 and ending at $91

The barrier type significantly influences the option price:

- P_2 has the lowest price due to the high initial knock-out risk from the low starting barrier.
- P_3 has the highest price because of the low initial knock-out risk from the high starting barrier.
- P_1, with a fixed barrier, falls between P_2 and P_3 as it presents a constant moderate risk.

In summary, time-dependent barriers alter the risk profile over time, with increasing barriers resulting in lower prices and decreasing barriers resulting in higher prices compared to a fixed barrier.

2 Lecture 8: Fixed Income Securities

2.1 Problem 3

2.1.1 (a)

Price of the Coupon Paying Bond is **$1019.588143**

2.1.2 (b)

Using Monte Carlo Simulation price of the European Call Option is **$0.377003**

2.1.3 (c)

Using Implicit Finite-Difference method price of the European call option price is **$0.393502**

Comparison of Different Call Option Prices

The prices obtained using the Monte Carlo Simulation and the Implicit Finite-Difference Method are relatively close, with the price from the Implicit Finite-Difference Method being slightly higher. This slight difference can be attributed to the following factors:

- **Monte Carlo Simulation:** This method relies on repeated random sampling to obtain numerical results. The accuracy of the result increases with the number of simulations, but it can be computationally intensive and may still introduce some variability due to its stochastic nature.
- **Implicit Finite-Difference Method:** This method solves the partial differential equation (PDE) governing the option price. It typically provides more stable and precise results compared to Monte Carlo simulations but requires discretizing the PDE and can be complex to implement.

In conclusion, while both methods provide similar option prices, the Implicit Finite-Difference Method tends to offer more precise and stable results due to its deterministic nature. However, it can be more challenging to implement compared to the Monte Carlo Simulation, which is more straightforward and flexible, especially for complex option pricing problems.

2.2 Problem 4

The price of the European put option using **Monte Carlo simulations** for $\rho = 0.7$ is **\$7.6068**.
The price of the European put option using **Closed form solution** for $\rho = 0.7$ is **\$6.6713**.

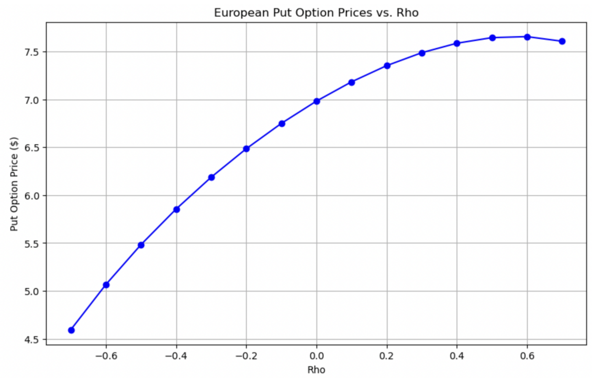


Figure 1: Using Simulations

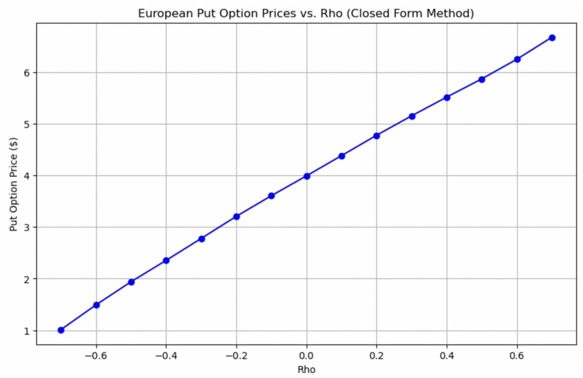


Figure 2: Using Closed Form Solution

Rho	Monte Carlo Price	Rho	Closed Form Price
-0.7	\$4.5945	-0.7	\$1.0132
-0.6	\$5.0667	-0.6	\$1.4928
-0.5	\$5.4830	-0.5	\$1.9407
-0.4	\$5.8542	-0.4	\$2.3539
-0.3	\$6.1867	-0.3	\$2.7778
-0.2	\$6.4841	-0.2	\$3.2053
-0.1	\$6.7487	-0.1	\$3.6045
0.0	\$6.9814	0.0	\$3.9887
0.1	\$7.1826	0.1	\$4.3794
0.2	\$7.3515	0.2	\$4.7748
0.3	\$7.4868	0.3	\$5.1506
0.4	\$7.5857	0.4	\$5.5128
0.5	\$7.6441	0.5	\$5.8654
0.6	\$7.6551	0.6	\$6.2451
0.7	\$7.6068	0.7	\$6.6713

Table 1: Comparison of Put Option Prices for Different Rho Values

The closed-form solution generally provides the exact answer under the model assumptions, offering precise and computationally efficient pricing for the European put option. On the other hand, the Monte Carlo simulation, while approximate and potentially less precise due to its reliance on random sampling, offers greater flexibility in handling complex derivatives and path-dependent options. The discrepancies between the Monte Carlo prices and the closed-form prices can highlight the strengths and limitations of each method. For example, the closed-form solution is highly accurate within its model assumptions, whereas the Monte Carlo method can capture more complex dynamics but may suffer from numerical errors. Hence, the choice between these methods should consider the specific context of the option being priced and the trade-offs between precision and flexibility.

3 Lecture 9, 10: MBS

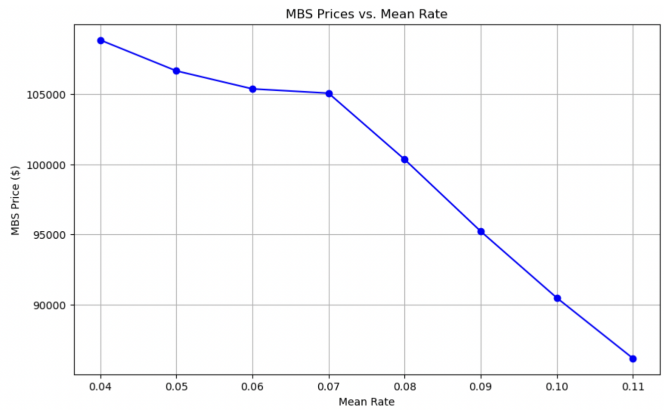
3.1 Problem 5

3.1.1 (a)

Price of the MBS for the original parameters is **$100342.59**.

Mean Rate	MBS Price ($)
0.04	108834.53
0.05	106641.29
0.06	105360.67
0.07	105054.86
0.08	100342.59
0.09	95229.10
0.1	90496.61

Table 2: MBS Prices for Different Mean Rates



The price of a Mortgage-Backed Security (MBS) generally decreases as the mean interest rate rises for several reasons:

1. Higher discount rates reduce the present value of the MBS's future cash flows, leading to a lower price.
2. When interest rates increase, prepayment rates typically decrease, extending the duration of the MBS and exposing it to greater interest rate risk over a longer period. This increased risk contributes to a lower MBS value.
3. As interest rates rise, investors demand higher returns from existing MBSs to match the yields of new investments, putting downward pressure on MBS prices.
4. The complex nature of MBSs, with embedded prepayment options, causes their duration to increase when interest rates rise, making them more sensitive to rate changes and further contributing to price declines.

In essence, higher mean interest rates negatively impact MBS prices by reducing the present value of cash flows, extending duration, increasing interest rate risk, and making MBSs less attractive compared to new, higher-yielding investments.

3.1.2 (b)

Given the market price of $98,000, the OAS is: **0.3686%**

3.1.3 (c)

The price of the IO tranche using the Numerix model is: **$53477.04**
The price of the PO tranche using the Numerix model is: **$46928.87**