Riding the Tails: The Alpha in Asymmetry

ABSTRACT

Forecasting market behavior is an important objective for policymakers and investors alike. We attempt to formulate a link between extra information carried by different players in the options market and the implied Risk Neutral Densities calculated using observed option prices. We study the relation between future returns and the term structure of risk-neutral skewness. We then try to exploit the mean reversal behavior of the premium between the forecasted VIX and the realized volatility to generate market timing signals. Lastly, we build a Treasury trading strategy using conditional skewness implied from Treasury options. Treasury dynamics differ from those in other markets as the Federal Reserve directly gives future rate guidance information, which gradually flows into RNDs. While we have explored a few trading strategies, it is important to note that the paper's objective is to study the predictive power of higher moments of RNDs across asset classes.

1. Introduction

The paper explores Risk-Neutral Densities (RNDs) and their capacity to forecast future market behavior, leveraging datasets from the Federal Reserve Bank of Minneapolis [12] that include market probability density functions derived from option trading data across various financial assets. We examined whether these moments can forecast future returns, volatility, or market reversals across different asset classes, including equities in the U.S. and Europe, fixed income markets, and the VIX. Our methodology incorporated a combination of regression and machine learning models. The findings suggested that the predictive power of RND moments, especially skewness and kurtosis, varied significantly across markets and asset types, showing substantial predictive capacity for Treasury futures and the VIX but less consistency in equity markets.

The investigation revealed that while broad market indices did not exhibit significant predictive power, analysis of individual securities suggested that higher moments can be informative, particularly for under-researched stocks with market frictions such as short-selling constraints. An examination of the term structure of RNDs yielded negligible predictive outcomes for large-cap equities.

In the Treasury market, a more definitive connection was established, especially with regard to conditional skewness. Applying techniques traditionally used for macroeconomic forecasting, our study uncovered their potential in predicting excess bond returns, with conditional skewness of ten-year Treasury futures signaling possible shifts in interest rate regimes.

For the VIX, it was observed that amalgamating RND tail attributes with realized volatility forecasts yielded a statistically significant predictor of future VIX values. This predictive insight could be utilized to formulate trading strategies, benefiting from the historically inverse relationship between the VIX and market trends.

The remainder of this paper is structured as follows. Section 2 reviews a selection of the existing literature on RNDs and their market forecasting applications. Section 3 presents an initial exploration of the predictability of the S&P 500 and FTSE indexes' as well as several individual stocks. Section 4 describes and applies our methodology to develop and evaluate three data-driven trading strategies. Section 5 summarizes our results and conclusions and suggests possible future research directions.

2. Literature Review

The application of RNDs in predicting future market behaviors has been a topic of some interest in financial research, particularly in understanding the relationship between RNDs and future realized probability density functions. The 1978 paper by Breeden and Litzenberger [7] set the foundation for RND calculation from option prices. Following this, Shimko (1993) [17] fitted the implied volatility at discrete strikes to represent option prices in a continuous strike space. This has led to ongoing research in constructing RNDs and relating them to real-world probability density functions.

Further studies have explored using higher RND moments to predict future returns. Harris and Qiao (2018) [13] analyzed moment risk premia's ability to predict stock returns, finding that variance and skew risk premia are negatively related to returns, unlike the kurtosis risk premium. Neumann and Skiadopoulos (2012) [15] identified predictable patterns in higher-order risk-neutral moments from S&P 500 index options, with 1-day-ahead skewness forecasts exhibiting significant utility. Fan, Xiao, and Zhou (2020) [11] discovered that while the second-moment risk premium predicts short-term market returns, the third and fourth-moment risk premia are more indicative of medium-term returns.

The literature on the relationship between option-implied skewness and future returns of single stocks presents mixed findings. Bali and Murray (2012) [2] and Conrad, Dittmar, and Ghysels (2013) [9] supported a skewness preference theory, observing a negative correlation between Risk-Neutral Skewness (RNS) and future equity returns. They presumed that option and stock markets reflected similar information. Thus, a higher option-implied skewness, coupled with underlying skewness preference, leads to lower expected returns. Contrary studies like Xing, Zhang, and Zhao (2010)[20] and Stilger, Kostakis, and Poon (2014) [18] demonstrated a positive correlation between future stock returns and RNS. Xing et al. (2010) argued that informed option traders purchasing OTM put options ahead of downward jumps increased the implied volatility of these options, resulting in a more negative RNS. Stilger et al. (2014) observed that such trading activities were concentrated in stocks perceived as overpriced and hard to short-sell. Over time, as this mispricing information moved into the stock market, stocks with low RNS, being relatively overpriced, tended to underperform. Borochin et al. (2020) [6] combined the two perspectives by examining the term structure of risk-neutral skewness, revealing that short-term skewness suggested positive returns via informed trading, while long-term skewness implied negative returns, in line with skewness preference.

The importance of predicting market-realized volatility for effective trading strategies and risk management is well recognized. Engle's (1982) [10] seminal work on Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models set the groundwork for such forecasts. Using VIX predictions to trade the S&P 500 has also been a significant topic in financial research. Whaley (1993) [19] noted the VIX's importance in understanding market uncertainty and investor sentiment, while Baker and Wurgler (2006) [1] observed that VIX spikes correlated with market instability and potential declines, underscoring its predictive value for market trends. Bollerslev et al. (2009) [5] confirmed the VIX's ability to forecast market returns and risk. More recently, the application of Long Short-Term Memory (LSTM) networks has attracted attention in time series predictive modeling, especially for forecasting VIX levels. Studies by Hirsa et al. (2021) [14] and Petrozziello et al. (2022) [16] highlighted LSTM's adeptness at decoding complex volatility patterns, marking a key progression in the field.

Bonds are often overlooked in risk-neutral probability research. Yet, Bauer and Chernov (2023) [4] demonstrated that the conditional skewness of Treasury yields can indicate macroeconomic risks and predict excess bond returns. This skewness is an important indicator because it can illustrate the balance of risks to Treasury yields and potential regime shifts in Federal Reserve policy.

3. Exploratory Data Analysis

3.1. OLS Regression

Our initial approach involved applying Ordinary Least Squares (OLS) to examine any linear relationships between the moments and future 6-month S&P 500 returns.

Specifically, we regress over individual moments of 6 months S&P 500 Minneapolis Fed Data against future 6 months S&P 500. The outcome

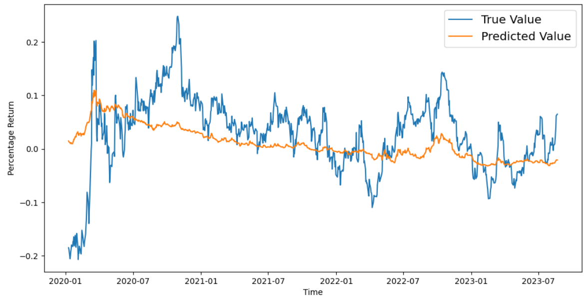

Figure 1: Prediction of Next 6-Month FTSE 100 Return (Out of Sample)

was underwhelming for the first two moments, with R^2 values of 0 and 0.027. However, higher moments had significant R^2 of 0.09 and 0.123. This motivated us to explore the predictive power of higher moments for forecasting future returns. In subsequent sections, we explored this idea further.

Parameter	Coefficient	Std. Error	t-stat	P>$\|t\|$	R^2
μ	0.0062	0.841	0.007	0.994	0.000
sd	0.6978	0.328	2.126	0.035	0.027
skew	0.1529	0.036	4.209	0.00	0.098
kurt	-0.045	0.009	-4.787	0.00	0.123

Table 1: Regression Summary of Returns vs. Individual Moments

3.2. Exploring LSTM with FTSE 100 Data

Given the limitations posed by the small dataset from the Minneapolis Fed and the inadequate performance of regression analysis, we sought to extend our analysis by both expanding our dataset and exploring more sophisticated modeling approaches.

We found that the Bank of England (BOE) also publishes estimates of risk-neutral probability density functions for future values of the FTSE 100 index. Based on this dataset, we developed a Long Short-Term Memory (LSTM) model aimed at predicting six-month returns. The choice of LSTM was motivated by our hypothesis that the financial market is influenced by both short-term fluctuations and long-term trends. LSTM's unique architecture enables it to capture these temporal dynamics more effectively than OLS, making it a promising alternative for our analysis.

In line with OLS, the LSTM model's performance was not significant (R^2 = 0.098) in predicting the next six months' return, highlighted in Figure 1. This outcome suggested that directly utilizing RNDs for future returns prediction may be less effective than anticipated.

3.3. Stock Level OLS

We further explored the relation between RND moments and future returns of individual stocks. To do so, we first calculated the RND moments of individual stocks using the Breeden-Litzenberger Approach (presented in Section 4.1). Sample results for 2 midcap stocks are shown in Table 2.

	PZZA			OLED		
	Coeff.	t-Stat	P-value	Coeff.	t-Stat	P-value
Constant	-1.62	-20.88	0.00	-1.39	-31.12	0.00
Skew	1.55	19.34	0.00	1.07	17.64	0.00
Kurtosis	0.34	0.74	0.46	0.34	1.42	0.16
R-Square		0.71			0.54	

Table 2: Regression Results

We found that the 30-day RND moments have statistically significant prediction power for many midcap stocks. The observed high correlation motivated us to further study skewness.

4. Empirical Study and Trading Strategies

Given the results of the exploratory data analysis in the previous section, we decided to explore higher moments' predictive power. In the subsequent sections, we explore this idea further.

4.1. Bi-Focal RNS Study

We started with studying the impact of risk-neutral skewness (RNS) on individual stock returns in search of a trading strategy. The idea was to draw insights from the term structure of RNS. For short-term options, we aligned with the Informed Trader Hypothesis: betting on the under-performance of overpriced stocks with negative RNS where short selling is restricted. Conversely, in longer-term maturity options (1 Year), we adopted the Skewness Preference View, expecting a market correction from overbidding on assets with potentially high payoffs (and hence higher RNS).

We first needed to generate the RND moments for individual stocks over time, as the Fed's data focuses only on large indexes.

Breeden and Litzenberger's Approach:
Under basic no-arbitrage restrictions, Breeden and Litzenberger (1978) showed that the first derivative of an option price with respect to the strike price is a function of its cumulative distribution

$$\frac{dC}{dK} = -e^{-rT}(1 - F_Q(S_T)) \tag{1}$$

,where T is time to maturity.

Then, the second derivative gives us the probability density function of the option's underlying discounted at the risk-free rate:

$$\frac{d^2C}{dK^2} = e^{-rT} f_Q(S_T) \tag{2}$$

Hence, for a call option, the risk-neutral probability distribution function for its underlying stock can be defined as f_Q:

$$f_Q(K,T) = e^{rT} \frac{d^2C(K,T)}{dK^2} \tag{3}$$

To evaluate this function, we use a second-order finite difference approximation:

$$f_Q(K,T) \approx e^{rT} \frac{C(K+\Delta K,T) - 2C(K,T) + C(K-\Delta K,T)}{(\Delta K)^2} \tag{4}$$

A significant limitation of this methodology is the discreteness of listed option strikes. To enable valuation across continuous strike levels, a cubic B-spline curve (Shimko 1993) was fitted to interpolate implied volatilities.

The sample results for Apple Inc. as of 25th July 2022 are as shown below:

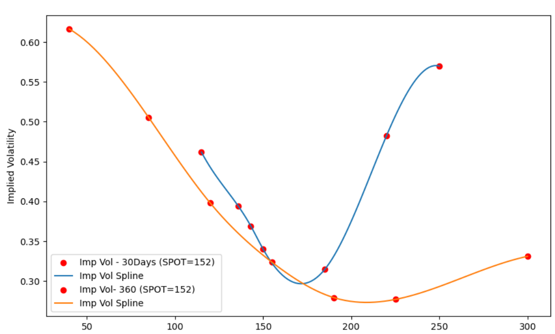

Figure 2: Cubic Spline Fitting of Volatility $AAPL

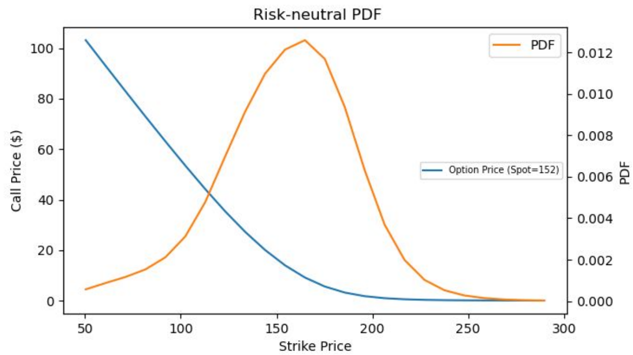

Figure 3: PDF and Call Price of $AAPL

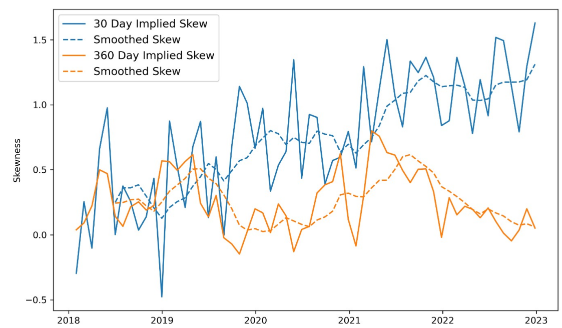

Figure 4: 30/360 Day Option Imp. Skewness for $AAPL

4.1.1 ***Data Description***:

The study utilizes daily stock options data from OptionMetrics, stock price data from CRSP, and risk-free rate information from the U.S. Department of the Treasury from 1 Jan 2018 to 31 Dec 2022.

4.1.2 ***Methodology***:

For each stock in our selected universe, we constructed the risk-neutral probability distribution and corresponding moments using the Breeden-Litzenberger approach. We performed this calculation for data at weekly intervals to capture trends over time of these moments. We then looked at the outperformance of a portfolio constructed by going long the top 5 stocks with the highest RNS (monthly rebalanced) versus the portfolio constructed by going long on the bottom 5 with the lowest RNS. We repeated this process across options with varying maturities of 90, 180, and 360 days. The performance of both these portfolios was benchmarked against an

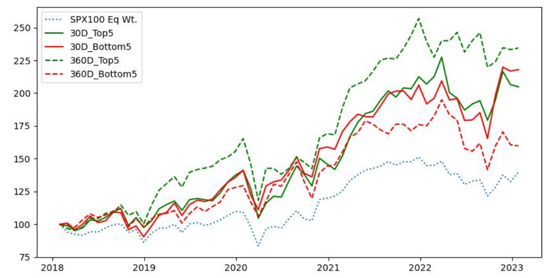

Figure 5: **30/360 Day RNS Top 5 vs. Bottom 5 Stocks**

equal-weighted S&P 100 portfolio.

4.1.3 ***Results****:*
Table 3 captures the mean alpha of the portfolio with the top 5 RNS stocks over the portfolio with the bottom 5 RNS stocks from the universe of Large Cap stocks in the S&P 100. We find a positive mean alpha in annual returns (6.44%) when using 90-day implied RNS (statistically significant NW t-statistic of 2.1), which aligns with the informed trader view. Using long-term RNS (360 Day), we observed a positive mean annual alpha (8.44%), also with a significant NW t-stat of 1.84. These findings imply Long-term RNS (360-Day) positively correlates with future stock returns in the S&P 100 Large Cap, contrary to the Skewness Preference view. Short-term RNS (30 & 90-Day) shows a positive impact, aligning with the Informed Trader Hypothesis.

	Mean Monthly Alpha	NW t-stat	Mean Annual Alpha	NW t-stat
30 Day	-0.10%	-0.09	0.26%	0.09
90 Day	0.27%	0.25	6.44%	2.1
180 Day	0.05%	0.05	0.27%	0.08
360 Day	0.64%	0.54	8.66%	1.84

Table 3: **Regression Statistics**

4.2. **Predictive VIX-Index Signal (PVIS)**

In this section, we further explore how an RND's higher moments and tail attributes can help identify trading signals for VIX forecasting.
Forecasting volatility holds immense value in guiding investment positioning and risk control in financial markets. The CBOE Volatility Index's (VIX) intricate linkage to the S&P 500 manifests in an inverse relationship where downturns in the S&P 500 foreshadow upticks in the VIX. [8][3]. This empirical study proposes and evaluates a trading strategy activated by $\widehat{VIX}_{t+30}$ forecasts from GARCH and LSTM models. The objective is to assess whether predictive analytics can effectively leverage the link between volatility expectations and market returns for systematic trading.

4.2.1 ***Data Description****:*
This study uses adjusted daily closing prices of the S&P 500 index and the CBOE Volatility Index (VIX) obtained from OptionMetrics spanning from January 2008 to November 2023. The RND moments for the S&P 500 were derived from the Federal Reserve Bank of Minneapolis website.

4.2.2 ***Methodology****:*
GARCH (1, 1) Model:
Predicting realized volatility is a cornerstone of our analytical framework because it captures past market behaviors, providing a statistical baseline for future market uncertainties. As a barometer of market sentiment, the VIX reflects aggregated investor expectations of volatility, making the prediction of realized volatility a logical precursor to understanding and anticipating VIX trajectories.

The GARCH model is widely utilized in financial econometrics. It captures the time-varying volatility characteristic of financial time series, allowing for both lagged squared returns and lagged variances to influence the current period's variance. Moreover, this model is particularly adept at modeling volatility clustering, a prevalent feature in financial markets.

To calibrate our GARCH model, we use insights from the Partial Autocorrelation Function (PACF). PACF allows us to identify the significant lags in realized volatility, thus shaping our volatility forecasts and pinpointing the nature of volatility persistence.

The PACF plot, as shown in Figure 6, indicates a significant partial autocorrelation at the first lag with a steep decline thereafter, suggesting a short memory process indicative of a potential GARCH(1, n) model application.

For our GARCH(1, 1) model, the mathematical

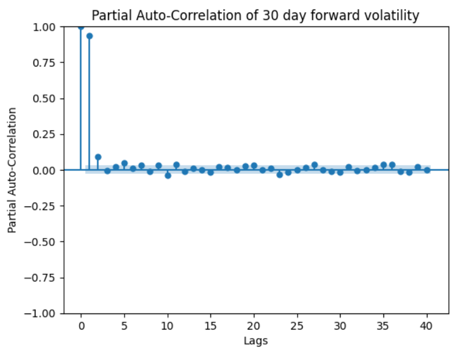

Figure 6: PACF of 30-day Forward Volatility

formulation is as follows:

$$\sigma_t^2 = \omega + \alpha_1 \cdot \sigma_{t-1}^2 + \beta_1 \cdot \epsilon_{t-1}^2 \tag{5}$$

Here, σ_t^2 is the conditional variance at time t, ϵ_{t-1} is the lagged error term, ω is the constant term, α_1 is the coefficient for the first lagged variance term, and β_1 and β_2 are the coefficients for the first and second lagged error terms, respectively.

All the coefficients generated by the GARCH (1, 1) model are statistically significant, as shown in the table below:

Parameter	Coefficient	Std. Error	t-stat	P>$\|t\|$
omega	0.206	0.036	5.788	0.00
alpha[1]	0.918	0.065	14.126	0.00
beta[1]	0.082	0.035	2.362	0.02

Metric	Value
Mean Squared Error (MSE)	4.437
Mean Absolute Error (MAE)	1.853

Table 4: GARCH (1, 1) Coefficients and Error Statistics

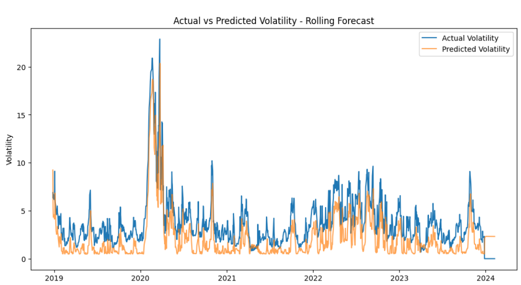

Figure 7: Actual vs. Predicted Volatility (Out of Sample)

Finding Predictive Variables for VIX:
Building on the realized volatility predictions, our next step incorporates supplementary predictive factors to enrich the VIX forecasting model using RND tail metrics, which are expected to contain information from informed investors. Employing linear regression analysis, these variables are tested to identify those exhibiting significant explanatory power for future VIX movements.

$$\begin{aligned}\widehat{VIX}_{\text{t+30}} = \beta_0 &+ \beta_1 \times \textit{Predicted Volatility} \\ &+ \beta_2 \times \textit{skew}+ \\ &+ \beta_3 \times p10 + \beta_4 \times p90 \\ &+ \beta_5 \times \textit{prInc} + \beta_6 \times \textit{prDec}\end{aligned} \tag{6}$$

,where:

- $\widehat{VIX}_{\text{t+30}}$ is the predicted VIX at t+30 days.
- $p10$ and $p90$ denote the 10^{th} and 90^{th} percentiles of the risk neutral distribution, respectively.
- *prInc* and *prDec* denote the probability of 20% increase and decrease, respectively, of S&P 500 over the next six months.

Parameter	Coefficient	Std. Error	t-stat	P>$\|t\|$
const	-2.2146	1.766	-1.254	0.210
Predicted Volatility	1.8513	0.048	38.779	0.000
skew	0.7456	0.500	1.492	0.136
p10	-27.8179	11.992	-2.320	0.020
p90	119.2893	11.973	9.963	0.000
prInc	-52.1553	11.248	-4.637	0.000
prDec	-8.5660	17.898	-0.479	0.632

Table 5: Regression Summary of Predictive Variables

The observation that *skew* and *prDec* lack statistical significance suggests the presence of multicollinearity among the predictors. Excluding *prDec* from the linear regression model yields statistical significance for all remaining variables at the 95% confidence level, with the model's R^2 value of 59%.

LSTM Model for VIX Prediction:
Having identified the essential predictive variables, we shift to a Long Short-Term Memory (LSTM) model to capture the complexities of the market. Regularization and dropout techniques enhance out-of-sample generalization by controlling overfitting. Moreover, its sequential configuration culminating in a dense output layer suits the continuous forecasting task. Table 6 lays out the detailed Long Short-Term Memory neural net-

work architecture underlying the VIX prediction model, including the optimized hyperparameters for each layer. Evaluating performance on the out-of-sample test set has a mean squared prediction error of 38.950. As shown in Figure 8, while the model broadly tracks volatility shifts, intermittent large divergences contribute disproportionately to the aggregate error.

Layer (Type)	Output Shape	Param #
LSTM 1	(None, 1, 30)	4440
Dropout 1	(None, 1, 30)	0
LSTM 2	(None, 30)	7320
Dropout 2	(None, 30)	0
Dense	(None, 1)	31

Table 6: **LSTM Neural Network Architecture**

Metric	Value
Out-of-Sample Test MSE	38.950
Out-of-Sample Test MAE	3.983

Table 7: **LSTM Model Performance Metrics**

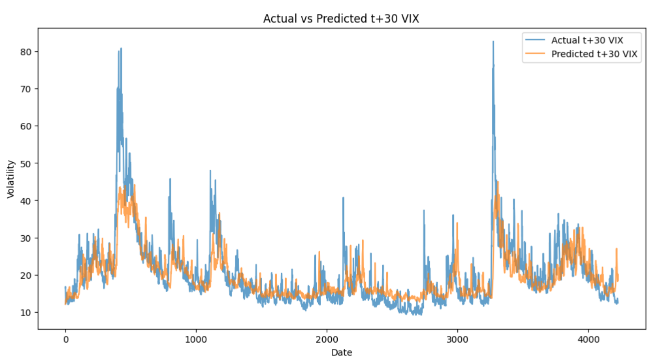

Figure 8: **Actual vs. Predicted t+30 VIX**

Signal Generation:
After deriving the forecasted VIX values, we can develop signals taking advantage of the well-documented inverse relation between VIX and market movements. Keeping a buffer of 20% increase in a single day due to some noise in the value of forecasted $V\hat{I}X_{t+30}$, the signals for trading were defined as:

Condition	**Position**	**Signal (L)**	**Signal (S)**
$\widehat{VIX}_{t+30} < 1.2 \times$ VIX MA	Long	1	0
$\widehat{VIX}_{t+30} > 1.2 \times$ VIX MA	Short	0	1
Otherwise	Neutral	0	0

Note: VIX MA = 30 Day VIX Moving Average
L: Long Only Strategy *S: Short Only Strategy*

Table 8: **Signal Generation Based on** $\widehat{VIX}_{\mathbf{t+30}}$

A dynamic trailing stop-loss approach was implemented to lock in accumulating returns and curtail losses as market conditions evolve. Specifically, for established long positions, the trailing stop triggers a liquidation threshold at 5% below the maximum attained price level since trade inception, thereby safeguarding profits. Conversely, for short positions, the algorithm institutes a symmetric repurchase threshold at an equal percentage above the minimum reached value, containing losses.

Trading signals from 2008-2023 were back-tested using daily S&P 500 return historical data. Signal returns assumed adjusted closing price execution. No transaction costs or slippage were incorporated.

	CAGR	**Annualized Volatility**	**Sharpe Ratio**	**Maximum Drawdown**
PVIS (L)	11.82%	21.83%	0.43	-20.15%
PVIS (S)	2.84%	17.66%	0.01	-26.86%
PVIS (L/S)	14.99%	23.63%	0.53	-26.97%
S&P 500	7.28%	20.57%	0.24	-53.25%

Table 9: **Comparison of Performance Metrics**

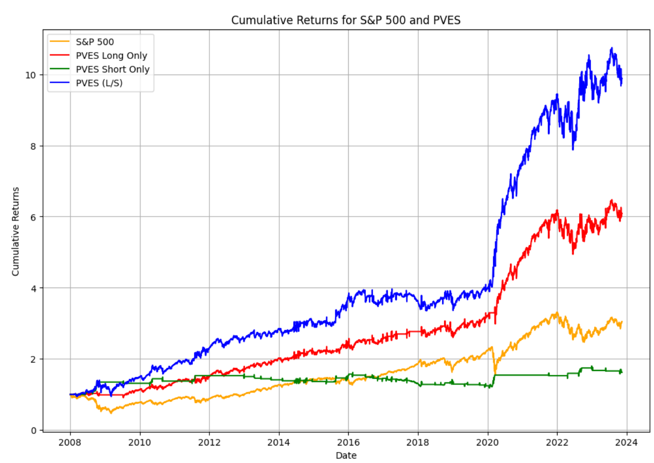

Figure 9: **Cumulative returns of PVES vs. S&P 500**

4.2.3 ***Results****:*
As per Table 9, both long-only and short-only strategies can be combined to create a more effective trading strategy with increased alpha. The Sharpe ratio of 0.53 for the combined strategy indicated healthy risk-adjusted performance exceeding the risk-free rate. Given the potential for auto-correlation and heteroskedasticity in the

time-series data, we employed the Newey-West t-test to assess the statistical significance of the backtested trading strategy's returns. The resulting t-statistic, t_{NW} (L/S: 10.526; L: 12.741; S: -6.897), significantly exceeded the critical value from the t-distribution for 95% confidence, confirming the statistical significance of the strategy's performance.

The trading strategy demonstrated consistent performance across diverse market regimes ranging from *bullish* to *bearish* states. This exemplified the model's proficiency in transcending market cycles, whether prolonged booms or periods of heightened volatility.

4.3. **Treasury Bond Strategy**

4.3.1 ***Data Description****:*

Our analysis of skewness in Treasury futures used data from the ten-year Treasury futures contract. We favored the ten-year Treasury future for its consistent use in macroeconomic outlooks and its popularity among market participants. To understand the skewness, we looked at the historical data from the Fed's dataset for 2013-2023. We gathered the risk-neutral probabilities from the Fed's dataset and took an average of the data. This data could then be modeled as our risk-neutral distribution and compared with empirical ten-year bond returns from the same period. We used a Kolmogorov-Smirnov (KS) test to statistically evaluate the similarity of these two distributions, which gave us a p-value of 0.05. Since this was close to a 95% confidence level rejection, we theorized that there could be some predictive power behind the data. Chernov and Bauer's analysis showed that skewness can inform investors about the future path of interest rates, which is what we wanted to explore by looking at the trend of skewness since we could not draw any conclusions based on just the average distribution.

4.3.2 ***Methodology****:*

Establishing the potential predictive value of the risk-neutral distribution, we focused on the skewness of the Treasury data. Our analysis used the skewness of Treasury futures prices instead of yields, as in Chernov and Bauer's analysis. Since skewness is scale-invariant, either choice is acceptable, but we chose to focus on futures prices to be consistent with the Fed's data. Over our period, we calculated an average skewness of -0.11. Yield curve models typically use symmetric distributions, which is why our results are so interesting. This analysis points to the impact of recent economic disruptions in Treasury volatility, focusing on the period from the COVID-19 pandemic through 2023, characterized by inflationary pressures affecting Treasury markets. Skewness metrics reveal two distinct phases: (1) January 2013 to December 2018, coinciding with the Federal Reserve's interest rate hiking cycle's conclusion, where skewness averaged -0.01, indicating relative market stability and modest interest rate adjustments; and (2) the subsequent period, averaging a skewness of -0.20, reflecting heightened market turbulence. This latter phase experienced a transition from significant positive skewness amidst the COVID-19 crisis to pronounced negative skewness by March 2022, as inflationary shocks influenced Treasury dynamics. The signaling of skewness in these trends makes it a powerful indicator of the future path of interest rates.

Figure 10: **10 Year Treasury Futures Skew**

We took the Fed's skewness data and computed monthly averages to reduce the volatility in the data series and make the data more comparable over time since the frequency of the data snapshot changed. We compared the ten-year Treasury futures skew data to the three-month return of the two-year, five-year, and ten-year bonds using the

WRDS Treasury Index data. We used linear regression with K-fold cross-validation with five folds. We used Principal Component Analysis (PCA) of the yield curve to isolate the effect of skewness, thus converting the skewness to a conditional skewness. We used the Bloomberg BVAL spot Treasury curve to get PCA factors and used them in our regressions. Our regressions established the statistical significance of conditional skewness for each maturity.

	2 Year	5 Year	10 Year
Skewness	0.011***	0.032***	0.054***
	(0.003)	(0.007)	(0.012)
Level	0.000	0.000	0.002
	(0.000)	(0.000)	(0.001)
Slope	0.000	0.004	0.009**
	(0.000)	(0.001)	(0.002)
Curvature	−0.004	−0.006	−0.012
	(0.002)	(0.005)	(0.009)
Constant	0.000	0.004	0.006
	(0.000)	(0.002)	(0.003)
Observations	119	119	119
R^2	0.16	0.18	0.22

Table 10: **Bond Index Return Regression**

Using the data from our regressions, we forecasted returns for each Treasury maturity based on the monthly skew data and PCA Treasury factors. We used quarterly data points matched with lagged quarterly returns to keep it consistent with the MPD data. We then built a strategy using these predicted returns to go either long or short the two-year, five-year, and ten-year Treasury maturities if the predicted return was positive or negative. We used equal weighting for the maturities, which constrains the portfolio to have a maximum absolute value weight of one while allowing different maturities to move independently. Each position was held throughout the three-month period and only rebalanced at the end of the three-months.

	Strategy	Benchmark
Annualized Excess Return	3.44%	0.01%
Std. Dev.	3.22%	7.35%
Sharpe Ratio	1.07	0
Max Drawdown	-4.75%	-25.61%

Table 11: **Summary of Strategy**

4.3.3 ***Results:***

Our strategy demonstrated strong excess returns and was substantially better than the equally weighted long-only benchmark we created using the same maturities. This portfolio provided surprisingly robust returns in a historically volatile period for bonds. The average net position over the period was 0.16, demonstrating the strategy did not have a consistent directional view and could capture both sides of the market. Furthering the robustness of the strategy is indicated by the fact that 66% of the quarterly returns were positive.

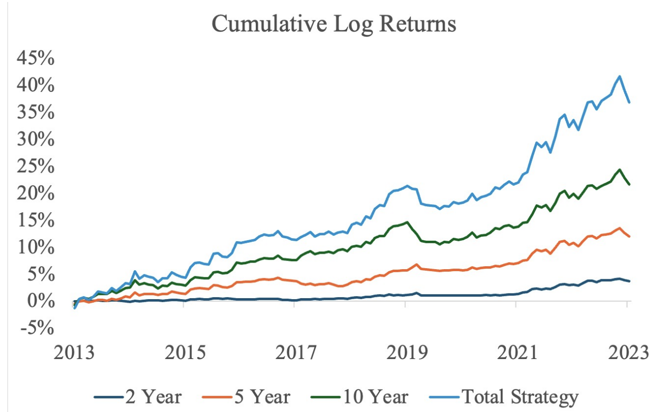

Figure 11: **Cummulative Returns**

Due to the limited dataset, there is a risk of overfitting in the data; however, the period includes multiple interest rate regimes in a compact time frame. This can be seen in the Figure 10, where the COVID-19 pandemic caused a significant shift in the interest rate regime. The use of K-fold cross-validation further tries to mitigate this issue. We could improve these shortcomings with further analysis, such as solving for more MPD data to expand the dataset and exploring different Treasury Futures maturities. We chose an equal-weighted portfolio weighting to make the model parsimonious, but many optimization methods could improve the performance. Essentially, this could serve as a lower bound.

5. Conclusions And Future Work

Much of the market probability literature focuses on equity returns and their inability to reliably forecast, which was confirmed through our explanatory analysis. We found that the prediction for large-cap stocks is ambiguous. At best, this could be attributed to the fact that these stocks are over-researched and practically did not have trading frictions. Further exploration of under-researched (less-liquid) stocks is required to gain more insight.

The tail metrics of the RNDs, along with the forecasted realized volatility, generated a statistically significant prediction of future VIX values. The well-documented negative correlation of the VIX movement with the S&P 500 was confirmed through the out/under-performance of long/short strategies built on this trend. The forecasted VIX offered additional avenues for systematic trading strategies beyond the equity market timing approach undertaken thus far. One promising extension involves exploring strategies in VIX futures that exploit the typical contango/backwardation shape of the VIX futures curve. While not pursued in this paper due to time constraints, this strategy warrants future research given the mispricings frequently observed within these instruments.

A potential differentiator for Treasuries is the exogenous factor of the Federal Reserve, which has the power to influence yields. Forward guidance is now one of the main tools for Fed policy and could be behind the effect in our analysis and our strategy performance. The Fed signals what it would like future interest rates to be, which is then reflected in the MPD. Since these processes take time, it is essential to look at these over a longer horizon and this is why we believe forecasting three-month returns is a reasonable interpretation of predictability using skewness.

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