Lock-In Theory
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PHYS-316 Advanced Lab*

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Introduction

The lock-in amplifier is a useful piece of instrumentation. Lock-ins are ubiquitous in research labs worldwide. The lock-in amplifier can detect AC signals buried in a lot of noise, so long as a reference signal at the same AC frequency as the signal of interest is provided to the instrument. Lock-ins provide phase-sensitive detection of the signal. Traditional lock-ins used analog mixer/modulators, while modern lock-ins use digital signal processing. This note summarizes the basic theory of lock-in operation, which applies to either analog or digital implementations.

A lock-in amplifier needs an AC reference signal $V_R(t)$ at the same frequency as the signal of interest, $V_S(t)$. Many experiments are designed from the outset to provide this strong reference signal at the same frequency as the weak noisy signal of interest, so that a lock-in can be used. For example, an experiment which seeks to measure the optical reflectivity of a sample as a function of wavelength, might deliberately "chop" (on off on off on off) the input light at some frequency, and scan wavelength slowly (compared to the chopping). A simple photodiode at the light source provides a clean strong reference signal at the chopper frequency. The actual experimental signal, the reflectivity, may be weak and obscured with a lot of noise. However, because it is AC, it can be fed into a lock-in amplifier along with the reference from the photodiode, and the interesting portion of the signal - reflectivity as a function of wavelength - is easily extracted.

Lock-ins provide phase sensitive detection, which allows many other interesting observations. For example, the resistivity and dielectric constant of a poor conducting sample might be of interest. If the material is driven with an AC current, the resistivity (real part of the impedance) produces an AC voltage that is in-phase, while the capacitance (imaginary part of the impedance) produces an AC voltage that is 90 degrees out-of-phase ("quadrature"). A lock-in separates these two so that they can be independently measured! This is more information than simply detecting the amplitude of the resultant AC signal, which would of course depend on only the magnitude of the impedance, which contains both the real and the imaginary parts.

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Building blocks of a lock-in

A lock-in amplifier accepts two inputs; the signal and the reference. The signal is the actual sinusoidal signal you are interested in with noise of other frequencies added into it. The reference signal is an AC signal at the same frequency as the signal you are trying to measure. The first stage of the lock-in is the Mixer. This multiplies the signal input with the reference.

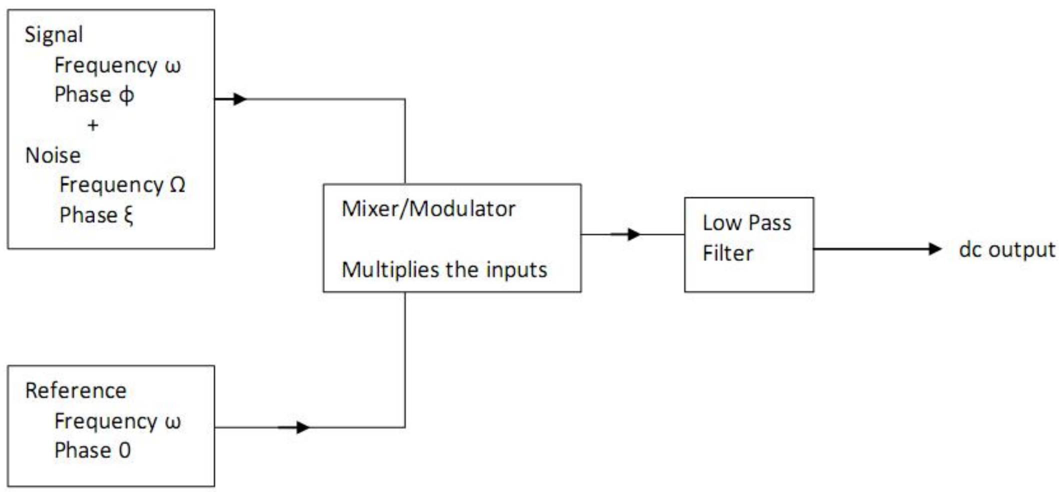

Figure 1: The basic building blocks of a lock-in.

The next stage of the lock-in is a low-pass filter. This filter allows DC and very low frequency to go through, and attenuates other freqencies severely.

The following sections show how the lock-in produces the DC signal which is proportional to your original AC signal of interest, while rejecting the noise, and how the DC signal also provides information as to the phase difference between your signal and the reference.

Trignometry review facts

In order to follow the theory, one needs to remember some high school trigonometry, notably:

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \tag{1}$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \tag{2}$$

and from these, the double-angle formulas

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 1 - 2\sin^2 A \end{aligned} \tag{3}$$

$$\sin 2A = 2 \sin A \cos A \tag{4}$$

Mixer output with a clean input

The mixer stage of a lock-in amplifier, be it digital or analog, takes the signal, and multiplies it by the reference. For now, let us consider a pure sinusoidal input signal (no noise) at the same frequency as the reference, though their phases may be different. Thus the input signals are

$$V_S(t) = V_{0S} \sin(\omega t + \phi) \qquad \text{Signal} \tag{5}$$
$$V_R(t) = V_{0R} \sin(\omega t) \qquad \text{Reference} \tag{6}$$

Using either analog amplifier circuitry or digital logic, these signals are multiplied together at the mixer stage. The resultant voltage at the output of the mixer is

$$V_{mixer,Signal}(t) = V_{0R} V_{0S} \sin(\omega t) \sin(\omega t + \phi) \tag{7}$$

First, expand the $\sin(\omega t + \phi)$ to get

$$V_{mixer,Signal} = V_{0R} V_{0S} [\sin \omega t \sin \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi] \tag{8}$$

Rewrite in terms of the double angle formulas,

$$V_{mixer,Signal} = \frac{V_{0R} V_{0S}}{2} [(1 - \cos 2\omega t) \cos \phi + \sin 2\omega t \sin \phi] \tag{9}$$

And now, expand to give

$$V_{mixer,Signal} = \frac{V_{0R} V_{0S}}{2} [\cos \phi - \cos 2\omega t \cos \phi + \sin 2\omega t \sin \phi] \tag{10}$$

which can be rewritten using the trig identities as:

$$V_{mixer,Signal} = \frac{V_{0R} V_{0S}}{2} [\cos \phi - \cos(2\omega t + \phi)] \tag{11}$$

The first term is a constant; it depends on the phase but not on time. The first term is proportional to the amplitude of the signal, V_{0S}. The second term is at double the original frequency. In a lock-in amplifier, a low pass filter is used to remove the AC signal at 2ω. What remains is the DC signal, the first term, which depends on phase as well as the original signal's amplitude. Hence the name "phase-sensitive detection".

Mixer output with a noisy input

The section above showed how a perfect sinusoidal input $V_S(t)$, along with a reference signal at the same frequency, is processed by the mixer. Realistically, one never has a perfect sinusoidal input signal. In fact, if you did, you wouldn't need the lock-in at all! A more realistic input signal includes both the desired signal and some noise. Thus you would have

$$V_{input}(t) = V_S(t) + V_{Noise}(t) \tag{12}$$

In the mixer, this entire signal is multiplied by the reference signal. We've already treated the first term, $V_S(t) = V_{0S} sin\omega t$. The noise is almost certainly comprised of many freqencies, however, we treat it one frequency at a time (Fourier decomposition). So, let us consider what happens to one frequency component of the noise:

$$V_{Noise} = V_{0N} \sin(\Omega t + \xi) \tag{13}$$

When the mixer multiplies this noise component by the reference signal, $V_R(t)$, it gives

$$V_{mixer,Noise} = V_{0R} V_{0N} \sin \omega t \sin(\Omega t + \xi) \tag{14}$$

Again, expand to yield

$$V_{mixer,Noise} = V_{0R} V_{0N} [\sin \omega t \sin \Omega t \cos \xi + \sin \omega t \cos \Omega t \sin \xi] \tag{15}$$

which can be simplified to

$$\begin{aligned} V_{mixer,Noise} = \frac{V_{0R} V_{0N}}{2} [\ &[\cos(\omega - \Omega)t - \cos(\omega + \Omega)t] \cos \xi \\ &+ [\sin(\omega - \Omega)t + \sin(\omega + \Omega)t] \sin \xi\ ] \end{aligned} \tag{16}$$

This messy form is the usual sum and difference frequencies, weighted by the phase of the noise. While there are a lot of different frequency terms here, you'll note there is no DC constant term!

The output from the mixer for a real signal is the sum of the mixer output for the signal and the noise, that is

$$V_{mixer} = V_{mixer,Signal} + V_{mixer,Noise} \tag{17}$$

where $V_{mixer,Signal}$ is given by Equation 11 and $V_{mixer,Noise}$ is given by Equation 16. This total output signal contains AC terms at twice the reference frequency as well as all the sum and difference frequencies between the reference and the noise. All of these AC components are removed, in a real lockin, with a low pass filter at the output. There is, however, one DC term, and that is the first part of Equation 11. This is the final output voltage of the lock-in amplifier:

$$V_{final} = \frac{V_{0R} V_{0S}}{2} \cos \phi \tag{18}$$

Thus we see that the lock-in, though mixing the signal and noise with the reference signal, and then filtering with a low pass filter, gets rid of signals at all frequencies except that of the reference. The DC output it produces depends on the amplitudes of the signal and reference as well as the phase angle between them, ϕ, as given in Equation 18.