Polarization Rotation AC
Rochester Institute of Technology

PHYS-316 Advanced Lab*

January 11, 2022

Goals

The relationship between the change in angle of polarization δ, the magnetic field B, the length of the sample L, and the material property known as the Verdet constant, ν, is the same regardless of whether a measurement is performed using a DC field or an AC one.

$$\delta = BL\nu \tag{1}$$

The DC measurement you completed previously would not be possible if a less-accurate polarizer mount were used, or if the sample had a smaller Verdet constant (flint glass has a LARGE rotation compared to most things). One solution to this problem is to use an AC measurement technique. The lock-in amplifier is a sophisticated piece of electronics designed specifically to isolate very small AC signals of known frequency from a lot of noise. Lock-in amplifiers are ubiquitous in condensed matter experimental physics. You will use a lock-in to measure the much smaller AC signal when we drive an oscillating magnetic field through our glass sample.

Theory

If the experiment is done with an AC field, the analysis of the rotation of the angle of polarization proceeds as follows. This theoretical analysis closely follows that presented by Jain et al., AJP 67 #8, 714, (August 1999).

Linearly polarized light incident on a sample is assumed to be propagating in the z-direction and initially polarized along the x-axis. The electric field of the beam can be written as

$$\mathbf{E_0} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} A\, e^{-i\omega t + ikz} \tag{2}$$

where A is the amplitude of the electric field. In this formalism, the upper number of the matrix indicates the amplitude of the field in the x-direction, and the lower number of the matrix is the amplitude of the field in the y-direction. After passing through a sample, the

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polarization is rotated by a small angle δ and the resultant field is given by

$$\mathbf{E_1} = \begin{pmatrix} \cos\delta \\ \sin\delta \end{pmatrix} A\, e^{-i\omega t + ikz}$$

When the light passes through the analyzer, which is set at an angle of ϕ with respect to the initial polarizer, its electric field is given by

$$\mathbf{E_2} = \begin{pmatrix} \cos(\phi-\delta)\, \cos\phi \\ \cos(\phi-\delta)\, \sin\phi \end{pmatrix} A\, e^{-i\omega t + ikz}$$

After some algebra and trig identities, the intensity measured at the detector, which is proportional to the magnitude of the electric field vector squared (say $I = aE^2$), is given by

$$I = aA^2\, \cos^2(\phi - \delta) \tag{3}$$

In order to have a maximum AC modulation of the light intensity, the analyzer needs to be set with an optimum angle ϕ, so that a small change in polarization due to the 'Faraday effect' within the sample, δ, produces a maximum AC component to the intensity I. That is, you want to work near the angle where the intensity is most strongly dependent on angle, that is, where $dI/d\delta$ is maximum. To find the optimum angle for the analyzer ϕ, the derivative $dI/d\delta$

$$\frac{dI}{d\delta} = aA^2\, \sin 2(\phi - \delta) \tag{4}$$

is a maximum at approximately ϕ=45° since the polarization rotation δ is very small.

The intensity given by Eq. 3 can be simplified for the special case of small δ and ϕ=45°,

$$I = \frac{1}{2}\, aA^2(1 + 2\delta) \tag{5}$$

Since the B field is being driven sinusoidally $B = B_0\, \sin(\omega\, t)$, and thus the rotation angle δ is also varying sinusoidally $\delta(t) = \delta_0\, \sin(\omega\, t)$, the total intensity is comprised of a DC part $I_0 = \frac{a}{2}A^2$ and an AC part $aA^2\, \delta(t)$

$$I = I_0 + \Delta I\, \sin\, \omega t \tag{6}$$

From this you should show that:

$$\delta_0 = \frac{1}{2}\frac{\Delta\, I}{I_0} \tag{7}$$

that is, the amplitude of the polarization rotation, δ_0, is half the ratio of the amplitude of the AC intensity to the DC intensity. Remember, with the polarizer at 45 degrees, there is a lot of DC signal, and only the small AC "wiggle" on top due to the polarization rotation. Note that the angle given by Equation 7 is in radians.

Throughout this discussion the AC intensity ΔI is referred to as the amplitude of the AC signal, but you should measure it as the RMS. If the AC intensity at the detector, ΔI, is measured as the RMS, the AC polarization rotation δ_0 is also RMS. In other words, you should be careful that the way that you describe your AC optical rotation ΔI (RMS) is the same way that you describe the AC B field (RMS).

The Verdet constant of the material is then determined using Equation 1, after you convert to degrees.

Before you proceed, you **must** work through these two training sections, in order to learn how to drive the solenoid using AC current, and then to learn how to use the lock-in. You must document these in your notebook as you work.

Training: Resonant Tank Circuit Setup and Characterization

In order to do measure the 'Faraday effect' using AC methods, the field must be driven in AC. The solenoid is a big coil, with substantial inductance. It is not simple to drive an inductive load in AC. In order to do so, one should work "at resonance", the frequency at which the maximum current can be pushed through the solenoid in AC.

Procedure

1. Construct the circuit shown in Figure 1. The 2-Ω resistor and 1-μF capacitor are installed in a small black box with banana jacks on the front for input; the back is removed so you can see what is inside. It is driven by the audio amp, which in turn is driven by a function generator.

   Note that with the components assembled as in this drawing, the "ground" side of the signal to the scope is the same as the ground (low) side of the audio amp output; that is, all the instrument grounds are tied to the same point. **You need to ensure the instrument grounds are arranged this way, or your scope will not read correctly. The ground side of the scope must be the same as the ground black lead from the audio amp. Additionally, be mindful of the ground side of the BNC-to-two-banana adaptor, which has a small bump-out over the banana plug that represents ground.**

2. Turn on the function generator. Use a BNC connector which allows you to choose a sinusoidal waveform of variable frequency and amplitude.

3. Using the scope and cursor controls, adjust the output of the function generator to be a sine wave of 400 mV peak-to-peak.

4. The audio amp allows variable amplification of this signal: gain is adjusted using the "input attenuator" knob on front of the audio amp. Set it to about halfway.

5. In order to measure the actual current going through your tank circuit, monitor the voltage across the 2-Ω resistor using the scope. It is a good idea to use either the scope cursor controls or the scope "Measure" function set for RMS voltage. Since $V = IR$ for this resistive element, you can calculate the current to the solenoid, and from that, the field.

6. Adjust the frequency of the function generator while watching the current through the circuit (via the voltage across the resistor). At a frequency near what you calculated

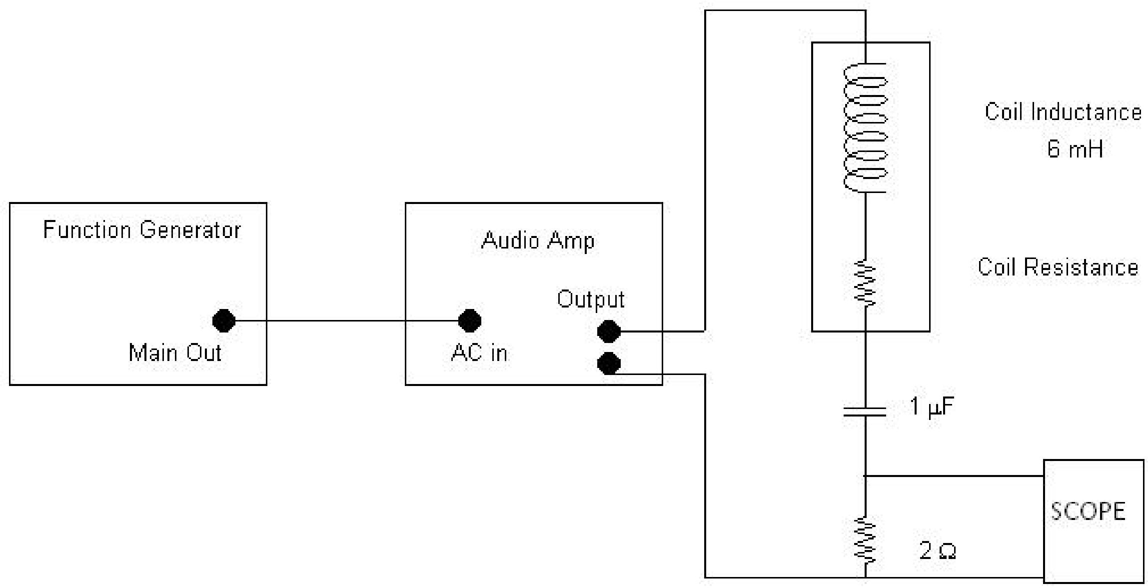

Figure 1: An RLC circuit. At resonance, the maximum amount of current is driven through the coil.

for the resonance you should see a strong increase in current; this maximum occurs at the resonant frequency.

7. Once you have found this, take quantitative data. Measure current through the circuit as a function of frequency, for about a factor of two in frequency above and below the resonance. Get enough data in the peak to clearly show the resonance. The most useful data is along the **sides** of the peak, not at the top! You've now found a frequency that allows you to 'push' a lot of current (thus a large alternating B field) through the inductive load.

8. Plot magnet current versus frequency.

9. Find or derive the theoretical functional form for this resonance curve from a trusted source on RLC circuits. Cite your source. Fit it to your data, and determine the resonant frequency.

10. The Q of an oscillator (short for "Quality") is related to the ratio of the energy stored by an oscillator to the energy lost in one cycle. The Q is defined for any resonator, mechanical, electronic, or otherwise. High-Q oscillators have little damping, and thus have a strong sharp resonance peak, narrow in frequency. Low Q resonators are lossy, and may have only a broad maxima in the general vicinity of the resonance. Look up the formula for the theoretical Q of the tank circuit. Compute it for your circuit, including both the coil resistance and the 2-Ω resistor.

11. Practically speaking, for most resonant systems, the parameters describing the device may not be well known, as we don't know the inductance of this solenoid very well. An ad-hoc definition of Q is given by

$$Q = \frac{f_0}{\Delta f} \tag{8}$$

where f_0 is the resonant frequency and Δf is the width of the resonance peak. The width of the peak is defined to be the full-width at half **power**. What do you need to do to compute power? Compute the experimental Q from the width of the resonance peak, and compare to theory.

12. Experimentally determined Q, from the measured resonance curve, seldom match theory very well, but are a very quick way to give you a guesstimate of the Q of a resonant system. Oftentimes one needs to know only if one has a "good resonance" (Q of 10 or 20 or better) or a "lossy one" (broad peak, low Q, or no peak at all). Or, if you are creating a system from scratch, you might need to know if the present version is better (higher Q) or worse (lower Q) than the last one. For these types of measurements, a Q determined roughly from the resonance peak is more than adequate, and can be done in seconds (adjust the oscillator frequency while watching the scope). Almost always, the Q you measure will be "worse" (lower), and the resonance not as strong, than the Q you compute from theory. Is yours?

13. Set up the Keithley microvolt meter in "4-wire ohms" mode, and measure the actual resistance of the nominal 2-Ω resistor, with uncertainty. Keep as many significant figures as you can. You will use this value throughout the lab to compute the actual current through the solenoid based on the voltage across this resistor. Ask your instructor if you are uncertain about your Keithley setup for 4-wire resistance measurement.

Training: Lock-In Amplifier

* You may refer to the the Lock-In Theory document in myCourses if you are interested in how the lock-in amplifier operates on the time-varying input signal and reference signal, to determine the small amplitude of the input signal.

A lock-in amplifier 'tunes' to a frequency you supply via a reference signal. The lock-in throws out all signals at other frequencies. Thus it can 'find' a very small AC signal amongst a lot of noise, so long as you have some source that provides a reference signal at the same frequency as the signal you want to investigate. As such, these instruments are real workhorses in physics instrumentation, for they allow you to pluck a tiny signal out of a mess!

In order to use the lock-in, you must supply a reference signal at the same frequency as the output signal you're interested in. Equipment designed for AC measurement of physical properties is often designed from the start with the goal of readily providing a reference signal! You must manually tune both the frequency and the phase, of the actual signal compared to the reference signal. This is truly 'phase sensitive detection'; the lock-in aligns your signal with the reference sine wave, both in frequency and in phase. Once this tuning is done, the lock-in provides an accurate measurement of a very, very small AC signal.

For many decades, Princeton Applied Research (PAR) built the premier lock-ins, instruments that have been used in physics labs worldwide for the measurement of small AC signals with high precision.

Familiarize yourself with the front panel of the Princeton Applied Research 122 Lock-In amplifier. Refer to Figure 2.

DO NOT ATTACH ANY SIGNAL OR REFERENCE INPUTS until you understand these

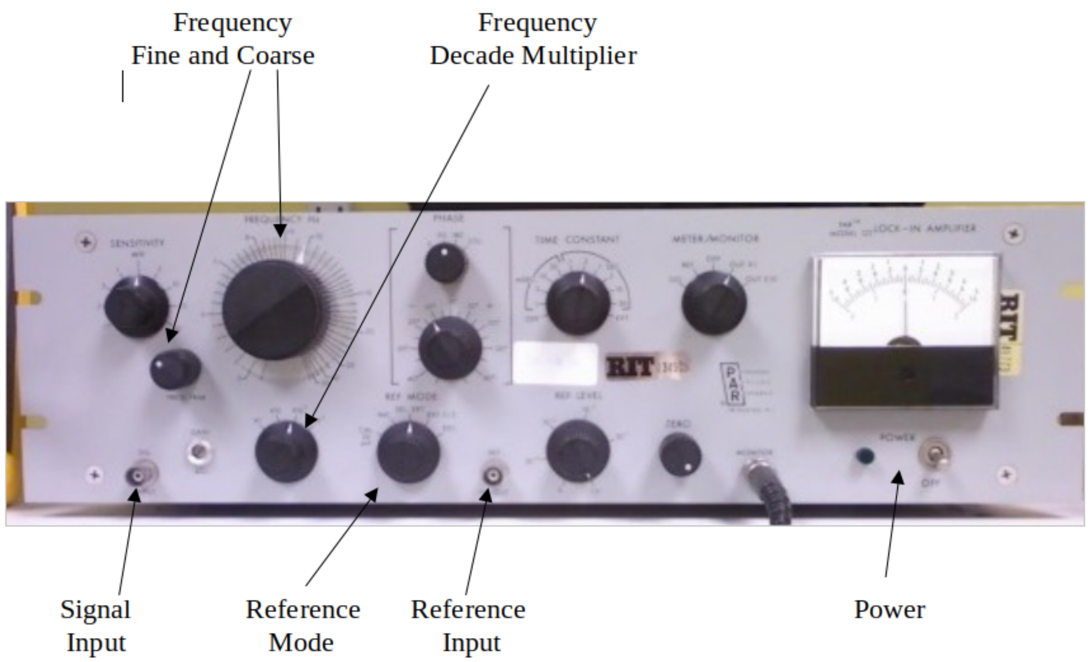

Figure 2: Front face of the PAR122.

settings.

> **Signal Input:** Attach the AC signal you want to monitor. The AC signal should not exceed 50 mV, zero-to-peak, at the frequency of interest. There may be more DC signal on top of the AC signal.
>
> **Reference Input:** Attach the AC reference signal, generated by your equipment, that is at the same frequency as the real signal. Reference signals must never exceed about 400 mV zero-to-peak. Exceeding this will destroy the lock-in.
>
> **Reference Mode:** Set to 'SEL EXT' to use an external reference provided by your equipment. This is the usual way a lock-in is used, rather than relying on the lock-in's internal oscillators as a reference. Again, the reference signal must be less than 400 mV in amplitude, and must be at the same frequency as the signal you are looking for.
>
> **Frequency Coarse and Fine:** Use to tune the lock-in, to look at SIGNAL at the same frequency as REFERENCE. The Frequency Decade Multiplier just multiplies the dial setting by different factors of ten. Always make sure you are in the right decade before you try to tune! Set 'FREQUENCY Hz' close to where you should be, and do fine tuning with 'FREQ. TRIM'.

Once you have appropriate signal and reference attached to the lock-in and it is powered up, you will need to tune frequency. Refer to Figure 3.

> **Meter/Monitor switch:** This controls what signal is displayed on the Analog Display and is fed to the Monitor ouput BNC jack. It should be set to OUT X1, the output times 1.

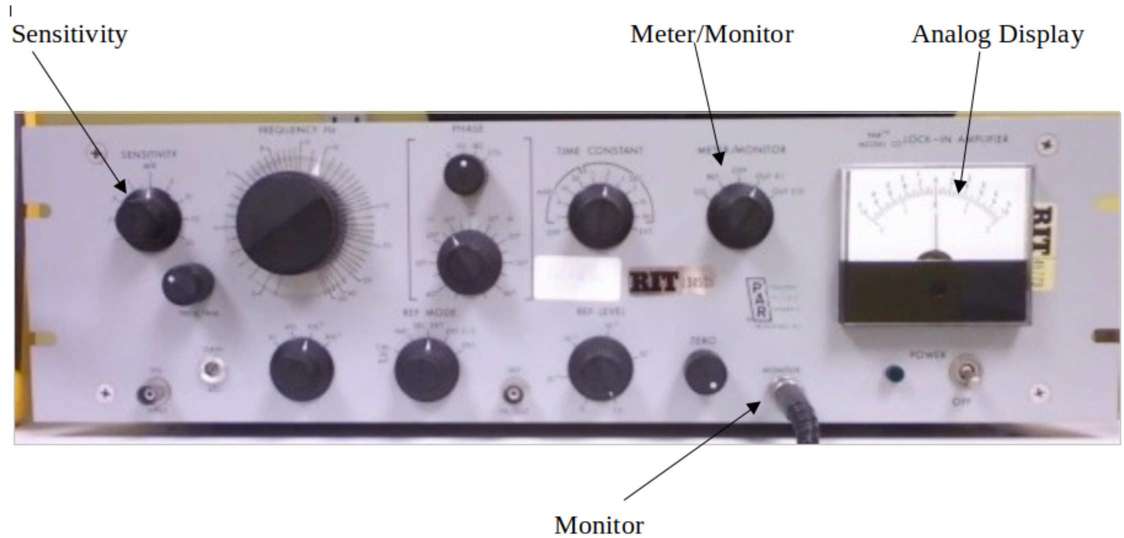

Figure 3: Front face of the PAR122.

Analog Display: Displays whatever is set by the Meter/Monitor switch.

Monitor: Gives a DC voltage to measure with a Fluke Multimeter, 0-10 Vdc, proportional to whatever signal is chosen by the Meter/Monitor switch. **This output is calibrated to report the RMS of your AC signal.**

Sensitivity: Sets the range for the input signal, and thus allows a calibrated output from the Analog Display or the Monitor jack. If the Sensitivity is set to '.5 mV', for example, a full scale reading (10) on the Analog Display means that the input AC signal at the Signal jack is 0.5 mV RMS. **To compute the RMS of your AC signal, divide the Monitor output by 10 V, then multiply by the Sensitivity knob setting.** For example, if the Monitor jack reads 6.48 Vdc, and the Sensitivity is set to '.5 mV', then the real AC signal is (6.48 V / 10 V) * (0.5 mV) = 0.324 mV.

To tune the frequency, you must monitor the output on the Analog Display and/or the Monitor jack, as you fiddle the Frequency Coarse, Fine, and Decade Multipliers. You are trying to tune for the maximum output signal, without maxing out the analog meter. This signal that you read at the Monitor jack or the Analog Display is the RMS of the input signal at the frequency of the reference; all other frequency components have been rejected, as discussed in the Lock-In Theory document.

In addition to tuning the frequency, one must also set the zero and the phase. Refer to Figure 4.

Zero: To adjust the zero, it is best to leave the Reference Input in, but turn the Signal Input down to zero. Then you know there is no actual signal, and anything the lock-in is displaying is a zero offset that must be removed. Trim the zero knob until the output is zero at the Monitor jack and the Analog Display. Then turn the signal back on for the phase adjustment.

Phase Quadrant: Adjusts the phase angle between signal and reference by 90 degree

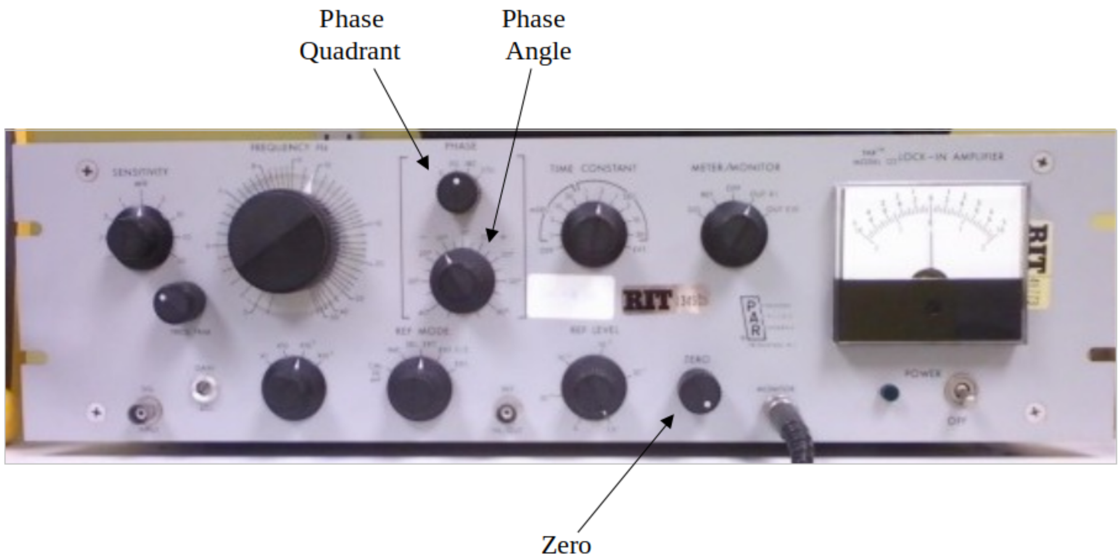

Figure 4: Front face of the PAR122.

steps.

Phase Angle: Adjusts the phase angle between signal and reference over a continuous range of ± 45 degrees.

Once you have the frequency tuning approximately right, the zero approximately right, and the signal on, you must adjust the phase. The best way to do this is “null in quadrature”:

1. It is easiest to locate a zero signal. Set the Plase quadrant knob to 90. Use the phase knob to adjust Monitor output to zero. You are then adjusted exactly 90 degrees out of phase. Flip the Quadrant knob to zero degrees and you should see the maximum magnitude Monitor signal (you’ve moved 90 degrees on the sine wave). This may be positive or negative voltage. Flip it the other way and you should see the opposite-sign maximum signal (90 degrees the other way on the sine wave).

2. If you flip through the Phase quadrant 0, 90, 180, 270 settings, you will see a positive signal, null, negative signal, and null, but not necessarily starting with the positive signal.

3. The Quadrant switch on this lock-in may be cranky and its knob may be loose, so the knob may turn a little before the switch actually switches. You can feel when the switch engages, though, so just bear with it. Phase adjustment is VERY tricky to get right.

Typically one iterates around, re-tuning frequency, zero, phase, and again, “tweaking” until you have a maximum signal. It can take an hour or more, as a beginner, to tune up a lock-in.

You have now tuned the lock-in to match the signal to the reference in frequency as well as phase. For a noisy input signal, of a fraction of a millivolt, this is difficult to do! Once tuned, the lock-in provides a stable large DC signalat the Monitor output which is proportional (in a calibrated way) to that small noisy input AC signal. Our calibration is chosen so that the

Monitor reports the RMS of the input AC signal.

There are two more things to learn on the lock-in front panel:

Time Constant: There is an obvious switch that sets a time constant. Find it. This places an RC filter on the DC output signal, to smooth out any wiggles if it is fluctuating. Typically we use 0.3 second or 1 second, but it is not a particularly critical setting for most applications. **When you change a parameter in your experiment (such as current and magnetic field) you must wait and take data only after a few time constants have passed, so the smoothed signal has caught up with any change you made.**

Overload: If the analog panel meter overloads, and the needle pins at one side, change ranges using the Sensitivity switch. You may then need to change the time constant then reset it back. Twiddling the time constant setting seems to allow the lock-in to get out of an overload situation.

Procedure

To learn to use the lock-in, you will use a simple circuit that makes a blinky light, and a photodiode detector. When the source and the detector are close together, the photodiode will produce a square-wave signal that is clear on a scope, and easily detected with the lock-in. When the source and detector are moved further apart, the signal degrades so that it is indistinguishable from noise when viewed directly on the scope, but it is still easily detected by the lock-in.

The inputs to the lock-in are the small AC signal of interest, and a second AC "reference" signal which is at the same frequency as the signal of interest. The following circuit has been built into a small black box.

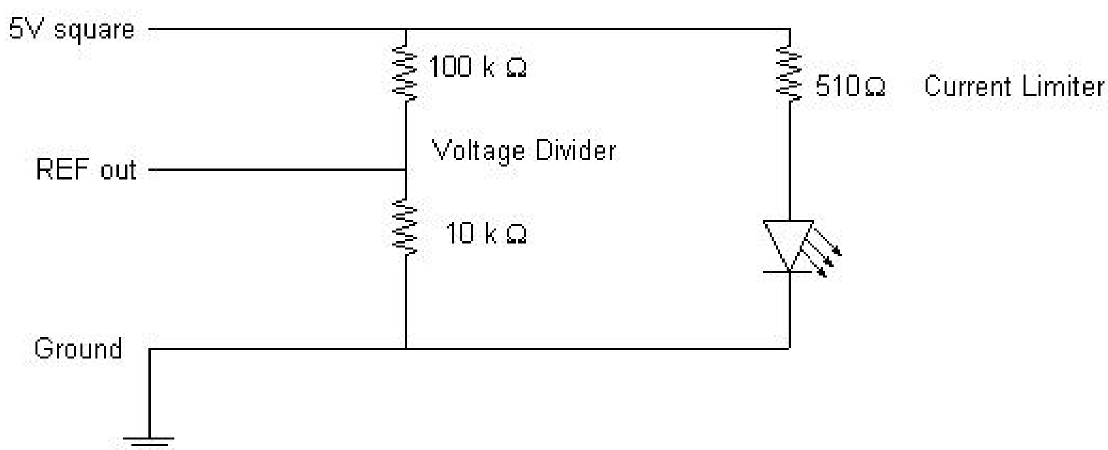

Figure 5: A simple circuit to produce a blinky light and reference signal.

1. Use a function generator with a TTL square-wave output signal. Note that the TTL output on any function generator is **always** a 5-V square wave. Set it up for some low frequency around 10 to 30 Hz, and verify this on the scope. It should be a square wave that alternates between 5 VDC and 0 VDC.

2. Once you have a nice signal from the function generator, move it to the blinky-light box with an LED mounted, at the "5 V square input" tap. The blinky light box is shown in Figure 6. The LED should blink. If you can't see it, slow the frequency down, above around 30 Hz your eye can no longer distinguish the blinking.

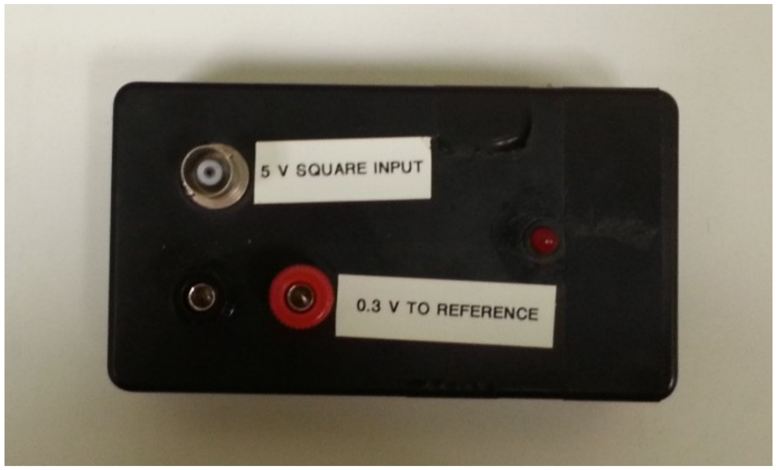

Figure 6: The circuit shown in the diagram 5 is built into a box, and makes a blinky light.

3. Attach the scope Channel 1 to the "0.3 V to reference" taps on the box. This signal will be used as a reference to the lock-in. Verify that this signal is also a square wave of amplitude <0.5 V. **YOU MUST NOT attach any signal to the lock-in reference channel that is larger than $\tilde{0}$.5 V, so it is important to check this.** This will be your reference signal.

4. Take the detector from the Faraday apparatus and set it on the most sensitve range, resistor 10 kΩ.

5. Mount the detector near the blinking LED. Create a mount with a table stand and clamp, as shown in Figure 7. View the detector signal on scope Channel 2. Your scope should now have two signals, the reference, and the signal from the detector, both square waves.

6. Move the detector away from the LED and until you have a noisy mess from the detector - room lighting is swamping the weak LED signal. Move it back in close for now.

7. Tee on the detector signal so you can go both to the lock-in and to the scope. Tee on the reference signal so it can go both to the lock-in and the scope.

8. **Have your instructor check these signals before you attach anything to the lock-in!** Attach the reference signal to the reference input of the lock-in, and the detector output to the signal input of the lock-in.

9. Tune the lock-in as described in the PAR 122 instructions above.

10. When you have a good lock-in signal, move the source/detector far apart, so you have a really flat signal as viewed on the scope. You can lift the detector a foot or more above the blinky LED. Change lock-in sensitivity (range) until this is visible. Check the frequency and phase tuning. Do you believe this is a "real" signal? How far away can

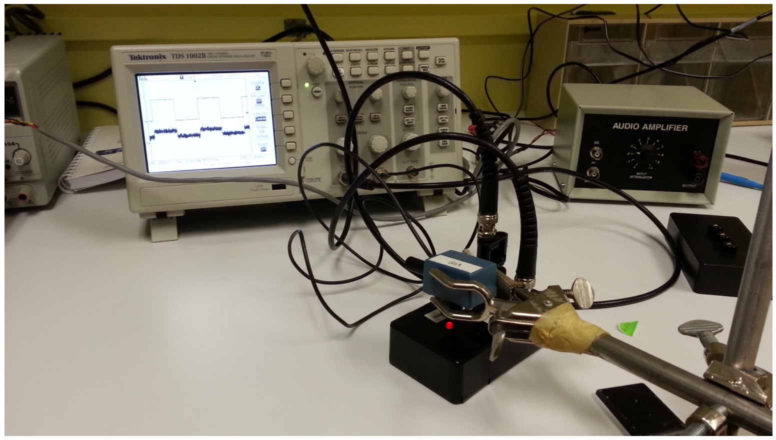

Figure 7: The detector mounted above the blinky light, adjustable in height. Note the scope trace shows the "clean" signal from the function generator and a "dirty" signal from the detector.

you get, and still "see" the signal with the lock-in? Block the optical path with your hand; does the lock-in output change? Can you see a change on the scope? Continue playing with signal amplitudes, lock-in tuning, etc, until you are confident that you really are seeing the weak signal from the LED, despite the noise, with the lock-in.

11. Change the function generator frequency to something very different, under 1 kHz (the detector has problems if you go faster than this). With the detector and LED close to each other, get good signals on the scope.

12. Re-tune the lock-in for this new frequency. Again, verify that you can move the LED away from the detector until you have a flat signal as viewed on the scope, but still measureable on the lock-in.

AC measurement of polarization rotation

When you have completed these two trainings, you are ready to actually measure the AC 'Faraday effect'.

Procedure

1. Reassemble the optics, and make sure the glass rod is centered in the solenoid. Set the detector switch to whatever you used for the DC measurement (probably 3 kΩ).

2. Align the laser and the detector as needed to obtain the exact same detector voltage that you measured with the rod centered in the coil during the DC experiment.

3. Build the tank circuit to drive the solenoid, and tune the frequency to resonance. Turn the audio amp gain down to zero. This gives you zero current through the magnet.

4. Verify the polarizer positions that give a maximum, and minimum field. **Make sure these numbers are consistent with what you measured in the DC experiment. Tweak your alignment until the numbers are identical, to your sat-**

isfaction. Set the polarizer to 45 degrees away from the max or min, that is, halfway between the maximum signal and the minimum signal. (This is the steepest part of the sinusoidal curve of output of the detector versus analyzer angle.) Tighten the locking screw to hold the polarizer in place.

5. Make sure there are no signals plugged into the lock in at this stage.

6. Record the DC signal intensity (with uncertainty) measured with the Keithley microvoltmeter, for the polarizer in this position. This is I_0 of Equation 7. You will divide everything you measure by this, so get it right.

7. Now compare this single measurement of I_0 with the relevant value from the functional fit to your Malus' Law data. It is related to the amplitude of your fit. If your measurement is not consistent with your fit, stop and resolve the conflict. Ask your instructor if they do not agree and you cannot determine why.

8. Set up the function generator output (usually the right most BNC jack) to sinusoidal 400-mV peak-peak measured by the oscilloscope. **Have your instructor check this before you proceed.** This is the raw signal from the function generator, before the audio amp. This will be a wonderful reference signal for the lock-in; it is guaranteed to be at the same frequency as the field and thus the optical variation in signal at the detector. **A reference signal that is too big will destroy the input stage of the lock-in. Do not exceed 400 mV peak-peak.**

9. Once your instructor has confirmed that you have a 400-mV peak-peak signal, you may attach this to the Reference input of the lock-in.

10. Attach the optical signal from the photodiode directly to the Signal input of the lock-in.

11. To measure the 'Faraday effect', the lock-in measures the optical signal. The amplitude of the B field must also be measured. Determine the solenoid current by measuring the **RMS** voltage across the in-line 2-Ω resistor using the scope. Set up the scope to measure the RMS voltage across this 2-Ω resistor.

12. Turn the audio amp gain to 0.9. You should immediately see the solenoid current on the scope.

13. Tune the lock-in. Note that you have done two different 'tunings' in this lab. You tuned the RLC circuit to resonance so as to drive a large current through the magnet. Now you are tuning the lock-in to that same frequency. You are not changing the circuit in any way when tuning the lock-in. You are simply adjusting the detection electronics to the frequency and phase at which you are operating.

14. Monitor the optical signal output by measuring the lock-in Monitor jack using a Fluke multimeter. This is a DC voltage proportional to the RMS of the AC signal! Make sure you understand how to read this and convert to signal level voltage. **Be sure to record the Sensitivity setting as well as the Monitor output.** The voltage you compute from the Monitor jack output and the Sensitivity switch setting (there is another factor of ten involved) is the RMS amplitude ΔI of the AC optical signal.

15. For audio amp gains from 0.9 down to 0.1, measure the RMS optical AC signal from

the lock-in Monitor, and the current through the magnet from the RMS voltage across the 2-Ω resistor using the scope. This will give you optical polarization rotation as a function of B. After spending some hours setting all this up, the actual physics measurement will take you around ten minutes. This is very typical of lock-in based instrumentation!

16. Before you leave the lab, 'work up' one data point (with audio amp near 0.9) to make sure you are getting approximately the correct Verdet constant. Convert lock-in voltage to actual AC voltage, then use Equation 7 to convert to radians of AC polarization rotation, then convert that to degrees. Use the RMS voltage you monitored on the scope to get the AC current in the solenoid, then convert that to B. With this 'one point' measurement of AC-rotation angle and AC-field, compute the Verdet constant. If it is not close to the expected value, you should get some help and repeat the measurement while everything is still set up.

Analysis

To analyze all your data, convert the RMS voltages you measured on the scope across the 2-Ω resistor to RMS current, and use the magnet calibration to convert to RMS B. If all your measurements are RMS, all is well. Otherwise, convert everything now to RMS.

Use Equation 7 to convert the the lock-in output (RMS volts) to angles. Note that I in this equation is **intensity**, which you measured in mV from the photodetector, via the lock in. Don't get I for current mixed up with I for intensity. You will need the DC intensity I_0 that you observed. Convert your RMS angle of rotation to degrees. Are these rotations large enough to see with the manual method, by rotating the polarizer?

Plot RMS rotation angle vs. RMS magnetic field. From the slope, determine the Verdet constant. Compare to the known value. Compare to the value you obtained using the DC technique.

Which technique do you trust more? Why? If you had a sample with a Verdet constant twenty times smaller than the Verdet constant of this piece of glass, which technique would you use? If you had a magnet twenty times stronger? What if you only had a paper-thin sample?