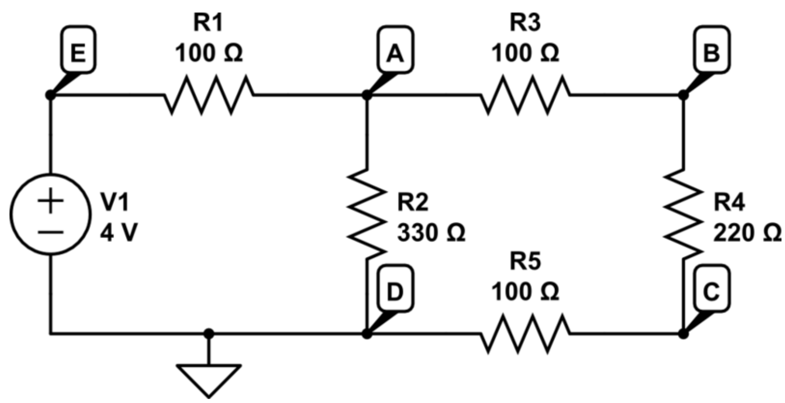
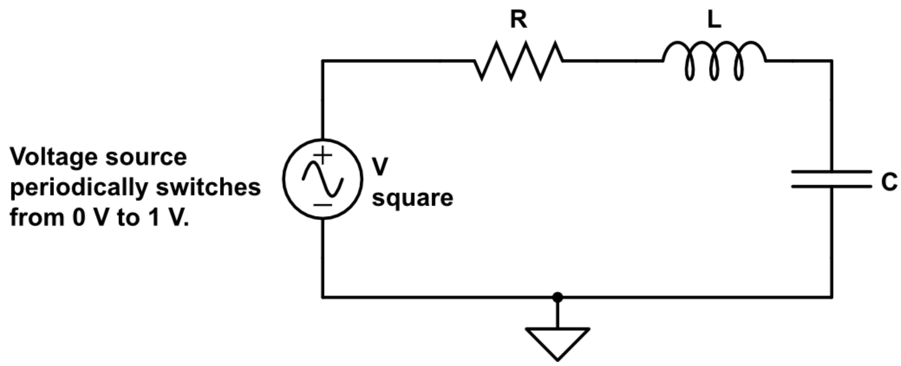
Question 1 Nodal Analysis (20 pts)

A. (16 pts) Use nodal analysis to find the voltage at each labeled node: V_A, V_B, V_C, V_D.
B. (4 pts) Find the currents going through R_1, R_2, R_3, R_4, and R_5.

You are required to set up a complete system of equations, but you can solve them however you'd like (an online solver, CircuitLab, etc).



Question 2 RLC Series circuit and its solution (20 points)



A. (6 point) Suppose at t = 0 the voltage source switches from 1V to 0V. Show that Kirchhoff's voltage law gives the following equation. Explain the meaning of each term.
   a. $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$

B. (10 points) Show that an exponentially decaying oscillation is a solution of the circuit equation from the previous task.
   a. $q(t) = Q_0 e^{-\frac{R}{2L}t}\cos(\omega t + \phi)$
   b. And show the solution only works if the oscillation frequency is given by
   $\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

C. (4 points) Note that Q_0 and ϕ could have any value and the decaying oscillation would still satisfy the equations. However, the solution must also match the initial conditions at t = 0 when the voltage jumps from 1 V to 0 V.
   a. Why does $q(t)$ have to be continuous at $t = 0$? Hint: consider each of the terms in the circuit equation. What would happen if $q(t)$ had a jump?
   b. Why does the current $I(t)$ have to be continuous at $t = 0$? Hint: consider each of the terms in the circuit equation. What would happen if $I(t)$ had a jump?
   c. What is the value of $q(t)$ at $t = 0$?
   d. What is the value of $I(t)$ at $t = 0$?

Question 3 Visualizing time domain response of RLC circuit (20 pts)

The question can be completed on the JupyterHub server or in any Python environment of your choice. If you want to use the JupyterHub server, you can make a Python notebook in your home directory. Your solution must include your code and final plot.

For the circuit in Question 2, assume L = 10 mH, C = 10 nF, and the resistor can be changed between R = 10 ohms, R = 100 ohms, and R = 1000 ohms. These component values are similar to what we will have in the lab. The voltage jumps from 1 V to 0 V at t = 0, as in Question 2.

Make plots of the solution $q(t) = Q_0 e^{-\frac{R}{2L}t} \cos(\omega t + \phi)$where

- Frequency $\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$
- Phase shift determined from initial conditions, $\phi = \arctan\left(-\frac{R}{2L\omega}\right)$
- Initial charge on the capacitor at t = 0, $Q_0 = \frac{CV_0}{\cos\phi}$

CODE: 10 points
- Code is Included in solution
- Easy to read and follow the calculation, some comments
- Accuracy

GRAPH: 10 points
- Graph is included
- Easy to read (includes labels and legend)
- Shows qualitative trend
- Accurate (matches the example)

To help you know if you are on the right track, here is an example of what your graph should look like.