Electron Motion in a Magnetic Field

Modern Physics Lab - Rochester Institute of Technology*

Introduction

Charged particles moving in a uniform magnetic field experience uniform circular motion. In our apparatus, electrons accelerate from rest by an accelerating voltage V and pass into a region of uniform magnetic field. The radius of the path they take is given by

$$r = \frac{\sqrt{2V}}{B\sqrt{e/m}} \tag{1}$$

A measurement of the radius of the electron's path at various values of the accelerating voltage V and the applied magnetic field B gives a direct measurement of the charge to mass ratio of the electron, e/m.

Pre-lab: Field Calibration and Uniformity Check

Before beginning the actual experiment, you need to calibrate the field as a function of both current and position for the Helmholtz coil. A second apparatus with the e/m tube removed is provided. Before using, read the guide to the gaussmeter and make sure you know how to power the coils.

Field Calibration

This calibration is extremely important for the rest of the lab, so make sure you take your time and gather enough data. Once you are ready use the FW Bell gaussmeter to calibrate the magnetic field strength:

1. Turn the probe on, and place the probe in the zero field chamber, and zero the meter.
2. Attach the bench power supply (DC Voltage Provider) with ammeter to the apparatus with the tube missing.
3. Hold (as steady as possible) the gaussmeter probe in the center of the coil and measure field strength as a function of current up to around 3 A (make sure to get both positive and negative currents). GET THE DATA!

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4. Equation 3 from the Theory section can be simplified as $B = AI$, where the constant A is a geometrical factor that contains the details of the coil geometry (number of turns and size). Determine the theoretical value of A for our apparatus. Use the equation.

5. Next determine your experimental value for A. Plot magnetic field as a function of current, and the slope of the line of best fit is your A value. How does the theoretical value compare to your measured one?

Uniformity Check

Now you will also verify that the magnetic field is uniform throughout the center of the helmholtz coils.

1. Set a current of 2.0 Amps, center the gaussmeter probe fore-and-aft (z=0), and measure the field strength as a function of position side to side (in the $x - y$ plane).

2. Helmholtz coils are known to provide spectacularly uniform B along their common axis. Verify this by centering the probe side-to-side, and measuring field strength as a function of fore-and-aft (z direction) position. [Note: You can also prove this uniformity theoretically; both dB/dz and d^2B/dz^2 vanish at the center for the Helmholtz configuration of coil spacing equal to coil radius.]

3. Plot measured B vs position for both the fore-and-aft (along z axis) and side-to-side (in $x - y$ plane) scans. For the size of electron beam paths you observe, how uniform is B, in all coordinate directions? By what percentage does it vary, over this region? Does the field uniformity along the z-direction of the Helmholtz geometry matter for this experiment?

Theory

You have previously derived the field of a loop of wire, where the loop is in the $x - y$ plane, and the field is measured along the symmetry axis out of that plane, the z axis. For a single loop of radius R_0, driven by current I, the magnitude of the field is given by

$$B(z) = \frac{\mu_0 I R_0^2}{2(R_0^2 + z^2)^{3/2}} \tag{2}$$

This result is multiplied by N, the number of turns in the coil. For the Helmholtz configuration, two such loops are placed a distance R_0 apart, so that the center point of the assembly is at z=0, with each coil centered at $z = \pm R_0/2$, and the fields generated by the two coils add in superposition. It is straightforward to show that the field at the center of this configuration is given by

$$B_{Helmholtz}(z = 0) = \frac{\mu_0 N I}{(5/4)^{3/2} R_0} \tag{3}$$

1. Derive Equation 1 for the electron orbital radius (hint: you will need to use two fundamental physics principles to do this).

2. Show that the orbital diameter D is given by

$$D = (C) * (e/m)^{-1/2} \frac{\sqrt{V}}{I} \tag{4}$$

   where C is a constant that depends on your specific equipment. Find the constant C for two cases:

   (a) Your experimental calibration of $B = AI$

   (b) Using the formula for a Helmholtz coil, Equation 3.

3. How do you convert Tesla to Gauss?

Apparatus

The Leybold-Heraeus apparatus consists of a gas-focused cathode ray tube in the form of a spherical evacuated glass bulb. Within the bulb, a cathode filament is heated to generate electrons, which are subjected to a variable accelerating potential V. The bulb contains a low pressure of hydrogen atoms which glow a faint blue when excited by the electron beam, making the path of the electrons visible.

The vacuum tube is mounted within a pair of coils that provide the applied magnetic field. This coil configuration is a common one when a uniform adjustable magnetic field is needed: it is called a Helmholtz coil pair. Helmholtz coils consist of two identical circular coils, driven with the same current, placed a distance apart that is equal to their radius. Our specific equipment has 130 coils and a radius of 150 mm.

The wiring for the apparatus is shown in Figure 2. The left-most jacks provide the high voltage used to accelerate the electrons. The second pair of jacks, labeled 6.3V, provide power to the filament, which causes thermionic emission of electrons. To the right, the current to the Helmholtz coil magnet is supplied by an ordinary bench power supply. It should be operated in Constant Current mode, and the current delivered monitored with an inline ammeter. Typically, currents ranging from 0-3 A are used.

A regulated high-voltage DC power supply unit is used to provide the accelerating voltage for the electron beam. See Figure 3. This unit also provides the 6.3 VAC for the heating of the filament. The filament voltage is supplied by the left taps, the voltage labeled "B" is the high voltage output for the accelerating potential, on the right, controlled by the rightmost knob. Note that the unit has two switches: an AC Power rocker switch to turn it on, and then a toggle switch (upper right) to turn the high voltage stage from standby to on.

In order to determine the diameter of the circular path of the electrons in the electron beam tube, a measuring device consisting of two parts is used. Behind the tube, a grey rail with a mirror is mounted. In front of the tube, another rail is mounted with two movable black

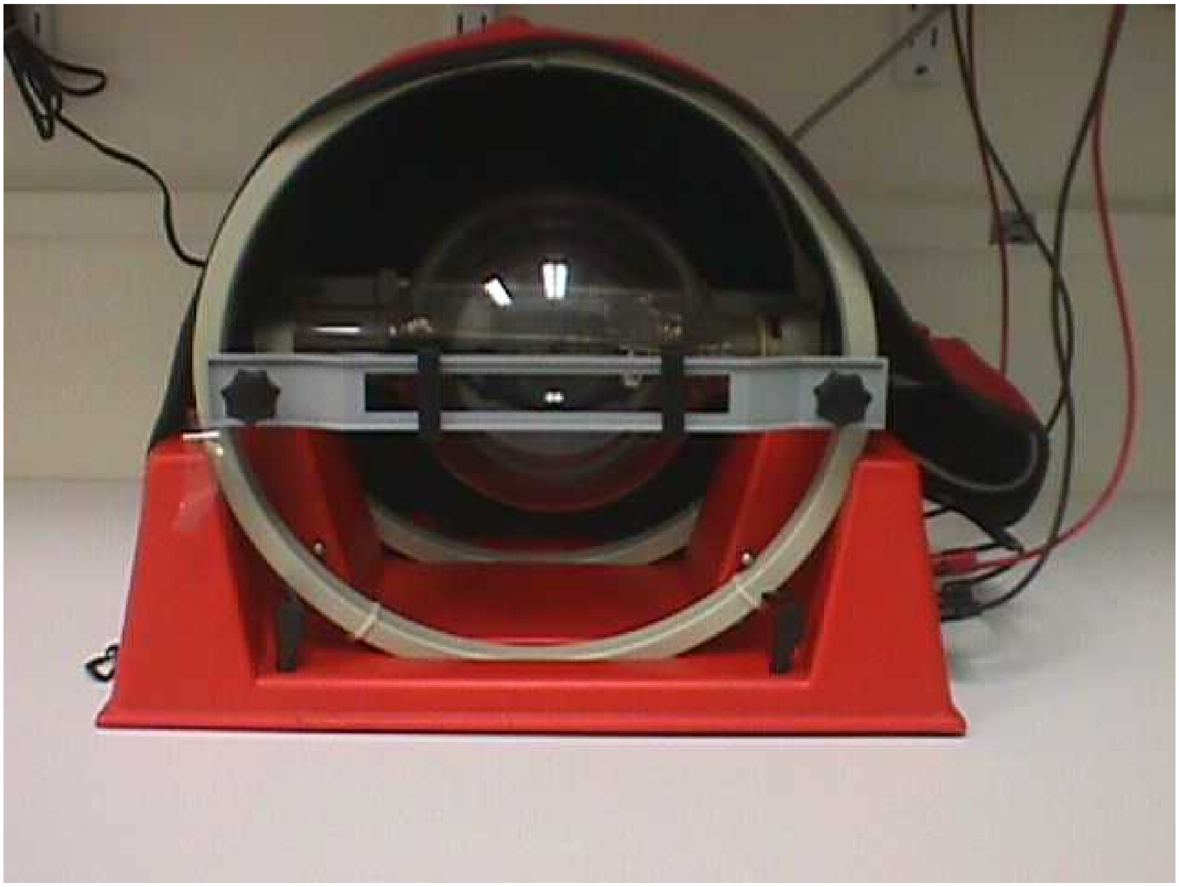

Figure 1: The Leybold-Heraeus e/m apparatus, showing the spherical evacuated tube and the Helmholtz coil pair used to supply the magnetic field. The two horizontal (grey) assemblies mounted on the Helmholtz coils are used to measure the radius of the electron path.

"riders" that can be positioned side-to-side. When a circular beam path is observed one must visually align three things: the beam path, its reflection in the back mirror, and the "rider" in front. This is done by moving the rider, for both the left and the right side of the circular path. Since the electron beam starts at the position of the right rider, you should only need to place it once for all of your measurements. Then the distance between the riders can be measured with an ordinary ruler, yielding the diameter of the electron path. See Figure 4.

Week 1 Check in: Determine μ_0 from Field Calibration

Plot B at the center versus I. Fit with a linear trend line, as predicted by your derived Equation 3. The equipment manufacturer gives nominal values of $N = 130$ turns and $R_0 = 0.15$ m. Extract μ_0 with uncertainty from the slope of your $B(I)$ data, and compare to the known value.

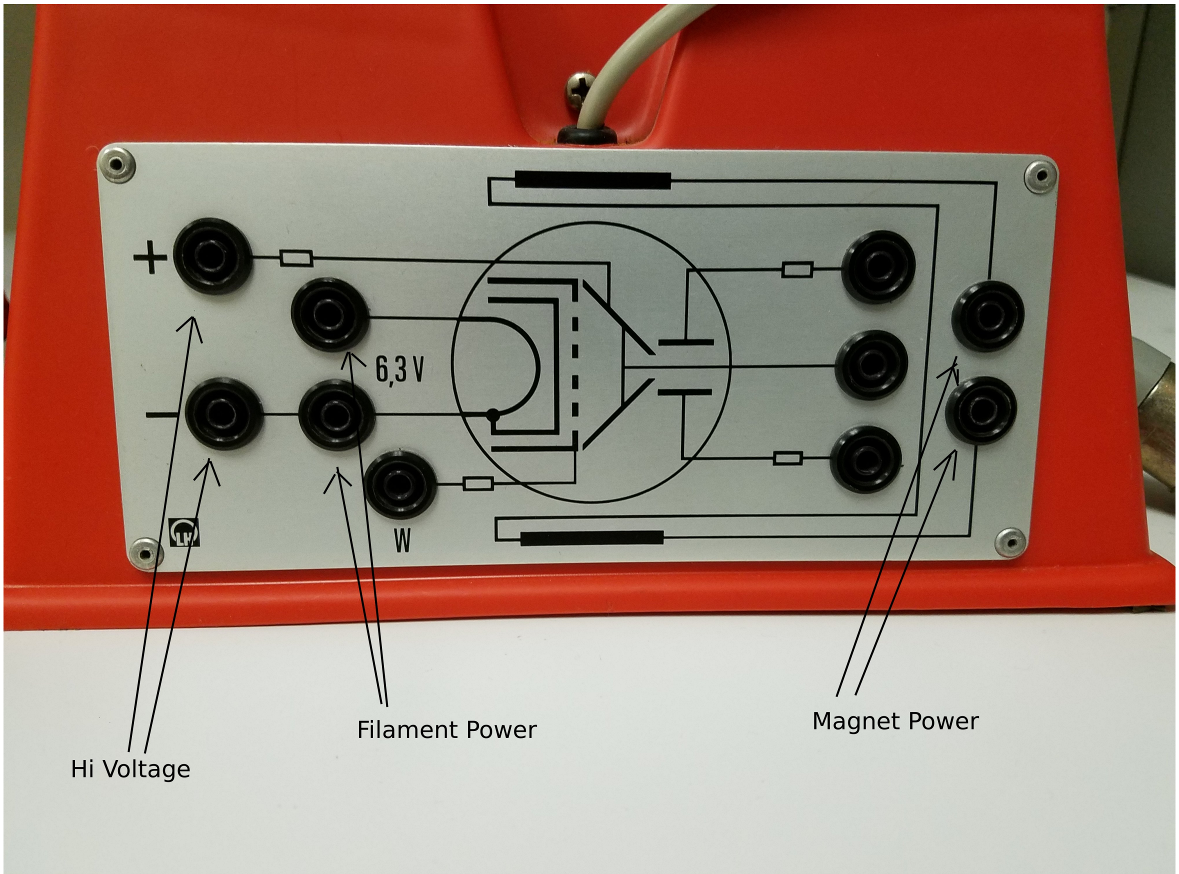

Figure 2: Wiring for the Leybold-Heraeus e/m apparatus

Week 2: Diameter as a function of V_{acc} and I

Put aside the apparatus with the tube removed and wire the full apparatus. Wire the apparatus to the power supply for the magnet (a typical bench supply) and for the high voltage and filament (one supply shown in Fig. 2). Have your instructor check this if you are unsure. Turn everything on, turn up the voltage until the tube "lights" (around 200 V). Investigate how changing the magnet current or the accelerating voltage changes the diameter of the circle. Take notes. You will need to work in the dark. Have a flashlight ready.

Next, figure out how the "sliders" work. Make sure you can see the circular electron orbit **and its image in the mirror**. Turn the equipment on with the magnet current set to zero and the accelerating voltage also set to zero. Allow the filament to warm up for a few minutes.

Slowly ramp up the accelerating voltage until you see a straight blue line emanating from the filament; this should occur around or above 200 V. **You will need to work in a dark room in order to see the electron orbit.** When you have an electron beam visible, turn up the magnet current until the beam goes in a circular path that fits inside the spherical bulb without hitting the bulb walls. Get a feel for how changing the accelerating voltage or the magnet current affects the diameter of the circle. Qualitatively, what does changing the

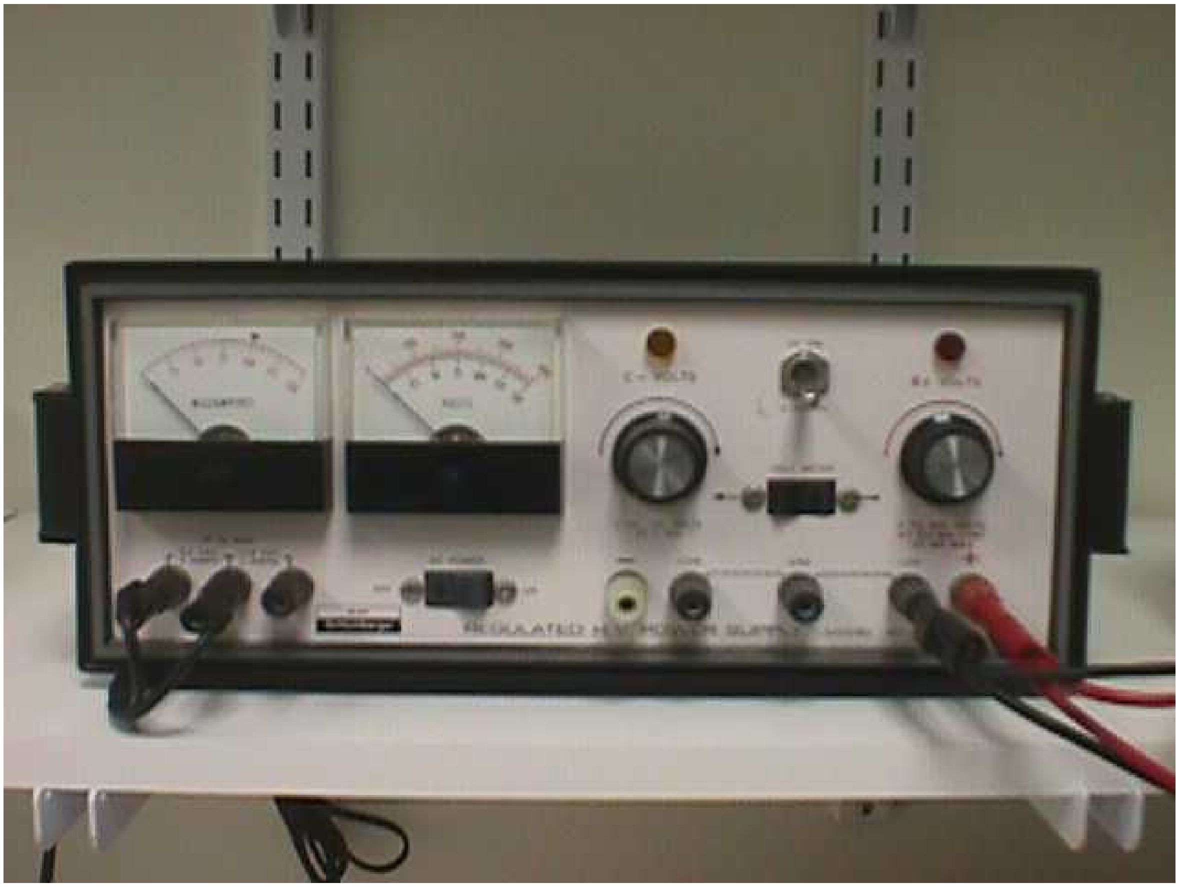

Figure 3: High-voltage power supply for the e/m apparatus

accelerating potential and the magnet current do?

At a fixed voltage, vary the magnet current (6-8 well-spaced values) and measure diameter of the path at each magnet current. Repeat this scan for five fixed voltages. Accelerating voltages typically range from about 150 V to 350 V, while magnet currents vary from about 0.5-3 A. At each accelerating voltage, try to span the entire range of beam paths from as-big-as-fits to quite small.

If you have time, investigate what other magnetic fields (not perpendicular to the initial velocity of the electrons) do to the path, qualitatively.

Analysis: Determine e/m of the Electron

Plot all of your data for D as a function of the variable $V^{1/2}/I$, as suggested by your derived Equation 4. The cool thing about this experiment is that all of your data can easily be combined, you don't have to separate out based on each voltage. From the slope, determine the ratio e/m, with uncertainty. Use your measured calibration constant that relates B to I instead of the theoretical one, since it incorporates any slight differences in your setup from the manufacturer's specs and the exact theoretical expression.

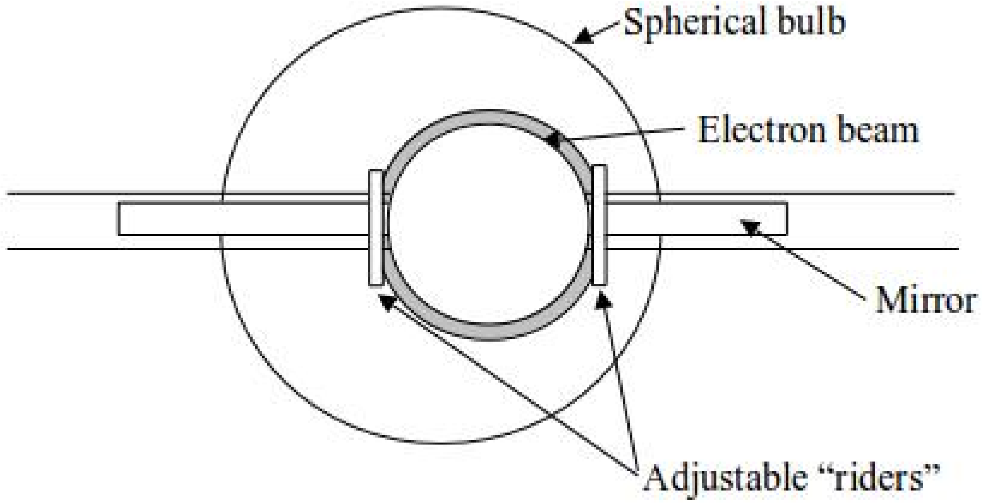

Figure 4: The riders used to measure the diameter of the electron beam circular path. The beam is aligned with its reflection in the mirror and with the rider, which minimizes parallax in the measurement.

Does your data pass through the origin? Should it? What would an intercept mean (a diameter of a circular orbit, at zero accelerating voltage)? Should you be fitting a line with an intercept, or a simple proportionality with the fit forced through the origin?

Compare the e/m you determined with the one computed from known value computed from the fundamental constants e and m.