Electron Diffraction

Modern Physics Lab - Rochester Institute of Technology*

Introduction

This experiment observes the wave-like properties of electrons. The wavelength of the electrons is measured, and is compared to the wavelength predicted by the DeBroglie relation.

Pre-lab

1. Read the lab write up.

2. The diffraction ring is made visible by the diffracted electrons impinging on a luminescent material coated on the inside surface of a glass tube. However, the relation between the diffraction angle θ and the diameter D of the ring assumes is observed on a flat surface. Thus, the measured diameter on the curved tube must be extrapolated in order to correct for both the radius of curvature r of the tube, and the thickness t of the glass tube.

   The tube parameters t, r and L are known, since they are supplied by the manufacturer. Show that the corrected diameter $D' = D + 2x$, which is the diameter you would observe if the front of the tube were flat, can be expressed in terms of these parameters as

   $$D' = D + 2x = \frac{DL}{L - t - r + \sqrt{r^2 - (D/2)^2}} \qquad (1)$$

   Hint: Calculate the distance a. Consider the tangent of the angle 2θ, and the two triangles formed with 2θ at one corner, one of which has opposite side $D/2$ and the other $(D/2) + x$.

3. Calculate the momentum of an electron accelerated from rest through a 4000 V potential difference both classically and relativistically. How do the values compare?

4. Calculate the deBroglie wavelength of an electron accelerated from rest through a 4000 V potential difference. Should the calculation be done relativistically?

*Prepared by P. Cardegna, L. Barton, L. McLane, rev 2021

5. Assuming the deBroglie relationship $\lambda = \frac{h}{p}$ is correct, show that the deBroglie wavelength of an electron accelerated from rest is inversely proportional to the square root of the accelerating voltage V;
$$\lambda \propto 1/\sqrt{V} \tag{2}$$

6. When electrons are diffracted through the graphite target (hexagonal symmetry), two diffraction rings are observed. Why are there two diffraction rings?

7. Why was the demonstration of electron diffraction important in the early development of quantum physics?

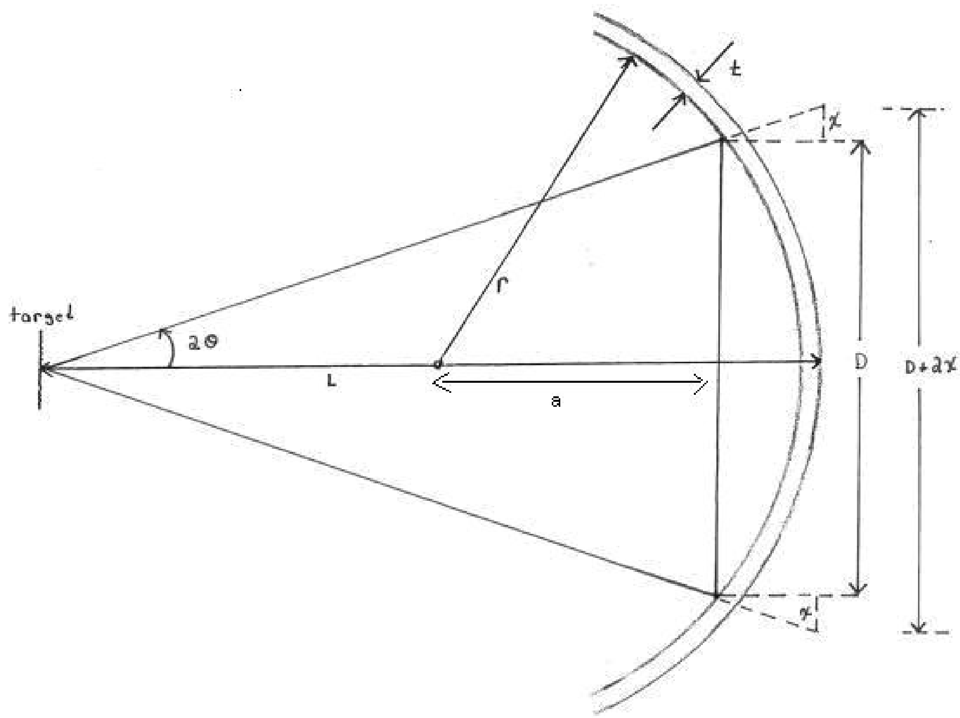

Figure 1: Geometry for diameter correction. If D is the measured diameter on the curved tube, then the corrected diameter is $D' = D + 2x$ on a projected flat surface.

X-ray Diffraction

The wavelength of an electromagnetic wave can determined by analyzing the diffraction pattern resulting from the wave passing through a grating of suitable spacing (spacing comparable to the wavelength of the wave). For x-rays, this spacing is of the order of the interatomic spacing.

Consider the two-dimensional model of a crystalline solid. Assume a square crystal structure in which the lattice spacing; that is, the spacing between adjacent atoms in the solid, is a. The model is shown in Figure 1.

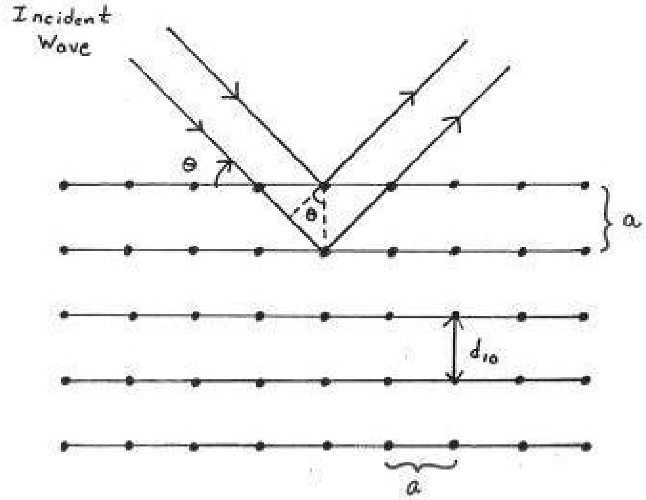

Figure 2: Geometry for Bragg's Law assuming a square lattice. The black dots represent the equilibrium positions of the atoms.

When an incident beam strikes the surface at an angle θ with respect to the surface, constructive interference will occur if the additional path length traveled by the wave reflecting from the adjacent surface is an integer multiple of the wavelength; specifically

$$2d_{hk}\sin\theta = m\lambda \qquad m = 1, 2, 3\ldots \tag{3}$$

where d_{hk} is the spacing between adjacent planes and θ is the angle between the incident beam and the reflecting plane. The angle between the incident and the reflected beam is therefore 2θ. Equation 3 is referred to as Bragg's law. For the planes shown in Figure 2, the spacing d_{10} is simply a.

However, there are many other possible planes from which the diffraction pattern can be constructed. For the square lattice, another possible set of planes is shown in Figure 3. For these planes, the spacing d_{11} is $a/\sqrt{2}$. The subscripts on the d_{hk} refer to the standard crystallographic identification of the planes used.

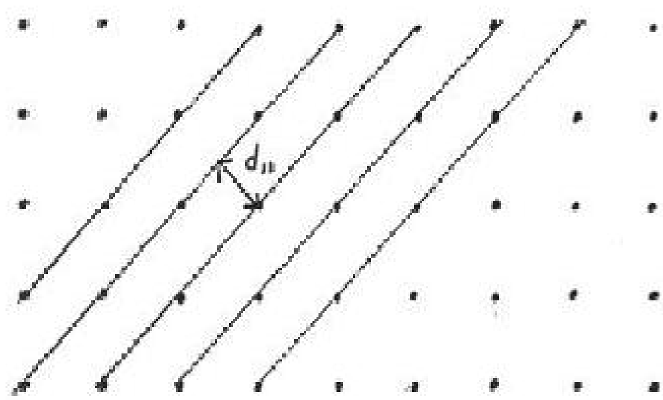

Figure 3: A second possible set of planes for Bragg reflection from a square lattice.

If a beam consisting of a broad range of wavelengths is incident on a single crystal, the interference pattern consists of a set of diffraction points. The points correspond to different wavelengths constructively interfering after reflecting from different planes.

If a monochromatic beam is incident on a polycrystalline solid, the resulting pattern consists of concentric diffraction rings. Each ring corresponds to diffraction from a set of planes of different spacing. The diameter of one ring D is related to the diffraction angle θ and distance L between the plane of the target and the viewing screen; specifically,

$$\tan 2\theta = \frac{D/2}{L} \tag{4}$$

The geometry is shown in Figure 3.

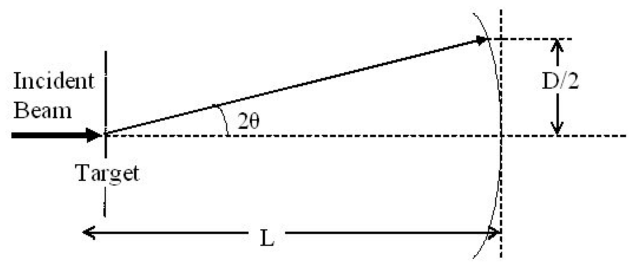

Figure 4: Relation between diffraction angle θ and ring diameter D.

Electron Diffraction

By the early 1920's, it was clearly established that electromagnetic radiation possessed both wave- and particle-like characteristics. In 1924, Louis deBroglie argued that symmetry encouraged, and the laws of physics did not forbid, that particles also possess both wave- and particle-like characteristics. The fundamental relation relating the magnitude of the particle's momentum p to its deBroglie wavelength λ is

$$\lambda = \frac{h}{p} \tag{5}$$

where h is Planck's constant. Davisson and Germer, scattering electrons from a nickel target, quantitatively verified this expression in 1925.

Apparatus: The TelAtomic 2555 Electron Diffraction Tube

The electron diffraction tube consists of an electron source, a grid to accelerate the electrons to a known energy, and a thin polycrystalline graphite target contained inside an evacuated glass tube. The source of the electrons is an indirectly heated cathode. An anode voltage V_a accelerates the electrons toward the target. The electrons diffract through the graphite target and impinge on a luminescent surface that has been deposited on the inside surface of the glass. A schematic of the hexagonal graphite lattice and the two possible diffraction planes is shown in Figure 5.

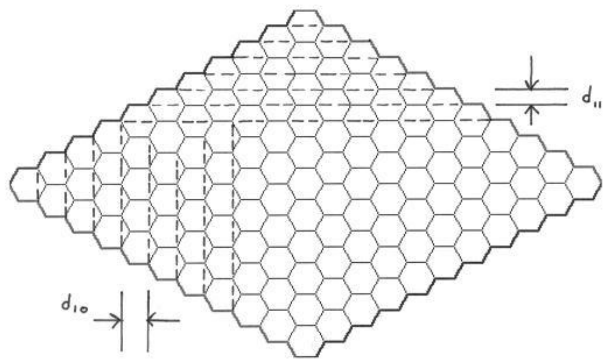

Figure 5: Two-dimensional hexagonal lattice with the two possible diffraction planes indicated.

Procedure

1. Wait a few minutes after turning on the heater supply to allow the cathode to reach thermal equilibrium. You will need low light (although not complete darkness) to do this experiment.

   There should be two “fuzzy” bright rings, so they are difficult to measure accurately. Your goal here is to measure the diameter for each of the two diffraction rings using vernier calipers.

2. Take at few measurements of the diameter of the smaller ring at different positions around the ring, taking these measurements at its innermost edge. The reason to take multiple measurements is so that you can average them in case the ring is not perfectly symmetric.

3. Repeat these measurements at the middle of the small ring.

4. Do the same measurements (innermost edge and middle) for the larger ring.

5. Repeat all these measurements for several different accelerating anode voltages between 2.0 and 5.0 kV. Thus at each anode voltage you should have readings for the diameter at the innermost edge of the smaller ring, the middle of the smaller ring, the innermost edge of the larger ring, and the middle of the larger ring. Tabulate your raw data results.

Analysis

1. For each anode voltage, use Equation 5 to calculate the deBroglie wavelength and its uncertainty.

2. Your measured diameter of the diffraction rings D must be corrected for the curvature and thickness of the glass, as in Equation 1.

   According to the manufacturer, the radius of curvature r of the inside surface of the glass is 6.6 cm, the thickness t of the glass is 1.5 mm, and the distance L between the target and the outside surface of the glass envelope is (0.130 ± 0.002) m. See Figure 1. The known interplanar spacings for hexagonal graphite are $d_{11} = 0.123$ nm and $d_{10} = 0.213$ nm.

   Calculate your corrected diameter D' for each measurement.

3. Note that the angle θ is small, so that $sin\theta = tan\theta \cong \theta$. Make this approximation for both equation 3 and equation 4, and solve each for θ. Combine the equations to get the Bragg wavelength as a function of your corrected diameter D'.

   The two rings are visible due to the two different lattice spacings d_{hk}. You must decide which ring corresponds to which d_{hk}; that is, decide whether a ring with smaller diameter (or angle θ) has the smaller, or the larger, lattice spacing d_{hk}. Note that all your observations are of first order Bragg diffraction, that is, $m = 1$ in Equation 3.

4. For each anode voltage, use the equation you found above, and the corrected diameter D' instead of the measured diameter D to calculate the Bragg wavelength and its uncertainty.

   Note that the propagation of uncertainty in D' is extremely difficult. Thus for the Bragg error calculations you should use the numerical error propagation demonstrated in lecture.

5. Next, compare the values of the wavelengths determined from the deBroglie relationship and those determined from the Bragg relationship. Your goal is to compare your measurements using the innermost edges of the rings as well as those from the middle of the bright band, and to determine which measurement is the correct one.

   The easiest way to see this is to plot the Bragg vs. deBroglie wavelength for each type of measurement. Make two plots for each ring, one where the Bragg vs deBroglie is plotted for the inner edge measement and the other where it is plotted for the middle measurement. Since the wavelength should be the same regardless how you calculated it, the slope of a line fit to this data should be exactly 1 (make sure to think about if you should include an intercept in your plot).

6. Would you advise the next student to measure the innermost edge of the ring or the middle?