Millikan Oil Drop

Modern Physics Lab - Rochester Institute of Technology*

Introduction

This historic experiment measures the charge on an electron. It is relatively simple to measure the charge to mass ratio, e/m, or the ratio of the charge to Planck's constant e/h, however, obtaining the charge e by itself is more challenging. Millikan first published his results in 1913; he received the Nobel prize for his work on measuring e and on the photoelectric effect, in 1923. His work provided a value for e that was accurate to better than 0.5%, and his results were limited primarily by a poor choice for the viscosity of air. More importantly, though, his results were convincing proof of the discrete, quantized nature of charge. When combined with the Bohr model, they gave an accurate prediction for the Rydberg constant in hydrogen, which was the first conclusive proof of the quantum nature of atoms.

While his experiment is simple in principle, it can be tedious and finicky to operate well. In order to get "good" data, many many drops must be observed. We will combine data from many groups to obtain statistically significant results.

Pre-Lab

1. Create a spreadsheet or other analysis script that will take as input your two measured distances and times, and calculate all intermediate quantities leading to a final determination of both q and q_c. You will eventually include columns for uncertainties, calculated by error propagation, on all measured and calculated quantities.

2. A student works with our Millikan apparatus using a plate voltage of 500 V. She observes an oil droplet which travels a vertical distance of 1.0 mm. The time it takes the drop to go 1.0 mm with the voltage off is 15.92 seconds, while the time it takes to go 1.0 mm with the voltage on is 6.82 seconds. Use the equations and constants given below (which are derived in the main body of the manual).

*Modified by L. McLane, 2021. Originally prepared by L. Barton, A. McGowan, M. Zemcov

$$
\begin{aligned}
M &= 2.04 \pm 0.05 \\
\rho_{oil} &= 872 \;\; kg/m^3 \\
\eta &= 1.85 \times 10^{-5} \;\; Ns/m^2 \\
\rho_{air} &= 1.2 \;\; kg/m^3 \\
d &= 5.91 \pm 0.05 \;\; mm \\
A &= 0.07776 \;\; \mu m
\end{aligned}
$$

$$
\begin{aligned}
v &= \frac{\Delta y}{M \Delta t} \\
r &= \sqrt{\frac{9\eta v_g}{2(\rho_{oil} - \rho_{air})g}} \\
q &= \frac{6\pi\eta d(v_g + v_u)r}{V} \\
q_c &= \frac{q}{\sqrt{(1 + \frac{A}{r})^3}}
\end{aligned}
$$

Calculate the corrected charge q_c on the droplet, using your spreadsheet or script. What is the charge expressed as a multiple of the electron charge e. In other words what is N in $q_c = Ne$?

3. Check the value with your instructor. You must have this correct before you begin to analyze real data.

4. Calculate the uncertainty on the charge. Because the final equation is so unweidly this is a prime candidate for the numerical variational method for error propagation. You may even want to calculate error on intermediate equations (e.g. δr, δv_g, δq, etc) before calcualting the final uncertainty, δq_c, to help with troubleshooting.

5. Take a look through the apparatus. Spritz in some oil drops. Try to "drive" them up and down by flipping the voltage off and on. See if you can identify both positive and negatively charged drops. Get familiar with the apparatus so you can work smoothly and quickly next week.

Theory

Very small oil drops are produced by spraying oil out of an "atomizer" (similar to those used in nasal sprays, it merely makes a very fine mist). These drops are typically one to two microns in diameter. In free fall, they would drift slowly down under the influence of gravity. Their motion is opposed by the bouyant force due to the air, and the viscous drag of the air. Since the drops are so small, they reach terminal velocity very quickly.

Some oil drops are charged. They become charged as they are sprayed out of the nozzle of the atomizer. The total charge they are carrying is some multiple of the fundamental charge e.

The oil drops are sprayed into the area between two horizontal plates of a parallel plate capacitor. The capacitor can be attached to a potential; when the potential is turned on there is an electric field pointing downward. A negatively charged oil drop will thus feel an electrical force directed upward, opposing the gravitational force.

Stokes derived an expression for the frictional force exerted on a spherical droplet with small Reynolds number in a viscous fluid,

$$F = 6\pi r\eta v \tag{1}$$

where v is the droplet's velocity and η is the viscosity of the air the droplet is falling through. Thus in free fall (no electric force present) an oil drop is under the influence of three forces; gravitational downward, bouyant and viscous drag upward. As with any system where there is a drag force (force = dv/dt) proportional to the velocity v, the exponential function which solves the equation of motion shows that the mass approaches a terminal velocity. For tiny drops like we will use, the terminal velocity is reached extremely quickly.

At the terminal velocity, the acceleration is zero, and Newton's second law gives

$$0 = 6\pi r\eta v_g + \frac{4}{3}\pi r^3 \rho_{air} g - \frac{4}{3}\pi r^3 \rho_{oil} g \tag{2}$$

where v_g is the speed of the drop moving downward, r is its radius where the drop is assumed to be spherical, ρ_{oil} is the density of the oil drop itself, and ρ_{air} is the density of the surrounding air which is providing some bouyancy.

Equation 2 can be solved for the radius of the drop;

$$r = \sqrt{\frac{9\eta v_g}{2(\rho_{oil} - \rho_{air})g}} \tag{3}$$

Since the densities of air ρ_{air} and oil ρ_{oil} are known, as is the viscosity of air η, the radius of the drop can be determined by measuring the free-fall speed v_g and using Equation 3.

When an electric field is present, due to the voltage V applied to the plates of the capacitor, the charged droplets have an additional force due to that field.

If a negatively charged drop has small enough mass that it moves upward (the electric force is stronger than the gravitational one), then there is the electric force upward, the bouyant force due to the air upward, the gravitational force downward, and the viscous drag force downward. At the terminal velocity the net force is zero,

$$0 = qE - 6\pi r\eta v_u + \frac{4}{3}\pi r^3 \rho_{air} g - \frac{4}{3}\pi r^3 \rho_{oil} g \tag{4}$$

where v_u is the speed of the drop moving upward in the field.

If we take Equation 3 and square it, substituting this into Equation 4 for r^2 in the density r^3 terms (leaving one factor of r), and re-arrange, we obtain

$$qE = [6\pi\eta v_u + 6\pi\eta v_g]r \tag{5}$$

Using the parallel plate geometry we know that $E = V/d$ where d is the spacing between the plates and V is the applied voltage. Thus we have, finally,

$$q = \frac{6\pi\eta d(v_g + v_u)r}{V} \tag{6}$$

Note that v_u is the speed (positive number) of the drop moving upward in the field, and v_g is the speed (positive number) moving downward in free fall.

Thus if the speed of a drop is measured in free fall, v_g, Equation 3 can be used to determine its radius. That same drop is then driven upward with a field and its speed upward is measured, v_u, and Equation 6 is used, along with the radius previously determined, to determine its charge. Only the speeds need be measured.

The charge must be corrected due to one additional term. This form corrects for the additional friction due to extremely small droplets. Now known as the Cunningham formula, it adds a radius-dependent term to the viscosity. The result of this is a correction to q given below:

$$q_c = \frac{q}{\sqrt{(1 + \frac{A}{r})^3}} \tag{7}$$

where $A = 0.07776\ \ \mu m$ is an empirically determined constant.

Apparatus: The Leybold Millikan Oil Drop equipment

The Leybold Millikan oil drop apparatus is shown in Figure 1. This section details the specifics of the apparatus, and has instructions for operating it. You should review it even though you are not taking the data yourself.

The power supply unit provides voltage to the capacitor plates as well as power to the scope illuminator lamp. Turn it on. Note that switch U turns on the power to the capacitor. The voltage is set with the knob and reads out on the LED display. Set a voltage of about 400 V to begin, toggle this with switch U. Turn the voltage off for now.

There is a micrometer scale within the view of the measuring microscope; it has a 10 mm scale divided into 0.1 mm increments. Sight down the scope and be sure you can read this scale and understand the units. The total spacing between the plates is 5.91 ± 0.05 mm.

The microscope objective inverts the image. An object in freefall, moving downward, appears to drift upward! However in the videos we recorded for you, this was adjusted by rotating the camera 180 degrees.

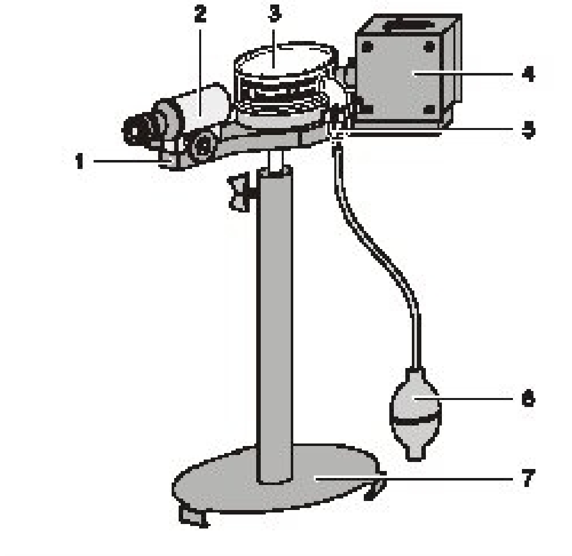

Figure 1: The Millikan apparatus: 1 base plate, 2 microscope and focus knob, 3 capacitor in plexiglas shield, 4 light source, 5 oil atomizer, 6 rubber ball, 7 base

The objective magnification is $M = 2.04 \pm 0.05$, thus the velocity you measure for a displacement Δy in the scope, over a time Δt, is

$$v = \frac{\Delta y}{M \Delta t} \tag{8}$$

Make sure the oil tube is aligned to send the oil into the little holes on the side of the cylindrical capacitor assembly. If it isn't, you're going to squirt oil all over the case rather than into the apparatus. With the voltage off, spritz the rubber bulb to produce some oil drops. The entire capacitor arrangement should be covered with a black cloth to minimize air drafts. Observe the drops. You will need the room to be dark. There will be a distressingly large number of drops. They should be drifting upward in your view, which is actually downward in free fall.

Toggle the voltage on briefly, while watching the drops. Find one that appears to move downward (actually up). This is a negatively charged drop, and you can control its motion. With the voltage on, it drifts up (looks like its going down), and with the voltage off, it drifts down (looks like its going up). Practice "holding" a drop in your field of view going up, and down, and up, and down, by toggling switch U.

Data acquisition

Turn on the apparatus, set the voltage to 500 V, and spray some drops into the chamber. Since there will initially be a large quantity of drops, turn the voltage on for a bit before trying to track a drop in order to remove the drops with large amounts of charge (since they will move extremely fast due to the large electrical force on them).

Notice that there are many sizes and speeds of drops. The really tiny ones drift extremely slowly with the voltage off. If they are so small they seem to "bounce around" they will probably not be good drops to study as their motion is being influenced by individual collisions with air molecules, via Brownian motion.

A drop that takes 10 to 30 seconds to move 1 mm (one major division on the scale) is a good candidate. Drops that are appreciably slower than this are too small, while drops that move much faster than this likely have too much charge. In order to measure Δy try to analyze drops near the center of the field of view, near the scale.

You will note that many drops are either uncharged or positively charged. These are the drops that either do not change speed when the voltage is applied, or fall faster (appear to drift up faster) with the voltage on. Since the positively charged drops rapidly get to the lower (appears upper) plate and are thus lost, we will ignore them. Take data only on negatively charged drops.

Ideally you will follow drops that are in the center of the field of view, near the scale. Track drops as they move 1 mm (one major division on the scale) both when the voltage is on and off. You may find it useful to use two stopwatches, one to time the drop with voltage on, and one to time with voltage off. That way you don't look away and lose track of the drop you were following.

Calculate the charge on each drop using your spreadsheet from the Pre-lab **imediately** after recording its time data. This way you can get a feel for what "kind" of drop has one electron on it. The last thing you want to do is record a bunch of data for drops that have multiple electrons on them, which is not useful for what we want to do.

In order to see anything useful, you will need data for 30-50 drops with small amounts of charge (1-2 electrons). Ideally what you want is data for drops with a charge close to one electron. The only useable drops are those that you can time going both up and down.

Data Analysis

Compute the average free fall velocity v_g and the average upward velocity v_u for each of your drops. Use v_g and Equation 3 to compute the radius of the drop. Then use Equation 6 to find the charge on the drop.

For this first part you should only analyze drops that one one electron, so their charge should be somewhat close to $1.6 \times 10^{-19}C$. Drops that have larger quantities of charge are useful in theory, but you are unlikely to have enough data for you to get anything useful from analyzing them (more on this later).

Do you observe quantized charge? Use the `Python` code in MyCourses to make a histogram of your observed charge. Initially try a "bin size" of perhaps 0.1 or 0.2 $\times$ 10^{-16} C, but you will need to adjust this value for your specific data. Much of the art of using histograms is modifying the bin size until features within the data appear.

Once you have your results in a histogram, it should hopefully look somewhat like a gaussian (see Equation (9). Fit a gaussian to your histogram. If all has gone well, the position of the center of the gaussian (the b value) will correspond to your value for the fundamental charge of the electron. If you are having issues with fitting gaussians (or your data doesn't look very gaussian) an alternative is just to find the mean and standard deviation for your data (again only for the drops that clearly have one electron).

$$f(x) = ae^{\frac{-(x-b)^2}{2c^2}} \tag{9}$$

Because you do not have a lot of drops, you probably have a rather poor histogram that barely, shows charge quantization. To work around this problem, you will also analyze the compedium of data taken by every RIT physics major who has taken this course since 2008. This text file is found on MyCourses.

This full dataset should have multiple "bumps" (recall the data you didn't analyze for your own set). You should fit each "bump" to its own gaussian. The first "bump" corresponds to drops that have one electron, the second "bump" are drops that have two electrons, and so on. Divide the charge for each "bump" (the center posistion of the gaussian) by the number of electrons that for each group. Then you can average these values to find an even better value for the fundamental charge of the electron. You will likely only be able to see 2-3 discreet "bumps". Try to think why this is (hint: it's related to uncertanties).

Email your data to Dr. McLane as a spreadsheet or table. You should have "time with voltage off", "time with voltage on", the distance traveled for voltage on and distance for voltage off, the voltage used, and the temperature, as well as your calculated r, q and q_c, with uncertainties. Make sure your columns are clearly labelled with units. I will add your data to everyone elses for the huge data set of all physics students since 2008 for future use.