Note: the project is due Monday, May 25th.

Distributed coordination of mobile robots

Introduction

Our focus in class so far has been almost exclusively on distributed averaging algorithms for first-order systems, i.e., each agent in the network has first-order dynamics $\dot{x}_i = u_i$, where $x_i \in \mathbb{R}$ is the state of agent i and $u_i \in \mathbb{R}$ is the control input for agent i, which can use information from neighboring agents. However, many important physical systems, such as mechanical and electrical systems, have second-order dynamics coming from, e.g., Newton's laws of motion or inductive-capacitive circuits. In this project, you will analyze and design distributed averaging algorithms in networks with second-order dynamics. You will then apply your results to some robotic coordination problems, including rendezvous, flocking, and formation shape control. Finally, you will derive and apply similar results to distributed formation control of a team of quadcopters, where the linearized dynamics feature both second- and fourth-order dynamics in different directions.

Task 1: Analysis of second-order distributed averaging dynamics

(a) Consider a network of n agents and suppose that the state of each agent in the network is governed by the second-order dynamics

$$\ddot{x}_i = u_i.$$

Derive state space equations for the network dynamics for the state $\mathbf{x} = [x_1, ..., x_n, \dot{x}_1, ..., \dot{x}_n]^T = [\mathbf{p}^T, \mathbf{v}^T]^T$ with input $\mathbf{u} = [u_1, ..., u_n]^T$.

(b) Suppose the agents can communicate according to a fixed graph $G = (V, E)$, where V is a vertex set corresponding to the agents and edge $(i, j) \in E$ if agent i can use relative state information from agent j. Consider the input

$$\mathbf{u} = -\alpha \mathbf{p} - \gamma_0 L \mathbf{p} - \beta \mathbf{v} - \gamma_1 L \mathbf{v},$$

where α, β, γ_0, and γ_1 are constants and L is the Laplacian matrix of G. Prove the following statements:

- Let $\alpha = 0$, $\gamma_1 = 0$, and suppose that G is symmetric and connected. Then the system reaches position consensus and velocity convergence, i.e., $(\mathbf{p}^*, \mathbf{v}^*) \in \text{span}(\mathbb{1}_n, \mathbb{0}_n)$, if and only if $\beta > 0$ and $\gamma_1 > 0$.
- Let $\alpha = 0$, $\gamma_1 = 0$, and suppose that G has a globally reachable node. Then the system reaches position consensus and velocity convergence if and only if

$$\beta > 0, \quad 0 < \gamma_0 < \beta^2 \min_{i \in \{2,...,n\}} \frac{r_i}{q_i^2},$$

  where $r_i = \mathbf{Re}(\mu_i)$, $q_i = \mathbf{Im}(\mu_i)$, and μ_i are the eigenvalues of L.
- Let $\alpha = 0$, $\beta = 0$, $\gamma_0 = 0$, and suppose that G has a globally reachable node. Then the system reaches velocity consensus if and only if $\gamma_1 > 0$.

- The system reaches position and velocity consensus if and only if

$$\beta > \max_{i \in \{2,\ldots,n\}} -r_i\gamma_1, \quad \alpha > \max_{i \in \{2,\ldots,n\}} \left(\frac{\gamma_0^2 q_i^2}{(\beta + r_i\gamma_1)^2} - \frac{\gamma_0\gamma_1 q_i^2}{\beta + \gamma_1 r_i} - \gamma_0 r_i \right).$$

(c) For each of the problems considered above, illustrate the resulting dynamics for $n = 2$ with a free body diagram of two masses, springs, and dampers.

Task 2: Rendezvous, flocking, and formation shape control

(a) Now suppose that each agent corresponds to a robot that can move in three-dimensional space, and that each component of the position vector is governed by second-order dynamics, i.e., $p_i = [x_i, y_i, z_i]^T$ and $\ddot{p}_i = [u_{xi}, u_{yi}, u_{zi}]^T$. Derive state space equations for the network dynamics for the state $\mathbf{x} = [p_1, ..., p_n, \dot{p}_1, ...\dot{p}_n]^T = [\mathbf{p}^T, \mathbf{v}^T]^T$ with input $\mathbf{u} = [u_{x1}, u_{y1} u_{z1}, ..., u_{xn}, u_{yn} u_{zn}]^T$.

(b) **Rendezvous**: Suppose it is desired for all agents to rendezvous at some fixed position in three dimensional space and that each agent should use only relative state information from neighboring agents. Choose appropriate parameters for the control law in Task 1 using the representation you derived in Task 2(a) to achieve this goal. Simulate the network to verify your results. Parameterize your code so that you can adjust the number of agents and the sensing/communication graph.

(c) **Flocking**: Now suppose it is desired for all agents to converge to the same velocity, but not necessarily to the same position. Choose appropriate parameters from the control law in Task 1 using the representation you derived in Task 2(a) to achieve this goal. Simulate the network to verify your results.

(d) **Rendezvous + Flocking**: Combine the previous two tasks and choose appropriate parameters from the control law so that all agents converge to a common position with a common velocity. Simulate the network to verify your results.

(e) **Formation shape control**: Now suppose it is desired for the agents to move together in a specified formation shape. The shape is specified by associating a certain desired relative position between agents who can share information; in particular, we would like that $\lim_{t\to\infty}(p_i(t) - p_j(t)) = p_{ij}^* \in \mathbb{R}^3$ whenever $(i, j) \in E$. Show that the control law

$$\mathbf{u} = -\gamma_0 L\mathbf{p} - \beta\mathbf{v} - \mathbf{c}$$

with $\gamma_0 > 0$, $\beta > 0$ and a suitably chosen constant vector $\mathbf{c} \in \mathbb{R}^n$ achieves the desired shape and velocity convergence under the same conditions as in the first two parts of Task 1(b).

(f) **Distributed integral controller**: Consider now a setting in which the dynamics of each agent is influenced by a constant unknown disturbance:

$$\ddot{p}_i = u_i + d_i$$

for some constant $d_i \in \mathbb{R}^3$. Design a distributed integral controller to augment the distributed formation controller from the previous task and show via analysis and simulation that your design achieves the desired formation shape despite the disturbances. (Note that since the disturbances are unknown, they cannot simply be cancelled.)

Task 3: Distributed formation control with quadcopters

(a) Repeat part 1(a) for a fourth-order dynamics model:

$$\frac{d^4 x_i}{dt^4} = u_i.$$

(b) Suppose that the agents can communicate according to a fixed undirected graph $G = (V, E)$, where V is a vertex set corresponding to the agents and edge $(i, j) \in E$ if agent i can use relative state information from agent j. Derive a necessary and sufficient condition for position consensus $\mathbf{x}^* \in \text{span}(\mathbb{1}_n)$ and convergence of $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$, and $\dddot{\mathbf{x}}$ to zero.

(c) Consider the quadcopter dynamics

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = R \begin{bmatrix} 0 \\ 0 \\ u_4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}, \quad \dot{R} = R \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}, \quad J\dot{\omega} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} - \omega \times J\omega,$$

where $\ddot{x}, \ddot{y}, \ddot{z}$ are accelerations in a fixed reference frame, g is gravitational acceleration, R is the rotation matrix from the body frame to the fixed reference frame, u_4 is a mass-normalized thrust input, $\omega = [p, q, r]^T$ are angular body rates about the body $x-$, $y-$, and $z-$axes, respectively, J is the mass moment of inertia matrix, and u_1, u_2, u_3 are moment inputs applied to the airframe about body $x-$, $y-$, and $z-$axes, respectively, via the rotor aerodynamics. The rotation matrix R can be parameterized in terms of $Z - X - Y$ Euler angles. To get from the fixed frame to the body frame, one can first rotate about the fixed z-axis by the yaw angle ψ, then rotate about the intermediate x-axis by the roll angle ϕ, and finally rotate about the body y axis by the pitch angle θ. The rotation matrix is then given by

$$R = \begin{bmatrix} c_\psi c_\theta - s_\phi s_\psi s_\theta & -c_\phi s_\psi & c_\psi s_\theta + c_\theta s_\phi s_\psi \\ c_\theta s_\psi + c_\psi s_\phi s_\theta & c_\phi c_\psi & s_\psi s_\theta - c_\psi c_\theta s_\phi \\ -c_\phi s_\theta & s_\phi & c_\phi c_\theta \end{bmatrix}.$$

Finally, the components of the angular velocity in the body frame, p, q, and r, are related to the roll, pitch, and yaw derivatives by

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c_\theta & 0 & -c_\phi s_\theta \\ 0 & 1 & s_\phi \\ s_\theta & 0 & c_\phi c_\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}.$$

Linearize the dynamics about a hover condition, in which $x = y = z = const$, $\dot{x} = \dot{y} = \dot{z} = 0$, $R = I_3$, $p = q = r = 0$, $u_1 = u_2 = u_3 = 0$, and $u_4 = g$, and derive an associated state space representation for the linearized dynamics for deviations from these nominal conditions with the state $\mathbf{x} = [\delta x, \delta y, \delta z, \delta \dot{x}, \delta \dot{y}, \delta \dot{z}, \delta \phi, \delta \theta, \delta \psi, \delta p, \delta q, \delta r]^T$. Show that the linearized dynamics consist of a set of decoupled higher-order linear systems: the altitude (z) and yaw dynamics are second-order and the lateral (x and y) positions are fourth-order, involving the moment inputs being integrated through pitch and roll states and rates.

(d) Now we would like to have multiple quadcopters cooperate in a distributed way to perform a rendezvous or formation shape control task. However, note that the fourth-order lateral quadcopter dynamics are not exactly fourth-order integrators as in Task 3(a) and (b). Modify the conditions you obtained in Task 3(b) using the linearized quadcopter model, and design a distributed controller that achieves a rendezvous or formation shape control task. Simulate a network of several quadcopters using the linearized model to verify your results.

(e) Using your control law based on the linearized model, simulate a rendezvous or formation shape control task on the full nonlinear system. Experiment with different initial conditions to see how sensitive the design is to the linearized model and get an idea of the region of attraction.