Foundations of Physics: Optics

Solutions

Name: _____

Date: _____

Note: $\arcsin(\theta)$ can be thought of as the same as $\sin^{-1}(\theta)$, as well as for the cosine.

1 Questions

1. The speed of light in ice is 2.29×10^8 m/s. What is the index of refraction of ice?
Solution:
$$n = \frac{c}{v} = \frac{3 \times 10^8 \text{m/s}}{2.28 \times 10^8 \text{m/s}} = 1.3$$

2. The speed of light in a certain substance is 82% of its value in water. What is the index of refraction of that substance?
Solution:
$$n_w = 1.33 = \frac{3 \times 10^8}{v} \implies v = 3 \times 10^8 / 1.33 = 2.26 \times 10^8 \text{ m/s}$$
$$n = \frac{3 \times 10^8 \text{ m/s}}{0.82 \times 2.26 \times 10^8 \text{ m/s}} = 1.6$$

3. Flashlight beam strikes surface of a pane of glass ($n = 1.56$) at an angle of 67 degrees to the normal. What is the angle of refraction?
Solution:
$$1 \sin 67 = 1.56 \sin\theta \implies \theta = \arcsin\left(\frac{\sin 67}{1.56}\right) = 36.2 \text{ deg}$$

4. Glass block with index of refraction 1.5 is immersed in water whose index of refraction is 1.33. What is the critical angle at the glass-water interface?
Solution:
$$1.5 \sin\theta_c = 1.33 \sin 90 \implies \theta_c = \arcsin\left(\frac{1.33}{1.5}\right) = 62.5 \text{ deg}$$

5. Diver shines flashlight upward from beneath the water at an angle of 35.2 degrees to the vertical. What is the angle at which light leaves water?
Solution:
$$1.33 \sin 35.2 = \sin\theta \implies \theta = \arcsin(1.33 \sin 35.2) = 50.1 \text{ deg}$$

6. A beam of light in air strikes a slab of glass and is partially reflected and partially refracted. Determine the angle of incidence if the angle of reflection is twice the angle of refraction. ($n = 1.51$)
Hint: $\sin 2x = 2 \sin x \cos x$
Solution:
$$\theta_i = \theta_r = 2\theta_R$$
$$\sin\theta_i = 1.51 \sin\theta_R \iff \sin(2\theta_R) = 1.51 \sin\theta_R$$
Using trigonometric identity,
$$2\cancel{\sin\theta_R}\cos\theta_R = 1.51\cancel{\sin\theta_R}$$
$$\cos\theta_R = \frac{1.51}{2} \implies \theta_R = \arccos(0.755) = 40.97$$
$$\theta_i = 2\theta_R = 82 \text{ deg}$$

7. A solar cooker, really a concave mirror pointed at the Sun, focuses the Sun's rays 18.8 cm in front of the mirror. What is the radius of the spherical surface from which the mirror was made?
Solution:
$$f = 18.8, f = \frac{R}{2} \implies R = 2f = 37.6 \text{ cm}$$

8. How far from a concave mirror (radius 21.0 cm) must an object be placed if its image is to be at infinity?
Solution: Conceptually, this means that the image cannot be seen in the mirror.
$$f = \frac{R}{2} = 10.5 \text{ cm}$$
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$\frac{1}{10.5} = \frac{1}{d_o} + \frac{1}{\cancel{\infty}}^{0}$$
$$d_o = 10.5$$

9. A small candle is 38 cm from a concave mirror having a radius of curvature of 24 cm.
   (a) What is the focal length of the mirror?
   Solution:
   $$f = \frac{R}{2} = \frac{24}{2} = 12 \text{ cm}$$
   (b) (1 point) Where will the image of the candle be located?
   Solution:
   $$\frac{1}{f} = \frac{1}{d_i}\frac{1}{d_o}$$
   $$\frac{1}{12} = \frac{1}{d_i} + \frac{1}{38} \iff \frac{1}{d_i} = \frac{1}{12} - \frac{1}{38}$$
   $$\frac{1}{d_i} = \frac{13}{228}$$
   $$d_i = \frac{228}{13} = 17 \text{ cm into the mirror}$$
   (c) What will the orientation of the image be?
   Solution: Reflected vertically.

10. A dentist wants a small mirror that, when 2.00 cm from a tooth, will produce a 4x upright image. What kind of mirror must be used and what must its radius of curvature be?
Solution:
$$m = -\frac{d_i}{d_o} \iff 4 = -\frac{d_i}{2} \implies d_i = -8$$
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \iff \frac{1}{f} = \frac{1}{2} + \frac{1}{-8} = \frac{3}{8}$$
$$f = \frac{8}{3} = 2.67$$
$$f = \frac{R}{2} \iff R = 2f = 2(2.67) = 5.34 \text{ cm}$$
The mirror used must be convex because the image is erect and the d_i is negative.

11. You are standing 3.4 m from a convex security mirror in a store. You estimate the height of your image to be half of your actual height. Estimate the radius of curvature of the mirror.
Solution:

$$m = \frac{h_i}{h_o} \iff m = \frac{\frac{1}{2}h_o}{h_o} \implies m = \frac{1}{2}$$

$$m = -\frac{d_i}{d_o} \iff \frac{1}{2} = -\frac{d_i}{3.4} \implies d_i = -3.4 \times \frac{1}{2} = -1.57$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{3.4} + \frac{1}{-1.57} = -0.34$$

$$f = \frac{1}{-0.34} = -2.92$$

$$f = -2.92 = \frac{R}{2} \implies R = -5.22$$

Although this measure of distance is negative, it still is valid and remains to have meaning.

12. The image of a distant tree is virtual and very small when viewed in a curved mirror. The image appears to be 19.0 cm behind the mirror. What kind of mirror is it, and what is its radius of curvature?
Solution: Assume $d_o \approx \infty$, which means,

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{f} = \frac{\overset{0}{\cancel{1}}}{\cancel{\infty}} + \frac{1}{19} \implies f = 19 \text{ cm}$$

$$f = \frac{R}{2} \iff R = 2f = 38 \text{ cm}$$

Because this image is virtual (upright) we conclude that the mirror is convex.

13. You look at yourself in a shiny 8.8-cm-diameter Christmas tree ball. If your face is 25.0 cm away from the ball's front surface, where is your image? Is it real or virtual? Is it upright or inverted?
Solution:

$$R = \frac{8.8}{2} = 4.4, d_o = 25$$

$$f = \frac{R}{2} = \frac{4.4}{2} = 2.2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{2.2} = \frac{1}{25} + \frac{1}{d_i} \iff \frac{1}{d_i} = \frac{1}{2.2} - \frac{1}{25} = 0.415$$

$$d_o = \frac{1}{0.415} = 2.4 \text{ cm}$$

This image is inverted and real because this is a convex mirror.

14. What is the critical angle for the interface between water and crown glass ($n = 1.52$) ? To be internally reflected, the light must start in which material?
Solution: First, we assume that the light enters from water into the crown glass:

$$1.33 \sin\theta_c = 1.52 \sin 90 \implies \sin\theta = \frac{1.52}{1.33}$$

This value does not have a definition in the $\arcsin\theta$ function, (try for yourself!), so we conclude that the light must travel from the crown glass into the water,

$$1.52 \sin\theta_c = 1.32 \sin 90 \iff \sin\theta_c = \frac{1.33}{1.52}$$

$$\theta_c = \arcsin\left(\frac{1.33}{1.52}\right) = 61 \text{ deg}$$

15. A sharp image is located 391 mm behind a 215-mm- focal-length converging lens. Find the object distance
Solution:

$$f = 215, d_i = 391$$
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \iff \frac{1}{215} = \frac{1}{d_o} + \frac{1}{391}$$
$$\implies \frac{1}{d_o} = \frac{1}{215} - \frac{1}{391} = 0.00209362$$

Thus $d_o = 478$ mm

16. What is the power of a 32.5-cm-focal-length lens?
Solution: We convert the focal length into SI units (m),

$$32.5 \text{ cm } = 0.325 \text{ m}$$

Thus, $P = \frac{1}{f} = \frac{1}{0.325} = 3.1$ D

17. What is the focal length of a -6.75 D lens? Are these lenses converging or diverging?
Solution:

$$\frac{1}{f} = -6.75 \iff f = -\frac{1}{6.75} = 1.5 \times 10^{-1} \text{ m}$$

Since this lens is for nearsightedness, the lens must be one that is diverging (light rays move apart to hit the focal point)

Notably eye defects are attributed to the size of the eyeball changing. Fascinating.

18. A 1.50-cm-high object is placed 20.0 cm from a concave mirror with radius of curvature 30.0 cm. Determine
(a) Position of the image
Solution: Given:

$$h_o = 1.50\,\text{cm}, \quad d_o = 20.0\,\text{cm}, \quad R = 30.0\,\text{cm}.$$

For a concave mirror, the focal length is

$$f = \frac{R}{2} = 15.0\,\text{cm}.$$

Using the mirror equation,

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i},$$

we solve for d_i:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15.0} - \frac{1}{20.0} = \frac{4-3}{60} = \frac{1}{60}.$$

Therefore,

$$d_i = 60.0\,\text{cm}.$$

Since $d_i > 0$, the image is **real** and formed **in front of the mirror**.

(b) Its size
Solution: The magnification is

$$m = -\frac{d_i}{d_o} = -\frac{60.0}{20.0} = -3.00.$$

Thus, the image height is

$$h_i = mh_o = (-3.00)(1.50\,\text{cm}) = -4.50\,\text{cm}.$$

The negative sign indicates the image is **inverted**.

$$\boxed{d_i = 60.0\,\text{cm}, \quad h_i = -4.50\,\text{cm}}$$

Therefore, the image is real, inverted, and 3 times larger than the object.