PHYS 1006

Foundations of Physics: Fundamental Principles

Week 1,2 & 3 Content

Name: Solutions

Date:

1 Measurements and Uncertainties

In life, we can never obtain a truly accurate, and 100 percent precise measurement, thus we say that every measurment has an "uncertainty" attached to it.

We define the **uncertainty of the measuring instrument to be one half of the smallest division**. For example, if a ruler has millimeter divisions, then the uncertainty associated with that ruler is ± 0.5 mm.

Treatment of Uncertainties

We want to be able to perform mathematical operations on values with uncertainties, where in most cases the uncertainties are distinct.

(1) Addition:

$$L = L_0 \pm \Delta L, \qquad W = W_0 \pm \Delta W$$
$$L + W = (L_0 + W_0) + (\Delta L + \Delta W)$$

Essnetially, you add the components piecewise.

(2) Multiplication/Division

Say we have a general expression in the form,

$A = \pi r^2$

$$x = \frac{A^n B^m}{C^p}$$

$\frac{\Delta A}{A} = 2\left(\frac{\Delta r}{r}\right)$

Then the uncertainty for the value x can be found via,

$$\frac{\Delta x}{x} = n\left(\frac{\Delta A}{A}\right) + m\left(\frac{\Delta B}{B}\right) + p\left(\frac{\Delta C}{C}\right)$$

Example 1. Pendulum

A student measures the length, L of a simple pendulum to be 1.217 $\pm$ 0.001 m, and the period, T to be 2.25 $\pm$ 0.04 s. By using the below formula, what is the measured value of gravity, g?

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Solution: We do the following algebra to obtain an expression independent of g,

$$\begin{aligned}
(T)^2 &= \left(2\pi\sqrt{\frac{L}{g}}\right)^2 \\
T^2 &= 2^2\pi^2\frac{L}{g} \\
\frac{T^2}{4\pi^2} &= \frac{L}{g} \\
\left(\frac{T^2}{4\pi^2}\right)^{-1} &= \left(\frac{L}{g}\right)^{-1} && \text{Power of minus 1 to both sides to flip fraction} \\
\frac{g}{L} &= \frac{4\pi^2}{T^2} \\
g &= \frac{4\pi^2 L}{T^2}
\end{aligned}$$

We can now put in the values of L and T (without uncertainties) into the equation, which gives:

$$g = 9.49$$

Now, we find an expression for Δg, which we use the equation, $x = \frac{A^n B^m}{C^p}$ to help us.

$$\begin{aligned}
g &= \frac{4\pi^2 L}{T^2} \\
&= 4\pi^2\frac{L}{T^2} \\
\implies \frac{\Delta g}{g} &= \frac{\Delta L}{L} + 2\frac{\Delta T}{T} \\
\therefore \Delta g &= g\left(\frac{\Delta L}{L} + 2\frac{\Delta T}{T}\right)
\end{aligned}$$

And by substituting the values in, we obtain,

$$\Delta g = 0.35$$

Meaning,

$$g \pm \Delta g = 9.49 \pm 0.35 \text{ ms}^{-2}$$

A good measurement for the gravity present on Earth.

2 Significant Figures

Significant figures have the following defining rules,

(1) All non-zero digits are significant

Example, 123.21 has 5 s.f. 21 has 2 s.f.

(2) All zeros within non-zero digits are significant

Example, 20005 has 5 s.f. 50042 has 5 s.f.

(3) Leading zeros are insignificant

Example, 00000.42 has 2 s.f.

(4) Trailing zeros are significant. Why? Because they show the precision of measurement.

Example, 2.300 has 4 s.f. 32000 has 5 s.f.

(5) A number in scientific notation, $A \times 10^x$, has significant figures only in the A term.

Example, 12.3×10^3 has 3 s.f. 3×10^{-4} has 1 s.f.

A general rule of thumb is to keep the significant numbers in your answers consistent with the number with the least amount of significant figures used in the question. If numbers aren't stated, you can assume to use 2 significant figures.

3 Scientific Notation

This is used for 2 reasons, to avoid the ambiguity of traling zeros as significant figures, and to express values in a concise manner, with use of standard (SI) units.

For example, the number 2304000 has 7 s.f, but the number 2.3×10^7 has 2 s.f.
Another example, could be the mass of a proton in kg, which is 1.7×10^{-27} kg. We write this in scientific notation because it is simply inconvinient to write out 29 digits on paper.

It is natural to ask why we want numbers in standard units (SI), and the reason why is because our equations that we use in physics are derived for only those specific units, so in a way, our equations would not work if we had inconsistencies in our units.

A brief note on SI units, prefixes, and conversions

4 Kinematics

We have the following equations of motion,

$$\begin{aligned} v_f &= v_i + at \\ x_f &= x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 &= v_i^2 + 2a(x_f - x_i) \\ \bar{v} &= \frac{v_f + v_i}{2} \end{aligned}$$

4.1 Vectors and Trigonometry

$v = \frac{x}{t} \Rightarrow x = vt$

4.1.1 Projectile Motion

If a body has initial velocity v_i, projected at an angle θ from the horizontal, then,

$$v_{ix} = v_i \cos\theta \Rightarrow x_x = vt = v_i t \cos\theta$$
$$v_{iy} = v_i \sin\theta - gt$$

At the peak of the motion, $v_{iy} = 0$. We use this principle to analyse the motion of a body undergoing projectile motion.

Football kicked with an intial velocity of 30 m/s at an angle of 35 deg to the horizontal. What can you say about the ball's motion?

5 Questions

1. How many significant figures are in each of the following numbers? (a) 214, (b) 81.60, (c) 0.0086, (d) 8700.

a) 3 sf c) 2 sf
b) 4 sf d) 4 sf

2. Write the following numbers in powers of 10 notation: (a) 1.156, (b) 0.0068, (c) 328.65, (d) 444.

a) 1.2×10^{0} c) 3.3×10^{2}
b) 6.8×10^{-3} d) 4.4×10^{2}

3. Convert the following measurements to full decimal numbers without prefixes: (a) 286.6 mm, (b) 760 mg, (c) 22.5 nm, (d) 2.50 gigavolts.

$10^{-1-2} = 10^{-3}$

a) $\underbrace{286.6 \times 10^{-1} \times 10^{-2}}$
286.6×10^{-3}
29×10^{-4} m
0.00029 m

b) $760 \times 10^{-3} \times 10^{-3}$
760×10^{-6}
0.0000076 kg

c) 22.5×10^{-9}
0.0000000225 m

d) 2.5×10^{9} V
2500000000 V

4. Using appropriate SI prefixes, express the following quantities: (a) 0.00056 m, (b) 1250 s, (c) 0.0000083 kg, (d) 7,500,000 V.

a) 0.56×10^{-3} m
0.56 mm

b) 1250 s
1.25×10^{3} ks
1.3×10^{3} ks

c) 8.3 μkg

d) 7.5 MV

5. A typical atom has a diameter of 1.0×10^{-10} m. Approximately how many atoms are along a 1.0-cm line?

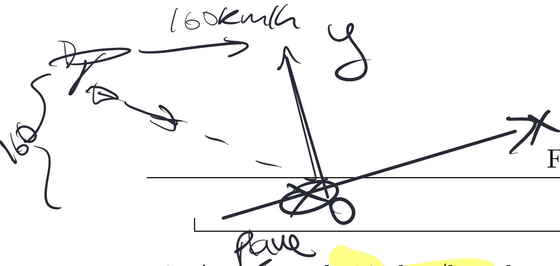
$\frac{a^n}{a^m} = a^{n-m}$

1 cm = 0.01 m = 1×10^{-2} m

$$\text{no.} = \frac{\cancel{1} \times 10^{-2}}{\cancel{1} \times 10^{-10}} = 1 \times 10^{-2--10}$$

$= 1 \times 10^{8}$ atoms

$a^n a^m = a^{n+m}$



6. A car travels 160 km/h and a package is dropped from it from a height of 160 m. How many seconds before being directly overhead should the package be released? (Neglect air resistance.)

$t = ?$

$a = -9.8\,m/s^2$

$x = 160\,m$

$x_f = x_i + v_i t + \frac{1}{2}at^2$

$0 = 160 + \cancel{0t} + \frac{1}{2}(-9.8)t^2$

$-160 = \frac{1}{2}(-9.8)t^2$

$\frac{-320}{-9.8} = t^2$

$t = \sqrt{\frac{320}{9.8}} = 5.71\,s$

7. The Sun is on average 93 million miles from Earth. Convert this distance into (a) meters using powers of 10, (b) kilometers using a metric prefix. Note 1 mile = 1.6 km.

$\frac{(93 \times 10^6)}{1.6} = 58 \times 10^6\,km$

$58 \times 10^9\,m$

b) 58 Mkm

8. A circle has radius $r = 1.57 \pm 0.05$ m. Calculate the area of the circle and its approximate uncertainty.

$A = \pi r^2$

$\frac{\Delta A}{A} = 2\left(\frac{\Delta r}{r}\right)$

$A = \pi(1.57)^2 = 7.74\,m^2$

$\Delta A = A \cdot 2\frac{\Delta r}{r} \longrightarrow \Delta A = (7.74) \cdot 2\left(\frac{0.05}{1.57}\right) = 0.49$

Thus

$A = 7.74 \pm 0.49\,m^2$

9. Determine how many acres are in one hectare, given that one hectare is defined as 10,000 m^2 and one acre is 4047 m^2.

$\text{no.} = \frac{10\,000\,m^2}{4047\,m^2} = 2.471$ acres/hectare

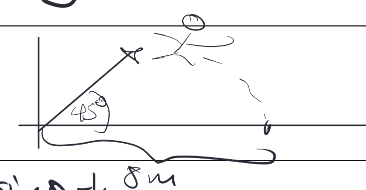
10. A light-year is the distance light travels in one year at speed $c = 3.00 \times 10^8$ m/s. (a) How many meters are in 1.00 light-year? (b) How many astronomical units (AU) are there in 1.00 light-year if 1 AU $= 1.50 \times 10^{11}$ m?

a) $m = 3\times10^8 \times 60 \times 60 \times 24 \times 365$
$= 9.46 \times 10^{15}$ m

b) $\frac{9.46\times10^{15}}{1.5\times10^{11}} = 6.31\times10^4$ AU/Ly

See add. page

11* A long jumper leaves the ground at 45° above the horizontal.



(1) She lands 8.0 m away. What is her "takeoff" speed?

$x_x = v_{ix} t$
$= v_i \cos\theta$

$v_{iy} = v_i \sin\theta - gt$
$x_y = v_i \sin\theta t - \frac{1}{2} g t^2$
$0 = v_i \sin\theta t - \frac{1}{2} g t^2$

$0 = t(v_i \sin\theta - \frac{1}{2} g t)$
$t = 0$, or $\underbrace{v_i \sin\theta - \frac{1}{2} g t = 0}$
$t = \frac{2 v_i \sin\theta}{g}$

(2) Now she is on a hike and comes to the left bank of a river. There is no bridge and the right bank is 10.0 m away horizontally and 2.5 m vertically below. If she long jumps from the edge of the left bank at 45° with the speed calculated in (a), how long, or short, of the opposite bank will she land?

$x_{yf} = x_{yi} + v_i \sin\theta t - \frac{1}{2} g t^2$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$0 = \underbrace{2.5}_{c} + \underbrace{8.7 \sin(45)}_{b} t - \underbrace{\frac{1}{2}(9.8)}_{a} t^2$

$x_x(t)$.

12. A light plane must reach a speed of 120 km/h for takeoff. How long a runway is needed if the constant acceleration is 1.8 m/s^2?

m/s

$v_f^2 = v_i^2 + 2a(x_f - x_i)$

$\frac{120\times1000}{60\times60} = 33.33$ m/s $= v_f$

$0 = v_i$
$x_i = 0$

$\frac{33.33^2}{3.6} = x_f = 31\times10$ m

$x_f = ?$
$a = 1.8$ m/s^2

$$x_x = v_{ix} t = v_i \cos\theta\, t = v_i \cos\theta \left(\frac{2 v_i \sin\theta}{g} \right)$$

$$\frac{2 v_i^2 \sin\theta \cos\theta}{g} \qquad \sin(2\theta) = 2 \sin\theta \cos\theta$$

$$\cancel{\sqrt{\frac{v_i^2 \sin(2\theta)}{\sin(2\theta)}}} = \sqrt{\frac{x_x g}{\sin(2\theta)}} = v_i$$

$$\sqrt{\frac{(8)(9.5)}{\sin(90)}} = 8.72 \, m/s = 8.7 \, m/s$$

13. A baseball pitcher throws a baseball with a speed of 43 m/s. Estimate the average acceleration of the ball during the throwing motion. In throwing the baseball, the pitcher accelerates it through a displacement of about 3.5 m, from behind the body to the point where it is released.

$$v_f^2 = \cancel{v_i^2}^{\,0} + 2a(x_f - \cancel{x_i}^{\,0}) \longrightarrow \frac{v_f^2}{2x_f} + \frac{\cancel{2}a\cancel{x_f}}{\cancel{2x_f}}$$

$v_i = 0$

$x_i = 0$

$$\therefore a = \frac{v_f^2}{2x_f} = \frac{43^2}{2(3.5)} =$$

$$= 26 \times 10 \text{ m/s}^2$$