PSWF-Radon approach to reconstruction from band-limited Hankel transform

Mikhail Isaev [*]

School of Mathematics
Monash University
Clayton, VIC, Australia

mikhail.isaev@monash.edu

Roman G. Novikov

CMAP, CNRS, Ecole Polytechnique
Institut Polytechnique de Paris
Palaiseau, France

IEPT RAS, Moscow, Russia

novikov@cmap.polytechnique.fr

Rodion Zaytsev[†]

Higher School of Economics
Moscow, Russia

rvzaytsev@edu.hse.ru

Abstract

We give new formulas for reconstructions from band-limited Hankel transform of integer or half-integer order. Our formulas rely on the PSWF-Radon approach to super-resolution in multidimensional Fourier analysis. This approach consists of combining the theory of classical one-dimensional prolate spheroidal wave functions with the Radon transform theory. We also the use relation between Fourier and Hankel transforms and Cormack-type inversion of the Radon transform. Our numerical examples illustrate super-resolution even with a considerable noise.

Keywords: Hankel transform, Fourier transform, Radon transform, prolate spheroidal wave functions, super-resolution
AMS subject classification: 42A38, 49K40

[main contents of this draft were removed, as this is a work in progress]

References

[1] R. Adelman, N.A. Gumerov, R. Duraiswami, Software for computing the spheroidal wave functions using arbitrary precision arithmetic, (2014), CoRR abs/1408.0074, arXiv:1408.0074.

[2] N.V. Alexeenko, V.A. Burov, O.D. Rumyantseva, Solution of the three-dimensional acoustical inverse scattering problem. The modified Novikov algorithm, *Acoustical Physics* **54**(3) (2008), 407–419.

[3] N. Alibaud, P. Maréchal, Y. Saesor, A variational approach to the inversion of truncated Fourier operators. *Inverse Problems*, **25**(4) (2009), 045002.

[4] G. Ambartsoumian, S. Roy, Numerical inversion of a broken ray transform arising in single scattering optical tomography, *IEEE Transactions on Computational Imaging*, **2**(2) (2016), 166–173.

[5] G. Ambartsoumian, R. Gouia-Zarrad, V. Krisnan, S. Roy, Image reconstruction from radially incomplete spherical Radon data. *European Journal of Applied Mathematics*, **29**(3) (2018), 470–493.

[6] G. Beylkin, L. Monzón, Nonlinear inversion of a band-limited Fourier transform, *Applied and Computational Harmonic Analysis*, **27**(3) (2009), 351–366.

[7] A. Bonami, A. Karoui, Uniform bounds of prolate spheroidal wave functions and eigenvalues decay, *Comptes Rendus de l'Académie des Sciences, Series I*, **352**(3) (2014), 229–234.

[8] A. Bonami, A. Karoui, Spectral decay of time and frequency limiting operator, *Applied and Computational Harmonic Analysis* **42**(1) (2017), 1–20.

[9] R.N. Bracewell, Strip Integration in Radio Astronomy, *Australian Journal of Physics*, **9**(2) (1956), 198–2017.

[10] V.A. Burov, N.V. Alekseenko, O.D. Rumyantseva, Multifrequency generalization of the Novikov algorithm for the two-dimensional inverse scattering problem, *Acoustical Physics* **55**(6) (2009), 843–856.

[11] E. J. Candès, C. Fernandez-Granda, Towards a mathematical theory of super-resolution, *Communications on Pure and Applied Mathematics*, **67** (2014), 906–956.

[12] A.M. Cormack, Representation of a function by its line integrals, with some radiological applications. *Journal of Applied Physics* **34** (9) (1963), 2722–2727.

[13] R.W. Gerchberg, Superresolution through error energy reduction, *Optica Acta: International Journal of Optics*, **21**(9) (1974), 709–720.

[14] R. Gouia-Zarrad, S. Roy, S. Moon, Numerical inversion and uniqueness of a spherical radon transform restricted with a fixed angular span, *Applied Mathematics and Computation*, **408** (2021), doi:10.1016/j.amc.2021.126338.

[15] P. Hähner, T. Hohage, New stability estimates for the inverse acoustic inhomogeneous medium problem and applications, *SIAM Journal on Mathematical Analysis*, **33**(3) (2001), 670–685.

[16] H.A. Hasanov, V.G. Romanov, *Introduction to inverse problems for differential equations*. Second edition, Springer (2021), 515 pp.

[17] T. Hohage, F. Weidling, Variational source conditions and stability estimates for inverse electromagnetic medium scattering problems, *Inverse Problems and Imaging*, **11**(1) (2017), 203–220.

[18] M. Isaev, R.G. Novikov, New global stability estimates for monochromatic inverse acoustic scattering, *SIAM Journal on Mathematical Analysis*, **45**(3) (2013), 1495–1504.

[19] M. Isaev, R.G. Novikov, Hölder-logarithmic stability in Fourier synthesis, *Inverse Problems* **36**(12) (2020), 125003.

[20] M. Isaev, R.G. Novikov, Stability estimates for reconstruction from the Fourier transform on the ball, *Journal of Inverse and Ill-posed Problems*, **29**(3) (2021), 421–433.

[21] M. Isaev, R.G. Novikov Reconstruction from the Fourier transform on the ball via prolate spheroidal wave functions, *Journal de Mathématiques Pures et Appliquées*, **163** (2022), 318–333.

[22] M. Isaev, R.G. Novikov, G.V. Sabinin, Numerical reconstruction from the Fourier transform on the ball using prolate spheroidal wave functions, *Inverse Problems*, **38**(10) (2022), 105002.

[23] M. Isaev, R.G. Novikov, G.V. Sabinin, Super-resolution reconstruction from truncated Fourier transform, *Extended abstracts MWCAPDE*, 2023, Birkhäuser, DOI: 10.1007/978-3-031-41665-1_7.

[24] S. Karnik, J. Romberg, M. A. Davenport, Improved bounds for the eigenvalues of prolate spheroidal wave functions and discrete prolate spheroidal sequences. *Applied and Computational Harmonic Analysis* **55**(1) (2021), 97–128.

[25] A. Karoui, T. Moumni, Spectral analysis of the finite Hankel transform and circular prolate spheroidal wave functions, *Journal of Computational and Applied Mathematics* **233**(2) (2009), 315–333.

[26] M.I. Kolobov, V.N. Beskrovnyy, Quantum theory of super-resolution for optical with circular apertures, *Optics Commun.* **264** (2006), 9–12.

[27] A. Lannes, S. Roques, M.-J. Casanove, Stabilized reconstruction in signal and image processing: I. partial deconvolution and spectral extrapolation with limited field. *Journal of modern Optics*, **34**(2) (1987), 161–226.

[28] F. Natterer, The Mathematics of Computerized Tomography. *Society for Industrial Mathematics*, (2001), 184 pp.

[29] C. Niven, On the conduction of heat in ellipsoids of revolution, *Philosophical transactions of the Royal Society of London*, **171**(1880) 117–151.

[30] A. Papoulis, A new algorithm in spectral analysis and band-limited extrapolation. *IEEE Transactions on Circuits and Systems*, **22**(9) (1975), 735–742.

[31] Alfred Puro, Cormack-type inversion of exponential Radon trans-form". *Inverse problems*, **17**(1) (2001), 179.

[32] J. Radon, Uber die Bestimmung von Funktionen durch ihre Integralwerte l´angs gewisser Mannigfaltigkeiten, *Ber. Saechs Akad. Wiss. Leipzig, Math-Phys*, **69** (1917), 262–267.

[33] R. Ramlau, O. Scherzer (eds), *The Radon Transform: The First 100 Years and Beyond*, Berlin, Boston: De Gruyter (2019), https://doi.org/10.1515/9783110560855.

[34] S. Roy, V.P. Krishnan, P. Chandrashekar, A.S. Vasudeva Murthy, An efficient numerical algorithm for the inversion of an integral transform arising in ultrasound imaging, *Journal of Mathematical Imaging and Vision*, **53** (2015), 78–91.

[35] T.A. Syed, V.P. Krishnan, J. Sivaswamy, Numerical Inversion of Circular arc Radon Transform, *IEEE Transactions on Computational Imaging*, **2**(4) (2016), 540–549.

[36] A.S. Shurup, Numerical comparison of iterative and functional-analytic algorithms for inverse acoustic scattering, *Eurasian Journal of Mathematical and Computer Applications* **10**(1) (2022), 79–99.

[37] D. Slepian, H.O. Pollak, Prolate spheroidal wave functions, Fourier analysis and uncertainty — I, *Bell System Technical Journal*, **40** (1) (1961), 43–63.

[38] A.N. Tikhonov, V. Y. Arsenin, *Solutions of ill-posed problems.* Washington : New York : Winston ; distributed solely by Halsted Press, (1977), 258 p.

[39] L. L. Wang, Analysis of spectral approximations using prolate spheroidal wave functions. *Mathematics of Computation* **79** (2010), no. 270, 807–827.

[40] H. Xiao, V. Rokhlin, N. Yarvin, Prolate spheroidal wavefunctions, quadrature and interpolation. *Inverse Problems* **17**(4) (2001), 805–838.