Physics Formula Sheet

402-0023-01L Physics

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Constants

Constant	Symbol	Value
Speed of light	c	3.00×10^8 m/s
Gravitational constant	G	6.674×10^{-11} N(m/kg)^2
Planck's constant	h	6.626×10^{-34} J.s
Mass of the electron	m_e	9.10939×10^{-31} kg
Mass of the proton	m_p	1.67262×10^{-27} kg
Charge of the electron	$-e$	-1.60218×10^{-19} C
Permittivity of free space	ϵ_0	8.85419×10^{-12} C^2/J m
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T m / A
Boltzmann constant	k_B	1.38066×10^{-23} J/ K
Avogadro's constant	N_A	6.022×10^{23} 1/mol

Oscillations

- **Natural Frequency**: $\sqrt{\frac{k}{m}}$

- **Damping Ratio** (ζ):

$$\zeta = \frac{b}{b_c}$$

where $b_c = 2\sqrt{mk}$

Quality Factor (Q factor)

The Q factor is a dimensionless parameter that describes the damping of an oscillator. It represents the energy stored to energy dissipated ratio.

$$Q = \frac{1}{2\zeta} = \frac{\omega_0}{\Delta\omega} = 2\pi f \times \frac{\text{energy stored}}{\text{power loss}}$$

where $\Delta\omega$ is the bandwidth over which the energy is stored.

Types of Oscillations

- **Critically Damped** ($\zeta = 1$): The system returns to equilibrium as quickly as possible without oscillating.

- **Overdamped** ($\zeta > 1$): The system returns to equilibrium without oscillating but slower than the critically damped case.

- **Underdamped** ($\zeta < 1$): The system oscillates about the equilibrium position with a frequency ω_d given by:

$$\omega_d = \omega_0\sqrt{1 - \zeta^2}$$

General Solution

For a driven damped harmonic oscillator, the general solution can be expressed as:

$$x(t) = e^{-\zeta\omega_0 t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right)$$

where A and B are constants determined by initial conditions.

Thermodynamics

0th law: If two objects are in thermal equilibrium with a third object, then all three objects are in thermal equilibrium with each other.

1st law: For any process concerning a given system, the change in internal energy ΔU of that system is equal to the sum of the heat Q transferred to that system and the work W performed on that system.

2nd law:

- **Carnot:** Wherever there exists a difference in temperature, motive power can be produced.
- **Kelvin:** It is impossible for a self-acting machine to convey heat from a colder body to a hotter one.
- **Clausius:** Heat cannot flow from a colder to a hotter body without another process occurring, connected therewith, simultaneously.

$$T = \left(\frac{\partial U}{\partial S}\right)_{V,N}$$

Energy per mode: $\langle E_{\text{mode}}\rangle = \frac{3}{2}k_B T$

$$Q = C\Delta T, \quad Q = \int_{T_1}^{T_2} C(T)\,dT$$

$$L = \frac{Q_{\text{latent}}}{m}, \quad \gamma = \frac{C_P}{C_V}, \quad dS = \frac{\delta Q_{\text{rev}}}{T}$$

Electrostatics and dynamics

$$\mathbf{F} = \sum_{i=1}^{N} \frac{q_0 q_i (\mathbf{r} - \mathbf{r_i})}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r_i}|^3}$$

Torque: $\tau = \mathbf{p} \times \mathbf{E}$

Energy of a dipole: $U(\theta) = -\mathbf{p} \cdot \mathbf{E}$

Gauss' law: $\phi = \oiint_S \mathbf{E} \cdot d\mathbf{A}$

Potential: $\Delta V \equiv \frac{\Delta U}{q} = -\int_C \mathbf{E} \cdot d\mathbf{l}$

Energy of a capacitor: $U = \frac{Q^2}{2C}$

Current: $I = \dot{Q}$

Potential: $V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$

Kirchhoff's rules: 1. $\sum_{j,\text{loop}} \Delta V_j = 0$ 2. $\sum_j I_{j,\text{into node}} = 0$

Magnetic force: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$

Cyclotron radius: $r = \frac{mv}{qB}$

Biot-Savart: $\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{q\mathbf{v} \times \hat{r}}{r^2}$

Faraday's Law: $\mathcal{E} = -\frac{d\phi_m}{dt}$

Self-inductance of a solenoid: $L = \mu_0 n^2 A l$

Mutual inductance: $\frac{\phi_{m1}}{N_1} = \frac{\phi_{m2}}{N_2}$

Impedance: $Z_R = R$, $Z_C = \frac{1}{i\omega C}$, $Z_L = i\omega L$

Waves

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \qquad \text{(Gauss's Law for Electricity)} \tag{1}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \qquad \text{(Gauss's Law for Magnetism)} \tag{2}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's Law)} \tag{3}$$

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0\epsilon_0 \frac{d\Phi_E}{dt} \qquad \text{(Ampère's Law with Maxwell's addition)} \tag{4}$$

In electromagnetic waves, the ratio $B_0 = \frac{E_0}{c}$ holds.

Wavenumber: $\omega = vk$

Compton wavelength: $\lambda_c = \frac{h}{m_e c}$

De Broglie wavelength: $\lambda_{\text{dB}} = \frac{h}{p}$

Heisenberg uncertainty relation: $\Delta x \Delta p \geq \frac{h}{4\pi}$

Energy of a particle in a 1D box: $E_n = \frac{h^2 n^2}{8L^2 m}$

Quantum Mechanics

Time-dependent Schrodinger's Equation : $i\hbar\frac{\partial}{\partial t}\Psi(\vec{x}, t) = [-\frac{\hbar^2}{2m}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) + V(x)]$

Special relativity

Postulates of relativity and inertial reference frames:
1: Absolute uniform motion cannot be detected.
2: The speed of light in a vacuum is independent of the motion of the source.

Doppler Shift

Non-relativistic Doppler Shift:

$$f' = f\left(\frac{c \pm v_{\text{observer}}}{c \pm v_{\text{source}}}\right) \quad \text{(for sound or slow-moving sources)} \tag{5}$$

Relativistic Doppler Shift:

$$f' = f\sqrt{\frac{1 + \beta}{1 - \beta}} \quad \text{(for motion towards the observer)} \tag{6}$$

$$f' = f\sqrt{\frac{1 - \beta}{1 + \beta}} \quad \text{(for motion away from the observer)} \tag{7}$$

where $\beta = \frac{v_{\text{source}}}{c}$.

Velocity Transformations in Special Relativity

For two observers in relative motion with velocity v along the x-axis:

$$u'_x = \frac{u_x + v}{1 + \frac{vu_x}{c^2}} \tag{8}$$

$$u'_y = \frac{u_y}{\gamma(1 + \frac{vu_x}{c^2})} \tag{9}$$

$$u'_z = \frac{u_z}{\gamma(1 + \frac{vu_x}{c^2})} \tag{10}$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ is the Lorentz factor.

Energy

$E_{\text{total}} = \gamma mc^2 = \sqrt{p^2 c^2 + m^2 c^4}$, $E_{\text{rest}} = mc^2$

Mathematical equations

Trigonometric functions:

$$\int \sin^n ax\,dx = -\frac{1}{a}\cos ax \,\, {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}; \cos^2 ax\right]$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax + C \tag{11}$$

$$\int x\sin^2 ax\,dx = \frac{x^2}{4} - \frac{x}{4a}\sin 2ax - \frac{1}{8a^2}\cos 2ax + C \tag{12}$$

$$\int x^2\sin^2 ax\,dx = \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3}\right)\sin 2ax - \frac{x}{4a^2}\cos 2ax + C \tag{13}$$

$$\int \tan ax\,dx = -\frac{1}{a}\ln|\cos ax| + C = \frac{1}{a}\ln|\sec ax| + C \tag{14}$$

$$\int \frac{\cos ax}{x}\,dx = \ln|ax| + \sum_1^\infty (-)^k \frac{(ax)^{2k}}{2k(2k)!} + C \tag{15}$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax + C \tag{16}$$

$$\int \sin^3 ax\,dx = \frac{\cos 3ax}{12a} - \frac{3\cos ax}{4a} + C \tag{17}$$

$$\int \tan^2 x\, dx = \tan x - x + C \tag{18}$$

$$\int \sin ax \cos ax\, dx = -\frac{\cos^2 ax}{2a} + C \tag{19}$$

$$\int x \cos ax\, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a} + C \tag{20}$$

$$\int \cos ax\, dx = \frac{1}{a} \sin ax + C \tag{21}$$

$$\int x \sin ax\, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + C \tag{22}$$

$$\int (\sin ax)(\cos^n ax)\, dx = -\frac{1}{a(n+1)} \cos^{n+1} ax + C \tag{23}$$

Exponential functions:

$$\int_{-\infty}^{\infty} e^{-ax^2}\, dx = \frac{\sqrt{\pi}}{\sqrt{a}} \ (a > 0) \tag{24}$$

$$\int_{-\infty}^{\infty} x e^{-ax^2 + bx}\, dx = \frac{\sqrt{\pi} b}{2a^{3/2}} e^{\frac{b^2}{4a}} \ (\Re(a) > 0) \tag{25}$$

$$\int_{-\infty}^{\infty} x^n e^{-ax}\, dx = \begin{cases} \dfrac{\Gamma(n+1)}{a^{n+1}} \ (n > -1, a > 0) \\ \dfrac{n!}{a^{n+1}} \ (n = 0, 1, 2, ..., a > 0) \end{cases} \tag{26}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}} \ (a > 0) \tag{27}$$

$$\int x e^{cx}\, dx = \left(\frac{x}{c} - \frac{1}{c^2}\right) e^{cx} \tag{28}$$

$$\int x^2 e^{cx}\, dx = \left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right) e^{cx} \tag{29}$$

$$\int x^4 e^{-ax^2}\, dx = \sqrt{\frac{\pi}{a}} \frac{3}{4a^2} \tag{30}$$

Spherical coordinates

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\phi$$

Volume fraction:

$$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

Solid angle:

$$d\Omega = \frac{dS_r}{r^2} = \sin\theta\, d\theta\, d\phi$$

Surface element:

$$dS_r = r^2 \sin\theta\, d\theta\, d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{1}{\sin\theta}\frac{\partial f}{\partial\phi}\vec{\phi} \tag{31}$$

$$\text{div } \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi}. \tag{32}$$

$$\nabla \times \mathbf{F} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right)\vec{r}$$
$$+ \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right)\vec{\theta} \tag{33}$$
$$+ \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}\right)\vec{\phi}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2} =$$
$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}f \tag{34}$$

Inner product and expectation

Expectation value (discrete)

$$\langle f_i \rangle = \sum_i P_i f_i$$

Expectation value (continuous)

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x) P(x)\, dx$$

$$\langle \hat{O} \rangle = \int \psi^*(\mathbf{r}) \hat{O} \psi(\mathbf{r})\, d^3 r$$

Inner product

$$\langle \psi | \phi \rangle = \int \psi^*(x) \phi(x)\, dx$$

Variance

$$\sigma_f^2 = \langle f^2 \rangle - \langle f \rangle^2$$

Periodic Table of Elements

Group																	
1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8 VIIIB	9 VIIIB	10 VIIIB	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA

Period 1: H (1, 1.0079); He (2, 4.0025)

Period 2: Li (3, 6.941); Be (4, 9.0122); B (5, 10.811); C (6, 12.011); N (7, 14.007); O (8, 15.999); F (9, 18.998); Ne (10, 20.180)

Period 3: Na (11, 22.990); Mg (12, 24.305); Al (13, 26.982); Si (14, 28.086); P (15, 30.974); S (16, 32.065); Cl (17, 35.453); Ar (18, 39.948)

Period 4: K (19, 39.098); Ca (20, 40.078); Sc (21, 44.956); Ti (22, 47.867); V (23, 50.942); Cr (24, 51.996); Mn (25, 54.938); Fe (26, 55.845); Co (27, 58.933); Ni (28, 58.693); Cu (29, 63.546); Zn (30, 65.39); Ga (31, 69.723); Ge (32, 72.64); As (33, 74.922); Se (34, 78.96); Br (35, 79.904); Kr (36, 83.8)

Period 5: Rb (37, 85.468); Sr (38, 87.62); Y (39, 88.906); Zr (40, 91.224); Nb (41, 92.906); Mo (42, 95.94); Tc (43, 96); Ru (44, 101.07); Rh (45, 102.91); Pd (46, 106.42); Ag (47, 107.87); Cd (48, 112.41); In (49, 114.82); Sn (50, 118.71); Sb (51, 121.76); Te (52, 127.6); I (53, 126.9); Xe (54, 131.29)

Period 6: Cs (55, 132.91); Ba (56, 137.33); La-Lu (57-71); Hf (72, 178.49); Ta (73, 180.95); W (74, 183.84); Re (75, 186.21); Os (76, 190.23); Ir (77, 192.22); Pt (78, 195.08); Au (79, 196.97); Hg (80, 200.59); Tl (81, 204.38); Pb (82, 207.2); Bi (83, 208.98); Po (84, 209); At (85, 210); Rn (86, 222)

Period 7: Fr (87, 223); Ra (88, 226); Ac-Lr (89-103); Rf (104, 261); Db (105, 262); Sg (106, 266); Bh (107, 264); Hs (108, 269); Mt (109, 268); Ds (110, 281); Rg (111, 280); Uub (112, 285); Uut (113, 284); Uuq (114, 289); Uup (115, 288); Uuh (116, 293); Uus (117, 292); Uuo (118, 294)

Lanthanides (57-71): La (57, 138.91); Ce (58, 140.12); Pr (59, 140.91); Nd (60, 144.24); Pm (61, 145); Sm (62, 150.36); Eu (63, 151.96); Gd (64, 157.25); Tb (65, 158.93); Dy (66, 162.50); Ho (67, 164.93); Er (68, 167.26); Tm (69, 168.93); Yb (70, 173.04); Lu (71, 174.97)

Actinides (89-103): Ac (89, 227); Th (90, 232.04); Pa (91, 231.04); U (92, 238.03); Np (93, 237); Pu (94, 244); Am (95, 243); Cm (96, 247); Bk (97, 247); Cf (98, 251); Es (99, 252); Fm (100, 257); Md (101, 258); No (102, 259); Lr (103, 262)

Legend: Z (mass); Symbol; Name. Categories: Alkali Metal, Alkaline Earth Metal, Metalloid, Non-metal, Halogen, Noble Gas, Metal, Lanthanide/Actinide. black: natural; gray: man-made.