Physics Formula Sheet

402-0023-01L Physics

2023/ 2024

Constants

Constant	Symbol	Value
Speed of light	c	3.00×10^{8} m/s
Gravitational constant	G	6.674×10^{-11} N(m/kg)^2
Planck's constant	h	6.626×10^{-34} J.s
Mass of the electron	m_e	9.10939×10^{-31} kg
Mass of the proton	m_p	1.67262×10^{-27} kg
Charge of the electron	$-e$	-1.60218×10^{-19} C
Permittivity of free space	ϵ_0	8.85419×10^{-12} C^2/J m
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T m / A
Boltzmann constant	k_B	1.38066×10^{-23} J/ K
Avogadro's constant	N_A	6.022×10^{23} 1/mol

Oscillations

- **Natural Frequency**: $\sqrt{\frac{k}{m}}$
- **Damping Ratio** (ζ):

$$\zeta = \frac{b}{b_c}$$

where $b_c = 2\sqrt{mk}$

Quality Factor (Q factor)

The Q factor is a dimensionless parameter that describes the damping of an oscillator. It represents the energy stored to energy dissipated ratio.

$$Q = \frac{1}{2\zeta} = \frac{\omega_0}{\Delta\omega} = 2\pi f \times \frac{\text{energy stored}}{\text{power loss}}$$

where $\Delta\omega$ is the bandwidth over which the energy is stored.

Types of Oscillations

- **Critically Damped** ($\zeta = 1$): The system returns to equilibrium as quickly as possible without oscillating.
- **Overdamped** ($\zeta > 1$): The system returns to equilibrium without oscillating but slower than the critically damped case.
- **Underdamped** ($\zeta < 1$): The system oscillates about the equilibrium position with a frequency ω_d given by:

$$\omega_d = \omega_0\sqrt{1-\zeta^2}$$

General Solution

For a driven damped harmonic oscillator, the general solution can be expressed as:

$$x(t) = e^{-\zeta\omega_0 t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right)$$

where A and B are constants determined by initial conditions.

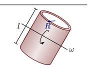
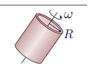
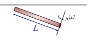
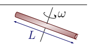
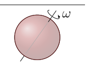
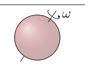
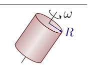
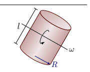
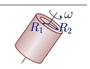
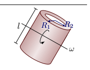
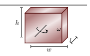
Moments of Inertia

Parallel axis theorem

$$I_O = I_{cg} + md^2 \tag{1}$$

Perpendicular axis Theorem:

$$I_z = I_x + I_y \tag{2}$$

Object	**Axis**	**Moment of Inertia (I)**	
Thin cylindrical shell	Diameter through centre	$\frac{1}{2}mr^2 + \frac{1}{12}ml^2$	
Thin cylindrical shell	Axis	mr^2	
Thin rod	End	$\frac{1}{3}ml^2$	
Thin rod	Centre	$\frac{1}{12}ml^2$	
Spherical shell	Centre	$\frac{2}{3}mr^2$	
Solid sphere	Centre	$\frac{2}{5}mr^2$	
Solid cylinder	Axis	$\frac{1}{2}mr^2$	
Solid cylinder	Diameter through the centre	$\frac{1}{4}mr^2 + \frac{1}{12}ml^2$	
Hollow cylinder	Axis	$\frac{1}{2}m(r_1^2 + r_2^2)$	
Hollow cylinder	Diameter through centre	$\frac{1}{4}m(r_1^2 + r_2^2) + \frac{1}{12}ml^2$	
Rectangular parallelpiped	Through centre, perpendicular to sides	$\frac{1}{12}m(h^2 + w^2)$	

Thermodynamics

0th law: If two objects are in thermal equilibrium with a third object, then all three objects are in thermal equilibrium with each other.

1st law: For any process concerning a given system, the change in internal energy ΔU of that system is equal to the sum of the heat Q transferred to that system and the work W performed on that system.

2nd law:

- **Carnot:** Wherever there exists a difference in temperature, motive power can be produced.
- **Kelvin:** It is impossible for a self-acting machine to convey heat from a colder body to a hotter one.
- **Clausius:** Heat cannot flow from a colder to a hotter body without another process occurring, connected therewith, simultaneously.

$$T = \left(\frac{\partial U}{\partial S}\right)_{V,N}$$

Energy per mode: $\langle E_{\text{mode}}\rangle = \frac{3}{2}k_B T$

$$Q = C\Delta T, \quad Q = \int_{T_1}^{T_2} C(T)\,dT$$

$$L = \frac{Q_{\text{latent}}}{m}, \quad \gamma = \frac{C_P}{C_V}, \quad dS = \frac{\delta Q_{\text{rev}}}{T}$$

Electrostatics and dynamics

$$\mathbf{F} = \sum_{i=1}^{N} \frac{q_0 q_i(\mathbf{r} - \mathbf{r_i})}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r_i}|^3}$$

Torque: $\tau = \mathbf{p} \times \mathbf{E}$

Energy of a dipole: $U(\theta) = -\mathbf{p}\cdot\mathbf{E}$

Gauss' law: $\phi = \oint_S \mathbf{E}\cdot d\mathbf{A}$

Potential: $\Delta V \equiv \frac{\Delta U}{q} = -\int_C \mathbf{E}\cdot d\mathbf{l}$

Energy of a capacitor: $U = \frac{Q^2}{2C}$

Current: $I = \dot{Q}$

Potential: $V_b - V_a = -\int_a^b \mathbf{E}\cdot d\mathbf{l}$

Kirchhoff's rules: 1. $\sum_{j,\text{loop}} \Delta V_j = 0$ 2. $\sum_j I_{j,\text{into node}} = 0$

Magnetic force: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$

Cyclotron radius: $r = \frac{mv}{qB}$

Biot-Savart: $\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{q\mathbf{v}\times\hat{r}}{r^2}$

Faraday's Law: $\mathcal{E} = -\frac{d\phi_m}{dt}$

Self-inductance of a solenoid: $L = \mu_0 n^2 A l$

Mutual inductance: $\frac{\phi_{m1}}{N_1} = \frac{\phi_{m2}}{N_2}$

Impedance: $Z_R = R$, $Z_C = \frac{1}{i\omega C}$, $Z_L = i\omega L$

Waves

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad \text{(Gauss's Law for Electricity)} \tag{3}$$

$$\oint_S \mathbf{B}\cdot d\mathbf{A} = 0 \quad \text{(Gauss's Law for Magnetism)} \tag{4}$$

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's Law)} \tag{5}$$

$$\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0\epsilon_0\frac{d\Phi_E}{dt} \quad \text{(Ampère's Law with Maxwell's addition)} \tag{6}$$

In electromagnetic waves, the ratio $B_0 = \frac{E_0}{c}$ holds.

Wavenumber: $\omega = vk$

Compton wavelength: $\lambda_c = \frac{h}{m_e c}$

De Broglie wavelength: $\lambda_{\text{dB}} = \frac{h}{p}$

Heisenberg uncertainty relation: $\Delta x\Delta p \geq \frac{h}{4\pi}$

Energy of a particle in a 1D box: $E_n = \frac{h^2 n^2}{8L^2 m}$

Quantum Mechanics

$$\text{Time-dependent Schrodinger's Equation} : i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t) = [-\frac{\hbar^2}{2m}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) + V(x)]$$

Special relativity

Postulates of relativity and inertial reference frames:
1: Absolute uniform motion cannot be detected.
2: The speed of light in a vacuum is independent of the motion of the source.

Doppler Shift

Non-relativistic Doppler Shift:

$$f' = f\left(\frac{c \pm v_{\text{observer}}}{c \pm v_{\text{source}}}\right) \quad \text{(for sound or slow-moving sources)} \tag{7}$$

Relativistic Doppler Shift:

$$f' = f\sqrt{\frac{1+\beta}{1-\beta}} \quad \text{(for motion towards the observer)} \tag{8}$$

$$f' = f\sqrt{\frac{1-\beta}{1+\beta}} \quad \text{(for motion away from the observer)} \tag{9}$$

where $\beta = \frac{v_{\text{source}}}{c}$.

Velocity Transformations in Special Relativity

For two observers in relative motion with velocity v along the x-axis:

$$u'_x = \frac{u_x + v}{1 + \frac{vu_x}{c^2}} \tag{10}$$
$$u'_y = \frac{u_y}{\gamma(1 + \frac{vu_x}{c^2})} \tag{11}$$
$$u'_z = \frac{u_z}{\gamma(1 + \frac{vu_x}{c^2})} \tag{12}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor.

Energy

$E_{\text{total}} = \gamma mc^2 = \sqrt{p^2c^2 + m^2c^4}, E_{\text{rest}} = mc^2$

Mathematical equations

Trigonometric functions:

$$\int \sin^n ax dx = -\frac{1}{a}\cos ax \ {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$
$$\int \sin^2 ax dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax + C \tag{13}$$
$$\int x\sin^2 ax dx = \frac{x^2}{4} - \frac{x}{4a}\sin 2ax - \frac{1}{8a^2}\cos 2ax + C \tag{14}$$
$$\int x^2\sin^2 xax dx = \frac{x^3}{6} - (\frac{x^2}{4a} - \frac{1}{8a^3})\sin 2ax - \frac{x}{4a^2}\cos 2ax + C \tag{15}$$
$$\int \tan ax dx = -\frac{1}{a}\ln|\cos ax| + C = \frac{1}{a}\ln|\sec ax| + C \tag{16}$$
$$\int \frac{\cos ax}{x} dx = \ln|ax| + \sum_1^\infty (-)^k \frac{(ax)^{2k}}{2k(2k)!} + C \tag{17}$$
$$\int \cos^2 ax dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax + C \tag{18}$$
$$\int \sin^3 ax dx = \frac{\cos 3ax}{12a} - \frac{3\cos ax}{4a} + C \tag{19}$$
$$\int \tan^2 x dx = \tan x - x + C \tag{20}$$
$$\int \sin ax \cos ax dx = -\frac{\cos^2 ax}{2a} + C \tag{21}$$
$$\int x\cos ax dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a} + C \tag{22}$$
$$\int \cos ax dx = \frac{1}{a}\sin ax + C \tag{23}$$
$$\int x\sin ax dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a} + C \tag{24}$$
$$\int (\sin ax)(\cos^n ax) dx = -\frac{1}{a(n+1)}\cos^{n+1} ax + C \tag{25}$$

Exponential functions:

$$\int_{-\infty}^{\infty} e^{-ax^2} \mathrm{d}x = \frac{\sqrt{\pi}}{\sqrt{a}} \ (a > 0) \tag{26}$$
$$\int_{-\infty}^{\infty} xe^{-ax^2+bx} \mathrm{d}x = \frac{\sqrt{\pi}b}{2a^{3/2}} e^{\frac{b^2}{4a}} \ (\Re(a) > 0) \tag{27}$$
$$\int_{-\infty}^{\infty} x^n e^{-ax} \mathrm{d}x = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}} \ (n > -1, a > 0) \\ \frac{n!}{a^{n+1}} \ (n = 0, 1, 2, ..., a > 0) \end{cases} \tag{28}$$
$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} \, dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}} \ (a > 0) \tag{29}$$
$$\int xe^{cx} dx = \left(\frac{x}{c} - \frac{1}{c^2}\right) e^{cx} \tag{30}$$
$$\int x^2 e^{cx} dx = \left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right) e^{cx} \tag{31}$$
$$\int x^4 e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \frac{3}{4a^2} \tag{32}$$

Spherical coordinates

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\phi$$

Volume fraction:

$$dV = r^2\sin\theta dr d\theta d\phi$$

Solid angle:

$$d\Omega = \frac{dS_r}{r^2} = \sin\theta d\theta d\phi$$

Surface element:

$$dS_r = r^2\sin\theta d\theta d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\vec{\phi} \tag{33}$$

$$\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi}. \tag{34}$$

$$\begin{aligned} \nabla\times\mathbf{F} = {} & \frac{1}{r\sin\theta}(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi})\vec{r} \\ & + \frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi))\vec{\theta} \\ & + \frac{1}{r}(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi})\vec{\phi} \end{aligned} \tag{35}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2} = (\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r})f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}f \tag{36}$$

Inner product and expectation

Expectation value (discrete)

$\langle f_i\rangle = \sum_i P_i f_i$

Expectation value (continuous)

$\langle f(x)\rangle = \int_{-\infty}^{\infty} f(x)P(x)\,dx$

$\langle\hat{O}\rangle = \int\psi^*(\mathbf{r})\hat{O}\psi(\mathbf{r})\,d^3r$

Inner product

$\langle\psi|\phi\rangle = \int\psi^*(x)\phi(x)\,dx$

Variance

$\sigma_f^2 = \langle f^2\rangle - \langle f\rangle^2$