Physics Formula Sheet

402-0023-01L Physics

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Constants

Constant	Symbol	Value
Speed of light	c	3.00×10^8 m/s
Gravitational constant	G	6.674×10^{-11} N(m/kg)^2
Planck's constant	h	6.626×10^{-34} J.s
Mass of the electron	m_e	9.10939×10^{-31} kg
Mass of the proton	m_p	1.67262×10^{-27} kg
Charge of the electron	$-e$	-1.60218×10^{-19} C
Permittivity of free space	ϵ_0	8.85419×10^{-12} C^2/J m
Boltzmann constant	k_B	1.38066×10^{-23} J/ K
Avogadro's constant	N_A	6.022×10^{23} 1/mol

Classical Physics

Title	**Equation**
Bragg's Reflection	$n\lambda = 2d\sin(\theta)$
Diffraction (Single Slit)	$\lambda = d\sin(\theta)$
Young's Double Slit	$\frac{\Delta x}{L} = \frac{\lambda}{d} \approxeq \sin\theta$
Heat Transfer (Fourier's Law)	$Q = mC_v\Delta T$
Continuity Equation	$\nabla \cdot J = -\frac{d\rho}{dt}$
Force of Gravity	$F = G\frac{m_1 m_2}{r^2}$
Coulomb Force	$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$
Special Relativity (Time Dilation)	$E^2 = (pc)^2 + (m_0 c^2)^2$

Nuclear and magnetic physics

Magnetic Field	$E_B = -\mu B,$
	$\mu = \frac{e}{2m}L$
	$F_z = -\frac{\partial V}{\partial z} = \mu\frac{\partial B}{\partial z}$
Rigid rotator	$E_{\text{rot}} = \frac{\mathbf{L}^2}{2I}$
	$I = \frac{m_1 m_2}{m_1+m_2}R^2$
Radioactive decay	$N(t) = N(0)\exp^{-\lambda t} = N(0)(\frac{1}{2})^{t/\tau_{1/2}}$
	$\tau_{1/2} = \ln(2)/\lambda$

Thermodynamics

0th law: If two objects are in thermal equilibrium with a third object, then all three objects are in thermal equilibrium with each other.
1st law: For any process concerning a given system, the change in internal energy ΔU of that system is equal to the sum of the heat Q transferred to that system and the work W performed on that system, or:
2nd law: Carnot: Wherever there exists a difference in temperature, motive power can be produced.

Kelvin: It is impossible for a self acting machine to convey heat from a colder body to a hotter.
Clausius: Heat cannot flow from a colder to hotter body without another process occurring, connected therewith, simultaneously.
$T = (\frac{\partial U}{\partial S})_{\text{V, N}}$

Energy per mode: $< E_{\text{mode}} >= \frac{3}{2}k_b T$
$Q = C\Delta T$ $Q = \int_{T_1}^{T_2} C(T)dT$
$L = \frac{Q_{\text{latent}}}{m}$
$\gamma = \frac{C_P}{C_V}$ $dS = \frac{\delta Q_{\text{rev}}}{T}$

Electrostatics and dynamics

$\vec{F} = \sum_{i=1}^{N} \frac{q_0 q_i(\vec{r}-\vec{r_i})}{4\pi\epsilon_0|\vec{r}-\vec{r_i}|^3}$ Torque: $\vec{\tau} = \vec{p} \times \vec{E}$
Energy of a dipole: $U(\theta) = -\vec{p}\cdot\vec{E}$
Gauss' law: $\phi = \oiint_S \mathbf{E}\cdot d\mathbf{A}$
Potential: $\Delta V \equiv \frac{\Delta U}{q} = -\int_C \vec{E}\cdot d\vec{l}$ Energy of a capacitor: $U = \frac{Q^2}{2C}$
Current: $I = \dot{Q}$
Potential: $V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$
Kirchhoff's rules: 1: $\sum_{j,\text{loop}} \Delta V_j$
2: $\sum_j I_{j,\text{into node}} = 0$
Magnetic force: $\vec{F} = q\vec{v} \times \vec{B}$

Black body:

$$D(k)dk = \frac{\partial N(k)}{\partial k}\frac{dk}{V} = \frac{k^2}{\pi^2}dk$$
$$D(\omega)d\omega = \frac{\omega^2}{\pi^2 c^3}d\omega$$
$$u(\omega)d\omega = \frac{\omega^2}{\pi^2 c^3}k_B T d\omega \text{ classical limit}$$
$$u(\omega)d\omega = \frac{\hbar\omega^3}{\pi^2 c^3}\frac{1}{\exp(\frac{\hbar\omega}{k_B T}) - 1}d\omega$$
$$I(\omega) = cu(\omega)d\omega$$

Quantum Mechanics

$$\text{Time-dependent Schrodinger's Equation}: i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t) = [-\frac{\hbar^2}{2m}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) + V(x)]$$
$$\text{Energy of a photon}: E = hf$$

$$\text{Time-independent Schrodinger's Equation}: E\phi = \hat{H}\phi = \Big(-\frac{\hbar^2}{2m}\nabla^2 + V(x)\Big)\cdot\phi$$
$$\text{Energy of a photon}: E = hf$$

$$\text{Infinite potential well}: E_n = \frac{\hbar^2}{2m}k_n^2 = \frac{\hbar^2\pi^2 n^2}{2mL^2} = n^2 E_0,\ \ \psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L}),\ \ E_0 = \frac{\hbar^2\pi^2}{2mL^2}$$

$$\text{Transmission through a barrier}: T = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2\sinh^2[\sqrt{2m(V_0 - E)}\frac{l}{\hbar}]}$$

$$T \approx \frac{16E(V_0 - E)}{V_0^2}e^{-2\rho_2 l},\ \ \text{with}\rho_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}},\ \ \rho_2\cdot l >> 1$$

$$\text{De Broglie wavelength}: \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

$$\text{Photoelectric effect}: h\nu - \phi_0 = \frac{1}{2}mv^2 = eV$$

$$\text{Bohr-Sommerfeldt condition}: \oint_C \mathbf{p}\cdot d\mathbf{s} = nh,\ \ 2\pi r = nh(\text{circular orbit})$$

$$\text{Probability current}: j = \frac{\hbar}{2mi}(\psi^*\frac{\partial\Psi}{\partial x} - \Psi\frac{\partial\Psi^*}{\partial x})$$

$$\text{Compton scattering}: \lambda_2 - \lambda_1 = \frac{h}{m_0 c}(1 - \cos\theta)$$
$$\mathbf{p}_{h\nu 1} = \mathbf{p}_{h\nu 2} + \mathbf{p}_e$$
$$hv_1 + m_0 c^2 = h\nu_2 + \sqrt{m_0^2 c^4 + p_e^2 c^2}$$

Mathematical equations

Trigonometric functions:

$$\int \sin^n ax dx = -\frac{1}{a}\cos ax\ {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$
$$\int \sin^2 ax dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax + C \tag{1}$$
$$\int x\sin^2 ax dx = \frac{x^2}{4} - \frac{x}{4a}\sin 2ax - \frac{1}{8a^2}\cos 2ax + C \tag{2}$$
$$\int x^2\sin^2 xax dx = \frac{x^3}{6} - (\frac{x^2}{4a} - \frac{1}{8a^3})\sin 2ax - \frac{x}{4a^2}\cos 2ax + C \tag{3}$$
$$\int \tan ax dx = -\frac{1}{a}\ln|\cos ax| + C = \frac{1}{a}\ln|\sec ax| + C \tag{4}$$
$$\int \frac{\cos ax}{x}dx = \ln|ax| + \sum_1^\infty (-)^k\frac{(ax)^{2k}}{2k(2k)!} + C \tag{5}$$
$$\int \cos^2 ax dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax + C \tag{6}$$
$$\int \sin^3 ax dx = \frac{\cos 3ax}{12a} - \frac{3\cos ax}{4a} + C \tag{7}$$
$$\int \tan^2 x dx = \tan x - x + C \tag{8}$$

$$\int \sin ax\cos ax dx = -\frac{\cos^2 ax}{2a} + C \tag{9}$$
$$\int x\cos ax dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a} + C \tag{10}$$
$$\int \cos ax dx = \frac{1}{a}\sin ax + C \tag{11}$$
$$\int x\sin ax dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a} + C \tag{12}$$
$$\int (\sin ax)(\cos^n ax)dx = -\frac{1}{a(n+1)}\cos^{n+1} ax + C \tag{13}$$

Exponential functions:

$$\int_{-\infty}^{\infty} e^{-ax^2}\mathrm{d}x = \frac{\sqrt{\pi}}{\sqrt{a}}\ (a > 0) \tag{14}$$
$$\int_{-\infty}^{\infty} xe^{-ax^2+bx}\mathrm{d}x = \frac{\sqrt{\pi}b}{2a^{3/2}}e^{\frac{b^2}{4a}}\ (\Re(a) > 0) \tag{15}$$
$$\int_{-\infty}^{\infty} x^n e^{-ax}\mathrm{d}x = \begin{cases} \dfrac{\Gamma(n+1)}{a^{n+1}}\ (n > -1, a > 0) \\ \dfrac{n!}{a^{n+1}}\ (n = 0, 1, 2, ..., a > 0) \end{cases} \tag{16}$$
$$\int_{-\infty}^{\infty} x^2 e^{-ax^2}\ dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}\ (a > 0) \tag{17}$$
$$\int xe^{cx}dx = \left(\frac{x}{c} - \frac{1}{c^2}\right)e^{cx} \tag{18}$$
$$\int x^2 e^{cx}dx = \left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right)e^{cx} \tag{19}$$
$$\int x^4 e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}\frac{3}{4a^2} \tag{20}$$

Spherical coordinates

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\phi$$

Volume fraction:

$$dV = r^2\sin\theta dr d\theta d\phi$$

Solid angle:

$$d\Omega = \frac{dS_r}{r^2} = \sin\theta d\theta d\phi$$

Surface element:

$$dS_r = r^2\sin\theta d\theta d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\vec{\phi} \tag{21}$$
$$\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi}. \tag{22}$$

$$\nabla \times \mathbf{F} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right)\vec{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right)\vec{\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}\right)\vec{\phi} \tag{23}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2} = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}f \tag{24}$$

Harmonic oscillator:

First four harmonic oscillator wavefunction	Hermite polynomials	E_n
$\psi_0(\xi) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{1}{2}\xi^2}$	1	$\frac{1}{2}\hbar\omega$
$\psi_1(\xi) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \sqrt{2}\xi e^{-\frac{1}{2}\xi^2}$	$2y$	$\frac{3}{2}\hbar\omega$
$\psi_2(\xi) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2}}(2\xi^2 - 1) e^{-\frac{1}{2}\xi^2}$	$4y^2 - 2$	$\frac{5}{2}\hbar\omega$
$\psi_3(\xi) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \frac{1}{\sqrt{3}}(2\xi^3 - 3\xi) e^{-\frac{1}{2}\xi^2}$	$8y^3 - 12y$	$\frac{7}{2}\hbar\omega$

Harmonic oscillator	$\psi_n(x) = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}(a^\dagger)^n e^{-\frac{1}{2}\frac{m\omega}{\hbar}x^2}\psi_0(x)$
Raising operator	$a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x - ip)$
Lowering operator	$a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$
$a^\dagger\vert n\rangle = \sqrt{n+1}\vert n+1\rangle$	$a\vert n\rangle = \sqrt{n}\vert n-1\rangle$
Number operator	$\hat{N} = a^\dagger a \hat{N}\vert n\rangle = n\vert n\rangle$
Commutation relation	$[a, a^\dagger] = aa^\dagger - a^\dagger a = 1$
Hamiltonian	$\hat{H} = \hbar\omega\left(\hat{N} + \frac{1}{2}\right)$

Inner product and expectation

Expectation value (discrete)

$$\langle f_i \rangle = \sum_i P_i f_i$$

Expectation value (continuous)

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x)P(x)\,dx$$

$$\langle \hat{O} \rangle = \int \psi^*(\mathbf{r})\hat{O}\psi(\mathbf{r})\,d^3r$$

Inner product

$$\langle \psi | \phi \rangle = \int \psi^*(x)\phi(x)\,dx$$

Variance

$$\sigma_f^2 = \langle f^2 \rangle - \langle f \rangle^2$$

Commutation relations

$$[A, B] = AB - BA$$
$$[AB, C] = A[B, C] - [A, C]B$$
$$[x, p_x] = i\hbar$$
$$[y, p_y] = i\hbar$$
$$[x, y] = [x, p_y] = [y, p_x] = 0$$

Hydrogen atom

Fine structure constant:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$$

Bohr radius:

$$a_0 = \frac{\hbar}{m_e c\alpha} \approx 0.529 \times 10^{-10}\mathrm{m}$$

Bohr energy:

$$E_n = -\frac{2\pi^2 k^2 e^4 m_e}{h^2 n^2}$$

Ground state energy:

$$E_1 = -13.6\mathrm{eV}$$

Wave function:

$$\psi_{n\ell m}(r, \theta, \phi) = R_{n\ell}(r)Y_{\ell m}(\theta, \phi)$$

Rydberg formula:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Rydberg constant:

$$R_H \approx 1.097 \times 10^7\mathrm{m}^{-1}$$

Radial wavefunctions:

$$R_{n\ell}(r) = N_{n\ell}r^\ell e^{-\rho/2}L_{n-\ell-1}^{2\ell+1}(\rho)$$

Legendre polynomials

Angular momentum

$$L_+ = L_x + iL_y$$

$$L_- = L_x - iL_y$$

$$L^2 = L_z^2 + \frac{1}{2}(L_+L_- + L_-L_+)$$

$$[L_x, L_y] = i\hbar L_z$$

$$[L^2, L_i] = 0 \quad \text{where } i = x, y, \text{or } z$$

$$L_x = -i\hbar\left(\sin\phi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right),\ L_y = i\hbar\left(\cos\phi\frac{\partial}{\partial\theta} - \cot\theta\sin\phi\frac{\partial}{\partial\phi}\right),\ L_z = -i\hbar\frac{\partial}{\partial\phi}$$

$$L_+ = \hbar e^{i\phi}\left(\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right),\ L_- = \hbar e^{-i\phi}\left(-\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right)$$

$$L^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)$$

Hund's rule

1: All other thing being equal, the state with the highest total spin (S), will have the lowest.
2: For a given spin, the state the highest total orbital angular momentum (L), consistent with overall anti-symmetrization, will have the lowest energy.
3: If a subshell (n, l) is no more than half filled, then the lowest energy level has $J = |L - S|$: if it is more than half filled, then $J = L + S$ has the lowest energy.

Spin

Two particle spin states

$|0, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ s = 0 singlet

$|1, 1\rangle = |\uparrow\uparrow\rangle$
$|1, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, s = 1 triplet
$|1, -1\rangle = |\downarrow\downarrow\rangle$

$$S_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \quad S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad S_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \quad S^2 = \frac{3}{4}\hbar^2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \quad S_+ = \hbar\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix} \quad S_- = \hbar\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$$

Two particle Hamiltonian:

$$\hat{H} = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V(\mathbf{r}_1, \mathbf{r}_2)$$

Hamiltonian with an atom with atomic number Z:

$$\hat{H} = -\frac{\hbar^2}{2m_e}\nabla^2 - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

Clebsch-Gordan coefficients

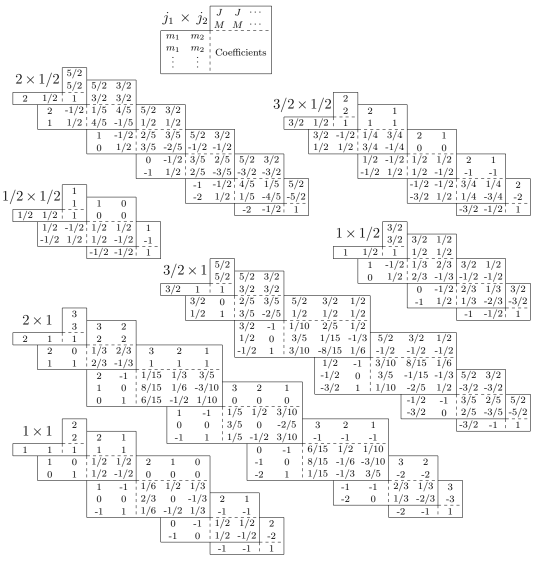

Figure 1: Clebsch-Gordan coefficients. A square root is understood on each coefficient, that is, $-1/3$ means $-\sqrt{1/3}$.

Condensed Matter

Free electron gas:

$$\psi_{n_x,n_y,n_z}(\mathbf{r}) = \sqrt{\frac{8}{V}}\sin\left(\frac{n_x\pi x}{L_x}\right)\sin\left(\frac{n_y\pi y}{L_y}\right)\sin\left(\frac{n_z\pi z}{L_z}\right), \quad E(\mathbf{k}) = \frac{\hbar^2k^2}{2m}$$

Fermi energy of a metal:

$$E_F = \frac{\hbar^2k_F^2}{2m}, \rho \equiv \frac{N_q}{V}, N_q = \text{number of electrons in volume V}, n_c = n_i\exp\left(\frac{E_F - E_i}{k_BT}\right)$$

Density of states:

3D: $g(E) = \frac{Vm}{2\pi^2\hbar^3}\sqrt{2mE}$, 2D: $g(E) = \frac{m}{\pi\hbar^2}$

Distribution functions:

Maxwell-Boltzmann	Fermi-Dirac	Bose-Einstein
$f(E) = e^{-\frac{E-\mu}{k_BT}}$	$f(E) = \frac{1}{e^{\frac{E-\mu}{k_BT}}+1}$	$f(E) = \frac{1}{e^{\frac{E-\mu}{k_BT}}-1}$

First order perturbation theory:

$\Delta E_n^{(1)} = \langle \psi_n^0 | \hat{H}' | \psi_n^0 \rangle$, $\psi_n^{(1)} = \sum_{m \neq n} \frac{\langle \psi_m^0 | \hat{H}' | \psi_n^0 \rangle}{E_n^{(0)} - E_m^{(0)}} \psi_m^{(0)}$