Physics Formula Sheet

402-0023-01L Physics

2023/ 2024

Constants

Constant	Symbol	Value
Speed of light	c	3.00×10^8 m/s
Gravitational constant	G	6.674×10^{-11} N(m/kg)^2
Planck's constant	h	6.626×10^{-34} J.s
Mass of the electron	m_e	9.10939×10^{-31} kg
Mass of the proton	m_p	1.67262×10^{-27} kg
Charge of the electron	$-e$	-1.60218×10^{-19} C
Permittivity of free space	ϵ_0	8.85419×10^{-12} C^2/J m
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T m / A
Boltzmann constant	k_B	1.38066×10^{-23} J/ K
Avogadro's constant	N_A	6.022×10^{23} 1/mol

Oscillations

Hook's Law:

$$F = kx$$

Equation of Motion

Undamped simple harmonic oscillator:

$$m\frac{d^2x}{dt^2} + kx = 0$$

Standard solution:

$$x(t) = A\sin(\omega_0 t + \phi)$$

The equation of motion for a damped simple harmonic oscillator is:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

General Solutions

Light damping: ($b^2 < 4mk$), the general solution is:

$$x(t) = e^{-\frac{b}{2m}t}\left(C_1\cos(\omega_d t) + C_2\sin(\omega_d t)\right)$$

Where:

$$\omega_d = \sqrt{\frac{k^2}{m^2} - \frac{b^2}{4m^2}}$$

Critical camping: ($b^2 = 4mk$), the general solution is:

$$x(t) = e^{-\frac{b}{2m}t}\left(C_1 + C_2 t\right)$$

Heavy camping: ($b^2 > 4mk$), the general solution is:

$$x(t) = e^{-\frac{b}{2m}t}\left(C_1 e^{\lambda_1} + C_2 e^{\lambda_2}\right)$$

where

$$\lambda_{1,2} = -\frac{b}{2m} \pm \sqrt{\frac{b^2}{4m^2} - \frac{k^2}{m^2}}$$

Amplitude of forced oscillations:

$$A = \frac{F_0}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + b^2\omega^2}}$$

Phase of forced oscillations:

$$\phi = \arctan[\frac{b\omega}{m(\omega_0^2 - \omega^2)}]$$

In the limit $\omega << \omega_0$

$$A \to \frac{F_0}{m\omega_0^2}, \phi \to 0$$

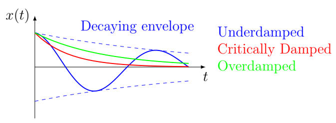
In the limit $\omega >> \omega_0$

$$A \to \frac{F_0}{m\omega^2}, \phi \to \pi$$

When $\omega = \omega_0$

$$A = \frac{F_0}{m\omega_0^2}Q, \phi = \pi/2$$

Graphs



Pendulum

Torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\frac{d^2\theta}{dt^2} + \frac{rMg}{I}\sin\theta = 0$$

$$\omega_0 = \sqrt{\frac{g}{l}}$$

Standard solution (for small angles):

$$A\sin(\omega_0 t + \phi)$$

Q Factor

The quality factor, or Q factor, describes the damping of the system:

$$Q = \frac{1}{\frac{b}{2\sqrt{mk}}} = \frac{\omega_0}{\Delta\omega}$$

A higher Q means the system is less damped.

Moments of Inertia

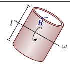
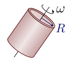
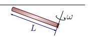
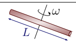
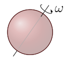
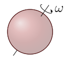
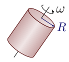
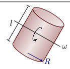
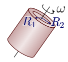
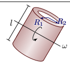
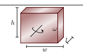
$$I = \int r^2 dm$$

Parallel axis theorem:

$$I_0 = I_{cg} + md^2 \quad (1)$$

Perpendicular axis Theorem:

$$I_z = I_x + I_y \quad (2)$$

Object	Axis	Moment of Inertia (I)	
Thin cylindrical shell	Diameter through centre	$\frac{1}{2}mr^2 + \frac{1}{12}ml^2$	
Thin cylindrical shell	Axis	mr^2	
Thin rod	End	$\frac{1}{3}ml^2$	
Thin rod	Centre	$\frac{1}{12}ml^2$	
Spherical shell	Centre	$\frac{2}{3}mr^2$	
Solid sphere	Centre	$\frac{2}{5}mr^2$	
Solid cylinder	Axis	$\frac{1}{2}mr^2$	
Solid cylinder	Diameter through the centre	$\frac{1}{4}mr^2 + \frac{1}{12}ml^2$	
Hollow cylinder	Axis	$\frac{1}{2}m(r_1^2 + r_2^2)$	
Hollow cylinder	Diameter through centre	$\frac{1}{4}m(r_1^2 + r_2^2) + \frac{1}{12}ml^2$	
Rectangular parallelpiped	Through centre, perpendicular to sides	$\frac{1}{12}m(h^2 + w^2)$	

Thermodynamics

0th law: If two objects are in thermal equilibrium with a third object, then all three objects are in thermal equilibrium with each other.

1st law: For any process concerning a given system, the change in internal energy ΔU of that system is equal to the sum of the heat Q transferred to that system and the work W performed on that system.

$$\Delta U = Q + W$$

,

$$dU = \delta Q + \delta W$$

2nd law:

- **Carnot:** Wherever there exists a difference in temperature, motive power can be produced.
- **Kelvin:** It is impossible for a self-acting machine to convey heat from a colder body to a hotter one.
- **Clausius:** Heat cannot flow from a colder to a hotter body without another process occurring, connected therewith, simultaneously.

Ideal gas law:

$$pV = NkT$$

$$(p + a(\frac{N}{V})^2)(V - bN) = Nk_BT$$

Van der Waals equation of state:

$$T = \left(\frac{\partial U}{\partial S}\right)_{V,N}$$

Average energy per particle for p degrees of freedom:

$$\langle E_{\text{mode}} \rangle = \frac{p}{2}k_BT$$

Heat capacity, C:

$$Q = C\Delta T, \quad Q = \int_{T_1}^{T_2} C(T)\,dT$$

Latent heat, L:

$$L = \frac{Q_{\text{latent}}}{m}$$

Isochoric process:

$$W = -\int PdV = 0$$

$$\Delta Q = mC_V\Delta T$$

Isothermal process:

$$\Delta T = 0$$

$$Q = \Delta U - W = -W = Nk_BT_A\ln(\frac{V_B}{V_A})$$

Adiabatic process:

$$dU = \delta W$$

Adiabatic component:

$$\gamma = \frac{C_P}{C_V}$$

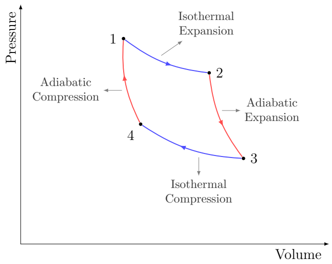
Where P, V indicate at constant pressure/ volume

Polytropic equation for an adiabatic process:

$$pV^\gamma = \text{constant}$$

Efficiencies:

Heat engine: $\epsilon = \frac{W}{Q_H}$ Heat pump: $\epsilon = \frac{Q_H}{W}$ Fridge: $\epsilon = \frac{Q_C}{W}$



Carnot cycle

Electrostatics and dynamics

$$\mathbf{F} = \sum_{i=1}^{N} \frac{q_0 q_i (\mathbf{r} - \mathbf{r_i})}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r_i}|^3}$$

Electric field: $\vec{F} = q\vec{E}$

Torque: $\vec{\tau} = \vec{p} \times \vec{E}$

Energy of a dipole: $U(\theta) = -\vec{p} \cdot \vec{E}$

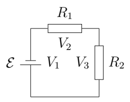
Gauss' law: $\phi = \oiint_S \vec{E} \cdot d\vec{A}$

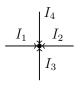
Potential: $\Delta V \equiv \frac{\Delta U}{q} = -\int_C \vec{E} \cdot d\vec{l}$

Energy of a capacitor: $U = \frac{Q^2}{2C}$

Current: $I = \dot{Q}$

Potential: $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$





Kirchhoff's rules: 1. $\sum_{j,\text{loop}} \Delta V_j = 0$ 2. $\sum_j I_{j,\text{into node}} = 0$

Magnetic force: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$

Cyclotron radius: $r = \frac{mv}{qB}$

Biot-Savart: $\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{q\mathbf{v} \times \hat{r}}{r^2}$

Faraday's Law: $\mathcal{E} = -\frac{d\phi_m}{dt}$

Self-inductance of a solenoid: $L = \mu_0 n^2 A l$

Mutual inductance: $\frac{\phi_{m1}}{N_1} = \frac{\phi_{m2}}{N_2}$

Impedance: $Z_R = R$, $Z_C = \frac{1}{i\omega C}$, $Z_L = i\omega L$

Impedance in series and parallel: $Z_{\text{series}} = \sum_{i=1}^{n} Z_i$, $\frac{1}{Z_{\text{parallel}}} = \sum_{i=1}^{n} \frac{1}{Z_i}$

Ampere's Law: $\oint_C \vec{B} \cdot \vec{ds} = \mu_0 I_C$

Waves & Quantum

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \qquad \text{(Gauss's Law for Electricity)} \tag{3}$$
$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \qquad \text{(Gauss's Law for Magnetism)} \tag{4}$$
$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's Law)} \tag{5}$$
$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0\epsilon_0 \frac{d\Phi_E}{dt} \qquad \text{(Ampère's Law with Maxwell's addition)} \tag{6}$$

In electromagnetic waves, the ratio $B_0 = \frac{E_0}{c}$ holds.

Wavenumber: $\omega = vk$

String wave velocity: $v = \sqrt{\frac{F_T}{\mu}}$

String wave power: $P_{\text{ave}} = \frac{1}{2}\mu v \omega^2 A^2$

Pressure wave speed:

Bulk: $v = \sqrt{\frac{B}{\rho}}$, Longitudinal wave: $v = \sqrt{\frac{E}{\rho}}$, Transverse/ shear wave: $v = \sqrt{\frac{G}{\rho}}$

Compton wavelength: $\lambda_c = \frac{h}{m_e c}$

Compton scattering: $\Delta\lambda = \lambda_c(1 - \cos\theta)$

De Broglie wavelength: $\lambda_{\text{dB}} = \frac{h}{p}$

Heisenberg uncertainty relation: $\Delta x \Delta p \geq \frac{h}{4\pi}$

Energy of a particle in a 1D box: $E_n = \frac{h^2 n^2}{8L^2 m}$

Energy of a photon: $h\nu = E_m - E_n$

Time-dependent Schrodinger's Equation : $i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t) = [-\frac{\hbar^2}{2m}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) + V(x)]\Psi(\vec{x},t)$

Special relativity

Postulates of relativity and inertial reference frames:
1: Absolute uniform motion cannot be detected.
2: The speed of light in a vacuum is independent of the motion of the source.

Time dilation: $\Delta t = \gamma \Delta t_0$

Length contraction: $L = \frac{L_0}{\gamma}$

Doppler Shift

Non-relativistic Doppler Shift:

$$f' = f\left(\frac{c \pm v_{\text{observer}}}{c \pm v_{\text{source}}}\right) \quad \text{(for sound or slow-moving sources)} \tag{7}$$

Relativistic Doppler Shift:

$$f' = f\sqrt{\frac{1+\beta}{1-\beta}} \quad \text{(for motion towards the observer)} \tag{8}$$
$$f' = f\sqrt{\frac{1-\beta}{1+\beta}} \quad \text{(for motion away from the observer)} \tag{9}$$

where $\beta = \frac{v_{\text{source}}}{c}$.

Velocity Transformations in Special Relativity

For two observers in relative motion with velocity v along the x-axis:

$$u'_x = \frac{u_x + v}{1 + \frac{vu_x}{c^2}} \tag{10}$$
$$u'_y = \frac{u_y}{\gamma(1 + \frac{vu_x}{c^2})} \tag{11}$$
$$u'_z = \frac{u_z}{\gamma(1 + \frac{vu_x}{c^2})} \tag{12}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor.

Energy

$\text{E}_{\text{total}} = \gamma mc^2 = \sqrt{p^2c^2 + m^2c^4}, \text{E}_{\text{rest}} = mc^2$

Spherical coordinates

$$x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\phi$$

Volume fraction:

$$dV = r^2 \sin\theta dr d\theta d\phi$$

Solid angle:

$$d\Omega = \frac{dS_r}{r^2} = \sin\theta d\theta d\phi$$

Surface element:

$$dS_r = r^2 \sin\theta d\theta d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\vec{\phi} \tag{13}$$

$$\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi}. \tag{14}$$

$$\begin{aligned}\nabla\times\mathbf{F} = &\frac{1}{r\sin\theta}(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi})\vec{r} \\ &+ \frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi))\vec{\theta} \\ &+ \frac{1}{r}(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi})\vec{\phi}\end{aligned} \tag{15}$$

$$\begin{aligned}\nabla^2 f = &\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2} = \\ &(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r})f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}f\end{aligned} \tag{16}$$

Inner product and expectation

Expectation value (discrete)

$\langle f_i\rangle = \sum_i P_i f_i$

Expectation value (continuous)

$\langle f(x)\rangle = \int_{-\infty}^{\infty} f(x)P(x)\,dx$

$\langle\hat{O}\rangle = \int \psi^*(\mathbf{r})\hat{O}\psi(\mathbf{r})\,d^3r$

Inner product

$\langle\psi|\phi\rangle = \int \psi^*(x)\phi(x)\,dx$

Variance

$\sigma_f^2 = \langle f^2\rangle - \langle f\rangle^2$

Trigonometry and Taylor

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\begin{aligned}\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \ldots \\ &= \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}\end{aligned}$$

Where $x \in \mathbb{R}$.

$$\begin{aligned}\frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + \dots \\ &= \sum_{n=0}^{\infty} x^n\end{aligned}$$

Where $x \in (-1, 1)$.

$$\begin{aligned}e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \\ &= \sum_{n=0}^{\infty} \frac{x^n}{n!}\end{aligned}$$

Where $x \in \mathbb{R}$.

$$\begin{aligned}\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}\end{aligned}$$

Where $x \in \mathbb{R}$.

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \dots$$

Substance	c, kJ/kg·K	c, kcal/kg·K or Btu/lb·F°	c', J/mol·K
Aluminium	0.900	0.215	24.3
Bismuth	0.123	0.0294	25.7
Copper	0.386	0.0923	24.5
Glass	0.840	0.20	—
Gold	0.126	0.0301	25.6
Ice (−10°C)	2.05	0.49	36.9
Lead	0.128	0.0305	26.4
Silver	0.233	0.0558	24.9
Tungsten	0.134	0.0321	24.8
Zinc	0.387	0.0925	25.2
Alcohol (ethyl)	2.4	0.58	111
Mercury	0.140	0.033	28.3
Water	4.18	1.00	75.2
Steam (at 1 atm)	2.02	0.48	36.4

Figure 2

Substance	MP, K	L_f, kJ/kg	BP, K	L_v, kJ/kg
Alcohol, ethyl	159	109	351	879
Bromine	266	67.4	332	369
Carbon dioxide	—	—	194.6*	573*
Copper	1356	205	2839	4726
Gold	1336	62.8	3081	1701
Helium	—	—	4.2	21
Lead	600	24.7	2023	858
Mercury	234	11.3	630	296
Nitrogen	63	25.7	77.35	199
Oxygen	54.4	13.8	90.2	213
Silver	1234	105	2436	2323
Sulfur	388	38.5	717.75	287
Water (liquid)	273.15	333.5	373.15	2257
Zinc	692	102	1184	1768

Figure 3