# Physics Formula Sheet

402-0023-01L Physics

2023/ 2024

## Constants

| Constant | Symbol | Value |
|---|---|---|
| Speed of light | $c$ | $3.00 \times 10^8$ m/s |
| Gravitational constant | $G$ | $6.674 \times 10^{-11}$ $\text{N(m/kg)}^2$ |
| Planck's constant | $h$ | $6.626 \times 10^{-34}$ J.s |
| Mass of the electron | $m_e$ | $9.10939 \times 10^{-31}$ kg |
| Mass of the proton | $m_p$ | $1.67262 \times 10^{-27}$ kg |
| Charge of the electron | $-e$ | $-1.60218 \times 10^{-19}$ C |
| Permittivity of free space | $\epsilon_0$ | $8.85419 \times 10^{-12}$ $\text{C}^2/\text{J m}$ |
| Permeability of free space | $\mu_0$ | $4\pi \times 10^{-7}$ T m / A |
| Boltzmann constant | $k_B$ | $1.38066 \times 10^{-23}$ J/ K |
| Avogadro's constant | $N_A$ | $6.022 \times 10^{23}$ 1/mol |

## Oscillations

- **Natural Frequency**: $\sqrt{\frac{k}{m}}$
- **Damping Ratio** ($\zeta$):
$$\zeta = \frac{b}{b_c}$$
where $b_c = 2\sqrt{mk}$

### Quality Factor (Q factor)

The Q factor is a dimensionless parameter that describes the damping of an oscillator. It represents the energy stored to energy dissipated ratio.

$$Q = \frac{1}{2\zeta} = \frac{\omega_0}{\Delta\omega} = 2\pi f \times \frac{\text{energy stored}}{\text{power loss}}$$

where $\Delta\omega$ is the bandwidth over which the energy is stored.

### Types of Oscillations

- **Critically Damped** ($\zeta = 1$): The system returns to equilibrium as quickly as possible without oscillating.
- **Overdamped** ($\zeta > 1$): The system returns to equilibrium without oscillating but slower than the critically damped case.
- **Underdamped** ($\zeta < 1$): The system oscillates about the equilibrium position with a frequency $\omega_d$ given by:
$$\omega_d = \omega_0\sqrt{1-\zeta^2}$$



### General Solution

For a driven damped harmonic oscillator, the general solution can be expressed as:

$$x(t) = e^{-\zeta\omega_0 t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right)$$

where $A$ and $B$ are constants determined by initial conditions.

### Moments of Inertia

| **Object** | **Axis** | **Moment of Inertia ($I$)** |
|---|---|---|
| Thin cylindrical shell | Diameter through centre | $\frac{1}{2}mr^2 + \frac{1}{12}ml^2$ |
| Thin cylindrical shell | Axis | $mr^2$ |
| Thin rod | End | $\frac{1}{3}ml^2$ |
| Thin rod | Centre | $\frac{1}{12}ml^2$ |
| Spherical shell | Centre | $\frac{2}{3}mr^2$ |
| Solid sphere | Centre | $\frac{2}{5}mr^2$ |
| Solid cylinder | Axis | $\frac{1}{2}mr^2$ |
| Solid cylinder | Diameter through the centre | $\frac{1}{4}mr^2 + \frac{1}{12}ml^2$ |
| Hollow cylinder | Axis | $\frac{1}{2}m(r_1^2 + r_2^2)$ |
| Hollow cylinder | Diameter through centre | $\frac{1}{4}m(r_1^2 + r_2^2) + \frac{1}{12}ml^2$ |
| Rectangular parallelpiped | Through centre, perpendicular to sides | $\frac{1}{12}m(h^2 + w^2)$ |

## Thermodynamics

**0th law:** If two objects are in thermal equilibrium with a third object, then all three objects are in thermal equilibrium with each other.

**1st law:** For any process concerning a given system, the change in internal energy $\Delta U$ of that system is equal to the sum of the heat $Q$ transferred to that system and the work $W$ performed on that system.

**2nd law:**

- **Carnot:** Wherever there exists a difference in temperature, motive power can be produced.
- **Kelvin:** It is impossible for a self-acting machine to convey heat from a colder body to a hotter one.
- **Clausius:** Heat cannot flow from a colder to a hotter body without another process occurring, connected therewith, simultaneously.

$$T = \left(\frac{\partial U}{\partial S}\right)_{V,N}$$

Energy per mode: $\langle E_{\text{mode}}\rangle = \frac{3}{2}k_B T$

$$Q = C\Delta T, \quad Q = \int_{T_1}^{T_2} C(T)\, dT$$

$$L = \frac{Q_{\text{latent}}}{m}, \quad \gamma = \frac{C_P}{C_V}, \quad dS = \frac{\delta Q_{\text{rev}}}{T}$$



### Electrostatics and dynamics

$$\mathbf{F} = \sum_{i=1}^{N} \frac{q_0 q_i(\mathbf{r} - \mathbf{r_i})}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r_i}|^3}$$

Torque: $\tau = \mathbf{p} \times \mathbf{E}$

Energy of a dipole: $U(\theta) = -\mathbf{p} \cdot \mathbf{E}$

Gauss' law: $\phi = \oiint_S \mathbf{E} \cdot d\mathbf{A}$

Potential: $\Delta V \equiv \frac{\Delta U}{q} = -\int_C \mathbf{E} \cdot d\mathbf{l}$

Energy of a capacitor: $U = \frac{Q^2}{2C}$

Current: $I = \dot{Q}$

Potential: $V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$

Kirchhoff's rules: 1. $\sum_{j,\text{loop}} \Delta V_j = 0$ 2. $\sum_j I_{j,\text{into node}} = 0$

Magnetic force: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$

Cyclotron radius: $r = \frac{mv}{qB}$

Biot-Savart: $\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{q\mathbf{v} \times \hat{r}}{r^2}$

Faraday's Law: $\mathcal{E} = -\frac{d\phi_m}{dt}$

Self-inductance of a solenoid: $L = \mu_0 n^2 Al$

Mutual inductance: $\frac{\phi_{m1}}{N_1} = \frac{\phi_{m2}}{N_2}$

Impedance: $Z_R = R$, $Z_C = \frac{1}{i\omega C}$, $Z_L = i\omega L$

### Waves

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad \text{(Gauss's Law for Electricity)} \tag{1}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \quad \text{(Gauss's Law for Magnetism)} \tag{2}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's Law)} \tag{3}$$

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0\epsilon_0\frac{d\Phi_E}{dt} \quad \text{(Ampère's Law with Maxwell's addition)} \tag{4}$$

In electromagnetic waves, the ratio $B_0 = \frac{E_0}{c}$ holds.

Wavenumber: $\omega = vk$

Compton wavelength: $\lambda_c = \frac{h}{m_e c}$

De Broglie wavelength: $\lambda_{\text{dB}} = \frac{h}{p}$

Heisenberg uncertainty relation: $\Delta x \Delta p \geq \frac{h}{4\pi}$

Energy of a particle in a 1D box: $E_n = \frac{h^2 n^2}{8L^2 m}$



## Quantum Mechanics

Time-dependent Schrodinger's Equation : $i\hbar\frac{\partial}{\partial t}\Psi(\vec{x}, t) = [-\frac{\hbar^2}{2m}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) + V(x)]$

## Special relativity

Postulates of relativity and inertial reference frames:
1: Absolute uniform motion cannot be detected.
2: The speed of light in a vacuum is independent of the motion of the source.

### Doppler Shift

**Non-relativistic Doppler Shift:**

$$f' = f\left(\frac{c \pm v_{\text{observer}}}{c \pm v_{\text{source}}}\right) \quad \text{(for sound or slow-moving sources)} \tag{5}$$

**Relativistic Doppler Shift:**

$$f' = f\sqrt{\frac{1+\beta}{1-\beta}} \quad \text{(for motion towards the observer)} \tag{6}$$

$$f' = f\sqrt{\frac{1-\beta}{1+\beta}} \quad \text{(for motion away from the observer)} \tag{7}$$

where $\beta = \frac{v_{\text{source}}}{c}$.

### Velocity Transformations in Special Relativity

For two observers in relative motion with velocity $v$ along the x-axis:

$$u'_x = \frac{u_x + v}{1 + \frac{vu_x}{c^2}} \tag{8}$$

$$u'_y = \frac{u_y}{\gamma(1 + \frac{vu_x}{c^2})} \tag{9}$$

$$u'_z = \frac{u_z}{\gamma(1 + \frac{vu_x}{c^2})} \tag{10}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor.

### Energy

$\text{E}_{\text{total}} = \gamma mc^2 = \sqrt{p^2c^2 + m^2c^4}, \text{E}_{\text{rest}} = mc^2$

## Mathematical equations

### Trigonometric functions:

$$\int \sin^n axdx = -\frac{1}{a}\cos ax \;{}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$

$$\int \sin^2 axdx = \frac{x}{2} - \frac{1}{4a}\sin 2ax + C \tag{11}$$

$$\int x\sin^2 axdx = \frac{x^2}{4} - \frac{x}{4a}\sin 2ax - \frac{1}{8a^2}\cos 2ax + C \tag{12}$$

$$\int x^2\sin^2 xaxdx = \frac{x^3}{6} - (\frac{x^2}{4a} - \frac{1}{8a^3})\sin 2ax - \frac{x}{4a^2}\cos 2ax + C \tag{13}$$

$$\int \tan axdx = -\frac{1}{a}\ln|\cos ax| + C = \frac{1}{a}\ln|\sec ax| + C \tag{14}$$

$$\int \frac{\cos ax}{x}dx = \ln|ax| + \sum_{1}^{\infty}(-)^k\frac{(ax)^{2k}}{2k(2k)!} + C \tag{15}$$

$$\int \cos^2 axdx = \frac{x}{2} + \frac{1}{4a}\sin 2ax + C \tag{16}$$

$$\int \sin^3 axdx = \frac{\cos 3ax}{12a} - \frac{3\cos ax}{4a} + C \tag{17}$$

$$\int \tan^2 xdx = \tan x - x + C \tag{18}$$

$$\int \sin ax\cos axdx = -\frac{\cos^2 ax}{2a} + C \tag{19}$$

$$\int x\cos axdx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a} + C \tag{20}$$

$$\int \cos axdx = \frac{1}{a}\sin ax + C \tag{21}$$

$$\int x\sin axdx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a} + C \tag{22}$$

$$\int (\sin ax)(\cos^n ax)dx = -\frac{1}{a(n+1)}\cos^{n+1}ax + C \tag{23}$$

### Exponential functions:

$$\int_{-\infty}^{\infty} e^{-ax^2}\,\mathrm{d}x = \frac{\sqrt{\pi}}{\sqrt{a}}\;(a>0) \tag{24}$$

$$\int_{-\infty}^{\infty} xe^{-ax^2+bx}\,\mathrm{d}x = \frac{\sqrt{\pi}b}{2a^{3/2}}e^{\frac{b^2}{4a}}\;(\Re(a)>0) \tag{25}$$

$$\int_{-\infty}^{\infty} x^n e^{-ax}\,\mathrm{d}x = \begin{cases} \dfrac{\Gamma(n+1)}{a^{n+1}}\;(n>-1, a>0) \\ \dfrac{n!}{a^{n+1}}\;(n=0,1,2,...,a>0) \end{cases} \tag{26}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2}\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}\;(a>0) \tag{27}$$

$$\int xe^{cx}dx = \left(\frac{x}{c} - \frac{1}{c^2}\right)e^{cx} \tag{28}$$



$$\int x^2e^{cx}dx = \left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right)e^{cx} \tag{29}$$

$$\int x^4e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}\frac{3}{4a^2} \tag{30}$$

## Spherical coordinates

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\phi$$

Volume fraction:

$$dV = r^2\sin\theta drd\theta d\phi$$

Solid angle:

$$d\Omega = \frac{dS_r}{r^2} = \sin\theta d\theta d\phi$$

Surface element:

$$dS_r = r^2\sin\theta d\theta d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\vec{\phi} \tag{31}$$

$$\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi}. \tag{32}$$

$$\begin{aligned}\nabla\times\mathbf{F} &= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right)\vec{r} \\ &+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right)\vec{\theta} \\ &+\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}\right)\vec{\phi}\end{aligned} \tag{33}$$

$$\begin{aligned}\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2} = \\ &\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}f\end{aligned} \tag{34}$$

## Inner product and expectation

Expectation value (discrete)

$$\langle f_i\rangle = \textstyle\sum_i P_i f_i$$

Expectation value (continuous)

$$\langle f(x)\rangle = \textstyle\int_{-\infty}^{\infty} f(x)P(x)\,dx$$

$$\langle\hat{O}\rangle = \textstyle\int \psi^*(\mathbf{r})\hat{O}\psi(\mathbf{r})\,d^3r$$



Inner product

$$\langle\psi|\phi\rangle = \textstyle\int \psi^*(x)\phi(x)\,dx$$

Variance

$$\sigma_f^2 = \langle f^2\rangle - \langle f\rangle^2$$