Physics Formula Sheet

Your Name

2023/ 2024

Constants

Constant	Symbol	Value
Speed of light	c	3.00×10^8 m/s
Gravitational constant	G	6.674×10^{-11} N(m/kg)^2
Planck's constant	h	6.626×10^{-34} J.s
Mass of the electron	m_e	9.10939×10^{-31} kg
Mass of the proton	m_p	1.67262×10^{-27} kg
Charge of the electron	$-e$	-1.60218×10^{-19} C
Permittivity of free space	ϵ_0	8.85419×10^{-12} C^2/J m
Boltzmann constant	k_B	1.38066×10^{-23} J/ K
Avogadro's constant	N_A	6.022×10^{23} 1/mol

Classical Physics

Title	**Equation**
Bragg's Reflection	$n\lambda = 2d\sin(\theta)$
Diffraction (Single Slit)	$\lambda = d\sin(\theta)$
Young's Double Slit	$\frac{\Delta x}{L} = \frac{\lambda}{d} \approxeq \sin\theta$
Heat Transfer (Fourier's Law)	$Q = mC_v\Delta T$
Continuity Equation	$\nabla \cdot J = -\frac{d\rho}{dt}$
Force of Gravity	$F = G\frac{m_1 m_2}{r^2}$
Coulomb Force	$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$
Special Relativity (Time Dilation)	$E^2 = (pc)^2 + (m_0c^2)^2$

Nuclear and magnetic physics

Magnetic Field : $E_B = -\mu B,$
$\mu = \frac{e}{2m}L$
$F_z = -\frac{\partial V}{\partial z} = \mu\frac{\partial B}{\partial z}$

Rigid rotator : $E_{\text{rot}} = \frac{\mathbf{L}^2}{2I}$
$I = \frac{m_1 m_2}{m_1+m_2}R^2$

Radioactive decay $N(t) = N(0)\exp^{-\lambda t} = N(0)(\frac{1}{2})^{t/\tau_{1/2}}$
$\tau_{1/2} = \ln(2)/\lambda$

Thermodynamics

Black body:

$$Insertorlinktoadetailedperiodictablehere. D(k)dk = \frac{\partial N(k)}{\partial k}\frac{dk}{V} = \frac{k^2}{\pi^2}dk$$

$$D(\omega)d\omega = \frac{\omega^2}{\pi^2 c^3}d\omega$$

$$u(\omega)d\omega = \frac{\omega^2}{\pi^2 c^3}k_B T d\omega \text{ classical limit}$$

$$u(\omega)d\omega = \frac{\hbar\omega^3}{\pi^2 c^3}\frac{1}{\exp(\frac{\hbar\omega}{k_B T}) - 1}d\omega$$

$$I(\omega) = cu(\omega)d\omega$$

Quantum Mechanics

$$\text{Time-dependent Schrodinger's Equation} : i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t) = [-\frac{\hbar^2}{2m}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) + V(x)]$$

$$\text{Energy of a photon} : E = hf$$

$$\text{Time-independent Schrodinger's Equation} : E\phi = \hat{H}\phi = \Big(-\frac{\hbar^2}{2m}\nabla^2 + V(x)\Big)\cdot\phi$$

$$\text{Energy of a photon} : E = hf$$

$$\text{Infinite potential well} : E_n = \frac{\hbar^2}{2m}k_n^2 = \frac{\hbar^2\pi^2 n^2}{2mL^2} = n^2E_0,\ \ \psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L}),\ \ E_0 = \frac{\hbar^2\pi^2}{2mL^2}$$

$$\text{Transmission through a barrier} : T = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2\sinh^2[\sqrt{2m(V_0 - E)}\frac{l}{\hbar}]}$$

$$T \approx \frac{16E(V_0 - E)}{V_0^2}e^{-2\rho_2 l},\ \ \text{with}\rho_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}},\ \ \rho_2 \cdot l >> 1$$

$$\text{De Broglie wavelength} : \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

$$\text{Photoelectric effect} : h\nu - \phi_0 = \frac{1}{2}mv^2 = eV$$

$$\text{Bohr-Sommerfeldt condition} : \oint_C \mathbf{p}\cdot d\mathbf{s} = nh,\ \ 2\pi r = nh(\text{circular orbit})$$

$$\text{Probability current} : j = \frac{\hbar}{2mi}(\psi^*\frac{\partial\Psi}{\partial x} - \Psi\frac{\partial\Psi^*}{\partial x})$$

$$\text{Compton scattering} : \lambda_2 - \lambda_1 = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\mathbf{p}_{h\nu 1} = \mathbf{p}_{h\nu 2} + \mathbf{p}_e$$

$$hv_1 + m_0c^2 = h\nu_2 + \sqrt{m_0^2c^4 + p_e^2c^2}$$

Mathematical equations

Trigonometric functions:

$$\int \sin^n ax dx = -\frac{1}{a}\cos ax \ {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax + C \tag{1}$$

$$\int x\sin^2 ax dx = \frac{x^2}{4} - \frac{x}{4a}\sin 2ax - \frac{1}{8a^2}\cos 2ax + C \tag{2}$$

$$\int x^2\sin^2 xax dx = \frac{x^3}{6} - (\frac{x^2}{4a} - \frac{1}{8a^3})\sin 2ax - \frac{x}{4a^2}\cos 2ax + C \tag{3}$$

$$\int \tan ax dx = -\frac{1}{a}\ln|\cos ax| + C = \frac{1}{a}\ln|\sec ax| + C \tag{4}$$

$$\int \frac{\cos ax}{x} dx = \ln|ax| + \sum_{1}^{\infty}(-)^k\frac{(ax)^{2k}}{2k(2k)!} + C \tag{5}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax + C \tag{6}$$

$$\int \sin^3 ax dx = \frac{\cos 3ax}{12a} - \frac{3\cos ax}{4a} + C \tag{7}$$

$$\int \tan^2 x dx = \tan x - x + C \tag{8}$$

$$\int \sin ax\cos ax dx = -\frac{\cos^2 ax}{2a} + C \tag{9}$$

$$\int x\cos ax dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a} + C \tag{10}$$

$$\int \cos ax dx = \frac{1}{a}\sin ax + C \tag{11}$$

$$\int x\sin ax dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a} + C \tag{12}$$

$$\int (\sin ax)(\cos^n ax) dx = -\frac{1}{a(n+1)}\cos^{n+1} ax + C \tag{13}$$

Exponential functions:

$$\int_{-\infty}^{\infty} e^{-ax^2}\,\mathrm{d}x = \frac{\sqrt{\pi}}{\sqrt{a}}\ (a>0) \tag{14}$$

$$\int_{-\infty}^{\infty} xe^{-ax^2+bx}\,\mathrm{d}x = \frac{\sqrt{\pi}b}{2a^{3/2}}e^{\frac{b^2}{4a}}\ (\Re(a)>0) \tag{15}$$

$$\int_{-\infty}^{\infty} x^n e^{-ax}\,\mathrm{d}x = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}}\ (n>-1, a>0) \\ \frac{n!}{a^{n+1}}\ (n=0,1,2,...,a>0) \end{cases} \tag{16}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2}\ dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}\ (a>0) \tag{17}$$

$$\int xe^{cx}dx = \left(\frac{x}{c} - \frac{1}{c^2}\right)e^{cx} \tag{18}$$

$$\int x^2e^{cx}dx = \left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right)e^{cx} \tag{19}$$

$$\int x^4e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}\frac{3}{4a^2} \tag{20}$$

Spherical coordinates

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\phi$$

Volume fraction:

$$dV = r^2\sin\theta drd\theta d\phi$$

Solid angle:

$$d\Omega = \frac{dS_r}{r^2} = \sin\theta d\theta d\phi$$

Surface element:

$$dS_r = r^2\sin\theta d\theta d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\vec{\phi} \tag{21}$$

$$\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi}. \tag{22}$$

$$\begin{aligned}\nabla\times\mathbf{F} &= \frac{1}{r\sin\theta}(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi})\vec{r} \\ &+\frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi))\vec{\theta} \\ &+\frac{1}{r}(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi})\vec{\phi}\end{aligned} \tag{23}$$

$$\begin{aligned}\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2} = \\ &(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r})f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}f\end{aligned} \tag{24}$$

Harmonic oscillator:

First four harmonic oscillator wavefunction	Hermite polynomials	E$_\mathrm{n}$
$\psi_0(\xi) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{1}{2}\xi^2}$	1	$\frac{1}{2}\hbar\omega$
$\psi_1(\xi) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \sqrt{2}\xi e^{-\frac{1}{2}\xi^2}$	$2y$	$\frac{3}{2}\hbar\omega$
$\psi_2(\xi) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2}}\left(2\xi^2-1\right) e^{-\frac{1}{2}\xi^2}$	$4y^2-2$	$\frac{5}{2}\hbar\omega$
$\psi_3(\xi) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \frac{1}{\sqrt{3}}\left(2\xi^3-3\xi\right) e^{-\frac{1}{2}\xi^2}$	$8y^3-12y$	$\frac{7}{2}\hbar\omega$

Harmonic oscillator	$\psi_n(x) = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}(a^\dagger)^n e^{-\frac{1}{2}\frac{m\omega}{\hbar}x^2}\psi_0(x)$
Raising operator	$a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x - ip)$
Lowering operator	$a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$
$a^\dagger\lvert n\rangle = \sqrt{n+1}\lvert n+1\rangle$	$a\lvert n\rangle = \sqrt{n}\lvert n-1\rangle$
Number operator	$\hat{N} = a^\dagger a \hat{N}\lvert n\rangle = n\lvert n\rangle$
Commutation relation	$[a, a^\dagger] = aa^\dagger - a^\dagger a = 1$
Hamiltonian	$\hat{H} = \hbar\omega\left(\hat{N} + \frac{1}{2}\right)$

Inner product and expectation

Expectation value (discrete)

$$\langle f_i \rangle = \sum_i P_i f_i$$

Expectation value (continuous)

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x)P(x)\,dx$$

$$\langle \hat{O} \rangle = \int \psi^*(\mathbf{r})\hat{O}\psi(\mathbf{r})\,d^3r$$

Inner product

$$\langle \psi | \phi \rangle = \int \psi^*(x)\phi(x)\,dx$$

Variance

$$\sigma_f^2 = \langle f^2 \rangle - \langle f \rangle^2$$

Commutation relations

$$[A,B] = AB - BA$$
$$[AB,C] = A[B,C] - [A,C]B$$
$$[x,p_x] = i\hbar$$
$$[y,p_y] = i\hbar$$
$$[x,y] = [x,p_y] = [y,p_x] = 0$$

Hydrogen atom

Fine structure constant:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$$

Bohr radius:

$$a_0 = \frac{\hbar}{m_e c\alpha} \approx 0.529 \times 10^{-10}\text{m}$$

Bohr energy:

$$E_n = -\frac{2\pi^2 k^2 e^4 m_e}{h^2 n^2}$$

Ground state energy:

$$E_1 = -13.6\text{eV}$$

Wave function:

$$\psi_{n\ell m}(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$$

Rydberg formula:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Rydberg constant:

$$R_H \approx 1.097 \times 10^7\text{m}^{-1}$$

Radial wavefunctions:

$$R_{n\ell}(r) = N_{n\ell} r^\ell e^{-\rho/2} L_{n-\ell-1}^{2\ell+1}(\rho)$$

Legendre polynomials

Angular momentum

$$L_+ = L_x + iL_y$$

$$L_- = L_x - iL_y$$

$$L^2 = L_z^2 + \frac{1}{2}(L_+L_- + L_-L_+)$$

$$[L_x, L_y] = i\hbar L_z$$

$$[L^2, L_i] = 0 \quad \text{where } i = x, y, \text{or } z$$

$$L_x = -i\hbar\left(\sin\phi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right),\; L_y = i\hbar\left(\cos\phi\frac{\partial}{\partial\theta} - \cot\theta\sin\phi\frac{\partial}{\partial\phi}\right),\; L_z = -i\hbar\frac{\partial}{\partial\phi}$$

$$L_+ = \hbar e^{i\phi}\left(\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right),\; L_- = \hbar e^{-i\phi}\left(-\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right)$$

$$L^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)$$

Hund's rule

1: All other thing being equal, the state with the highest total spin (S), will have the lowest.
2: For a given spin, the state the highest total orbital angular momentum (L), consistent with overall antisymmetrization, will have the lowest energy.
3: If a subshell (n, l) is no more than half filled, then the lowest energy level has $J = |L - S|$: if it is more than half filled, then $J = L + S$ has the lowest energy.

Spin

Two particle spin states

$|0,0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ s = 0 singlet

$|1,1\rangle = |\uparrow\uparrow\rangle$
$|1,0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, s = 1 triplet
$|1,-1\rangle = |\downarrow\downarrow\rangle$

$$S_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \quad S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad S_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \quad S^2 = \frac{3}{4}\hbar^2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \quad S_+ = \hbar\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix} \quad S_- = \hbar\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$$

Two particle Hamiltonian:

$$\hat{H} = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V(\mathbf{r}_1, \mathbf{r}_2)$$

Hamiltonian with an atom with atomic number Z:

$$\hat{H} = -\frac{\hbar^2}{2m_e}\nabla^2 - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

Clebsch-Gordan coefficients

Condensed Matter

Free electron gas:

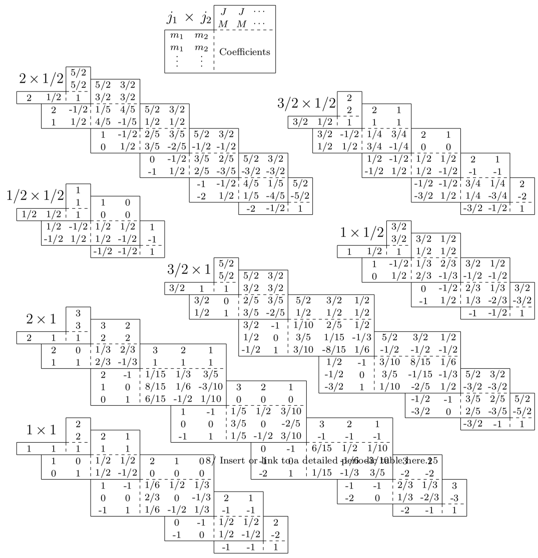

Figure 1: Clebsch-Gordan coefficients. A square root is understood on each coefficient, that is, $-1/3$ means $-\sqrt{1/3}$.

$$\psi_{n_x,n_y,n_z}(\mathbf{r}) = \sqrt{\tfrac{8}{V}}\sin\left(\tfrac{n_x\pi x}{L_x}\right)\sin\left(\tfrac{n_y\pi y}{L_y}\right)\sin\left(\tfrac{n_z\pi z}{L_z}\right), \quad E(\mathbf{k}) = \tfrac{\hbar^2k^2}{2m}$$

Fermi energy of a metal:

$E_F = \frac{\hbar^2k_F^2}{2m}, \rho \equiv \frac{N_q}{V}, N_q =$ number of electrons in volume V, $n_c = n_i \exp(\frac{E_F - E_i}{k_BT})$

Density of states:

3D: $g(E) = \frac{Vm}{2\pi^2\hbar^3}\sqrt{2mE}$, 2D: $g(E) = \frac{m}{\pi\hbar^2}$

Distribution functions:

Maxwell-Boltzmann	Fermi-Dirac	Bose-Einstein
$f(E) = e^{-\frac{E-\mu}{k_BT}}$	$f(E) = \frac{1}{e^{\frac{E-\mu}{k_BT}}+1}$	$f(E) = \frac{1}{e^{\frac{E-\mu}{k_BT}}-1}$

First order perturbation theory:

$\Delta E_n^{(1)} = \langle\psi_n^0|\hat{H}'|\psi_n^0\rangle$, $\psi_n^{(1)} = \sum_{m\neq n}\frac{\langle\psi_m^0|\hat{H}'|\psi_n^0\rangle}{E_n^{(0)}-E_m^{(0)}}\psi_m^{(0)}$

Periodic Table

Insert or link to a detailed periodic table here.

Periodic Table of Elements

	1 IA	2 IIA	3 IIIA	4 IVB	5 VB	6 VIB	7 VIIB	8 VIIIB	9 VIIIB	10 VIIIB	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 V
1	1 1.0079 **H** Hydrogen																
2	3 6.941 **Li** Lithium	4 9.0122 **Be** Beryllium											5 10.811 **B** Boron	6 12.011 **C** Carbon	7 14.007 **N** Nitrogen	8 15.999 **O** Oxygen	9 1 **F** Fluor
3	11 22.990 **Na** Sodium	12 24.305 **Mg** Magnesium											13 26.982 **Al** Aluminium	14 28.086 **Si** Silicon	15 30.974 **P** Phosphorus	16 32.065 **S** Sulphur	17 3 **C** Chlor
4	19 39.098 **K** Potassium	20 40.078 **Ca** Calcium	21 44.956 **Sc** Scandium	22 47.867 **Ti** Titanium	23 50.942 **V** Vanadium	24 51.996 **Cr** Chromium	25 54.938 **Mn** Manganese	26 55.845 **Fe** Iron	27 58.933 **Co** Cobalt	28 58.693 **Ni** Nickel	29 63.546 **Cu** Copper	30 65.39 **Zn** Zinc	31 69.723 **Ga** Gallium	32 72.64 **Ge** Germanium	33 74.922 **As** Arsenic	34 78.96 **Se** Selenium	35 7 **B** Brom
5	37 85.468 **Rb** Rubidium	38 87.62 **Sr** Strontium	39 88.906 **Y** Yttrium	40 91.224 **Zr** Zirconium	41 92.906 **Nb** Niobium	42 95.94 **Mo** Molybdenum	43 96 **Tc** Technetium	44 101.07 **Ru** Ruthenium	45 102.91 **Rh** Rhodium	46 106.42 **Pd** Palladium	47 107.87 **Ag** Silver	48 112.41 **Cd** Cadmium	49 114.82 **In** Indium	50 118.71 **Sn** Tin	51 121.76 **Sb** Antimony	52 127.6 **Te** Tellurium	53 **I** Iodi
6	55 132.91 **Cs** Caesium	56 137.33 **Ba** Barium	57-71 **La-Lu**	72 178.49 **Hf** Hafnium	73 180.95 **Ta** Tantalum	74 183.84 **W** Tungsten	75 186.21 **Re** Rhenium	76 190.23 **Os** Osmium	77 192.22 **Ir** Iridium	78 195.08 **Pt** Platinum	79 196.97 **Au** Gold	80 200.59 **Hg** Mercury	81 204.38 **Tl** Thallium	82 207.2 **Pb** Lead	83 208.98 **Bi** Bismuth	84 209 **Po** Polonium	85 **A** Astat
7	87 223 **Fr** Francium	88 226 **Ra** Radium	89-103 **Ac-Lr** Actinide	104 261 **Rf** Rutherfordium	105 262 **Db** Dubnium	106 266 **Sg** Seaborgium	107 264 **Bh** Bohrium	108 277 **Hs** Hassium	109 268 **Mt** Meitnerium	110 281 **Ds** Darmstadtium	111 280 **Rg** Roentgenium	112 285 **Uub** Ununbium	113 284 **Uut** Ununtrium	114 289 **Uuq** Ununquadium	115 288 **Uup** Ununpentium	116 293 **Uuh** Ununhexium	117 **Uu** Unuse

57 138.91	58 140.12	59 140.91	60 144.24	61 145	62 150.36	63 151.96	64 157.25	65 158.93	66 162.50	67 164.93	68 167.26	69 168.93	70

Alkali Metal
Alkaline Earth Metal