Physics Formula Sheet

402-0023-01L Physics

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Constants

Constant	Symbol	Value
Speed of light	c	3.00×10^{8} m/s
Gravitational constant	G	6.674×10^{-11} N(m/kg)^2
Planck's constant	h	6.626×10^{-34} J.s
Mass of the electron	m_e	9.10939×10^{-31} kg
Mass of the proton	m_p	1.67262×10^{-27} kg
Charge of the electron	$-e$	-1.60218×10^{-19} C
Permittivity of free space	ϵ_0	8.85419×10^{-12} C^2/J m
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T m / A
Boltzmann constant	k_B	1.38066×10^{-23} J/ K
Avogadro's constant	N_A	6.022×10^{23} 1/mol

Classical Physics

Title	**Equation**
Bragg's Reflection	$n\lambda = 2d\sin(\theta)$
Diffraction (Single Slit)	$\lambda = d\sin(\theta)$
Young's Double Slit	$\frac{\Delta x}{L} = \frac{\lambda}{d} \approxeq \sin\theta$
Heat Transfer (Fourier's Law)	$Q = mC_v\Delta T$
Continuity Equation	$\nabla \cdot J = -\frac{d\rho}{dt}$
Force of Gravity	$F = G\frac{m_1 m_2}{r^2}$
Coulomb Force	$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$
Special Relativity (Time Dilation)	$E^2 = (pc)^2 + (m_0c^2)^2$

Nuclear and magnetic physics

Magnetic Field	$E_B = -\mu B$, $\mu = \frac{e}{2m}L$ $F_z = -\frac{\partial V}{\partial z} = \mu\frac{\partial B}{\partial z}$
Rigid rotator	$E_{\text{rot}} = \frac{\mathbf{L}^2}{2I}$ $I = \frac{m_1 m_2}{m_1+m_2}R^2$
Radioactive decay	$N(t) = N(0)\exp^{-\lambda t} = N(0)(\frac{1}{2})^{t/\tau_{1/2}}$ $\tau_{1/2} = \ln(2)/\lambda$

Thermodynamics

0th law: If two objects are in thermal equilibrium with a third object, then all three objects are in thermal equilibrium with each other.
1st law: For any process concerning a given system, the change in internal energy ΔU of that system is equal to the sum of the heat Q transferred to that system and the work W performed on that system, or:

2nd law: Carnot: Wherever there exists a difference in temperature, motive power can be produced.
Kelvin: It is impossible for a self acting machine to convey heat from a colder body to a hotter.
Clausius: Heat cannot flow from a colder to hotter body without another process occurring, connected therewith, simultaneously.
$T = (\frac{\partial U}{\partial S})_{\text{V, N}}$

Energy per mode: $< E_{\text{mode}} >= \frac{3}{2}k_b T$
$Q = C\Delta T$ $Q = \int_{T_1}^{T_2} C(T)dT$
$L = \frac{Q_{\text{latent}}}{m}$
$\gamma = \frac{C_P}{C_V}$ $dS = \frac{\delta Q_{\text{rev}}}{T}$

Electrostatics and dynamics

$\vec{F} = \sum_{i=1}^{N} \frac{q_0 q_i(\vec{r}-\vec{r_i})}{4\pi\epsilon_0|\vec{r}-\vec{r_i}|^3}$ Torque: $\vec{\tau} = \vec{p} \times \vec{E}$
Energy of a dipole: $U(\theta) = -\vec{p}\cdot\vec{E}$
Gauss' law: $\phi = \oiint_S \mathbf{E}\cdot d\mathbf{A}$
Potential: $\Delta V \equiv \frac{\Delta U}{q} = -\int_C \vec{E}\cdot d\vec{l}$ Energy of a capacitor: $U = \frac{Q^2}{2C}$
Current: $I = \dot{Q}$
Potential: $V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$
Kirchhoff's rules: 1: $\sum_{j,\text{loop}} \Delta V_j$
2: $\sum_j I_{j,\text{into node}} = 0$
Magnetic force: $\vec{F} = q\vec{v}\times\vec{B}$
Cyclotron radius: $r = \frac{mv}{qB}$
Biot-Savart: $\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{q\vec{v}\times\hat{r}}{r^2}$
Faraday's Law: $\mathcal{E} = -\frac{d\phi_m}{dt}$
Self-inductance of a solenoid: $L = \mu_0 n^2 Al$
Mutual inductance: $\frac{\phi_{m1}}{N_1} = \frac{\phi_{m2}}{N_2}$
Impedance: $Z_R = R, Z_C = \frac{1}{i\omega C}, Z_L = i\omega L$

Waves

$$\oint_S \mathbf{E}\cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad \text{(Gauss's Law for Electricity)} \tag{1}$$

$$\oint_S \mathbf{B}\cdot d\vec{\mathbf{A}} = 0 \quad \text{(Gauss's Law for Magnetism)} \tag{2}$$

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's Law)} \tag{3}$$

$$\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{enc}} + \mu_0\epsilon_0\frac{d\Phi_E}{dt} \quad \text{(Ampère's Law with Maxwell's addition)} \tag{4}$$

In electromagnetic waves the ratio: $\mathrm{B}_0 = \frac{\mathrm{E}_0}{c}$ holds
Wavenumber: $\omega = vk$
Compton wavelength: $\lambda_c = \frac{h}{m_e c}$
De Broglie wavelength: $\lambda_{\text{dB}} = \frac{h}{p}$
Heisenberg uncertainty relation: $\Delta x\Delta p \geq \frac{h}{4\pi}$
Energy of a particle in a 1D box: $E_n = \frac{h^2 n^2}{8L^2 m}$

Quantum Mechanics

$$\text{Time-dependent Schrodinger's Equation} : i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t) = [-\frac{\hbar^2}{2m}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) + V(x)]$$

Special relativity

Postulates of relativity and inertial reference frames:
1: Absolute uniform motion cannot be detected.
2: The speed of light in a vacuum is independent of the motion of the source.

Doppler Shift

Non-relativistic Doppler Shift:

$$f' = f\left(\frac{c \pm v_{\text{observer}}}{c \pm v_{\text{source}}}\right) \quad \text{(for sound or slow-moving sources)} \tag{5}$$

Relativistic Doppler Shift:

$$f' = f\sqrt{\frac{1+\beta}{1-\beta}} \quad \text{(for motion towards the observer)} \tag{6}$$

$$f' = f\sqrt{\frac{1-\beta}{1+\beta}} \quad \text{(for motion away from the observer)} \tag{7}$$

where $\beta = \frac{v_{\text{source}}}{c}$.

Velocity Transformations in Special Relativity

For two observers in relative motion with velocity v along the x-axis:

$$u'_x = \frac{u_x + v}{1 + \frac{vu_x}{c^2}} \tag{8}$$

$$u'_y = \frac{u_y}{\gamma(1 + \frac{vu_x}{c^2})} \tag{9}$$

$$u'_z = \frac{u_z}{\gamma(1 + \frac{vu_x}{c^2})} \tag{10}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor.

Energy

$\mathrm{E}_{\text{total}} = \gamma mc^2 = \sqrt{p^2c^2 + m^2c^4}, \mathrm{E}_{\text{rest}} = mc^2$

Mathematical equations

Trigonometric functions:

$$\int \sin^n ax dx = -\frac{1}{a}\cos ax \;{}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax + C \tag{11}$$

$$\int x\sin^2 ax dx = \frac{x^2}{4} - \frac{x}{4a}\sin 2ax - \frac{1}{8a^2}\cos 2ax + C \tag{12}$$

$$\int x^2\sin^2 xax dx = \frac{x^3}{6} - (\frac{x^2}{4a} - \frac{1}{8a^3})\sin 2ax - \frac{x}{4a^2}\cos 2ax + C \tag{13}$$

$$\int \tan ax dx = -\frac{1}{a}\ln|\cos ax| + C = \frac{1}{a}\ln|\sec ax| + C \tag{14}$$

$$\int \frac{\cos ax}{x}dx = \ln|ax| + \sum_1^\infty (-)^k\frac{(ax)^{2k}}{2k(2k)!} + C \tag{15}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax + C \tag{16}$$

$$\int \sin^3 ax dx = \frac{\cos 3ax}{12a} - \frac{3\cos ax}{4a} + C \tag{17}$$

$$\int \tan^2 x dx = \tan x - x + C \tag{18}$$

$$\int \sin ax\cos ax dx = -\frac{\cos^2 ax}{2a} + C \tag{19}$$

$$\int x\cos ax dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a} + C \tag{20}$$

$$\int \cos ax dx = \frac{1}{a}\sin ax + C \tag{21}$$

$$\int x\sin ax dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a} + C \tag{22}$$

$$\int (\sin ax)(\cos^n ax)dx = -\frac{1}{a(n+1)}\cos^{n+1} ax + C \tag{23}$$

Exponential functions:

$$\int_{-\infty}^{\infty} e^{-ax^2}\mathrm{d}x = \frac{\sqrt{\pi}}{\sqrt{a}}\ (a > 0) \tag{24}$$

$$\int_{-\infty}^{\infty} xe^{-ax^2+bx}\mathrm{d}x = \frac{\sqrt{\pi}b}{2a^{3/2}}e^{\frac{b^2}{4a}}\ (\Re(a) > 0) \tag{25}$$

$$\int_{-\infty}^{\infty} x^n e^{-ax}\mathrm{d}x = \begin{cases} \dfrac{\Gamma(n+1)}{a^{n+1}} & (n > -1, a > 0) \\ \dfrac{n!}{a^{n+1}} & (n = 0, 1, 2, ..., a > 0) \end{cases} \tag{26}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2}\ dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}\ (a > 0) \tag{27}$$

$$\int xe^{cx}dx = \left(\frac{x}{c} - \frac{1}{c^2}\right)e^{cx} \tag{28}$$

$$\int x^2 e^{cx}dx = \left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right)e^{cx} \tag{29}$$

$$\int x^4 e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}\frac{3}{4a^2} \tag{30}$$

Spherical coordinates

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\phi$$

Volume fraction:

$$dV = r^2\sin\theta dr d\theta d\phi$$

Solid angle:

$$d\Omega = \frac{dS_r}{r^2} = \sin\theta d\theta d\phi$$

Surface element:

$$dS_r = r^2\sin\theta d\theta d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\vec{r} + \frac{1}{r}\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\vec{\phi} \tag{31}$$

$$\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi}. \tag{32}$$

$$\begin{aligned}\nabla\times\mathbf{F} = \frac{1}{r\sin\theta}&(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi})\vec{r}\\ &+\frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi))\vec{\theta}\\ &+\frac{1}{r}(\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi})\vec{\phi}\end{aligned} \tag{33}$$

$$\begin{aligned}\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2} =\\ (\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r})f + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})f + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}f\end{aligned} \tag{34}$$

Inner product and expectation

Expectation value (discrete)

$$\langle f_i\rangle = \sum_i P_i f_i$$

Expectation value (continuous)

$$\langle f(x)\rangle = \int_{-\infty}^{\infty} f(x)P(x)\,dx$$

$$\langle\hat{O}\rangle = \int \psi^*(\mathbf{r})\hat{O}\psi(\mathbf{r})\,d^3r$$

Inner product

$$\langle\psi|\phi\rangle = \int \psi^*(x)\phi(x)\,dx$$

Variance

$$\sigma_f^2 = \langle f^2\rangle - \langle f\rangle^2$$