Post-Toll Traffic Study

transportation investment corporation

BRITISH COLUMBIA

Travel Time

Analysis

Throughout this section, travel times and their related parameters are separated into Westbound and Eastbound. Additionally, the data analysis is presented by the hour of day (from 12 AM-12 PM). Unfortunately, data for 8-11 PM for August and September 2017 are unavailable due to recording hardware malfunction, which prevents a complete analysis to be done. The coverage scope in this section is travel time for the stretch of Highway 1 from Langley to Grandview, Vancouver.

Post-toll removal results in average travel time increase for both West- and Eastbound. This is easily seen in the figure below in which September and October 2017 average travel time towers the other time periods. As well, the roundtrip travel times increase by 12 minutes from August to September 2017. August and September 2017 travel time variance analysis indicates there is an average increase of 7.82% and 2.45% for westbound and eastbound respectively. Specifically, there is an increase of 29.5% during westbound peak travel time hours (6-9 AM), and an increase of 7.75% during eastbound peak travel time hours (3-6 PM).

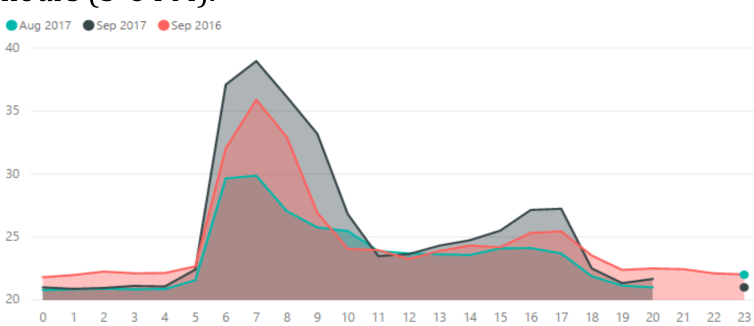


Figure 1:Average travel time for Westbound

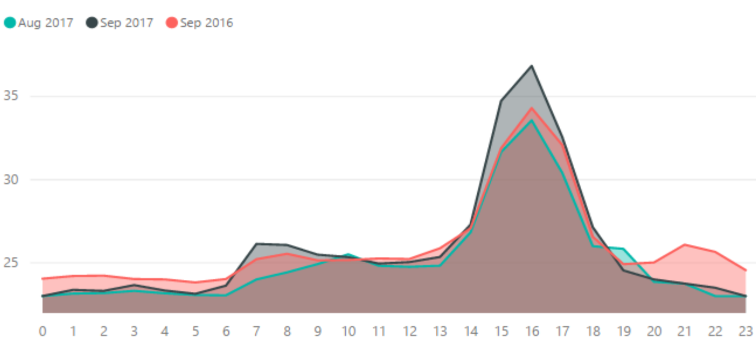


Figure 2: Average travel time for Eastbound

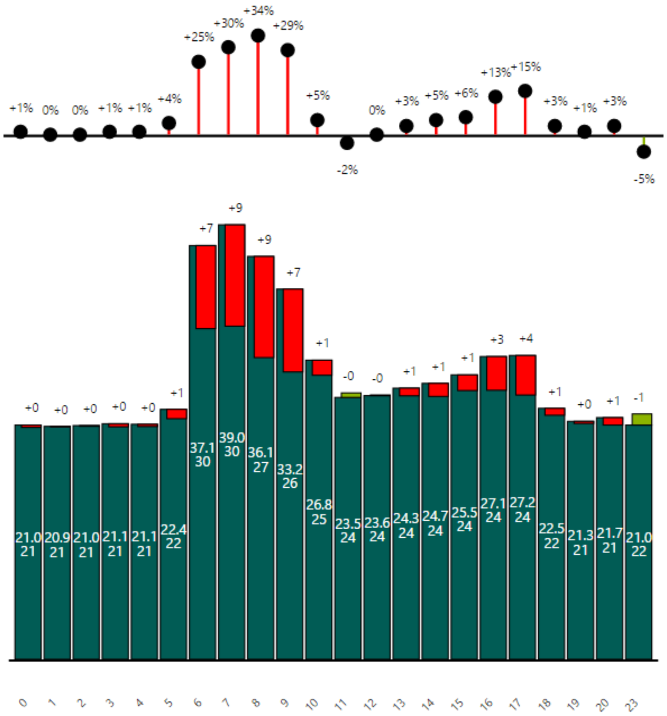


Figure 4:Travel time variances for August and September 2017 (Westbound)

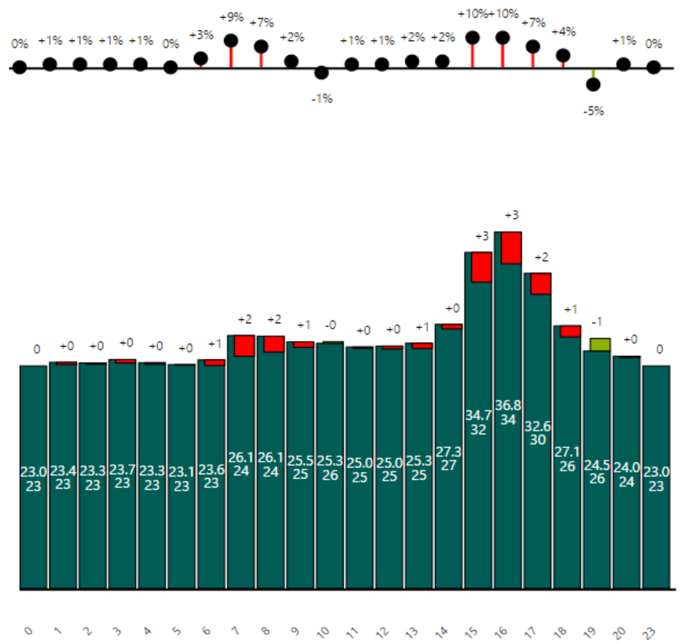


Figure 3:Travel time variances for August and September 2017 (Eastbound)

Cost of Travel

The increase in travel time entails economic costs for travellers. These costs of travel are made up of operational and time value costs. The former includes fuel, insurance, licensing and registration, and vehicle maintenance as well as depreciation. This cost is estimated through the cost per minute of travel for car sharing services (Car2Go and Evo). The latter is the product of extra time spent travelling and monetary cost of travel time per unit of time. Specifically, it is the sum of prevailing commercial wages and personal value of time that is valued at 25%-50% of prevailing commercial wages depending on commuting purpose. The table below details the break down of the cost of travels.

Vancouver-Langley					
Travel Time Increase	12 minutes	Operational costs per min	$0.35	Total Operational Costs	$4.20
		Value of time per minute	$0.28	Total Value of Time	$3.30

Table 2: Breakdown of cost of travel associated with increased travel time

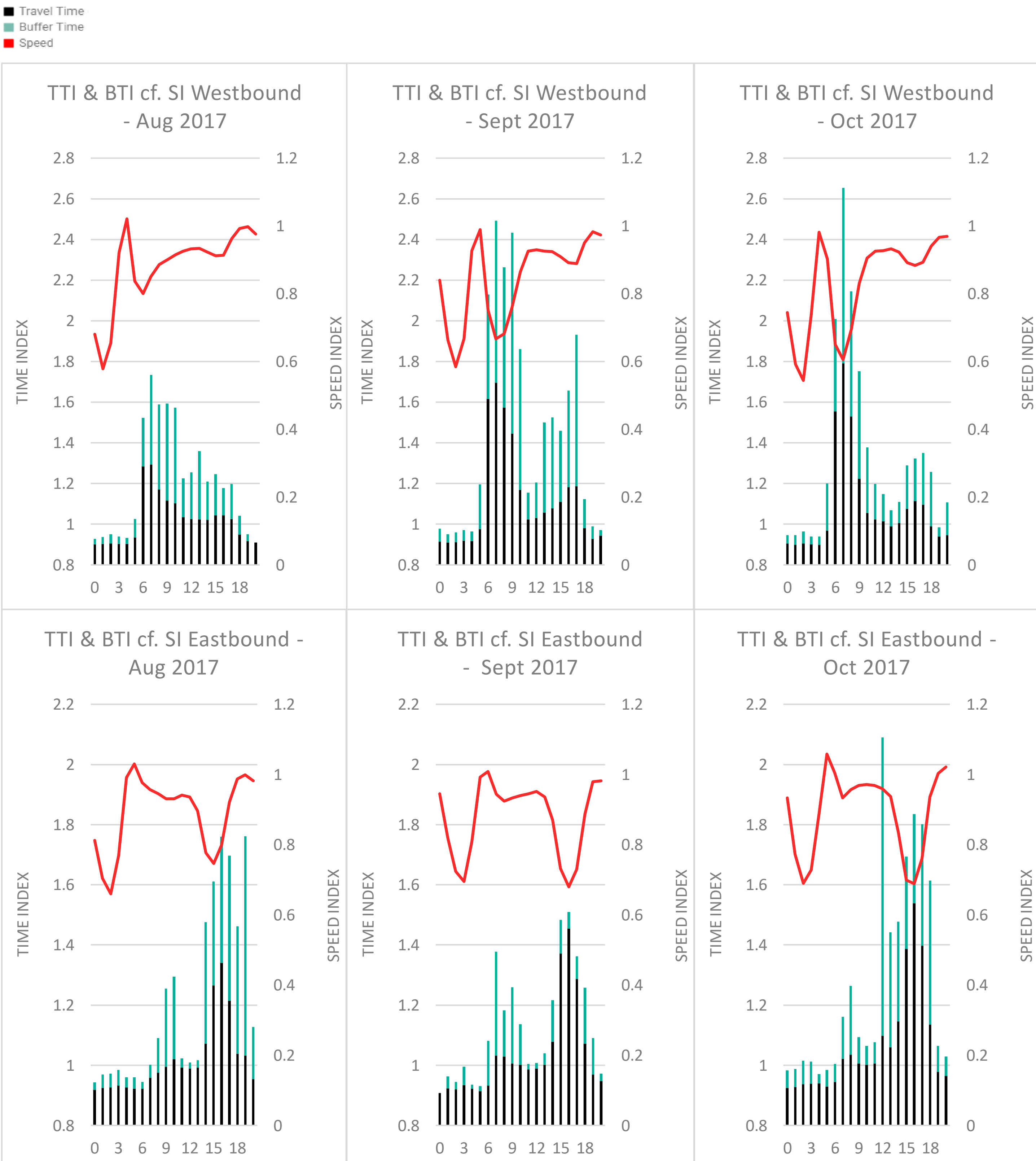
Traffic Indexes

Aside from solely analyzing travel time based on observed data as is, an improvement to the current method of analysis can be found in the construction of three indexes: travel time, buffer time and speed indexes. These indexes are calculated for each hour of the day. The reason why indexes are useful is that they produce a compact yet extensive visualization that enable simultaneous cross-analysis of the traffic variables of interest in a single chart.

Travel Time Index (TTI) is simply a ratio of travel time of a particular hour of day to the free-flow travel time. Likewise, Speed index (SI) is a ratio of speed of a particular hour of day to the free-flow speed. The free-flow denominators are obtained by averaging the pertinent values from 11 AM-2 PM. Free-flow is chosen for the base of the indexes because it best represents the ideal traffic condition in which travellers are not impeded by congestion under normal road usage capacity.

Buffer Time Index (BTI) uses extra travel time needed to ensure on-time arrival. It is defined as the difference between 95th percentile travel time and average travel time divided by average travel time (Russell, 2014). However, a research done by Pu (2011) discovered that using median travel time is more appropriate compared to using average travel time. He found that average-based buffer could underestimate unreliability and generate unrealistic result that is misleading. Thus, buffer index calculation presented in this report uses median instead of average travel time.

Based on various research, travel time follow approximately a lognormal distribution (Russell, 2014). A test via constructing histograms was performed to test whether the observed data are approximately lognormal and it was found to be the case. Interestingly, the histograms for speed are approximately inverse lognormal. For consistency and calculation convenience, the observed data for speed are mathematically transformed accordingly. All of the histograms for the periods and traffic variables of interest are presented in Appendix (C). While lognormality does not impact the calculations for TTI and SI, an adjustment is needed for BTI since it involves identifying the 95th percentile travel time. For more information regarding the formula used for the calculation of BTI, please refer to Appendix (D). To reflect on lognormality of the observed data, all of the subsequent calculations use normalized data by taking their natural logarithm (ln).



Figures 16: Traffic indexes for August to October 2017 for west- and eastbound

Correlations

Complementary to traffic indexes presented previously, another useful measure that is worth keeping track is the correlations of the traffic variables. Although individually they might not be of much value, since they are rather obvious intuitively, monitoring their changes over time can serve as crucial signalling information. Additionally, they can provide an overarching, general summary for the traffic variables. For example, one can easily deduce a very strong positive correlation between travel and buffer time for a specific time period, so noting their correlation seems to be useless. However, knowing how such correlation changes over the next time period(s) can tell if there is any improvement in reliability in the road network. In the case of this report, the change in correlations can measure how overall traffic is impacted post-toll. Below are the correlations for the traffic variables for August to October 2017. The calculation for the correlations have been adjusted accordingly considering how the observed data follow approximately lognormal distribution. For more detail, please refer to Appendix (E).

	Westbound			Eastbound		
	Aug 2017	Sept 2017	Oct 2017	Aug 2017	Sept 2017	Oct 2017
Travel & Buffer Time	0.954	0.962	0.985	0.888	0.889	0.876
Travel Time & Speed	0.132	-0.312	-0.361	-0.161	-0.542	-0.512
Buffer Time & Speed	0.171	-0.226	-0.325	-0.022	-0.341	-0.337

Table 3: Correlations for all traffic variables for August to October 2017

In general, an increase in travel time and buffer time correlation indicates a worsening traffic condition since the two variables are more strongly correlated than the previous period. A decrease in correlation of travel time or buffer time with speed also indicates worsening traffic condition. This is because a smaller correlation translates to slower speed for travellers, effectively increasing travel and buffer time, *ceteris paribus*.

Comments on Traffic Indexes and Correlations

From the results shown above, it is clear that toll removal negatively impacts traffic all around. For westbound, traffic indexes showcase how there is an increase in travel time and buffer time and a decrease in speed. The indexes charts clearly show higher travel and buffer time index as well as lower speed index. Similarly, the correlations are in the same page as there is an increase in travel time and buffer time coefficients as well as decreasing travel time or buffer time with speed coefficients. Travel and buffer time correlations are shown to be continuously rising. Moreover, the difference for travel time or buffer time with speed between August and September 2017 is rather significant for Westbound and Eastbound, – 0.444 and 0.397, respectively. This means that on top of less reliability (allocating additional extra travel time than before) for travellers to arrive on time, they are travelling at a quite significantly slower speed post-toll — practically the worst-case scenario possible.

Interestingly, for eastbound, travel and buffer time correlation remains virtually unchanged. However, the traffic indexes charts seem to show that there is an improvement in traffic condition throughout September 2017 but substantially degrades for the next month. The discrepancy between the charts and the correlations is due to buffer time being the extra time to ensure on-time arrival relative to travel time. Since the correlation remains almost the same, reliability for Eastbound is about the same, although travellers have to spend more time commuting post-toll. Considering that travel time and speed correlation falls by 0.463 when comparing August 2017 to September 2017, it makes sense that travel time is longer.

From the above, both traffic indexes and correlations are complementary to each other, so they are best used in tandem. Each specializes in different aspect of summarization and presentation of the traffic variables. Thus, combining the two provides a comprehensive analysis from a terse and high-level numerical perspective to a detailed breakdown of traffic variables interaction trends. Further, it was shown earlier that exclusively relying on one measure can lead to an inaccurate conclusion. By making use of both measures, each can fill in the gap to produce a more complete analysis to inform readers better.

Appendix

(C)

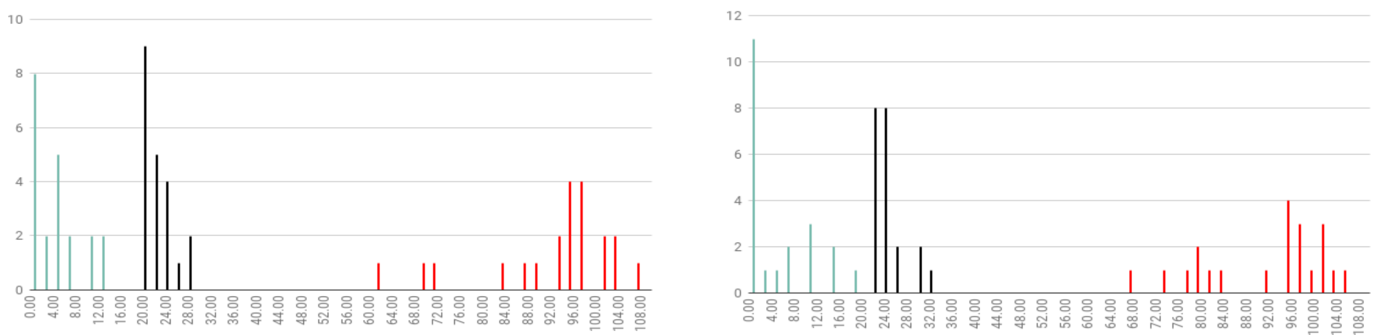


Figure 18: Histograms for August 2017 (Left is westbound and right is eastbound)

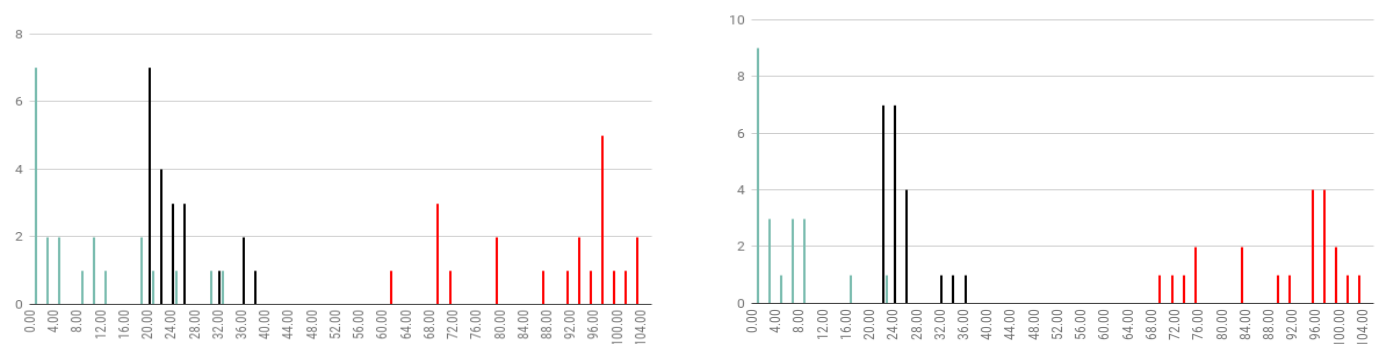


Figure 19: Histograms for September 2017 (Left is westbound and right is eastbound)

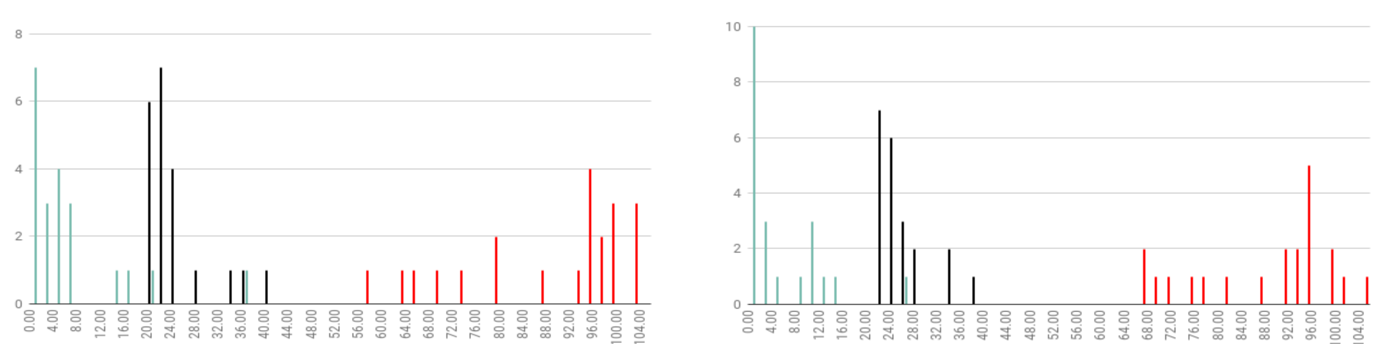


Figure 20: Histograms for October 2017 (Left is westbound and right is eastbound)

(D)

To reflect on the fact that travel time and buffer time is lognormally distributed, a new formula to calculate the 95th percentile travel time is derived. In a paper by Arezoumandi (2011), he showed that the 95th percentile of normally distributed travel time is

$$Travel\ Time\ 95\% = \mu + 1.6449\sigma \tag{1}$$

Where μ is mean and σ is standard deviation

After lognormally transforming equation (1), the 95th percentile travel time is defined as

$$Travel\ Time\ 95\% = \mu^* * (\sigma^*)^{1.6449} \tag{2}$$

Where $\mu^* = \frac{\mu}{\sqrt{1+\left(\frac{\sigma}{\mu}\right)^2}}$ and σ^*= $\exp\left(\sqrt{\ln\left(1+\left(\frac{\mu}{\sigma}\right)^2\right)}\right)$

Therefore,

$$Buffer\ Time\ Index = \frac{\frac{\mu}{\sqrt{1+\left(\frac{\sigma}{\mu}\right)^2}} * \left(\exp\left(\sqrt{\ln\left(1+\left(\frac{\mu}{\sigma}\right)^2\right)}\right)\right)^{1.6449} - Median\ Travel\ Time}{Median\ Travel\ Time} \tag{3}$$

(E)

Let $y_1, \dots, y_n$ be the sample from a bivariate lognormal distribution (i.e. observed traffic data), and $X_i = \ln Y_i$ where $i = 1, \dots, n$. Therefore, $x_1, \dots, x_n$ follows a normal distribution with mean vector of $\mu = (\mu_1, \mu_2)'$ and covariance matrix Σ, i.e.,

$$x_i \sim N\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}, \Sigma = \begin{pmatrix}\sigma_{11}^2 & \sigma_{12}^2\\ \sigma_{12}^2 & \sigma_{22}^2\end{pmatrix} \tag{4}$$

With the correlation between Y_{i1} and Y_{i2} to be $\rho = \mathrm{cov}(Y_{i1}, Y_{i2}) \,/\, \sqrt{Var(Y_{i1})Var(Y_{i2})}$ (5)

Using the facts that $E(Y_{i1}) = e^{\mu_{11}+\frac{\sigma_{11}^2}{2}}$, $E(Y_{i2}) = e^{\mu_{22}+\frac{\sigma_{22}^2}{2}}$, $\mathrm{Var}(Y_{i1}) = e^{2\mu_{11}+\sigma_{11}^2}(e^{\sigma_{11}^2}-1)$ and $\mathrm{Var}(Y_{i2}) = e^{2\mu_{22}+\sigma_{22}^2}(e^{\sigma_{22}^2}-1)$, it can be shown that

$$\mathrm{cov}(Y_{i1}, Y_{i2}) = e^{\mu_{11}+\mu_{22}+\frac{\sigma_{11}^2+\sigma_{22}^2}{2}}(e^{\sigma_{12}^2}-1) \tag{6}$$

$$\sqrt{Var(Y_{i1})Var(Y_{i2})} = e^{\mu_{11}+\mu_{22}+\frac{\sigma_{11}^2+\sigma_{22}^2}{2}}\sqrt{(e^{\sigma_{11}^2}-1)(e^{\sigma_{22}^2}-1)} \tag{7}$$

Therefore,

$$\rho = \frac{e^{\sigma_{12}^2}-1}{\sqrt{(e^{\sigma_{11}^2}-1)(e^{\sigma_{22}^2}-1)}} \tag{8}$$

Equation 1: Derived correlation formula for bivariate lognormal variables adapted from Zhang and Chen (2014)

Let S be the sum of squares and cross products for the observed data, i.e.,

$$S = \sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x}) = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \tag{9}$$

The matrix for S above has a Wishart distribution with $\Sigma = \sigma_{ij}$ and $df = n - 1$. Further, let $\sigma^* = \sigma_{11} - \sigma_{12}^2 / \sigma_{22}$ and $S^* = S_{11} - S_{12}^2 / S_{22}$. The following variables then follow either normal or chi-squared distribution:

$$V_{22} = \frac{S_{22}}{\sigma_{22}} \sim x_{n-1}^2 \ \ (10) \ \ , \ \ V_{11}^* = \frac{S^*{}_{11}}{\sigma^*{}_{11}} \sim x_{n-2}^2 \ \ (11) \ \ , \ \ z_1 = \left(S_{12} - \frac{\sigma_{12}}{\sigma_{22}}S_{22}\right) / \sqrt{\sigma_{11}^* S_{22}} \sim N(0,1) \ \ (12)$$

Equations 2: Variables set up for defining generalized pivots for parameter of interest adapted from Johnson and Wichern (2008)

Define

$$b_{22} = \frac{s_{22}}{V_{22}} \ \ (13) \ \ , \ \ b_{12} = \frac{s_{12}}{V_{12}} - \sqrt{s *_{11} s_{22}} \frac{z_1}{\sqrt{V*_{11}}} \frac{1}{V_{22}} \ \ (14) \ \ , \ \ b_{11} = \frac{s*_{11}}{V*_{11}} + \frac{b_{12}^2}{b_{22}} \ \ (15)$$

Where s_{ij} is the observed S

Equations 3: Derived variables for generalized pivots for parameter of interest adapted from Johnson and Wichern (2008)

To define a generalized pivot for a parameter, these two conditions must be satisfied:

1. The distribution of the generalized pivot must be free of unknown parameters
2. The observed generalized pivot is the parameter of interest

The b_{ij}s above are free of unknown parameters and the observed values of b_{ij}s can be easily seen to equal to σ_{ij}s. Therefore, $b_{ij}s$ are the generalized pivots for $\Sigma = \sigma_{ij}$.

To calculate the generalized pivot for ρ, G_ρ, individual σ_{ij}s for each of the data points needs to be derived. This is done by comparing Y_{i1} and Y_{i2} through their means and variances. To do this, the following parameters are constructed:

$$\theta = (\mu_1 - \mu_2) + \frac{1}{2}(\sigma_{11} - \sigma_{22}) \tag{16}$$

Where θ is the log-normal-ratio of population means

$$\delta = e^{2\theta} \times \frac{e^{\sigma_{11}}-1}{e^{\sigma_{22}}-1} \tag{17}$$

Where δ is ratio of the variances

Define

$$T_{11} = (\bar{x}_1 - \bar{x}_2) - \frac{(\overline{X_1} - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_{11} - 2\sigma_{12} + \sigma_{22}}{n}}} \sqrt{\frac{b_{11} - 2b_{12} + b_{22}}{n}} = \frac{z_2}{\sqrt{n}} \sqrt{b_{11} - 2b_{12} + b_{22}} \quad (18)$$

Where $z_2 = \frac{(\overline{X_1} - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_{11} - 2\sigma_{12} + \sigma_{22}}{n}}} \sim N(0,1)$

Equations 4: Variables set up and derived variables for generalized pivots for parameter of interest adapted from Bebu and Matthew (2008)

T_{11} is free of unknown parameters so it qualifies as generalized pivots for $\mu_1 - \mu_2$. Likewise, $T_{12} = \frac{1}{2}(b_{11} - b_{22})$ is a generalized pivot for $\frac{1}{2}(\sigma_{11} - \sigma_{22})$. Since b_{ij}s have been established as generalized pivots for $\Sigma = \sigma_{ij}$ and that the objective for the calculation of θ is to calculate G_ρ, T_{12} can be expressed as $T_{12} = \frac{1}{2}(b_{11} - b_{22})$. Using (16), (17) and T_{12}, individual σ_{ij}s can be calculated through the following method:

Let $2\times T_{12} = k = \sigma_{11} - \sigma_{22}$ and $\frac{\sigma_{11}}{\sigma_{22}} = \sqrt{\delta}$ so that $\sigma_{11} = \sqrt{\delta}\sigma_{22}$,

By algebraic manipulation, individual σ_{ij}s are found to be:

$G\sigma_{11} = \frac{k}{\left(1 - \frac{1}{\sqrt{\delta}}\right)}$ (19) and $G\sigma_{22} = \frac{k}{(\sqrt{\delta} - 1)}$ (20)

Therefore, G_ρ can be defined with the following formula:

$$G_\rho = \frac{e^{G\sigma_{12}^2} - 1}{\sqrt{(e^{G\sigma_{11}^2} - 1)(e^{G\sigma_{22}^2} - 1)}} \quad (21)$$

Equations 5: Adaptation of ρ for individual correlations

The generalized test variable for ρ is $G_\rho - \rho$ and is used to define the confidence intervals of the correlations of the traffic variables. Alternatively, taking the appropriate percentiles from the series of calculated G_ρ also yield the sought confidence intervals. In this case, the usual normal confidence interval formula can be used. This is because the cumulative distribution function of the lognormal distribution follows normal distribution after taking the natural logarithm (ln) of the traffic variables. Below is the derivation:

If $Y \sim LN(\mu, \sigma^2)$, then

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right]$$

Where $f(x)$ is the probability distribution function of lognormal distribution

To find the cumulative distribution function, integrate $f(x)$, i.e.,

$$\int f(x)\,dx = \int_{-\infty}^{x_1} \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right] dx$$

Let $z = \frac{\ln x - \mu}{\sigma}$ so that $dx = x\sigma\,dz$ and substitute these terms to $\int f(x)\,dx$ to perform integration by substitution.

Therefore,

$$\int f(x)\,dx = \int_{-\infty}^{x_1} \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right] dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{z^2}{2}}\,dz$$

The function above is the cumulative distribution function of a normal distribution which proves the statement made above.

Works Cited

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Citation in text: (Bebu and Matthew, 2014)

Johnson, R. A., & Wichern, D. W. (2008). Applied multivariate statistical analysis, 6th edition. New York: Wiley.

Citation in text: (Johnson and Wichern, 2008)

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Russell, Marlene Beverley. (2014). *Travel Time Reliability and Level of Service*. LSU Master's Theses. 3747. Retrieved from http://digitalcommons.lsu.edu/gradschool_theses/3747

Citation in text: (Russell, 2014)

Zhang, G., & Chen, Z. (2014). *Inferences on correlation coefficients of bivariate log-normal distributions*. Journal of Applied Statistics, 42(3), 603-613. doi:10.1080/02664763.2014.980786

Citation in text: (Zhang and Chen, 2014)