CST8506 – Advanced Machine Learning

Week 4: Recurrent Neural Networks (RNNs)

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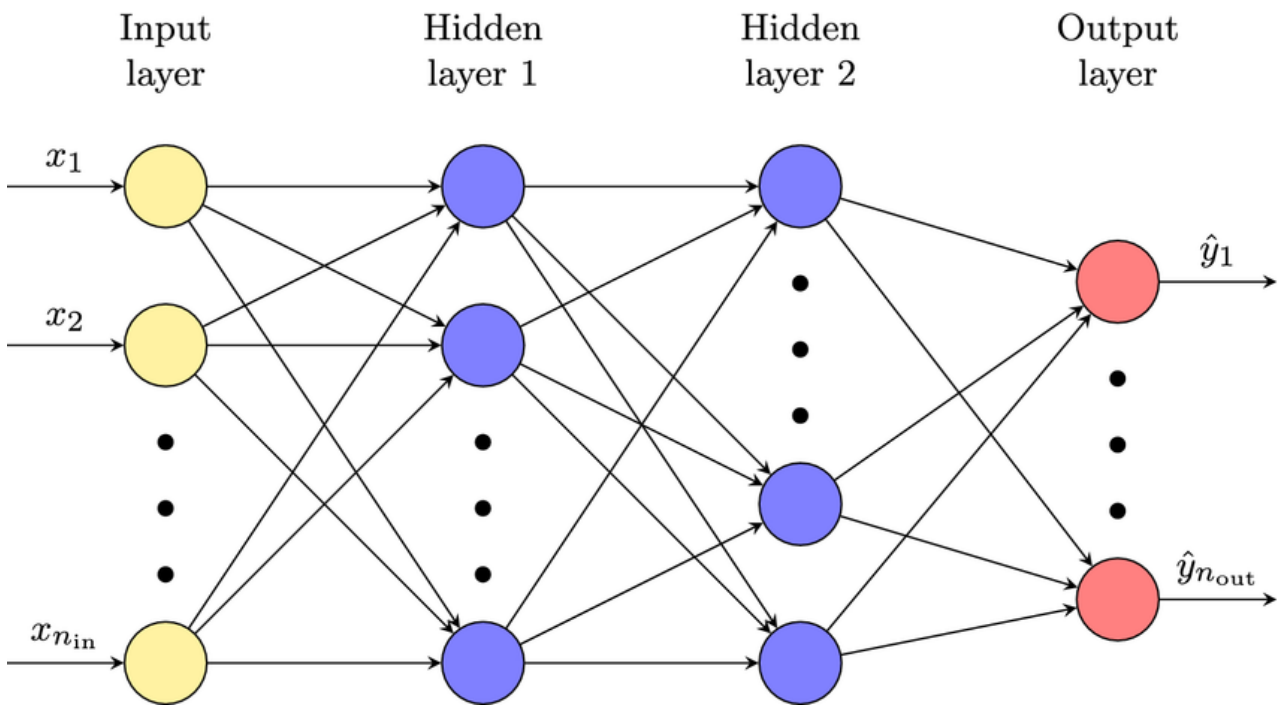
Winter 2025

- FNN– Review
- Motivation
- Usages of Sequential Data
- Time Series
- Time Series – Components
- Recurrent Neural Networks (RNNs)
- Backpropagation Refresher
- Backpropagation Through Time (BTT)
- Vanishing Gradient Problem
- Long-Short Term Memory

Review on Feed Forward Network

- Information flows only in the **forward direction**. No **cycles or Loops**.
- Decisions are based on current input, **no memory** about the past
- Doesn't know how to handle **sequential data**



Motivation

Questions:

- How Google's autocomplete feature predicts the next word when a user is typing?
- How Translators converting sentences from English to French?
- How Siri or Google Assistant converting spoken words into text?
- How AI composes melodies or generates background music?
- How it is possible to predict the future prices based on historical trends?
- Etc.

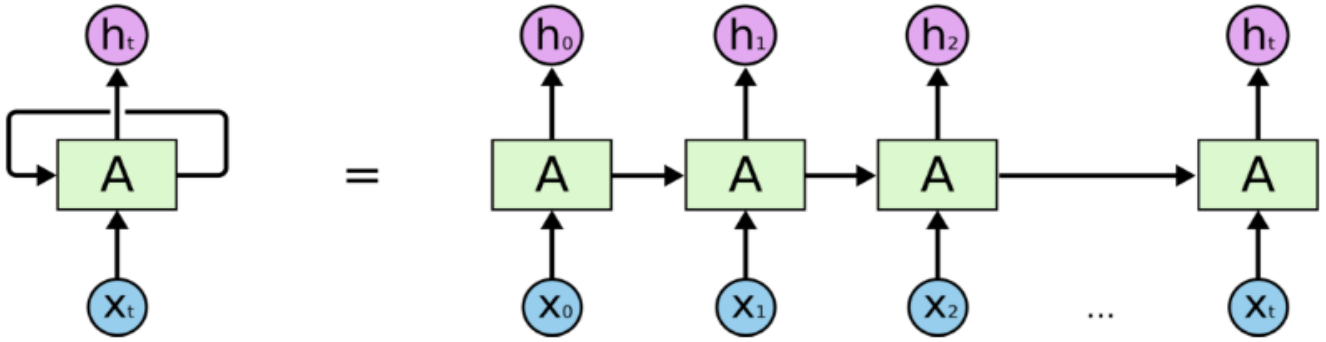
Motivation …

We need a model:

- To handle sequential data.
- Able to consider the **current input** also the **previously received** inputs.
- Able to memorize history in its internal memory.

FFNs cannot process the sequential data!

What is the solution?



Usages of Sequence Data - Examples

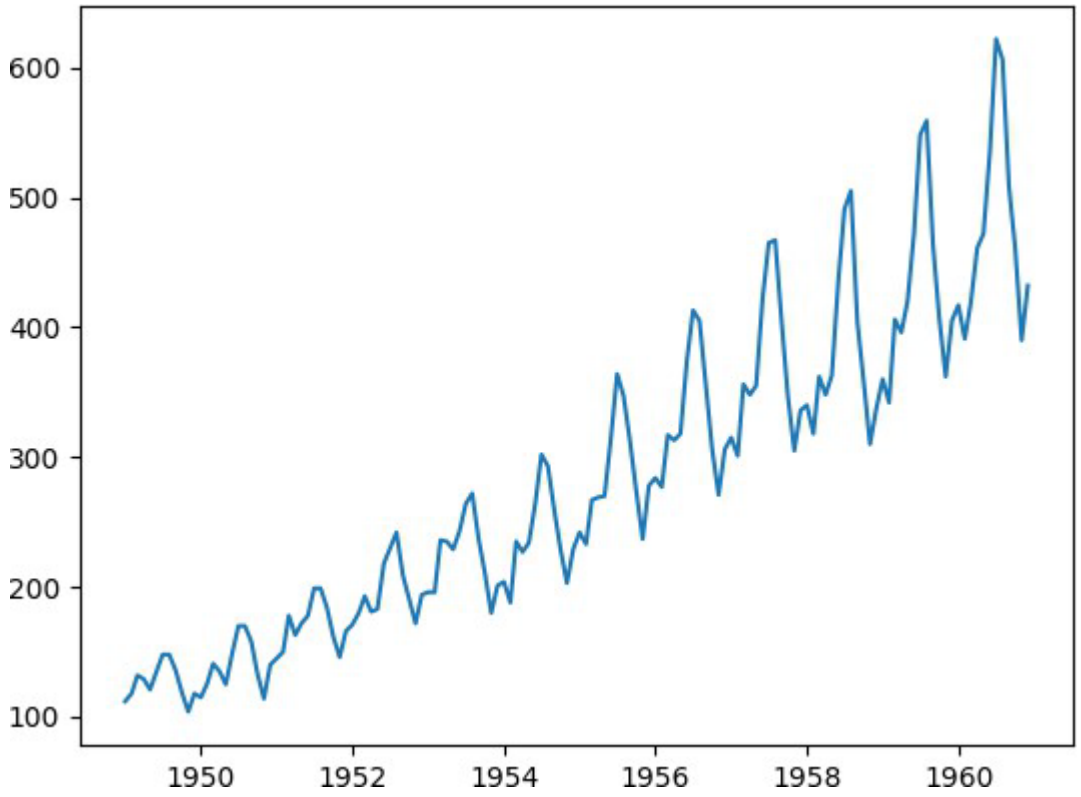
- Speech recognition (audio clip to text)
- Sentiment analysis (sequence of text to number of stars)
- DNA Sequence analysis
- Machine translation (sequence of text in one language translated to another)
- Video activity recognition (detect the activity from a sequence of video frames)
- **Time Series Forecasting**

Time Series

- A **Time Series** is a sequence of data points collected or recorded at specific time intervals.
- Unlike standard "cross-sectional" data (where you look at a snapshot of many things at once), time series focuses on **one(more) thing over a duration.**
  - **The X-Axis:** Almost always represents time (seconds, days, years).
  - **The Y-Axis:** The variable you are measuring (Price, Temperature, Population).
  - **The Goal:** To understand the past and, ideally, peer into the future (Forecasting).

Time Series - example

- Air Passengers
- Non-stationary data
  - Mean & sd changes with time
- Seasonal data
- Data from Jan 1949-Dec 1960



Taken from: https://www.kaggle.com/datasets/rakannimer/air-passengers

Time Series – Components

1. **Trend:** The long-term "direction." Is it generally going up, down, or staying flat?
2. **Seasonal :** Patterns that repeat over a fixed period (e.g., retail sales spiking every December).
3. **Cycle:** A cycle is a long-term fluctuation in a time series that repeats, but NOT at a fixed, regular interval.
4. **Noise (Residuals):** The random "hiccups" in the data that can't be explained by the other three.

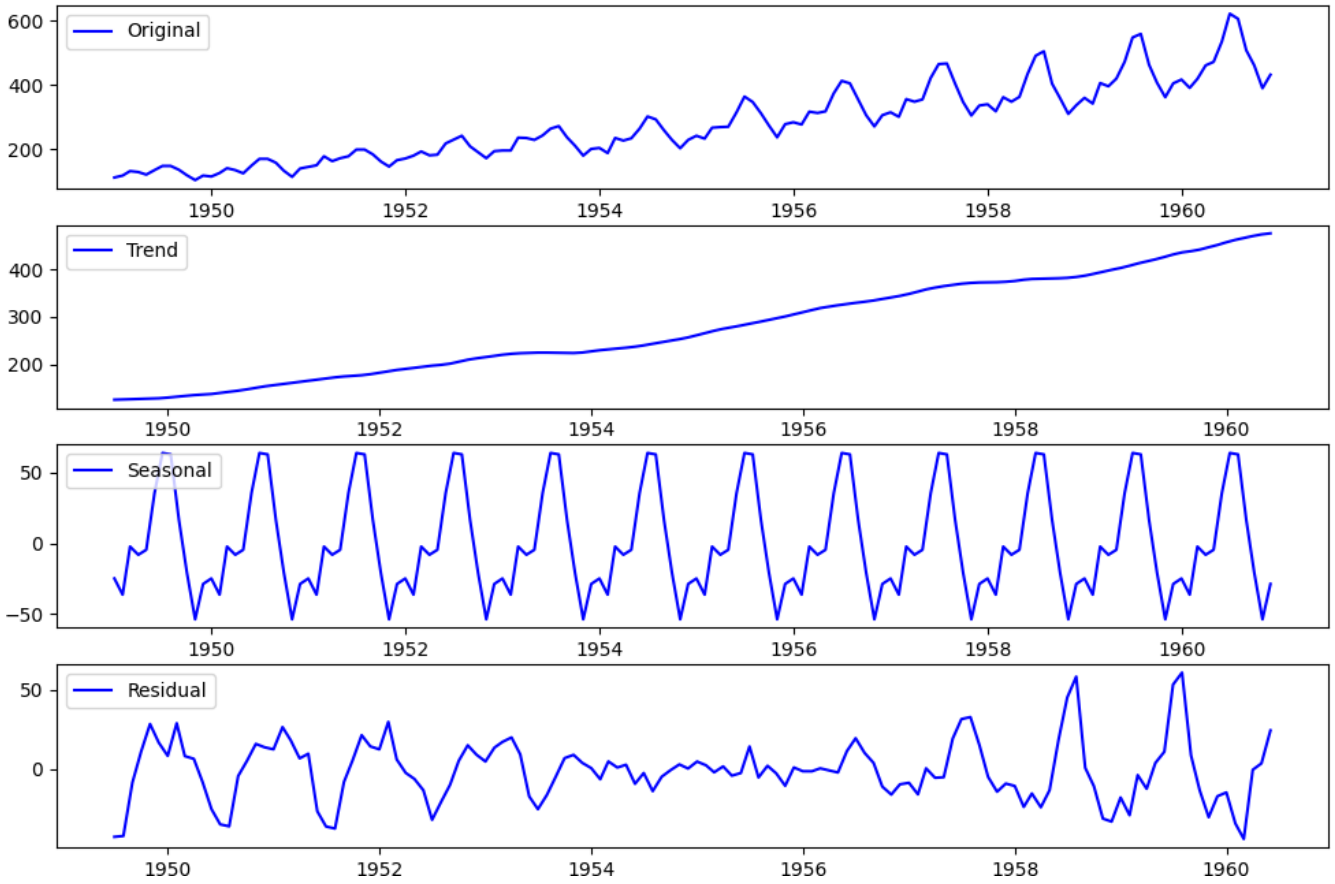
Time Series – Components

Original plot

Trend

Seasonal

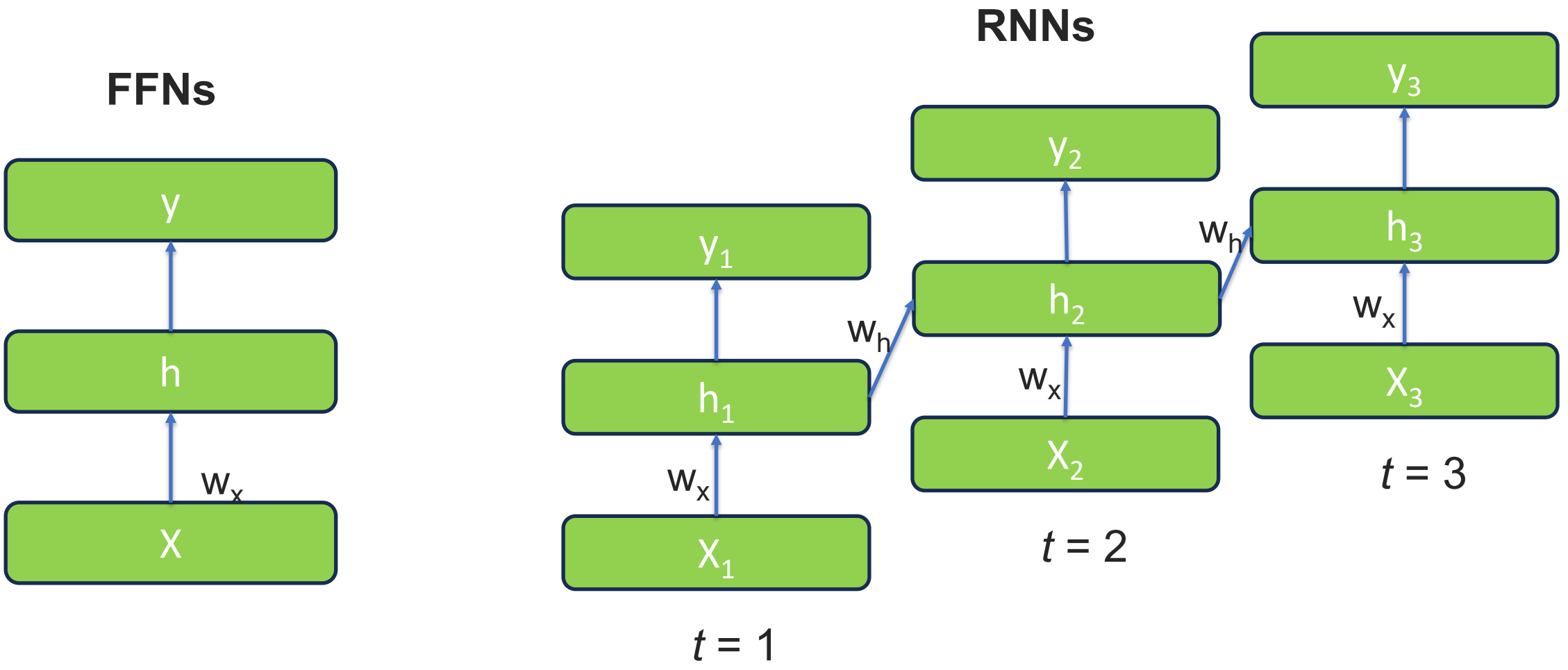
Residual



Recurrent Neural Networks (RNNs)

- RNNs are kind of DL models that takes the **previous output or hidden states** as inputs. i.e. the **composite input** at time *t* has some historical information about the happenings at time T < *t*.
- RNNs are useful as their **intermediate states** can store information about **past inputs** for a time that is not fixed.
- In RNNs, each input vector (e.g. word vector) is typically fed into the network **one at a time**, not all at once.

RNNs ...



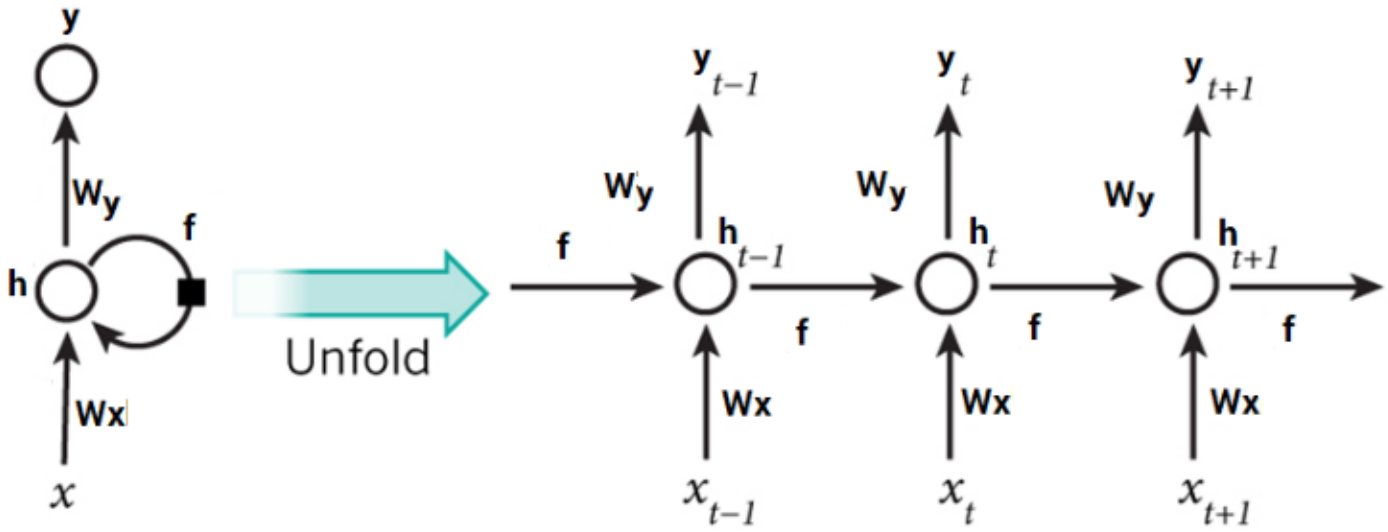
RNNs …

$h_t = f(W_x\, x_t + W_y\, h_{t-1})$

h_t : *hidden state at time step t*

x_t : *input at time step t*

W_x *and* W_y : *Weight matrices. Filters that determine the importance of the present input and past information.*



- Note that the weights are **shared over time**

  copies

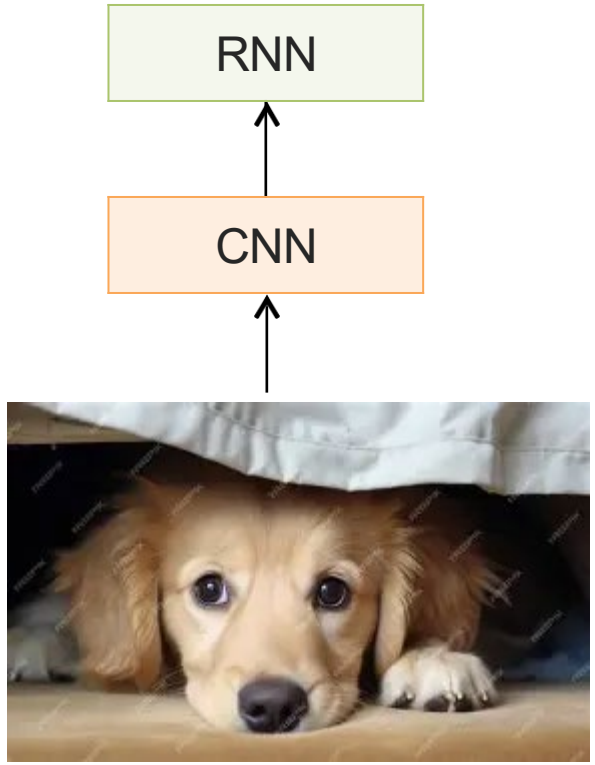
Example (Image captioning)

Problem: Given an image, produce a sentence describing its contents

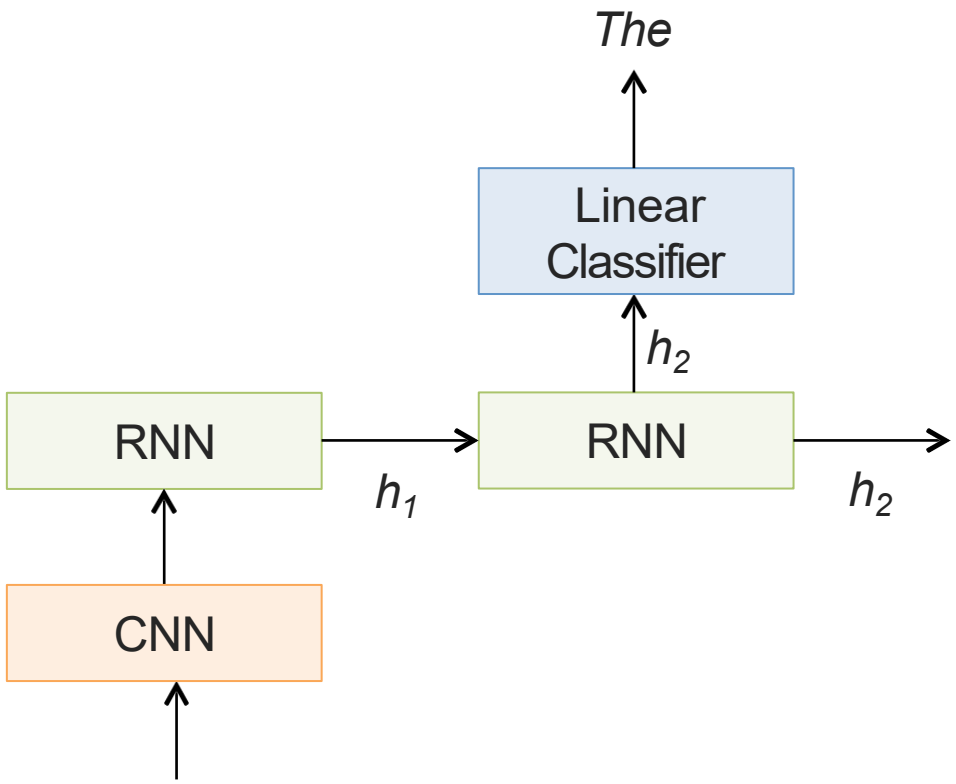
- **Inputs:** Image feature (from a CNN)
- **Outputs:** Multiple words

The dog is hiding

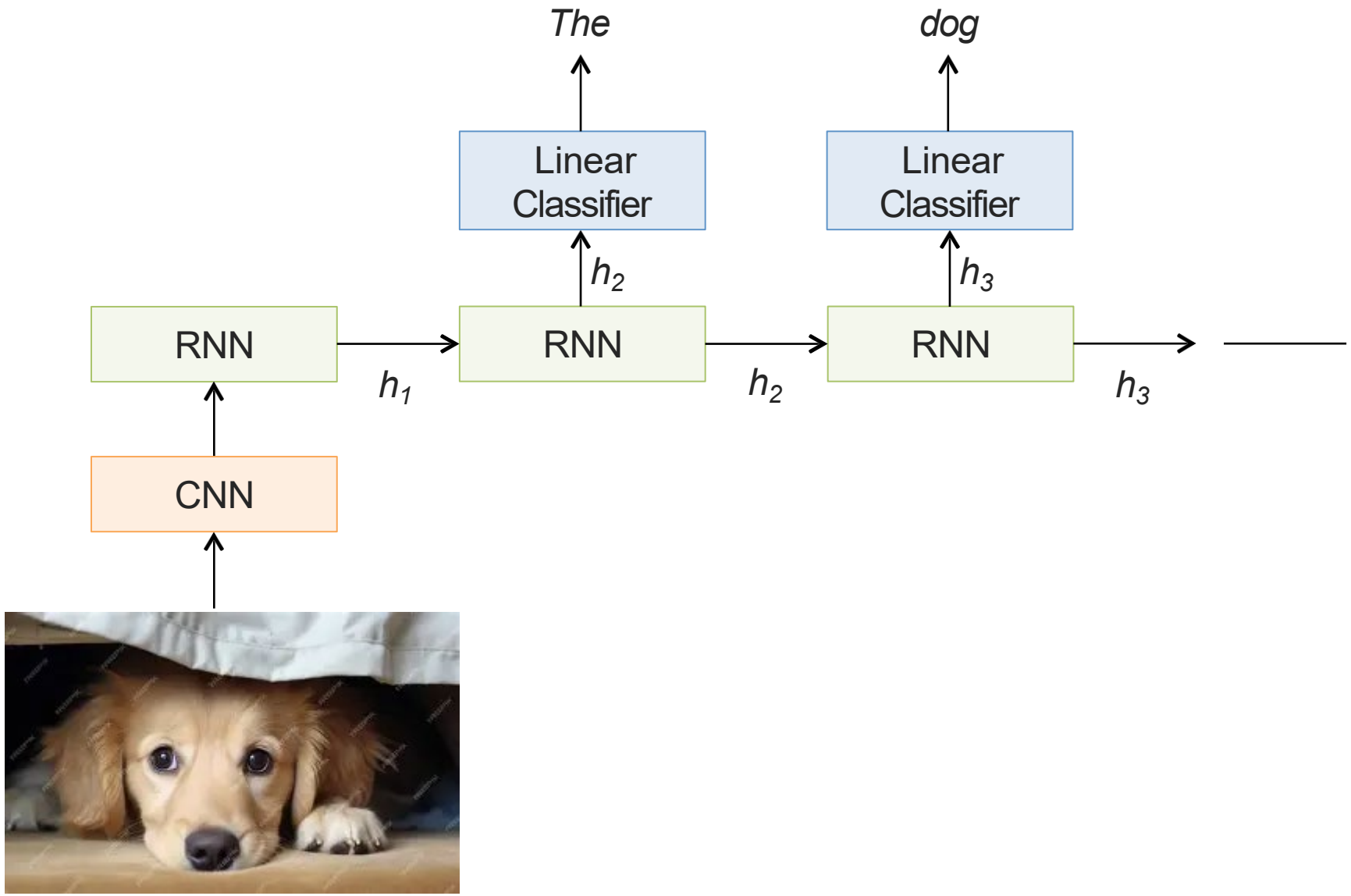
Example (Image Captioning)



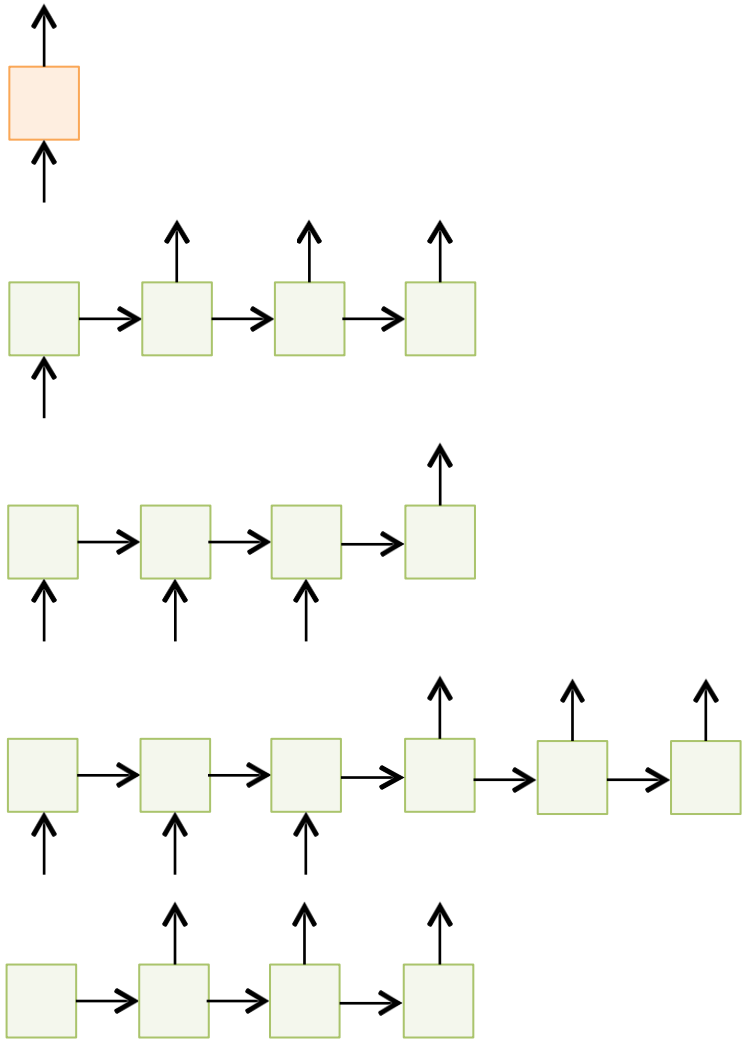
Example (Image Captioning)



Example (Image Captioning)



Input-output Scenarios



Loss Functions

- Method to evaluate how well an algorithm models the given data
- Quantifies the error between the output and the target
- Also known as cost function or error function
  - Regression Losses
  - Probabilistic Losses
  - Hinge Losses for maximum-margin classification
- https://keras.io/api/losses/

Regression Loss Functions

- Mean Square Error (MSE) / Quadratic Loss / L2 Loss
  - Average of the sum of the squared differences between actual value and the predicted value
- Mean Absolute Error (MAE) / L1 Loss
  - Average of the sum of the absolute differences between actual value and the predicted value
  - Robust to outliers since it does not make use of square
- Mean Bias Error
  - Average of the sum of the differences between actual value and the predicted value
  - Positive and negative values may cancel out – less accurate in practice
  - Can be used to see whether model has positive or negative bias

MSE & MAE

$w = -266.5344 + 6.1376h$

		x	y_i	y_i'	$y_i - y_i'$	$(y_i - y_i')^2$
-266.53	6.1376	63	127	120.1388	6.8612	47.076065
-266.53	6.1376	64	121	126.2764	-5.2764	27.840397
-266.53	6.1376	66	142	138.5516	3.4484	11.891463
-266.53	6.1376	69	157	156.9644	0.0356	0.0012674
-266.53	6.1376	69	162	156.9644	5.0356	25.357267
-266.53	6.1376	71	156	169.2396	-13.2396	175.28701
-266.53	6.1376	71	169	169.2396	-0.2396	0.0574082
-266.53	6.1376	72	165	175.3772	-10.3772	107.68628
-266.53	6.1376	73	181	181.5148	-0.5148	0.265019
-266.53	6.1376	75	208	193.79	14.21	201.9241
					Total	597.38627

Prediction Error	Absolute Error	Squared Error	
6.8612	6.8612	47.07607	
-5.2764	5.2764	27.8404	
3.4484	3.4484	11.89146	
0.0356	0.0356	0.001267	
5.0356	5.0356	25.35727	
-13.2396	13.2396	175.287	
-0.2396	0.2396	0.057408	
-10.3772	10.3772	107.6863	
-0.5148	0.5148	0.265019	
14.21	14.21	201.9241	
Mean Absolute Error	5.92384	7.729077	**Root Mean Squared Error**
		59.73863	**Mean Square Error (MSE)**

Probabilistic Loss Functions

Used when a model predicts probabilities for different classes instead of class labels

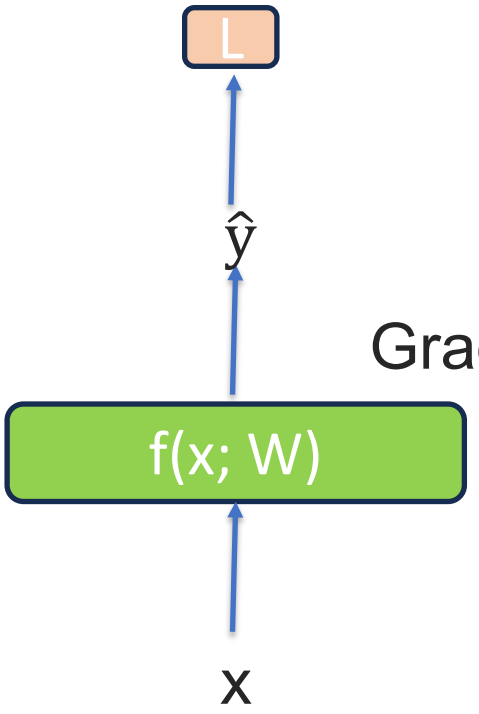
- Cross Entropy (also known as log loss): measure of the difference between two probability distributions (predicted vs actual).
  - Binary Cross Entropy (two classes – 0 and 1 as class labels)
  - Categorical Cross Entropy (one-hot encoded class labels)
  - Sparse Categorical Cross Entropy (integers as class labels)

https://www.youtube.com/watch?v=Pwgpl9mKars&ab_channel=AdianLiusie
https://www.youtube.com/watch?v=Md4b67HvmRo&ab_channel=DigitalSreeni
https://www.youtube.com/watch?v=6ArSys5qHAU&ab_channel=StatQuestwithJoshStarmer

Hinge Loss

- Primarily for classification tasks, especially with SVMs
- Helps maximizes the margin between different classes
- Loss is 0 when the correct class is confidently predicted, but penalizes predictions that are too close to the decision boundary
- Requires labels to be -1 and +1 (instead of 0 and 1)
- For multi-class classification: Categorical Hinge Loss
- Can be used in NN

Backpropagation Refresher

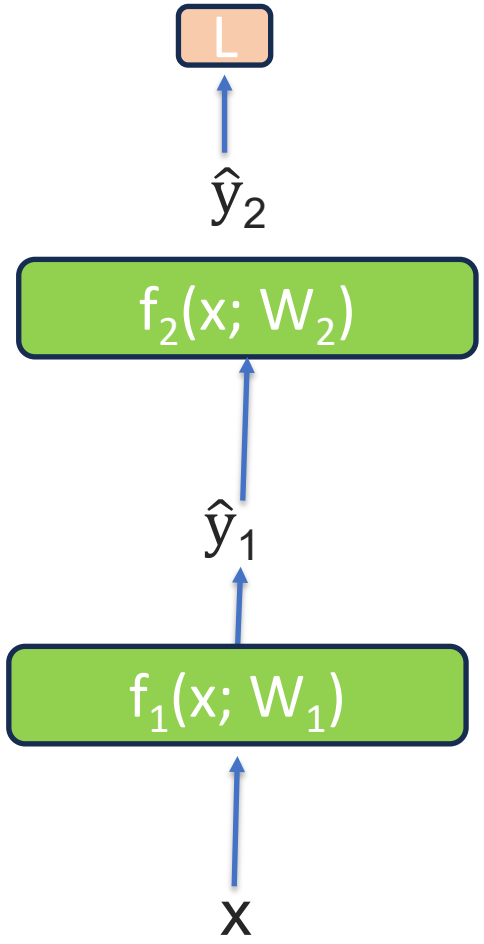


$$\hat{y} = f(x; W)$$
$$L = Loss(y, \hat{y})$$

Gradient Descent:

$$W = W - \alpha \frac{\partial L}{\partial W}$$

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial W}$$



$$\hat{y}_1 = f_1(x; W_1)$$
$$\hat{y}_2 = f_2(\hat{y}_1; W_2)$$
$$L = Loss(y, \hat{y}_2)$$

Gradient Descent:

$$W_1 = W_1 - \alpha \frac{\partial L}{\partial W_1}$$

$$W_2 = W_2 - \alpha \frac{\partial L}{\partial W_2}$$

$$\frac{\partial L}{\partial W_2} = \frac{\partial L}{\partial \hat{y}_2} \cdot \frac{\partial \hat{y}_2}{\partial W_2}$$

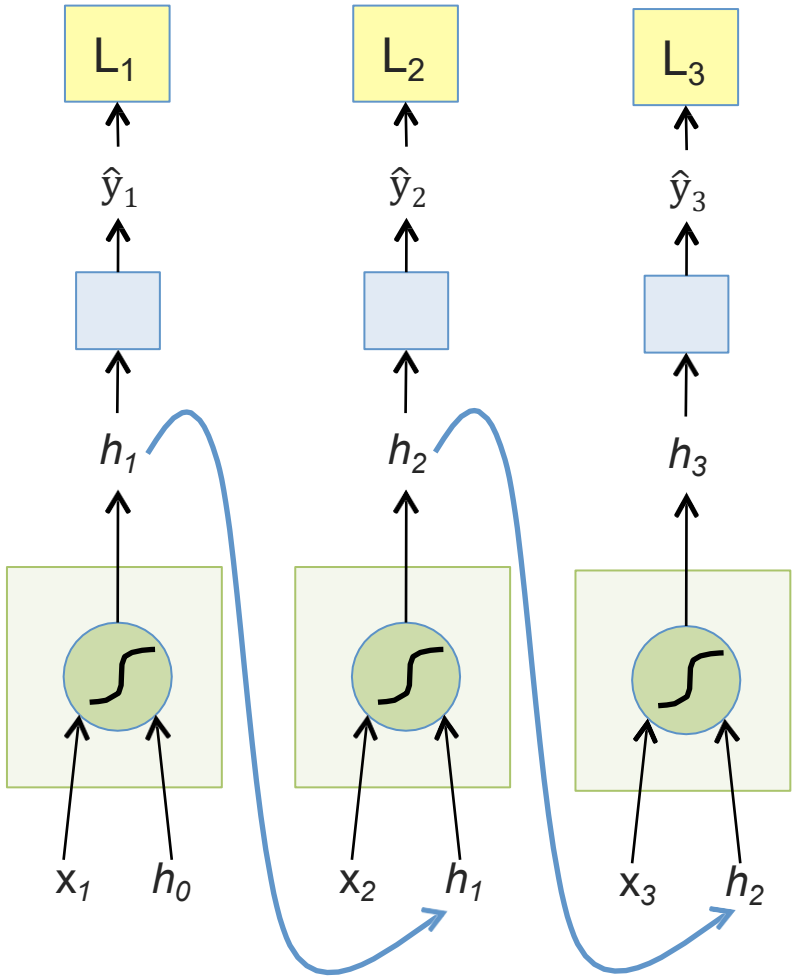
$$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial \hat{y}_1} \cdot \frac{\partial \hat{y}_1}{\partial W_1}$$

$$= \frac{\partial L}{\partial \hat{y}_2} \cdot \frac{\partial \hat{y}_2}{\partial \hat{y}_1} \cdot \frac{\partial \hat{y}_1}{\partial W_1}$$

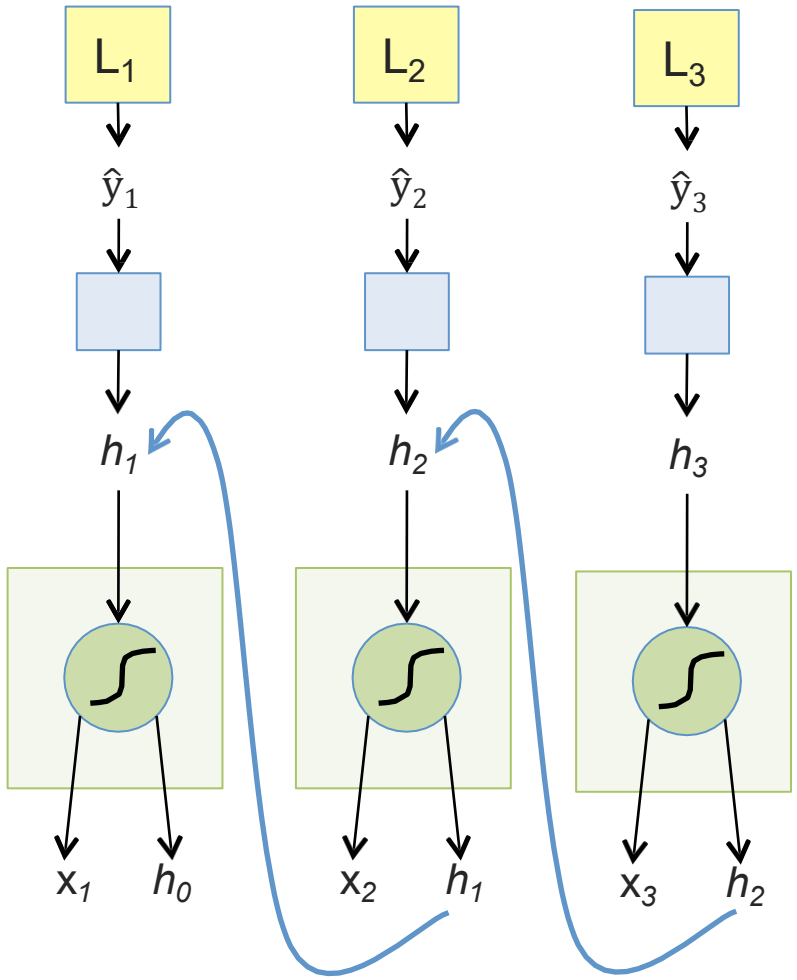
Backpropagation Through Time (BPTT)

- In a normal neural network, we use **backpropagation** to update weights by calculating gradients layer by layer.
- In an RNN, the same weights are used at every time step, and the network is "unrolled" across time steps.
- **BPTT means we compute gradients across all these time steps and update the shared weights.**
- The **weight updates** are computed for **each copy** in the **unfolded network**, then **summed** (or averaged) and then applied to the RNN weights.

BPTT – Unfolded RNN - Forward



BPTT – Unfolded RNN - Backward



$$\frac{\partial Lt}{\partial h_1} = \frac{\partial Lt}{\partial \hat{y}_t} * \frac{\partial \hat{y}_t}{\partial h_1}$$

$$= \frac{\partial Lt}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial h_t} \cdot \frac{\partial h_t}{\partial h_{t\_1}} \cdots \frac{\partial h_2}{\partial h_1}$$

Problems with the Vanilla RNN

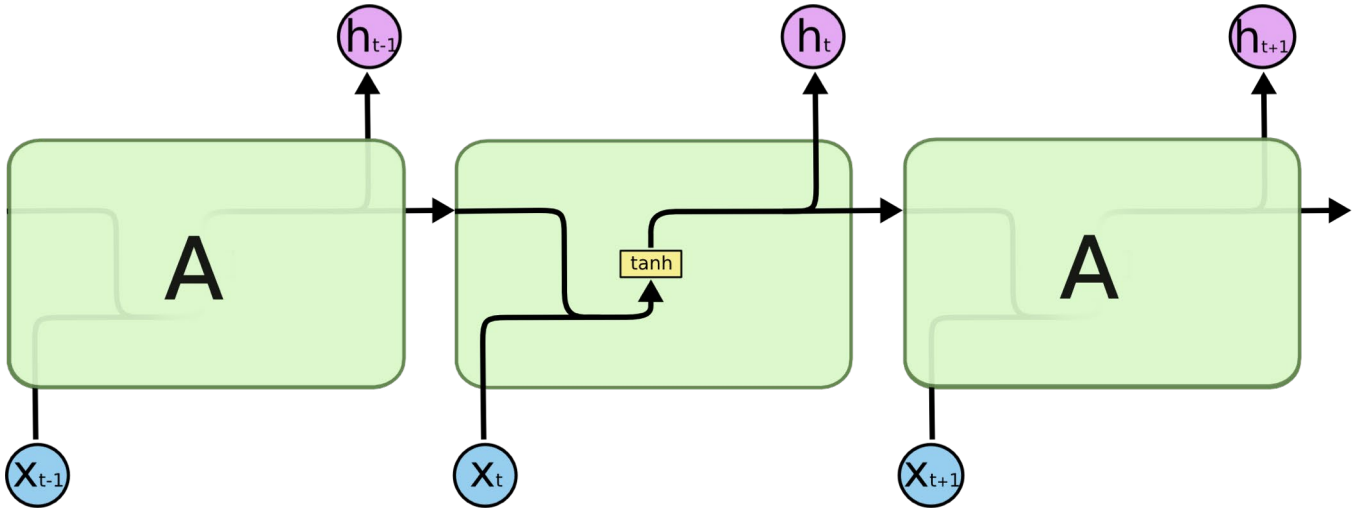
- In the same way a product of k real numbers can shrink to zero or explode to infinity, so can a product of matrices
- Vanishing gradient causes:
  - Gradients become extremely small as they propagate backward.
  - The first layers (or earliest time steps in RNN) receive almost no updates.
  - The network fails to learn long-term dependencies.

Solutions to Avoid Vanishing Gradient Problem

1) **Use Gated Architectures (LSTM / GRU)**
2) **Gradient Clipping** - Prevents gradients from becoming too small or too large.
3) **Use Activation Functions Carefully** - functions like **ReLU** (instead of tanh or sigmoid) do not squash values as much
4) **Layer Normalization / Batch Normalization** - Normalizes activations to keep values in a stable range.
5) **Use Shorter Sequences** - Backpropagating through fewer time steps reduces gradient decay.

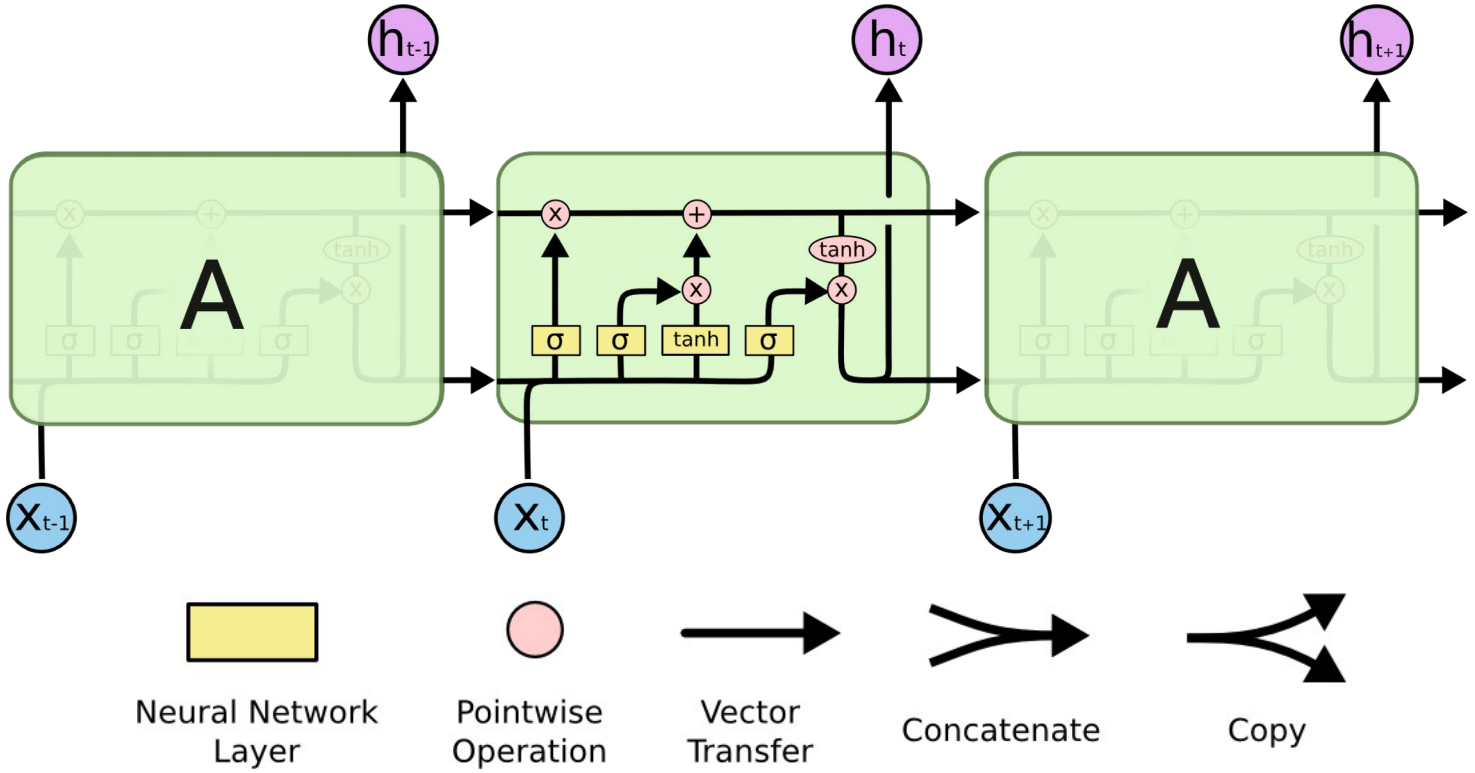
Long Short Term Memory (LSTM)

- **Long Short Term Memory networks** – usually just called "**LSTMs**" – are a special kind of RNN, capable of learning **long-term dependencies**. *Hochreiter & Schmidhuber (1997)*



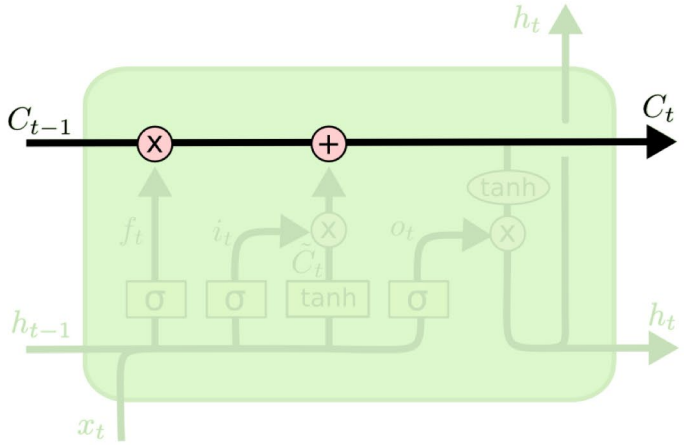
Long Short-Term Memory (LSTM)

The repeating module in a standard LSTM contains a single layer

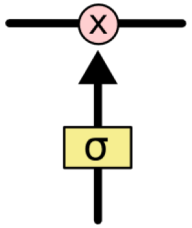


Long Short-Term Memory (LSTM)

- The core idea behind LSTMs is the **cell state**.



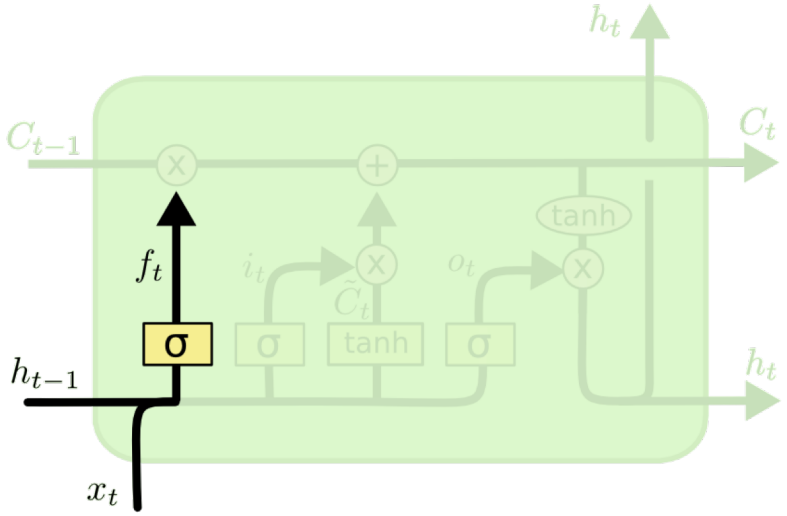
- The LSTM has the ability to **remove** or **add** information to the cell state : thanks to **gates**



- Gates are composed out of a sigmoid neural net layer and a pointwise multiplication operation

Long Short-Term Memory (LSTM)

- Step-by-Step LSTM Walk Through
  - **Step 1:** Decide what information to **throw away** from the cell state, **forget layer.**

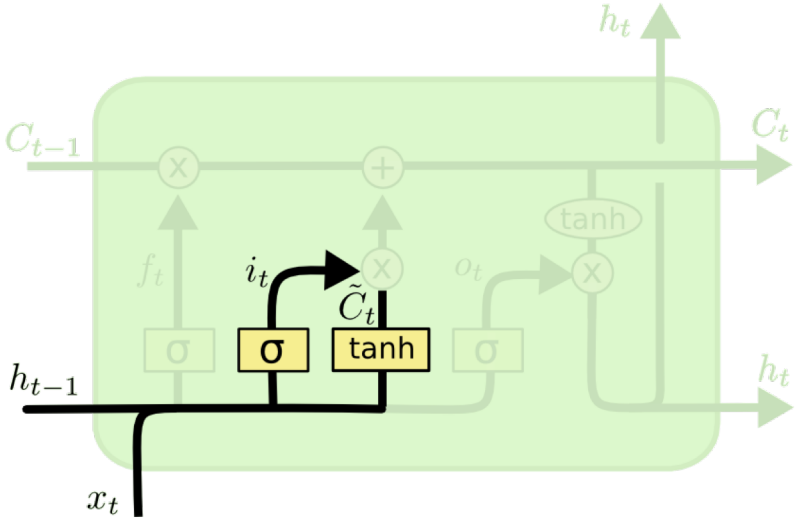


$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \; + \; b_f\right)$$

  - **1** represents “completely keep this”
  - **0** represents “completely get rid of this.”

Long Short-Term Memory (LSTM)

- Step-by-Step LSTM Walk Through
  - **Step 2**: Decide what new information we're going to store in the cell state



$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \; + \; b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \; + \; b_C)$$

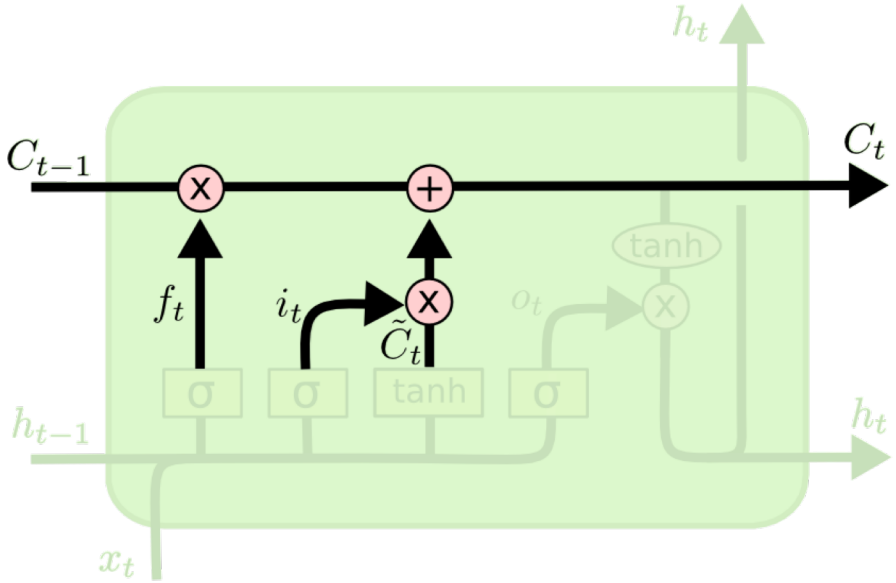
  - **Input gate layer** : decides which values we will update
  - **Tanh layer** : creates a vector of new candidate values

- **Example** : "I grew up in France… I speak fluent *French*."

Long Short-Term Memory (LSTM)

- Step-by-Step LSTM Walk Through

- **Step 3**: Update the cell state

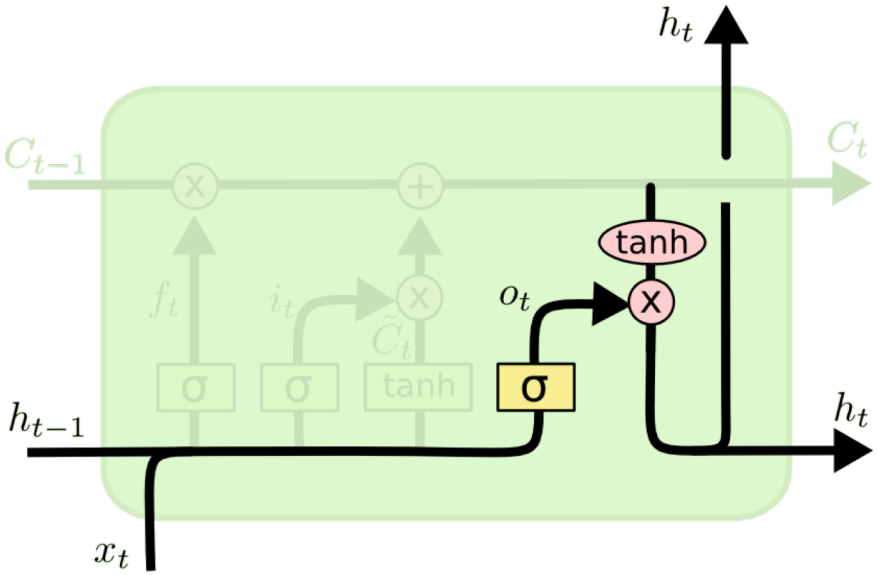


$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

- **Example** : "I grew up in France… I speak fluent *French*."

Long Short-Term Memory (LSTM)

- Step-by-Step LSTM Walk Through
  - **Step 4**: Decide what is the output



$$o_t = \sigma\left(W_o\left[h_{t-1}, x_t\right] + b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

- **Example** : "I grew up in France… I speak fluent *French*."

Summary

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- Backpropagation Through Time (BTT)
- Vanishing Gradient Problem
- Long-Short Term Memory


WHY?
WHAT?
WHERE?
WHEN?
WHO?
HOW?
WHICH?
WHEN? WHERE?
WHAT? HOW? WHY?
WHY? HOW?
WHO? WHERE? WHAT? HOW?
WHY? HOW? WHERE?
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