CST8506 ADVANCED MACHINE LEARNING

Week 2

Support Vector Machines

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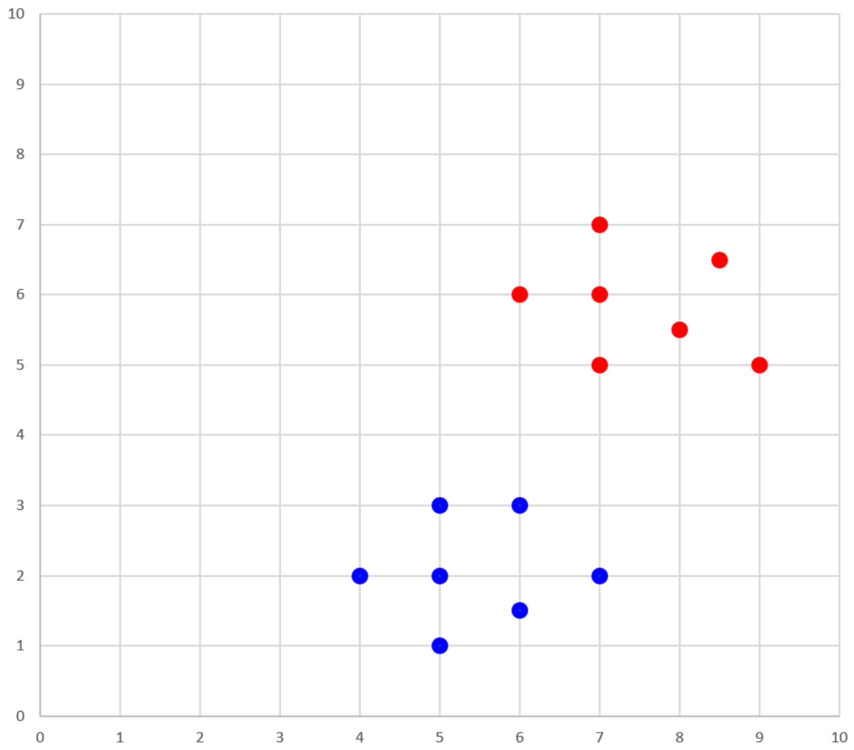
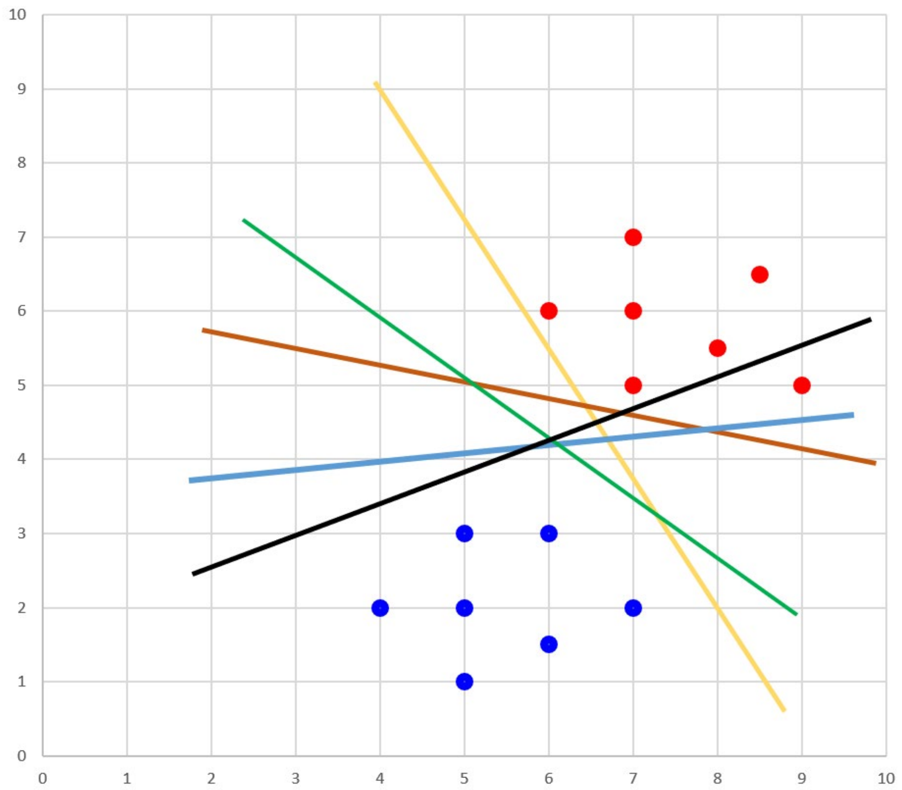
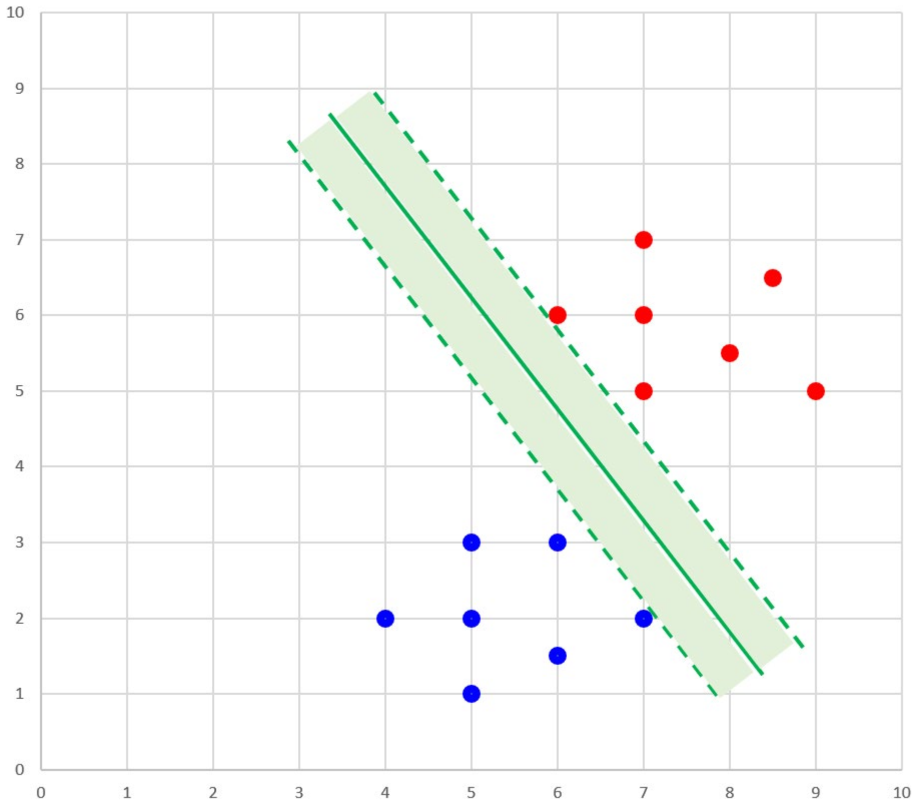
Linear Separators



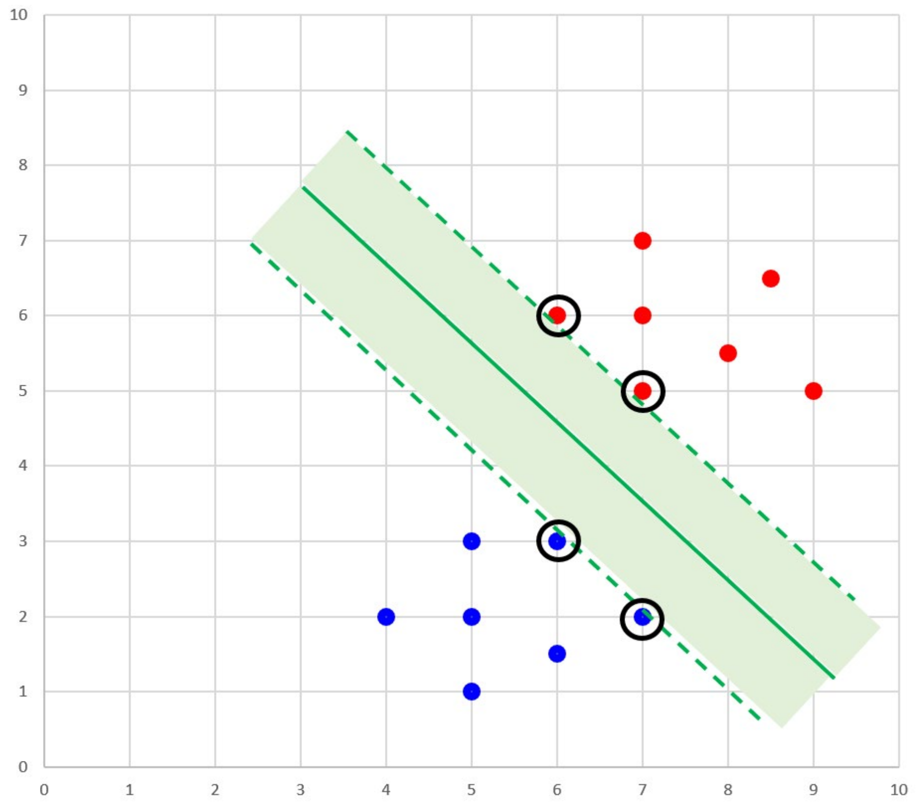
Linear Separators – which one is optimal?



Classification Margin



Classification Margin



Support vectors marked in circle

Classification Margin

Distance between the hyperplane and the vectors closest to the hyperplane (support vectors)

Support Vectors

Vectors (data points) that :

- Are closer to the hyperplane
- Can influence the position and the orientation of the hyperplane

Using the support vectors, we maximize the classification margin

Support Vector Machine (SVM)

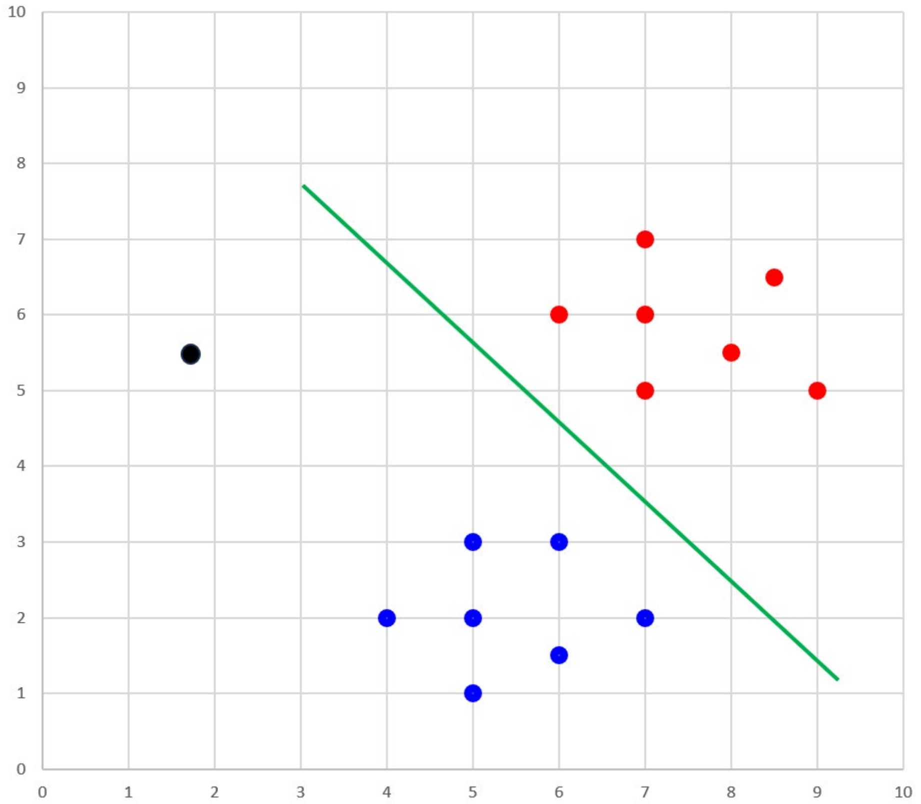
Objective: find a hyperplane in an n-dimensional space (n is the number of features) that has the maximum margin (that can distinctly classify the instances)

- If n is 1, classifier will be a dot
- If n is 2, classifier will be a line
- If n is 3, classifier will be a 2d plane
- If n>3, classifier will be a hyperplane in the n-dimensional space

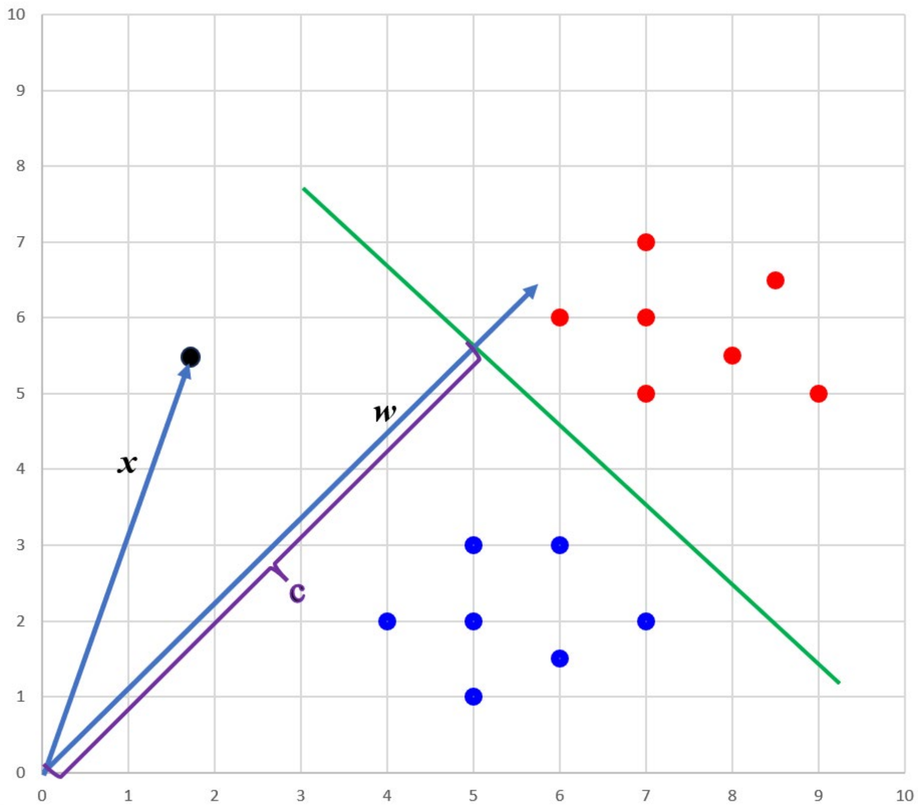
SVM is a supervised algorithm that works best on small complex datasets.

SVM can be used for classification and regression tasks but generally used more for classification.

Example – How to predict for black point?



Example – How to predict for black point?



Vector **w** is perpendicular to the green line.

The projection of any vector or another vector is called dot-product.

Vector $\boldsymbol{x}$ is projected on vector $\boldsymbol{w}$.

If $\boldsymbol{x} \cdot \boldsymbol{w} = c, then\ the\ point\ x\ lies\ on\ the\ line,$
If $\boldsymbol{x} \cdot \boldsymbol{w} > c, then\ the\ point\ x\ lies\ on\ right\ side$ of line
If $\boldsymbol{x} \cdot \boldsymbol{w} < c, then\ the\ point\ x\ lies\ on\ right\ side$ of line

For **positive samples,** $\boldsymbol{x} \cdot \boldsymbol{w} > c$

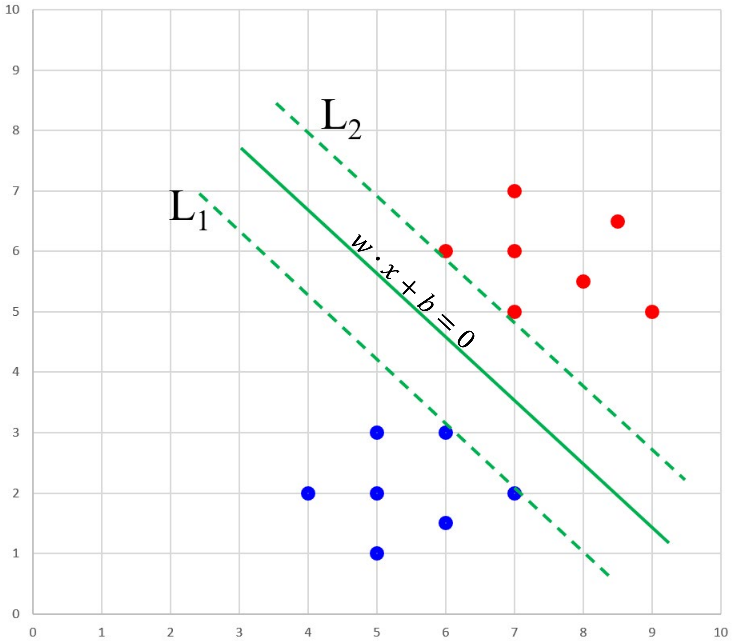
$$\boldsymbol{x} \cdot \boldsymbol{w} - c > 0$$

Set b = –c, $\boldsymbol{x} \cdot \boldsymbol{w} + b > 0$

$$y = \begin{cases} +1 \quad if \quad w \cdot x + b \geq 0 \\ -1 \quad if \quad w \cdot x + b < 0 \end{cases}$$

Optimization Function and its Constraints

We need to find a **w** and **b** for the hyperplane $\boldsymbol{w} \cdot \boldsymbol{x} + \boldsymbol{b} = 0$ such that the margin d is maximum.



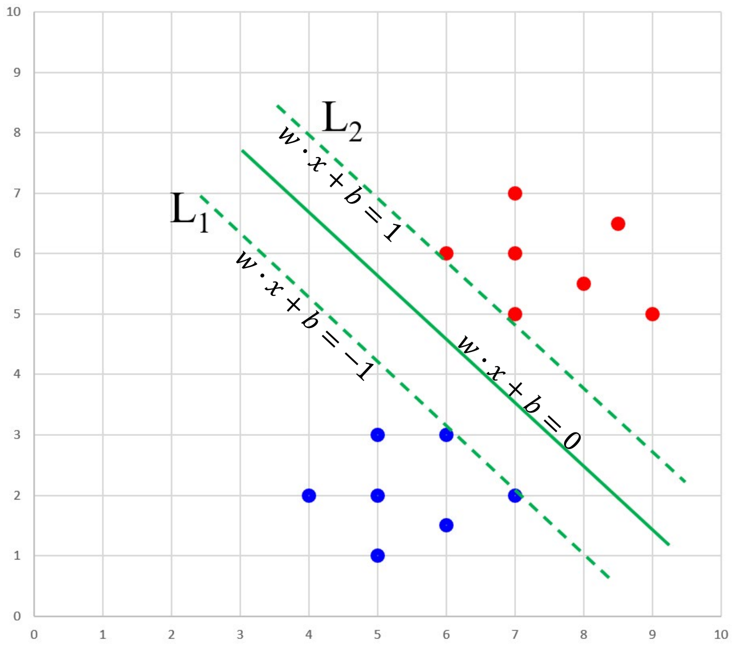
$$y = \begin{cases} +1 & if \quad w \cdot x + b \geq 0 \\ -1 & if \quad w \cdot x + b < 0 \end{cases}$$

Let L_1 be $w \cdot x + b = -1$

Let L_2 be $w \cdot x + b = \ \ 1$

Optimization Function and its Constraints

Let's consider blue points as +1 and red points as -1.



L_1: $w \cdot x + b = -1$
L_2 : $w \cdot x + b = \quad 1$

For red points, $w \cdot x + b \;\geq\; 1$
For blue points, $w \cdot x + b \;\leq -1$

$y_i(w \cdot x + b) \geq 1$, where $y_i = 1\ for\ red$
$and\ -1\ for\ blue$

Distance between two hyperplanes

Distance between two parallel hyperplanes $w \cdot x + b = c$ and $w \cdot x + b = -c$ is $d = \frac{|c_2 - c_1|}{\|w\|}$, where w is the Euclidean norm of the weight vector w.

Euclidean norm measures the "length" or "magnitude" of a vector in Euclidean space.

$$\|w\| = \sqrt{w_1^2 + w_2^2 + w_3^2 + \cdots + w_n^2}$$

Distance between $w \cdot x + b = 1$ and $w \cdot x + b = -1$ is $\frac{2}{\|w\|}$.

Optimization Function and its Constraints (Contd.)

The goal when training an SVM is

- Maximize $\frac{2}{\|w\|}$
- Subject to the constraint $y_i(w \cdot x + b) \geq 1$

Maximize $\frac{2}{\|w\|}$ means minimize $\|w\|$.

In this, we are not allowing any misclassifications – hard margin.
This method is called Maximum Margin Classifier (MMC).

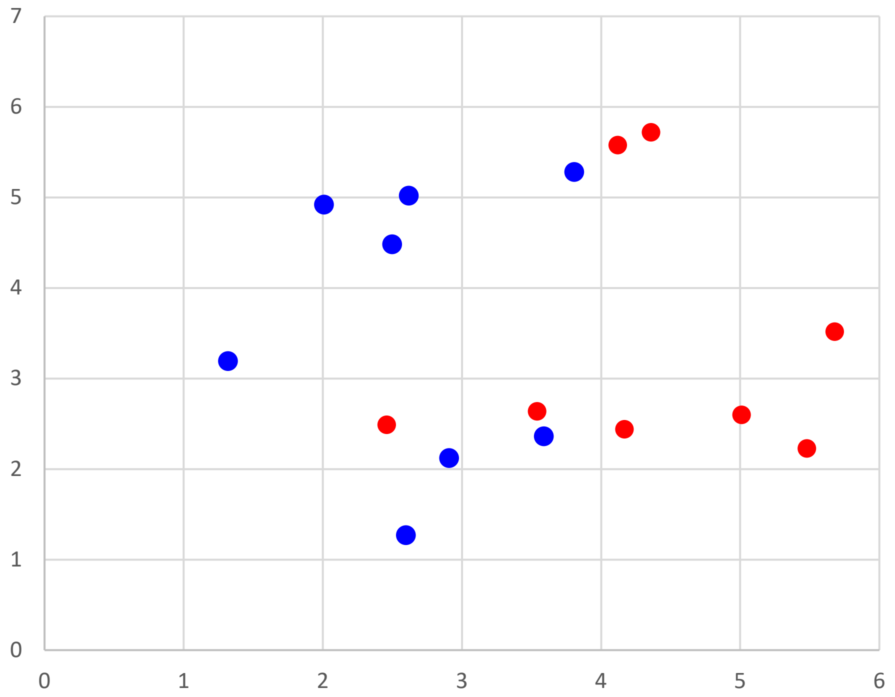
Types of SVM

- Linear SVM (LSVM) – when the data is linearly separable
- Non-linear SVM – data cannot be separated into 2 classes using a straight line.

Non-linear & Inseparable classes

When the data is not separable, we cannot separate them with linear classifiers.

We need to use soft-margin instead of hard margin – by allowing a few misclassifications.

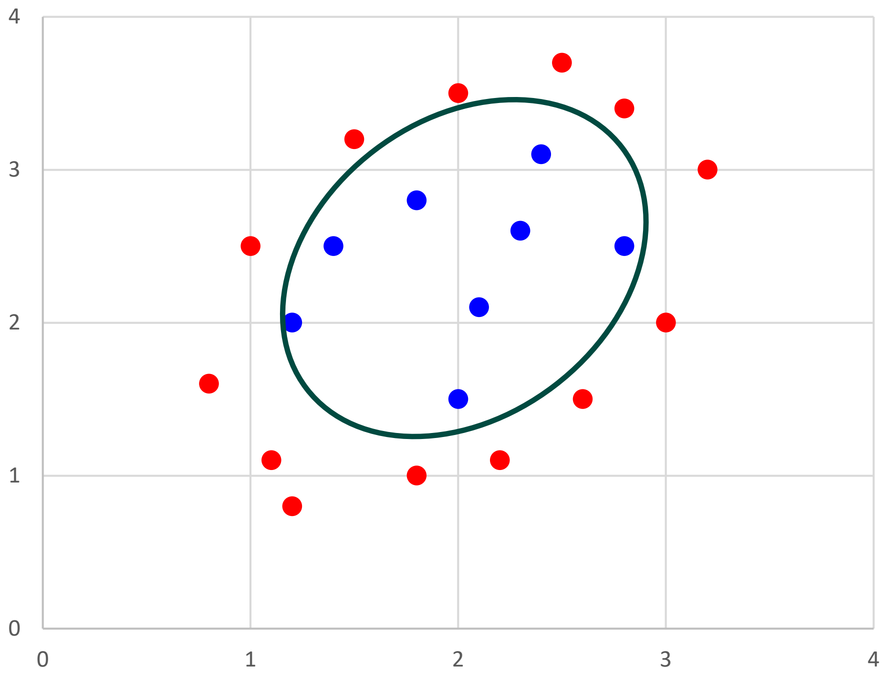


This method is called Support Vector Classifier (SVC).

Non-linear & Inseparable classes

When the data is not separable like this, we cannot separate them with linear classifiers.

We need to transform the low-dimensional data into a higher dimensional space, but this is computationally expensive. We can achieve similar results using kernels.

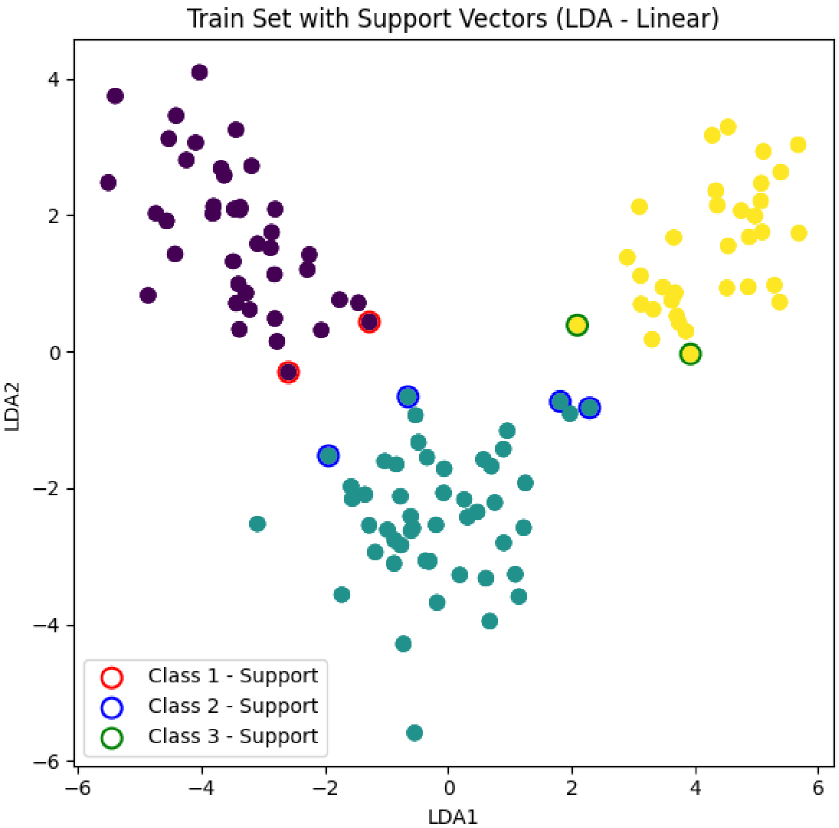


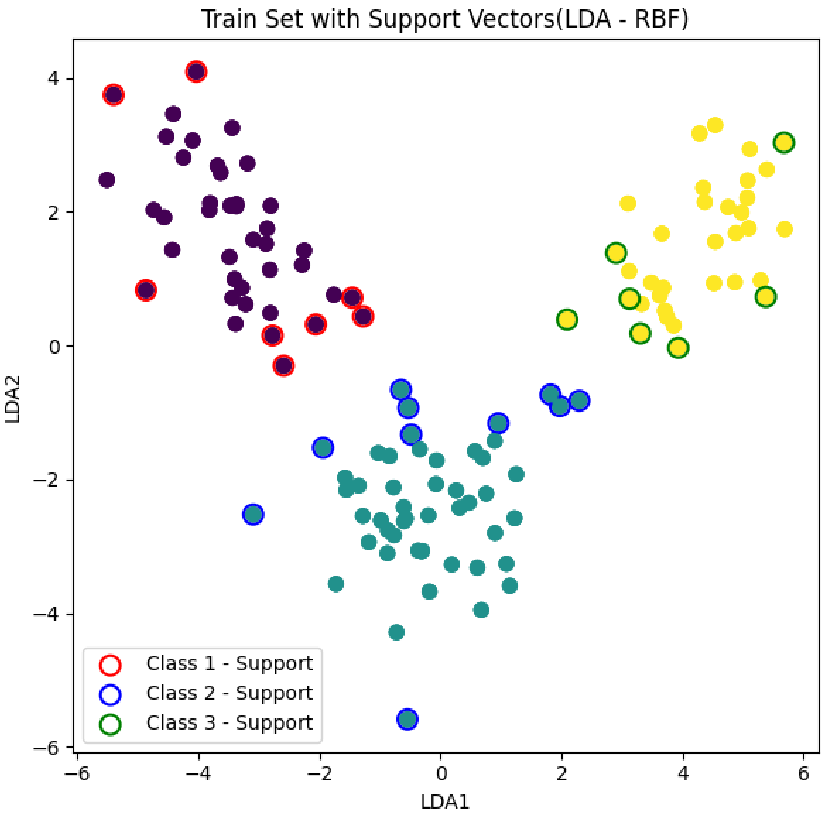
Kernel

Kernel is a function that quantifies the similarities between observations by summarizing the relationship between every instance in the dataset. This will transform data into higher dimensions without going into higher dimensions by computing dot products in a high-dimensional feature space without explicitly mapping the data to that space.

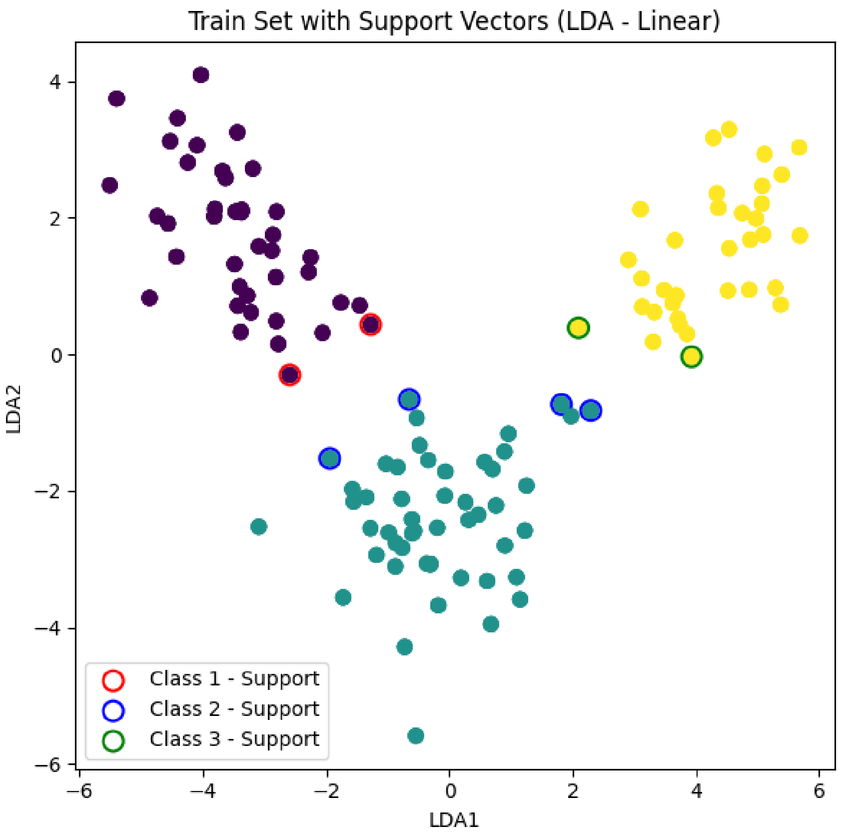
1. Polynomial: generalized form of linear kernel. Useful for non-linear hyperplane.
2. Radial Basis Function (Gaussian): can map an input space to infinite dimensional space (widely used)
3. Sigmoid: rarely used, sometimes, works for specific datasets

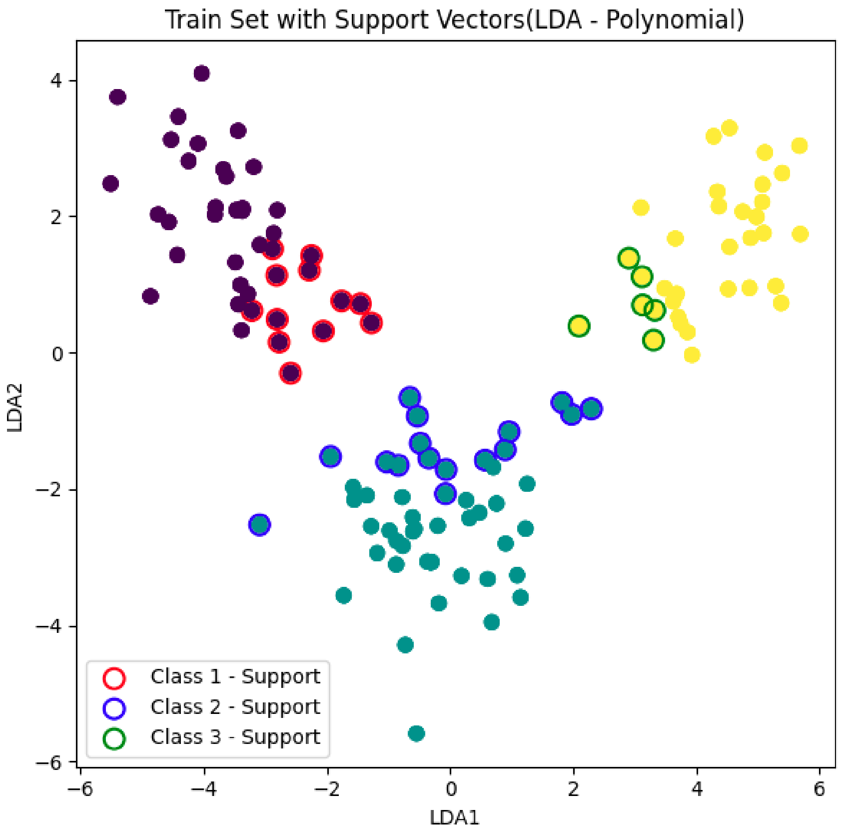
Example: Linear kernel vs RBF kernel



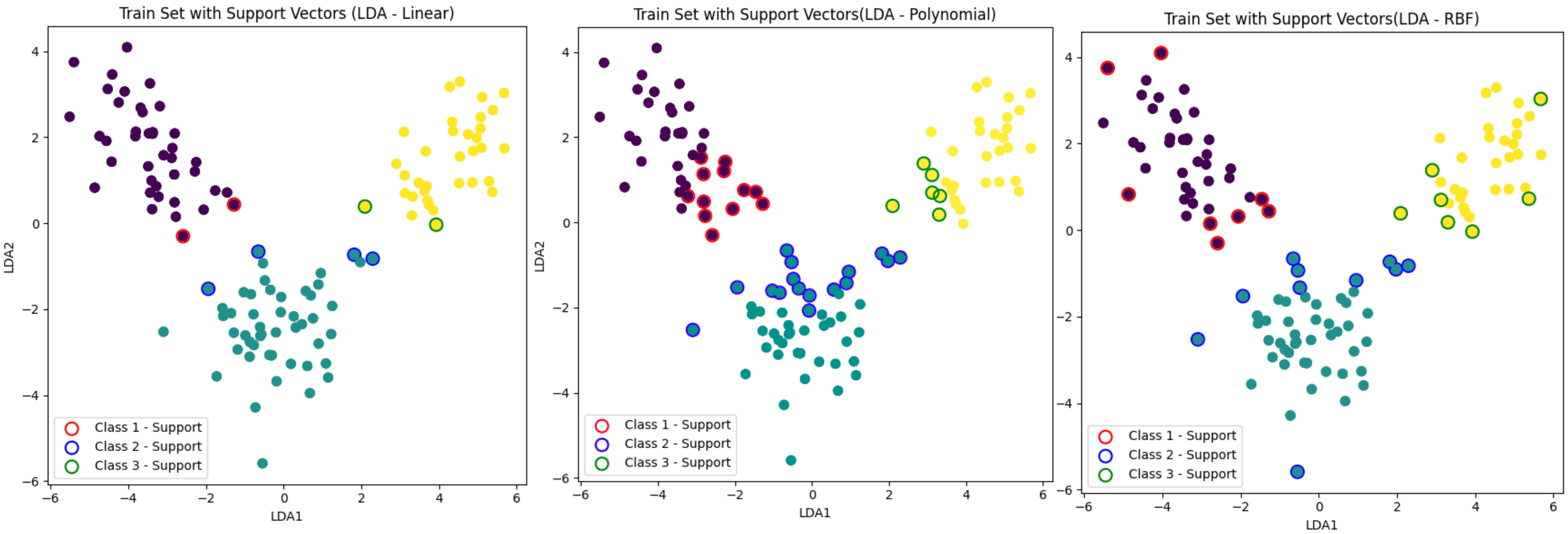


Example: Linear vs Polynomial kernel





Example: Linear vs Polynomial kernel



MMC vs SVC vs SVM

- Maximum margin Classifier (MMC) – with hard margin
- Support Vector Classifier (SVC) – with soft margin and linear kernel
- Support Vector Machine (SVM) – SVC + non-linear kernel

Other important parameters for SVM

- C – (inversely proportional to the Regularization parameter)
  - represents the acceptable amount of misclassification or error.
  - A smaller C value (high regularization) creates a wider margin hyperplane, allows more misclassifications (large margin - high misclassifications)
  - larger value creates small-margin hyperplane (forcing the algorithm to classify every training point correctly. (Larger value of C can cause overfitting).
- Gamma – factor that control how the model fit on the training data.
  - Lower value: loosely fit the train data, more data points will influence the decision boundary. So, decision boundary will be more generic (may cause underfitting)
  - Higher. value: fewer data points will influence the decision boundary. So, this may cause overfitting

Advantages & Disadvantages

<u>Advantages</u>

- High accuracy, faster prediction
- Memory efficient
- Works well if the dataset is small, separable
- Effective in high-dimensional space
- Effective when number of dimensions greater than the number of instances
- Variety of kernel functions

<u>Disadvantages</u>

- Not suitable for larger datasets
- Poor performance on overlapping classes
- Highly sensitive to the type of kernel

References

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