PyLith v3.0 Tutorial

Quasi-static Poroelasticity with No Fault

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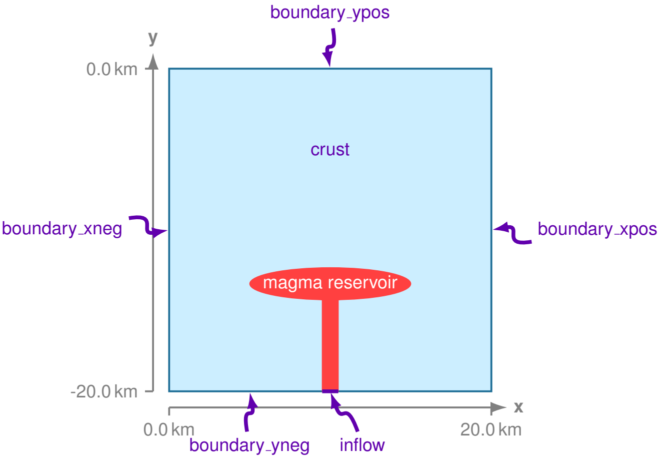
Matthew Knepley
Brad Aagaard

June 21, 2022

2D Magma Reservoir Using Poroelasticity: `examples/magma-2d`



Model flow and deformation for a magma reservoir with poroelastic properties that differ from the surrounding domain.

PyLith

Steps in example

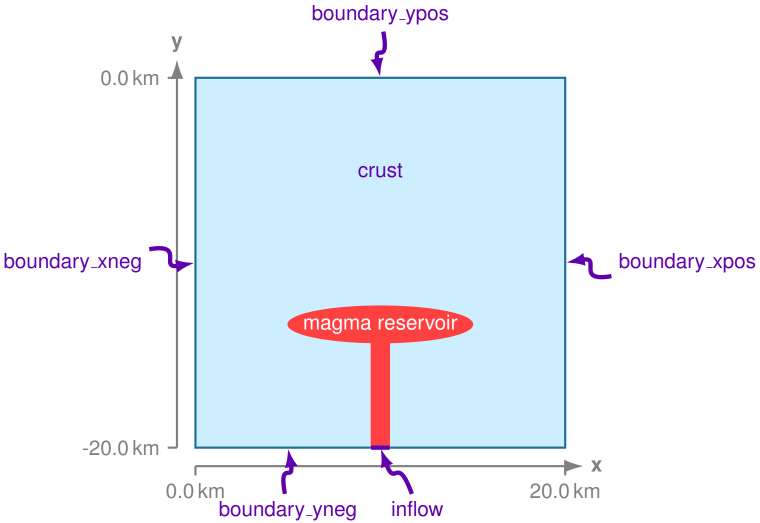
Step 1 **Magma influx with displacement and pressure boundary conditions**

Step 2 **Magma influx with updating porosity, displacement and pressure boundary conditions**

Concepts covered

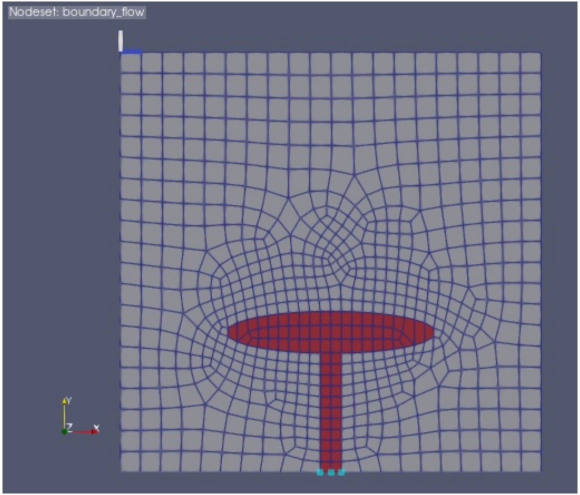
- Generation of mesh using Cubit
- Variable mesh size with distance from the magma reservoir
- Quasistatic simulations for poroelasticity
- Elastic bulk rheology
- Dirichlet fluid pressure boundary condition
- Initial condition for fluid pressure

Cubit: Geometry



Cubit: Creating the finite-element mesh

We create the mesh using geometric primitives



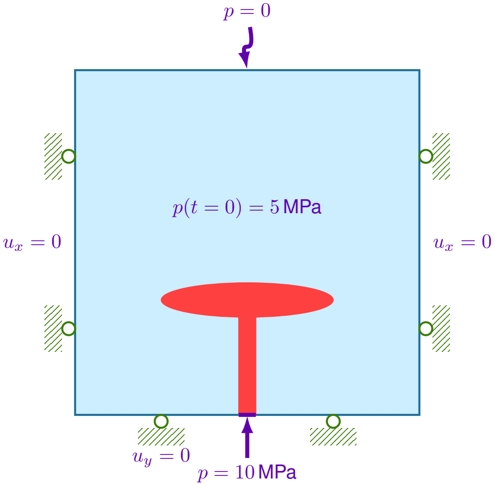
Files used in simulations

Files are in directory `examples/magma-2d`

README.md	Brief description of the various examples
*.cfg	PyLith parameter files
*.jou	Cubit Journal scripts used to generate the mesh
*.exo	Finite-element mesh files generated by Cubit
*.spatialdb	Spatial database files
viz	Directory containing ParaView Python scripts
output	Directory containing simulation output; created automatically when running the simulations

Step 1: Overview

Inflation of the magma reservoir



Step 1: Physics

$$\vec{s} = (\vec{u} \quad p \quad \epsilon_v)^T$$

$$\nabla \cdot \boldsymbol{\sigma}(\vec{u}, p) = \vec{0}$$

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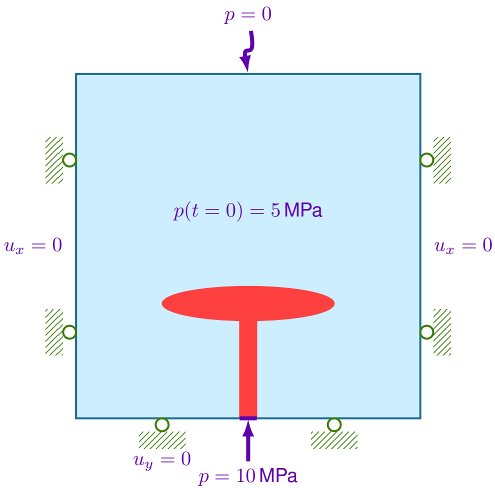
$p = 10\,\text{MPa}$ on boundary_flow

$p(t = 0) = 5\,\text{MPa}$ in domain



Auxiliary Fields

for Quasistatic Linear Isotropic Poroelasticity

Origin	Variable	Description	Position	Notes
Material	ρ_b	Rock Density	0	
	ρ_f	Fluid Density	1	
	μ_f	Fluid Viscosity	2	
	ϕ	Porosity	3	
	$\vec{f}_b$	Body Force	$+1$	
	$\vec{g}$	Gravity	$+1$	
	γ	Fluid Source	$+1$	
Rheology	$\boldsymbol{\sigma}_R$	Reference Stress	NumAux - 7	
	$\boldsymbol{\epsilon}_R$	Reference Strain	NumAux - 6	
	G	Shear Modulus	NumAux - 5	
	K_d	Drained Bulk Modulus	NumAux - 4	
	α	Biot Coefficient	NumAux - 3	
	M	Biot Modulus	NumAux - 2	$\frac{K_f}{\phi} + \frac{K_s}{\alpha - \phi}$
	$\boldsymbol{k}$	Permeability	NumAux - 1	
Input	K_s	Solid Grain Bulk Modulus	-	
	K_f	Fluid Bulk Modulus	-	

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[pylithapp.problem]
solution = pylith.problems.SolnDispPresTracStrain

[pylithapp.problem.solution.subfields]
displacement.basis_order = 2
pressure.basis_order = 1
trace_strain.basis_order = 1

[pylithapp.problem]
normalizer = spatialdata.units.NondimElasticQuasistatic
normalizer.length_scale = 100.0*m
normalizer.relaxation_time = 0.2*year
normalizer.shear_modulus = 10.0*GPa
```

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[pylithapp.problem]
materials = [crust, intrusion]
materials.crust = pylith.materials.Poroelasticity
materials.intrusion = pylith.materials.Poroelasticity

[pylithapp.problem.materials]
crust.bulk_rheology = pylith.materials.IsotropicLinearPoroelasticity
intrusion.bulk_rheology = pylith.materials.IsotropicLinearPoroelastic

[pylithapp.problem.materials.crust]
description = crust
label_value = 1

db_auxiliary_field = spatialdata.spatialdb.UniformDB
db_auxiliary_field.description = Poroelastic properties for the crust
db_auxiliary_field.values = [solid_density, fluid_density, fluid_visco
db_auxiliary_field.data   = [ 2500*kg/m**3,  1000*kg/m**3,       0.001*
...
```

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bc.bc_xpos = pylith.bc.DirichletTimeDependent
bc.bc_yneg = pylith.bc.DirichletTimeDependent
bc.bc_ypos = pylith.bc.DirichletTimeDependent
bc.bc_flow = pylith.bc.DirichletTimeDependent

...
[pylithapp.problem.bc.bc_flow]
constrained_dof = [0]
label = boundary_flow
field = pressure
db_auxiliary_field = spatialdata.spatialdb.UniformDB
db_auxiliary_field.description = Flow into external boundary of condu
db_auxiliary_field.values = [initial_amplitude]
db_auxiliary_field.data = [10.0*MPa]
```

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[pylithapp.problem]
ic = [domain]
ic.domain = pylith.problems.InitialConditionDomain

[pylithapp.problem.ic.domain]
db = spatialdata.spatialdb.UniformDB
db.description = Initial conditions for domain
db.values = [displacement_x, displacement_y, pressure, trace_strain]
db.data = [0.0*m, 0.0*m, 5.0*MPa, 0.0]
```

Step 1: Input files

mesh_quad.exo Finite-element mesh generated using Cubit

pylithapp.cfg PyLith parameter file common to all steps

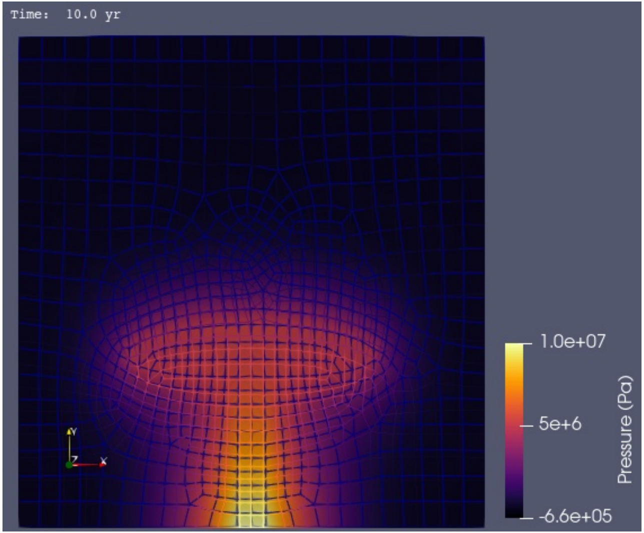
step01_inflation.cfg PyLith parameter file

Step 1: Run the simulation

```
pylith step01_inflation.cfg

# Output
 >> /Users/baagaard/software/unix/py39-venv/pylith-debug/lib/python3.9/site-packages/pylith/meshio/MeshIOObj.py:44:read
 -- meshiocubit(info)
 -- Reading finite-element mesh
 >> /Users/baagaard/src/cig/pylith/libsrc/pylith/meshio/MeshIOCubit.cc:157:void pylith::meshio::MeshIOCubit::_readVertices(pylith::mesh
 -- meshiocubit(info)
 -- Component 'reader': Reading 747 vertices.
 >> /Users/baagaard/src/cig/pylith/libsrc/pylith/meshio/MeshIOCubit.cc:217:void pylith::meshio::MeshIOCubit::_readCells(pylith::meshio:
 -- meshiocubit(info)
 -- Component 'reader': Reading 705 cells in 2 blocks.

# -- many lines omitted --

50 TS dt 1. time 49.
    0 SNES Function norm 3.049429649018e-03
    Linear solve converged due to CONVERGED_ATOL iterations 1
    1 SNES Function norm 5.567219918314e-16
  Nonlinear solve converged due to CONVERGED_FNORM_ABS iterations 1
51 TS dt 1. time 50.
 >> /Users/baagaard/software/unix/py39-venv/pylith-debug/lib/python3.9/site-packages/pylith/problems/Problem.py:201:finalize
 -- timedependent(info)
 -- Finalizing problem.
```

Step 1: Visualize results

Run the `viz/plot_dispwarp.py` Python script from within ParaView



Step 2: Physics

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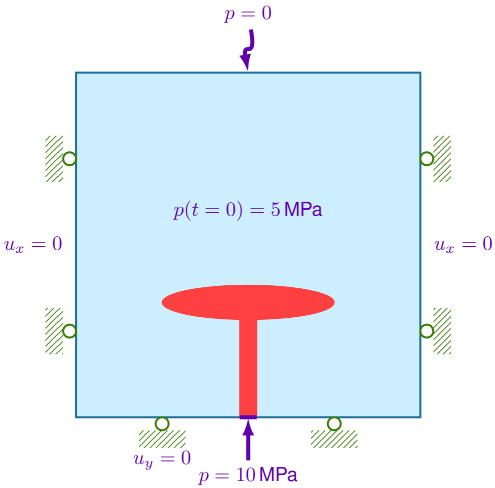

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$$\frac{\partial \phi}{\partial t} = (\alpha - \phi)\,\dot{\epsilon}_v + \frac{(1-\alpha)(\alpha - \phi)}{K_{dr}}\dot{p}$$

$$\phi^{n+1} = \phi^n + \left[(\alpha - \phi^n)\,\dot{\epsilon}_v^n + \frac{(1-\alpha)(\alpha - \phi^n)}{K_{dr}}\dot{p}^n\right]\Delta t$$

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[pylithapp.problem.solution.subfields]
displacement.basis_order = 2
pressure.basis_order = 1
trace_strain.basis_order = 1
velocity.basis_order = 2
pressure_t.basis_order = 1
trace_strain_t.basis_order = 1

[pylithapp.problem]
normalizer = spatialdata.units.NondimElasticQuasistatic
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