543T Course Project: Disease Spread Optimization Model

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Abstract

This project enhances the traditional Susceptible, Infected, Recovered, Dead (SIRD) epidemiological model by incorporating quarantine and vaccination mechanisms to simulate public health responses to infectious disease outbreaks more accurately. The extended model introduces a quarantine system that temporarily reduces the infectivity of newly infected individuals and a vaccination module that evaluates various rollout strategies, including delayed high-efficacy vaccines, continuous distribution, and early use of lower-efficiency vaccines.

Simulation results suggest that to minimize both mortality and societal disruption, populations should adopt shorter quarantine durations alongside early, rapid vaccine deployment to vulnerable populations, even if the vaccines are less effective. These findings imply that prompt, imperfect interventions can outperform slower, more optimized responses in managing outbreaks, particularly in the early stages of disease spread.

Introduction

Epidemiological models are mathematical frameworks used to understand and predict the spread of infectious diseases within populations. First devised in the early 20th century, these models have become essential tools for public health planning and disease management. One of the earliest examples is the Reed-Frost model, which estimates how quickly an infection spreads and how rapidly individuals recover based on the size of the susceptible population and the rate of infection. While simple, such early models laid the groundwork for more advanced simulations. [1]

In contemporary epidemiology, more sophisticated models have emerged to account for the complex dynamics of disease progression. Among these, the Susceptible, Infected, Recovered, and Dead (SIRD) model stands out for its ability to incorporate mortality as an outcome of infection, providing researchers with a more complete picture of disease impact. This model divides the population into four different segments: susceptible people who have not yet been infected, infected individuals who can spread disease to the susceptible, recovered people who have had the disease and gained immunity, and the dead who did not survive the infection. Through the use of the model, we can easily track how individuals move between these states over time.

Our project builds on the SIRD model by introducing two major extensions. First, we have introduced population interactions, where each subpopulation represents different geographic or social regions, allowing for cross-infection through individual movement. Second, we evaluate the impact of two intervention strategies: quarantine and vaccination. We use these two strategies to minimize death rates while considering broader social and economic consequences.

Problem Statement

The challenge we address is how to reduce mortality from infectious diseases in a connected population system, while also minimizing disruptions to daily life. Public health interventions such as quarantines and mass vaccinations can reduce disease spread, but they come with costs such as economic impacts and psychological harm. Our objective is not only to reduce the number of deaths caused by a hypothetical disease outbreak, but to do so in a way that balances safety with societal function.

Scope of the Project

Our project focuses on a modified SIRD model that incorporates cross-population travel, variable infection rates, and intervention strategies. We simulate various scenarios involving different levels of quarantine strictness and vaccination coverage to assess their effectiveness in limiting mortality. This scope is limited to theoretical modeling and simulation for both hypothetical outbreaks and the real-world results from the COVID-19 pandemic. Our results aim to provide insights into the trade-offs involved in disease management strategies and identify optimal approaches to minimize the overall costs both in terms of life and societal impact.

Optimization Problem

Let:

- $L \in Z_+$ be the quarantine length (in days)
- $D(L)$ be the total number of deaths resulting from a quarantine of length L
- $C(L)$ be the total social and economic cost of a quarantine of length L
- $\alpha \in R_+$ be the total weight assigned to deaths (e.g., reflecting the societal cost of a life lost)
- $\beta \in R_+$ be the linear quarantine cost rate (e.g., cost per day of quarantine)
- $\gamma \in R_+$ be the quadratic quarantine costs rate (e.g., compounding costs like mental health degradation and economic stagnation)

We define the cost function as:

$$C(L) = \beta \cdot L + \gamma \cdot L^2$$

The objective is to minimize the total societal burden, defined as a weighted sum of deaths and quarantine costs:

$$\min_{L \in \{L_{\min}, L_{\max}\}} \alpha \cdot D(L) + C(L)$$

Subject to:

- $L_{\min} \geq 1$ (minimum quarantine length)
- $L_{\max} \leq 50$ (maximum tested quarantine length)
- $D(L)$ is obtained via simulation using the extended SIRD model
- $C(L)$ is a known convex function of L

Related Theory and Practice

Our project builds upon a long-standing practice of mathematical modeling in epidemiology, particularly compartmental models such as SIRD, which are used for both theoretical analysis and to inform real-world policy decisions. The SIRD model is a deterministic framework governed by differential equations that describe how individuals transition between different health states over time. In matters of public health and epidemiology, such models are routinely used to evaluate and predict the effectiveness of interventions like social distancing, quarantines, and vaccines. [2]

Regarding optimization techniques, this project is rooted in operations research, where resource allocation under various constraints is a consistent decision point. In particular, prior work has explored how to minimize infections and/or deaths given limited healthcare capacity or vaccine availability. Models such as optimal control theory and cost-benefit analysis are frequently applied to balance competing priorities, such as minimizing mortality while preserving economic stability. Our approach integrates this type of analysis by introducing cost functions to quantify the societal burden of quarantine and vaccine distribution, allowing us to study trade-offs between public health outcomes and socio-economic disruptions. [3]

On the practical side, our project models real-world decision-making during pandemics like COVID-19, where governments have to decide when to impose quarantines, how long to sustain them, and how to prioritize vaccine development and distribution. Epidemiological simulations were used extensively during the COVID-19 pandemic to inform policies such as phased reopenings, tiered restrictions, and vaccine rollout. Our simulated vaccine distribution strategies, which include constant rollout, delayed high-efficacy rollout, and group prioritization, mirror these real-world debates and illustrate their impact within a controlled, theoretical environment. [4]

By combining classical disease modeling, modern optimization principles, and real-world public health strategies, our project not only reproduces known outcomes, it also enables the exploration of new questions around disease mitigation, policy timing, and trade-offs in health interventions.

Technical Details

In order to minimize the number of deaths in our extended SIRD model, we implemented two major intervention systems: quarantine and vaccination. These mechanisms introduce control over disease spread by limiting exposure and promoting immunity, respectively. Additionally, our implementation allows for multi-population interaction and the simulation of intervention trade-offs using custom cost functions and optimization.

Quarantine Implementation

Quarantine is modeled by delaying the infectivity of individuals after infection. When someone becomes infected, they are stored in a dictionary, *quarantine*, keyed by the day they exit quarantine. Each day, infected individuals who are quarantined either recover or die at rates defined by parameters *gamma* and *mu*, respectively. Only individuals who have completed quarantine or refused to do so (modeled with a "leak" factor) can infect others. The *quarantineLeak* parameter simulates the portion of the population that does not comply with quarantine rules, while *quarantineLength* determines how long each new case remains isolated before becoming infectious.

The figure below depicts an example of the quarantine functionality. We ran two identical populations through our simulation but only included a quarantine in one of the simulations. The leftmost graph shows the simulation result with a perfect quarantine implemented (*quarantineLeak* set to 0 and *quarantineLength* set to 14). The rightmost figure shows the same population without no quarantine at all. The left side has a much smaller peak on the infection curve and, at the conclusion of the simulation, nearly a third of the population remains completely uninfected. The simulation with no quarantine, however, leaves nearly every individual infected by the end of the simulation and has a significantly higher infected peak.

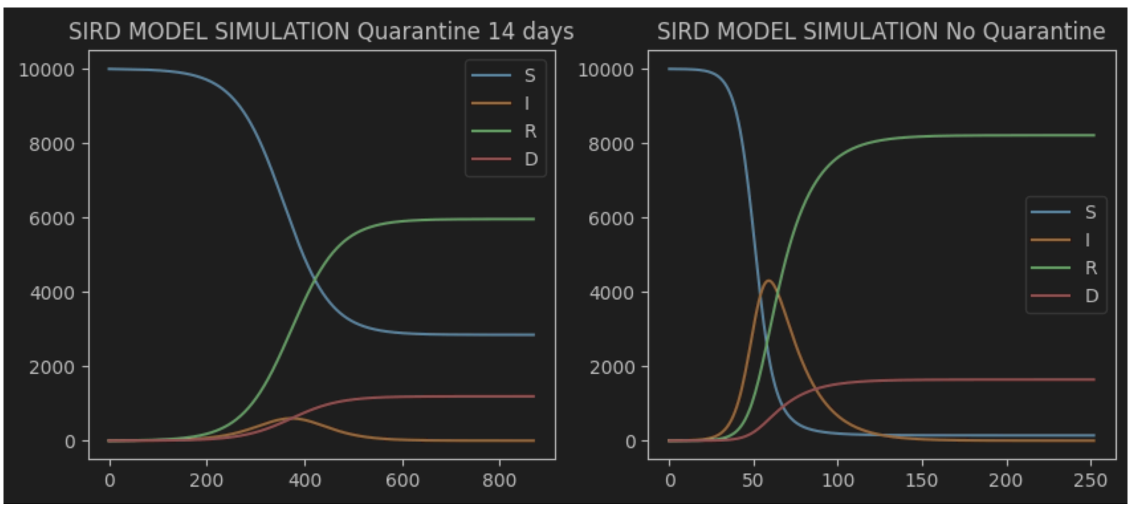

Figure 1: Comparison of Extended SIRD Model Simulation of Identical Populations With (Left) and Without (Right) Quarantines

Vaccination Implementation

Vaccination is modeled through an *apply_vaccination* method as a member of the *Population* class. This function removes a specified number of individuals from the susceptible pool, *S*, and adds them to the recovered pool, *R*, based on a defined vaccine efficacy. This approach assumes that vaccinated individuals, when successfully immunized, gain immediate and permanent immunity. Vaccination strategies are modeled by calling this function daily according to a predefined rollout plan, allowing us to simulate real-world policies such as constant rollouts, delayed vaccinations with increased efficacy, or prioritizing high-risk populations.

The figure below depicts an example simulation of the various vaccination policies in effect. Four identical populations were simulated with different vaccine policies, including no vaccination at all. It clearly shows that the policies have a drastically different impact on the overall mortality rate in our simulations.

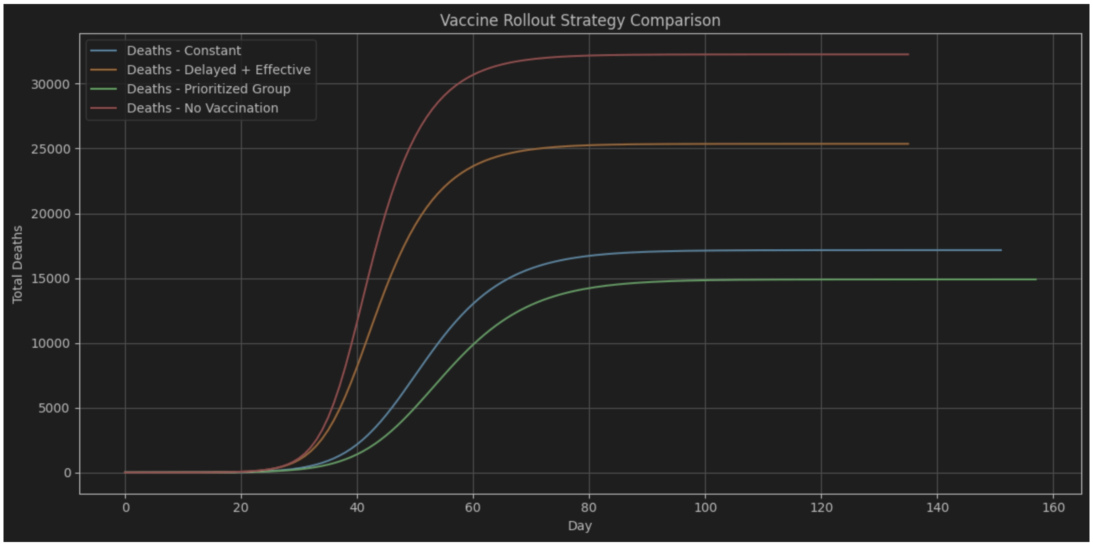

Figure 2: Comparison of Extended SIRD Model Simulations With Different Vaccine Distribution Policies

Cross-Population Infection

To simulate disease spread between connected populations, like cities or demographic groups, we include a *crossInfect* function. This function computes how many individuals from each population travel to others and adjusts infection rates accordingly. The *crossInfectivity* parameter defines the probability of inter-population interaction and transmission, enabling the study of disease spread with various levels of interaction between population groups.

Cost Functions and Optimization

To evaluate quarantine policies, we introduced cost functions that reflect the social and economic burden of quarantining large segments of a population. These costs grow both linearly and polynomially with quarantine length, representing cumulative disruptions like missed appointments or schooling and compounding hardships like mental health degradation or economic stagnation. We also defined objective functions that balance death tolls with quarantine impact, allowing us to simulate trade-offs between minimizing mortality and minimizing societal costs. We utilized the built-in *scipy.optimize.minimze_scalar* to find optimal quarantine lengths under different assumptions, and we plotted total deaths, total costs, and objective values to visualize the effects of policy decisions.

The figure below depicts the cost function and its components in an example simulation of an arbitrary disease infecting an arbitrary population. The leftmost curve graphs the cost of a quarantine policy as a function of quarantine length. The center graph plots the absolute number of deaths based on quarantine length. Finally, the rightmost figure shows the objective function we seek to minimize, a weighted sum of deaths and costs, as a function of quarantine length. In this example, the optimal quarantine length (highlighted by the vertical dashed red line), was 8 days.

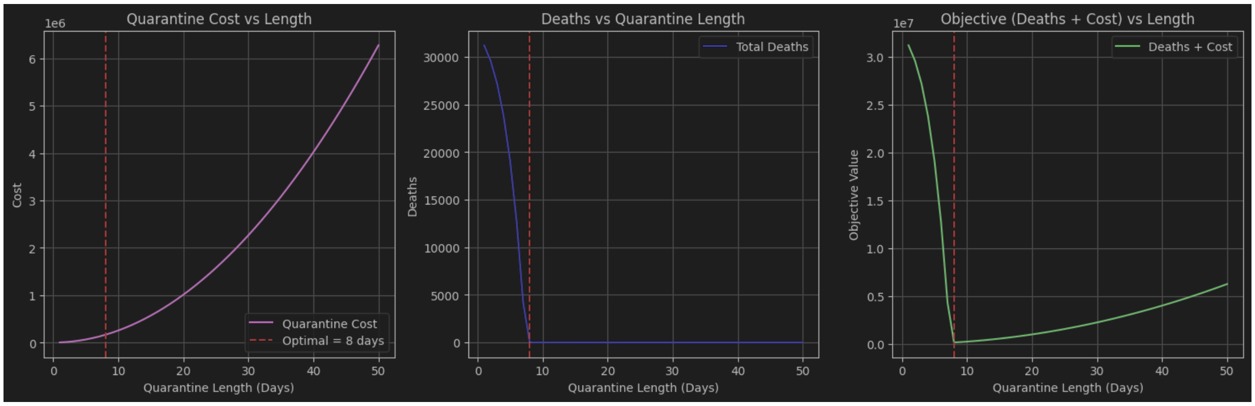

Figure 3: Cost, Deaths, and Objective Function Value Graphed as a Function of Quarantine Length for an Arbitrary Example Disease and Population

Simulation Design

Our simulation proceeds in daily time steps. At each step, we:

1. Apply vaccinations based on the current strategy
2. Run the SIRD update step to model disease progression
3. Apply the *crossInfect* function for connected populations
4. Record the state of the system for analysis

We use this setup to test and compare intervention policies over various time periods. The entire system is implemented in Python using object-oriented principles, with separate functions for disease progression, policy implementation, and plotting results.

Experimental Results

To ground our simulations in a realistic scenario, we calibrated our model using data from the COVID-19 pandemic. Drawing from published studies and public health sources, we identified parameter values that approximate the early behavior of the disease:

- **Infection rate (β):** 0.32 per day, based on estimates of COVID-19's basic reproduction number in uncontrolled environments. [5]
- **Recovery rate (γ):** Approximately 99% of individuals recovered over a two-week period. Normalizing this to a daily time scale yields an approximate recovery rate of 0.07 per day.
- **Death rate (μ):** With an estimated 1.1% mortality over two weeks, the normalized daily death rate was set at an approximate 0.0008. [6]
- **Quarantine leak (delinquency rate):** Real-world compliance with quarantine protocols was imperfect. We modeled this using a 40% quarantine delinquency, meaning 40% of infected individuals did not adhere to isolation measures. [7]

Using these parameters, we simulated a variety of outbreak scenarios. These included both single and multi-population models, where the latter allowed us to explore the effect of geographic spread and population density. We also adjusted infection rates and cross-infectivity to reflect urban versus rural population mixing. For each experiment we ran the SIRD algorithm for each of our vaccination strategies to identify both the optimal strategy and the ideal amount of quarantine time for the strategy.

Across all simulations, we consistently found that:

- **Shorter quarantine durations** (approximately one week) led to significantly lower death tolls when combined with vaccination, and did so with reduced social and economic costs compared to long-term lockdowns.
- **Delayed vaccine rollouts**, even with higher efficacy, underperformed due to the rapid early spread of infection.
- **Early vaccination of priority groups**, even with rushed and moderately effective vaccines, was the most effective strategy in minimizing deaths. This approach flattened the infection curve early and prevented runaway outbreaks.

Figure 4 below demonstrates the outcomes from these experiments, including a cost vs. quarantine length curve, total deaths as a function of quarantine length, and the objective function plotted against quarantine length. Figure 5 depicts a comparison of the vaccination strategies we tested. This simulation resulted in an optimal quarantine length of 7 days, and the results reinforce the conclusion that timely interventions (even imperfect ones) outperform slower, more polished responses, both in terms of health outcomes and societal burdens.

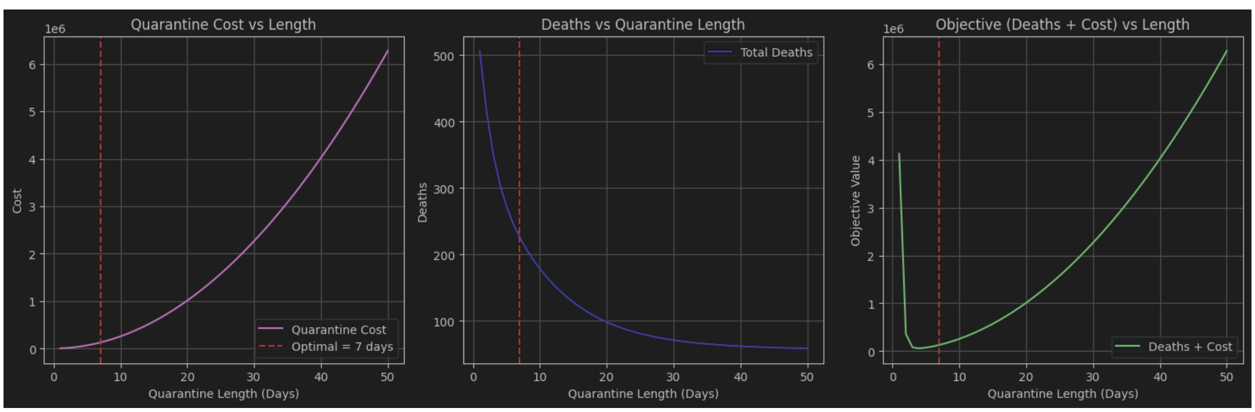

Figure 4: Cost, Deaths, and Objective Function Value Graphed as a Function of Quarantine Length for the COVID-19 Disease

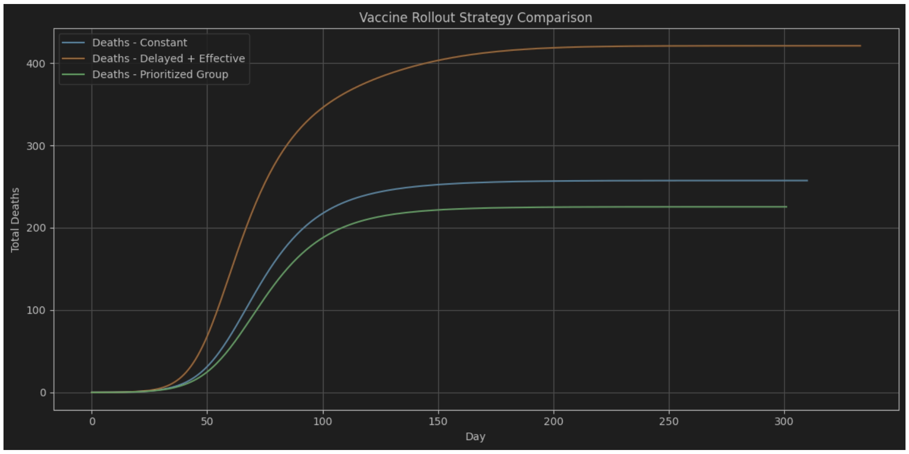

Figure 5: Comparison of Extended SIRD Model Simulations With Different Vaccine Distribution Policies for the COVID-19 Disease

Discussions, Observations, and Comparisons

Through simulation and optimization of our extended SIRD model, we observed several important dynamics that mirror real-world public health challenges. Each intervention strategy – quarantine, vaccination, and their combinations – had distinct effects on the progression of the disease and total mortality, as well as on the associated social and economic costs.

Effectiveness of Quarantine

One of the first observations was that longer quarantine lengths significantly reduce the number of deaths by isolating newly infected individuals before they can spread the disease. However, this benefit begins to plateau beyond a certain length, after which the additional reduction in deaths is minimal. At the same time, the cost of quarantine begins to increase rapidly, especially when modeled with a polynomial penalty to reflect compounded social and economic strain. This highlights the classic phenomenon of diminishing returns, where increasingly strict policies bring smaller health benefits at increasing higher societal costs.

Optimizing Quarantine Duration

By introducing a cost function and performing scalar minimization over quarantine length, we found that the optimal quarantine policy balances both outcomes: reducing deaths while avoiding overly burdensome lockdown durations. In particular, we observed that when deaths are weighted heavily in the objective function, the model favors longer quarantines. However, increasing the relative weight of quarantine costs shifts the optimum toward shorter, more practical lockdowns. This mirrors real-world policy trade-offs regarding how long lockdowns should be maintained and under what conditions they should be relaxed.

Vaccination Strategy Comparisons

We also tested three distinct vaccination strategies:

1. **Constant Rollout:** This strategy delivered a consistent number of doses per day with relatively high efficacy. It produced a steady decline in deaths and served as a solid baseline.
2. **Delayed Rollout with Increased Efficacy:** While intuitively appealing, this strategy performed the worst. The delay in distributing the vaccine allowed the disease to spread aggressively before vaccination began, and the increased efficacy could not compensate for the early exponential growth.
3. **Prioritized Rollout:** Vaccinating a high-risk subgroup early yielded the best results. It flattened the infected curve early and prevented peak infections from becoming unmanageable.

These results strongly suggest that early action, even with moderate efficacy, is more valuable than delayed perfection. Prioritization of vulnerable populations also provides clear epidemiological benefits with limited resources.

Cross-Population Infection and Mobility

Simulations involving multiple populations connected by mobility showed that even a small cross-infectivity rate can lead to synchronized outbreaks. This demonstrates the importance of regional coordination in public health interventions, as isolated policies are less effective when individuals move between communities.

Recommendations

Based on the outcomes of our modified SIRD model simulations, which incorporated both vaccination and quarantine strategies, we propose the following recommendations to effectively minimize the total cost of death and economic disruption:

Prioritized and Early Vaccination Deployment

Our findings strongly support the implementation of a prioritized vaccine distribution system. Even when the efficacy of a vaccine is reduced due to rushed production, early deployment, especially when targeting high-risk groups, significantly reduces the overall mortality rate. We recommend that public health groups prioritize early accessibility of vaccination over waiting for more effective formulations.

Short Duration Quarantines

Our model indicates that shorter quarantines, possibly of less than one week depending on the disease, applied to infected individuals can effectively suppress further transmission without imposing severe economic or social burdens. Prolonged quarantines tend to offer diminishing returns relative to their societal costs. Therefore, we recommend implementing short, focused quarantine periods for infected individuals as an effective strategy.

Future Work

Looking ahead, future work could expand the model to include age-based risk stratification, healthcare capacity limits, and/or behavioral changes over time. Incorporating stochastic elements or real-time policy switching could also improve realism and decision-making value.

Contributions of Team

Robert Walsh

Robert created the initial SIRD models and modified them to support quarantine functionality and cross-infectivity among multiple populations. Robert experimented with multiple different configurations of this model, including versions based on statistics from the COVID-19 pandemic. Robert was an active participant in writing this entire report, but was most instrumental in the Abstract, Introduction, Experimental Results, and Recommendations sections.

Josh Sapira

Josh implemented the initial vaccination functionality and performed the initial comparison of the various vaccination strategies. Josh also wrote the README in the GitHub repository, developed the cost function for quarantining, and formalized the optimization problem as a weighted average of deaths and societal costs. Josh was an active participant in writing this entire report, but was most instrumental in the Related Theory and Practices, Technical Details, Experimental Results, and Discussions, Observations, and Comparisons sections.

Sydney Seder

Sydney was instrumental in designing the implementation of this project in the early, pre-coding stages. Sydney helped implement extensions to the SIRD model and helped to generate many of the plots used in this report, as well as those not included. Sydney also took on a leading role in the presentation of this project and participated in writing various sections of this report.

Code

The code used for this research paper can be found in the GitHub repository linked below. Follow the instructions in the README file to install the necessary packages and reproduce the results in this paper.

Link to code: https://github.com/rwalsh7231/543T-Final-Project/tree/main

References

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