Definitions

N_e = Inbreeding effective population size, a measure of how the average inbreeding coefficient changes from one generation to the next.

N = Census size of the population during the parental generation.

N_p = Number of parents contributing at least one gamete to the next generation.

k = Number of offspring (gametes) contributed by an individual from the parental generation.

k_i = Vector of the k values for i individuals. This vector Can include zeroes when indexed by 1-¿N, or exclude zeroes when indexed by 1-¿N_p.

S = Number of (observed) offspring. Each offspring has two parents, so $S = \sum k_i/2$

P_{same} = Chance that two random gametes in the offspring generation come from the same parent. The chance that two random gametes are identical by descent in the previous generation is $P_{same}/2$.

Equations

Crow and Denniston (1988):

$$N_e = \frac{\overline{k}_i N - 1}{\overline{k}_i - 1 + Var_k/\overline{k}_i} \tag{1}$$

In (1) k_i indexes **all** possible parents N, not just those that successfully contributed to the next generation.

Waples (2011), equation 2a:

$$N_e = \frac{\sum k_i - 1}{\frac{\sum k_i^2}{\sum k_i} - 1} \tag{2}$$

Waples (2011) noted that (1) holds even when excluding individuals from the parental population that do not contribute offspring. They also note that a sample of offspring can be used to estimate N_e by estimation of of k_i.

Wang (2009) addresses a very similar situation: "Equations for the effective size (N_e) of a population were derived in terms of the frequencies of a pair of offspring taken at random from the population being sibs sharing the same one or two parents".

Wang (2009) equation 10:

$$\frac{1}{N_e} = \frac{1+3\alpha}{4}(Q_1 + Q_2 + 2Q_3) - \frac{\alpha}{2}(\frac{1}{N_1} + \frac{1}{N_2}) \tag{3}$$

where α is F_{IS}, Q_1, Q_2, and Q_3 are the probabilities of a pair of offspring being paternal, maternal half-sibs, and full-sibs, and N_1 and N_2 are the number of male and female parents.

If we assume no inbreeding ($\alpha = 0$), (3) becomes:

$$\frac{1}{N_e} = \frac{1}{4}(Q_1 + Q_2 + 2Q_3) \tag{4}$$

Wang (2009) equation 8 shows that:

$$Q_1 + Q_2 + 2Q_3 = 4P_{same} \tag{5}$$

Substitution into (4) leads to

$$N_e = \frac{1}{P_{same}} \tag{6}$$

I.e. N_e is equal to the one-generation identity by descent within the the offspring gametes.

We can also estimate P_{same} from k_i, recall $N_p = \text{length}(k_i)$:

$$P_{same} = \frac{1}{N_p(N_p - 1)} * \sum_{i=1}^{N_p}(k_i * (k_i - 1)) \tag{7}$$