Definitions

N_e = Inbreeding effective population size, a measure of how the average inbreeding coefficient changes from one generation to the next.

N = Census size of the population of potential parents.

N_p = Number of parents contributing at least one gamete to the next generation.

k = number of offspring gametes contributed by a single individual from the parental population.

k_i = vector of the k values for i individuals.

S = number of observed offspring. Each offspring has two parents, so $S = \sum k_i/2$

P_{same} = the chance that two random gametes selected from the offspring generation are from the same parent. So the chance that two random gametes are identical by descent is $P_{same}/2$.

Equations

Crow and Denniston (1988):

$$N_e = \frac{\overline{k}_i N - 1}{\overline{k}_i - 1 + Var_k/\overline{k}_i} \tag{1}$$

In (1), k_i indexes **all** possible parents, not just those that successfully contributed to the next generation.

Waples (2011), equation 2a:

$$N_e = \frac{\sum k_i - 1}{\frac{\sum k_i^2}{\sum k_i} - 1} \tag{2}$$

Waples (2011) noted that the Crow and Denniston equation holds even when excluding individuals from the parental population that do not contribute offspring. They also noted that this also holds for a sample of offspring and can be used to estimate the N_e from a sample of offspring by inferring the k_i vector for the parents contributing to the observed offspring.

Wang (2009), equation 10:

$$\frac{1}{N_e} = \frac{1+3\alpha}{4}(Q_1 + Q_2 + 2Q_3) - \frac{\alpha}{2}(\frac{1}{N_1} + \frac{1}{N_2}) \tag{3}$$

where Q_1, Q_2, and Q_3 are the probabilities of a pair of offspring being paternal, maternal half-sibs and full-sibs respectively.

assuming no inbreeding, $\alpha = 0$ and (3) becomes:

$$\frac{1}{N_e} = \frac{1}{4}(Q_1 + Q_2 + 2Q_3) \tag{4}$$

Wang (2009) equation 8 shows that:

$$(Q_1 + Q_2 + 2Q_3) = 4 * P_{same} \tag{5}$$

Substitution into (4) leads to

$$N_e = \frac{1}{P_{same}} \tag{6}$$

I.e. N_e is equal to the one-generation identity by descent within the the offspring gametes.

Given a vector k_i, we can also calculate P_{same}:

recall $N_p = \text{length}(k_i)$

$$P_{same} = \frac{1}{N_p(N_p - 1)} * \sum_{i=1}^{N_p}(k_i * (k_i - 1)) \tag{7}$$