BSOS

CCJS 200 Introduction to Statistics for Criminology and Criminal Justice

Friday Discussion Sessions

Week 7: 03/14/2025

COLLEGE OF BEHAVIORAL & SOCIAL SCIENCES

Today's Agenda

PRACTICE QUESTIONS FOR CHAPTER 7 (7.1-7.5)

REMINDERS FOR ASSIGNMENT 2 & OTHERS

Practice Question 7.1

Calculate the probability for each of the following:

a) Two tails in two tosses of a fair coin.

b) Three heads in three tosses of a fair coin.

c) Four heads in four tosses of an unfair coin where the probability of a head is 0.75.

d) Three sixes in three rolls of a fair die.

e) Five fours in five rolls of an unfair die where the probability of a four is 0.25.

Practice Question 7.1a-c KEY

- **7.1a:** the two tosses are considered to be independent experiments.
  - p(T & T) = p(T)*p(T) = 1/2*1/2 = 1/4
- **7.1b:** the three tosses are considered to be independent experiments:
  - p(H & H & H) = p(H)*p(H)*p(H) = 1/2 x 1/2 x 1/2 = 1/8
- **7.1c:** the 4 tosses are considered to be independent experiments, but the coin is unfair, p(H) = 3/4
  - p(H & H & H & H) = 3/4 x 3/4 x 3/4 x 3/4 = 3^4/4^4 = 81/256

Practice Question 7.1d-e KEY

- **7.1d:** each roll of the die is an independent experiment, p(get a 6) = 1/6
  - p(6 & 6 & 6) = 1/6 * 1/6 * 1/6 = 1/216
- **7.1e:** each roll of the die is an independent experiment, p(get a 4) = 1/4
  - p(4 & 4 & 4 & 4 & 4) = (1/4)^5 = 1/1024

Practice Question 7.2

All of Kate's children are boys.

a) Intuitively, how many boys do you think Kate would have to have in succession before you would be willing to say with some certainty that, for some biological reason, she is more likely to give birth to boys than girls?

b) Now calculate the number of successive births required before you could make such a decision statistically with a 5% risk of error.

c) How many successive boys would have to be born before you would be prepared to come to this conclusion with only a 1% risk of error?

Practice Question 7.2a KEY

- Intuitively, how many boys do you think Kate would have to have in succession before you would be willing to say with some certainty that, for some biological reason, she is more likely to give birth to boys than girls?

- Answer: this is subjective for each child

Practice Question 7.2b KEY

- Now calculate the number of successive births required before you could make such a decision statistically with a 5% risk of error.

- Answer: each birth is an independent experiment

  risk of error threshold: 1/20

  p(1 birth, 1 boy) = 1/2; 1/2 > 1/20
  p(2 births, 2 boys) = 1/2*1/2 = 1/4; 1/4 > 1/20
  p(3 births, 3 boys) = 1/2*1/2*1/2 = 1/8; 1/8 > 1/20
  p(4 births, 4 boys) = 1/2*1/2*1/2*1/2 = 1/16; 1/16 > 1/20
  p(5 births, 5 boys) = 1/2*1/2*1/2*1/2*1/2 = 1/32; 1/32 < 1/20

  So, 5 successive births of boys would be **very unlikely** (p < 1/20) if each birth had an **equal** chance of resulting in the birth of a boy.

Practice Question 7.2c KEY

- How many successive boys would have to be born before you would be prepared to come to this conclusion with only a 1% risk of error?

- Answer: continuing with problem 7.2b:

  new risk of error threshold: 1/100

  p(6 births, 6 boys) = (1/2)^6 = 1/64; 1/64 > 1/100
  p(7 births, 7 boys) = (1/2)^7 = 1/128; 1/128 < 1/100

  So, 7 successive births of boys would be *very very unlikely* (p < 1/100) if each birth had an *equal* chance of resulting in the birth of a boy.

Practice Question 7.3

The Federal Bureau of Investigation trains sniffer dogs to find explosive material. At the end of the training, Lucy, the FBI's prize dog, is let loose in a field with four unmarked parcels, one of which contains Semtex explosives. The exercise is repeated three times, and on each occasion, Lucy successfully identifies the suspicious parcel.

a) What is the chance of an untrained dog performing such a feat? (Assume that the untrained dog would always approach one of the parcels at random.)

b) If there had been five parcels instead of four and the exercise had been carried out only twice instead of three times, would the chances of the untrained dog finding the single suspicious parcel have been greater or less?

Practice Question 7.3a KEY

What is the chance of an untrained dog performing such a feat? (Assume that the untrained dog would always approach one of the parcels at random.)

Answer:

- 3 independent experiments; p(successful outcome on each experiment) = ¼
- p(3 successes) = 1/4 * 1/4 * 1/4 = 1/64

Practice Question 7.3b KEY

- If there had been five parcels instead of four and the exercise had been carried out only twice instead of three times, would the chances of the untrained dog finding the single suspicious parcel have been greater or less?

Answer:

- 2 independent experiments; p(successful outcome on each experiment) = 1/5
- p(2 successes) = 1/5 x 1/5 = 1/25

- Note: 1/25 is greater than 1/64.

Practice Question 7.4

Alex, an attorney, wishes to call eight witnesses to court for an important case. In his mind, he has categorized them into three “strong” witnesses and five “weaker” witnesses. He now wishes to make a tactical decision on the order in which to call the strong and the weaker witnesses.

For example, one of his options is:

Strong Weak Weak Strong Weak Weak Weak Strong

a) In how many different sequences can he call his strong and weaker witnesses?

b) If Alex decides that one of his three strong witnesses is in fact more suited to the weaker category, how many options does he now have?

Practice Question 7.4a KEY

- In how many different sequences can he call his strong and weaker witnesses?

Answer:

- Overview: total of 8 witnesses, 3 strong + 5 weak
- The question is asking about the number of possible distinct sequences, so this is a combination question.

Practice Question 7.4a KEY (continued)

- Number of combinations

  = N!/[r!(N-r)!]
  = 8!/[3!(8-3)!]
  = 8!/[3! x 5!]
  = 40320/[6 x 120]
  = 40320/720
  = 56

- OR

  = N!/[r!(N-r)!]
  = 8!/[5!(8-5)!]
  = 8!/[5! x 3!]
  = 40320/[120 x 6]
  = 40320/720
  = 56

- Please use Appendix 1 at the back of the textbook to look up factorials

Practice Question 7.4b KEY

- If Alex decides that one of his three strong witnesses is in fact more suited to the weaker category, how many options does he now have?

Answer:

- Change the mix of witnesses to 2 strong + 6 weak

$$
\begin{aligned}
&= N!/[r!(N-r)!] \\
&= 8!/[2!(8-2)!] \\
&= 8!/[2! \times 6!] \\
&= 40320/[2 \times 720] \\
&= 40320/1440 \\
&= 28
\end{aligned}
$$

- Note if the denominator was r=6 and N=8, we would get the same answer.

Practice Question 7.5

In a soccer match held at a low-security prison, the inmates beat the guards 4 to 2.

a) How many different arrangements are there for the order in which the goals were scored?

b) What would your answer be if the final score were 5 to 1?

Practice Question 7.5a KEY

- How many different arrangements are there for the order in which the goals were scored?

Answer:

- inmates beat the guards in soccer by score of 4 to 2; this means a total of 6 points were scored.
- Number of orderings:

$$
\begin{aligned}
&= N!/[r!(N-r)!] \\
&= 6!/[2!(6-2)!] \\
&= 720/[2! \times 4!] \\
&= 720/[2 \times 24] \\
&= 720/48 \\
&= 15
\end{aligned}
$$

Practice Question 7.5b KEY

- What would your answer be if the final score were 5 to 1?
- Answer:

  = N!/[r!(N-r)!]
  = 6!/[1!(6-1)!]
  = 720/[1! x 5!]
  = 720/[1 x 120]
  = 720/120
  = 6

- Let's see what the options are:

  GGGGGI
  GGGGIG
  GGGIGG
  GGIGGG
  GIGGGG
  IGGGGG

Reminders

Assignment #2 has been distributed yesterday, and it is due on **March 26th (Wednesday, after spring break) at 11:59pm**

Let us know if you have any questions or concerns, have a great spring break!