

CCJS200 Friday Discussion Sections

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Week 9- 3/28/2025

Today's Agenda

- How was your spring break??
- Did you turn in assignment #2 (it was **DUE** on Wednesday 3/26, please see course webpage for late submission penalties)
- We will be going through practice problems 7.6-7.8 from your textbook
- Before we start, does anyone need a refresher on the practice problems from week 7 (week before spring break)?

But first, some helpful formulas

How to obtain the total number of combinations with N trials and R outcomes (aka **N choose R**)

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Calculating a **Binomial Probability** (you will also have to use the total number of combinations in this formula)

$$p(\text{x outcome in N trials}) = \# \text{ of combinations} \times p^x \times (1-p)^{(N-x)}$$

Practice Problem 7.6

At the end of each year, the police force chooses its “Police Officer of the Year.” In spite of the fact that there are equal numbers of men and women on the force, in the last 15 years, 11 of the winners have been men and 4 have been women. Paul has been investigating whether women and men are treated differently in the police force.

Practice Problem 7.6a

Do these figures provide Paul with a reasonable basis to suspect that the sex of the officer is an active factor? Explain your answer.

11 winners have been men, 4 have been women.

15 total trials; 4 events.

$$p_0 = \frac{1}{2}$$

$${}^{15}C_4 = \frac{15!}{4!(15-4)!} = \frac{15!}{4!11!} = \frac{1307674368000}{(24 \cdot 39916800)} = \frac{1307674368000}{958003200} = 1365$$

Practice Problem 7.6a cont.

Binomial probability

$$= 1365 * (1/2)^4 * (1 - 1/2)^{11}$$

$$= 1365 * 1/16 * 1/2048$$

$$= 1365 * 1/32768$$

$$= 1365/32768$$

$$= 0.04165649$$

Practice Problem 7.6a cont.

The probability of getting 4 women and 11 men (assuming $p=1/2$) is a small number (0.042)

The fraction of women in our sample is $4/15=0.267$

Look up 95% confidence interval on table and find that the interval is [0.097,0.517]

Since 0.5 is inside the interval the evidence is not strong enough to reject H_0 :
 $p_0 = 1/2$

Practice Problem 7.6b

Looking back further into the records, Paul discovers that for the three years before the 15-year span initially examined, a woman was chosen each time. Does this affect his conclusion? Explain your answer.

11 winners have been men; 7 winners have been women.

18 total trials; 7 events.

$p_0 = 1/2$

$18c_7 =$

$18!/(7!(18-7)!)=18!/(7!11!)=6402373705728000/(5040*39916800)=6402373705728000/201180672000=31824$

Practice Problem 7.6b cont.

Binomial probability

$$= 31824 * (1/2)^7 * (1 - 1/2)^{11}$$

$$= 31824 * 1/128 * 1/2048$$

$$= 31824 * 1/262144$$

$$= 31824/262144 \text{ (greatest common factor is 16)}$$

$$= 1989/16384$$

$$= 0.1213989$$

Practice Problem 7.6b cont.

This probability is higher than we estimated before, suggesting that this (7 women and 11 men) is a more common scenario if $p = \frac{1}{2}$

The estimated fraction of women in our sample is $\frac{7}{18} = 0.389$

Look up 95% confidence interval on table and find that the interval is $[0.194, 0.617]$

Since 0.5 is inside the interval the evidence is not strong enough to reject H_0 : $p = \frac{1}{2}$

We would reach the same conclusion as part a.

Practice Problem 7.7

Use the binomial distribution to calculate each of the following probabilities:

Practice Problem 7.7a

Three heads in eight tosses of a fair coin.

$$8C3 = 8!/(3!(8-3)!)$$

$$= 40320/(6*5!)$$

$$= 40320/(6*120)$$

$$= 40320/720$$

$$= 56$$

Binomial Probability

$$= 56 * (1/2)^3 * (1-1/2)^5$$

$$= 56 * 1/8 * 1/32$$

$$= 56 * 1/256$$

$$= 56/256 \text{ (greatest common factor=8)}$$

$$= 7/32$$

$$= 0.21875$$

Practice Problem 7.7b

Six tails in thirteen tosses of a fair coin.

$${}^{13}C_6 = 13!/(6!(13-6)!)$$

$$= 6227020800/(6!*7!)$$

$$= 6227020800/(720*5040)$$

$$= 6227020800/3628800$$

$$= 1716$$

Binomial probability

$$= 1716 * (1/2)^6 * (1-1/2)^7$$

$$= 1716 * 1/64 * 1/128$$

$$= 1716 * 1/8192$$

$$= 1716/8192 \text{ (greatest common factor} = 4)$$

$$= 429/2048$$

$$= 0.2094727$$

Practice Problem 7.7c

Four fives in five rolls of a fair die.

$${}^5C_4 = \frac{5!}{4!(5-4)!}$$

$$= \frac{120}{(24 \cdot 1)}$$

$$= 5$$

Binomial probability

$$= 5 \cdot \left(\frac{1}{6}\right)^4 \cdot \left(1 - \frac{1}{6}\right)^1$$

$$= 5 \cdot \frac{1}{1296} \cdot \frac{5}{6}$$

$$= \frac{25}{7776}$$

$$= \frac{25}{7776}$$

$$= 0.003215021$$

Practice Problem 7.7d

Two ones in nine rolls of a fair die.

$$9C2 = 9!(2!(9-2)!)$$

$$= 362880/(2*7!)$$

$$= 362880/(2*5040)$$

$$= 362880/10080$$

$$= 36$$

Binomial probability:

$$= 36 * (1/6)^2 * (5/6)^7$$

$$= 36 * 1/36 * (78125/279936)$$

$$= 36 * (78125/10077696)$$

$$= 2812500/10077696$$

$$= 0.003215021$$

Practice Problem 7.7e

Five sixes in seven rolls of a fair die.

$${}^7C_5 = 7!/(5!*(7-5)!)$$

$$= 5040/(5!*2!)$$

$$= 5040/(120*2)$$

$$= 5040/240$$

$$= 21$$

Binomial Probability:

$$= 21 * (1/6)^5 * (5/6)^2$$

$$= 21 * 1/7776 * 25/36$$

$$= 21 * 25/279936$$

$$= 525/279936 \text{ (greatest common factor = 3)}$$

$$= 175/93312$$

$$= 0.001875429$$

Practice Problem 7.8

Tracy, a teacher, gives her class a ten-question test based on the homework she assigned the night before. She strongly suspects that Mandy, a lazy student, did not do the homework. Tracy is surprised to see that of the ten questions, Mandy answers seven correctly. What is the probability that Mandy successfully guessed seven of the ten answers to the questions if:

Practice Problem 7.8a

The questions all required an answer of true or false?

$$10C7 = 10!(7!(10-7)!)$$

$$= 3628800/(5040*6)$$

$$= 3628800/30240$$

$$= 120$$

Binomial probability:

$$= 120*(1/2)^7*(1-1/2)^3$$

$$= 120*1/1024$$

$$= 120/1024 \text{ (greatest common factor} = 8)$$

$$= 15/128$$

$$= 0.1171875$$

Practice Problem 7.8b

The questions were all in multiple-choice format, with students having to circle one correct answer from a list of five choices?

This leads to the assumption that the correct answer is chosen with probability = $\frac{1}{5}$

$$\mathbf{10c7 = 10!(7!(10-7)!)}$$

$$\mathbf{= 3628800/(5040*6)}$$

$$\mathbf{= 3628800/30240}$$

$$\mathbf{= 120}$$

Binomial probability

$$\mathbf{= 120*(\frac{1}{5})^7*(1-\frac{1}{5})^3}$$

$$\mathbf{= 120* \frac{64}{9765625}}$$

$$\mathbf{= \frac{7680}{9765625} \text{ (greatest common factor = 5)}}$$

$$\mathbf{= \frac{1536}{1953125}}$$

$$\mathbf{= 0.000786432}$$

Reminders

- R is now available on the web! Please see Dr. Brame's announcement on ELMS from Wednesday for more information.
- Marshae's office hours will be held on Thursday **4/3** from 9:30-11:00AM—Please note she will not be holding office hours from 12:30-2:00 this day.
- Next week's discussion sessions on **4/4** will be held via **Zoom**. Please pay attention to an announcement from your designated TA to get the zoom link. Remember, you must get approval to attend a different discussion session that you are not assigned to before doing so. Attendance will be monitored.
- Your second EXAM is on the horizon— scheduled for **4/15**
- Reach out if you have any questions or concerns