

## Symbols and Formulas for Exam 2

- $H_0$ : null hypothesis
- $p_0$ : null hypothesis value of binomial probability parameter (in our examples, we have set this value to 0.5).
- significance level ( $\alpha$ ): the probability of making a Type 1 error that we are willing to tolerate.
- p-value: the probability of getting a result at least as extreme as the one we got assuming  $H_0$  is true.
- $!$  = factorial so that  $3! = 3 \times 2 \times 1$ ; common factorials are listed in Appendix 1 (attached).
- Multiplication rule: if A and B are independent events then the probability of both A and B occurring is:  $p(A \text{ and } B) = p(A) \times p(B)$ .
- Addition rule: if A and B are independent events then the probability of either A or B occurring is:  $p(A \text{ or } B) = p(A) + p(B)$ .
- Confidence Interval for Binomial Probability Parameter: use look-up table (attached).
- Binomial Probability Formula:

$$\begin{aligned} p(x|N, p_0) &= \binom{N}{x} \times p_0^x \times (1 - p_0)^{N-x} \\ &= \frac{N!}{x!(N-x)!} \times p_0^x \times (1 - p_0)^{N-x} \end{aligned}$$

- Test statistic ( $T$ ) for equal number of cases in  $k$  categories:

$$T = \sum \frac{(f_O - f_E)^2}{f_E}$$

where the summation is over each cell of the table. Please keep in mind that if there are  $k$  categories then the test has  $k - 1$  degrees of freedom. You should look up the critical value of the chi-square distribution for the appropriate significance level ( $\alpha$ ) using the table in Appendix 2 (attached).

# Appendix 1

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## Factorials

$0!$	$= 1$
$1!$	$= 1$
$2!$	$= 2$
$3!$	$= 6$
$4!$	$= 24$
$5!$	$= 120$
$6!$	$= 720$
$7!$	$= 5,040$
$8!$	$= 40,320$
$9!$	$= 362,880$
$10!$	$= 3,628,800$
$11!$	$= 39,916,800$
$12!$	$= 479,001,600$
$13!$	$= 6,227,020,800$
$14!$	$= 87,178,291,200$
$15!$	$= 1,307,674,368,000$
$16!$	$= 20,922,789,888,000$
$17!$	$= 355,687,428,096,000$
$18!$	$= 6,402,373,705,728,000$
$19!$	$= 121,645,100,408,832,000$
$20!$	$= 2,432,902,008,176,640,000$
$21!$	$= 51,090,942,171,709,440,000$
$22!$	$= 1,124,000,727,777,607,680,000$
$23!$	$= 25,852,016,738,884,976,640,000$
$24!$	$= 620,448,401,733,239,439,360,000$
$25!$	$= 15,511,210,043,330,985,984,000,000$

## Appendix 2

### Critical Values of $\chi^2$ Distribution

df	$\alpha$					
	0.20	0.10	0.05	0.02	0.01	0.001
1	1.642	2.706	3.841	5.412	6.635	10.827
2	3.219	4.605	5.991	7.824	9.210	13.815
3	4.642	6.251	7.815	9.837	11.341	16.268
4	5.989	7.779	9.488	11.668	13.277	18.465
5	7.289	9.236	11.070	13.388	15.086	20.517
6	8.558	10.645	12.592	15.033	16.812	22.457
7	9.803	12.017	14.067	16.622	18.475	24.322
8	11.030	13.362	15.507	18.168	20.090	26.125
9	12.242	14.684	16.919	19.679	21.666	27.877
10	13.442	15.987	18.307	21.161	23.209	29.588
11	14.631	17.275	19.675	22.618	24.725	31.264
12	15.812	18.549	21.026	24.054	26.217	32.909
13	16.985	19.812	22.362	25.472	27.688	34.528
14	18.151	21.064	23.685	26.873	29.141	36.123
15	19.311	22.307	24.996	28.259	30.578	37.697
16	20.465	23.542	26.296	29.633	32.000	39.252
17	21.615	24.769	27.587	30.995	33.409	40.790
18	22.760	25.989	28.869	32.346	34.805	42.312
19	23.900	27.204	30.144	33.687	36.191	43.820
20	25.038	28.412	31.410	35.020	37.566	45.315
21	26.171	29.615	32.671	36.343	38.932	46.797
22	27.301	30.813	33.924	37.659	40.289	48.268
23	28.429	32.007	35.172	38.968	41.638	49.728
24	29.553	33.196	36.415	40.270	42.980	51.179
25	30.675	34.382	37.652	41.566	44.314	52.620
26	31.795	35.563	38.885	42.856	45.642	54.052
27	32.912	36.741	40.113	44.140	46.963	55.476
28	34.027	37.916	41.337	45.419	48.278	56.893
29	35.139	39.087	42.557	46.693	49.588	58.302
30	36.250	40.256	43.773	47.962	50.892	59.703

Source: From Table IV of R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research* (London: Longman Group Ltd., 1974). (Previously published by Oliver & Boyd, Edinburgh.) Reprinted by permission of Pearson Education Ltd.

TABLE 5  
95% Limits of the Jeffreys prior interval

$x$	$n = 7$		$n = 8$		$n = 9$		$n = 10$		$n = 11$		$n = 12$	
0	0	0.292	0	0.262	0	0.238	0	0.217	0	0.200	0	0.185
1	0.016	0.501	0.014	0.454	0.012	0.414	0.011	0.381	0.010	0.353	0.009	0.328
2	0.065	0.648	0.056	0.592	0.049	0.544	0.044	0.503	0.040	0.467	0.036	0.436
3	0.139	0.766	0.119	0.705	0.104	0.652	0.093	0.606	0.084	0.565	0.076	0.529
4	0.234	0.861	0.199	0.801	0.173	0.746	0.153	0.696	0.137	0.652	0.124	0.612
5					0.254	0.827	0.224	0.776	0.200	0.730	0.180	0.688
6									0.270	0.800	0.243	0.757
$x$	$n = 13$		$n = 14$		$n = 15$		$n = 16$		$n = 17$		$n = 18$	
0	0	0.173	0	0.162	0	0.152	0	0.143	0	0.136	0	0.129
1	0.008	0.307	0.008	0.288	0.007	0.272	0.007	0.257	0.006	0.244	0.006	0.232
2	0.033	0.409	0.031	0.385	0.029	0.363	0.027	0.344	0.025	0.327	0.024	0.311
3	0.070	0.497	0.064	0.469	0.060	0.444	0.056	0.421	0.052	0.400	0.049	0.381
4	0.114	0.577	0.105	0.545	0.097	0.517	0.091	0.491	0.085	0.467	0.080	0.446
5	0.165	0.650	0.152	0.616	0.140	0.584	0.131	0.556	0.122	0.530	0.115	0.506
6	0.221	0.717	0.203	0.681	0.188	0.647	0.174	0.617	0.163	0.589	0.153	0.563
7	0.283	0.779	0.259	0.741	0.239	0.706	0.222	0.674	0.207	0.644	0.194	0.617
8					0.294	0.761	0.272	0.728	0.254	0.697	0.237	0.668
9									0.303	0.746	0.284	0.716
$x$	$n = 19$		$n = 20$		$n = 21$		$n = 22$		$n = 23$		$n = 24$	
0	0	0.122	0	0.117	0	0.112	0	0.107	0	0.102	0	0.098
1	0.006	0.221	0.005	0.211	0.005	0.202	0.005	0.193	0.005	0.186	0.004	0.179
2	0.022	0.297	0.021	0.284	0.020	0.272	0.019	0.261	0.018	0.251	0.018	0.241
3	0.047	0.364	0.044	0.349	0.042	0.334	0.040	0.321	0.038	0.309	0.036	0.297
4	0.076	0.426	0.072	0.408	0.068	0.392	0.065	0.376	0.062	0.362	0.059	0.349
5	0.108	0.484	0.102	0.464	0.097	0.446	0.092	0.429	0.088	0.413	0.084	0.398
6	0.144	0.539	0.136	0.517	0.129	0.497	0.123	0.478	0.117	0.461	0.112	0.444
7	0.182	0.591	0.172	0.568	0.163	0.546	0.155	0.526	0.148	0.507	0.141	0.489
8	0.223	0.641	0.211	0.616	0.199	0.593	0.189	0.571	0.180	0.551	0.172	0.532
9	0.266	0.688	0.251	0.662	0.237	0.638	0.225	0.615	0.214	0.594	0.204	0.574
10	0.312	0.734	0.293	0.707	0.277	0.681	0.263	0.657	0.250	0.635	0.238	0.614
11					0.319	0.723	0.302	0.698	0.287	0.675	0.273	0.653
12									0.325	0.713	0.310	0.690
$x$	$n = 25$		$n = 26$		$n = 27$		$n = 28$		$n = 29$		$n = 30$	
0	0	0.095	0	0.091	0	0.088	0	0.085	0	0.082	0	0.080
1	0.004	0.172	0.004	0.166	0.004	0.160	0.004	0.155	0.004	0.150	0.004	0.145
2	0.017	0.233	0.016	0.225	0.016	0.217	0.015	0.210	0.015	0.203	0.014	0.197
3	0.035	0.287	0.034	0.277	0.032	0.268	0.031	0.259	0.030	0.251	0.029	0.243
4	0.056	0.337	0.054	0.325	0.052	0.315	0.050	0.305	0.048	0.295	0.047	0.286
5	0.081	0.384	0.077	0.371	0.074	0.359	0.072	0.348	0.069	0.337	0.067	0.327
6	0.107	0.429	0.102	0.415	0.098	0.402	0.095	0.389	0.091	0.378	0.088	0.367
7	0.135	0.473	0.129	0.457	0.124	0.443	0.119	0.429	0.115	0.416	0.111	0.404
8	0.164	0.515	0.158	0.498	0.151	0.482	0.145	0.468	0.140	0.454	0.135	0.441
9	0.195	0.555	0.187	0.537	0.180	0.521	0.172	0.505	0.166	0.490	0.160	0.476
10	0.228	0.594	0.218	0.576	0.209	0.558	0.201	0.542	0.193	0.526	0.186	0.511
11	0.261	0.632	0.250	0.613	0.239	0.594	0.230	0.577	0.221	0.560	0.213	0.545
12	0.295	0.669	0.282	0.649	0.271	0.630	0.260	0.611	0.250	0.594	0.240	0.578
13	0.331	0.705	0.316	0.684	0.303	0.664	0.291	0.645	0.279	0.627	0.269	0.610
14					0.336	0.697	0.322	0.678	0.310	0.659	0.298	0.641
15									0.341	0.690	0.328	0.672

Source: Lawrence D. Brown, T. Tony Cai, and Anirban DasGupta (2001). Interval estimation for a binomial proportion. *Statistical Science*, 16:101-133.