Lesson 15

Tuesday 3/26/24

A Judge Hands Out 10 Prison Sentences

data = 7,2,1,3,8,4,5,5,3,7

<u>Given</u> these 10 cases, what is the probability that a case drawn at random is sentenced to more than 5 years in prison?

Which cases in this dataset received a prison sentence exceeding 5 years?

data = <u>7</u>,2,1,3,<u>8</u>,4,5,5,3,<u>7</u>

$$p(\text{sentence} > 5 \text{ years}) = \frac{3}{10} = 0.3$$

Seven people are released from prison. Each of these 7 people are followed for 3 years. At the end of the 3 years, we can classify each person as a success (S) or a failure (F) in terms of recidivism (a set of Bernoulli trials). Here is the data:

data = S,F,F,S,F,S,F

If we assume these people are independent of each other (like coin flips are independent), we can use this information to calculate the probability that someone selected at random from these data is observed to fail:

$$p(\text{fail}) = \frac{4}{7} = 0.571$$

With p(fail) in hand (0.571), we are now asked to calculate the probability distribution of the number of failures for 5 new prison releasees.

$$p(k \text{ failures out of 5 releasees}) = \frac{n!}{k!(n-k)!} \times p^k \times (1-p)^{n-k}, k = 0, 1, ..., 5$$

k	p(k)
0	0.015
1	
2	
3	
4	
5	

k = 0

$$n! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$k! = 0! = 1$$

$$(n-k)! = (5-0)! = 5! = 120$$

$$p^k = 0.571^0 = 1$$

$$(1-p)^{n-k} = (1-0.571)^{5-0} = 0.429^5 = 0.015$$

$$\frac{120}{1 \times 120} \times 1 \times 0.015 = 0.015$$

k = 1

$$p(k \text{ failures out of 5 releasees}) = \frac{n!}{k!(n-k)!} \times p^k \times (1-p)^{n-k}, k = 0, 1, ..., 5$$

k	p(k)
0	0.015
1	0.097
2	
3	
4	
5	

$$n! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$k! = 1! = 1$$

$$(n-k)! = (5-1)! = 4! = 24$$

$$p^k = 0.571^1 = 0.571$$

$$(1-p)^{n-k} = (1-0.571)^{5-1} = 0.429^4 = 0.034$$

$$\frac{120}{1 \times 24} \times 0.571 \times 0.034 = 0.097$$

k = 2

$$p(k \text{ failures out of 5 releasees}) = \frac{n!}{k!(n-k)!} \times p^k \times (1-p)^{n-k}, k = 0, 1, ..., 5$$

k	p(k)
0	0.015
1	0.097
2	0.258
3	
4	
5	

$$n! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$k! = 2! = 2$$

$$(n-k)! = (5-2)! = 3! = 6$$

$$p^k = 0.571^2 = 0.326$$

$$(1-p)^{n-k} = (1-0.571)^{5-2} = 0.429^3 = 0.079$$

$$\frac{120}{2 \times 6} \times 0.326 \times 0.079 = 0.258$$

k = 3

$$p(k \text{ failures out of 5 releasees}) = \frac{n!}{k!(n-k)!} \times p^k \times (1-p)^{n-k}, k = 0, 1, ..., 5$$

k	p(k)
0	0.015
1	0.097
2	0.258
3	0.342
4	
5	

$$n! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$k! = 3! = 6$$

$$(n-k)! = (5-3)! = 2! = 2$$

$$p^k = 0.571^3 = 0.186$$

$$(1-p)^{n-k} = (1-0.571)^{5-3} = 0.429^2 = 0.184$$

$$\frac{120}{6 \times 2} \times 0.186 \times 0.184 = 0.342$$

k = 4

$$p(k \text{ failures out of 5 releasees}) = \frac{n!}{k!(n-k)!} \times p^k \times (1-p)^{n-k}, k = 0, 1, ..., 5$$

k	p(k)
0	0.015
1	0.097
2	0.258
3	0.342
4	0.227
5	

$$n! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$k! = 4! = 24$$

$$(n-k)! = (5-4)! = 1! = 1$$

$$p^k = 0.571^4 = 0.106$$

$$(1-p)^{n-k} = (1-0.571)^{5-4} = 0.429^1 = 0.429$$

$$\frac{120}{24 \times 1} \times 0.106 \times 0.429 = 0.227$$

k = 5

$$p(k \text{ failures out of 5 releasees}) = \frac{n!}{k!(n-k)!} \times p^k \times (1-p)^{n-k}, k = 0, 1, ..., 5$$

k	p(k)
0	0.015
1	0.097
2	0.258
3	0.342
4	0.227
5	0.061

$$n! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$k! = 5! = 120$$

$$(n-k)! = (5-5)! = 0! = 1$$

$$p^k = 0.571^5 = 0.061$$

$$(1-p)^{n-k} = (1-0.571)^{5-5} = 0.429^0 = 1$$

$$\frac{120}{120 \times 1} \times 0.061 \times 1 = 0.061$$

With p(fail) in hand (0.571), we are now asked to calculate the probability distribution of the number of failures for 5 new prison releasees.

$$p(k \text{ failures out of 5 releasees}) = \frac{n!}{k!(n-k)!} \times p^k \times (1-p)^{n-k}, k = 0, 1, ..., 5$$

k	p(k)
0	0.015
1	0.097
2	0.258
3	0.342
4	0.227
5	0.061

Now that we have this distribution, we are in a position to estimate quantities like:

p(3 or 4 or 5 people fail) =
p(3 or more people fail) =

0.342 + 0.227 + 0.061 = 0.63

or, p(no failures) = 0.015

Unconditional vs. Conditional Probability

Suppose we observe a sample of 3200 people who are 25 years old. For each of these people, we check their criminal history records and we determine that 448 of them had attained at least one criminal conviction after turning 18 years old. Then,

$$p(\text{Convicted at least once}) = \frac{\#\text{ of times an event occurred}}{\#\text{ of times it could have occurred}} = \frac{448}{3200} = 0.14$$

Now, suppose we ask a slightly different question: among the 679 people who were arrested at least one time after turning 18, what is the probability of at least one conviction?

p(Convicted at least once | Arrested) = 448/679 = 0.660

Let's Consider the Same Two Calculations in Table Form

	Arrest= No	Arrest= Yes	Total
Convict = No	2521	231	2752
Convict = Yes	0	448	448
Total	2521	679	3200

p(convict) = 448/3200 = 0.140

p(arrest|convict) = 448/448 = 1

p(arrest) = 679/3200 = 0.212

$$p(\text{Convict}|\text{Arrest}) = \underset{0.140}{p(\text{convict})} \times \underset{1}{p(\text{Arrest}|\text{Convict})} \,/\, \underset{0.212}{p(\text{arrest})}$$

Reduces to: p(convict|arrest) = 0.140/0.212 = 0.660

***Very important to pay attention to the conditioning.

Hypothesis Testing

**In lay terms, a hypothesis is an idea, explanation, or statement that can be tested; truth value of the statement can be assessed with empirical evidence.

Hypothesis Testing

My definition: In classical probability and statistics, we think of hypotheses in terms of the value or magnitude of a scientifically interesting parameter (i.e., a mean, a proportion, a correlation, etc.).

Slightly different from the book definition on p. 165: "a hypothesis is simply a scientific "hunch" or assumption about the relationship between two variables that is tested empirically."

Statistical Inference

Based on the evidence we have, a statement is made about a scientifically interesting but unobserved parameter (an extrapolation). Inferences are comprised of two components: (1) an estimate; and (2) a measure of uncertainty to accompany the estimate.

Deductive Science

1. State hypothesis

2. Collect appropriate evidence

3. Use a valid probability calculation to test the hypothesis

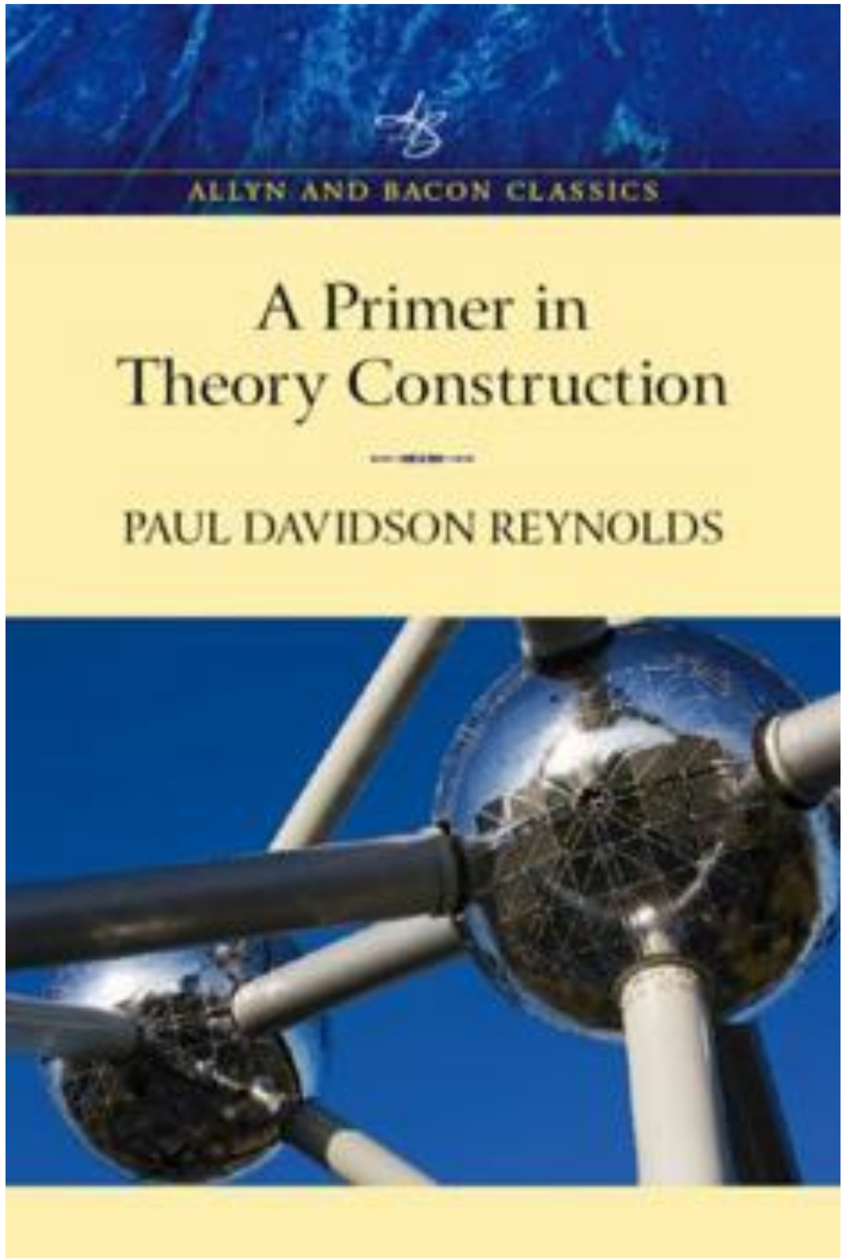
4. Draw conclusion

5. Go back to #1

(Scientific Method)

Inductive Science

Reynolds (1977) distinguishes between deductive and inductive science: (1) deductive means that the hypothesis is stated before data are collected; (2) inductive means that data are collected and then a hypothesis is developed based on the evidence. Both are valid and necessary!



Hypotheses and Evidence

$$p(H|E) = \frac{p(E|H)p(H)}{p(E)}$$

"Inverse Probability" or Bayesian Inference

$$p(E|H) = \frac{p(H|E)p(E)}{p(H)}$$

Classical Inference

At top of page 167, your book says "In a hypothesis test, the null hypothesis is the hypothesis that is intially *assumed to be true.*" Thus, classical inference focuses on the probability of the evidence looking the way it does conditional on the truth of a particular hypothesis.

Page 169 of your book:

"Recall that we begin our hypothesis test by assuming that <u>the null hypothesis is true</u>..."

"What we do, then, is <u>maintain our belief in the null hypothesis</u> until we are informed by the data that this belief is improbable given what we have observed." [Emphasis added]

<u>For now</u>, let's ignore the "null" adjective and just consider the word "hypothesis."

Cook and Zarkin (1985)

CRIME AND THE BUSINESS CYCLE

*PHILIP J. COOK and GARY A. ZARKIN**

THE business cycle has a pervasive effect on the structure of economic opportunity and hence on behavior. The effect is reflected in social indicators as diverse as school enrollments, birthrates, and labor force participation.[1] It would be surprising indeed if crime rates were immune to general business conditions, and certainly the conventional wisdom asserts that ''street'' crime is countercyclical. A recession always provides police chiefs with a comfortable explanation for their failure to prevent increases in the crime rate.

Question: when the economy contracts, what happens to crime?

Let's think about robbery.

From the 1930's to the early 1980's, there were 9 complete business cycles (NBER Business Cycle Dating Committee)

August 1929 (1929Q3)	March 1933 (1933Q1)	43	21	64	34
May 1937 (1937Q2)	June 1938 (1938Q2)	13	50	63	93
February 1945 (1945Q1)	October 1945 (1945Q4)	8	80	88	93
November 1948 (1948Q4)	October 1949 (1949Q4)	11	37	48	45
July 1953 (1953Q2)	May 1954 (1954Q2)	10	45	55	56
August 1957 (1957Q3)	April 1958 (1958Q2)	8	39	47	49
April 1960 (1960Q2)	February 1961 (1961Q1)	10	24	34	32
December 1969 (1969Q4)	November 1970 (1970Q4)	11	106	117	116
November 1973 (1973Q4)	March 1975 (1975Q1)	16	36	52	47
January 1980 (1980Q1)	July 1980 (1980Q3)	6	58	64	74
July 1981 (1981Q3)	November 1982 (1982Q4)	16	12	28	18

When each business cycle ends, the economy transitions from a growth phase to a contraction phase (a recession).

For each of the 9 times the economy tipped into a recession, C&Z asked the question, "Did robbery rates move higher relative to their yearly trends during the growth phase?"

Notice this is a yes or no question.

Suppose we assume that there is <u>no relationship</u> between changes in the economy and changes in crime.

What would we expect to see?

We might expect to see that when the economy tips into recession, robberies would be equally likely to increase or decrease (relative to their trend in the growth phase). Like a coin flip would be equally likely to come up heads or tails.

This a hypothesis -- what your book would call a null hypothesis.

C&Z collected their data and found that in 8 out of the 9 business cycles that robbery increased relative to its growth phase trend when the economy tipped into recession.

In other words, they found what they called a strong countercyclical pattern.

The question is whether the evidence is strong enough to reject the hypothesis of equally likely outcomes.

What is the probability distribution for the number of "countercyclical movements"?

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assuming that a growth or decline in robberies is equally likely when the economy moves into a recession.

$$p(k \text{ countercyclical movements}|p = 0.5, N = 9) = \frac{N!}{k!(N-k)!} p^k (1-p)^{N-k}$$

$$p(k \text{ countercyclical movements}|p = 0.5, N = 9) = \frac{N!}{k!(N-k)!}p^k(1-p)^{N-k}$$

k	p(k)
0	0.002
1	0.018
2	0.070
3	0.164
4	0.246
5	0.246
6	0.164
7	0.070
8	0.018
9	0.002

p(0 or 1 CC Movements) = 0.002 + 0.018 = 0.020

p(2-7 CC Movements) = 0.070+0.164+0.246+0.246+0.164+0.070 = 0.96

p(8 or 9 CC Movements) = 0.018 + 0.002 = 0.020

In words, if *p* is really 0.5, it would be surprising to see data that look like this; this is the kind of evidence that would lead us to reject the hypothesis that *p* is 0.5.